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Value-at-Risk: measure and manage VaR in a portfolio composed of bonds and stocks

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Master in Finance

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Department of Finance

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Resumo

Os principais temas deste estudo são a medição e gestão do Value-at-Risk (VaR) de uma carteira composta por ações e obrigações Europeias e Americanas, utilizando diversos modelos de VaR e estratégias de cobertura para mitigar potenciais perdas. A análise inclui o cálculo de diferentes modelos de VaR e backtesting rigoroso para garantir que o modelo com o melhor desempenho é escolhido. Após definir o VaR máximo, é aplicada uma técnica de cobertura no modelo selecionado. Comparando a técnica de cobertura com uma carteira sem cobertura, observamos que esta foi eficaz em preservar o VaR dentro de um limite específico, diminuindo assim o potencial de perdas excessivas. A análise diária dos lucros e perdas revelou que a carteira coberta flutuou menos e sofreu perdas menores, o que contribuiu para a estabilidade geral. Para os investidores e gestores de risco que procuram estratégias para reduzir o risco, estas conclusões destacam a importância da implementação de técnicas eficazes de gestão de risco para aumentar a resistência da carteira contra as flutuações do mercado.

Palavras-chave: Value-at-Risk, gestão de risco, lucros e perdas, cobertura, backtesting.

Classificação JEL: C53, G32.

Abstract

The main topics of this study are the measurement and management of Value-at-Risk (VaR) for a portfolio composed of European and American stocks and bonds, employing various VaR models and hedging strategies to mitigate potential losses. The analysis includes computing different VaR models and rigorous backtesting to ensure the best-performing model is chosen. After setting the VaR limit, a hedging technique is employed in the model selected. Comparing the hedging technique to an unhedged portfolio, we observed it was effective in preserving the VaR within the specified limit, hence lowering the potential for excessive losses. The daily profit and loss analysis revealed the hedged portfolio fluctuated less and suffered smaller losses, which added to overall stability. For investors and risk managers pursuing strategies to reduce risk, these findings highlight the importance of implementing effective risk management techniques to enhance the portfolio's resistance against market fluctuations.

Keywords: Value-at-Risk, risk management, profit and loss, hedging, backtesting.

JEL Classification: C53, G32.

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List of Abbreviations

BCP: Berkowitz, Christoffersen and Pelletier

CAPM: Capital Asset Pricing Model

EC: Economic Capital

ETF: Exchange-Traded Funds

EUR: Euro

EWMA: Exponential Weighted Moving Average

FRED: Federal Reserve Economic Data

FTSE: Financial Times Stock Exchange

GBP: Great Britain Pound

OLS: Ordinary Least Squares

PV: Present Value

PV01: Present Value of a Basis Point

P&L: Profit and Loss

QR: Quantile Regression

RAROC: Risk-Adjusted Return On Capital

RORAC: Return On Risk-Adjusted Capital

RM: RiskMetrics

S&P 500: Standard & Poor's 500

SGSt: Skewed Generalized Student-t

UC: Unconditional Coverage

U.S.: United States

USD: United States Dollar

VaR: Value-at-Risk

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CHAPTER 1

Introduction

Since the beginning of financial history, investors in the market have been interested in risk measurement. Yet, numerous earlier attempts have shown themselves to be overly complicated and impractical. For instance, the rigorous data requirements of Harry Markowitz's Nobel Prize-winning portfolio risk measurement theory (Markowitz, 1952) prevented it from being implemented in practice when it was first proposed. Sharpe expanded on Markowitz's approach in 1963 when he presented the Capital Asset Pricing Model (CAPM), which addresses the issue of determining an asset's accurate, arbitrage-free, fair price (Sharpe, 1963). Bill Sharpe and his team established the standard for measuring financial risk in real-world applications by adopting the simplifying premise that all risk could be divided into two categories: residual and systematic. Systematic risk, according to Sharpe, is the volatility the market index can explain, in other words, the risk tied to the broader market, such as economic, political, and social factors. Relative variance, often known as unsystematic (residual, specific, or idiosyncratic) risk, is the unexplained variance, specific to a company or industry.

The resulting CAPM claimed just the market risk measurement β is needed, significantly decreasing the amount of data inputs, since well-diversified investments are just required to be compensated for the securities' systematic risk. This method produced an easily quantifiable risk assessment effective in a real-time market setting. The main issue was β's identification as the true risk measure was called into question because it turned out to have only an uncertain connection with actual security returns. Given the doubts surrounding β and the broader state of confusion around asset pricing, market experts sought an alternative risk measure that would be both relatively affordable to estimate and highly accurate: Value-at-Risk (VaR). It has become widely used, even though many alternative models and measurements have been considered. The decision by J.P. Morgan to develop RiskMetrics, an explicit VaR measuring approach, contributed to the general acceptance of VaR. A free-to-use database including the essential inputs needed to estimate the model was employed to enable this method. A crucial first step toward using economic capital-based criteria to calculate the minimum capital requirement was the public disclosure of RiskMetrics (Allen et al., 2004).

All financial institutions are vulnerable, at some level, to market risk. They should evaluate the Economic Capital (EC) needed to sustain their risks, even though their principal financial risk exposure may vary. EC is the capital at risk because of the investment activity (Porteous and Tapadar, 2005). The distribution of EC can be influenced by company risks; thus, a metric may be estimated by VaR, stated as the entire amount of capital needed to operate as a

safeguard against unanticipated losses (Jorion, 2007). Risk management techniques can be developed around this benchmark if the EC has a predetermined maximum value.

This project seeks to measure and manage the VaR of a portfolio for a year. The portfolio contains bonds and stocks from the European and American markets. After setting a VaR limit, we will monitor the portfolio using a hedging strategy. Lastly, we will compare the year-end return earned to what we would have achieved had we not managed the VaR.

The first step is to select the appropriate model to measure the VaR. Choosing the best approach to use depends on several factors. In this study, we will examine various VaR models in addition to the pioneering RiskMetrics VaR model developed by J.P. Morgan and Reuters (1996). As a result, we will consider the following VaR models: RiskMetrics (RM), Skewed Generalized Student-t (SGSt), Historical and Quantile Regression (QR). Using historical data, we will do a backtest to determine which model is better able to estimate the portfolio VaR. The backtest will cover a period of five years, starting on December 30, 2016, and ending on August 15, 2022. The Unconditional Coverage (UC) test (Kupiec, 1995) and the Berkowitz, Christoffersen, and Pelletier test (Berkowitz et al., 2011) will be used to assess each model's performance.

Once the best-performing model has been identified, the portfolio’s VaR will be monitored and managed daily for a year using an equity exposure hedging strategy. The daily VaR limit will be set according to historical ranges. The portfolio is then hedged daily to ensure the VaR stays within this limit. We will decompose the VaR into components related to important risk factors, using a Marginal decomposition. Finally, the effects of restricting the VaR with the defined equity exposure hedging method on the portfolio's one-year performance will be investigated. For that, we will use advanced metrics such as risk-adjusted return on capital (RAROC) and return on risk-adjusted capital (RORAC). These metrics provide deeper insights into the performance of a portfolio by considering the amount of risk taken to generate returns.

A comprehensive review of the literature is given in Chapter 2, which delves into the earlier studies and underlying theories relevant to VaR and risk management. The selection and allocation of assets within the portfolio are covered in detail in Chapter 3. The methodology is described in Chapter 4, along with the procedures and methods used to measure and analyze the VaR. In Chapter 5, we focus on model selection and compare different models to find the best fit for accurately estimate the VaR. VaR management is the subject of Chapter 6, also examining the efficiency of the managing technique applied to mitigate the portfolio’s risk. Lastly, a summary of the main findings and conclusion is provided in Chapter 7.

CHAPTER 2

Literature Review

During the early 1990s, several notable companies collapse, which were ascribed to inadequate internal controls and improper use of derivatives (Krause, 2003). Even though the entire financial system failed decades before these events, it highlighted the risks companies and financial institutions faced. It was no longer possible to only blame inadequate supervision and deficient government policy for losses. It became evident better risk management was required, particularly for financial institutions. Value-at-Risk (VaR), an innovative method for calculating risk that focuses on losses companies can incur with a reasonable likelihood, was created in reaction to these incidences. J.P. Morgan was the first to introduce this concept, through their product RiskMetrics (1996), complemented by CreditMetrics (1997), and CorporateMetrics (1999). The industry standard for measuring risk was promptly established by RiskMetrics. The combined impact of exposure to financial risks and underlying volatility is captured by this approach. Dennis Weatherstone, at the time the Chairman of J.P. Morgan, stated the basic question to explain what VaR is: "How much can we lose on our trading portfolio by tomorrow's close?".

According to Jorion (2007), risk is the volatility of unforeseen events, which might be perceived as the assets, earnings, or equity value. Active financial risk management is the main function of financial institutions. They exist to take on, manage, or offer advice on financial risks (potential losses resulting from financial market activity). Organizations comprehend to manage and price financial risk effectively, they must measure it as precisely as possible. Financial managers can be more adequately prepared for unavoidable uncertainty by proactively planning for the potential consequences of unfavorable outcomes when they better understand risk. Since the early 1970s, financial markets have experienced greater volatility, directly contributing to the expansion of the risk management industry. Losses can occur through a combination of two factors: the volatility in the underlying financial variable and the exposure to this source of risk. For many years, with the portfolio diversification theory, the standard deviation, also known as volatility, served as the traditional measurement of portfolio risk in textbooks. Considering the standard deviation can only accurately represent the risk for returns in a multivariate normal distribution context, it is seen as insufficient. By the 1990s, banks had started using VaR, with an emphasis on market risk and the impact of unfavorable fluctuations on an investment portfolio (Damodaran, 2008).

There are three types of risk: market, credit, and operational (Allen et al., 2004). Operational risk is defined as the "risk of loss caused by failures in operational processes or the systems that support them, including those adversely affecting reputation, legal

enforcement of contracts and claims" by Kingsley et al. (1998). Market risk is an indicator of the level of uncertainty surrounding the future value of financial assets resulting from fluctuations in market pricing (Alexander, 2009). Lastly, the potential loss arising from the default of a borrower to pay back a loan or fulfill contractual commitments is known as credit risk (Gubareva and Borges, 2017). Although the foundational ideas of VaR originated in the field of market risk measurement, they have been expanded to other areas in the past years. Specifically, credit and operational risks are now frequently assessed with VaR models.

In this work, we will deal with market risk. Following the original Basel Accord, banks experienced a rapid increase in their trading activities, or trading for their account, which was not initially subject to a capital charge (Shakdwipee and Mehta, 2017). In 1996, the Basel Accord was revised to include a market risk fee to address this omission. The primary sources of market risk can be divided into four distinct types: interest rate, exchange rate, equity, and commodity risk (Jorion, 2007). The risk of higher volatility brought on by a shift in interest rates is known as interest rate risk. Equity price risk refers to the possibility of a decrease in a portfolio's value resulting from changes in the stock’s price. Systematic and unsystematic are two types of equity price risk. Diversification can help reduce the unsystematic but not the systematic risk. During a global economic crisis, equity price risk is systematic due to how it affects various types of assets. One type of risk that appears when exchange rates fluctuate is currency risk. Inadequate currency hedges may expose multinational corporations to this type of risk. Commodity price risk is the probability a rise in the price of raw materials would impact a company's profitability.

VaR is a risk indicator currently used by nearly all financial institutions, following the example set by regulators and big international banks in the mid-1990s. The Basel Accords specify the primary goals of bank capital: a metric for measuring the level of risk associated with bank assets, the minimum capital requirements a credit institution must comply with, mechanisms for risk analysis and coverage, supervision, and market discipline (Shakdwipee and Mehta, 2017). The 1996 Market Risk Amendment to the Basel Accord granted regulatory approval to VaR as an internal measure of risk. According to this arrangement, banks are allowed to base their internal VaR predictions on regulatory capital requirements. VaR is currently used as a management tool, despite its original purpose as a risk measurement (J.P. Morgan and Reuters, 1996).

In practice, traders receive a fixed amount of capital from the bank management, and each trader is normally given a trading limit to control overall risk. These trading restrictions offer a framework for managing possible losses and are often based on the portfolio's VaR. As a result, VaR is used by dealers and portfolio managers to guide decisions regarding the portfolio, such as closing positions when an increase in VaR indicates an unfavorable increase in risk.

Financial institutions are not the only organizations using VaR; a growing number of private companies are also choosing it as their preferred approach for enterprise-wide risk management. Its ease of understanding by non-specialists is a clear facilitator of this trend, as it is not the case with many alternative methodologies.

