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VaR Estimation and Risk Management: Evidence from a Multi-Asset Portfolio

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Master in Finance

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Resumo

De forma a que exista uma alocação eficiente de capital face à volatilidade do mercado, as instituições financeiras necessitam de recorrer a ferramentas quantitativas de gestão de risco. O Value at Risk (VaR) é um destes instrumentos essenciais, uma vez que funciona como um indicador do capital económico e como métrica regulamentar. Esta dissertação investiga a aplicação do VaR como um instrumento de medição de risco, mas também como um mecanismo dinâmico de controlo do risco de portfólios, apoiado por estratégias de cobertura.

Nesta dissertação, é realizada uma análise empírica a um portfólio composto por 20 ações e 4 obrigações soberanas, com exposição a vários mercados internacionais e moedas. Foram testados 4 modelos para estimar o VaR, de forma a que se percebesse qual seria o mais indicado para o portfólio. Os 4 modelos foram os seguintes: Parametric Normal, Skewed Generalized Student-t (SGSt), Historical Simulation (HS) e Quantile Regression (QR). São realizadas decomposições do VaR total, permitindo a avaliação das contribuições stand-alone e marginais de cada ativo, de cada tipo de ativo e de cada moeda. Os modelos são avaliados através de procedimentos estatísticos de backtesting, utilizando 2 testes, o Unconditional Coverage test e o Berkowitz Conditional Probability test, a fim de identificar a especificação mais robusta.

O modelo selecionado é posteriormente aplicado num contexto dinâmico, no qual é definido um valor-alvo de VaR e são introduzidas várias posições de cobertura sempre que o risco estimado excede o limiar predefinido. Por fim, o desempenho da carteira é comparado em dois cenários: uma estratégia passiva sem cobertura e uma estratégia de cobertura ativa, na qual foram adicionadas posições de cobertura.

Os resultados sugerem que um quadro de gestão de risco dinâmico baseado no VaR pode reduzir eficazmente o risco da carteira, melhorando simultaneamente os retornos ajustados ao risco (RORAC).

Palavras-chave: Value-at-Risk, Gestão de Risco, Backtesting, Estratégias de Cobertura

Classificação JEL: G11, G17, G32.

Abstract

In order to ensure the efficient allocation of capital in the face of market volatility, financial institutions need to make use of quantitative risk management tools. Value at Risk (VaR) is one such essential tool, as it serves as an indicator of economic capital and as a regulatory metric. This dissertation investigates the application of VaR as a risk measurement tool, but also as a dynamic mechanism for portfolio risk control, supported by hedging strategies.

In this dissertation, an empirical analysis is carried out on a portfolio comprising 20 equities and 4 sovereign bonds, with exposure to various international markets and currencies. Four models were tested to estimate VaR, in order to determine which would be most suitable for the portfolio. The four models were as follows: Parametric Normal, Skewed Generalised Student-t (SGSt), Historical Simulation (HS) and Quantile Regression (QR). Decompositions are performed for both total VaR and systematic VaR, allowing for the assessment of the stand-alone and marginal contributions of each asset, each asset class and each currency. The models are evaluated using statistical backtesting procedures, employing two tests, the Unconditional Coverage test and the Berkowitz Conditional Probability test, in order to identify the most robust specification.

The selected model is subsequently applied in a dynamic context, where a target VaR value is set and various hedging positions are introduced whenever the estimated risk exceeds the predefined threshold. Finally, the portfolio's performance is compared across two scenarios: a passive strategy without hedging and an active hedging strategy where hedging positions have been added.

The results suggest that a dynamic risk management framework based on VaR can effectively reduce portfolio risk, whilst improving risk-adjusted returns (RORAC).

Keywords: Value-at-Risk, Risk Management, Backtesting, Hedging Strategy

JEL Classification: G11, G17, G32.

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LIST OF ABBREVIATIONS

ARCH	Autoregressive Conditional Heteroskedasticity
BCP	Berkowitz–Christoffersen–Pelletier test
EC	Economic Capital
ES	Expected Shortfall
EUR	Euro
EVT	Extreme Value Theory
EWMA	Exponentially Weighted Moving Average
FHS	Filtered Historical Simulation
FTSE 100	Financial Times Stock Exchange 100 Index
FRTB	Fundamental Review of the Trading Book
FX	Foreign Exchange
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GBP	British Pound Sterling
GSPC	S&P 500 Index
HKD	Hong Kong Dollar
HSI	Hang Seng Index
HS	Historical Simulation
JPY	Japanese Yen
N225	Nihon Keizai Shimbun 225 Index (Nikkei 225)
P&L	Profit and Loss
PV01	Price Value of a Basis Point
QR	Quantile Regression
RORAC	Return on Risk Adjusted Capital
SGSt	Skewed Generalised Student-t
UC	Unconditional Coverage
USD	United States Dollar
VaR	Value-at-Risk

Chapter 1.

INTRODUCTION

In a time when global markets are increasingly volatile, efficient market risk management mechanisms are necessary. In this sense, VaR has become one of the most popular risk management metrics, both in the internal control systems of financial institutions and in European regulations.

In addition to being a statistical tool, VaR also functions as an indicator of economic capital, being defined as the amount of capital needed to absorb unexpected losses for a given confidence level and over a specific time horizon. Thus, controlling VaR is equivalent to controlling the level of capital that must be allocated to support portfolio risk. Managing VaR dynamically therefore involves managing economic capital actively and systematically, having direct implications for capital efficiency and risk-adjusted performance measures, such as RORAC, defined as Risk-Adjusted Return on Capital.

This dissertation evaluates and implements VaR within a dynamic portfolio risk management framework. Empirical analysis is conducted on a portfolio composed of 20 equities and 4 sovereign debt securities. The equities come from five major markets, namely, the United States, the United Kingdom, Germany, Japan, and Hong Kong, while the public debt securities comprise sovereign debt issued by the United States and Germany. The portfolio is denominated in multiple currencies, including USD, GBP, EUR, JPY, and HKD.

Since VaR serves as a proxy for economic capital, the dynamic hedging strategy can be interpreted as an attempt to optimize the relationship between return and required capital, assessing whether active risk management improves capital efficiency without unduly sacrificing performance.

By integrating VaR model assessment with a dynamic hedging process, this dissertation contributes to the literature by bridging the gap between risk measurement and practical risk control. It provides empirical evidence on the extent to which VaR can be transformed from a regulatory compliance metric into a strategic instrument for managing economic capital and improving the performance of the risk-adjusted portfolio. The research is structured in several chapters. In Chapter 2, we can find all the literature relevant to the realization of this thesis. Chapter 3, are the sources from which all the relevant data was taken, as well as the composition of the portfolio. Chapter 4 the mapping of the portfolio by the main risk factors is described, as well as the description of the models that were tested and what they consist of. In Chapter 5, the backtesting process and the process of choosing the model and parameterization that works best

for the model were described. In Chapter 6 the hedging process is described and the comparison of the portfolio with and without it is made. Finally, Chapter 7 describes the results obtained.

Chapter 2.

LITERATURE REVIEW

The importance of regularized risk assessment and management in the context of financial decision-making has been recognized for a long time both in academic and applied contexts (Jorion, 2007). As the world's financial markets continued to go global, the concept of risk was first associated with uncertainty and the likelihood of negative results (Dowd, 2002). One of the most influential developments in this context was the development of Modern Portfolio Theory by Markowitz (1952), who claimed investors should construct portfolios based on balancing expected returns and variance of those returns. However, this framework treated deviations symmetrically, penalizing gains and losses equally, which subsequent literature showed does not reflect investors' greater sensitivity to losses (Roy, 1952; Fishburn, 1977; Kahneman and Tversky, 1979). Roy (1952)'s safety-first principle, which centered on minimizing the probability of returns falling short of an imposed minimum level, developed a downside-risk orientation foreshadowing more recent loss-based approaches to risk measurement.

In the following decades, work has confirmed the view that investors prefer to focus on issues of extreme loss over variability in general (Bawa, 1975; Fishburn, 1977). But it was in the 1990s when widespread application of market risk measurement methods occurred, subsequent to J.P. Morgan's development of the RiskMetrics™ framework, which also enshrined VaR as a commonly applied measure of financial risk (J.P. Morgan/Reuters, 1996). VaR became an overnight favorite with widespread application due to ease of application and comparability, as it provided a single number for the maximum expected loss over a given time interval and level of confidence (Duffie and Pan, 1997; Jorion, 2007). Regulatory endorsement of VaR helped boost widespread application, especially following the Basel Committee (1996) allowing internal VaR models to be used in calculation of capital amounts for market risk (Basel Committee on Banking Supervision, 1996).

Even though VaR became an international standard, theoretical flaws were soon recorded. As a quantile of the loss distribution, VaR does not respond to the size of losses beyond the threshold level and does not necessarily meet the subadditivity property, hence contravening the principle of diversification (Artzner et al., 1999; Daníelsson, 2002). As a replacement of these flaws, Artzner et al. (1999) proposed coherent risk measures, which are required to meet the tests of monotonicity, subadditivity, homogeneity, and translation invariance. One notable alternative is the Expected Shortfall (ES), or also referred to as the Conditional VaR, which is defined as the expected loss given losses exceeding the level of VaR, hence reflecting the

size of the tails (Acerbi and Tasche, 2002; Yamai and Yoshiba, 2005). Coherence axioms were shown to be satisfied by ES, which has been adopted in the Basel III Fundamental Review of the Trading Book (FRTB) as an alternative measure to VaR in deriving the regulatory capital requirements (Basel Committee on Banking Supervision, 2016). The calculation of Value at Risk can be attempted using the application of parametric and non-parametric models. The Parametric Normal VaR, also referred to as the variance–covariance or the Delta-Normal method, assumes a known probability distribution of returns and estimates risk using the mean, the variance, and the covariance structure corresponding to portfolio returns (J.P. Morgan/Reuters, 1996; Jorion, 2007). The RiskMetrics model, first published in 1994, applied an Exponentially Weighted Moving Average (EWMA) to capture volatility clustering effectively and hence became a pragmatic standard for parametric VaR (Longerstaey and Zangari, 1996). Extensions of the original model followed through the contributions of Engle's (1982) Autoregressive Conditional Heteroskedasticity (ARCH) and Bollerslev's (1986) Generalized ARCH (GARCH), allowing conditional volatility to evolve dynamically (Engle, 1982; Bollerslev, 1986). An integration of GARCH with Extreme Value Theory (EVT) by (McNeil and Frey, 2000) demonstrated the intentional modeling of tails in return distributions considerably improves the accuracy of quantile predictions of both VaR and ES. As an attempt to better reflect skewness and fat tails in empirical distributions, Theodossiou (1998) introduced the Skewed Generalized Student-t distribution. Empirics have confirmed that SGSt-based VaR offers a better measure of tail risk compared with Normal VaR, especially during periods of increased market volatility (Giot and Laurent, 2003). These evolutions reflect the adaptability of parametric models in response to more complex return distributions (Christoffersen, 2003).

By contrast, non-parametric approaches dispense with distributional assumptions and rely instead on the empirical distribution of past returns. The most common method is Historical Simulation (HS), which calculates VaR directly from ordered historical returns (Hendricks, 1996). The reliability of HS, however, depends heavily on the choice of window length: short samples yield unstable estimates, while long samples may neglect recent shifts in volatility (Pritsker, 2001). Hendricks (1996), testing twelve different VaR variants on 1,000 foreign exchange portfolios, concluded that no method consistently dominated, though adaptive approaches tended to perform better. To address these issues, Barone-Adesi et al. (1999) developed Filtered Historical Simulation (FHS), in which returns are filtered using GARCH or EWMA models before applying HS. Gurrola-Perez and Murphy (2015) confirmed that FHS improves responsiveness to volatility shocks compared with plain HS.

Another parametric alternative is also Quantile Regression (QR), constructed by Koenker and Bassett (1978), as an estimator of conditional quantiles without likelihood-based assumptions of a known distribution. For VaR, QR has been found to capture asymmetry and heteroscedasticity of financial returns (Koenker and Bassett, 1978; Taylor, 2008). QR's adaptability makes it particularly suitable in modeling risk in cases where distributions of returns are non-normal (Christoffersen, 2003).

The adequacy of VaR forecasts must be empirically verified through backtesting, which compares realized losses with model predictions. The most widely used is the Unconditional Coverage Test proposed by Kupiec (1995), which evaluates whether the observed proportion of exceptions is statistically consistent with the expected frequency given the confidence level. However, this test does not address whether exceptions occur independently. Berkowitz et al. (2009) developed the the Berkowitz, Christoffersen and Pelletier (BCP) Test, which combines unconditional coverage with independence testing, thereby identifying potential clustering of exceptions. Together, these procedures form the foundation of regulatory and academic practice for VaR model validation (Christoffersen, 2003; Basel Committee on Banking Supervision, 2005).

Beyond statistical validation, VaR has evolved into a tool for active risk management. Jorion (2007) emphasizes that VaR supports daily monitoring, limit-setting, and hedging strategies, while Alexander (2008) points out its function of synchronizing risk exposures with institutional risk tolerance. Implementation of hedging policies, however, causes the unavoidable questioning of the trade-off between risk minimization and profitability.

To assess such an equilibrium, Value at Risk must be complemented with measures of performance based on risk-adjusted returns. This requires the integration of the concept of Economic Capital, intended as the level of capital required that a financial institution must retain in order to remain solvent at a predetermined level of confidence over a given time interval (Dowd, 2002; Tasche, 1999). Unlike externally set regulatory capital, economic capital is set internally with the help of risk measurement models and better reflects the institution's actual risk profile (Crouhy et al., 2001).

By allocating economic capital across portfolios or business units, banks can evaluate profitability relative to the risks actually undertaken. Within this framework, the Risk-Adjusted RORAC emerges as a consistent measure of value creation. Stahl (1997) introduced RORAC as the ratio between net return and economic capital allocated to absorb potential losses. Schroeck (2002) argued that RORAC links risk management with value creation, while Matten (2000) documented its widespread adoption in banking practice as a measure of both profitability and resilience.

Chapter 3.

DATA AND PORTFOLIO COMPOSITION

The data set ranges from November 1, 2005, until April 30, 2025, with daily frequency. The long time series guarantees an appropriate length best suited to purposes of backtesting, model calibration, and future risk measurement.

The portfolio under consideration is composed of 20 international stocks and four sovereign bonds, issued in various currencies. The equity component is composed of stocks traded on the United States, United Kingdom, Eurozone, Japan, and Hong Kong. The fixed-income component is composed of sovereign bonds issued in USD and EUR, selected so as to capture interest rate risk with low credit risk, given the fact that they correspond to paper from highly rated governments, both having Aaa rating by Moodys.

Since the portfolio is composed of foreign-currency-denominated assets, there was a need to incorporate the exchange rates with reference to the euro (EUR), which is the reporting currency of the study. The exchange rates considered include USD/EUR, GBP/EUR, JPY/EUR, and HKD/EUR.

