



A refracted process in options: A credit valuation application

Andrew Clare^a, Carlos Manuel Pinheiro^{b,c,d,e} ^{*}, Alberto Franco Pozzolo^{f,g} , João Miguel Reis^{b,c}

^a Bayes Business School, City St Georges, London, United Kingdom

^b Universidade Europeia, Portugal

^c Instituto Universitário de Lisboa (ISCTE-IUL), Business Research Unit (BRU-IUL), Portugal

^d ISCAL - Lisbon Accounting and Business School, Portugal

^e Center for Transdisciplinary Development Studies (CETRAD), Portugal

^f Roma Tre University, Italy

^g Centro Studi Luca d'Angliano, Italy

ARTICLE INFO

Keywords:

Options
Regime-shift
Loans
Bonds

ABSTRACT

Borrowing from the principle of refraction in optics, we develop an option pricing model in which the underlying asset's process changes upon touching a barrier, alongside a structural credit risk model. To achieve this, we extend the Black–Scholes and Merton models to incorporate regime shifts when a barrier is crossed. Our credit model demonstrates that if a drop in firm value below a certain threshold triggers a structural change in its policies — that in our framework is captured by a shift in the model governing its time evolution — this affects the initial value of its loans. As the policy change alters the underlying asset process, loan values decrease, and credit spreads widen. Our findings underscore the importance of accounting for regime shifts and their impact on loan pricing in dynamic market conditions.

1. Introduction

Our study presents a novel approach on how regime-shifts influence loan valuation, by exploring a barrier option framework and refracted processes, a concept borrowed from physics.¹ Just as light bends when passing through mediums with varying characteristics, resulting in a refracted process, we assume that the parameters of the underlying asset process change when crossing a barrier. This introduces a regime-shift which is triggered when the value of the underlying asset (the firm value) crosses a barrier.

The Black and Scholes (1973) model is the fundamental framework for pricing options in financial markets. Merton (1974) extends the approach to value a firm, especially its liabilities.

The Black–Scholes framework assumes that the price of a financial asset follows a geometric Brownian motion (GBM) process, derived from two components: a deterministic drift, and a stochastic component, doing so with constant parameters over time. However, markets can switch between different regimes and exhibit varying volatilities (as in Hull, 2018).

Academics have addressed various assumption violations, particularly the mismatch between assumed normal distributions of returns and the skewness in real-world data (see, for example, Peiro, 1999

and Hull et al., 2005). For instance, in terms of credit valuation, Jang et al. (2016), introduce stochastic volatility as in Heston (1993).

Regime-shifting models have gained traction, notably due to the framework of Hamilton (1989), which introduces the possibility that key parameters can vary over time. Recent studies such as He and Zhu (2016) incorporate stochastic volatility. In a similar vein, Costabile et al. (2014) develop jump-diffusion models. Meenagh et al. (2007) test market efficiency while assuming different regime-switching regimes and Kirby (2023) explores skewness in asset prices between bull and bear markets. Our approach differs from these by pointing to the crossing of a barrier by the underlying asset value as the trigger to the regime-switch.

Our results show that higher barrier values entail greater possibilities of regime-shifts and reduce loan values, whilst increasing the respective spreads.

We begin by laying down the formal model of the refracted process. Section 3 extends the Merton model introducing a refracted effect. Section 4 conducts a numerical analysis. Section 5 concludes.

2. Options with refracted process

A refracted process is a stochastic model where a path changes its dynamics upon reaching a threshold, similar to how light bends when

^{*} Corresponding author at: Universidade Europeia, Portugal.

E-mail address: carlos.manuel.pinheiro@universidadeeuropeia.pt (C.M. Pinheiro).

¹ We thank Zbigniew Palmowski for the suggestion of this framing of our research question.

passing through different mediums in optics. In particular, we assume that it is the process of the underlying asset that changes when a barrier is crossed, and so we denote the process as “Refracted”. This refraction occurs through a change in the volatility of the underlying asset and/or in the components of the drift.

We depart from the Black–Scholes model with dividends for pricing options. In this regime-change, upon crossing a barrier, the process of the underlying asset, V , is changed.² The instantaneous change can occur in one or more of the parameters and each of these is piecewise continuous. Barrier options, being path-dependent derivatives, are particularly well-suited to frame the proposed regime-shift.