While VaR is widely used for managing portfolio risk, its implementation relies on several underlying statistical assumptions simplifying its estimation (Allen et al., 2004). The most important premise is the distributional assumption of the portfolio returns, which is usually assumed to be a normal distribution. Yet, evidence shows most securities prices are not normally distributed, typically exhibiting kurtosis and negative skewness (Fama, 1965; Peiro, 1994). Another usual assumption is the stationarity criterion which states the chance of observing any given return remains the same at any point in time, making calculations more accessible. In addition, returns are considered unrelated and the financial assets with restricted responsibility cannot have negative values, also known as the random walk assumption and non-negativity requirement, respectively. Lastly, all single-period assumptions must be held over the multiperiod time horizon, according to the time consistency criterion.

Despite the assumptions required to estimate VaR, it offers several advantages. VaR provides a clear, quantifiable amount that could be lost with a specified probability, facilitating the comparison of potential risks with the capital available to cover them. This straightforwardness enhances its utility as a communication tool among stakeholders. Additionally, VaR not only captures the overall risk exposure but also accounts for the sensitivities of various risk factors, allowing for consistent comparisons across different markets. Whether applied to a single trade or an entire organization, VaR is a versatile and comprehensive risk indicator that can be used across various activities and types of risk, making it a valuable asset in enterprise-wide risk management.

Notwithstanding its widespread application, VaR is subject to limitations. Some authors considered VaR not a coherent risk measure (Krause, 2003). As mentioned earlier, the distribution of returns is not always normal. Significant deviations are occasionally noted, particularly in the tails assessed for the VaR. It is greatly affected by the standard deviation. Therefore, it is implicitly assumed the distribution remains constant over time when historical data is used. It is evident from empirical data this assumption is inaccurate as the variance changes over time - heteroscedasticity. Consequently, VaR is not appropriate to calculate the loss in extreme events and it is subject to inaccuracy since estimates made for the same data at different time scales sometimes result in different values. Krause, in 2003, investigated how VaR performs for significance levels of 1% and 5%. He concluded the VaR was not accurately capturing the risk associated with the two investments. An additional limitation on VaR is it is not sub-additive, as previously mentioned, meaning that VaR(V_1 + V_2) = VaR(V_1) + VaR(V_2) is

not normally valid unless certain requirements on the distribution of outcomes are met. Yet, VaR is coherent in many circumstances, and the conditions for the validity of subadditivity are met. Surprisingly, though, most of these criteria are not satisfied in the payout distributions of options and other derivatives.

As a result of trade-offs and simplifications inherent in all existing approaches, achieving reliable VaR estimations is challenging. It becomes an empirical question to determine the optimal way for VaR estimation. It may depend on the specific objective of the computation, the quantity and nature of the underlying assets, and other factors (Boudoukh et al., 1998). Hence, it is crucial to choose the most appropriate approach of this model for the portfolio it is being used to measure the risk. Several models have been developed since RM VaR creation to increase prediction accuracy. One of the main differences between the VaR methods is how the returns distribution is inferred. While parametric models impose the density function of risk factors of asset returns to follow a normal distribution, non-parametric models do not require any statistical assumptions beyond the returns distribution being stationary. This is because non-parametric models do not take market reality into account (Barone-Adesi & Giannopoulos, 2001).

The RM VaR model, also named Parametric Normal VaR, supposes returns will ultimately follow a normal distribution (J.P. Morgan and Reuters, 1996). One viable solution to deal with the non-normality of returns and weaknesses in the RM model is to replace the Normal distribution with the SGSt distribution (Theodossiou, 1998). The SGSt VaR is able to take into account fat tails and skewness commonly observed in financial return distributions. Lin and Shen (2006) claimed the SGSt VaR can produce more precise VaR estimates at lower significance levels.

Using a nonparametric approach, such as the Historical VaR, it is possible to not impose a parametric structure on the returns. The Historical VaR employs the characteristics of the empirical return distribution, making it feasible to accurately represent the empirical skewness and kurtosis of the returns distribution. For this reason, selecting the appropriate sample size is essential. Larger sample sizes broaden the range of outcomes, but they also reduce the ability to reflect the current levels of market volatility, as Pritsker (2006) explored. Barone-Adesi et al. (1998) and Boudoukh et al. (1998) improved the conventional methods to address this issue by giving more weight to recent observations. Alternatively, Hull and White (1998) proposed, in scenarios of changing market volatility, the magnitude of past returns should be modified to align the volatility of the sample of returns with the current market volatility environment. To do so, past returns are scaled by the ratio between the current and contemporaneous volatility estimates. The most popular models for estimating VaR in financial institutions are the RM and the Historical simulation (Alexander, 2009).

Building on the concept of using empirical data for VaR estimation, the QR model, introduced by Koenker and Bassett Jr. (1978), offers an alternative option for estimating the VaR by allowing it to be expressed as a conditional quantile (Xiao et al., 2015). One of its greatest features is the freedom it offers in terms of selecting explanatory variables. It produces accurate outcomes for most commodities at the 5% significant level (Steen et al., 2015). Although their results depend on the model's configuration, QR VaR estimates have the potential to produce more robust outputs than the RM and the Historical VaR.

The VaR model used on two different portfolios may yield drastically different results considering each portfolio is unique. Therefore, in a technique known as backtesting, widely used to assess model performance, statistical tests are conducted using a historical sample of portfolio returns with comparable VaR estimates. In their paper, Zhang and Nadarajah (2018) comprehensively reviewed all the recent backtesting methods for VaR. One approach is the Unconditional Coverage (UC) test (Kupiec, 1995). It compares the observed to the projected exceedance rate and counts the number of exceedances, or situations in which the actual loss was more than the VaR estimate. The Berkowitz, Christoffersen and Pelletier (BCP) test (Berkowitz et al., 2011) is an alternative backtesting method. This test examines the autocorrelation between the instances when exceedances are found. Thus, exceedance clustering can be captured more efficiently than in other tests.

Enhancing risk management performance is the aim of risk measurement. One of the primary policies in any economy is to pay attention to risk management (Das, 2006). Risk management combines hedging techniques to change the probability distribution of future asset values. Risk has prompted assorted studies focused on the impact of financial risk management on the profits the banking sector will make in the future (Greuning and Sonja, 1999). Indeed, most financial institutions use active risk management. This entails controlling the risks still present, transferring others not required to be taken in, and limiting needless risks.

One of the most frequently implemented risk management techniques is hedging. As it is impossible to tackle all risks without removing the opportunities to create value, the choice of the hedging strategy is crucial. It entails taking a position to offset risk exposure, including changes in interest rates, exchange rates, or price fluctuations. Yet, numerous factors are involved, including the goal of risk management. Financial instruments or non-instrumental strategies, such as asset allocation adjustments or diversification, can be used to reduce potential losses.

Risk performance measurements may be employed to evaluate the effectiveness of a hedging strategy. The most well-known integrated assessment techniques are return on risk-adjusted capital (RORAC) and risk-adjusted return on capital (RAROC). These metrics are used to assess how well investments performed by taking risks and returns into account. The ratio of returns to the risk taken to attain them is provided by RORAC (Matten, 1996). This ratio contrasts a non-adjusted return with a risk-adjusted capital base. An alternative to risk-adjusting capital is to consider the capital as fixed and compute the RAROC. The maximum loss on an investment within a predetermined confidence interval calculated using total volatility is commonly used to define the capital at risk, divided by returns using the RAROC approach. As a result, RAROC assigns a risk-adjusted value to each investment (Froot and Stein, 1998).

CHAPTER 3

Portfolio Composition

The portfolio selected to be studied is split between bonds and stocks. The Great Britain Pound (GBP) and the United States Dollar (USD) will be classified as the foreign currencies, and the Euro (EUR) as the local currency. We used daily data for over five years, from December 30, 2016, to August 15, 2022.

The portfolio's equity component consists of twenty positions in stocks (three short and the remaining long) from the European and American markets. The USD/EUR conversion rate and the daily adjusted closing prices for each stock were obtained from YahooFinance (https://finance.yahoo.com). The GBP/USD exchange rate was collected through the Federal Reserve Economic Data (FRED) website (https://www.federalreserve.gov/).

Six fixed-coupon government bonds with varying maturities, coupon payments, and coupon dates represent the fixed-income portion. The selected bonds were issued in the American and German markets and Bloomberg provided the bond's attributes (Table 3.2) (https://www.bloomberg.com/europe). The daily interest rate data was taken from the FRED website for the USD and from the European Central Bank website for the EUR (https://data.ecb.europa.eu/data).

Table 3.1 lists all portfolio assets and their corresponding investments, converted as needed from foreign to local currency. As of August 15, 2022, the exchange rate GBP/EUR is 1.1869, and USD/EUR is 0.9817. The total present value (PV) of all projected cash flows, discounted to August 15, 2022, and, if required, converted to EUR, is the bond's fair value.

Table 3.1. **Portfolio composition on August 15, 2022.** This table outlines the portfolio's composition, including stocks and bonds, with details on the amount invested in each in EUR.

Asset	Ticker/ISIN	No. Shares/ Face Value	Price	Currency	Value (EUR)	Allocation
Vodafone	VOD.L	1 500 000	1.16	GBP	2 054 229	3.42%
Compass Group PLC	CPG.L	200 000	19.23	GBP	4 565 229	7.60%
United Utilities Group PLC	UU.L	400 000	11.10	GBP	5 268 577	8.77%
Intertek Group plc	ITRK.L	(50 000)	41.98	GBP	(2 491 632)	(4.15%)
Tesco PLC	TSCO.L	1 200 000	2.61	GBP	3 718 210	6.19%
JD Sports Fashion Plc	JD.L	2 500 000	1.29	GBP	3 831 293	6.38%
Sanofi	SAN.PA	10 500	85.12	EUR	893 760	1.49%
Thales S.A.	HO.PA	25 000	120.76	EUR	3 018 886	5.02%
Orange S.A.	ORA.PA	(10 000)	9.85	EUR	(98 501)	(0.16%)
Airbus SE	AIR.PA	12 500	108.80	EUR	1 360 000	2.26%
Sodexo S.A.	SW.PA	7 000	82.60	EUR	578 200	0.96%
L'Oréal S.A.	OR.PA	5 500	358.26	EUR	1 970 423	3.28%
LVMH Moët Hennessy - Louis Vuitton	MC.PA	750	694.76	EUR	521 067	0.87%
BNP Paribas SA	BNP.PA	10 000	50.08	EUR	500 800	0.83%
JPMorgan Chase & Co	JPM	5 000	119.55	USD	586 815	0.98%
Amazon.com, Inc.	AMZN	14 500	143.18	USD	2 038 199	3.39%
HP Inc.	HPQ	120 000	33.48	USD	3 944 035	6.56%
Pfizer Inc.	PFE	(11 000)	48.36	USD	(522 195)	(0.87%)
Netflix, Inc.	NFLX	6 000	249.11	USD	1 467 367	2.44%
McDonald's Corporation	MCD	9 500	259.63	USD	2 421 458	4.03%
Total Equity					**35 626 220**	**59.29%**
German Bund 2025	DE0001102374	1 250 000	96.61%	EUR	1 253 639	2.09%
German Bund 2030	DE000BU27006	6 000 000	100.03%	EUR	6 836 439	11.38%
German Bund 2040	DE0001135366	5 500 000	126.53%	EUR	8 741 966	14.55%
US Treasury 2025	US912828ZW33	2 000 000	92.30%	USD	1 810 111	3.01%
US Treasury 2030	US912828ZQ64	4 000 000	79.40%	USD	3 335 383	5.55%
US Treasury 2040	US912810SQ22	3 500 000	55.17%	USD	2 483 721	4.13%
Total Bonds					**24 461 259**	**40.71%**
Total Portfolio					**60 087 479**	**100%**

Table 3.2. **Bond's attributes.** Key information about the bonds in the portfolio is given in this table, essential for assessing interest rate risk.

Bond no.	Bond	ISIN	Maturity	Coupon Rate	Coupon Periodicity	Face Value	Currency
1	German Bund 2025	DE0001102374	15/02/2025	0.500%	annually	1 250 000	EUR
2	German Bund 2030	DE000BU27006	15/11/2030	2.400%	annually	6 000 000	EUR
3	German Bund 2040	DE0001135366	04/07/2040	4.750%	annually	5 500 000	EUR
4	US Treasury 2025	US912828ZW33	30/06/2025	0.250%	semi-annually	2 000 000	USD
5	US Treasury 2030	US912828ZQ64	15/05/2030	0.625%	semi-annually	4 000 000	USD
6	US Treasury 2040	US912810SQ22	15/08/2040	1.125%	semi-annually	3 500 000	USD

CHAPTER 4

Methodology

Several decisions must be made before we can calculate a VaR figure. These include mapping the risk factors, deciding on the resolution method (Historical or Parametric), the return model (Normal, SGSt, Historical, or QR distribution), the estimation approach used to compute the model parameters and the sample size. The quality of these choices affects how accurate the VaR estimate is. Therefore, there is a risk associated with each one of those decisions.

This chapter delves into the process of estimating our portfolio's VaR. The methodology is a critical component of this work, as it provides the systematic framework necessary to quantify potential risks and assess their impact on the portfolio. The following chapters will cover the rationale behind the model selection process and the VaR management hedging technique used to prevent the VaR from surpassing the limit established.

