INSTITUTO
UNIVERSITÁRIO
DE LISBOA

A multidimensional auction model for privatizations with complementarities

Pedro Miguel Rocha Galha da Silva Bártolo

Master in Economics

Supervisor:
Ph.D. Thomas Greve, Assistant Professor, Iscte-Iul

November, 2022

BUSINESS SCHOOL

CIÊNCIAS SOCIAIS E HUMANAS

Department of Economics

Department of Political Economy

A multidimensional auction model for privatizations with complementarities

Pedro Miguel Rocha Galha da Silva Bártolo

Master in Economics

Supervisor:

Ph.D. Thomas Greve, Assistant Professor, Iscte-Iul

November, 2022

Acknowledgement

I would like to thank everyone that was present and helped me during the elaboration of this dissertation. In particular, I would like to thank my family that has always helped me and given me amazing support throughout my entire academic journey and to my supervisor, professor Thomas Greve, for all his tireless help and support, for sharing his valuable knowledge and for all the time he patiently dedicated to help me make this dissertation a reality.

Resumo

A existência de períodos de numerosas privatizações é comum à maioria dos países. Durante estes períodos, não é incomum que várias firmas que apresentam sinergias entre si sejam privatizadas num curto espaço de tempo. Esta dissertação pretende providenciar pela primeira vez uma moldura teórica para enfrentar estas situações, assim como providenciar alguma orientação sobre potenciais políticas a adotar. Nós conseguimos estender o trabalho pioneiro sobre leilões multidimensionais de Che (1993) para um contexto de leilões combinatórios com possibilidade de licitação sobre pacotes, otimizando assim o desfecho do cenário analisado. Apesar de a dissertação se focar em privatizações, os nossos resultados são aplicáveis à maior parte dos itens que o governo pretenda vender, desde que complementaridades estejam presentes.

Palavras-Chave: Leilões Multidimensionais; Licitação sobre pacotes; Privatização.
Classificação JEL: C72; D44

Abstract

Big privatization momentums in countries are commonplace. During these times, it is not uncommon for several firms presenting synergies among them to be privatized in a short time frame. This dissertation aims to provide, for the first time, a theoretical framework and some policy guidance for these situations. We managed to extend the pioneering work on multidimensional auctions of Che (1993) to a combinatorial package bidding setting, thus optimizing the outcome of our analysed scenario. While the dissertation focuses on privatizations, our results hold for most items the government is looking to sell, as long as they present complementarities.

Keywords: Multidimensional Auctions; Package Bidding; Privatization.
JEL Classification: C72; D44

Table of Contents

Resumo iii

Abstract v

Table of Contents vii

List of Tables ix

1.Introduction 1

2.Literature Review 3

3.A two-route three-bidder example 15

4.The Model 21

5.Policy recommendation 25

6. Concluding remarks 27

References 29

Appendix A-A two-firm three-bidder example 33

List of Tables

Table 3. 1 - Procurement auction bids 16
Table 3. 2 - Scoring system for single lots 17
Table 3. 3 - Scoring system for a package 18
Table 3. 4 - Submitted bids 18
Table 3. 5 - Auction Scores 19

Table A. 1 - Scoring system for single lots in privatization auction 34
Table A. 2 - Scoring system for a package in privatization auction 34
Table A. 3 - Submitted Privatization Bids 35
Table A. 4 - Privatization Scores 35

1.Introduction

The classic auction model involves a single seller with many buyers, with the auctioned item being assigned to the highest bidder. A multidimensional/scoring auction works in a similar way, but bidders will bid on different aspects for every one item instead of just price and the item will be assigned to the bidder with the highest score. The score is determined through a scoring rule that evaluates all these different aspects of the bids. The auctioneer needs to decide the weights assigned to each of the different dimension to determine the winner.

While auctions dominate other similar procedures for privatization, they are not the most common method used (Fluck et al., 2007). This may contribute to the fact that, while there have been several cases of firms that likely exhibit synergies being privatized within a short time-frame of one another, multidimensional auctions to privatize these firms at the same time are, to the best of my knowledge, virtually unheard of.

Most countries go through at least one (most of the time more) big wave of privatization as they grow and develop and in these situations a lot of potential complementary companies are sold. Countries like Germany (post-unification), Russia (after the USSR collapse) or the UK (Margaret Thatcher era) have gone through huge privatization waves in the past, but this phenomenon is as relevant as ever today. For instance, China is going through a big privatization process, Portugal did the same a few years ago after the need for foreign intervention and Angola is currently amidst a huge privatization wave as well. And, in almost all such cases, it is possible to find privatizations close together in time of complementary companies: REN and EDP in Portugal, BAI and ENSA (still under completion) in Angola or Jaguar and Rolls-Royce in the UK are among possible examples. While not all cases of complementarities can be privatized through the method that will be suggested in this dissertation due to, for instance, monopoly concerns or, in the case of REN and EDP specifically, due to EU energy market liberalization regulations, as shown here, there are many instances where it can be applied.

However, the literature review shows (see section 2), that there is currently a gap in terms of how to best handle these types of privatization scenarios and conduct the privatization process as efficiently as possible that this dissertation looks to fill.

In order to address the aforementioned limitation, this thesis' main aim is to analyse a setting where a government wishes to privatize several state-owned enterprises who exhibit complementarities and evaluate the best course of action. It intends to demonstrate that

multidimensional auctions are a good tool to face the issue at stake going forward and to provide some guidelines and additional information for policy regarding future privatizations that fit the criteria.

In this dissertation we use a scoring auction instead of the classic price only auction model, since it has been shown that, for similar circumstances, a scoring auction with an appropriate scoring rule is superior to alternative auction designs (Matteucci, 2021). Additionally, there is also a long history of governments giving up revenue in privatizations in favour of other factors: Carlin and Mayer (1994) report that, during the peak of German privatization post-unification, the German privatization agency (Treuhandstalt) accepted a revenue reduction in the sale process of 12,000 to 50,000 Deutschmarks per retained worker position; López-de-Silanes (2002) reports a 16% sale price reduction in exchange for labour retention guarantees from a worldwide privatization sample; and Wang et al. (2007) mention the trade-offs the Chinese government has been willing to make in its privation processes.

As such, based on the review of the literature (see section 2), Che's (1993) paper was chosen as the basis/starting point for this dissertation. The author pioneered the field of multidimensional auctions and his results have already been shown to hold under different settings. Hence, a key research objective of this thesis is to contribute to the current state of the art by extending Che's (1993) model to a setting with package bidding, making an application to the area of privatization. Auctions are characterized by information asymmetries and this thesis studies a setting where bidders hold private information about their cost structure. Although our focus is on privatizations, the results of this thesis can apply to most items the government intends to sell, like land or real estate, providing they show complementarities and the government is interested in establishing some quality setting about them.

Besides this section that describes the context of the research and presents key research aims, the dissertation comprises 5 other sections. Section 2 provides a comprehensive review of the literature, followed by a practical example of a multidimensional auction in section 3. The model is then presented in section 4 and a policy recommendation is made in section 5. Finally, section 6 presents the concluding remarks and discusses some possible extensions of this dissertation. Furthermore, an additional example of a practical application is provided in the appendix.

2.Literature Review

This dissertation is connected to several lines of literature, the first and most important one being the literature on government-run auctions and on multidimensional auctions. While auction theory has been an important field of economic study for a long time, research on multidimensional auctions only started appearing towards the end of the 20th century, focusing mostly on procurement contracts, where the government is the buyer.