Consider an option framework and define r as the risk-free rate, q as the dividend yield, σ as the volatility of the underlying asset, and T as the maturity. For one regime-shift, we can define the rate of change in V_t as:

$$\frac{dV_t}{V_t} := \begin{cases} (r_1 - q_1)dt + \sigma_1 dW_V(t) & \text{if } t < \tau_i \\ (r_2 - q_2)dt + \sigma_2 dW_V(t) & \text{if } t \geq \tau_i \end{cases} \quad (1)$$

where (r_1, q_1, σ_1) is the set of asset's process parameters in the first regime and (r_2, q_2, σ_2) is the set in the second regime. $W_V(t)$ is the Brownian motion, τ_i is the time when the regime-shift happens, as the underlying asset crosses a barrier, and i represents either the lower ($i = L$) or upper barrier ($i = U$). The following defines the barrier hitting times.

Definition 1. For barrier option contracts, we define

$$\tau_L := \inf \{u > t_0 : V_u = L\}, \quad (2)$$

as the first hitting time of the lower barrier L , by the asset price V_u , for $u \in [0, T]$ and

$$\tau_U := \inf \{u > t_0 : V_u = U\}, \quad (3)$$

as the first hitting time of the upper barrier U , by the asset price V_u , also for $u \in [0, T]$.

Allowing for the possibility of a regime-shift upon crossing a barrier B , and considering that the barrier might or not be crossed before T , the price of a European-style option is given by P_{rc} :

$$\begin{aligned} P_{rc}(V_0, K, B, T; \phi) &= \mathbb{E}_Q \left[e^{-\int_0^T r(l)dl} (\phi K - \phi V_T)^+ \left(\mathbb{1}_{\{\tau_i > T\}} + \mathbb{1}_{\{\tau_i \leq T\}} \right) \mid \mathcal{F}_0 \right] \\ &= \mathbb{E}_Q \left[e^{-\int_0^T r(l)dl} (\phi K - \phi V_T)^+ \mathbb{1}_{\{\tau_i > T\}} + e^{-\int_0^T r(l)dl} \right. \\ &\quad \times (\phi K - \phi V_T)^+ \mathbb{1}_{\{\tau_i \leq T\}} \mid \mathcal{F}_0 \Big] \end{aligned} \quad (4)$$

wherein $\phi = 1$ corresponds to a put option and $\phi = -1$ to a call option. This expression can be split into two parts:

$$\begin{aligned} P_{rc}(V_0, K, B, T; \phi) &= \\ &= \underbrace{\mathbb{E}_Q \left[e^{-\int_0^T r(l)dl} (\phi K - \phi V_T)^+ \mathbb{1}_{\{\tau_i > T\}} \mid \mathcal{F}_0 \right]}_{P_{ko}} \\ &\quad + \underbrace{\mathbb{E}_Q \left[e^{-\int_0^T r(l)dl} (\phi K - \phi V_T)^+ \mathbb{1}_{\{\tau_i \leq T\}} \mid \mathcal{F}_0 \right]}_{P_{ki}}. \end{aligned} \quad (5)$$

The first component, P_{ko} , a knock-out option, is always ruled by the initial regime, being deactivated when the barrier is crossed.³

The second component,

$$P_{ki}(V_0, K, B, T; \phi) = \mathbb{E}_Q \left[e^{-\int_0^T r(l)dl} (\phi K - \phi V_T)^+ \mathbb{1}_{\{\tau_i \leq T\}} \mid \mathcal{F}_0 \right], \quad (6)$$

² Here we use V for the asset “spot” price, given our focus on the firm value.

³ Its well known closed-formula can be found in Hull (2021).

is also a barrier option, knock-in, ruled by different regime settings in its process. It is ruled by the first regime until the barrier is crossed, but upon doing so, a vanilla option is activated, with the initial spot price for the underlying asset coinciding with the value of the barrier. The second regime rules the underlying asset process until the maturity date.

Thus, by using a Stopping Time (ST) approach, we can separate the two regimes. The first is used until the hitting of the barrier and the second afterwards. In order to obtain the values for Eq. (6), we use (Dias et al., 2015, Proposition 3.1), with $\Psi(V_T) = (\phi K - \phi V_T)^+$, considering the pure diffusion process without the possibility of hitting the absorbing barrier at zero.⁴ Thus:

$$\begin{aligned} P_{ki} &= \mathbb{E}_Q \left[e^{-\int_0^T r(l)dl} (K - V_T)^+ \mathbb{1}_{\{\tau_i \leq T\}} \mid \mathcal{F}_0 \right] \\ &= \int_0^T e^{-\int_0^u r(l)dl} P^{(2)}(B(u), K, T, \phi) Q^{(1)}(\tau_i \in du \mid \mathcal{F}_0), \end{aligned} \quad (7)$$

where the $j = \{1, 2\}$ superscript in $P^{(j)}$ and $Q^{(j)}$, allows us to identify which regime from Eq. (1) is used, $Q^{(1)}(\tau_i \in du \mid \mathcal{F}_0)$ represents the passage time density of τ_i , $P^{(2)}(B(u), K, T, \phi)$ is a vanilla option and $B(u)$ is the value of the barrier crossed at τ_i . This separates the two regimes: the first regime lasts until the barrier is crossed, and the second regime begins thereafter. By calculating the probabilities of crossing a given barrier separately, the ST approach distinguishes between the V_i regimes shown in Eq. (1).