4.1. Risk Factor Mapping

To measure the VaR of the portfolio under study, first we need to identify the risk factors having a direct impact on the portfolio's value. Then we need to map each portfolio position into the corresponding risk factors. As a result, we obtain the risk factor loadings associated to each portfolio position.

Two distinct types of mapping will be done: Total and Systematic. These two mapping methods vary in how they approach risk assessment and estimate the corresponding VaR. Total mapping provides a thorough analysis of possible losses by considering all risk factors and exposures in the portfolio. Because this method considers a wider range of risks, it frequently yields a greater VaR. It has the benefit of being extensive, assisting in comprehending a portfolio's entire risk profile. Alternatively, Systematic mapping only concentrates on systemic risks, those that have an impact on a significant portion of the market or the entire market. Systematic mapping usually results in a lower VaR as it removes idiosyncratic risks specific to individual assets. It allows to control and isolate risks across the market, being especially valuable when implementing hedging methods.

4.1.1. Stocks

In the case of Total VaR, the risk factors for stocks are the fluctuations in the market price of each in the currency in which it is denominated, as well as the exchange rate of the foreign currency against the EUR (if the price is denominated in a foreign currency). The exposure to both risk factors corresponds to the amount invested in the stock, converted to EUR at the spot exchange rate. In other words, it is the number of shares multiplied by the stock price and, when required, by the appropriate exchange rate.

Systematic VaR measures a stock's exposure to an index by multiplying its beta relative to the reference index by the value of its position in EUR. This process replaces changes in the stock price with changes in the reference index's value. To accomplish this, stocks were mapped into the corresponding market indexes. The correlation between an asset's risk and projected return was predicted using the CAPM.

4.1.2. Bonds

Before proceeding to mapping, we must first introduce a measure of the sensitivity of bond prices to interest rate fluctuations, the present value of a basis point (PV01). The change in the PV of a set of cash flows (C) when the yield curve (R) shifts downward by one basis point is measured by the PV01, which is as follows

$$PV01(C,R) \equiv PV(C,R-0.01\%) - PV(C,R), \tag{4.1}$$

where *PV(C, R)* denotes the present value of *C* when spot interest rates are given by *R*. A one-basis-point change in the yield curve indicates a 0.01% drop in all spot rates.

Using the first-order Taylor expansion and assuming continuous compounding, the PV01 is approximately given by

$$PV01(C_T,R_T) = T \times PV(C_T,R_T) \times 0.01\%, \tag{4.2}$$

where R_T represents the interest rate annually compounded and annualized for a maturity of *T* years. The step-by-step demonstration of the derivations can be found in Section III.1.8 (Alexander, 2009, p. 41-48).

The variation in the present value of a cash flow is calculated as the product of the change in the corresponding discount rate and the PV01 multiplied by -1. The - PV01 indicates the risk factor's exposure. The change in the PV of a cash flow reflects the behavior of the risk factor and is then given by

$$\Delta PV(C_T,R_T) = -PV01(C_T,R_T) \times \Delta R_T(b.p.) \tag{4.3}$$

As opposed to stocks, it is not reasonable to use a bond's historical price evolution as a risk factor. This is due to the fact that, as time goes on, the bond's maturity reduces, having an impact on its price. Bond prices fluctuate only due to changes in discount rates, unlike stocks, whose price shifts may also reflect changes in the expected value of future cash flows. Thus, bonds may be valued by applying the proper discount rate to their future cash flows. This approach works very efficiently in this type of asset as long as we are aware of the bond's cash flows, when they are paid, and the appropriate discount rates. Therefore, when assessing the risk of bonds, we can focus exclusively on the discount rates, as they are the primary risk factor. The price of the bond decreases as discount rates increases. Consequently, the volatility of the interest rates used to discount the bond's cash flows impacts the price as well.

We anticipate the bond price return volatility will gradually decline and eventually converge to zero at the bond's maturity. So, we will forecast the volatility of bond prices in the future based on the historical volatility of the discount rates and the bond's present price sensitivity to changes in the discount rates. A few additional benefits arise from this method: numerous bonds will have the same risk characteristics, and we may estimate the future volatility of bond returns by using historical interest rate data instead of historical bond price data.

The challenge is that a bond with many cash flows will have the same number of distinct interest rates acting as risk factors (one for each cash flow maturity). Since cash flows from an extensive bond portfolio can occur almost daily, changes in interest rates at almost all maturities will impact the bond portfolio. This implies we would have to monitor interest rate correlations and volatilities for a very large set of maturities. To solve this problem, we mapped the cash flows to a range of common maturities (for instance 6 months, 1 year, 3 years, etc.), also known as mapping into vertices (Alexandre, 2009).

There are several approaches to perform this mapping while maintaining various aspects of the bond, including volatility, PV, and changes in interest rates. We will use the one preserving PV01 and PV, given by the following equation

$$\begin{cases} x_1 = \frac{T_2 - T}{T_2 - T_1} PV_{C_T} \\ x_2 = (1 - \frac{T_2 - T}{T_2 - T_1}) PV_{C_T} \end{cases}, \tag{4.4}$$

where PV_{C_T} stands for the PV of the cash flow with maturity T being mapped; T_1 and T_2 correspond to the standard maturity vertices immediately below and above T, respectively, to which there is accessible interest rate data. The proportions of PV_{C_T} mapped into vertices T_1 and T_2 are represented by the variables x_{T_1} and x_{T_2}, respectively.

Every cash flow and bond in our portfolio is mapped with this procedure preserving the PV and PV01 of the original cash flow. Each standard maturity interest rate represents a risk factor in the risk factor mapping. The exposure to each risk factor is equal to the symmetric of the sum of the cash flows PV01 mapped into that maturity.

The last step was to use the exchange rate as of August 15, 2022, to convert the exposures to EUR for the bonds from the United States (U.S.) market. Regarding bonds, there is no difference between Total and Systematic mapping because both strategies aim to capture the sensitivity of the bond to key risk factors, such as changes in interest rates.

4.1.3. Portfolio Exposures

In summary, the number of shares multiplied by the amount invested (and by the exchange rate, if applicable) yields the risk factor sensitivity of the stocks. The risk factor associated is the returns on the market index. The PV01 multiplied by –1 is the risk factor sensitivity for the cash flows (bonds), where the risk factor is the absolute basis point change in the interest rates. The amount of capital invested in the portfolio's assets denominated in foreign currencies, namely USD and GBP, is the risk factor for the currency.

Table 4.1 and Table 4.2 show the stocks and bonds risk factor exposures, respectively, as of August 15, 2022, in the local currency (EUR). The vertices chosen for the maturities are displayed in Table 4.2. Exposures to the risk factors resulting from each position in the portfolio are consolidated in Table 4.3.

Table 4.1. **Stocks risk factor exposures on August 15, 2022.** This table displays the exposures of each risk factor to the respective stock, exchange rate, and index. The stocks are grouped by market.

Stocks							
Risk factor	**Value (EUR)**	**Exposures (EUR)**					
		Stock	**USD/EUR**	**GBP/EUR**	**^FTSE**	**^FCHI**	**^GSPC**
VOD.L	2 054 229	2 054 229	-	2 054 229	624 610	-	-
CPG.L	4 565 229	4 565 229	-	4 565 229	3 122 907	-	-
UU.L	5 268 577	5 268 577	-	5 268 577	999 560	-	-
ITRK.L	(2 491 632)	(2 491 632)	-	(2 491 632)	(960 010)	-	-
TSCO.L	3 718 210	3 718 210	-	3 718 210	2 135 188	-	-
JD.L	3 831 293	3 831 293	-	3 831 293	5 015 018	-	-
SAN.PA	893 760	893 760	-	-	-	(26 190)	-
HO.PA	3 018 886	3 018 886	-	-	-	1 030 221	-
ORA.PA	(98 501)	(98 501)	-	-	-	19 450	-
AIR.PA	1 360 000	1 360 000	-	-	-	1 389 071	-
SW.PA	578 200	578 200	-	-	-	438 124	-
OR.PA	1 970 423	1 970 423	-	-	-	1 838 603	-
MC.PA	521 067	521 067	-	-	-	666 902	-
BNP.PA	500 800	500 800	-	-	-	580 133	-
JPM	586 815	586 815	586 815	-	-	-	531 506
AMZN	2 038 199	2 038 199	2 038 199	-	-	-	3 952 984
HPQ	3 944 035	3 944 035	3 944 035	-	-	-	4 898 572
PFE	(522 195)	(522 195)	(522 195)	-	-	-	(256 187)
NFLX	1 467 367	1 467 367	1 467 367	-	-	-	2 908 780
MCD	2 421 458	2 421 458	2 421 458	-	-	-	921 155

Table 4.2. **Bonds risk factor exposures on August 15, 2022.** The exposures of each bond to the corresponding vertice and the exchange rate are provided in this table. See Table 3.2 for the description of each bond.

	Bonds					
Risk Factor	**1**	**2**	**3**	**4**	**5**	**6**
USD/EUR	-	-	-	1 810 110.90	3 335 382.86	2 483 720.85
EUR3M	-	(3.60)	-	-	-	-
EUR6M	(0.31)	-	(2.97)	-	-	-
EUR1Y	(0.31)	(10.76)	(23.08)	-	-	-
EUR2Y	(124.74)	(28.55)	(51.80)	-	-	-
EUR3Y	(186.18)	(63.60)	(115.39)	-	-	-
EUR5Y	-	(139.15)	(252.47)	-	-	-
EUR7Y	-	(2 492.08)	(430.03)	-	-	-
EUR10Y	-	(2 391.77)	(2 408.79)	-	-	-
EUR20Y	-	-	(8 560.33)	-	-	-
USD3M	-	-	-	(0.03)	(0.30)	-
USD6M	-	-	-	(0.09)	(0.30)	(0.95)
USD1Y	-	-	-	(0.35)	(1.77)	(2.79)
USD2Y	-	-	-	(45.82)	(4.61)	(7.26)
USD3Y	-	-	-	(472.32)	(9.98)	(15.72)
USD5Y	-	-	-	-	(21.25)	(33.47)
USD7Y	-	-	-	-	(1 685.09)	(55.07)
USD10Y	-	-	-	-	(793.36)	(562.50)
USD20Y	-	-	-	-	-	(3 279.87)

Table 4.3. **Portfolio and mapped portfolio risk factor exposures on August 15, 2022.** This table presents the consolidation of exposures to different risk factors arising from all positions in the portfolio. On the left is displayed the risk factor loading vector used to estimate the Total VaR. The risk factor loading vector used to estimate the Systematic VaR can be seen on the right.

Portfolio Exposures		Mapped Portfolio Exposures	
Risk Factor	**Value (EUR)**	**Risk Factor**	**Value (EUR)**
VOD.L	2 054 228.80	VOD.L	-
CPG.L	4 565 228.78	CPG.L	-
UU.L	5 268 577.49	UU.L	-
ITRK.L	(2 491 631.57)	ITRK.L	-
TSCO.L	3 718 209.66	TSCO.L	-
JD.L	3 831 293.38	JD.L	-
SAN.PA	893 759.97	SAN.PA	-
HO.PA	3 018 885.59	HO.PA	-
ORA.PA	(98 500.90)	ORA.PA	-
AIR.PA	1 360 000.05	AIR.PA	-
SW.PA	578 200.01	SW.PA	-
OR.PA	1 970 423.47	OR.PA	-
MC.PA	521 067.27	MC.PA	-
BNP.PA	500 799.99	BNP.PA	-
JPM	586 814.74	JPM	-
AMZN	2 038 199.39	AMZN	-
HPQ	3 944 034.97	HPQ	-
PFE	(522 195.04)	PFE	-
NFLX	1 467 366.98	NFLX	-
MCD	2 421 458.21	MCD	-
^FTSE	-	^FTSE	10 937 273.25
^FCHI	-	^FCHI	5 936 312.57
^GSPC	-	^GSPC	12 956 811.01
USD/EUR	17 564 893.86	USD/EUR	17 564 893.86
GBP/EUR	16 945 906.53	GBP/EUR	16 945 906.53
EUR3M	(3.60)	EUR3M	(3.60)
EUR6M	(3.28)	EUR6M	(3.28)
EUR1Y	(34.14)	EUR1Y	(34.14)
EUR2Y	(205.08)	EUR2Y	(205.08)
EUR3Y	(365.17)	EUR3Y	(365.17)
EUR5Y	(391.62)	EUR5Y	(391.62)
EUR7Y	(2 922.11)	EUR7Y	(2 922.11)
EUR10Y	(4 800.54)	EUR10Y	(4 800.54)
EUR20Y	(8 560.33)	EUR20Y	(8 560.33)
USD3M	(0.34)	USD3M	(0.34)
USD6M	(1.34)	USD6M	(1.34)
USD1Y	(4.92)	USD1Y	(4.92)
USD2Y	(57.70)	USD2Y	(57.70)
USD3Y	(498.03)	USD3Y	(498.03)
USD5Y	(54.71)	USD5Y	(54.71)
USD7Y	(1 740.16)	USD7Y	(1 740.16)
USD10Y	(1 355.86)	USD10Y	(1 355.86)
USD20Y	(3 279.87)	USD20Y	(3 279.87)

4.2. Value-at-Risk Models

Following the development of the RM VaR, a variety of other models emerged intending to improve the prediction accuracy of the risk that comes with investing. All known methodologies have trade-offs and simplifications, contributing to a complex computation of a reliable VaR estimate. As a result, choosing the best model for VaR estimation might not be intuitive. In this work, four distinct VaR models were computed to make an informed decision when choosing the best-performing model for the portfolio.