The data were collected from reputable financial databases. Specifically, daily adjusted closing prices for equities, indices, and exchange rates were retrieved from Yahoo Finance `https://finance.yahoo.com`. Comprehensive data involving sovereign bonds were extracted from Börse Frankfurt `https://www.boerse-frankfurt.de/bonds`. Daily interest rate data involving instruments with USD denominations were provided by the Federal Reserve Economic Data (FRED) `https://www.federalreserve.gov/datadownload/Choose.aspx?rel=H15`, while the European Central Bank Statistical Data Warehouse (YC-ECB Data Portal) supplied the corresponding interest rate data for EUR-denominated securities.

The main variables include stock prices, bond cash flows and specifications, benchmark interest rates, and exchange rates. All series were harmonised into euro terms, enabling the construction of consistent time series of portfolio returns. These constitute the basis for the subsequent procedures of volatility estimation, risk factor mapping, and VaR computation.

Table 3.1 presents the stock and bond composition of the portfolio.

Asset	Ticker/ISIN	Currency	Quantity / Face Value	Price	Value (EUR)	Allocation (%)
Apple Inc.	AAPL	USD	1 000	189.85	174 301.02	2.38
JPMorgan Chase	JPM	USD	800	151.46	111 245.89	1.52
Pfizer Inc.	PFE	USD	600	26.30	14 487.51	0.20
Procter & Gamble Co.	PG	USD	900	147.13	121 573.62	1.66
Unilever	ULVR.L	GBP	70 000	37.79	3 067 221.07	41.95
Rolls-Royce	RR.L	GBP	50 000	2.77	160 420.15	2.19
HSBC Holdings	HSBA.L	GBP	150 000	6.04	1 049 717.17	14.36
BP	BP.L	GBP	200 000	4.78	1 108 851.49	15.17
SAP SE	SAP.DE	EUR	2 500	144.87	362 168.24	4.95
Siemens AG	SIE.DE	EUR	1 800	148.21	266 784.66	3.65
BASF SE	BAS.DE	EUR	300	38.31	11 493.36	0.16
Allianz SE	ALV.DE	EUR	400	214.10	85 640.28	1.17
Sony Group	6758.T	JPY	500	2 528.50	7 837.10	0.11
Fast Retailing	9983.T	JPY	1 000	36 592.02	226 833.93	3.10
Tokyo Electron	8035.T	JPY	300	23 458.28	43 625.36	0.60
Toyota Motor Corp	7203.T	JPY	600	2 710.07	10 079.82	0.14
Hang Seng Bank Ltd.	0011.HK	HKD	2 000	76.51	17 986.39	0.25
Ping An Insurance	2318.HK	HKD	2 500	33.35	9 801.35	0.13
China Mobile	0941.HK	HKD	1 800	58.31	12 337.67	0.17
CLP Holdings Ltd	0002.HK	HKD	1 000	54.84	6 445.55	0.09
Total Equity					**6 868 851.62**	**93.95**
US Treasury 2041	US912810TA60	USD	100 000	67.51%	61 979.53	0.85
German Bund 2026	DE000BU22049	EUR	150 000	101.36%	152 041.16	2.08
US Treasury 2030	US91282CJG78	USD	50 000	104.25%	47 857.04	0.65
German Bund 2030	DE0001030559	EUR	200 000	90.14%	180 288.30	2.47
Total Bonds					**442 166.03**	**6.05**
Total Portfolio					**7 311 017.65**	**100.00**

Table 3.1. Portfolio composition on the mapping date. The table presents details for each asset, including the weights in the total portfolio.

Chapter 4.

METHODOLOGY

The objective of this study is to measure and manage the market risk of a multi-asset portfolio using several hedging positions in order to respect a predefined Economic Capital constraint.

The methodology follows two main stages. First, the portfolio was mapped into its underlying market risk factors by translating each position into exposures to equity prices, interest rate term structures and exchange rates. These exposures were expressed in euros on the mapping date and provided a consistent representation of portfolio sensitivities across asset classes. The resulting set of risk factor loadings formed the basis for constructing portfolio returns, estimating risk dynamics and computing VaR. Second, several VaR model specifications were estimated and evaluated through a structured backtesting procedure. The purpose of this stage was to identify the model that provides the most reliable risk forecasts for the portfolio. The selected model was then projected forward from the mapping date of 1 December 2023 and implemented in a framework that evaluates the same portfolio under two alternative settings: hedged and unhedged.

Backtesting was conducted over the period from 16 December 2013 to 1 December 2023, covering 2,600 daily observations. For each model configuration, one-day-ahead VaR forecasts were generated and compared with the realized portfolio P&L. An exceedance was recorded whenever the realized P&L was lower than the negative of the corresponding VaR estimate.

Model performance was evaluated using two statistical tests. The Unconditional Coverage (UC) test of Kupiec (1995) was applied to assess whether the observed frequency of exceedances was consistent with the nominal 2.5% confidence level. In addition, the BCP test, proposed by Berkowitz et al. (2009) was used to evaluate both coverage and independence of exceedances.

Model selection followed an elimination approach. Specifications failing the Unconditional Coverage (UC) test for the full sample or displaying unstable behaviour across individual years were discarded. Although the COVID-19 period generated significant market stress, priority was given to models that demonstrated stable performance across subperiods. Based on these criteria, the Historical Simulation VaR was selected for the forward risk management analysis.

4.1 Risk Factor Mapping

As described above, the first step consists of mapping the portfolio into its underlying market risk drivers. Since VaR is calculated using the factors that explain changes in asset prices rather than the securities themselves, this step is essential. The mapping process serves as the foundation for constructing portfolio returns, estimating volatility and calculating VaR by expressing each position in terms of its underlying risk exposures.

The reference structure used for both the forward analysis and the backtesting procedure corresponds to the portfolio composition on 1 December 2023.

Exposures were divided into three main categories of risk factors: equity risk, exchange rate risk and interest rate risk. Equity positions were represented through their individual stock price exposures. Exchange rates, as USD/EUR, GBP/EUR, JPY/EUR and HKD/EUR were used to capture currency risk. Bonds were mapped into sovereign yield curves using a term-structure approach, with maturity vertices at 3M, 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y and 20Y for USD and the same set plus 15Y for EUR.

By converting foreign-denominated positions using the spot exchange rates observed on the mapping date, all exposures were expressed in euros. As a result, individual assets may be exposed to multiple sources of risk, such as interest rate and currency risk for bonds or equity and currency risk for foreign stocks.

The main input for estimating VaR is the vector of risk factor loadings generated by this mapping step, denoted by

$$\Theta = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_n \end{bmatrix}^\top,$$

representing the exposures to each of the n risk factors affecting the portfolio.

VaR can therefore be expressed as a function of this vector:

$$\text{VaR} = f(\Theta).$$

4.1.1 Stocks

Given the multi-currency and multi-market composition of the portfolio, each stock position is represented through its sensitivity to two main sources of risk: equity price risk and exchange rate risk.

The exposure of stock i at time t is defined as the market value of the position expressed in the portfolio base currency. This value measures the sensitivity of portfolio wealth to changes in the stock price and is obtained as

$$M_{i,t} = N_{i,t} P_{i,t} FX_{i,t}, \tag{4.1}$$

where $N_{i,t}$ denotes the number of shares held, $P_{i,t}$ the market price of the stock and $FX_{i,t}$ the spot exchange rate converting the asset's denomination currency into euros.

Currency risk is incorporated by converting each foreign-denominated equity position into its euro market value. This reflects the sensitivity of portfolio value to exchange rate fluctuations. Positions already denominated in euros generate no currency exposure.

Given these exposures, the profit and loss generated by equity price movements is computed as

$$P\&L_{i,t} = M_{i,t}\left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right), \tag{4.2}$$

which represents the contribution of each equity position to overall portfolio returns.

4.1.2 Bonds

Bonds valuation depends on the present value of predetermined future cash flows, consisting of periodic coupon payments and the repayment of face value at maturity. Their value depends on the entire term structure of interest rates used to discount each cash flow. As a result, changes in bond prices are driven by shifts in interest rates across different maturities. For the bond mapping, we had to represent bond risk through sensitivities to multiple interest rate risk factors along the yield curve, that reflect the dependence of discounted cash flows on movements in interest rates (Alexander (2008); Jorion (2007).

For a bond with face value N, coupon rate c, coupon frequency n, and maturity T, the sequence of cash flows C_i is discounted using the corresponding continuously compounded zero-coupon rates. The present value of a cash flow payable at time T_i is therefore given by

$$PV_{C_i,t} = C_i e^{-r_{T_i,t}T_i} FX_{i,t}, \tag{4.3}$$

where $r_{T_i,t}$ denotes the spot interest rate for maturity T_i and $FX_{i,t}$ converts the cash flow into the portfolio reporting currency (EUR) when mapping treasuries. The bond price is obtained by aggregating the discounted cash flows,

$$PV_t = \sum_{i=1}^{n} PV_{C_i,t}. \tag{4.4}$$

To quantify interest rate sensitivity, the Present Value of a Basis Point (PV01) is used. PV01 measures the change in present value resulting from a one basis point shift in the yield curve. Using a first-order Taylor approximation, the PV01 of a cash flow can be expressed as

$$PV01_{C_i,t} \approx -\frac{\partial PV_{C_i,t}}{\partial r_{T_i,t}} \times 0.01\% = T_i\, PV_{C_i,t} \times 0.01\%. \tag{4.5}$$

Under this linear approximation, the change in the value of a bond can be written as the sum of the sensitivities of its individual cash flows to changes in interest rates,

$$\Delta PV_t \approx -\sum_{i=1}^{n} PV01_{C_i,t} \frac{\Delta r_{T_i,t}}{0.01\%}. \tag{4.6}$$

In practice, representing each cash flow maturity as a distinct risk factor would lead to an excessively large set of interest rate drivers. To ensure tractability, cash flows with non-standard maturities are mapped onto a predefined set of standard yield curve maturities, referred to as vertices (Alexander, 2008). Each cash flow is allocated to the two adjacent vertices T_1 and T_2 using interpolation weights,

$$w_{T_1} = \frac{T_2 - T}{T_2 - T_1}, \qquad w_{T_2} = 1 - w_{T_1}. \tag{4.7}$$

The mapping is constructed to preserve both the present value and the first-order interest rate sensitivity of the original cash flow. This PV+PV01 invariant mapping ensures that the aggregated exposures reproduce the bond's price response to small yield curve movements, while acknowledging that higher-order effects such as volatility are not fully preserved.

After mapping, the exposure of each bond to the interest rate risk factors corresponds to the negative PV01 allocated to each yield curve vertex.

Given these exposures, the P&L generated by interest rate movements was computed as the product between the PV01 exposure and the corresponding change in interest rates,

$$P\&L_t = \sum_{j=1}^{m} -PV01_{j,t} \frac{\Delta r_{j,t}}{0.01\%}, \tag{4.8}$$

where m denotes the number of yield curve vertices and $\Delta r_{j,t}$ represents the change in the interest rate at vertex j.

Interest Rate Risk Factor	Exposure (EUR)
USD 3M	−0.03
USD 6M	−0.06
USD 1Y	−0.27
USD 2Y	−0.70
USD 3Y	−1.50
USD 5Y	−3.85
USD 7Y	−26.54
USD 10Y	−15.65
USD 20Y	−69.61
EUR 3M	−0.09
EUR 6M	−0.07
EUR 1Y	−0.33
EUR 2Y	−20.80
EUR 3Y	−13.43
EUR 5Y	−28.19
EUR 7Y	−83.24
EUR 10Y	0.00
EUR 15Y	0.00
EUR 20Y	0.00

Table 4.1. Interest rate risk factor exposures on 1 December 2023. The table reports the aggregated PV01 exposures across standard yield curve vertices, representing the portfolio sensitivity to shifts in USD and EUR interest rate term structures.

4.1.3 Currency

The portfolio includes positions denominated in multiple foreign currencies (USD, GBP, JPY and HKD), while the reporting currency is the euro. Since exchange rate fluctuations represent an additional source of market risk that affect both equity and fixed-income positions, we have to compute the currency exposures for each asset. To do this, we converted the value of all foreign-denominated assets into euros using the spot exchange rates observed on the mapping date and aggregating positions by currency. This procedure provides a direct measure of the portfolio's sensitivity to movements in the USD/EUR, GBP/EUR, JPY/EUR and HKD/EUR exchange rates and ensures consistency with the factor-based VaR framework.

Given these exposures, the profit and loss generated by exchange rate movements was computed as the product between the currency exposure and the corresponding exchange rate return,

$$P\&L_{i,t} = M_{i,t}\left(\frac{FX_{i,t}}{FX_{i,t-1}} - 1\right), \tag{4.9}$$

where $M_{i,t}$ denotes the euro value of the position exposed to currency i and $FX_{i,t}$ represents the exchange rate relative to the euro.

Currency Risk Factor	Exposure (EUR)
USD/EUR	531 444.61
GBP/EUR	5 386 209.88
JPY/EUR	288 376.20
HKD/EUR	46 570.96

Table 4.2. Exchange rate risk factor exposures on 1 December 2023. The table reports the euro-denominated sensitivity of portfolio value to exchange rate movements, obtained by aggregating currency exposures from both equity and fixed-income positions.

4.2 Volatility

The estimation of volatility constitutes a central component of market risk measurement, as VaR depends directly on the variability and co-movement of the underlying risk factors. Following the mapping of the portfolio into equity, interest rate and exchange rate drivers, the corresponding time series of risk factor realizations were constructed and used as inputs for volatility modelling.

For equity and exchange rate factors, we had to define realizations as percentage returns, whereas interest rate factors were represented by absolute changes in yields expressed in basis points. This factor-based representation reflects the dynamic behaviour of the sources of portfolio profit and loss and enables a consistent multi-asset volatility estimation framework.

A basic measure of volatility can be obtained using an equal-weighted historical standard deviation. However, this approach assumes constant volatility and ignores the empirical evidence that financial market volatility varies over time and exhibits persistence and clustering. The presence of time-varying conditional heteroskedasticity has been extensively documented in the literature, beginning with the ARCH framework introduced by Engle (1982) and its extension to the GARCH model by Bollerslev (1986).

To capture these dynamics while maintaining computational simplicity, this study adopts the EWMA model. EWMA assigns greater weight to recent observations and gradually down-weights older data, allowing volatility estimates to react to changing market conditions. The approach was popularised by the RiskMetrics methodology, which established EWMA as a standard tool for market risk measurement (J.P. Morgan/Reuters, 1996).

Let x_t denote the realization of a generic risk factor at time t. The EWMA variance is estimated recursively as

$$\hat{\sigma}_t^2 = (1 - \lambda)x_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2, \tag{4.10}$$

where $\hat{\sigma}_t^2$ denotes the conditional variance forecast and $\lambda \in (0, 1)$ is the decay factor controlling the persistence of volatility.