In the field of auctions, the literature relevant to this thesis is vast, starting with Riley & Samuelson (1981), who provided a very important contribution to the progress in auction study by establishing an equivalence between English and Dutch style auctions under risk neutrality and that, when multiple items are to be auctioned, auctioning them in sequence is inefficient unless buyer valuations are additive and income independent. Maskin & Riley (1989) add to their contribution by showing that this equivalence between English and Dutch auctions holds under multiunit scenarios and that Riley & Samuelson's (1981) analysis of optimal auctions can be generalised to multiunit auctions.

Laffont & Tirole (1987) also provided significant advancements to the field by drawing a bridge between auction and incentive theory and showing that the optimal auction can be implemented using a dominant strategy that employs information about both the first and second bids while also characterizing the optimal mechanism when information is one dimensional. Dasgupta & Spulber (1989) designed an alternative to the standard fixed quantity procurement auction that allowed the buyer to change the quantity or the quality purchased based on the bids of the competing suppliers.

Maskin (1992) argues that no auction can be fully efficient under multidimensional private information but that the English and second-price auctions tend to be more efficient than first price sealed bidding under this scenario. The author also proves that under symmetric independent unidimensional private values, English auctions, second-price auctions and first price sealed bidding are all efficient but only the English auction remains efficient under common values.

Che (1993) was a very important pioneer in the field of multidimensional auctions, developing for the first time a model of two-dimensional auctions in which firms bid both on quality and prices and are evaluated according to a scoring rule. The author focuses on procurement contracts (more specifically defence contracts) and analyses three different auction methods: a first-score auction, a second-score auction and a second preferred offer auction, with firms having (unidimensional) private information about the costs of improving

quality. Che's (1993) paper shows that a scoring auction implements the optimal mechanism and provides an argument for why scoring auctions are better than price-only auctions with minimum quality standards. In his paper, the author also shows that the optimal scoring rule systematically discriminates quality in relation to the buyer's utility function (otherwise a case of excessive quality would happen) and that the optimal outcome can be achieved under first and second-score auctions. The excessive quality outcome from a naive scoring rule originates from its failure to take into account information costs associated with increased quality. However, without the commitment power to enforce the level of quality agreed, the only feasible scoring rule corresponds to the buyer´s utility function and, with this scoring rule, although all three of the analysed auction methods achieve similar results for the buyer, the second preferred offer auction provides strictly smaller profits for the winning firm compared to the other two methods.

Building upon the work of Che (1993), Branco (1997a) builds a model that allows for correlation between the firms' costs where, when correlation is present, contrary to the independent costs model, a two-stage auction is needed in order to achieve an optimal outcome (in the first stage the procurer selects a firm from the auction and in the second stage they bargain to adjust the level of quality contracted). Branco (1997a) also shows that when correlation between the cost functions is present, the optimal level of quality is a function of all the firms' private parameters, and that the solutions reached by Che (1993) are not optimal.

Jehiel et al. (1999) expand on auction literature by trying to incorporate possible externalities into the outcome of auctions where a buyer's type is multidimensional. These externalities relate to the fact that non-winner bidders are not indifferent towards the identity of the winner and that the auctioned good might play a role in future interactions among bidders. The authors manage to reach a multidimensional incentive compatible optimal mechanism that maximizes the seller's expected utility, reaching the conclusion that a second-price auction maximizes revenue if the buyers submit scalar bids and the object being sold is for sure allocated to a buyer. Bidders are, for the most part, unable to escape these externalities by refusing to participate in the auction, so the agents' reservation utilities are endogenous and type-dependant (Jehiel et al.,1999).

In an innovative study, Bichler (2000) set out to empirically study multidimensional auctions and conducted an experiment using two-attribute auctions, finding that the auctions generally have modest gains over single-attribute auctions. Building upon his work, Chen-Ritzo et al. (2005) tried to find whether, in an empirical context, the higher theoretical utility

that multiattribute auctions provide to the buyer is enough to offset the increased complexity over single-attribute auctions (only price) in the context of reverse auctions for procurement and consider both quality and lead time in addition to price (three-attribute auction). Their results suggest their multiattribute mechanism design provides both increased utility to the buyer and similar or increased profits to the supplier over price only auctions. In their experiment, Chen-Ritzo et al. (2005) provided the bidders with limited information about the buyer´s utility function and state that it is possible that this limited information may have actually improved auction performance, consistently with the results from the literature that suggest simple information can be effective at generating efficient outcomes.

Asker & Cantillon (2008) expand on Che's (1993) work by introducing multidimensional private information about the firms costs in their model, in order to represent the likely situation that firms differ in their fixed and variable costs of production (the firm with lowest cost at a low level of quality is not necessarily the firm with lowest cost as quality increases). The authors study the properties of scoring auctions where the scoring rule is linear in price and show that this multidimensional private information can be reduced to a single dimension (their pseudotype) sufficient to characterize the equilibrium outcomes in the auction, as long as the scoring rule is quasilinear and types are independently distributed. They also show that a correspondence can be established between these scoring auctions and a standard single-object one-dimensional independent private value auction.

Asker & Cantillon (2008) also show that the results obtained by Che (1993) in terms of the optimal scoring rule underweighting the buyer's true quality value and in terms of the first and second-score auction achieving equivalent utility results when the scoring rule corresponds to the buyer´s true preferences both still hold under a multidimensional private information setting. However, the proof that a scoring auction with a scoring rule linear in price results in an optimal mechanism from Che (1993) isn´t robust under multidimensional private information, with this scoring rule not allowing the optimal scheme to be implemented, though it performs almost as well as said optimal scheme (Asker & Cantillon, 2008; Asker & Cantillon, 2010).

Nakabayashi (2013) adds to the literature by using empirical data on Japanese public construction projects to evaluate the impact on public procurement of the introduction of distortions meant to provide opportunities for disadvantaged bidders. The author finds that the increased competition that these distortions create offsets the inefficiencies created, hence

removing these distortions would actually increase government procurement costs, similar to what had been previously suggested by Ayres and Cramton (1996).

Di Corato et al. (2018) explore multidimensional auctions that involve long term commitments and how early termination penalties should be handled. The authors establish that bidders' payoffs are lower when competing for contracts which do not provide for enforceable time commitments and that neglecting the risk of opportunistic behaviour and contract breaches leads to the inability to maximize the auctioneer´s potential payoff. The results from Di Corato et al. (2018) are consistent with the one´s from Che (1993), as the authors show that Che´s (1993) results still hold even when bidders incorporate into their bidding strategies potential early terminations of supply, as they do not face binding enforcement constraints. Kawasaki et al. (2012) had previously also scrutinized auction performance under imperfect monitoring and incentive for noncompliance, finding that uniform-price auctions tend to be more efficient and cost effective than discriminatory-price auctions.

Sant'Anna (2018) empirically studies multidimensional private information scoring auctions used to sell oil exploration rights in Brazil where an interdependent and non-linear scoring rule determined the winner, meaning that this is a type of scoring auction that cannot be characterized by pseudotypes. The author proposes a method to estimate underlying model primitives and finds that a first-price auction brings the highest possible revenue and that a scoring mechanism performs better than a mandate system in terms of the trade-off between revenue and exploration/other government objectives.

Takahashi (2018) shows that the uncertainty present in the design evaluations of Design-Build auctions leads to an exacerbation of the cost of quality compared to auctions where the evaluation of bids contains less uncertainty. This means that higher uncertainty and less transparency when running multidimensional auctions likely leads to a higher cost for the procurer to obtain the same level of quality. In the same vein, Allen et al. (2019) use the resolution process of failing banks in the USA to empirically analyse a multidimensional auction environment where the rule used to evaluate bids is proprietary and consequentially there is uncertainty about the weight attributed to the different components of the bid. The authors reach the conclusion that removing uncertainty leads to less bid shading by large acquirers and reduces the costs of resolution and thus is beneficial to disclose the quality components weight.