VaR is the present value of losses, for a certain significance level of α, we are confident it will not surpass if we retain the portfolio as is for the following h trading days, given by

$$VaR_{h,\alpha} = -C^{-1}(\alpha) = -x_{\alpha}, \tag{4.5}$$

where α indicates the α-quantile of the h-day distribution of a continuous random variable X, $x\alpha$ defines the α-quantile for any given value of α, and C^{-1} denotes the inverse cumulative distribution function of X. We will employ a significance level of α = 1% and a horizon of h = 1. In other words, we will estimate the 1-day $VaR_{1\%}$.

We will compute the RM, SGSt, Historical, and QR VaR. We chose these models because they offer a diverse range of approaches to risk measurement, capturing different aspects of market dynamics and portfolio characteristics.

4.2.1. RiskMetrics VaR

The Parametric Normal VaR model, more commonly referred to as the RM VaR model, assumes returns will eventually have a normal distribution. Considering this, we can rewrite equation 4.5 as

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha) \times \sigma_h - \mu_h, \tag{4.6}$$

where μ_h is the estimated mean, σ_h represents the standard deviation, and $\Phi^{-1}(\alpha)$ indicates the α-quantile of the standard normal distribution.

Considering we are dealing with daily data and VaR estimates with a future time horizon of one day, Alexander (2009) advises using μ_h = 0. Consequently, the previous equation can be rewritten as

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha) \times \sigma_h, \tag{4.7}$$

where σ_h is computed using the exponential weighted moving average (EWMA) volatility model. We recursively estimate the EWMA variance from the historical daily returns as

$$\sigma_t^2 = (1 - \lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2, \tag{4.8}$$

where σ_t^2 is the variance estimated for day t on the previous day (t − 1), r_{t-1} represents the return observed on day t − 1 and $\lambda \in (0, 1)$ denotes the smoothing factor.

4.2.2. Skewed Generalized Student-t VaR

The returns are assumed to have a parametric distribution by the SGSt VaR. However, in order to explain the fat tails and skewness frequently seen in financial return distributions, it attempts to fit the shape of the distribution to the empirical returns. The flexibility of this unimodal distribution allows it to capture deviations from normality in the return distribution of the portfolio. It has five parameters controlling the mean, standard deviation, asymmetry, and shape of the central zone and ends of the distribution. These parameters were estimated using the Maximum Likelihood approach. The parameters estimation was done in Excel using the function SGTFIT, a component of the RMTools add-in provided by the Risk Management professor António Barbosa.

We re-estimate the model's parameters every trading month to better represent the market. Four distinct SGSt VaR series were computed. The size of the rolling sample of portfolio returns used for the parametrization is the only distinction between them. The variants used were 200, 400, 600, and 800 daily observations.

Similar to a normal distribution, the VaR estimate will be the negative of the quantile of the estimated SGSt distribution. Thus, the SGSt VaR was calculated in Excel using the SGTINV function from the RMTools add-in.

4.2.3. Volatility-Adjusted Historical VaR

In contrast to the previous models, as mentioned in Chapter 2, the Historical VaR does not impose a parametric structure on the returns. It can accurately depict the empirical skewness and kurtosis of the returns distribution since it uses the returns' empirical distribution.

The first step was to choose the sample size *n*. To accurately calculate the Historical $VaR_{h,\alpha}$, a minimum of *h*-day portfolio profit and loss (P&L) observations is needed: $1/\alpha$. In this scenario, the VaR estimate will align with the lowest P&L within the sample. Therefore, we require at least 1/1% = 100 observations to compute the VaR at the 1% significance level. The Historical VaR estimate is highly vulnerable to the existence of outliers in the sample when working with the minimum sample size. An increased sample size reduces the effect of outliers. However, it requires us to go farther back in time, and older data is less likely to accurately reflect the current market conditions. Therefore, selecting a sample size involves making a trade-off between relevance (favoring shorter samples) and statistical accuracy (benefiting larger samples).

To address this hurdle, we will assign equal weight to each observation and modify the series of returns' volatility to reflect the present volatility. In other words, we change the sample to reflect the overall current market conditions, rather than having a sample representing different market conditions over time. To accomplish this, we will use the EWMA to generate a series of variance estimates, and then modify the series of returns as

$$r_t = \frac{\sigma_{T+1}}{\sigma_{t+1}} r_t, \tag{4.9}$$

where $t < T$ and T is the VaR measurement date. The T+1 volatility-adjusted Historical VaR is how we refer to this model.

We will use the same variants as in the SGSt VaR: 200, 400, 600, and 800 daily observations.

4.2.4. Quantile Regression VaR

An alternative method for estimating VaR involves the application of quantile regression. The QR is a linear model that attempts to explain the quantile of a variable (in this case, the portfolio returns) based on a set of explanatory variables. Its most appealing feature is the flexibility it provides in choosing the explanatory variables, as we may use any variable. The estimations of QR VaR have the potential to yield more reliable results than the previous models, notwithstanding their dependence on the model configuration. As a result, we will calculate four distinct model variants with the following explanatory variables: EWMA volatility, volatility lag 5, 15 days and 30 days average volatility.

Given that VaR is the symmetric of the quantile, this model allows direct estimation of the VaR_α as

$$VaR_\alpha = -\left(\hat{a} + \hat{b}x_i\right), \tag{4.10}$$

where the estimated parameters of the returns to x α-quantile regression are denoted by $\hat{a}$ and $\hat{b}$.

While QR focuses on estimating specific quantiles of the distribution, the more conventional Ordinary Least Squares (OLS) method aims to explain the average value of a variable. This fundamental difference highlights the distinction between the two approaches: though both are linear models, their methodologies diverge in terms of how they estimate the coefficients. The QR parameters can be estimated by solving the following equation

$$\left(\hat{a}, \hat{b}\right) = \arg\min_{a,b} \sum_{i=1}^{n} e_i(\alpha - I_{e_i<0}), e_i = y_i - (\hat{a} + \hat{b}x_i), \tag{4.11}$$

where I_e is an indicator function of an event, assuming a value of 1 if the event is true and 0 otherwise.

The parameters are re-estimated on the last trading day of each month based on a sample size of 800 daily observations. In the first variant of the model, we employed the volatility estimates for day *t* computed using the EWMA volatility as the explanatory variable. The remaining variants were computed also using the EWMA volatility as one of the explanatory variables. The difference between them lies in the second explanatory variable: volatility lag 5, 15 days and 30 days average volatility.

The VaR can be now computed as follows

$$VaR_{\alpha,t} = -\left(\hat{a} \times v_t + \hat{b} \times f_t\right), \tag{4.12}$$

where v_t and f_t are the explanatory variables.

4.3. Backtesting

The best-performing model for our portfolio cannot be simply determined given the return processes are not directly observable. This can be accomplished by comparing the VaR estimates with the actual returns of a test portfolio. As a result, we must validate, or backtest, the risk model and the estimation of its parameters.

The backtesting methodology includes choosing a test portfolio (the portfolio under study), selecting the test period, and computing the risk factor returns for that period. It also involves reserving the initial observations for estimating the first *h*-day VaR during the test period, proceeding *h* days ahead of time and calculating the second *h*-day VaR. We must repeat this process until we reach the end of the test period. Then, we can compare the series of *h*-day VaR projections with the series of *h*-day actual returns for the test portfolio and record the number of exceedances. We consider an exceedance when the loss is worse than the estimated VaR. We evaluate the accuracy of the VaR model by comparing the projected number of exceedances with the observed exceedance rate. To perform a backtesting, several distinct tests can be done. We will conduct the UC and BCP tests.

4.3.1 Unconditional Coverage Test

The UC test determines if the exceedance rate observed (π_{obs}) and the expected exceedance rate (π_{exp} ≥ *α*) are equal. The test hypotheses are

$$\begin{cases} H_0 : \pi_{obs} = \pi_{exp} \equiv \alpha \\ H_1 : \pi_{obs} \neq \pi_{exp} \end{cases}, \tag{4.13}$$

where *α* is the probability of an exceedance.

Considering n_1 the number of exceedances and n_0 the number of non-exceedances, the test statistic is

$$UC = -2[n_1(ln\alpha - \ln \pi_{obs}) + n_0(\ln[1-\alpha] - \ln[1-\pi_{obs}])], \quad (4.14)$$

where n is the total number of observations, $n_0 = n - n_1$, $\pi_{obs} = \frac{n_1}{n}$ and $\pi_{exp} = \alpha$.

The distribution of the test statistic under the null hypothesis is

$$UC \sim X_1^2 \quad (4.15)$$

If $n_1 = 0 \Rightarrow \pi_{obs} = 0$ the test statistic converges to

$$UC = -2\ln(1-\alpha) \quad (4.16)$$

4.3.2. Berkowitz, Christoffersen and Pelletier Test

The BCP test establishes whether the occurrences of exceedances are autocorrelated. The test relies on the first *K* autocorrelations of exceedances. If the VaR model is well-specified the time series of exceedances will not display autocorrelation at any lag, meaning it is impossible to predict when an exceedance will occur. Compared to other tests (for example, the Conditional Coverage test), this one is better at capturing exceedance clustering.

The test hypotheses are

$$\begin{cases} H_0 : \rho_k = 0, \forall k \in \{1, \dots, K\} \\ H_a : \exists_k \in \{1, \dots, K\}\, s.t.\ \rho_k \neq 0 \end{cases}, \quad (4.17)$$

where ρ_k is the lag *k* sample autocorrelation of the series $I_{t,\alpha} - \alpha$. $I_{t,\alpha}$ is the exceedance indicator.

Considering *K* the maximum autocorrelation lag in the test and *T* the sample size, the test statistic is

$$BCP(K) = T(T+2)\sum_{k=1}^{K} \frac{\rho_k^2}{T-k} \quad (4.18)$$

Under the null hypothesis, the asymptotic distribution of the test statistic is

$$BCP(K) \sim X_K^2 \quad (4.19)$$

The choice of the *K* is crucial. A larger *K* assists in identifying non-independence at higher order lags by providing details about the autocorrelations up to lag *K*. The test has more power when the *K* value is lower (a smaller number of degrees of freedom in the test statistic's distribution under the null hypothesis). Due to this tradeoff, we should consider several *K*. In this project, we will perform the test with lags ranging from 1 up to 5.

CHAPTER 5
Model Selection

The important VaR analysis process for choosing a model is covered in the following chapter. This stage is crucial for efficient risk management since the model selected has a significant impact on the accuracy and dependability of VaR estimation. To determine the VaR model with the best estimate for our portfolio, we will assess a range of models, considering factors such as prediction accuracy and computational efficiency. We intend to enhance our risk management strategy resilience and effectiveness by choosing the best VaR model, which would protect the portfolio against unfavorable market conditions.

The findings of the Unconditional Coverage (UC) and Berkowitz, Christoffersen and Pelletier (BCP) tests are shown in the first and second sub-chapters, respectively. The rationale for the model selection is presented in the third sub-chapter.

In total, we tested thirteen variants of four VaR models. Table 5.1 below presents the number corresponding to each model's description. The backtesting was performed for the period starting on December 30, 2016, and ending on August 15, 2022, corresponding to 1 467 observations. The daily VaR estimates were computed for this period. As mentioned before, we used two types of tests to choose the best-fitting model for the portfolio under study: UC and BCP tests. Since we are estimating the VaR with a significance level of 1%, we consider a model with around 15 ($1467 \times 0.01 = 14.67$) exceedances to be acceptable for the portfolio, as this is the number of exceedances we expect to occur. The exceedance details for all models are displayed in Appendix C.

5.1. Unconditional Coverage Test Results

As explored in previous chapters, the Total VaR considers a portfolio's overall risk, incorporating all sources of risk, including both systematic and idiosyncratic factors. In contrast, Systematic VaR isolates the risk associated with market-wide factors, ignoring risks specific to individual assets.