In a multi-factor environment, risk measurement requires modelling not only individual variances but also the dependence structure between risk factors. The EWMA covariance between risk factors i and j is estimated recursively as

$$\hat{\sigma}_{ij,t} = (1 - \lambda)x_{i,t-1}x_{j,t-1} + \lambda\hat{\sigma}_{ij,t-1}, \tag{4.11}$$

and stacking these estimates across all factors yields the conditional covariance matrix Σ_t, which summarises the joint dynamics of the portfolio's market drivers.

4.3 VaR Models

VaR represents the central quantitative tool used in this study to measure the potential downside risk of the mapped portfolio. Building on the risk factor exposures and the volatility dynamics estimated in the previous sections, VaR provides a forward-looking assessment of the maximum expected loss that may occur over a specified time horizon under normal market conditions.

VaR summarizes the lower tail of the portfolio profit and loss distribution. For a given time horizon h and significance level α, the h-day VaR corresponds to the negative α-quantile of the projected return (or P&L) distribution. In this dissertation, we will adopt a one-day horizon ($h = 1$) together with a significance level of $\alpha = 2,5\%$, which corresponds to a 97,5% confidence level. Under this definition, the VaR represents the loss that is expected to be exceeded only 2,5% of the time.

Let X_t denote the portfolio return or profit and loss at time t. The one-day VaR at level α is defined as

$$P(X_t < -VaR_{1,\alpha}) = \alpha, \tag{4.12}$$

which implies that VaR can be expressed as the negative of the inverse cumulative distribution function evaluated at the chosen significance level,

$$VaR_{h,\alpha} = -F^{-1}(\alpha), \tag{4.13}$$

where $F^{-1}(\alpha)$ denotes the α-quantile of the portfolio P&L distribution over the horizon h.

Because financial return distributions often display time-varying volatility, asymmetry and heavy tails, the estimation of VaR depends critically on the assumptions made about the conditional distribution of portfolio returns. For this reason, the analysis considers multiple modelling approaches that differ in their treatment of distributional shape and volatility dynamics.

In this dissertation, we implemented four VaR methodologies: a parametric model based on conditional normality, a parametric specification relying on the Skewed Generalized Student-t distribution, a non-parametric Historical Simulation approach, and a Quantile Regression framework that directly models tail behavior. Each model was estimated using the same set of mapped risk factor returns and volatility forecasts to ensure comparability across specifications.

The following subsections describe the theoretical foundations and implementation details of each VaR model before their predictive performance is evaluated through a structured backtesting exercise.

4.3.1 Parametric Normal VaR

The Parametric Normal VaR model assumes that portfolio returns follow a normal distribution, providing an analytically tractable benchmark for market risk measurement (Jorion, 2007; Alexander, 2008). The portfolio profit and loss is represented as a linear function of the mapped risk factors, implying that the distribution of portfolio returns can be approximated by a normal distribution characterized by its mean and variance.

Let X_h denote the portfolio return over a holding period h. Under the parametric normal assumption,

$$X_h \sim \mathcal{N}(\mu_h, \sigma_h^2), \tag{4.14}$$

where μ_h represents the expected portfolio return and σ_h the portfolio volatility.

From the general definition of VaR introduced earlier, the h-day VaR at significance level α corresponds to the negative α-quantile of the return distribution. Under normality, this yields

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha)\sigma_h - \mu_h, \tag{4.15}$$

where $\Phi^{-1}(\alpha)$ denotes the inverse cumulative distribution function of the standard normal distribution (Jorion, 2007).

For short horizons such as daily risk measurement ($h = 1$), the expected return is typically assumed negligible relative to volatility, leading to the simplified expression

$$VaR_{1,\alpha} = -\Phi^{-1}(\alpha)\sigma_h, \tag{4.16}$$

an approximation widely adopted in practical risk management applications (Alexander, 2008).

Within the mapping framework, portfolio volatility is obtained from the exposure vector Θ derived in Section 4.1 and the covariance matrix of risk factor returns Σ estimated using the EWMA methodology described in Section 4.2. This approach follows the RiskMetrics paradigm, where portfolio risk is computed through the quadratic form of exposures and the conditional covariance matrix (J.P. Morgan/Reuters, 1996). The portfolio standard deviation is therefore

$$\sigma_h = \sqrt{\Theta^\top \Sigma \Theta}. \tag{4.17}$$

Substituting this into the VaR expression yields the factor-based Parametric Normal VaR:

$$VaR_{1,\alpha} = -\Phi^{-1}(\alpha)\sqrt{\Theta^\top \Sigma \Theta}. \tag{4.18}$$

This specification links the mapping of exposures, the estimation of volatility and the statistical definition of risk into a single analytical expression, providing a transparent benchmark

against which more flexible VaR models can be evaluated (Duffie and Pan, 1997).

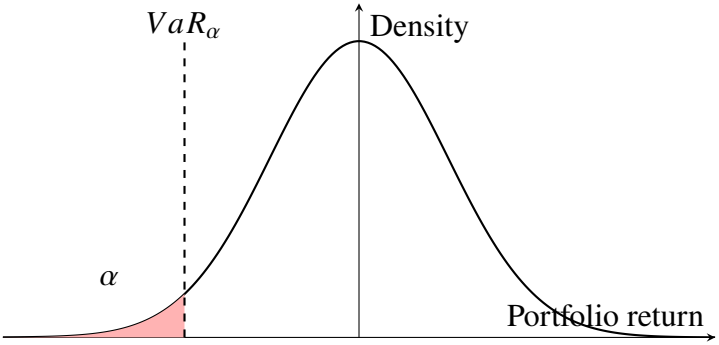


Figure 4.1. Parametric Normal VaR. The shaded region represents the probability α of losses exceeding the VaR threshold under the normality assumption.

4.3.2 Skewed Generalized Student-t VaR

Financial returns often exhibit skewness and excess kurtosis, indicating asymmetric distributions characterized by heavier tails than those anticipated by the normal distribution. To better capture this, a more flexible parametric specification can be adopted based on the Skewed Generalized Student-t (SGSt) distribution.

The SGSt distribution, introduced by Theodossiou (1998), extends the traditional Student-t distribution by allowing skewness and allowing different levels of tail thickness. From the flexibility provided by this distribution, we can have a more realistic representation of financial return distributions, particularly in the tails where extreme losses occur. Let X_h denote the portfolio return over a holding period h. Under the SGSt specification, portfolio returns follow

$$X_h \sim SGSt(\mu_h, \sigma_h, \lambda, p, q), \tag{4.19}$$

where μ_h denotes the location parameter and σ_h represents the scale parameter associated with portfolio volatility. The remaining parameters control the shape of the distribution. The parameter $\lambda \in (-1, 1)$ controls the degree of asymmetry in the distribution, with $\lambda = 0$ corresponding to a symmetric distribution, $\lambda > 0$ indicating positive skewness and $\lambda < 0$ negative skewness.

The parameters $p > 0$ and $q > 0$ control the shape of the distribution. The parameter q controls how thick the tails are. When we have lower values this implies heavier tails and therefore a higher probability of extreme outcomes. As q increases, the tails become thinner and the distribution approaches the Gaussian case.

The parameter p controls the shape of the centre of the distribution. When $p = 2$, the central part as the same aspect as a normal distribution, while different values of p change how peaked the distribution is around the centre.

This flexibility allows the SGSt distribution to capture both skewness and excess kurtosis commonly observed in financial return series.

From the general definition of VaR introduced previously, the h-day VaR at significance level α corresponds to the negative α-quantile of the return distribution. Under the SGSt specification this can be written as

$$VaR_{h,\alpha} = -T^{-1}_{\mu_h,\sigma_h,\lambda,p,q}(\alpha), \tag{4.20}$$

where $T^{-1}_{\mu_h,\sigma_h,\lambda,p,q}(\alpha)$ denotes the inverse cumulative distribution function of the SGSt distribution evaluated at probability level α.

For short holding periods such as the one-day horizon considered in this study ($h = 1$), the expected return component is typically assumed negligible relative to the volatility of returns (Alexander, 2008). Under this approximation, the VaR expression simplifies to

$$VaR_{1,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha)\sigma_h \tag{4.21}$$

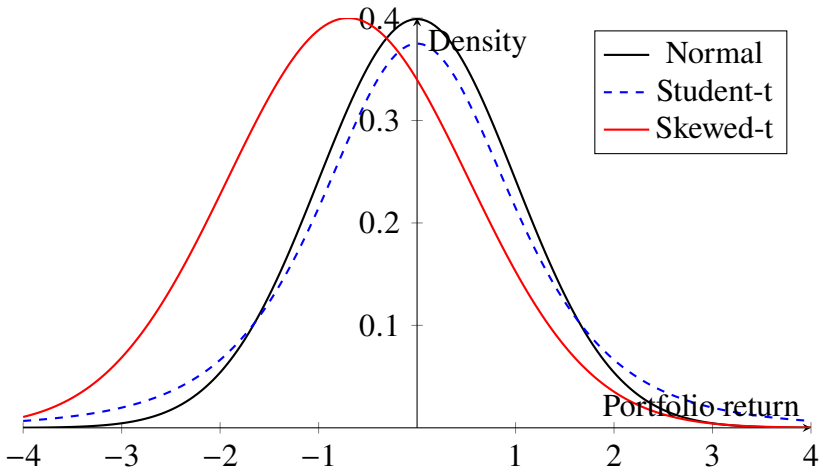


Figure 4.2. Comparison of Normal, Student-t and Skewed-t return distributions. The Student-t distribution captures fat tails relative to the Gaussian case, while the skewed-t distribution additionally allows for asymmetry in returns, providing a more flexible representation of financial risk.

4.3.3 Volatility-Adjusted Historical VaR

While the models Parametric Normal and SGSt assume that returns follow a specific distribution, the non-parametric model Historical Simulation provides a distribution-free method for estimating VaR. This model assigns equal weight to all past observations regardless of the prevailing volatility regime. As a result, historical observations generated under periods of low volatility may have the same influence on the risk estimate as observations from periods of heightened market turbulence. This limitation can lead to VaR estimates that are slow to adapt to changing market conditions Hull and White (1998).

To address this issue, Hull and White (1998) proposed the Volatility-Adjusted Historical Simulation method. The key idea is to rescale historical returns so that they reflect the level of

current market volatility while preserving the empirical structure of the return distribution.

Within the factor-based framework developed earlier, portfolio profit and loss is generated using the vector of risk factor returns and the portfolio exposure vector. Let r_t denote the vector of risk factor returns at time t, and let Θ represent the vector of risk factor loadings obtained in the mapping stage. The portfolio profit and loss at time t can therefore be expressed as

$$P\&L_t = \Theta^\top r_t. \tag{4.22}$$

Using a historical sample of T observations, this procedure generates a time series of historical portfolio returns that represents the empirical distribution of possible portfolio outcomes.

In the volatility-adjusted framework, each historical return is rescaled according to the ratio between the current volatility estimate and the volatility prevailing at the time the observation occurred. Let $\hat{\sigma}_t$ denote the volatility estimate associated with return r_t, and let $\hat{\sigma}_T$ represent the most recent volatility estimate available at the VaR measurement date T. Following Hull and White (1998), the adjusted return is obtained as

$$\hat{r}_t = \frac{r_t}{\hat{\sigma}_t}\hat{\sigma}_T. \tag{4.23}$$

This change first normalizes each historical return by its current volatility and then rescales it by the current level of volatility. This way, the adjusted return series shows the volatility conditions that were in place on the VaR evaluation date while keeping the historical distribution's shape.

After this adjustment, a new series of volatility-scaled portfolio returns is obtained,

$$\{\hat{r}_1, \hat{r}_2, \ldots, \hat{r}_T\}.$$

The volatility-adjusted Historical VaR is then computed as the empirical quantile of this adjusted distribution. For a given significance level α, the one-day VaR corresponds to the negative α-quantile of the ordered adjusted return series,

$$VaR_{1,\alpha} = -Q_\alpha(\hat{r}_t), \tag{4.24}$$

where $Q_\alpha(\cdot)$ denotes the empirical quantile operator.

By incorporating time-varying volatility through the scaling procedure, the volatility-adjusted Historical Simulation method preserves the non-parametric nature of Historical VaR while allowing the risk estimate to adapt more rapidly to changes in market volatility.

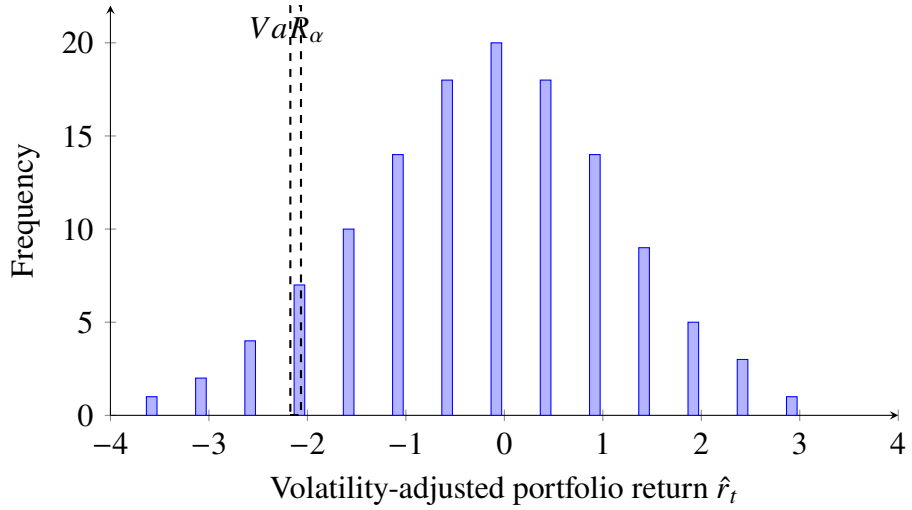


Figure 4.3. Volatility-Adjusted Historical VaR. The histogram represents the empirical distribution of volatility-adjusted portfolio returns $\hat{r}_t$ obtained after rescaling historical returns according to the method proposed by Hull and White (1998). The VaR corresponds to the α-quantile of this adjusted distribution.

4.3.4 Quantile Regression VaR

The quantile regression framework was introduced by Koenker and Bassett (1978) and allows the conditional α-quantile of a variable to be estimated as a function of a set of explanatory variables. In the context of market risk measurement, this approach enables the direct estimation of the lower tail behaviour of portfolio returns without imposing a specific distributional assumption.

Let y_t denote the portfolio return at time t and x_t represent a vector of explanatory variables. The conditional α-quantile of portfolio returns can be expressed as a linear function of these variables:

$$VaR_{\alpha,t} = -q_{\alpha,y} = -(\hat{a} + \hat{b}x_t), \tag{4.25}$$

where $\hat{a}$ and $\hat{b}$ denote the estimated parameters of the α-quantile regression. Under this specification, the VaR corresponds to the negative of the estimated conditional quantile of portfolio returns.