Laffont et al. (2020) establish a structural framework to analyse scoring procurement auctions where both bidder's quality and bid price are taken into account, develop a model in which bidding firms are characterized by costs dependant on their cost inefficiencies and exogenous quality, nonparametrically identify the model primitives under mild functional assumptions and describe the optimal mechanism when quality is exogenous, showing that the scoring rule doesn't need to be linear in the bid price.

Kong et al. (2021) contribute to the evolution of the literature on multidimensional private information by conducting a structural analysis of multi-attribute auctions of contracts with royalty payments when private information is multidimensional. The authors nonparametrically identify bidders' private information from their bids, develop a method to examine endogenous multivariate bids under multidimensional private information and, contrary to what would be expected based on results from previous literature, find than in this situation fixed-royalty auctions perform better than scoring auctions, since the bidder's incentive to produce is affected by its royalty bid. Kong et al. (2021) also reach the conclusion that single-attribute bidding can outperform multi-attribute bidding in the presence of adverse selection.

Quiroga (2021) extends the standard nonparametric method for independent private cost sealed-bid auctions to encompass multiattribute bidding in score auctions where bids are evaluated using a preannounced quasilinear scoring rule, accomplishing nonparametric identification of bidders' pseudotypes.

Chen (2021a), following the framework laid by Che (1993), researches how reference-based utility of a buyer affects the outcomes of multidimensional procurement auctions and determines the optimal reference-based utility of a buyer, extending the results from the standard analysis of multidimensional procurement auctions to more economically reasonable cases. The results from Che (1993) in terms of suppliers' bidding strategies and the utility equivalence between first-score and second-score auctions still hold under reference-based utility (Chen, 2021a). Shu & Durango-Cohen (2021) also follow the framework of Che (1993), but instead analyse procurement auctions with quality variables and study the setting of road concessions. The authors consider a multidimensional auction and derive a scoring function that permits the implementation of the optimal direct revelation mechanism in a first-score auction where the multidimensional bids consist of toll, capacity and government payment

levels, with the scoring function including a term that controls for firms´ incentives to distort information.

Chen (2021b) surveys the issue of bid orchestration (bribing the auctioneer) in scoring procurement auctions, analysing first-score and second-score auctions with honest and dishonest suppliers and reaching the conclusion that both types of suppliers find the first-score auction more profitable while the government buyer should opt for a second-score auction. The author also uncovers that in a second-score auction, buyers might be better off under limited bid orchestration than in a corruption free environment.

Kersten et al. (2022) analyse the trade-off that designers of procurement multi-attribute mechanisms face between the buyer´s preferences revelation and their ability to construct effective offers. The authors construct a procedure that transforms the preferences information into information on reservation levels perturbed by two parameters jointly representing the buyer´s trade-off between maintaining preference secrecy and achieving an efficient winning bid.

Wu et al. (2022) conduct research on loss aversion in multidimensional procurement auctions in the context of online labour markets, which have an all pay component. The authors show that bidders' risk aversion always decreases their equilibrium expected utility but may increase or decrease the buyer's equilibrium expected utility, being more detrimental to the buyer when bidding competition is more intense. The authors propose two reimbursement policies- percentage and flat reimbursement policy- that, according to their results, will improve social welfare without either side becoming worse off.

Another strand of literature relevant to this thesis is the one that comprises complementarity effects on auctions. Maskin (1992) states that the presence of complementarities without proper allocation of capital can create inefficiencies, suggesting that items who are likely to be complementary should be auctioned simultaneously. Klemperer (1999) states that, if complementarities are important, some form of auction in which a buyer can place bids in one or more package of items or make contingent bids should be used.

Branco (1997b) describes the equilibrium behaviour in English auctions when some bidders have superadditive values for the objects and argues that the presence of complementarities in sequential auctions leads to intense competition in early stages and causes a declining price over the sequence of auctions, providing a possible explanation for the declining price anomaly in these types of auctions.

Elmaghraby (2005) evaluates multi-unit auctions with complementarities, using a complete information one-shot framework applied to electricity auctions, arguing for horizontal simultaneous auctions over vertical auctions that allow more than one winner per demand lot.

Brusco & Lopomo (2009) consider ascending auctions of two objects with financially constrained bidders who have superadditive values for the objects and increasing marginal values: they characterize non-collusive equilibria and find that higher budget bidders are reluctant to bid since lower budget bidders may end up pursuing a single object and prevent the realization of complementarities for higher budget bidders. Thus, at equilibrium, a high budget bidder may leave the auction earlier than a low budget bidder with the same valuation. Simultaneous ascending auctions have been a popular method for the sale of multiple objects, particularly when a significant portion of the objects being auctioned exhibit complementarities but, while this type of auction leads to an efficient non-collusive equilibrium without budget constraints, the presence of such budget constraints generates several inefficiencies that prevent the efficient outcome (Brusco & Lopomo, 2009).

Moreover, combinatorial auctions literature also plays an important role in the development of this dissertation. Combinatorial auctions have been an important part of a multitude of different fields of study, like economics, artificial intelligence or logistics and have also been considered by several authors in the context of multidimensional procurement (Xia et al., 2005).

Cantillon & Pesendorfer (2004) study the results from the combinatorial auctions used to sell London bus routes, presenting a new method for inferring bidders' cost structures and finding no impact of auction size on bidding behaviour.

Muller et al. (2007) analyse a multi-attribute procurement environment where suppliers possess multidimensional private information and the procurer wants to procure several items at once, with offers evaluated according to a quasilinear scoring rule. The authors analyse the equilibria in such situations and show the link between combinatorial scoring auctions and combinatorial price-only auctions and how such link can be used to apply existing knowledge of equilibrium behaviour in price-only auctions to the strategic analysis of combinatorial score auctions, therefore establishing a foundation for equilibrium analysis in combinatorial score auctions under multidimensional private information.

Xiao (2021) studies the crowdsourced delivery market and constructs a multiattribute Vickrey-Clarke-Groves auction where bidders submit vector bids which are converted into a

vector score by the auctioneer. This auction is shown by the author to outperform the single-attribute auction and achieve both truthfulness and efficiency under an appropriate scoring rule.

Additionally, for the purpose of this dissertation, it is also very relevant to consider how auctions in general stack up to other similar procedures and what mechanisms can be used for privatization.

Several authors have been able to show that, in general, (score) auctions dominate other similar procedures: Lu & McAfee (1996) show standard auctions dominate bargaining, constituting the unique standard equilibrium, while Kultti (1999) demonstrates that standard auctions and posted prices sales are equivalent in most scenarios, and both dominate bargaining.

Cornelli and Li (1997) investigate optimal schemes for privatization cases where large risk neutral foreign investors are potential buyers and argue that the highest bidders may not be the best shareholders for the state enterprise, since their bid may only be reflecting their high private benefit of control. The authors also state that the governments face a trade-off between trying to obtain the highest revenue possible, allocation efficiency and maximizing the company´s future potential and valuation, so ordinary auctions are not appropriate. As such, they suggest that the government must carefully choose the buyer and how large of a controlling stake will be sold and that the optimal privatization mechanism is one where the number of shares sold is connected to the bidding price, where the higher the winning bid, the more shares are allocated. Cornelli and Li (1997) argue that an efficient investor would like to obtain as many shares as possible, as it believes in the company´s potential and in multiplying its valuation, while an investor whose interest lies mainly in the private benefit of control will attach minimal value to any shares in excess to those required to control the firm and, as such, the proposed mechanism minimizes the identified problem.