We will begin by analyzing if using the Systematic VaR is a viable approach. Firstly, the backtesting was done by comparing the Systematic VaR with the real returns. As we can see in Table 5.1, all models performed terribly in this backtesting. When comparing the Systematic VaR against the mapped returns (Table 5.2), models 11, 12, and 13 performed well in the backtesting. In this case, there is a significant discrepancy between the P&Ls generated by the mapped portfolio (on which the Systematic VaR computation is based) and the real P&Ls generated by the portfolio under study. Therefore, the Systematic VaR backtesting against the two sets of P&L (Table 5.1 and Table 5.2) are not similar, leading to the conclusion the portfolio is not well diversified.

Table 5.1. **Unconditional Coverage test results using the Systematic VaR against the portfolio returns.** A model is considered to pass the test if the p-value is higher than 5%.

Model no.	Model description	Exceedances no.	p-value (%)
1	RiskMetrics	68	0.00
2	Skewed Generalized Student-t, sample size 200	49	0.00
3	Skewed Generalized Student-t, sample size 400	48	0.00
4	Skewed Generalized Student-t, sample size 600	49	0.00
5	Skewed Generalized Student-t, sample size 800	48	0.00
6	T+1 volatility-adjusted Historical, sample size 200	57	0.00
7	T+1 volatility-adjusted Historical sample size 400	57	0.00
8	T+1volatility-adjusted Historical sample size 600	55	0.00
9	T+1volatility-adjusted Historical sample size 800	71	0.00
10	Quantile Regression, explanatory variable: EWMA volatility	62	0.00
11	Quantile Regression, explanatory variables: EWMA volatility and volatility lag 5	29	0.09
12	Quantile Regression, explanatory variables: EWMA volatility and 15 days average volatility	30	0.04
13	Quantile Regression, explanatory variables: EWMA volatility and 30 days average volatility	23	4.36

Table 5.2. **Unconditional Coverage test results using the Systematic VaR against the mapped portfolio returns.** Rows that have been bolded indicate which models pass the test (p-value > 5%).

Model no.	Model description	Exceedances no.	p-value (%)
1	RiskMetrics	39	0.00
2	Skewed Generalized Student-t, sample size 200	28	0.19
3	Skewed Generalized Student-t, sample size 400	26	0.73
4	Skewed Generalized Student-t, sample size 600	26	0.73
5	Skewed Generalized Student-t, sample size 800	26	0.73
6	T+1 volatility-adjusted Historical, sample size 200	35	0.00
7	T+1 volatility-adjusted Historical sample size 400	37	0.00
8	T+1volatility-adjusted Historical sample size 600	40	0.00
9	T+1volatility-adjusted Historical sample size 800	40	0.00
10	Quantile Regression, explanatory variable: EWMA volatility	33	0.00
11	**Quantile Regression, explanatory variables: EWMA volatility and volatility lag 5**	**13**	**65.50**
12	**Quantile Regression, explanatory variables: EWMA volatility and 15 days average volatility**	**17**	**55.09**
13	**Quantile Regression, explanatory variables: EWMA volatility and 30 days average volatility**	**15**	**93.12**

We intended to create a model flexible and able to easily accommodate future investment in more shares. However, given that the portfolio is not sufficiently well diversified and is still considerably small, it makes sense to measure the Total VaR. In other words, the Systematic VaR is not recommended because the portfolio is poorly diversified, whereas the Total VaR is still viable as the portfolio is relatively small and it has a manageable number of risk factors. For this reason, the backtesting will be now performed using the Total VaR against the real portfolio returns.

From Table 5.3 we can conclude the models which passed the UC test using the Total VaR are all variants from the SGSt and QR models. We excluded the RM model, as expected. In fact, this type of model supposes returns follow a normal distribution. This is not the case as the portfolio returns have a slightly asymmetric (skewness: -0.06675) and considerable heavy-tailed (kurtosis: 10.10835) distribution.

It can also be observed the performance for all the variants from the Historical model fell short of expectations in the UC test. Given the Historical VaR does not impose a predefined structure on returns, it leverages the empirical distribution's characteristics, allowing it to accurately capture the skewness and kurtosis inherent in the returns distribution. Therefore, it is surprising this type of model did not pass the UC test. The sample size producing the best outcome was 600 (model 8). Still, it generated a higher number of exceedances than it was predicted and, consequently, failed the test.

The best-performing model type was the QR since it is the one closest to the expected exceedances for the UC test. Among the variants, model 13 is the model with better results in the test, followed by model 11, with a p-value of 93.12% and 73.09%, respectively (Table 5.3).

Table 5.3. **Unconditional Coverage test using the Total VaR against the portfolio returns.** The bolded rows represent the models that pass the test (p-value > 5%).

Model no.	Model description	Exceedances no.	p-value (%)
1	RiskMetrics	30	0.04
2	**Skewed Generalized Student-t, sample size 200**	**19**	**27.71**
3	**Skewed Generalized Student-t, sample size 400**	**19**	**27.71**
4	**Skewed Generalized Student-t, sample size 600**	**18**	**39.88**
5	**Skewed Generalized Student-t, sample size 800**	**18**	**39.88**
6	T+1 volatility-adjusted Historical, sample size 200	29	0.09
7	T+1 volatility-adjusted Historical sample size 400	27	0.38
8	T+1volatility-adjusted Historical sample size 600	23	4.36
9	T+1volatility-adjusted Historical sample size 800	25	1.38
10	**Quantile Regression, explanatory variable: EWMA volatility**	**13**	**65.50**
11	**Quantile Regression, explanatory variables: EWMA volatility and volatility lag 5**	**16**	**73.09**
12	**Quantile Regression, explanatory variables: EWMA volatility and 15 days average volatility**	**17**	**55.09**
13	**Quantile Regression, explanatory variables: EWMA volatility and 30 days average volatility**	**15**	**93.12**

5.2. Berkowitz, Christoffersen and Pelletier Test Results

We will now examine the BCP test to select the VaR model among those who passed the UC test (Table 5.3). When the corresponding p-value for a given lag is greater than 5%, the model succeeds in this test. As can be seen in Table 5.4, almost all variants passed the BCP test in lag 1 (models 10 and 13 did not pass). Yet only models 11 and 12 passed all lags tested (until lag 5). The BCP test results for all models can be observed in Appendix B.

Table 5.4. **BCP test results.** This table presents the results of the BCP test using the Total VaR against the portfolio returns for the models that passed the UC test. The models passing the test for the five lags are marked by the bolded rows. P-values above 5% are indicated by the values in bold.

Model no.	Model description	p-value (%)				
		Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
2	Skewed Generalized Student-t, sample size 200	**12.30**	0.10	0.00	0.00	0.00
3	Skewed Generalized Student-t, sample size 400	**12.30**	0.00	0.00	0.00	0.00
4	Skewed Generalized Student-t, sample size 600	**9.30**	0.00	0.00	0.00	0.00
5	Skewed Generalized Student-t, sample size 800	**9.30**	0.00	0.00	0.00	0.00
10	Quantile Regression, explanatory variable: EWMA volatility	0.80	0.10	0.00	0.00	0.00
11	**Quantile Regression, explanatory variables: EWMA volatility and volatility lag 5**	**67.20**	**83.60**	**91.00**	**94.90**	**97.00**
12	**Quantile Regression, explanatory variables: EWMA volatility and 15 days average volatility**	**6.70**	**16.90**	**7.50**	**13.00**	**19.80**
13	Quantile Regression, explanatory variables: EWMA volatility and 30 days average volatility	2.90	8.50	2.10	4.30	7.40

Even so, the SGSt model variants only had 2 consecutive days of exceedances, 4 exceedances separated by 2 days, and 3 exceedances separated by 3 days (Figure 5.1). As the backtesting period is relatively short (five years) and the significance level is low (1%), we are basing rejections on very few cases. Although a trend cannot be inferred in a few cases, rejecting the SGSt model variants makes sense since the other models had better results in the BCP test. From the QR variants, models 10 and 13 were rejected by this test, as they had the most cases of close exceedance days.

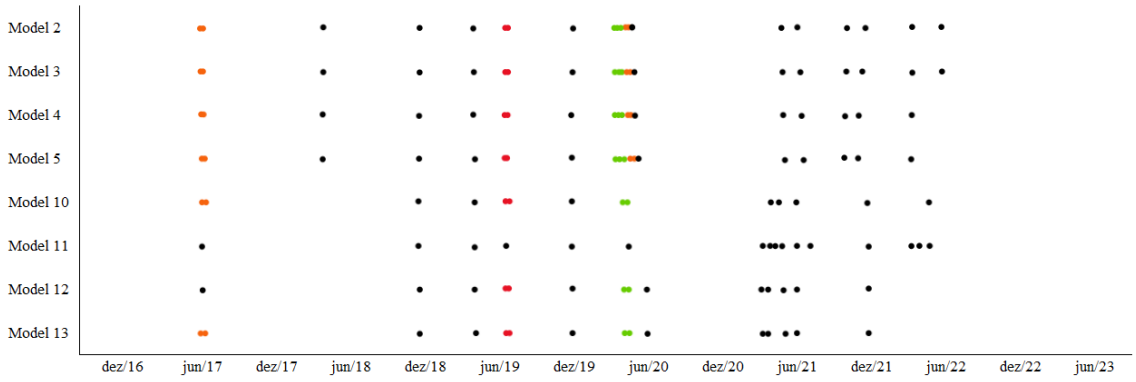


Figure 5.1. BCP test results for SGSt and QR VaR models. The black dots represent an exceedance. The red, orange, and green dots indicate a consecutive exceedance, 2 days separated exceedances, and 3 days separated exceedances, respectively.

5.3. Annual Sub-periods Backtesting and Model Selection

It appears model 11 is the best option. However, as model 12 also passed this test, we will now check the annual sub-periods to make a better-informed selection of the model that best fits the portfolio.

Both models exhibit good and highly similar behavior, except for the sub-period 2017-2018 (Table 5.5). Given the stock market dive in 2018, especially in the U.S., it is understandable the models underperformed in both tests. Model 11 performed better in the overall sub-periods of both tests. Particularly, in the BCP test for the sub-periods 2019-2020 and 2018-2019 this model was significantly more successful. Therefore, model 11 seems to be the best-performing model for the portfolio under study and it will be the one chosen.

Table 5.5. UC and BCP tests results for the annual sub-periods. The values in bold designate p-values lower than 5% and, as a result, the ones which failed the corresponding test.

	UC (p-value in %)		**BCP** (p-value in %)									
			Lag 1		**Lag 2**		**Lag 3**		**Lag 4**		**Lag 5**	
Model no.	**11**	**12**	**11**	**12**	**11**	**12**	**11**	**12**	**11**	**12**	**11**	**12**
2021-2022	41.90	80.80	79.90	85.90	93.70	96.40	97.80	99.10	99.20	99.70	99.70	99.90
2020-2021	18.40	18.40	75.00	75.00	90.30	90.30	95.80	95.80	98.10	98.10	99.10	99.10
2019-2020	80.00	41.90	84.90	79.90	96.40	93.70	99.10	**0.20**	99.70	**0.40**	99.90	**0.90**
2018-2019	80.00	41.90	84.90	**0.00**	96.40	**0.10**	99.10	**0.20**	99.70	**0.40**	99.90	**1.00**
2017-2018	**2.20**	**2.20**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**

CHAPTER 6

Value-at-Risk Management

The primary aim of this work is to measure and manage the VaR of a portfolio. In the preceding chapters, we covered VaR measurement, providing a strong basis for comprehending potential risks and effectively measuring them. Having established the model we will use to compute the VaR (model 11), we now turn our attention to VaR management.

The focus of Chapter 6 is on VaR management strategies. To guarantee the portfolio stays within the specified risk threshold, this chapter will introduce the hedging approach. The success of the employed strategy will also be evaluated. The VaR decomposition is given in sub-chapter 1. Sub-chapters 2 and 3 present the hedging technique and the results of the VaR management strategy, respectively.

We can observe in Figure 6.1 the VaR from the model chosen ranged mostly between €0.5 million and €1.5 million, for the backtest period. Furthermore, the VaR estimate as of August 15, 2022, was approximately €1 million, or nearly 1.7% of the portfolio's value. Based on these historical ranges, we will set a daily maximum value of EC of €750 000, for the upcoming year. Thereby, the maximum amount of capital at risk from keeping our portfolio for the next day should not exceed €750 000.