The parameters of the quantile regression are obtained by solving the minimization problem proposed by Koenker and Bassett (1978):

$$(\hat{a}, \hat{b}) = \arg\min_{a,b} \sum_{i=1}^{n} (y_i - (a + bx_i)) \left(\alpha - I_{y_i-(a+bx_i)<0}\right). \tag{4.26}$$

where $I_{y_i-(a+bx_i)<0}$ is an indicator function defined as

$$I_{y_i-(a+bx_i)<0} = \begin{cases} 1, & \text{if } y_i - (a + bx_i) < 0 \\ 0, & \text{otherwise.} \end{cases} \tag{4.27}$$

This asymmetric loss function assigns different weights to positive and negative residuals, ensuring that the estimated regression corresponds to the selected conditional quantile rather than the conditional mean.

In this study, the dependent variable y_t corresponds to the portfolio return, while the explanatory variables are constructed from volatility measures derived from the portfolio's return dynamics. A baseline specification relates portfolio returns to a single explanatory variable:

$$y_t = bx_t + \varepsilon_t, \tag{4.28}$$

where x_t represents a volatility-based indicator capturing the level of market uncertainty at time t.

More general specifications may include multiple explanatory variables in order to capture different aspects of volatility dynamics:

$$y_t = b_1 x_{1,t} + b_2 x_{2,t} + \varepsilon_t, \tag{4.29}$$

where $x_{1,t}$ and $x_{2,t}$ denote alternative volatility measures derived from the portfolio's historical return series.

The parameters of the quantile regression are estimated using a rolling estimation window, allowing the model to adapt to changes in market conditions over time. This framework produces a time series of conditional quantile estimates which can be directly interpreted as VaR forecasts.

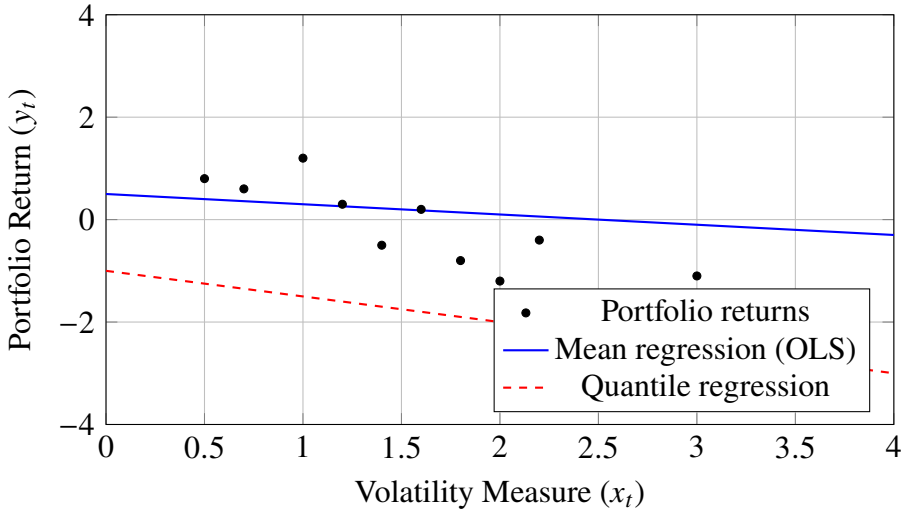


Figure 4.4. Illustration of Quantile Regression VaR. While traditional regression estimates the conditional mean of portfolio returns, quantile regression estimates a specific conditional quantile of the distribution. The lower quantile line corresponds to the VaR estimate, capturing the tail behaviour of portfolio losses.

Chapter 5.

BACKTESTING AND MODEL SELECTION

After estimating the different VaR models described in the previous chapter, we evaluated their performance through a formal backtesting procedure. The purpose of backtesting is to assess whether the VaR estimates produced by each model provide an accurate representation of the portfolio's downside risk when compared with realized outcomes.

The backtesting exercise was conducted using daily portfolio profit and loss observations over the period from 16 December 2013 to 1 December 2023, corresponding to a total of 2,600 trading days. For each model specification, a time series of one-day-ahead VaR forecasts was generated and compared with the realized portfolio returns observed on the corresponding day. This procedure ensures that the VaR estimate available at the end of day t is evaluated against the portfolio outcome observed on day $t + 1$, preventing the use of information that would not have been available at the time the forecast was made.

Given the selected significance level of $\alpha = 2.5\%$, a correctly specified model should generate exceedances with approximately the same frequency over time. Deviations from this expected frequency or patterns of clustering in exceedances may indicate that the model does not adequately capture the risk dynamics of the portfolio.

In the backtesting, two statistical procedures were implemented. First, the Unconditional Coverage test verifies whether the observed proportion of exceedances is consistent with the nominal confidence level. Second, the BCP test proposed by evaluates the joint properties of coverage and independence of exceedances, providing additional information about potential clustering in violation events.

Backtesting was performed over the full evaluation period as well as on an annual basis in order to assess the stability of model performance across different market conditions. The model selection procedure follows an elimination approach. Specifications that failed the UC test for the entire period were discarded, while the BCP test is used as an additional diagnostic tool. Finally, models that failed the UC test for any individual year were excluded, and in cases where several models exhibit similar performance, preference is given to the specification whose exceedance frequency was closer to the theoretical level of 2.5%.

The following subsections describe the statistical tests used in the backtesting procedure and present the empirical results used to identify the best-performing VaR model for the portfolio under analysis.

5.1 Unconditional Coverage Test

To evaluate the accuracy of VaR forecasts, we employ the Unconditional Coverage (UC) test proposed by Kupiec (1995). This test assesses whether the observed frequency of VaR exceedances is consistent with the probability implied by the chosen confidence level.

An exceedance occurs when the realized portfolio loss is larger than the predicted VaR threshold. Let r_t denote the portfolio return at time t and $VaR_{t,\alpha}$ the VaR forecast at significance level α. The occurrence of an exceedance can be represented by the indicator variable

$$I_t = \begin{cases} 1, & \text{if } r_t < -VaR_{t,\alpha} \\ 0, & \text{otherwise.} \end{cases} \tag{5.1}$$

Since VaR represents the maximum loss that is not expected to be exceeded with probability $1 - \alpha$, the probability that the realized loss exceeds the VaR estimate is α. Therefore, in a correctly specified model the proportion of exceedances should be approximately equal to α.

In this study, the backtesting sample used has $n = 2600$ daily observations and a VaR significance level of $\alpha = 2.5\%$. The expected number of exceedances is

$$n \times \alpha = 2600 \times 0.025 = 65. \tag{5.2}$$

Hence, we expect approximately 65 exceedances during the entire backtesting period. A substantial deviation from this expected number may indicate that the VaR model is incorrectly specified.

Let n denote the total number of observations and n_1 the number of exceedances observed during the backtesting period. The observed exceedance rate is defined as

$$\pi_{obs} = \frac{n_1}{n}, \tag{5.3}$$

while the expected exceedance rate is

$$\pi_{exp} = \alpha. \tag{5.4}$$

The UC test evaluates the following hypotheses:

$$H_0 : \pi_{obs} = \pi_{exp} = \alpha \tag{5.5}$$

$$H_a : \pi_{obs} \neq \pi_{exp}. \tag{5.6}$$

Under the null hypothesis, the VaR model produces the correct proportion of exceedances. Kupiec (1995) derives a likelihood ratio statistic based on the binomial distribution of violations, given by

$$LR_{uc} = \left(\frac{\pi_{exp}}{\pi_{obs}}\right)^{n_1} \left(\frac{1-\pi_{exp}}{1-\pi_{obs}}\right)^{n_0}, \tag{5.7}$$

where $n_0 = n - n_1$ represents the number of non-exceedances.

For numerical stability, the statistic is usually computed in logarithmic form:

$$-2\ln(LR_{uc}) = -2\left[n_1(\ln\alpha - \ln\pi_{obs}) + n_0(\ln(1-\alpha) - \ln(1-\pi_{obs}))\right]. \tag{5.8}$$

Under the null hypothesis, the test statistic asymptotically follows a chi-squared distribution with one degree of freedom,

$$-2\ln(LR_{uc}) \sim \chi^2_1. \tag{5.9}$$

If the computed statistic exceeds the corresponding critical value, the null hypothesis is rejected, indicating that the VaR model produces either too many or too few exceedances relative to the nominal confidence level.

5.2 BCP Test

While the Unconditional Coverage test verifies whether the total number of exceedances is consistent with the nominal confidence level, it does not consider the temporal dependence between these violations. In practice, VaR models may produce the correct number of exceedances while still failing to capture the dynamics of risk if these exceedances occur in clusters during periods of market stress.

To address this limitation, Berkowitz et al. (2009) propose the BCP test, which evaluates the independence of exceedances by analysing the autocorrelation structure of the exceedance indicator series.

If the VaR model is correctly specified, exceedances should occur randomly over time and therefore be independent. In this case, the autocorrelation of the exceedance series should be equal to zero for all lags.

The BCP test evaluates this property by analysing the first K autocorrelations of the exceedance series. Let $\hat{\rho}_k$ denote the sample autocorrelation of order k of the indicator series I_t. In this study, the test considers autocorrelation lags ranging from $k = 1$ to $k = 10$, so that $K = 10$. The test considers the following hypotheses:

$$H_0 : \rho_k = 0, \quad \forall k \in \{1, \ldots, K\} \tag{5.10}$$

$$H_a : \exists k \in \{1, \ldots, K\} \text{ such that } \rho_k \neq 0. \tag{5.11}$$

Under the null hypothesis, the exceedances are independent over time and the VaR model correctly captures the dynamics of risk. The BCP test statistic is defined as

$$BCP_K = T(T+2)\sum_{k=1}^{K}\frac{\hat{\rho}_k^2}{T-k}, \tag{5.12}$$

where $\hat{\rho}_k$ is the k-th order autocorrelation of the exceedance series, K is the maximum lag considered in the test, and T denotes the sample size.

Under the null hypothesis that the autocorrelations are equal to zero, the test statistic asymptotically follows a chi-squared distribution with K degrees of freedom:

$$BCP_K \sim \chi_K^2. \tag{5.13}$$

If the calculated statistic is greater than the critical value, the null hypothesis is false. This means that there is autocorrelation in the exceedance series, suggesting that violations tend to occur in clusters, which implies that the VaR model fails to fully capture the time-varying nature of risk.

5.3 VaR Model Selection

The selection of the VaR model was based on the structured backtesting procedure described above. This analysis was conducted over the period from 16 December 2013 to 1 December 2023, covering approximately 2,600 daily observations.

A significance level of 2.5% (corresponding to a 97.5% confidence level) was adopted for the VaR estimation. This choice is mainly motivated by computational convenience in models that rely on empirical quantiles, such as Historical Simulation, as it corresponds to approximately one observation in every forty ($1/0.025 = 40$). When the sample size is a multiple of forty, the empirical quantile can be obtained directly from the ordered data without interpolation, ensuring an exact correspondence between the theoretical significance level and the empirical exceedance rate in backtesting. This avoids the distortions that arise when the significance level does not align with the sample size, which may otherwise lead to biased VaR estimates and incorrect exceedance frequencies.

In this dissertation, the model selection process followed a sequential filtering approach. Initially, a large set of model parametrizations was explored for each VaR methodology considered in this study, namely the Parametric Normal VaR, the SGSt VaR, the HS VaR and the QR VaR. For each model, different combinations of estimation window lengths and EWMA smoothing factors were evaluated in order to identify robust specifications.

The first stage of the backtesting procedure focused on the Unconditional Coverage (UC) test proposed by Kupiec (1995). For each parametrization, the UC test was computed both for the full backtesting period. A model was considered acceptable only if the null hypothesis of correct exceedance frequency could not be rejected at the 5% significance level. Consequently, parametrizations for which the UC test produced a p-value lower than 5% in the full sample were immediately discarded. This requirement ensures that the selected model produces the

correct overall frequency of exceedances.

Within the family of Parametric Normal VaR models, several EWMA smoothing factors were evaluated. The only specification that satisfied the UC test criterion for both the full sample and all subperiods corresponded to a smoothing factor of $\lambda = 0.98$.

For the SGSt VaR model, an extensive grid of parametrizations was examined by combining estimation windows ranging from 200 to 520 observations, in increments of forty, with smoothing factors between 0.90 and 0.99. Among all tested configurations, only the specification using a sample size of 520 observations and a smoothing factor of $\lambda = 0.95$ satisfied the UC test criterion across the entire sample and all annual subperiods.

As mentioned in subsection 4.3.3 the Historical Simulation VaR was evaluated under two alternative specifications: the standard historical approach and a volatility-adjusted version incorporating volatility scaling. When the standard historical approach was applied, none of the tested parametrizations satisfied the UC test criterion. In contrast, once volatility adjustment was introduced, multiple parametrizations passed the UC test. Most of the successful specifications were associated with EWMA smoothing factors between 0.95 and 0.96 across the range of estimation window lengths considered, although in a few cases smoothing factors of 0.94, 0.97, 0.98 and 0.99 also produced acceptable results.

Finally, the Quantile Regression VaR was estimated without a constant term and initially using a single explanatory variable representing the volatility dynamics of portfolio returns. The same grid of estimation window lengths and smoothing factors used in the previous models was explored. Subsequently, a second explanatory variable was introduced in the specification. Because each variable requires a separate smoothing factor, testing all possible combinations across the full range of estimation windows would lead to an extremely large number of model configurations. To maintain computational tractability while preserving robustness, the analysis of the two-variable specification was restricted to the largest estimation windows, namely 480 and 520 observations.

ID	VaR Model	Sample Size	EWMA λ_1	EWMA λ_2	Exceedances	Exceedance %	UC p-value
1	Normal VaR	–	0.98	–	78	3.00%	11.3%
2	SGSt VaR	520	0.95	–	80	3.08%	6.88%
Historical Simulation VaR (Volatility Adjusted)							
3	Historical VaR	200	0.98	–	73	2.81%	32.42%
4	Historical VaR	200	0.99	–	71	2.73%	45.76%
5	Historical VaR	240	0.95	–	72	2.77%	38.73%
6	Historical VaR	240	0.96	–	72	2.77%	38.73%
7	Historical VaR	240	0.97	–	73	2.81%	38.42%
8	Historical VaR	280	0.95	–	73	2.81%	32.42%
9	Historical VaR	280	0.96	–	73	2.81%	32.42%
10	Historical VaR	280	0.97	–	71	2.73%	45.76%
11	Historical VaR	440	0.97	–	75	2.88%	22.00%
12	Historical VaR	480	0.96	–	72	2.77%	38.73%
Quantile Regression VaR (1 explanatory variable)							
13	QR VaR	240	0.95	–	74	2.85%	26.85%
14	QR VaR	240	0.97	–	73	2.81%	32.42%
15	QR VaR	360	0.98	–	75	2.88%	22.00%
16	QR VaR	440	0.95	–	75	2.88%	22.00%
17	QR VaR	440	0.96	–	75	2.88%	22.00%
Quantile Regression VaR (2 explanatory variables)							
18	QR VaR	480	0.90	0.98	79	3.04%	8.87%
19	QR VaR	480	0.92	0.97	80	3.08%	6.88%
20	QR VaR	480	0.93	0.97	80	3.08%	6.88%

Table 5.1. Best-performing VaR model parametrizations passing the UC test. The table reports the VaR specifications that produced exceedance frequencies consistent with the theoretical 2.5% level according to the Unconditional Coverage (UC) test. Only the best-performing parametrizations are presented.