Fang & Parreiras (2002) investigate privatizations conducted through affiliated value second-price auctions with two financially constrained bidders and prove the existence of a symmetric equilibrium.

It has also been shown that scoring auctions dominate other procedures often used for buying differentiated products, such as menu auctions (where the auctioneer lets suppliers submit menus of price and nonmonetary attributes and then chooses the combination that best suits its needs), beauty contests (where the auctioneer informs bidders he cares about attributes other than price but doesn't specify a specific decision rule to pick the winning bidder) and price-only auctions where a minimum quality threshold is implemented (Asker & Cantillon,

2008; Yoganarasimhan, 2015). The fact that scoring auctions dominate these procedures (who combine competition with the flexibility of being able to make decisions for all different dimensions of the product) implies that score auctions are a very useful mechanism for the purchase of differentiated products, with the authors speculating it may be due to them being relatively simple and straightforward procedures when compared to these alternatives (Asker & Cantillon, 2008).

Asker & Cantillon (2010) add to their previous contribution on multidimensional private information by analysing scenarios where quality is contractible and endogenously determined as part of the procurement process and comparing score auctions and bargaining under multidimensional private information in these scenarios. They demonstrate that score auctions can extract a significant part of the buyer´s strategic surplus and that, for this reason, bargaining is less efficient and yields poorer results. The authors also show that scoring auctions can implement an efficient mechanism and can often replicate the allocation probabilities of the optimal mechanism but fall short of the optimal mechanism due to their inflexibility in terms of quality evaluations. While bargaining allows more flexibility in product design, it does so at the cost of lower competition and is inherently inefficient, since it can't replicate the allocation probabilities of the optimal mechanism (Asker & Cantillon, 2010).

Irdam et al. (2015) analyse all the nine different methods of privatization used in eastern Europe after the fall of the USSR (competitive auctions, non-competitive auctions, management and employee buyouts, employee shared ownership schemes, foreign direct investment, cross-ownership, greenfield investment, vouchers/coupons and restitutions) with competitive auctions performing remarkably well.

Chen (2021a) shows that multidimensional auctions with reference-based utility also dominate several other procedures commonly used to buy differentiated products.

Bae & Kagel (2021) explore the differences in efficiency and revenue between a proportional auction and a first-price auction with budget constrained bidders, looking at Russian privatizations in their analysis. In the case of privatizations, a proportional auction leads to several bidders obtaining a portion of the privatized company´s shares, according to their bid. The authors results are consistent with theoretical predictions in that, under a strong budget constraint, proportional auctions can achieve higher efficiency and revenue than a first-price auction and under a loose budget constraint these results are reversed.

For a practical and empirical analysis, Matteucci (2021) evaluates the results of auctions in Italy using the European Union scoring framework (MEAT-Most Economically

Advantageous Tender). The author asserts this framework generated vibrant bidding competition and price performance and that the results suggest that, consistently with theoretical results, a scoring auction with an appropriate scoring rule is superior to alternative auction design like lowest price auctions and beauty contests.

The proven poor performance of bargaining contrasts with its big popularity among practitioners in complex procurement settings and in privatization situations, something that has intrigued a multitude of authors.

Lu & McAfee (1996) speculate that, in order for bargaining to persist in markets, there must be strong enough advantages to override the structural advantages of auctions, stating that the ability of a seller to price discriminate based on observations about the buyers' characteristics might be one of the strongest advantages. The authors also recognize that in situations when buyers arrive stochastically over time and it is thus costly to assemble them at the same time in the same place, bargaining may be advantageous.

Fluck et al. (2007) were perplexed by the frequent use of private negotiations/bargaining in privatizations in eastern Europe, an apparently suboptimal mechanism, and in their paper conclude that this suboptimal method selection is mostly due to political constraints. Fluck et al. (2007) also compare auctions with bargaining and other privatization methods, reaching the conclusion that, while the auction mechanism is generally more efficient and yields better results, when political constraints moderately distort the agent in charge´s incentives and that same agent highly values staying in office, surprisingly bargaining can lead to higher revenue than auctions.

Similarly, Bang (2007) was also intrigued by the comparably low popularity of auctions in privatization situations, with only 36 out of a sample of 139 telecommunication privatizations having been realised through an auction, and finds that, in countries where state involvement has been more pervasive, the method of privatization chosen is often not the best available option and that when the government's budget is more constrained and existing capital markets are deeper, efficient mechanisms become more likely. The author identifies government deficits, income per capita and depth and openness of capital markets as three main predictors of the method of sale, with high deficits assuming particular importance as a predictor.

Asker & Cantillon (2010) speculate about possible reasons behind this unexplained bargaining popularity, possibly attributing it to some nonmodeled hazard-type issues or to the fact that procurement practitioners prefer procedures over which they have more control.

Nevertheless, literature applying multidimensional auctions to privatization scenarios is still very scarce: to the best of my knowledge, only three papers by Wang et al. (2007), Wang and Wang (2007) and De Frutos & Pechlivanos (2010) study similar situations, all within the context of the trade-off between revenue and retained labour in privatizations.

In State-Owned Enterprise privatization auctions in China the government has different aspect of concern, such as maximizing revenue and minimizing induced unemployment so, on their paper, Wang et al. (2007) analyse a mechanism where the government predetermines a minimum quantity of positions the winning bid must assure to the former employees and the potential investors compete only over price in a first-price auction, considering possible asymmetries between investors. Unsurprisingly, the authors found that an increase in the amount of retained labour required leads to a decrease in revenue and strengthens the advantage of investors with lower labour accommodation costs, which will restrain their bids. Wang and Wang (2007) reach similar conclusions but add that, with a strong commitment ability, if the government sets a compensation standard for the layoffs that originate from the privatization to eliminate the investor's labour accommodation cost dispersion, while also appearing strong against unemployment and signalling it out to the potential investors, it can increase its revenue while simultaneously protecting the workers. The importance of a strong commitment enforcement capability is not surprising: Perotti (1995) had already shown that a government´s lack of credibility can diminish the value of the firm after privatization.

De Frutos & Pechlivanos (2010) also worked on the objectives of maximizing revenue and minimizing unemployment, studying an environment with bidimensional private signals (a signal about the estimation of profitability of the firm under their new management, and another about their cost of accommodating excess labour) and analysing both a first-price auction where bidders compete in price and amount of retained labour and another first-price auction where they only compete over price and commit to keep a predetermined number of workers, deriving optimal strategies for both cases. Like Wang and Wang (2007), de Frutos & Pechlivanos (2010) also reached the conclusion that when bidders differ in their costs of accommodating excess labour, the government should appear strong against labour redundancies, but this demands strong commitment power. The authors also found that local firms generally had a comparative advantage for accommodating excess labour and, as such,

without strong commitment ability by the government, foreign firms might shy away from investing.

At last, it is also important to note that there is a wide array of different views within the literature in regard to what the objectives during the privatization process should be. While authors like Maskin (1992) argue that the primary objective in privatization is allocation efficiency, and all other objectives will only bring new distortions, others like Wang et al. (2007) defend that we must look at several possible government objectives when considering privatizations, namely revenue maximization, choosing the investor that will maximize the value of the company and other political factors like social cost.

3.A two-route three-bidder example

In this section, an example will be provided in order to illustrate how a potential multidimensional auction can look like. Given the scarcity of practical examples of firms being privatized simultaneously, an example from an existent procurement situation will be provided first and later, in the appendix, it will be shown how the details can be extrapolated to a privatization situation. The tendering process of London bus routes was the one chosen for this purpose due its significance as an early example of the use of a combinatorial auction format for public procurement and the availability of literature on the topic.