Fig. 1 presents four simulations of the refracted process described by Eq. (1) for a stock-price option assuming a barrier at 80.

While in Panel 1 the volatility of the stock price is governed by a unique process and remains roughly constant through time, in all other cases it changes significantly after crossing the threshold, showing clear path-dependency.

Overall, an option which changes regime when a barrier is breached can be decomposed into two barrier options: a knock-out option and knock-in option. In particular, in the latter case, the process of the underlying asset is ruled by the first regime before the barrier is crossed, and by the second regime afterwards.

3. A Merton model application

In the Merton model, the face value of a firm's debt is represented by D , maturing at date T . The parameter r stands for the risk-free rate, and q represents the firm's dividend yield. As for the underlying asset, V , it represents the firm value — the sum of the equity and the market value of the debt on the firm. We represent a loan taken by a firm as a zero-coupon bond. In the case of default, when the firm value is below the face value of debt at maturity, $V_T < D$, the remainder of the firm's value V_T is appropriated by the lender. In the original Merton model V follows the (risk-neutral) GBM.

We express the discounted value of the loan as $G(V_0, D, T)$, as

$$B(V_0, D, T) = D e^{-rT} - G(V_0, D, T) \quad (8)$$

where it is represented by a put option, $G(V_0, D, T) = P(V_0, D, T; 1)$.

Here we assume that V follows a refracted process, as described in Eq. (1), modeling a scenario where the firm changes its policy, represented by a change in its diffusion process, upon crossing a lower barrier. While the firm value path is what triggers the regime-shifts, the value (barrier) at which this regime-shift occurs is exogenous, defined a priori. In other words, the firm changes its policy when an exogenous barrier is hit.

⁴ The ST approach developed by Dias et al. (2015) also includes the Constant Elasticity of Variance (CEV) model and the Jump to Default CEV (JDCEV) model of Carr and Linetsky (2006).

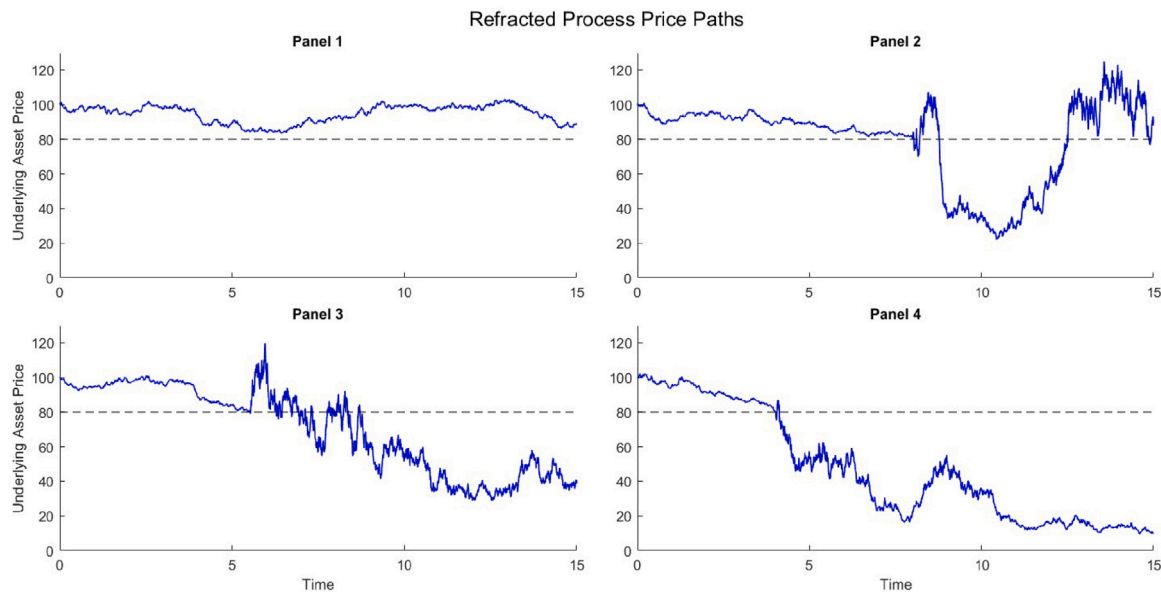


Fig. 1. Simulated possibilities for the refracted process for the underlying asset. Initial asset price of 100 and barrier of 80.