From the results reported in Table 5.1 we can see that the Parametric Normal VaR produces an exceedance rate of 3.00%, which is slightly above the theoretical benchmark. Although the UC test does not reject the model, the higher exceedance frequency suggests that the normal distribution tends to underestimate extreme losses. This outcome is consistent with the empirical evidence reported in the literature, which shows that financial return distributions often display fat tails and asymmetry relative to the Gaussian case (Jorion, 2007; Alexander, 2008).

The SGSt VaR specification produces an exceedance rate of 3.08%, which remains above the expected level but still passes the UC test. The SGSt distribution allows for skewness and heavier tails (Theodossiou, 1998), which generally improves the modelling of extreme returns. However, the results indicate that distributional flexibility alone is not sufficient to fully align the exceedance frequency with the theoretical target.

The volatility-adjusted Historical Simulation VaR produces exceedance rates between approximately 2.73% and 2.88%, which are considerably closer to the theoretical 2.5% level. The

corresponding UC p-values are also relatively high, indicating that the empirical exceedance frequency is statistically consistent with the theoretical benchmark. This result supports the findings of Hull and White (1998), who show that incorporating volatility scaling improves the performance of historical simulation models by accounting for time-varying volatility.

Finally, the best-performing Quantile Regression VaR models where obtained using only 1 variable, generating exceedance rates between 2.81% and 2.88% and using 2 variables generate exceedance rates of 3.04% and 3.08%.

From the UC test, it's clear that the best performing models come from the Historical Simulation VaR with the Volatility Adjustment.

Model	Total	22–23	21–22	20–21	19–20	18–19	17–18	16–17	15–16	14–15	13–14
1	3.00	3.46	3.08	3.46	4.62	1.92	2.69	1.54	1.92	3.46	3.85
2	3.08	2.69	2.69	4.62	4.62	1.54	3.46	1.54	1.92	3.46	4.23
3	2.81	3.46	1.92	4.23	2.69	1.54	3.46	1.92	2.31	3.08	3.46
4	2.73	3.85	2.31	2.69	2.69	1.92	4.23	1.92	1.54	2.69	3.46
5	2.77	3.08	1.92	4.23	2.69	2.69	3.85	2.31	1.15	3.08	2.69
6	2.77	3.08	1.92	3.85	3.08	2.69	3.85	2.31	1.15	3.08	2.69
7	2.81	3.08	2.31	3.85	3.08	2.31	3.46	2.69	1.15	3.08	3.08
8	2.81	3.08	1.92	3.85	3.08	2.69	3.85	2.31	1.54	3.08	2.69
9	2.81	2.69	2.31	3.85	3.08	2.69	3.46	2.31	1.54	3.46	2.69
10	2.73	2.69	2.31	3.46	3.08	2.31	3.46	2.31	1.15	3.08	3.46
11	2.88	3.46	2.31	3.08	3.85	2.31	3.46	2.31	1.54	3.46	3.08
12	2.77	2.69	2.31	3.46	3.85	2.31	3.46	1.92	1.15	3.46	3.08
13	2.85	3.08	1.54	3.46	3.85	2.31	4.23	2.31	1.92	3.08	2.69
14	2.81	3.08	2.31	3.46	3.46	2.31	3.85	2.69	1.15	3.08	2.69
15	2.88	3.46	3.08	2.69	3.46	1.92	3.46	2.31	1.92	3.46	3.08
16	2.88	3.46	2.31	3.08	3.85	3.08	3.46	1.92	1.92	3.08	2.69
17	2.88	3.46	2.69	3.08	3.85	2.69	3.85	1.54	1.92	3.08	2.69
18	3.04	3.08	2.69	4.62	4.23	2.31	3.46	1.92	1.54	3.46	3.08
19	3.08	3.08	2.69	4.23	4.62	2.31	3.46	1.54	2.31	3.46	3.08
20	3.08	3.08	2.69	4.23	4.62	1.92	3.85	1.92	1.54	3.46	3.46

Table 5.2. Exceedance rates for each VaR model across the backtesting subperiods.

Table 5.2 presents the exceedance rates for each VaR specification that passed the UC test across the different backtesting subperiods. Several consistent patterns emerge from the results. First, the overall exceedance frequencies for all models are relatively close to the theoretical 2.5% level, ranging approximately between 2.73% and 3.08% for the overall period. This indicates that, on average, the VaR models provide a reasonably accurate calibration of risk over the full sample period.

However, a clear increase in exceedances is observed during the 2019–2021 subperiod, where several models report exceedance rates above 4%. This period coincides with the market turmoil associated with the COVID-19 crisis, suggesting that extreme market movements led

to a temporary underestimation of risk by the models. Conversely, during relatively stable market conditions, such as the 2015–2017 period, exceedance rates are frequently below the theoretical level, often falling between 1% and 2%. This indicates that the VaR estimates tend to be relatively conservative during calm market regimes.

Model	Worst p-value	Lag	Clustering
1	0.00%	6	39
2	0.00%	3	21
3	0.00%	3	18
4	0.00%	4	22
5	0.03%	1	7
6	0.00%	3	17
7	0.00%	3	17
8	0.00%	3	17
9	0.00%	3	18
10	0.00%	3	18
11	0.00%	3	20
12	0.00%	3	19
13	0.00%	3	18
14	0.00%	3	19
15	0.00%	10	53
16	0.00%	3	16
17	0.00%	3	17
18	0.00%	3	18
19	0.00%	3	22
20	0.00%	3	23

Table 5.3. Results of the BCP test for the full backtesting period.

In the second stage of the backtesting, the BCP test described in the subsection 5.2 was implemented. The results of this test for the full backtesting period are reported in Table 5.3. The results indicate that all VaR specifications are rejected at the 5% significance level, as the worst p-values are extremely small, with most models presenting values equal to 0%. This leads to a clear rejection of the null hypothesis of independence and correct conditional coverage of exceedances. The failure of all models in the BCP test is largely explained by the clustering of exceedances observed during the period of heightened market volatility associated with the COVID-19 crisis. During this period, several large market movements occurred in close succession, generating multiple consecutive VaR violations and creating strong dependence between exceedances.

Model	Total	22–23	21–22	20–21	**19–20**	18–19	17–18	16–17	15–16	14–15	13–14
1	0.00	15.92	11.53	50.61	**0.00**	0.03	5.35	79.92	0.28	8.70	1.57
2	0.00	1.07	5.35	43.08	**0.08**	0.00	19.95	79.92	42.70	8.70	1.11
3	0.00	17.62	0.28	47.19	**0.00**	0.00	19.95	74.95	0.00	8.02	0.29
4	0.00	30.04	0.73	24.54	**0.00**	0.03	1.83	74.95	0.01	2.92	0.29
5	0.03	5.88	14.28	47.19	**0.00**	5.33	0.65	70.05	84.93	8.02	5.35
6	0.00	5.88	14.28	51.48	**0.00**	5.33	0.65	70.05	84.93	8.02	5.35
7	0.00	5.88	1.72	51.48	**0.00**	0.41	19.95	65.24	84.93	8.02	8.02
8	0.00	5.88	14.28	51.48	**0.00**	5.33	0.65	70.05	79.92	8.02	5.35
9	0.00	1.07	1.72	51.48	**0.00**	5.33	19.95	70.05	79.92	8.70	5.35
10	0.00	1.07	1.72	50.61	**0.00**	0.41	19.95	70.05	84.93	8.02	15.36
11	0.00	15.92	1.72	38.77	**0.00**	1.28	19.95	70.05	79.92	8.70	8.02
12	0.00	1.07	1.72	50.61	**0.00**	1.28	19.95	74.95	84.93	8.70	8.02
13	0.00	5.88	79.92	50.61	**0.00**	0.41	1.83	70.05	74.95	8.02	5.35
14	0.00	4.14	1.72	50.61	**0.00**	0.41	0.65	65.24	84.93	8.02	5.35
15	0.00	19.27	11.53	14.85	**0.00**	0.03	19.95	70.05	0.28	8.70	4.66
16	0.00	15.92	1.72	38.77	**0.00**	0.19	19.95	74.95	74.95	8.02	5.35
17	0.00	15.92	5.35	38.77	**0.00**	5.33	7.92	74.95	79.92	8.02	5.35
18	0.00	4.14	0.03	52.92	**6.52**	0.41	19.95	74.95	79.92	8.70	4.66
19	0.00	4.14	0.03	41.20	**0.08**	0.41	19.95	79.92	70.05	8.70	0.00
20	0.00	4.14	0.03	41.20	**0.08**	0.03	7.92	74.95	79.92	8.70	0.00

Table 5.4. Worst p-values of the BCP test for each VaR model across the backtesting subperiods. The COVID-19 crisis period (2019–2020) is highlighted in bold.

In the third stage of the model selection procedure, the focus shifts to the stability of the exceedance rates across the different annual subperiods. Since all remaining models successfully passed the UC test, the objective was to identify the specifications that produce the most consistent exceedance behaviour over time.

Following this analysis, presented in Table 5.2, the models that displayed the most stable exceedance behaviour across the subperiods are Model 9 and Models 13 to 17. These specifications presented exceedance rates that remain closer to the significance level of 2,5% across a larger number of subperiods compared to the remaining models.

In the fourth stage, the same evaluation procedure was repeated using a shorter backtesting window. The sample size was reduced to half of the original period, corresponding to approximately 260 weeks. This step serves as a robustness check aimed at breaking potential ties between models that display similar performance in the previous stage.

Model	Total	22–23	21–22	20–21	19–20	18–19
9	**2.92**	**2.69**	**2.31**	**3.85**	**3.08**	**2.69**
13	2.85	3.08	1.54	3.46	3.85	2.31
14	2.92	3.08	2.31	3.46	3.46	2.31
15	2.92	3.46	3.08	2.69	3.46	1.92
16	3.15	3.46	2.31	3.08	3.85	3.08
17	3.15	3.46	2.69	3.08	3.85	2.69

Table 5.5. Exceedance rates for the finalist VaR models using the reduced backtesting window (260 weeks).

Model 9 was selected as the final specification since it exhibits exceedance rates within the 2.2%–2.8% interval across the largest number of subperiods.

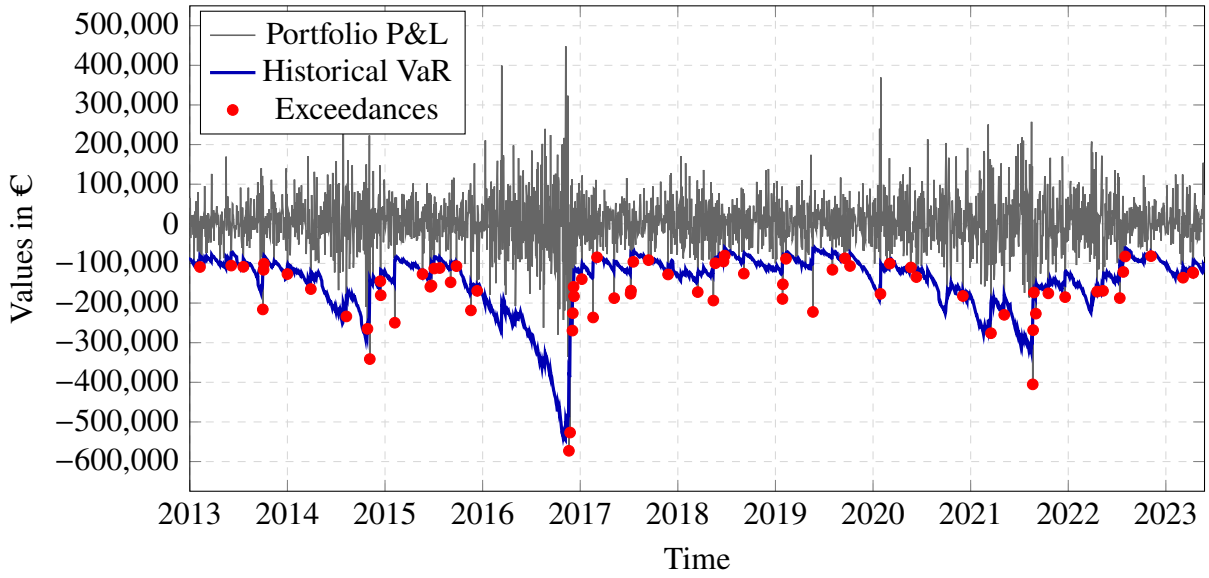


Figure 5.1. Backtesting results of the Historical VaR model.

Figure 5.1 shows that the VaR series broadly tracks the evolution of portfolio risk, adjusting to changes in market conditions and deepening during periods of higher volatility. Exceedances are generally well distributed over time, with a higher concentration during stress periods, particularly at the end of 2016 and between 2021 and 2022. Most exceedances are not far from the VaR threshold, suggesting that the model captures the magnitude of typical losses reasonably well. Overall, the alignment between the VaR estimates and the lower tail of the P&L supports its adequacy as a measure of downside risk.

Chapter 6.

VAR-BASED RISK MANAGEMENT AND HEDGING STRATEGY

6.1 VaR Management Framework

Using the VaR model selected in the backtesting, namely the volatility-adjusted Historical Simulation VaR, with a smoothing factor $\lambda = 0.96$ and a rolling estimation window of 280 observations, a dynamic risk control mechanism was implemented, defining a target level for the portfolio VaR. The target was set as the median of the distribution of VaR estimates obtained over the forward estimation period of 330 trading days.

The resulting threshold is equal to €100,066.27, which ensures robustness by reducing the influence of extreme observations.

The portfolio is managed dynamically on a daily basis over the 330-day period according to the following rule:

- If $VaR_t \leq VaR^{target}$, no action is taken and the portfolio remains unchanged.
- If $VaR_t > VaR^{target}$, a hedging strategy is implemented to reduce the VaR to the target level.

This rule ensures that risk is actively controlled while avoiding unnecessary trading when the portfolio remains within acceptable risk limits.

Throughout this period, bond cash flows (coupon payments) were reinvested in the portfolio to preserve the economic value of the portfolio and to avoid artificial reductions in risk due to uninvested cash positions. This reinvestment can be observed in Appendix A.1.

Transaction costs are not considered in this analysis, allowing the focus to remain on the effectiveness of the risk management strategy.

6.2 VaR Decomposition and Risk Drivers

While VaR provides a measure of the total level of portfolio risk, it does not offer information about the underlying sources of that risk. We need to identify the contribution of each risk factor for an effective risk management strategy.

