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Portfolio Risk Management Through Value-at-Risk (VaR) Measurement

Margarida Silva Fialho

Master in Finance

Supervisor:
PhD, António Manuel Rodrigues Guerra Barbosa, Assistant Professor, ISCTE-IUL

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BUSINESS
SCHOOL

Department of Finance

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Resumo

O Value-at-risk (VaR) é a métrica padrão utilizada no sector financeiro para medir o risco, estimando a perda máxima esperada decorrente das flutuações do mercado, ao longo de um determinado horizonte temporal e para um dado nível de significância. O VaR serve para avaliar o capital em risco associado às atividades de investimento, estando associado ao conceito de Capital Económico (EC). Ao definir um valor máximo para o EC, os decisores podem impor limites de risco que estejam em conformidade com os seus objetivos estratégicos. Neste estudo, o VaR de uma carteira diversificada — composta por ações de Hong Kong, Japão, Europa e EUA, bem como por obrigações da Europa e dos EUA — é medido e gerido de forma a garantir que permanece dentro do EC definido. Dada a diversidade de modelos existentes para o cálculo do VaR, são avaliados, através de *backtesting*, dezanove modelos distintos, com o objetivo de determinar o seu desempenho e selecionar o mais adequado com base nas suas características. Com base no modelo com melhor desempenho, o VaR da carteira é calculado diariamente, sendo o risco gerido através de uma estratégia de cobertura da exposição de ações ao longo do período de um ano. A eficácia desta estratégia é avaliada utilizando a métrica RORAC (rendibilidade do capital ajustada ao risco). Os resultados demonstram que, apesar de a estratégia de *hedging* ter conseguido limitar o VaR da carteira, a diminuição da rendibilidade foi superior à redução do risco, resultando numa rendibilidade inferior face à carteira que não sofreu *hedging*.

Key words: Value-at-Risk (VaR), Economic Capital (EC), Backtesting

Abstract

The industry-standard metric for risk measurement in the financial industry is the Value-at-Risk (VaR), which estimates the maximum expected loss due to market fluctuations over a specified future time horizon, at a given significance level. VaR is used to gauge the capital at risk from investment activities, which aligns with the concept of Economic Capital (EC). By establishing a pre-defined maximum value for the EC, decision-makers can set risk limits that align with their strategic objectives. In this study, the VaR of a diversified portfolio – consisting of equities from Hong Kong, Japan, Europe, and the U.S., as well as bonds from Europe and U.S. markets - is measured and managed to ensure it remains within the target EC. Given the range of models available for calculating VaR, nineteen different models are evaluated through backtesting to determine their performance, selecting the most suitable one based on model characteristics. Using the best-performing model, VaR of the portfolio is calculated daily, and risk is managed through an equity exposure hedging strategy over a one-year period. The effectiveness of this strategy is measured using the Return on risk-Adjusted Capital (RORAC) metric. The results indicate that while the hedging strategy successfully limited the portfolio's VaR, the reduction in return outweighed the reduction in risk, leading to lower profitability compared to the unhedged portfolio.

Key words: Value-at-Risk (VaR), Economic Capital (EC), Backtesting

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List of Abbreviations

BCP - Berkowitz, Christoffersen, and Pelletier

EC – Economic Capital

ECB – European Central Bank

ETL – Expected Tail Loss

EUR - Euro

EWMA – Exponentially Weighted Moving Average

FRED - Federal Reserve Economic Data

HKD – Hong Kong Dollar

i.d.d. – Independent and identically distributed

JPY – Japanese Yen

P&L – Profit and Loss

PIIGS – Portugal, Italy, Ireland, Greece, and Spain

PV – Present Value

PV01 – Present Value of a basis point

QR – Quantile Regression

RM - RiskMetrics

RORAC – Return on Risk-Adjusted Capital

SGSt – Skewed Generalized Student-t

UC – Uncoverage test

U.S. – United States of America

USD – United States Dollar

VaR – Value-at-Risk

Chapter 1

INTRODUCTION

Recognizing the significant impact that financial institutions have on the stability of the financial system and the broader economy, regulators implemented the Basel Accords in the last years of the 1980s (Drumond, 2009). These accords, along with subsequent updates (Drumond, 2009), were designed to bolster minimum capital requirements and improve risk monitoring and reporting, thereby promoting financial stability. For financial institutions, effective risk management is not only essential for the safeguard of shareholders but also for meeting regulatory obligations.