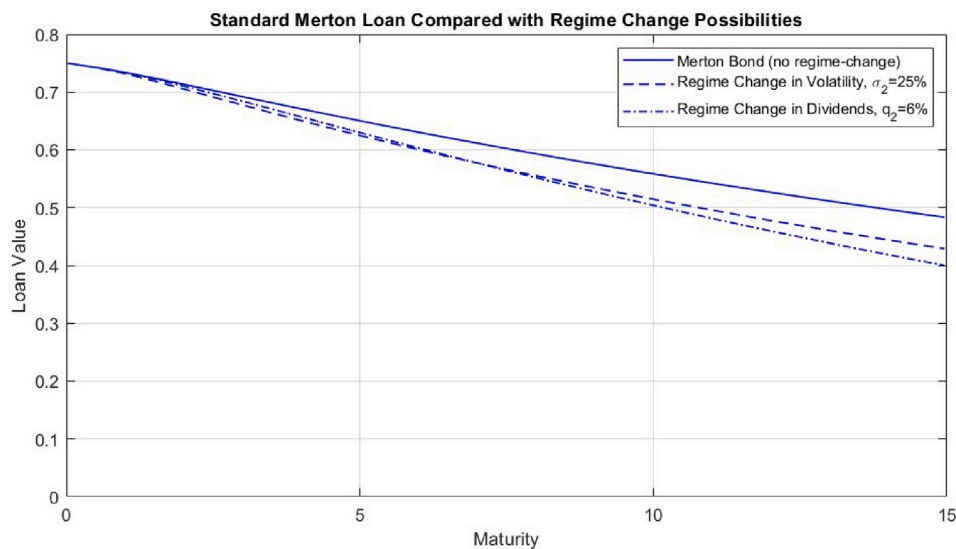


Fig. 2. Risk-shifting from $\sigma_2 = 15\%$ to $\sigma_2 = 25\%$ and from $q_2 = 2\%$ to $q_2 = 6\%$.

For instance, in the case of volatility, the observed skewness can be modeled as a one-time increase when the firm crosses a lower barrier. We can also model increases in dividends to shareholders, with a one time augmentation.⁵

By defining the discount value of the loan as a put option, as outlined in Eq. (5), we are able to replicate various scenarios where the firm's policy changes upon crossing a lower barrier, thereby affecting the value of the loan. So, the loan value under a refracted process is written as

$$B_{rc}(V_0, D, T) = De^{-rT} - P_{rc}(V_0, D, B, T; 1). \quad (9)$$

⁵ Acharya et al. (2022) highlight the case of banking during the 2007–2008 financial crisis, where distressed banks increased dividend payments.

4. Numerical analysis

We compute loan values by incorporating the refracted process in the firm's value and compare it with the baseline Merton model. Upon crossing a lower barrier, the volatility and dividends are increased.

As base parameters, we set the initial value of the firm, V_0 , to 1 and the face value of the loan, D is 0.75. The barrier triggering the regime-shift, L , is set at 80% of the firm's initial value, $L = 0.80$. The risk-free rate, r is 2% and the (initial) dividend yield, q_1 , is 2%. The initial volatility, σ_1 , is 15%. Maturity values will extend to 15 years, $T = 15$.

In Fig. 2, we compare three scenarios. The first uses the classical Merton model (no regime-shift); in the second scenario, volatility increases to $\sigma_2 = 25\%$ upon crossing the barrier; in the third, the dividend yield increases to $q_2 = 6\%$. The changes, as percentage points, are denoted as $\Delta\sigma = 10pp$ and $\Delta q = 4pp$.