To address this limitation, the total VaR is decomposed into contributions associated with each risk factor through the Marginal VaR. This measure captures the sensitivity of the portfolio

VaR to small changes in the exposure to a given risk factor. Formally, let Θ denote the vector of portfolio exposures. The Marginal VaR of factor i is defined as:

$$MVaR_i = \frac{\partial VaR}{\partial \theta_i}. \tag{6.1}$$

Collecting these sensitivities for all risk factors yields the gradient of the VaR function:

$$\nabla VaR(\Theta) = \begin{bmatrix} \frac{\partial VaR}{\partial \theta_1} \\ \vdots \\ \frac{\partial VaR}{\partial \theta_n} \end{bmatrix}. \tag{6.2}$$

Given the absence of a closed-form expression for VaR under the volatility-adjusted Historical Simulation approach, these sensitivities were computed numerically. Each exposure is perturbed by a small amount ϵ, and the resulting change in VaR is used to approximate the corresponding partial derivative:

$$\frac{\partial VaR}{\partial \theta_i} \approx \frac{VaR(\Theta + \epsilon) - VaR(\Theta)}{\epsilon}. \tag{6.3}$$

From this process, we can express the total VaR as a sum of individual contributions:

$$VaR = \sum_{i=1}^{n} \theta_i \frac{\partial VaR}{\partial \theta_i}. \tag{6.4}$$

This decomposition transforms VaR from a single aggregate measure into a detailed representation of how risk is distributed across the different factors affecting the portfolio.

Category	VaR (EUR)	Contribution (%)
Risk Factor Type Decomposition		
Stock	82 874.25	89.95
Currency	8 689.12	9.43
Interest Rate	571.23	0.62
Asset Class Decomposition		
Stock	91 781.20	99.62
Bond	353.40	0.38
Currency Decomposition		
USD	−366.08	−0.40
GBP	78 394.56	85.09
EUR	6 338.04	6.88
JPY	7 166.21	7.78
HKD	601.87	0.65

Table 6.1. Marginal VaR decomposition on 4 December 2023. The table presents the contribution of each category to the total portfolio VaR.

Asset	Marginal VaR (EUR)
AAPL	−1 193.38
JPM	−337.45
PFE	201.14
PG	845.23
ULVR.L	49 533.43
RR.L	5 029.15
HSBA.L	24 517.71
BP.L	−685.72
SAP.DE	2 663.22
SIE.DE	2 195.29
BAS.DE	166.70
ALV.DE	1 077.82
6758.T	106.82
9983.T	6 697.31
8035.T	644.88
7203.T	−282.79
0011.HK	219.87
2318.HK	253.31
0941.HK	173.48
0002.HK	−44.80
US912810TA60	94.88
DE000BU22049	68.78
US91282CJG78	23.51
DE0001030559	166.24

Table 6.2. Marginal VaR decomposition by asset on 4 December 2023. The table reports the marginal contribution of each asset to the total portfolio VaR. Positive values indicate an increase in portfolio risk, while negative values reflect diversification effects.

The marginal VaR decomposition presented in Tables 6.1 and 6.2 reveals a strong concentration of portfolio risk in equity exposures. As shown in Table 6.1, equities account for approximately 90% of total VaR, while interest rate risk is negligible. At the asset class level, this dominance becomes even more pronounced, with equities representing 99.62% of total risk, confirming that fixed income positions contribute only marginally to the overall risk profile.

From a currency perspective, Table 6.1 highlights a significant concentration of risk in GBP, which alone represents more than 85% of total VaR. This concentration is directly linked to the portfolio's exposure to UK equities. From Table 6.2, we can see that Unilever (ULVR.L) is by far the largest contributor to marginal VaR, accounting for a substantial share of total portfolio risk, explaining the dominant role of GBP in the currency decomposition of our portfolio.

From the individual asset perspective, we can see that the results in Table 6.2 indicate that risk is highly concentrated in a small number of positions, notably ULVR.L and HSBA.L, while most other assets contribute only marginally. Additionally, some assets such as AAPL, JPM and BP.L exhibit negative marginal VaR contributions, indicating that an increase in the exposure to

these assets leads to a reduction in total portfolio risk.

Overall, the marginal VaR decomposition reveals a highly concentrated risk structure in the UK market and GBP currency, providing a clear economic rationale for focusing the hedging strategy around these geography. The evolution of the total VaR marginal contribution by index for the unhedged portfolio can be observed in Appendix B. 1.

6.3 Hedging Strategy

The hedging strategy used in this work follows a structured process. For each day in which the VaR exceeds the predefined threshold, the excess risk was computed as:

$$Excess\ VaR_t = VaR_t - VaR^{target}. \tag{6.5}$$

This excess was allocated between the FTSE and Nikkei indices proportionally to their marginal contributions to total VaR. Let $MVaR_t^{FTSE}$ and $MVaR_t^{NIKKEI}$ denote the marginal VaR contributions of the two indices. The corresponding weights were defined as:

$$w_t^{FTSE} = \frac{MVaR_t^{FTSE}}{MVaR_t^{FTSE} + MVaR_t^{NIKKEI}}, \quad w_t^{NIKKEI} = 1 - w_t^{FTSE}. \tag{6.6}$$

The hedging strategy was implemented using forward contracts on the indices mentioned above. To find the size of the hedging positions we had to use a sensitivity-based approach. Let $\Delta VaR_t = VaR_t - VaR^{target}$ denote the excess risk to be eliminated. The hedge position associated with each risk factor was approximated as:

$$\theta_{i,t} \approx \frac{\Delta VaR_t}{\frac{\partial VaR_t}{\partial RF_i} + \frac{\partial VaR_t}{\partial FX_i}}. \tag{6.7}$$

Using this process, we could ensure that the hedge directly targets the reduction of portfolio risk rather than simply offsetting nominal exposures. It can be interpreted as a first-order approximation of the VaR function, where marginal VaR represents the sensitivity of the risk measure to each underlying factor.

Positions were iteratively adjusted until the difference between the estimated VaR and the target falls within a tolerance interval of [-2, 2], resulting in an almost complete hedging of excess risk.

The total VaR marginal contribution by index for the hedged portfolio can be observed in Appendix B.2.

6.4 Results: Hedged vs Unhedged Portfolio

In this section we will evaluate the effectiveness of the VaR risk management strategy by comparing the performance of the hedged and unhedged portfolios over the 330 days.

6.4.1 Hedging Positions

Figure 6.1 illustrates the evolution of the hedging positions implemented through forward contracts over the sample period.

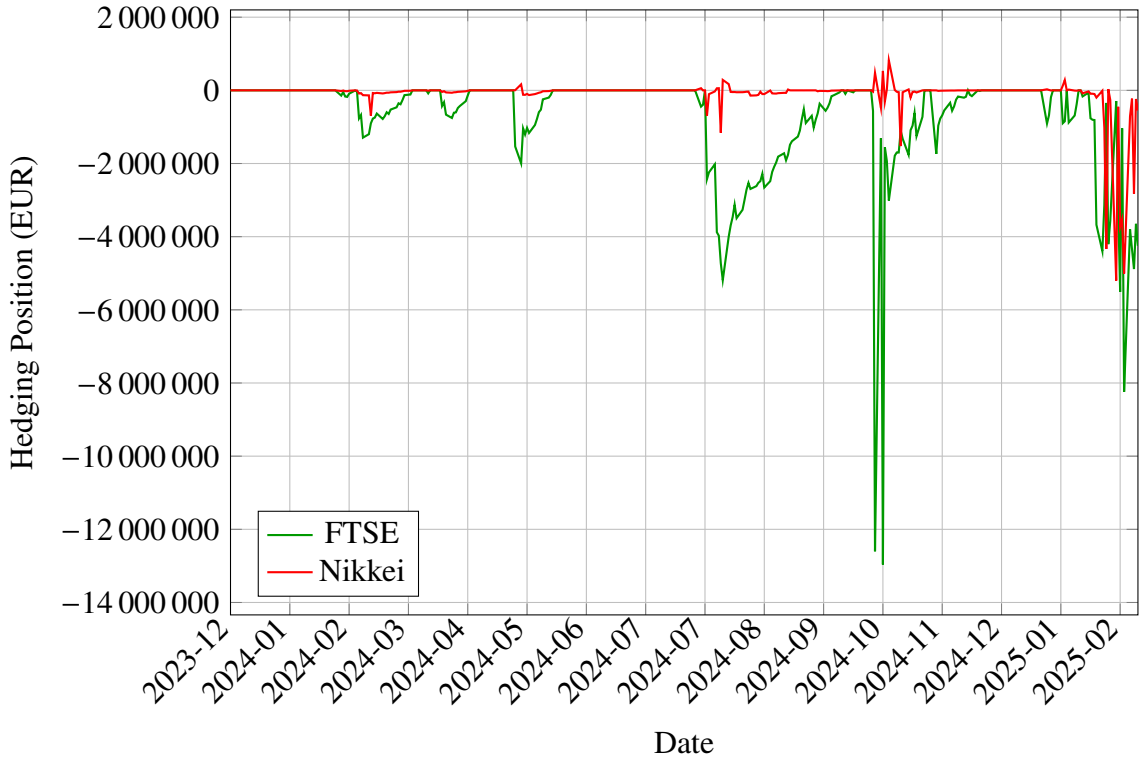


Figure 6.1. Evolution of hedging positions using FTSE and Nikkei forward contracts.

It is clear that the majority of positions that we took were short, reflecting the need to reduce the portfolio's exposure to the main sources of risk whenever the VaR exceeds the predefined threshold. Only a limited number of long positions are observed, only in the Nikkei index, were some adjustments to changes in the relative contribution of this risk factor needed to be done.

The magnitude of the hedging positions were significantly higher for the FTSE compared to the Nikkei, which is consistent with the marginal VaR decomposition by asset presented in Table 6.2, where assets of the FTSE index account for the largest share of portfolio risk. To understand better how the total VaR marginal decomposition changed with the hedging positions that we took, we can analyse Appendix B. 3.

Figure 6.2 presents the evolution of the total VaR for the hedged and unhedged portfolios, highlighting the periods in which the VaR exceeds the predefined threshold and the dynamic hedging strategy was implemented.

The final bond portfolio composition is presented in Appendix C.2 and the final stock portfolio composition is presented in Appendix C.3.

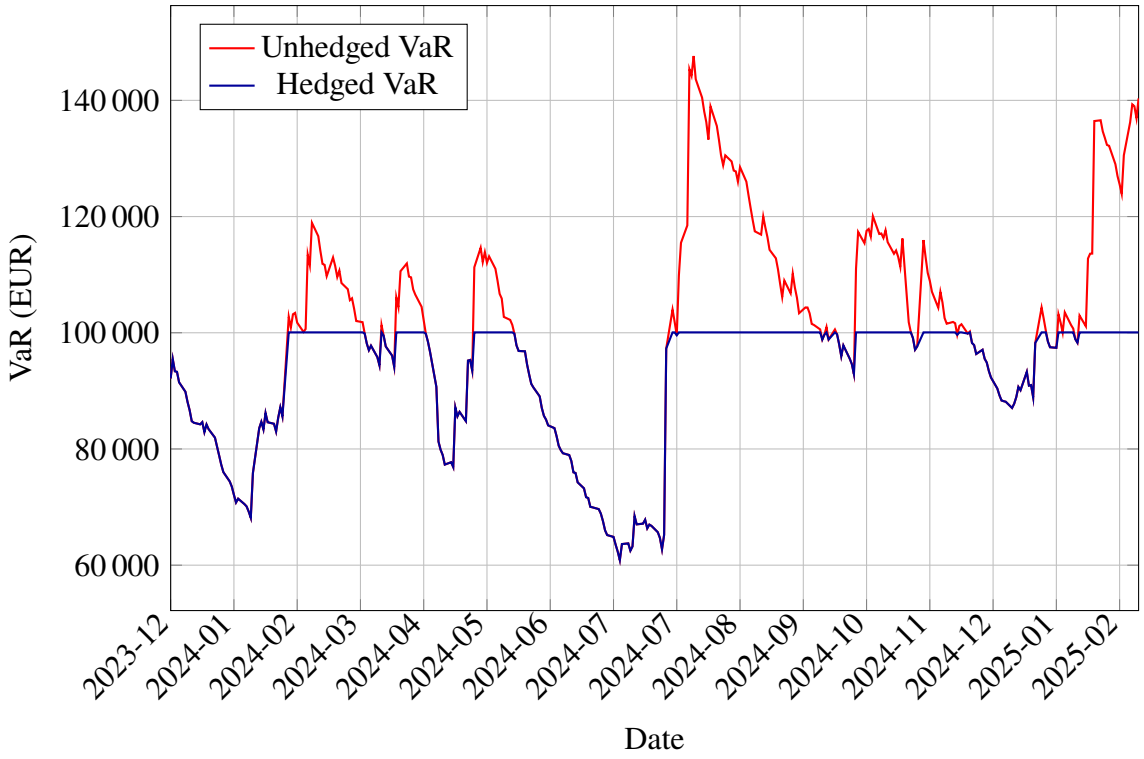


Figure 6.2. Comparison between hedged and unhedged VaR over the analysis period.

6.4.2 Portfolio P&L

Figure 6.3 presents the evolution of the portfolio P&L evolution of the hedged and unhedged portfolios. In the hedged portfolio value, the P&L generated by the forward positions is included but not reinvested.

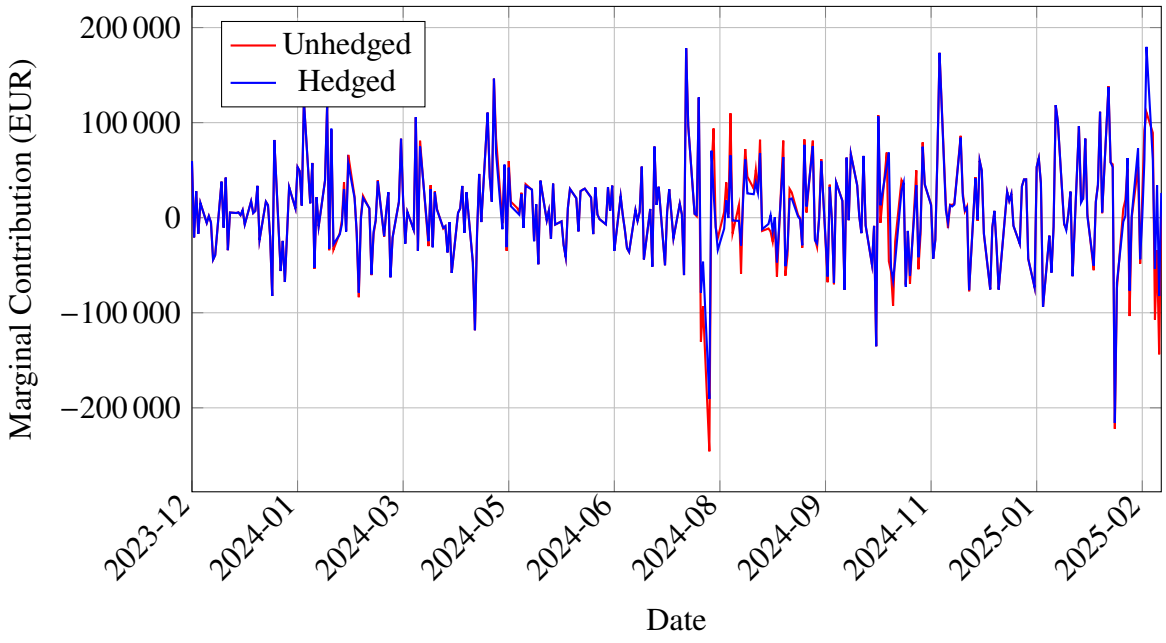


Figure 6.3. Comparison of portfolio P&L with and without hedging.