Consider a principal (London Regional Transport), hereafter auctioneer, who wishes to procure the operation of several bus routes from an agent. In their work, Cantillon & Pesendorfer (2004) observe that, during their sample period, 51 pre-qualified operators were able to participate in the auctions and the average auction involved 3.77 routes even though the number of routes auctioned at once varied, from a single route to up to 21 routes. However, for simplicity's sake, suppose only two routes are being auctioned simultaneously and only three bidders are participating in the auction: bidder 1 is only interested in operating one route while bidders 2 and 3 are interested in both routes. The bidders can bid on a single route or submit a package bid for both. Suppose as well that the auctioneer is interested in privatizing both routes, preferably to the same bidder due to the efficiency gains associated: for illustration and for this example only, one could assume a rule where a package bid is chosen if at least one bidder has submitted a bid on a package, such that any single lot bid is disqualified. Assume that bidders 2 and 3 have submitted a package bid while bidder 1 only submitted a bid for one route, meaning that, according to the assumption, the bid of bidder 1 is disqualified.

In the London bus routes tender discounted prices/costs for package bids were observed, attributed to the possibility to share spare vehicles and depot overhead costs and of more efficiently organizing and coordinating working schedules (Cantillon & Pesendorfer, 2004).

In the aforementioned procurement process, it was decided to establish the design of the bus network and all related aspects of the transport supply such as frequencies, bus types, exact routing and cross route coordination at the network level, so the bidders only needed to bid in terms of cost in a combinatorial first-price auction (Cantillon & Pesendorfer, 2004). As such, this dissertation will first present a price-only auction which will then be extrapolated into a hypothetical score auction example. Bids represent the payment required by the bidder to carry out the specific service and all revenue was remitted to London Regional Transport.

For this example, the dissertation will use routes 4 and 210, whose concession was made through a joint package bid in 2017 (Transport for London, n.d.). Table 3.1 shows the submitted bids, which should be seen as the suggested compensation (hereafter cost) of operating a bus route.

Table 3. 1 - Procurement auction bids

	Route 4	**Route 210**
Bidder 1	£4,600,000	
Bidder 2	£8,655,000 (for the package of both routes)	
Bidder 3	£8,172,000 (for the package of both routes)	

As mentioned previously, for this example, a rule where a package bid is chosen if at least one bidder has submitted a bid on a package is assumed, such that any single lot bid is disqualified. Given the submitted bids, it is easy to see that bidder 3 would win the procurement contract: due to the preferential treatment given to package bids, bidder 1 is excluded, and bidder 3's bid presents a lower cost than bidder 2's. In a first-price auction, bidder 3 would win the auction and receive a compensation of £8.172m for the operation of routes 4 and 210.

Now, suppose that Transport for London does not want to establish any fixed conditions on the way the contract was carried out and instead preferred for bidders to submit bids that contained several variables other than price regarding the way the operators pretended to execute the contract. All the information and rules regarding the auction above still apply, except now bidders will have to submit multidimensional bids and the winner will be the one that achieves the highest score instead of the one who presents the lowest cost. Each bid will be evaluated in terms of price, quality of buses and level of driver training, where the quality and driver training should be seen as indexes. For example, the Driver Training categorization of "Medium" takes an index 100 and can represent for instance an average of 10 years of experience, such that "Low" is below index 100 (less than 10 years of experience) and "High" is above index 100 (more than 10 years of experience). Both the sub criteria and scoring system are knowledge available pre-auctioning and therefore signal the preferences of the auctioneer.

Single lot bids will be scored according to the following table:

Table 3. 2 - Scoring system for single lots

Criteria	Weight (%)		Sub criteria for route 4	Sub criteria for route 210	Points
1. Cost	50	1.1	x ≥ £10m	x ≥ £10m	Disqualified
		1.2	£7.5m≤ x < £10m	£7.5m ≤ x <£10m	0
		1.3	£5m< x < 7.5m	£5m < x < £7.5m	10
		1.4	x≤ £5m	x≤ £5m	10-20 (proportional)
		1.5	0	0	20
2. Bus Quality	25	2.1	Low	Low	Disqualified
		2.2	Medium	Medium	10
		2.3	High	High	15
		2.4	Very High	Very High	20
3. Driver training	25	3.1	Low	Low	Disqualified
		3.2	Medium	Medium	10
		3.3	High	High	15
		3.4	Very High	Very High	20

Where Table 3.2 shows the criteria and scores when bidding on a single lot (either routes 4 or 210),Table 3.3 shows the criteria and scores when bidding for a combined contract of both route 4 and 210 (defined as a package), where we have assumed that the cost of running two routes is greater than running one route.

Table 3. 3 - Scoring system for a package

Criteria	Weight (%)		Sub criteria for route 4 + 210	Points
1. Cost	50	1.1	x ≥ £15m	Disqualified
		1.2	£12.5m ≤ x < £15m	0
		1.3	£10m < x < 12.5m	5
		1.4	x≤ £10	10-20 (proportional)
		1.5	0	20
2. Bus Quality	25	2.1	Low	Disqualified
		2.2	Medium	10
		2.3	High	15
		2.4	Very High	20
3. Driver training	25	3.1	Low	Disqualified
		3.2	Medium	10
		3.3	High	15
		3.4	Very High	20

Assume that bidders 2 and 3 have submitted a package bid, meaning that, according to the assumption, the bid of bidder 1 is disqualified. The bids of bidder 2 and 3 follow Table 3.4, where, for example, bidder 2 requests a compensation of £8.655m, but commits itself to deliver a "High" level when it comes to "Bus Quality", and a "Very High" level for "Driver training".

Table 3. 4 - Submitted bids

	Bidder 2	**Bidder 3**
	Route 4 + Route 210	Route 4 + Route 210
1.	£8,655,000	£8,172,000
2.	High	Very High
3.	Very High	Medium

Based on the bids submitted, each of the qualified bidders is given a score. For example, the bid of £8,655m gives a score of 11.345 points, "High" bus quality gives 15 points, and "Very High" driver training gives 20 points. Bidder 2's total score is therefore: 11.345*0.5+15*0.25+20*0.25 = 14.4225.

The scores of bidders 2 and 3 follow on Table 3.5.

Table 3. 5 - Auction Scores

	Bidder 2	**Bidder 3**
	Route 4+Route 210	Route 4+Route 210
1. (50%)	11.345	11.828
2. (25%)	15	20
3. (25%)	20	10
Total	14.4225	13.414

Using a first-score auction, bidder 2 wins the contract, receives a compensation of £8.655m and has to deliver on the bus quality and driver training it committed to in its bid.

As can be seen, in a first-score auction, given these bids, the operation of the two routes would be assigned to bidder 2. This contrasts with the first-price auction shown before, even though the bid price (cost) remained unchanged.

4.The Model

This section describes the proposed model. A government wants to privatize a set of distinct companies/items $K = \{1, ..., k\}$. There is a set of potential risk neutral bidders $N = \{1, ..., n\}$. Bidders are able to bid on any company, however, the buyer never wishes to split a company to more than one bidder.

Losing firms earn reservation profits, normalized at zero.

Each item can be contractualized for different levels of quality, q, with quality being modelled as a one-dimensional attribute for simplicity's sake: in reality, quality will encompass several different key characteristics considered important by the government. Additionally, for each k, each firm knows it cost parameter for providing quality, θ_i^k, as private information, and the distribution of θ is common knowledge. Therefore, each bidder's true cost, c_i^k ,depends on its cost parameter and the quality provided.

In a typical privatization auction, bidders submit a bid in terms of price and quality and government utility is usually dependant on its valuation of contracted quality and on revenue received, while bidder utility is generally considered to be dependent on price paid, quality contracted, the firm´s cost structure for providing quality, and the expected value of the firm under the bidder´s new management according to the optimal business plan.