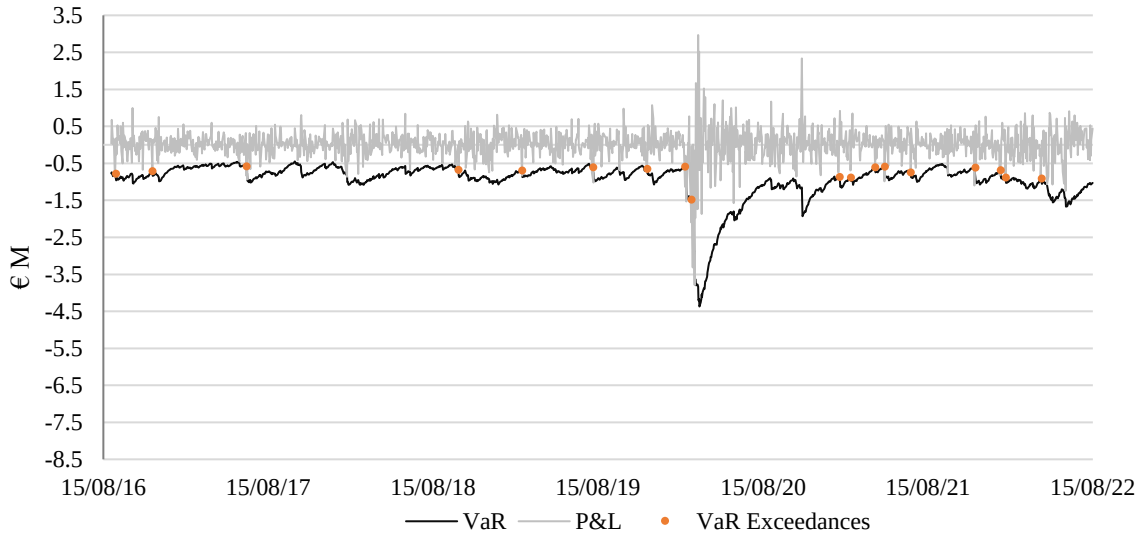


Figure 6.1. **Model 11 VaR and P&L (returns).** The darker line corresponds to the daily VaR estimates for model 11 multiplied by -1. The orange dots represent the VaR estimate for the days an exceedance occurred.

6.1. Value-at-Risk Decomposition

The purpose of the hedging technique is to reduce the portfolio's risk. We must first evaluate how each risk factor contributes to the portfolio's VaR. To achieve this, a marginal VaR decomposition was employed. It establishes how each slice (θ^s) contributes to the portfolio's VaR in terms of percentage, as was addressed in Chapter 2. Decomposing the vector of risk factor loadings (vector of exposures to each of the risk factors affecting the portfolio) underlying the VaR is the first step. The VaR is a function of this vector of risk factor loadings

$$VaR = f(\theta), \tag{6.1}$$

where θ is the risk factor loadings.

The Marginal VaR for a slice θ^s is provided by

$$Marginal\ VaR^s = \nabla f(\theta)^T \theta^s, \tag{6.2}$$

where $\nabla f(\theta)$ is the gradient vector for the portfolio's VaR and it is given by

$$\nabla f(\theta) = \frac{\delta f(\theta)}{\delta \theta} = \begin{bmatrix} \frac{\delta f(\theta)}{\delta \theta_1} \\ \frac{\delta f(\theta)}{\delta \theta_2} \\ \cdots \\ \frac{\delta f(\theta)}{\delta \theta_n} \end{bmatrix} \tag{6.3}$$

To perform marginal decomposition, we computed the gradient vector using the algebraic approach, assuming the regression coefficient is constant.

August 16, 2022, was the first day the VaR exceeded the set limit. The breakdown by asset class on that day is shown in the following table (Table 6.1). We can conclude the equity exposure represents a significant portion of the risk, raising the VaR.

***Table 6.1*. VaR decomposition by asset class as of August 16, 2022.** The contribution of each asset type to the VaR estimate is illustrated in this table.

	Equity	Currency	Interest Rates	Total
VaR (€)	739 080	55 055	225 230	1 019 365
VaR Weight (%)	72.50	5.40	22.10	100

Decomposing the above by market is valuable for identifying an efficient hedging strategy. Table 6.2 displays the equity and interest rates breakdown from markets and segmented by risk. This table shows the equity VaR weights for the U.S. and Europe are quite similar. As noted earlier in Table 6.1, interest rates and currency factors contribute far less to the VaR compared to equities. Consequently, we can conclude the primary drivers of the higher VaR on August 16, 2022, are the stocks from the American and European markets.

Table 6.2. **VaR decomposition by market as of August 16, 2022.** This table demonstrates the degree to which each asset type by market (U.S. and Europe) contributes to the VaR estimate.

	Equity			Interest Rates			Total
	U.S.	Europe	Currency	U.S.	Europe	Currency	
VaR (€)	351 934	387 146	64 057	54 371	170 859	(9 002)	1 019 365
VaR Weight (%)	34.52	37.98	6.28	5.33	16.76	(0.87)	100

6.2. Hedging Technique

One strategy to lower the VaR is to decrease the equity exposure by hedging both U.S. and European equities, according to the findings in the previous sub-chapter regarding the VaR decomposition by asset class and market. We will use, as the hedging technique, positions in Exchange-Traded Funds (ETF). The ETFs chosen were the Standard & Poor's 500 (S&P 500) for the U.S. and the Financial Times Stock Exchange (FTSE) for Europe. The reason behind the choice of the ETFs was using broad stock indexes currently widely accessible. The European developed countries' large and mid-cap equities are tracked by the FTSE index. The S&P 500 index includes five hundred leading companies on the NASDAQ stock exchange.

This is the approach we will take each time the VaR surpasses the fixed maximum (€750 000). We will also maintain the ratio of the marginal VaR between U.S. and European stocks (Table 6.2). As a result, we will use the hedging approach on the first day by taking a short position in the S&P 500 equal to the amount of the position we will hold in the FTSE. To maintain this ratio, on the day of exceedance, we will compute the VaR decomposition and adjust the ratio based on the outcomes.

The incremental VaR will be used to measure the approximate decrease in the VaR from the slight change in the risk factor loadings vector. Based on the first-order Taylor expansion of $f(\theta')$ around θ, the new VaR can be computed by

$$f(\theta') \approx f(\theta) + \nabla f(\theta)^T(\theta' - \theta), \tag{6.4}$$

where $(\theta' - \theta)$ represents the change in exposure and $f(\theta)$ the original VaR.

We will use this process every day VaR exceeds €750 000 until August 15, 2023. We will not add the short positions to the risk factor loadings vector whenever the VaR does not surpass this limit.

6.3. Value-at-Risk Management Results

Figure 6.2 displays the outcome of the hedging strategy for the one-year period, in terms of daily VaR. We designate two portfolios: the hedged portfolio, which has the hedging strategy applied, and the unhedged portfolio, corresponding to the initial portfolio.

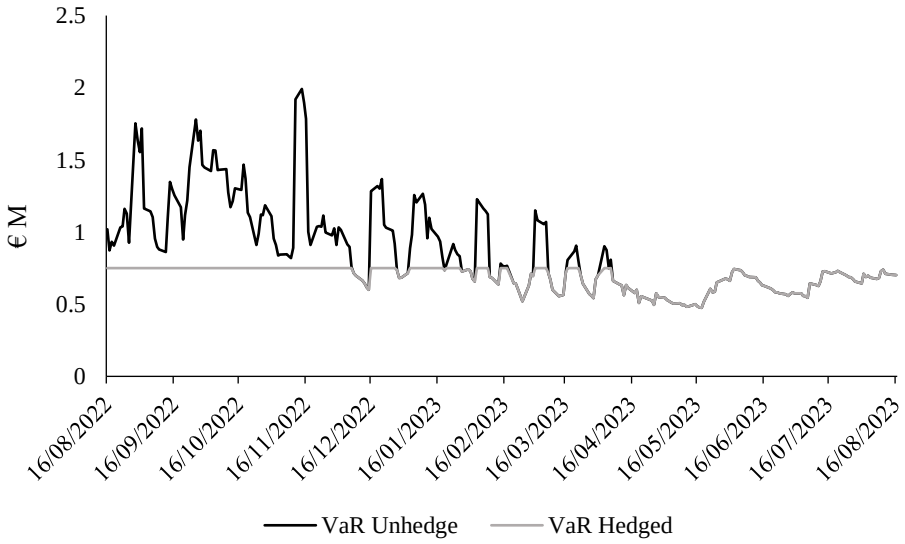


Figure 6.2. **Daily VaR estimates of the hedged and unhedged portfolios.** The graph highlights the potential losses between the two portfolios, emphasizing the impact of using the hedging technique.

The short positions introduced to the hedged portfolio in the FTSE and S&P 500 indexes are shown in Figure 6.3.

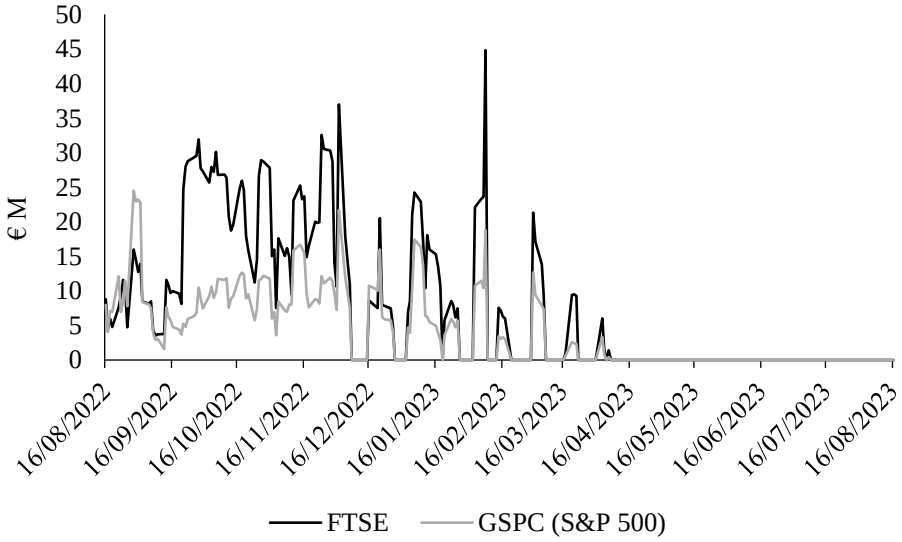


Figure 6.3. **Daily short positions added to the hedged portfolio.** The frequency and size of the short positions added to the portfolio to achieve the desired hedging strategy can be seen in this figure.

Figure 6.4 shows the P&L for each day, illustrating the daily fluctuations in the portfolios' returns over the analysis period. Appendix A illustrates the histogram of the daily P&L for both portfolios.

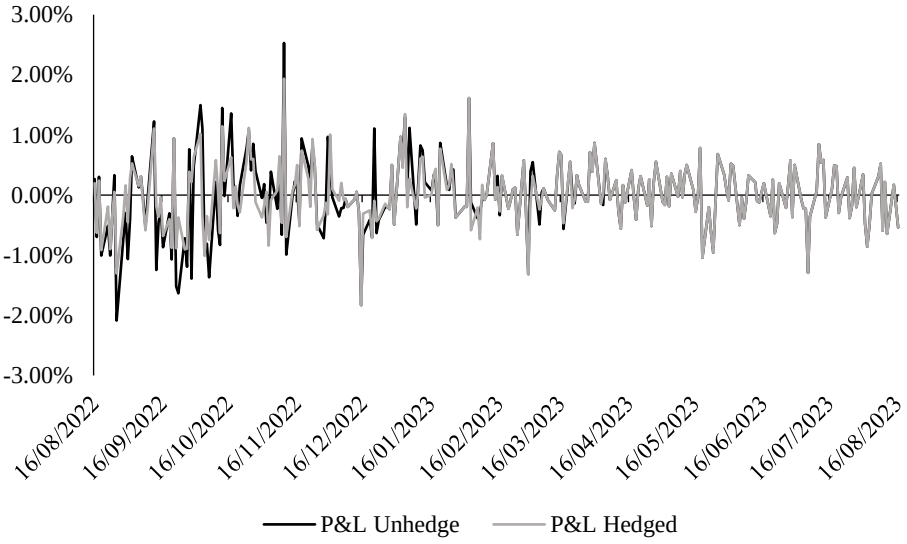


Figure 6.4. **Daily P&L (returns) of the hedged and unhedged portfolios.** The daily returns of the two portfolios during the analysis period are shown in this figure. The unhedged portfolio exhibited more volatile daily P&Ls compared to the hedged portfolio.

From the above figures, it is noticeable the hedging strategy effectively restricted the VaR. The hedged portfolio maintained a maximum VaR of €750 000 throughout the year, while the unhedged portfolio exceeded this limit multiple times, peaking at a maximum VaR of €1.99 million on November 14, 2022. Additionally, the unhedged portfolio exhibited more volatile daily P&Ls compared to the hedged portfolio, particularly at the beginning of the analysis period when a significant number of short positions were added.

The hedged portfolio had one less exceedance than the unhedged portfolio on August 26, 2022 (Table 6.3). On this day, the hedging technique was applied, resulting in a lower VaR estimate. Consequently, the loss did not exceed the estimated VaR, therefore not being considered an exceedance. The hedging technique was not applied on the remaining days in the below table (when an exceedance happened in both the hedged and the unhedged portfolios). Therefore, the exceedance size was the same for both portfolios. However, the magnitude of the exceedances remained low.

Table 6.3. **Exceedances of the unhedged and hedged portfolios.** This table displays the magnitude of exceedances observed in both the unhedged and hedged portfolios, highlighting whether the hedging strategy was applied.