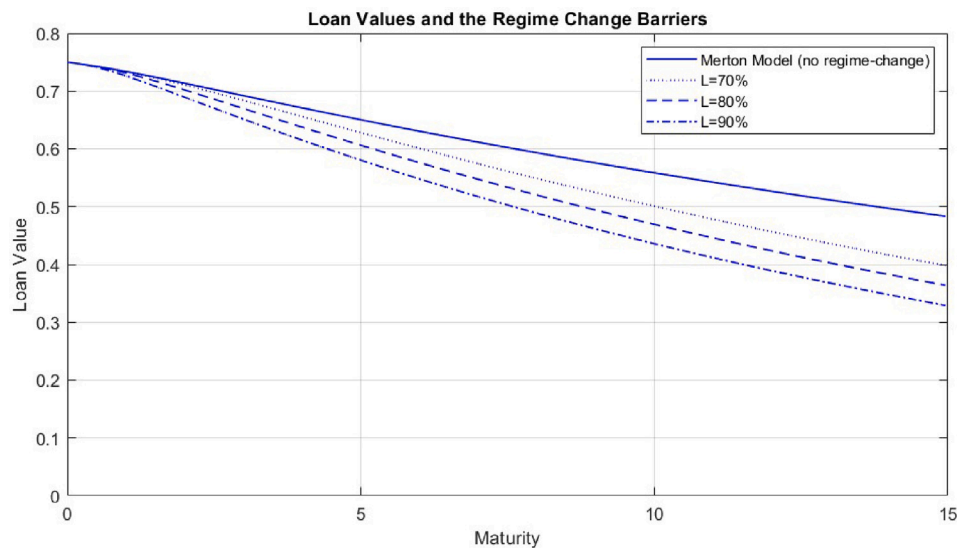


Fig. 3. Risk-shifting in both σ and q , from $\sigma_1 = 15\%$ and $q_1 = 2\%$ to $\sigma_2 = 25\%$ and $q_2 = 6\%$.

Table 1
Spreads values for loans with the possibility of risk-shifting for different parameters.

	L	$\Delta\sigma = 10pp$ $\Delta q = 0pp$	$\Delta\sigma = 0pp$ $\Delta q = 4pp$	$\Delta\sigma = 5pp$ $\Delta q = 2pp$	$\Delta\sigma = 10pp$ $\Delta q = 2pp$	$\Delta\sigma = 5pp$ $\Delta q = 4pp$	$\Delta\sigma = 10pp$ $\Delta q = 4pp$
$\sigma_1 = 15pp$	0.90	1.9816	2.5687	2.1482	2.6833	3.0103	3.4976
	0.80	1.7318	2.1980	1.8701	2.2519	2.4961	2.8285
	0.70	1.4821	1.8497	1.5986	1.8451	2.0259	2.2304
$\sigma_1 = 0.25$	0.90	3.5615	4.0210	3.6968	4.3729	4.6278	5.2730
	0.80	3.3611	3.7863	3.4919	4.0464	4.2720	4.7862
	0.70	3.1469	3.5407	3.2749	3.7082	3.9079	4.2977
$\sigma_1 = 0.35$	0.90	5.3137	5.6020	5.3826	6.1764	6.3359	7.1077
	0.80	5.1407	5.4314	5.2174	5.9071	6.0615	6.7192
	0.70	4.9517	5.2465	5.0374	5.6187	5.7692	6.3121

When $T = 10$, the loan value decreases from 0.56 to 0.51 when we admit the possibility of the volatility increase. When the dividends face the possibility of being augmented, the value of the loan declines to 0.50. In sum, the regime-shift potential reduces the market value of the loan.

In Fig. 3, we compare the baseline Merton case with different parameters for the barrier that triggers the regime-shift. In this scenario, when the regime changes, we combine the cases where $\Delta\sigma = 10pp$ and $\Delta q = 4pp$.

At $T = 10$, without the possibility of a regime-shift, the loan value is 0.56 again. When we admit that regime-shift can occur if the firm value crosses 70% of the initial firm value, the loan value is 0.50. In cases when the regime-shift becomes more likely to occur, only needing to cross the lower barrier at 80% and 90%, the value of the loans decrease to, respectively, 0.47 and 0.44. Overall, higher barrier values lead to greater reductions in the loan values, as smaller decreases in the firm value trigger regime-changes.

Table 1 presents the loan spreads, placing the refracted process loan as the evaluated asset:

$$S = -\frac{1}{T} \log\left(\frac{B_{rs}(V_0, D, L, T)}{De^{-r(T)}}\right).$$

As expected, when the initial volatility, σ_1 , and regime-shift parameters, $\Delta\sigma$ and Δq , increase, the spreads on the loans increase. As L decreases, the spreads decline.

5. Conclusions

We have developed a method to evaluate option values which incorporates changes in process parameters when a barrier is crossed.

Applying this framework to a Structural Credit Risk Model, we show that crossing a lower barrier leads to a decrease in loan value. This means a higher risk for the lender and translates into higher credit spreads on the company's debt.

Overall, our results highlight the importance of accounting for regime shifts in loan pricing in dynamic market conditions, which may introduce significant non-linearities into the firm's cost of capital.

Data availability

No data was used for the research described in the article.

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