In this thesis we will consider a new approach that will allow us to transform the complex combinatorial auction at hand into a more manageable know procurement auction. For our model, we will consider a situation where, for each company, k, there is an initial price set by the government to outright buy the company, but this price can be brought down through several quality provision possibilities in a scoring auction, with the government trading off revenue for quality. In this auction, the bidders will submit bids on p (price reduction) and q (quality provided). We can see this situation as analogous to the government setting up a procurement auction where it pays more for more quality of an item.

Two crucial assumptions for our model are that, for all bidders interested in an item, k, none has a cost structure, θ, such that they are better off just paying the initial price and not bidding on quality at the auction, and all have the same expectation of value for the firm under their management.

Consequently, for our modelled situation, we can write both the government utility and the bidder utility in a similar way to a procurement auction.

We also know from Che (1993) that instead of a (p, q) combination, the strategy of each firm can be equivalently described as an (s, q) score-quality combination and that in first and second-score auctions, there is no loss of generality in restricting attention to q when searching for equilibria, since the equilibrium quality bid can be determined independently from the choice of score.

As such, for our analysed setting where the government wants to privatize a set of K companies, it is possible to say that the full description of projects is given by (k, q), where k ϵ K is a single firm and q ϵ Q is the level of quality provided. We can then consider $\Delta \subseteq K \times Q$ as the set of all feasible projects (k, q). This means a bidder $i \epsilon$ N can submit a single-firm bid $b_i(k,q) \epsilon R \; for \; (k,q) \epsilon \Delta$ and/or a package bid b_i(A), where A ϵ Δ and (k, q), (k', q') ϵ A for k $\neq$ k', assuring each company can only be allocated once.

It is assumed that if b_i is defined on all projects in A', then it is also defined on A'. Furthermore, if b_i is defined on the two packages A' and A'', then it is defined as well on A' $\cap$ A''. It is also assumed that $b_i(\emptyset) = 0$.

Let the auctioneer have a utility function $u(\cdot) \epsilon R$, assigning a value to each firm in Δ, and $\mathbf{b} = \{b_1, ..., b_N\}$ be the array of bids of the N potential bidders.

We note that b_i and u are partial functions and defined on a subset of **S.** Using q as a partial function on **S**, we can use that Dom q is its domain.

An allocation is a map $a = (a_1, a_2)$: K $\rightarrow$ (N, Q) $\cup$ {$*$}, where {(k, q) | a_2 (k) =q} $\subset$ Δ. Hence, $a_1(k) = i$ gives that bidder i has been allocated company k that will be delivered on a level of quality q. If $a(k) = *$, then a company k is not allocated. Ω denotes the set of allocations.

An allocation a is feasible if:

1. {(k, q) | a (k) $\neq *$} ϵ $Dom\, u$
2. For each bidder i, {(k ϵ K) | a_1 (k) $= i$} belongs to Dom b_i

Moreover, the value of an allocation a is:

$$u(a) = u(\{k \mid a\,(k) \neq *\}) \tag{1}$$

and

$$b_i(a) = b_i(\{k \mid a_1\,(k) = i\}) \tag{2}$$

Therefore, we have derived the utility function of the auctioneer:

$$U(a; u, b) = u(a) - \sum_{i=1}^{N} b_i(a) \tag{3}$$

This is an equivalent to the utility function of the buyer that Che (1993) reaches in his paper. We have thus, through a reformulation of Greve and Rocha (2020) adapted to the present setup, reworked Che's (1993) model into a framework of package bidding.

In the same vein, we reach the utility of the individual bidder i (also equivalent to the one present in Che (1993)):

$$\pi_i(a) = b_i(a) - c_i(a) \tag{4}$$

where $c_i(a)$ is the true cost for bidder i of the allocation a.

The rest of the analysis follows Che (1993).

5.Policy recommendation

As seen above, in the literature review, while other methods like bargaining have been more popular for conducting privatizations, auctions remain the dominant alternative. Fluck et al. (2007) and Asker & Cantillon (2010) state this preference for inferior alternatives is mostly due to political constraints and individual bias and, as such, it is this dissertation´s recommendation that any government driven by the best interest of its country should opt for an auction for privatizing state-owned enterprises. It has also been shown above that generally government have other concerns in mind in addition to maximizing revenue, that the highest bidder may not be the best shareholder for the state-owned enterprise and that score auctions with an appropriate scoring rule are superior to alternative price-only auction designs, thereby it follows that a score auction should be the preferred privatization method.

As also seen, privatizations close together in time of firms that exhibit complementarities with each other are not that uncommon and, as Maskin (1992) and Klemperer (1999) state, complementary items should be auctioned simultaneously in an auction in which a buyer can place bids in one or more package of items, so using a score auction that allows for package bidding is the most obvious and best choice for governments in this situation.

The auction process should be conducted as transparently as possible and the scoring rule should be disclosed before the auction begins, reducing uncertainty (Allen et al.,2019; Takahashi, 2018).

As identified previously, government commitment power is very important: Che (1993) shows that, in the absence of commitment power by the government, the naive scoring rule, corresponding to the government's true preferences, will be used, with the winning bidder producing an excessive level of quality. With commitment power, the government should commit to the optimal scoring rule which systematically discriminates against quality: as Che (1993) shows, the optimal scoring rule that implements the optimal outcome, given by a revelation mechanism, distorts quality down to limit the information rents accruing to relatively efficient firms. Thus, in this scenario of strong commitment power, the government should use a scoring rule that reflects its true utility plus an additional term[1] identified by Che (1993) that

[1] This term has the following form in Che (1993): $\Delta(q) = \int_k^q \frac{F(q_0^{-1}(s))}{f(q_0^{-1}(s)} \, c_{q\theta}\left(s, q_0^{-1}(s)\right) ds$, where q represents the quality, s represents the score, k represents any arbitrary real number, $\frac{F(\cdot)}{f(\cdot)}$ represents the hazard rate, θ represents the bidder's private information cost parameter, q_o represents the optimal quality and c represents the bidder's cost.

distorts down its valuation of quality. First and second-score auctions can implement the optimal outcome.

While in the Department of Defence scenario analysed by Che strong commitment power is highly unlikely, I believe it is a reasonable assumption for most privatizations if the state acts with the nation´s best interests in mind, where clear quality objectives and penalties for breaching the contract can be set out for most quality components. Besides, for most of these components, verification of compliance also should not be very hard: for example, it is fairly easy for the government to know how many workers were laid off or how many new factories or stores were established.

Summarizing, when privatizing state-owned enterprises that manifest complementarities, governments should use a first or second-score auction that allows for package bidding, announce the scoring rule *a priori*, maximize transparency and commit to one of the scoring rules mentioned above depending on their commitment power. Under strong commitment power, the optimal scoring rule distorts down quality.

As outlined previously, this dissertation´s results, and therefore this recommendation, can also be extended to most items exhibiting complementarities that the government intends to sell.

6. Concluding remarks

This thesis' main goal was to help optimize the privatization of firms who display complementarities, which is particularly useful in periods of big privatization waves. It accomplishes this by managing to show that Che's (1993) results can be extended to this setting and that a multidimensional auction with a scoring rule that systematically discriminates against quality achieves the optimal outcome in a unidimensional private information and strong commitment power scenario, just like in the author's paper. An important gap in the literature in terms of how to best handle these specific situations is therefore being filled.

Additionally, this thesis´ results can also be extended to most items that the government intends to sell and are not limited to privatizations, although they are the main focus throughout.