Exceedance date		Hedging?	Exceedance Size (€)	Exceedance Size (%)
Unhedged	Hedged			
06/07/2023	06/07/2023	No	192 179	0.34
19/05/2023	19/05/2023	No	95 726	0.16
01/03/2023	01/03/2023	No	38 683	0.07
15/12/2022	15/12/2022	No	387 382	0.72
26/08/2022		Yes	269 089	0.47

Figure 6.5 shows both portfolios had a negative annual return. The fact that the loss was reduced for the hedged portfolio, however, suggests the strategy effectively prevented additional losses while reducing the portfolio's VaR. Further details of the hedging strategy are presented in Appendix D.

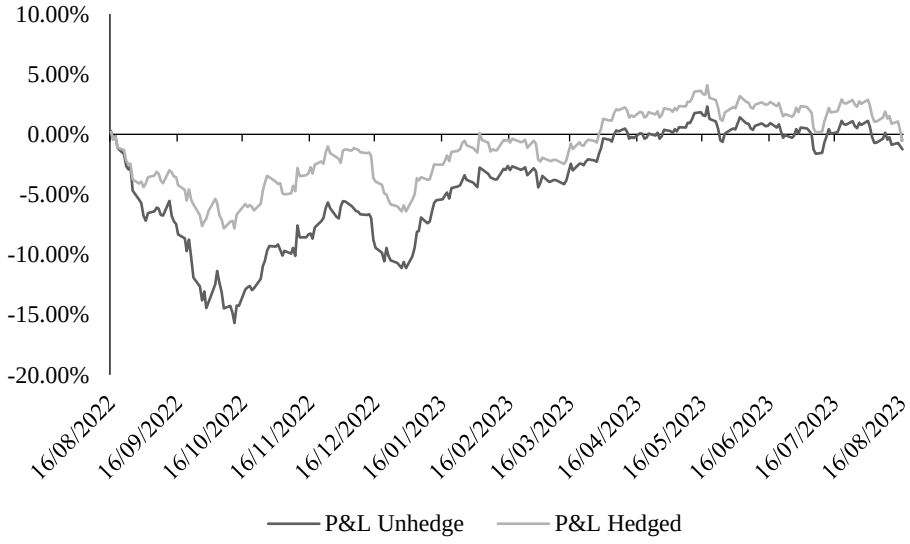


Figure 6.5. **Cumulative daily P&L (returns) of the hedged and unhedged portfolios.** This figure shows negative annual returns in both portfolios. However, the hedged portfolio reduced losses indicating the hedging strategy's effectiveness.

Nevertheless, it might not be fair to compare returns directly without considering each portfolio's risk profile as the strategy employed resulted in portfolios with distinct risk characteristics. Thus, we will use metrics to provide deeper insights into the portfolios' performance by considering the amount of risk taken to generate the returns: RAROC and RORAC. By incorporating these metrics into our analysis, we can better evaluate and manage the trade-offs between risk and return.

RORAC is calculated using the following equation

$$RORAC = \frac{P\&L}{EC}\ , \tag{6.5}$$

where *P&L* represents the annual returns, and the EC is the average VaR over one year.

The RORAC for the portfolios is displayed in the following table (Table 6.4).

Table 6.4. **RORAC for the unhedged and hedged portfolios**. This table demonstrates the hedged portfolio performed better in this metric.

	Unhedged	Hedged
P&L (%)	(1.25)	(0.52)
P&L (€)	(579 609)	(365 667)
Average VaR (€)	868 342	687 920
RORAC (%)	(66.75)	(53.16)

Additionally, the metric RAROC was also used. The RAROC is the RORAC minus the cost of capital. We computed the RAROC multiplied by the VaR to give us the P&L deducted from the costs associated with remunerating the capital at risk. In other words, instead of computing the profitability, we will calculate this profitability multiplied by the capital at risk (VaR), given by

$$RAROC \times VaR\ =\ P\&L - k \times VaR, \tag{6.6}$$

where *k* is the cost of capital (we used *k* = 7%).

Table 6.5 displays the RAROC for the year for the unhedged and hedged portfolios.

Table 6.5. **RAROC for the unhedged and hedged portfolios.** The hedging portfolio outperformed on this metric, as the table illustrates.

	Unhedged	Hedged
P&L (%)	(1.25)	(0.52)
P&L (€)	(579 609)	(365 667)
Average VaR (€)	868 342	687 920
RAROC × VaR	(640 392)	(413 821)

Both metrics results lead us to the conclusion: in comparison to the unhedged portfolio, the hedged portfolio suffered losses of lower magnitude. Overall, the hedging strategy proved successful in mitigating risks and reducing the magnitude of losses as a proportion of the capital at risk. This confirms its efficacy in enhancing the portfolio's resilience against market fluctuations and minimizing the overall risk incurred.

CHAPTER 7
Conclusion

Intending to measure and manage a portfolio's VaR, this work offers a thorough framework for comprehending and reducing financial risk. A hedging technique was employed to retain the VaR at an established maximum for a year.

The portfolio selected is composed of bonds and stocks from the European and American markets. To effectively diversify the market risk, we aimed to cover a range of industries. Measuring the portfolio's VaR was the first phase. Thirteen variants of four different VaR models (RM, Historical, SGSt, and QR) were computed to achieve this. Backtesting, through the UC and BCP tests, was carried out to assess the performance of each model.

The model that suited the portfolio under consideration the best was the QR VaR, with EWMA volatility and volatility lag 5 serving as the explanatory variables (model 11). Based on past ranges and to manage the VaR, a daily maximum of €750 000 was set as the limit. We computed the daily VaR estimations beginning on August 15, 2022. The VaR would have to be decreased by implementing a risk management approach when the VaR estimation for the following day exceeded the pre-established maximum. Adding short positions in ETFs to our portfolio — namely, the S&P 500 and FTSE indexes — was the hedging method used to reduce the equity exposure. The ratio between the marginal VaR for the U.S. and European stocks was preserved. To do so, the VaR marginal decomposition was computed every day the limit was exceeded. This methodology was conducted for one year, ending on August 15, 2023.

The outcomes of our study indicate, in contrast to the unhedged portfolio, whose VaR peaked at €1.99 million, the chosen hedging approach was effective in limiting the VaR below a predetermined limit of €750 000. Using metrics providing more in-depth information about the performance of the hedged portfolio, such as RORAC and RAROC, we evaluated the hedging strategy's performance by considering the exposed level of risk that produces returns.

In conclusion, even though the yearly returns from the hedged and unhedged portfolios were both negative, the thorough analysis of VaR and P&L indicators demonstrates clearly the hedged portfolio was able to reduce the amount of losses and manage risks in comparison to the unhedged portfolio. This confirms the applied hedging technique was successful in enhancing the portfolio's resilience against market fluctuations, thereby reducing the overall risk incurred. These results underline the relevance of using risk management strategies to improve portfolio resilience against market fluctuations, offering insightful information to risk managers and investors who desire to reduce risk.

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Appendices

Appendix A. Histogram of daily P&L for the hedged and unhedged portfolios.

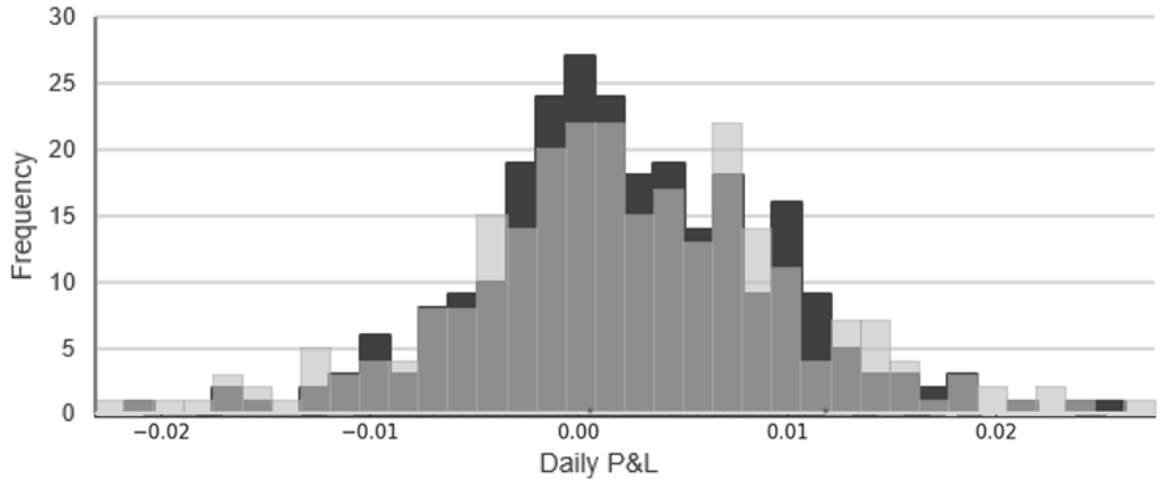


Figure 7. Histogram of daily P&L for the hedged and unhedged portfolios. The distribution of returns is displayed visually in this illustration. The light grey bars represent the unhedged portfolio, while the black ones reflect the hedge portfolio. The dark grey indicates coincident zones between the two portfolios.

Appendix B. BCP test results for all models

Table 14. BCP test results for all models. The time frame is between August 15, 2022, and December 30, 2016.

Model no.	Model description	p-value (%)				
		Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
1	RiskMetrics	61.40	17.20	0.40	0.80	1.50
2	Skewed Generalized Student-t, sample size 200	12.30	0.10	0.00	0.00	0.00
3	Skewed Generalized Student-t, sample size 400	12.30	0.00	0.00	0.00	0.00
4	Skewed Generalized Student-t, sample size 600	9.30	0.00	0.00	0.00	0.00
5	Skewed Generalized Student-t, sample size 800	9.30	0.00	0.00	0.00	0.00
6	T+1 volatility-adjusted Historical, sample size 200	0.10	0.10	0.00	0.10	0.00
7	T+1 volatility-adjusted Historical sample size 400	0.00	0.00	0.00	0.00	0.00
8	T+1volatility-adjusted Historical sample size 600	27.90	1.20	0.10	0.10	0.20
9	T+1volatility-adjusted Historical sample size 800	1.40	0.20	0.00	0.10	0.00
10	Quantile Regression, explanatory variable: EWMA volatility	0.80	0.10	0.00	0.00	0.00
11	Quantile Regression, explanatory variables: EWMA volatility and volatility lag 5	67.20	83.60	91.00	94.90	97.00
12	Quantile Regression, explanatory variables: EWMA volatility and 15 days average volatility	6.70	16.90	7.50	13.00	19.80
13	Quantile Regression, explanatory variables: EWMA volatility and 30 days average volatility	2.90	8.50	2.10	4.30	7.40

Appendix C. Exceedance details for all models

Table 15. RiskMetrics (model 1) exceedance size. The percentage column represents the proportion of the exceedance compared to the portfolio value. The time frame is between August 15, 2022, and December 30, 2016.

Date	Model 1	
	€ M	%
16/06/2022	(0.87)	(2.01%)
13/06/2022	(0.75)	(1.73%)
06/05/2022	(0.65)	(1.49%)
22/04/2022	(0.56)	(1.28%)
03/02/2022	(0.54)	(1.26%)
21/01/2022	(0.42)	(0.98%)
26/11/2021	(0.38)	(0.87%)
28/09/2021	(0.39)	(0.91%)
08/07/2021	(0.46)	(1.06%)
11/05/2021	(0.36)	(0.84%)
20/04/2021	(0.38)	(0.87%)
29/01/2021	(0.53)	(1.23%)
11/06/2020	(1.09)	(2.52%)
16/03/2020	(1.78)	(4.10%)
12/03/2020	(1.27)	(2.92%)
09/03/2020	(0.92)	(2.13%)
27/02/2020	(0.58)	(1.34%)
24/02/2020	(0.37)	(0.86%)
02/12/2019	(0.43)	(0.99%)
02/10/2019	(0.46)	(1.05%)
05/08/2019	(0.49)	(1.13%)
02/08/2019	(0.39)	(0.91%)
28/02/2019	(0.43)	(1.00%)
10/10/2018	(0.42)	(0.98%)
04/09/2018	(0.34)	(0.78%)
25/06/2018	(0.45)	(1.04%)
05/02/2018	(0.51)	(1.18%)
29/06/2017	(0.47)	(1.09%)
27/06/2017	(0.36)	(0.82%)
12/06/2017	(0.31)	(0.70%)

Table 16. Skewed Generalized Student-t (models 2 to 5) exceedance size. The percentage column represents the proportion of the exceedance compared to the portfolio value. The time frame is between August 15, 2022, and December 30, 2016.