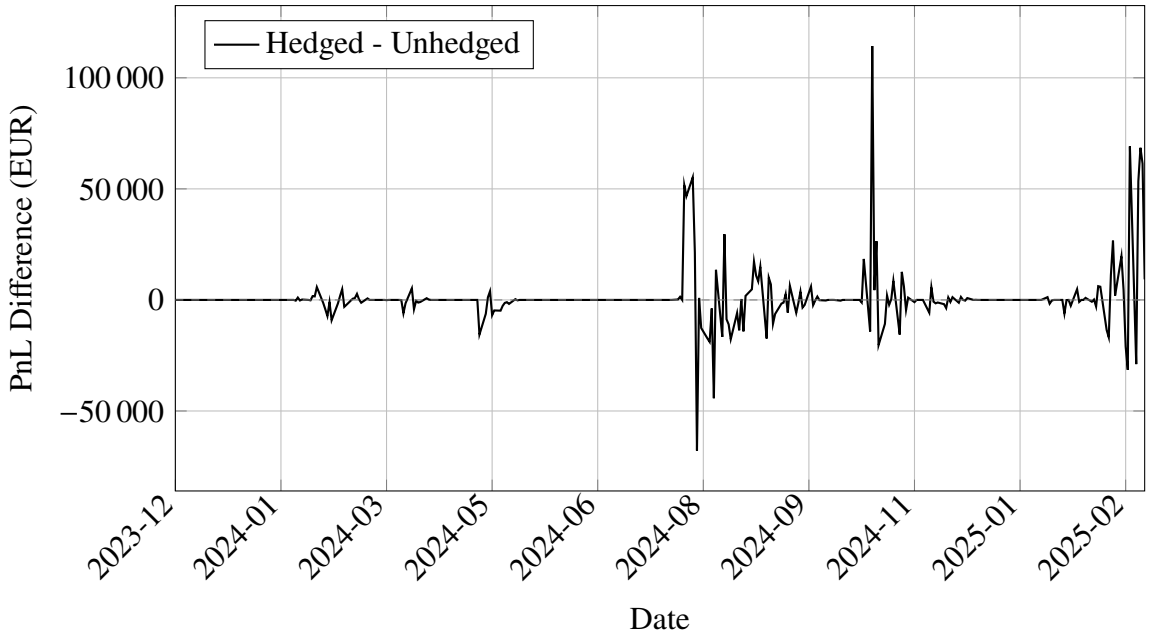


Figure 6.4. Evolution of the difference between hedged and unhedged P&L.

Analyzing Figure 6.3 jointly with Figure 6.4, it becomes clear that the implementation of the hedging strategy reduces the magnitude of portfolio P&L fluctuations, bringing outcomes closer to zero.

In periods of positive P&L, the hedged portfolio exhibited lower gains, while in periods of negative P&L, losses were mitigated, resulting in less adverse outcomes. This behaviour is consistent with the risk management literature, that highlights that hedging strategies reduce both downside risk and upside potential, leading to less volatile returns (Dowd, 2002).

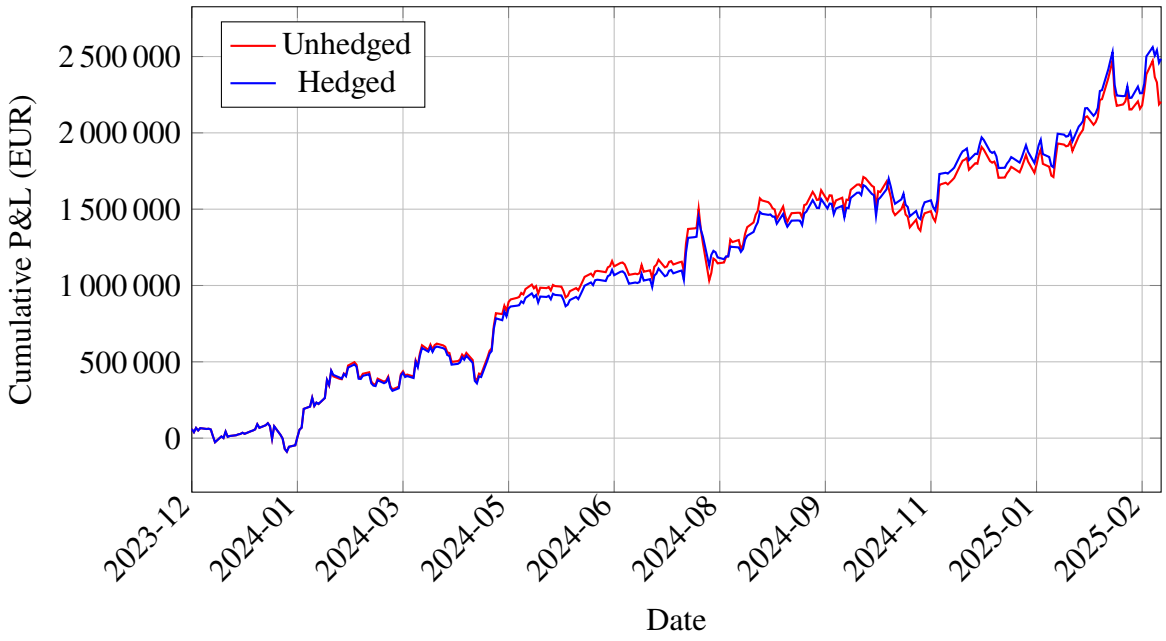


Figure 6.5. Evolution of cumulative P&L for the hedged and unhedged portfolios.

From the Figure 6.5 it's possible to understand that the hedged portfolio had a superior performance, that can be attributed to the timing of the hedging strategy, which was activated during periods of adverse market conditions. In these periods, the underlying portfolio tended to perform poorly, and the hedge effectively reduced exposure to these negative movements. As a result, the mitigation of large losses may have outweigheted the reduction in upside potential, leading to improved overall performance over the sample period.

As a result, the strategy implemented in this work not only mitigated downside risk but may also improved the portfolio performance (although this outcome is partly dependent on market conditions).

Statistic	**Unhedged**	**Hedged**
Mean	6 677.76	7 531.07
Median	5 940	5 625.81
Std. Deviation	53 291.09	49 906.07
Minimum	−245 767.86	−215 903.60
Maximum	178 351.30	179 730.17
5th Percentile	−74 841.12	−68 371.95

Table 6.3. Summary statistics of P&L for hedged and unhedged portfolios. The table reports key distributional measures based on the observed sample.

The results from Table 6.3 show that the hedged portfolio achieved a higher average return, with the mean increasing from 6,677.76€ to 7,531.07€, indicating an overall improvement in performance. However, the median slightly decreased, suggesting that the typical daily outcome remains broadly similar across both strategies.

When we analyze the standard deviation of both strategies, it's easy to identify a decrease from the unhedged to the hedged portfolio volatility, from 53,291.09€ to 49,906.07€, being this a logical outcome and in line with the literature mentioned above. In addition, the minimum P&L was significantly less negative in the hedged portfolio, highlighting the effectiveness of the strategy in limiting extreme losses.

This improvement in downside protection was further confirmed by the 5th percentile, which became less negative (from -74,841.12€ to -68,371.95€), suggesting a reduction in tail risk.

At the same time, the maximum P&L remained largely unchanged, indicating that the hedging strategy does not materially limited upside potential.

In addition to analysing returns and risk separately, it is also important to assess how efficiently risk was used. For this purpose, we used RORAC, which is a very useful metric, that relates the return generated by the portfolio to the amount of risk taken.

According to Matten (2000), RORAC is widely applied in risk management as a way of evaluating performance in a capital allocation framework. Intuitively, it captures how much return is generated for each unit of risk.

It is defined as:

$$RORAC = \frac{P\&L}{EC} \tag{6.8}$$

The portfolio P&L is the cumulative P&L value and the economic capital is the cumulative VaR value over the 330 days.

Portfolio	Cumulative P&L (EUR)	EC (EUR)	RORAC (%)
Unhedged	2 203 661.47	32 857 508.18	6.70
Hedged	2 485 252.41	30 521 403.48	8.14

Table 6.4. RORAC comparison between hedged and unhedged portfolios. The table reports cumulative returns and total economic capital over the sample period.

The results presented in Table 6.4 show that the hedging strategy significantly enhanced capital efficiency, delivering higher returns while requiring less risk, as evidenced by the increase in RORAC from 6.7% to 8.1%.

6.4.3 Exceedances

Figure 6.6 shows that the unhedged portfolio exhibited a total of 8 VaR exceedances over the 330 days period, indicating several instances where realized losses surpassed the defined risk threshold.

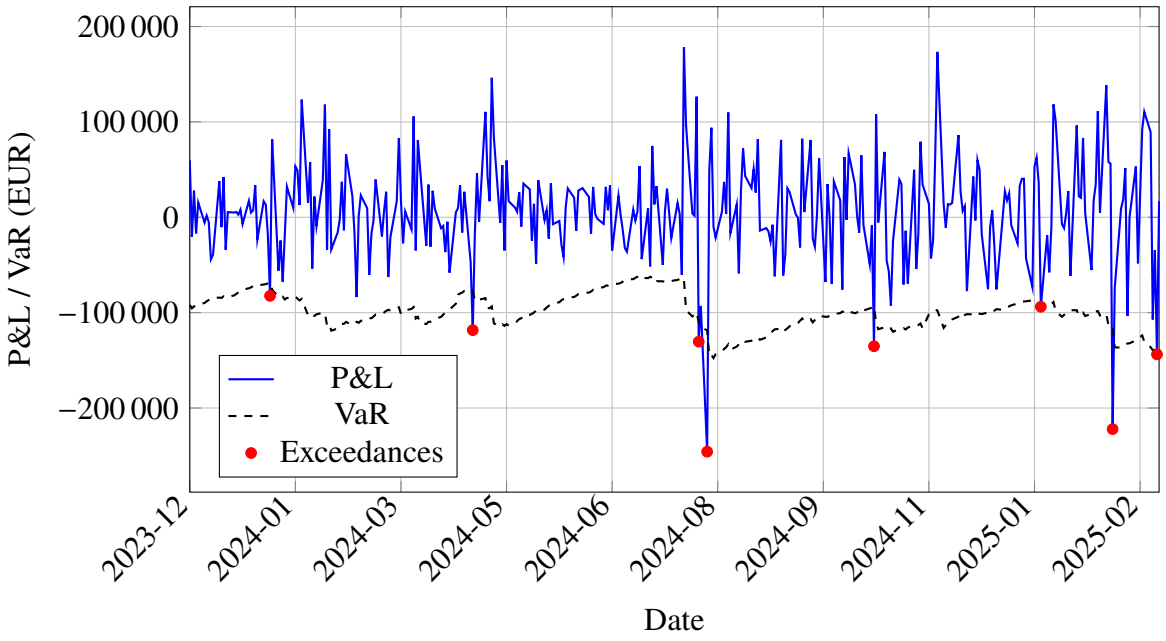


Figure 6.6. Unhedged portfolio: VaR exceedances.

Date	P&L	VaR Unhedged	Exc. Unhedged	VaR Hedged	Exc. Hedged
2024-01-11	−82 350.82	−68 166.41	14 184.42	−68 166.41	14 184.42
2024-04-16	−118 348.30	−76 889.77	41 458.53	−76 889.77	41 458.53
2024-08-01	−130 438.97	−109 855.48	20 583.49	-	-
2024-08-05	−245 767.86	−118 471.58	127 296.28	−100 067.61	90 514.63
2024-10-23	−135 215.04	−92 909.93	42 305.11	−92 909.93	42 305.11
2025-01-10	−93 821.18	−90 113.03	3 708.14	−90 113.03	3 708.14
2025-02-13	−222 094.39	−113 630.38	108 464.01	−100 066.27	115 837.33
2025-03-06	−143 731.57	−136 925.47	6 806.11	-	-

Table 6.5. Comparison of VaR exceedances for the hedged and unhedged portfolios. The table reports the exceedance dates and magnitudes for both portfolios. A dash indicates that no exceedance occurred for the corresponding portfolio on that date.

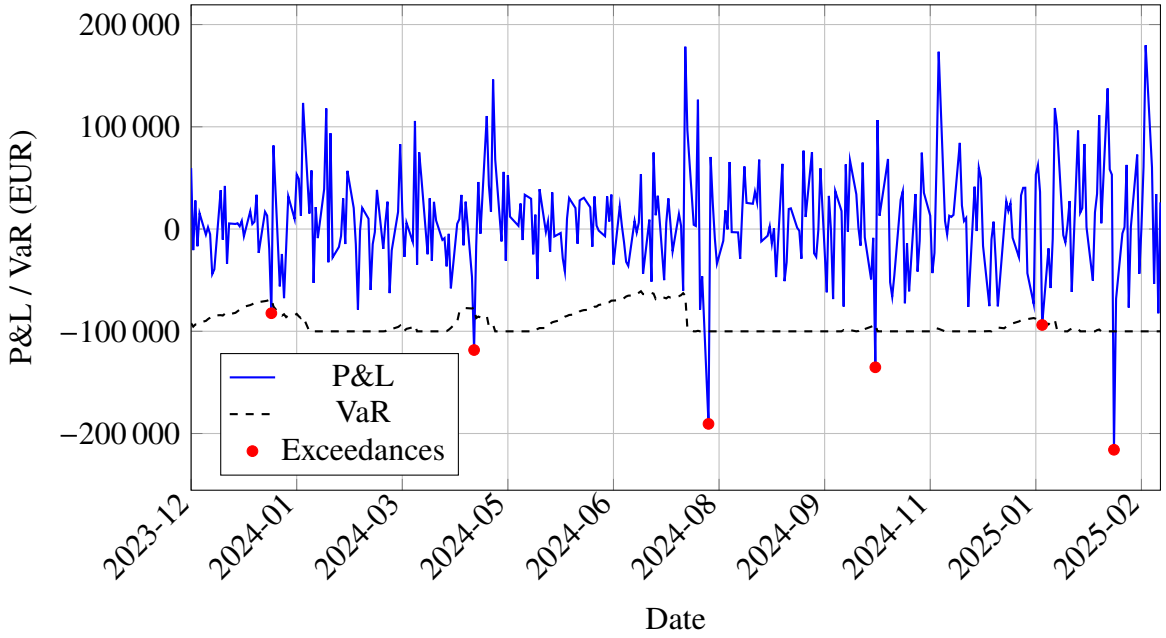


Figure 6.7. Hedged portfolio: VaR exceedances.