We have also showed that it is not unusual for firms displaying complementarities to be privatized within a short time frame of one another in periods of a big privatization movement and that, historically, governments have many times given up revenue on auctions in favour of other factors considered important.

One potential problem with implementing this dissertation's suggestions is that, in general, developing countries go through more privatizations than developed ones but as seen in this thesis, developed capital markets and income per capita are two of the main factors leading to the choosing of auctions as the method of privatization. In the absence of such conditions, a subpar method being chosen is much more likely, with the political constraints present in many developing countries making such a choice even more likely.

Further research could focus on different types of information asymmetry and its impacts or analyse how a similar scenario would play out under multidimensional private information.

Another possible avenue of research would be exploring how this work could be extended to scenarios of potential monopolies, eventually through the use of only partial privatization for some of the firms involved.

References

Allen, J., Clark, R., Hickman, B. R., & Richert, E. (2019). Resolving failed banks: Uncertainty, multiple bidding & auction design. Queen's Economics Department Working Papers, 1422;

Asker, J., & Cantillon, E. (2008). Properties of Scoring Auctions. *The RAND Journal of Economics*, *39*(1), 69–85;

Asker, J., & Cantillon, E. (2010). Procurement when price and quality matter. *The RAND Journal of Economics*, *41*(1), 1–34;

Ayres, I., & Cramton, P. (1996). Deficit reduction through diversity: How affirmative action at the FCC increased auction competition. *Stanford Law Review*, 48, 761-815;

Bae, J., & Kagel, J. H. (2021) Selling Shares to Budget-Constrained Bidders: An Experimental Study of the Proportional Auction. *Journal of the Economic Science Association*, 8, 45-55;

Bang, J. T. (2007). Determinants of the method of sale in privatization. *Economic Systems*, *31*(3), 272–291;

Bichler, M. (2000). An experimental analysis of multi-attribute auctions. *Decision support systems*, 29(3), 249-268;

Branco, F. (1997a). The Design of Multidimensional Auctions. *The RAND Journal of Economics*, *28*(1), 63–81;

Branco, F. (1997b). Sequential auctions with synergies: An example. *Economics Letters*, 54(2), 159–163;

Brusco, S., & Lopomo, G. (2009). Simultaneous ascending auctions with complementarities and known budget constraints. *Economic Theory*, 38(1), 105–124;

Cantillon, E., & Pesendorfer, M. (2004). Chapter 22 Auctioning Bus Routes: The London Experience. in Peter Cramton, Yoav Shoham, and Richard Steinberg (eds), *Combinatorial Auctions* (pp. 573–592). The MIT Press;

Carlin, W., & Mayer, C. (1994). The Treuhandanstalt: Privatization by state and market. In O. Blanchard, K. Froot & J. Sachs (Ed.), *The Transition in Eastern Europe, Volume 2: Restructuring* (pp. 189-214). Chicago: University of Chicago Press;

Che, Y.-K. (1993). Design Competition Through Multidimensional Auctions. *The RAND Journal of Economics*, *24*(4), 668–680;

Chen, Z. (2021a). The multidimensional procurement auctions with reference-based utility. Managerial and Decision Economics, 42(2), 319-325;

Chen, Z. (2021b). The bid orchestration and competitions in scoring procurement auctions. *Managerial and Decision Economics*, 42(7), 1718-1729;

Chen-Ritzo, C.-H., Harrison, T. P., Kwasnica, A. M., & Thomas, D. J. (2005). Better, Faster, Cheaper: An Experimental Analysis of a Multiattribute Reverse Auction Mechanism with Restricted Information Feedback. *Management Science*, 51(12), 1753-1762;

Cornelli, F., & Li, D. D. (1997). Large Shareholders, Private Benefits of Control, and Optimal Schemes of Privatization. *The RAND Journal of Economics*, *28*(4), 585–604;

Dasgupta, S., & Spulber, D. F. (1989). Managing procurement auctions. *Information Economics and Policy*, 4(1), 5–29;

de Frutos, M. A., & Pechlivanos, L. (2010). Auctioning Off with a Split Mind: Privatization Under Political Constraints. *Social Science Research Network Electronic Journal*;

Di Corato, L., Dosi, C., & Moretto, M. (2018). Multidimensional auctions for long-term procurement contracts with early-exit options: The case of conservation contracts. *European Journal of Operational Research*, *267*(1), 368–380;

Elmaghraby, W. J. (2005). Multi-unit auctions with complementarities: Issues of efficiency in electricity auctions. *European Journal of Operational Research*, 166(2), 430–448;

Fang, H., & Parreiras, S. O. (2002). Equilibrium of Affiliated Value Second Price Auctions with Financially Constrained Bidders: The Two-Bidder Case. *Games and Economic Behavior*, 39(2), 215–236;
Fluck, Z., John, K., & Ravid, S. A. (2007). Privatization as an agency problem: Auctions versus private negotiations. *Journal of Banking & Finance*, *31*(9), 2730–2750;
Greve, T., & Rocha, M. (2020). Policy and theoretical implications of the zero-subsidy bids in the German offshore wind tenders. *The Energy Journal*, 41(4), 89-104;
Irdam, D., Scheiring, G., & King, L. (2015). Mass Privatization. In J. Hölscher & H. Tomann (Eds.), *Palgrave Dictionary of Emerging Markets and Transition Economics* (pp. 488–507). Palgrave Macmillan UK;
Jehiel, P., Moldovanu, B., & Stacchetti, E. (1999). Multidimensional Mechanism Design for Auctions with Externalities. *Journal of Economic Theory*, *85*(2), 258–293;
Laffont, J.-J., Perrigne, I., Simioni, M., & Vuong, Q. (2020). Econometrics of Scoring Auctions. In T. Li, M. H. Pesaran, & D. Terrell (Eds.), *Advances in Econometrics* (pp. 287–322). Emerald Publishing Limited;
Laffont, J.-J., & Tirole, J. (1987). Auctioning Incentive Contracts. *Journal of Political Economy*, 95(5), 921–937;
López-de-Silanes, F. (1997). Determinants of privatization prices. *The Quarterly Journal of Economics*, 112(4), 965-1025;
Lu, X., & McAfee, R. P. (1996). The Evolutionary Stability of Auctions over Bargaining. *Games and Economic Behavior*, 15(2), 228–254;
Kawasaki, K., Fujie, T., Koito, K., Inoue, N., & Sasaki, H. (2012). Conservation Auctions and Compliance: Theory and Evidence from Laboratory Experiments. *Environmental and Resource Economics,* 52(2), 157–179;
Kersten, G. E., Szapiro, T., & Wu, S. (2022). A Procedure for Multiattribute Reverse Auctions with two Strategic Parameters. In: Szapiro, T., Kacprzyk, J. (eds) *Collective Decisions: Theory, Algorithms And Decision Support Systems. Studies in Systems, Decision and Control* (pp. 53-81)*,* Springer;
Klemperer, P., (1999). Auction Theory: A Guide to the Literature. *Journal of Economic Surveys*, 13(3), 227-286;
Kong, Y., Perrigne, I., & Vuong, Q. (2021) Multidimensional Auctions of Contracts: An Empirical Analysis. *American Economic Review,* 112(5), 1703-1736;
Kultti, K. (1999). Equivalence of Auctions and Posted Prices. *Games and Economic Behavior*, *27*(1), 106–113;
Maskin, E. (1992). Auctions and Privatization In Siebert H (eds) *Privatization* (pp. 115-136), J.C.B. Mohr Publisher;
Maskin, E., & Riley, J.G. (1989) Optimal Multi-Unit Auctions. In *The Economics of Missing Markets, Information, and Games* (pp.312-335), Oxford University Press;
Matteucci, N. (2021). Procuring NGA infrastructure: The performance of EMAT auctions in Italy. *Telecommunications Policy*, 45(1), 1020-1074;
Muller, R., Perea, A., & Wolf, S. (2007). Combinatorial Scoring Auctions. Maastricht Research School of Economics of Technology and Organization Research Memorandum, 20;
Nakabayashi, J. (2013). Small business set-asides in procurement auctions: An empirical analysis. Journal of Public Economics, 100, 28–44;
Perotti, E. C. (1995). Credible privatization. The American economic review, 85(4) 847-859;
Quiroga, B. F. (2021). Nonparametric identification and estimation of score auctions in multi-attribute procurement. *Operations Research Letters*, 49(5), 748-751;
Riley, J. G., & Samuelson, W. F. (1981). Optimal auctions. *The American Economic Review*, 71(3), 381-392;