This work focuses on market risk - the uncertainty surrounding the future value of financial assets due to market price fluctuations, among the various types of risk. Effective risk management starts with accurate measurement, and the industry standard for market risk assessment is Value-at-Risk (VaR). VaR is a statistical measure that estimates the maximum expected loss over a specified time horizon, with a given significance level (Alexander, 2009). In the context of financial institutions, Economic Capital (EC) represents the capital at risk from their investment activities and is calculated using VaR. EC is, therefore, numerically equivalent to VaR (Jorian, 2007), enabling risk management strategies to be structured around a predefined maximum EC value.

The primary objective of this project is to measure and manage the VaR of a portfolio comprising equities belonging to Hong Kong, Japan, Europe and United States (U.S.) and European and U.S. bonds, over a one-year period from January 30, 2023, to February 2, 2024 (estimation period). The goal is to ensure that the portfolio's VaR remains within a predetermined EC limit, and to compare the actual return achieved through VaR management, using a hedging strategy, with those that would have been realized without risk management.

The first step involves selecting an appropriate model for measuring VaR. The RiskMetrics VaR model, introduced by J. P. Morgan and Reuters around mid-1990s, was one of the first standardized solutions for market risk measurement, considered one variant of the Normal VaR (J. P. Morgan and Reuters, 1996). In this work, the Normal model but also several other approaches, specifically, four types of VaR models: Normal, Skewed Generalized Student-t (SGSt), Historical, and Quantile Regression (QR), covering a total of nineteen different models. A backtest is conducted using historical data to estimate VaR for each model, based on the portfolio's composition as of January 27, 2023. The backtest covers ten years, from February 11, 2013, to January 27, 2023, and assesses each model's performance using the Unconditional Coverage (UC) test (Kupiec, 1995) and the Berkowitz, Christoffersen, and Pelletier (BCP) test (Berkowitz et al., 2009).

Once the most suitable model is selected, the next phase involves measuring and managing the portfolio's VaR on a daily basis over one-year period through an equity exposure hedging strategy. The daily EC limit is set at €425000, approximately 1.4% of the portfolio's value as of January 27, 2023. The objective is to ensure that the daily VaR estimate remains within this limit and to evaluate how this VaR constraint, implemented through the hedging strategy, affects the portfolio's annual return. To facilitate comparison, two portfolios are analyzed: the Hedged portfolio, which incorporates the hedging strategy, and the Unhedged portfolio, which does not include the hedging strategy.

The results demonstrate that the hedging strategy effectively kept the portfolio's VaR at or below the €425.000 limit throughout the year. In contrast, without this risk management, the VaR would have exceeded this limit approximately 50% of the year, with higher VaR values concentrated in the first half of the year. When comparing returns using the Return on Risk-Adjusted Capital (RORAC) performance metric, while both the Hedged and Unhedged portfolios achieved positive returns, the Hedged portfolio generated lower profits but took on less risk. This indicates that while managing VaR effectively mitigated risk, it also led to reduced profitability compared to the Unhedged portfolio during the period analyzed.

The structure of the work is planned as follows: Chapter 1 will provide the introduction, setting the context for the study. Chapter 2 will review the relevant literature. Chapter 3 will detail the data, time frame, and portfolio specifics. Chapter 4 will explain the methodology and outline the research approach. Chapter 5 will present the results of the backtesting and model selection. Chapter 6 will analyze the impact of the equity exposure hedging strategy on portfolio returns. Finally, Chapter 7 will summarize the overall findings and conclusions.

Chapter 2
LITERATURE REVIEW

The measurement of risk has been a central concern for financial market participants since the earliest days of financial history (Allen et al., 2004). The possibility of distress in financial institutions can have severe negative consequences not only for financial markets but also for the broader economy (Hoggarth et al., 2002). To mitigate these risks, minimum capital requirements have been imposed alongside enhanced risk monitoring and reporting regulations, particularly since many financial institutions operate privately. The first significant regulatory framework addressing these issues was the Basel Accord in 1988 (commonly referred to as Basel I), which has undergone several revisions (Drumond, 2009). The regulation rests on three core pillars, namely minimum capital standards, which define the level of eligible capital a bank most hold to insure against the risks in its trading and banking books; supervisory review, which establishes guidelines for bank inspections and reporting requirements; and public disclosure and market discipline, which encourage market scrutiny from competitors, clients and shareholders to support regulatory oversight.

Risk management, which typically follows an initial risk analysis, involves evaluating a portfolio's risk exposure, identifying the sources of risk, and determining whether the investor is comfortable with the level of risk being taken. This process is an important tool for decision-making (Aven, 2015). Once the risk is assessed, various tools are applied to reduce the exposure to more acceptable levels by managing the primary sources of risk. Risk management is generally divided into three main categories: strategic risk, which is influenced by external factors such as mergers and acquisitions, political conditions, or regulatory changes; financial risk, which is linked to market fluctuations that impact the value of investments; and operational risk, which pertains to safety and security issues within the organization (Aven, 2015).