Table 6.5 and Figure 6.7 highlight the reduction in the number of VaR exceedances following the implementation of the hedging strategy, from 8 in the unhedged case to 6 in the hedged portfolio, namely those observed on 1 August 2024 and 6 March 2025.

The magnitude of the exceedances is reduced in specific cases, particularly on 5 August 2024, where the difference between the realized loss and the VaR threshold decreases significantly after hedging. However, this effect is not uniform across all observations, as the exceedance on 13 February 2025 increase.

Chapter 7.

CONCLUSION

This dissertation evaluated the role of VaR as both a statistical measure of market risk and an active tool for portfolio risk management. The analysis was structured in sequential stages, combining risk factor mapping, model selection through backtesting, and the implementation of a dynamic hedging strategy.

In the first stage of the dissertation, the portfolio was mapped into its underlying risk factors, allowing the decomposition of exposures into equity, interest rate and exchange rate components, in order to link the portfolio P&L to its main sources of risk. The results revealed a strong concentration of risk in equity exposures, particularly in UK assets, as well as a significant contribution from GBP risk.

In the second stage, alternative VaR models were estimated and evaluated using a structured backtesting procedure based on the Unconditional Coverage test and the BCP test. Among the models considered, the volatility-adjusted Historical Simulation VaR, with an EWMA parameter of $\lambda = 0.96$ and a rolling window of 280 observations, provided the most robust performance, producing exceedance rates close to the theoretical 2.5% significance level. However, the results showed that all models failed the BCP test due to clustering of exceedances during periods of market stress, highlighting a limitation of VaR in capturing extreme events.

The third stage involved the implementation of a dynamic risk management strategy based on a predefined VaR target (that we chose as the median of the forward VaR distribution), corresponding to €100,066.27. Whenever this threshold was exceeded, hedging positions were introduced using forward contracts on the FTSE and Nikkei indices. The hedging strategy was guided by the marginal VaR decomposition, which showed that equities account for approximately 90% of total risk, with GBP exposure alone representing more than 85%. In particular, assets such as Unilever and HSBC were identified as the main contributors to portfolio risk, justifying the focus on FTSE-based hedging instruments.

The results from comparing the hedged and unhedged portfolios show that the VaR-based strategy effectively reduces risk. The number of VaR exceedances decreased from 8 to 6, while the magnitude of extreme losses was also reduced, with the minimum P&L improving from -245,767.86€ to -215,903.60€ and the 5th percentile becoming less negative. At the same time, and this was a surprise, the hedged portfolio achieved a higher cumulative P&L, increasing from €2,203,661.47 to €2,485,252.41.

This improvement in performance was accompanied by a reduction in economic capital,

measured as cumulative VaR, from €32,857,508.18 to €30,521,403.48. As a result, the RORAC increased from 6.7% to 8.1%, indicating a clear improvement in capital efficiency.

From an economic perspective, these findings support the interpretation of VaR as a proxy for Economic Capital. By dynamically controlling VaR, the portfolio keeps track of how much money it needs to cover possible losses, which improves the balance between risk and return.

Although the results are in line with our expectations, there are some limitations that should be recognized. The analysis does not include transaction costs that could lower the benefits of the hedging strategy. On the other hand, VaR also doesn't show how bad losses are beyond the chosen quantile, which could lead to an underestimation of tail risk.

Future research could enhance this framework by integrating Expected Shortfall as an additional risk metric tool.

In conclusion, this dissertation shows that VaR can be transformed from a passive risk measure into an effective tool for dynamic risk management. By combining model selection, risk decomposition and targeted hedging, the proposed framework improves both risk control and capital efficiency in a multi-asset portfolio context.

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APPENDIX A

Bond Cash Flows Reinvestment

Date	Company	Code	Stocks	Amount (€)
15/02/2024	JPMorgan Chase	JPM	5	812.27
20/03/2024	Rolls-Royce	RR.L	8	3750.00
16/04/2024	Fast Retailing	9983.T	5	1274.00
02/05/2024	Ping An Insurance	2318.HK	280	1142.59
16/08/2024	HSBC Holdings	HSBA.L	1	794.50
01/11/2024	Tokyo Electron	8035.T	8	1126.32
17/02/2025	Pfizer Inc.	PFE	34	814.99

Table 1. Cash reinvestments. Allocation of bond cash flows into equity positions over the sample period.

APPENDIX B

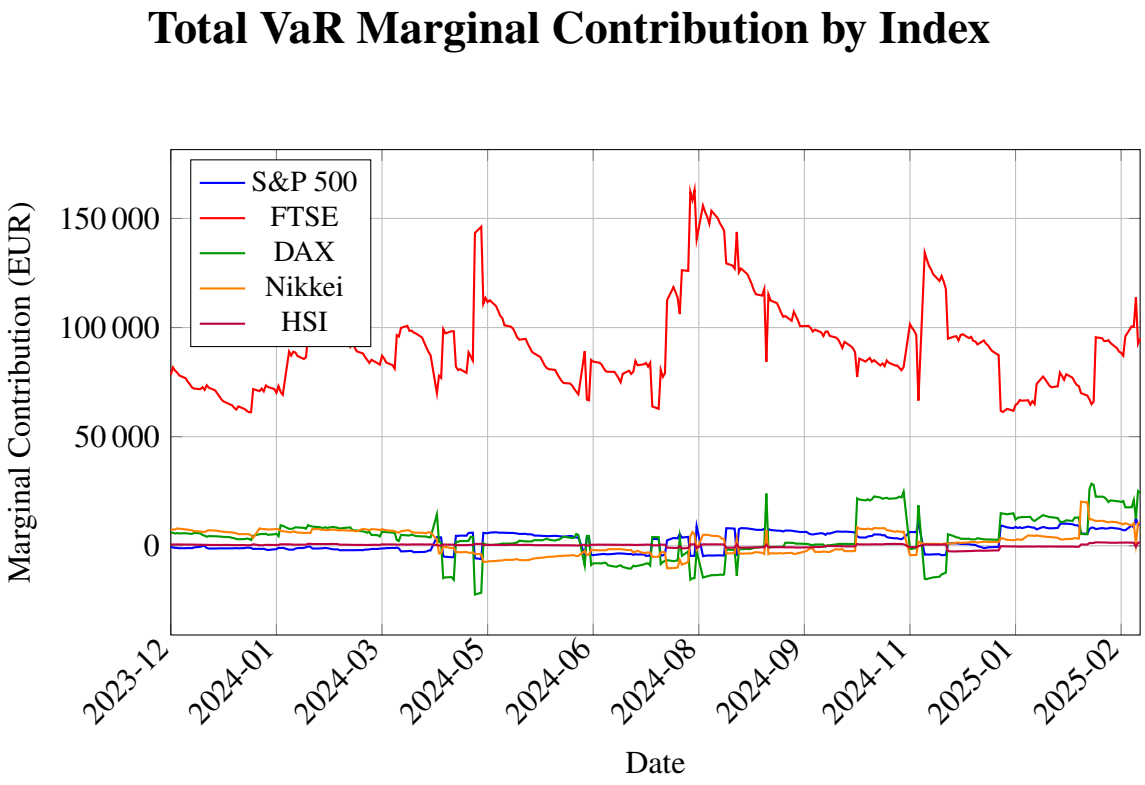


Figure 1. Time evolution of marginal risk contributions (unhedged portfolio). The figure shows the evolution of each index's contribution to portfolio risk over time.

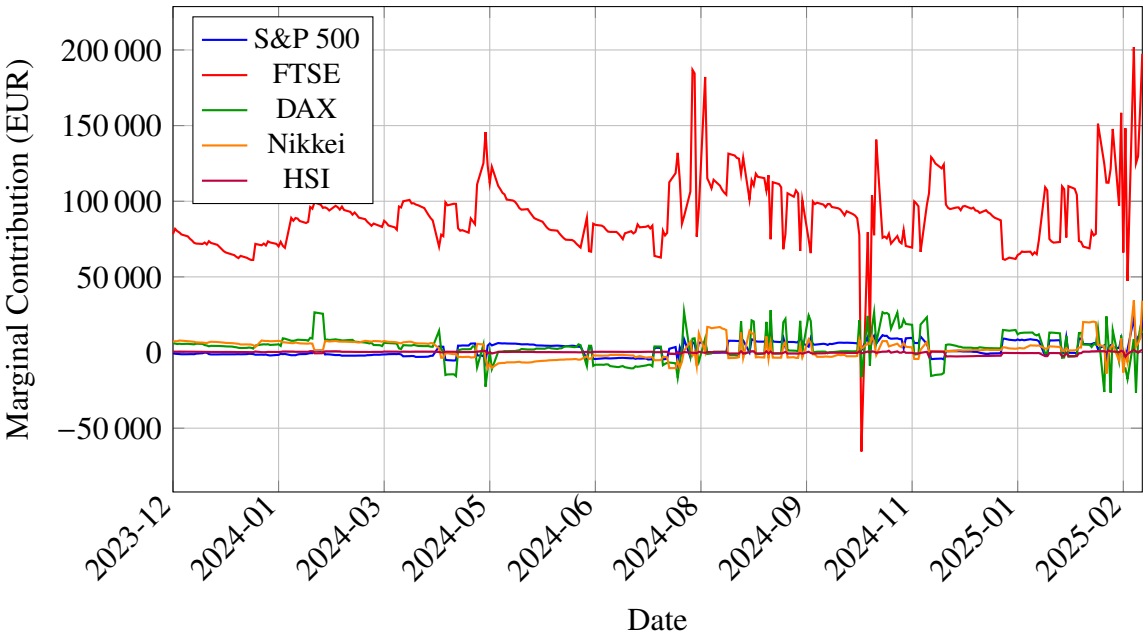


Figure 2. Time evolution of marginal risk contributions (hedged portfolio). The figure shows the evolution of each index's contribution to portfolio risk after hedging.

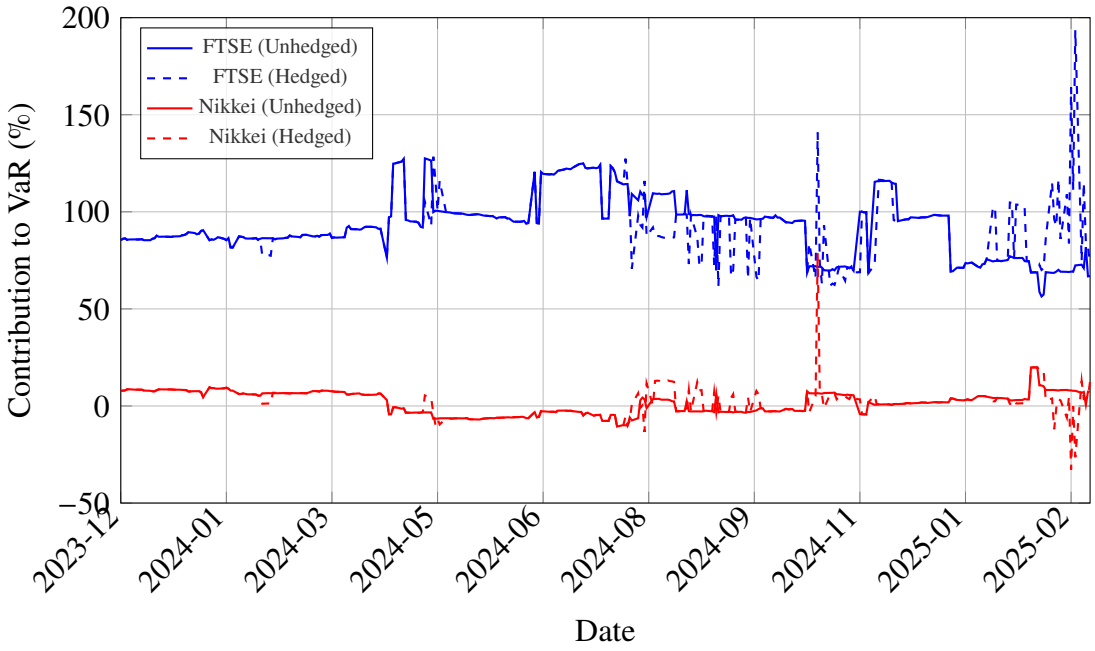


Figure 3. Comparison of marginal VaR contributions before and after hedging.

APPENDIX C

Final Portfolio Composition

Bond	ISIN	Face Value	Currency	Face Value (EUR)	Fair Value (EUR)
Bond 1	US912810TA60	100000	USD	92636.00	63567.65
Bond 2	DE000BU22049	150000	EUR	150000.00	154022.29
Bond 3	US91282CJG78	50000	USD	46318.00	48913.38
Bond 4	DE0001030559	200000	EUR	200000.00	182875.53
Total					**449 378.85**

Table 2. Final bond portfolio composition. The table reports the main characteristics of each bond position, including nominal value, currency denomination, and valuation in euros, on 7 March 2025

Stock	Ticker	Currency	Ref Mkt	Quantity	Stock Price	Value (EUR)
Apple Inc.	AAPL	USD	GSPC	1000	235.02	217714.80
JPMorgan Chase	JPM	USD	GSPC	805	245.03	182724.26
Pfizer Inc.	PFE	USD	GSPC	634	25.75	15122.58
Procter & Gamble Co.	PG	USD	GSPC	900	173.58	144716.17
Unilever	ULVR.L	GBP	FTSE	700	4537.61	3792854.88
Rolls-Royce	RR.L	GBP	FTSE	508	804.73	488153.50
HSBC Holdings	HSBA.L	GBP	FTSE	1501	880.90	1578867.90
BP	BP.L	GBP	FTSE	2000	409.01	976807.49
SAP SE	SAP.DE	EUR	GDAXI	2500	261.35	653375.02
Siemens AG	SIE.DE	EUR	GDAXI	1800	243.00	437400.00
BASF SE	BAS.DE	EUR	GDAXI	300	51.79	15535.51
Allianz SE	ALV.DE	EUR	GDAXI	400	331.50	132599.99
Sony Group	6758.T	JPY	N225	500	3717.35	11559.10
Fast Retailing	9983.T	JPY	N225	1005	47740.00	298379.54
Tokyo Electron	8035.T	JPY	N225	308	21780.00	41718.55
Toyota Motor Corp	7203.T	JPY	N225	600	2772.61	10345.70
Hang Seng Bank Ltd.	0011.HK	HKD	HSI	2000	106.36	25355.58
Ping An Insurance	2318.HK	HKD	HSI	2780	46.58	15435.78
China Mobile	0941.HK	HKD	HSI	1800	82.20	17637.13
CLP Holdings Ltd	0002.HK	HKD	HSI	1000	64.99	7746.94
Total						**9 064 050.40**

Table 3. Final stock portfolio composition. The table reports the equity positions held in the portfolio, including market of reference, number of stocks, and valuation in euros, on 7 March 2025