Date	Model 2		Model 3		Model 4		Model 5	
	€ M	%	€ M	%	€ M	%	€ M	%
06/05/2022	(0.75)	(1.73)	(0.75)	(1.73)	(0.79)	(1.82)	(0.77)	(1.78)
21/01/2022	(0.50)	(1.15)	(0.50)	(1.14)	(0.51)	(1.17)	(0.50)	(1.16)
26/11/2021	(0.43)	(1.00)	(0.43)	(1.00)	(0.45)	(1.04)	(0.45)	(1.03)
28/09/2021	(0.44)	(1.02)	(0.49)	(1.12)	(0.47)	(1.09)	(0.47)	(1.09)
11/05/2021	(0.41)	(0.95)	(0.43)	(1.00)	(0.42)	(0.98)	(0.43)	(0.99)
20/04/2021	(0.39)	(0.91)	(0.44)	(1.02)	(0.44)	(1.02)	(0.44)	(1.02)
16/03/2020	(2.18)	(5.03)	(2.14)	(4.94)	(2.08)	(4.81)	(2.06)	(4.76)
12/03/2020	(1.55)	(3.58)	(1.52)	(3.52)	(1.48)	(3.43)	(1.47)	(3.39)
09/03/2020	(1.13)	(2.61)	(1.11)	(2.57)	(1.08)	(2.50)	(1.07)	(2.47)
27/02/2020	(0.63)	(1.46)	(0.65)	(1.51)	(0.65)	(1.49)	(0.64)	(1.49)
24/02/2020	(0.41)	(0.94)	(0.42)	(0.97)	-	-	-	-
02/12/2019	(0.48)	(1.10)	(0.49)	(1.12)	(0.48)	(1.10)	(0.48)	(1.11)
05/08/2019	(0.53)	(1.23)	(0.54)	(1.25)	(0.54)	(1.25)	(0.54)	(1.26)
02/08/2019	(0.43)	(0.99)	(0.43)	(1.00)	(0.44)	(1.00)	(0.44)	(1.01)
28/02/2019	(0.50)	(1.16)	(0.48)	(1.11)	(0.49)	(1.12)	(0.51)	(1.17)
10/10/2018	(0.46)	(1.07)	(0.47)	(1.08)	(0.48)	(1.12)	(0.48)	(1.12)
05/02/2018	(0.58)	(1.35)	(0.58)	(1.33)	(0.60)	(1.38)	(0.60)	(1.38)
29/06/2017	(0.53)	(1.23)	(0.57)	(1.31)	(0.57)	(1.31)	(0.56)	(1.29)
27/06/2017	(0.40)	(0.93)	(0.43)	(0.99)	(0.43)	(0.99)	(0.42)	(0.98)

Table 17. T+1 volatility-adjusted Historical (models 6 to 9) exceedance size. The percentage column represents the proportion of the exceedance compared to the portfolio value. The time frame is between August 15, 2022, and December 30, 2016.

Date	Model 6		Model 7		Model 8		Model 9	
	€ M	%	€ M	%	€ M	%	€ M	%
06/05/2022	(0.73)	(1.68)	(0.72)	(1.66)	-	-	(0.75)	(1.73)
03/02/2022	-	-	(0.60)	(1.40)	-	-	-	-
21/01/2022	(0.56)	(1.30)	(0.45)	(1.04)	(0.52)	(1.19)	(0.49)	(1.13)
26/11/2021	(0.42)	(0.98)	(0.36)	(0.84)	(0.44)	(1.02)	(0.42)	(0.98)
28/09/2021	(0.42)	(0.97)	(0.38)	(0.88)	(0.46)	(1.07)	(0.44)	(1.01)
08/07/2021	(0.49)	(1.12)	-	-	-	-	(0.51)	(1.17)
11/05/2021	(0.32)	(0.74)	(0.44)	(1.02)	(0.42)	(0.97)	(0.40)	(0.92)
26/11/2021	-	-	-	-	-	-	-	-
28/09/2021	-	-	-	-	-	-	-	-
20/04/2021	(0.33)	(0.76)	(0.46)	(1.06)	(0.43)	(1.00)	(0.41)	(0.95)
25/02/2021	(0.49)	(1.13)	-	-	-	-	-	-
29/01/2021	(0.46)	(1.07)	-	-	-	-	-	-
16/03/2020	(2.33)	(5.38)	(2.17)	(5.00)	(1.96)	(4.53)	(1.96)	(4.53)
12/03/2020	(1.66)	(3.83)	(1.47)	(3.39)	(1.39)	(3.22)	(1.39)	(3.22)
09/03/2020	(1.21)	(2.79)	(1.02)	(2.35)	(0.98)	(2.27)	(0.98)	(2.27)
27/02/2020	(0.67)	(1.55)	(0.64)	(1.47)	(0.60)	(1.38)	(0.60)	(1.38)
24/02/2020	(0.41)	(0.95)	(0.40)	(0.91)	(0.38)	(0.88)	(0.38)	(0.88)
02/12/2019	(0.47)	(1.09)	(0.46)	(1.06)	(0.40)	(0.93)	(0.42)	(0.98)
02/10/2019	-	-	(0.49)	(1.12)	(0.49)	(1.12)	(0.47)	(1.08)
05/08/2019	(0.43)	(0.99)	(0.46)	(1.06)	(0.46)	(1.06)	(0.46)	(1.06)
02/08/2019	(0.34)	(0.79)	(0.37)	(0.85)	(0.37)	(0.85)	(0.37)	(0.85)
28/02/2019	(0.41)	(0.94)	(0.39)	(0.90)	(0.41)	(0.94)	(0.41)	(0.94)
17/12/2018	(0.53)	(1.23)	-	-	-	-	-	-
10/10/2018	(0.39)	(0.91)	(0.39)	(0.91)	(0.42)	(0.96)	(0.39)	(0.91)
04/10/2018	(0.36)	(0.84)	(0.36)	(0.84)	(0.38)	(0.89)	(0.36)	(0.84)
03/10/2018	(0.33)	(0.76)	(0.33)	(0.76)	-	-	-	-
05/09/2018	(0.36)	(0.83)	(0.36)	(0.83)	-	-	-	-
04/09/2018	(0.32)	(0.73)	(0.32)	(0.73)	(0.33)	(0.77)	(0.35)	(0.81)
25/06/2018	(0.39)	(0.91)	(0.42)	(0.97)	(0.44)	(1.02)	(0.47)	(1.08)
05/02/2018	(0.58)	(1.33)	(0.50)	(1.16)	(0.50)	(1.16)	(0.54)	(1.26)
29/06/2017	(0.46)	(1.07)	(0.46)	(1.07)	(0.50)	(1.16)	(0.50)	(1.16)
27/06/2017	(0.33)	(0.77)	(0.33)	(0.77)	(0.38)	(0.88)	(0.37)	(0.85)
12/06/2017	(0.30)	(0.69)	(0.30)	(0.69)	(0.32)	(0.75)	(0.32)	(0.73)

Table 18. Quantile Regression (models 10 to 13) exceedance size. The percentage column represents the proportion of the exceedance compared to the portfolio value. The time frame is between August 15, 2022, and December 30, 2016.

Date	Model 10		Model 11		Model 12		Model 13	
	€ M	%	€ M	%	€ M	%	€ M	%
22/04/2022	-	-	(0.56)	(1.29)	(0.48)	(1.10)	-	-
03/02/2022	-	-	(0.58)	(1.34)	-	-	-	-
21/01/2022	-	-	(0.36)	(0.83)	(0.38)	(0.87)	(0.41)	(0.95)
26/11/2021	(0.58)	(1.33)	(0.39)	(0.90)	(0.36)	(0.83)	(0.42)	(0.96)
28/09/2021	-	-	-	-	-	-	(0.55)	(1.27)
11/05/2021	(0.56)	(1.29)	(0.36)	(0.83)	(0.32)	(0.74)	(0.38)	(0.88)
20/04/2021	-	-	-	-	(0.30)	(0.69)	(0.31)	(0.73)
25/02/2021	-	-	-	-	(0.48)	(1.10)	(0.59)	(1.36)
29/01/2021	-	-	-	-	(0.50)	(1.15)	-	-
21/10/2020	-	-	-	-	(0.41)	(0.94)	-	-
11/06/2020	-	-	-	-	-	-	(1.07)	(2.48)
12/03/2020	(1.95)	(4.51)	(2.39)	(5.52)	(0.57)	(1.29)	-	-
09/03/2020	(1.42)	(3.29)	(1.11)	(2.57)	-	-	-	-
27/02/2020	(0.71)	(1.65)	(1.11)	(2.55)	(0.94)	(2.17)	(0.84)	(1.94)
24/02/2020	(0.46)	(1.06)	(0.34)	(0.78)	(0.31)	(0.71)	(0.37)	(0.01
02/12/2019	(0.46)	(1.07)	(0.44)	(1.02)	(0.51)	(1.18)	(0.46)	(0.01
05/08/2019	(0.57)	(1.31)	(0.73)	(1.70)	(0.73)	(1.68)	(0.64)	(0.01
02/08/2019	(0.45)	(1.05)	(0.42)	(0.96)	(0.46)	(1.07)	(0.44)	(1.02)
28/02/2019	(0.47)	(1.08)	(0.47)	(1.09)	(0.44)	(1.01)	(0.44)	(1.02)
10/10/2018	(0.45)	(1.05)	(0.52)	(1.19)	(0.53)	(1.23)	(0.49)	(1.12)
29/06/2017	(0.61)	(1.41)	(0.71)	(1.64)	-	-	-	-
27/06/2017	(0.46)	(1.07)	(0.45)	(1.05)	(0.48)	(1.10)	(0.48)	(1.11)

Appendix D. Hedging Strategy Details

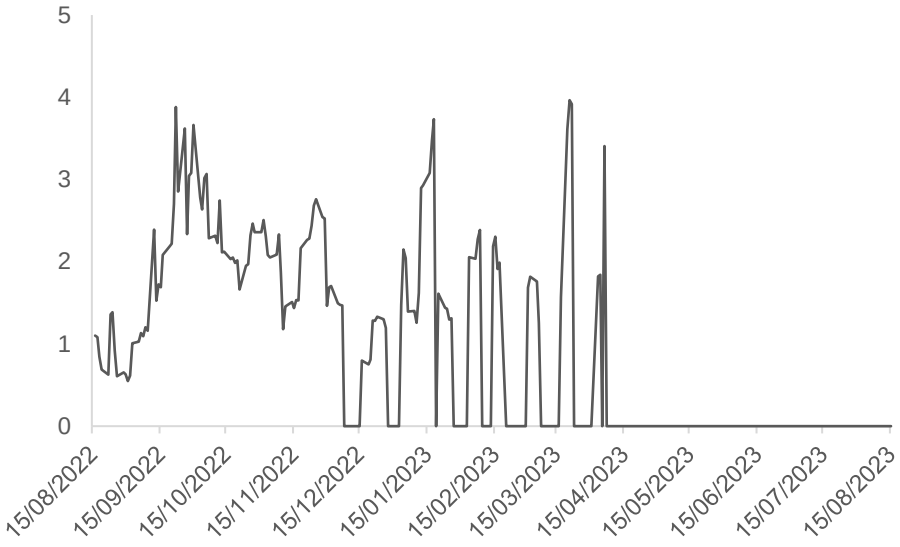


Figure 8. FTSE/S&P 500 ratio according to marginal decomposition by market.

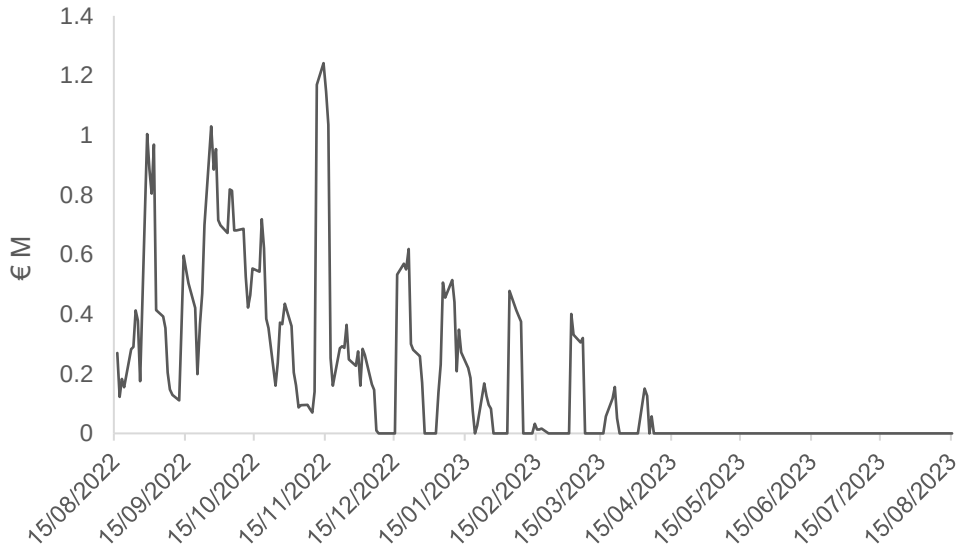


Figure 9. VaR decrease (in € M), resulting from the implementation of the hedging strategy.