Sant'Anna, M. (2018). Empirical Analysis of Scoring Auctions for Oil and Gas Leases. *FGV EPGE Economics Working Papers,*789;
Shu, H., & Durango-Cohen, P. L. (2021). On the design of optimal auctions for road concessions: Firm selection, government payments, toll and capacity schedules with imperfect information. *Transportation Research Part B: Methodological*, 146, 210–236;
Suyama, T. (2005). Strategy/False-name Proof Protocols for Combinatorial Multi-Attribute Procurement Auction. In: Deng, X., Ye, Y. (eds) *Internet and Network Economics. WINE* (pp. 278-287), Springer;
Takahashi, H. (2018). Strategic design under uncertain evaluations: Structural analysis of design-build auctions. *The RAND Journal of Economics*, 49(3), 594–618;
Transport for London. Bus tender search - Transport for London. Tfl.gov.uk. Retrieved 10 July 2022, from https://tfl.gov.uk/forms/13923.aspx;
Xia, M., Stallaert, J., & Whinston, A. B. (2005). Solving the combinatorial double auction problem. *European Journal of Operational Research*, 164(1), 239–251;
Xiao, F. (2021). Efficient and truthful multi-attribute auctions for crowdsourced delivery. *International Journal of Production Economics*, 240;
Yoganarasimhan, H. (2015). Estimation of Beauty Contest Auctions. *Marketing Science*,35(1), 27-54;
Wang, H. & Wang, J., (2007). How to Protect the Employees' Benefits in the SOE Auction in China. *International Journal of Business and Management*, 2(5), 90-96;
Wang, H., Zhang, W., & Wang, J. (2007). Auctioning the state owned enterprise in China: The trade-off between maximizing revenue and minimizing unemployment. *Economic Change and Restructuring*, *40*(3), 267–280;
Wu, X., Li, S., & Chen, K.-Y. (2022). Multidimensional Procurement Auction with Loss-Averse Workers in Online Labor Markets. *Social Science Research Network Electronic Journal*.

Appendix A-A two-firm three-bidder example

While it has not been a popular practice to this day, it is easy to see how a similar auction as the one presented in section 3 could be run by a government desiring to privatize two companies who present synergies. Therefore, the example that follows represents a possible hypothetical conversion of the section 3 example based on the theoretical model presented in section 4. We will keep the analysis to a two-item scenario both for simplicity's sake and because it makes practical sense (situations where the government is privatizing more than two synergistic companies at around the same time are not very common). The privatization of Rolls-Royce (1987) is perhaps the best company to use for an example since, given its important business in terms of both car manufacturing and airplane engine manufacturing, it presents undeniable synergies with two companies privatized in the same period: Jaguar (1984) and British Aerospace (1981 and 1985). Both would be suitable for the next example, but this dissertation will use Jaguar (and Rolls-Royce) for this hypothetical scenario.

A value of £2,000,000,000 is set out as the price for which the government is willing to sell the package of both companies without any quality restrictions. However, consistently with our model's assumptions, no bidder is better off just paying that price and not participating in the auction, so the firms are sold through auction and all bidders make a bid on price and quality.

Suppose once again that only two items (companies) are being auctioned simultaneously, bidders can bid on a single company or submit a package bid for both and the auctioneer still has the same preference for selling both, disregarding all other offers in the presence of package bids. Once more, only three bidders are participating in the auction, with one of them interested in just one company and the others interested in both. In this case, bid price represents government revenue so, unlike in the previous example where it represented cost, a higher value is preferred.

Suppose this time around, bids are evaluated in terms of price, retained labour compromise and further investment in new factories in the UK. Just like in the previous example, the sub criteria and scoring system are knowledge available pre-auctioning.

Single lot bids will be scored following the evaluation method laid out in table A.1 below, while the scoring method for package bids on both firms can be observed in table A.2.

Table A. 1 - Scoring system for single lots in privatization auction

Criteria	Weight (%)		Sub criteria for Rolls-Royce	Sub criteria for Jaguar	Points
1. Price	40	1.1	x < £900m	x < £250m	Disqualified
		1.2	£900m ≤ x < £1,000m	£250m ≤ x < £300m	0
		1.3	£1,000m ≤ x < £1,100m	£300m ≤ x < £400m	10-15 (proportional)
		1.4	x ≥£1,100m	x ≥£400m	20
2. Retained labour	30	2.1	y < 90%	y < 90%	Disqualified
		2.2	90% ≤ y < 95%	90% ≤ y < 95%	10
		2.3	95% ≤ y < 100%	95% ≤ y < 100%	15
		2.4	y=100%	y=100%	20
3. Further investment	30	3.1	z < £200m	z < £100m	10
		3.2	£200m ≤ z < £300m	£100m ≤ z < £150m	15
		3.3	z ≥£300m	z ≥£150m	20

Table A. 2 - Scoring system for a package in privatization auction

Criteria	Weight (%)		Sub criteria for Rolls-Royce + Jaguar	Points
1. Price	40	1.1	x < £1,100m	Disqualified
		1.2	£1,100m ≤ x < £1,300m	0
		1.3	£1,300m ≤ x < £1,400m	10-15 (proportional)
		1.4	x ≥£1,400m	20
2. Retained labour	30	2.1	y < 90%	Disqualified
		2.2	90% ≤ y < 95%	10
		2.3	95% ≤ y < 100%	15
		2.4	y=100%	20
3. Further investment	30	3.1	z < £150m	10
		3.2	£150m ≤ z < £250m	15
		3.3	z ≥£250m	20

The significant gap between the price value that awards maximum points (£1,400,000,000) and the initial value for the sale without quality restrictions (£2,000,000,000) illustrates how highly the government values the ability to set out quality obligations.

Given this scoring mechanism, the bidders submit their bids, which are exemplified in table A3 as follows, with the resulting scores in table A.4 directly below it. Bidder 1 only submitted a bid for one company hence, due to the presence of package bids in the auctions, its bid will not be considered and thus will be absent from tables A.3 and A.4.

Table A. 3 - Submitted Privatization Bids

	Bidder 2	**Bidder 3**
	Rolls-Royce + Jaguar	Rolls-Royce + Jaguar
1.	1.3 £1,300,000,000	1.4 £1,400,000,000
2.	2.4 100%	2.3 95%
3.	3.3 £250,000,000	3.2 £150,000,000

Table A. 4 - Privatization Scores

	Bidder 2	**Bidder 3**
	Rolls-Royce + Jaguar	Rolls-Royce + Jaguar
1. (40%)	10	20
2. (30%)	20	15
3. (30%)	20	15
Total	16	17

As can be observed in table A4, in a first-score auction, bidder 3 wins both firms, paying £1,400,000,000 and committing to investing a further £150,000,000 in the companies' UK production activities and retaining 95% of the current workforce.