Risk itself is defined as the uncertainty surrounding the future value of a portfolio, typically evaluated through potential profits, losses, or returns. The sources of risk can be decomposed into market risk, which involves fluctuations in the market price of financial assets; credit risk, which deals with defaults and changes in the probability of default on bonds and credits; and operational risk, which relates to problems in a company's business operations (Allen et al., 2004). This project focuses on market risk, which refers to the uncertainty about the future value of financial assets caused by fluctuations in market prices (Alexander, 2009). Before risk can be managed, however, it must first be measured. Value at Risk (VaR) is the industry-standard method for measuring market risk, despite its limitations (Krause, 2003). VaR is often compared to Expected Tail Loss (ETL), also known as conditional VaR, which is theoretically superior but mainly in conceptual terms, alongside other metrics like standard deviation and semi-standard deviation. As a statistical measure, VaR estimates the maximum expected loss that is unlikely to be exceeded, given a specific confidence level and time horizon (Alexander, 2009). The key parameters for VaR calculation are the confidence level (1 minus the significance level) and the risk horizon over which the portfolio's value changes are assessed (Alexander, 2009).

According to the Basel Accord II, financial institutions must maintain a minimum regulatory capital for market risk, calculated to cover potential losses, as recommended by stress tests recommended under the Basel I Amendment (Alexander, 2009). These stress tests help regulators assess an institution's capital adequacy, through various methods for calculating the necessary capital proposed by regulators (Bank for International Settlements, 2024). While financial institutions must meet Basel Accord requirements, they retain discretion in allocating capital to specific investments, allowing them to choose their activities and the associated capital allocation (Alexander, 2009). From a financial institution's perspective, Economic Capital (EC) refers to the capital at risk from its investments (Porteous and Tapadar, 2005), which is distributed across the firm's specific activities using a top-down approach. It is crucial that the capital at risk from each activity does not exceed the EC allocated to maintain financial stability. Since EC is measured by VaR, it is numerically equivalent to VaR (Jorian, 2007). This predefined EC limit serves as a benchmark for developing risk management strategies.

Changes in Basel II regulations prompted the adoption of VaR as the standard metric for market risk, which reflects the potential for value fluctuations in response to price changes in the market (Drumond, 2009). The first major approach to measuring market risk was the RiskMetrics (RM) VaR model, established by J. P. Morgan and Reuters (1996). This model, while evolving to incorporate the maximum information possible in the data sets, became a foundation for economic-based metrics to determine minimum capital requirements (Allen et al., 2004).

The Normal VaR model is parametric and assumes a normal distribution of risk factors or asset returns over a given time horizon (J. P. Morgan and Reuters, 1996). This parametric assumption simplifies analysis, requiring only the estimation of distribution parameters (Alexander, 2009). However, the model is limited to linear portfolios and can only be extended to a few parametric forms, such as a Student-t distributions or a mixtures of normal or Student-t distributions (Alexander, 2009).The model provides rough VaR estimates and fails to capture real-world characteristics like skewness and kurtosis. While a normal VaR assumes both skewness and kurtosis, are, approximately, zero, empirical evidence from financial as demonstrated by Fama (1965) and Peiró (2010), reveals that portfolios typically exhibit negative skewness and excess kurtosis – indicating a non-normal distribution. This negative skewness and excess kurtosis result in a leptokurtic distribution, which is characterized by a density function with a sharper peak and heavier tails compared to a normal distribution with the same variance. Leptokurtosis, marked by excess positive kurtosis, further highlights the limitations of the Normal VaR model. At lower significance levels (e.g., 5%, or a 95% confidence level), the assumption of normality may overestimate VaR, while at higher significance levels (e.g., 0.5%, or a 99.5% confidence level), it can significantly underestimate VaR (Alexander, 2009).

To addresss these limitations, the Skewed Generalized Student-t (SGSt) model, introduced by Theodossiou (1998), improves upon the Normal model by incorporating asymmetry and greater tail flexibility. Based on the Generalized Student-t (SGSt) distribution by McDonald and Newey (1988), the SGSt VaR model, like Normal, assumes a parametric return distribution but adapts better to empirical returns, particularly capturing the fat tails often seen in financial data. This makes it more effective at modeling deviations from normality. Lin and Shen (2006) compared SGSt VaR to Normal VaR using daily data from the S&P500, NASDAQ, DAX, and FTSE 100 indices over a three-year period. The normality assumption was rejected for all indices based on the Jarque-Bera test, and at significance levels ranging from 55% to 0.1%, the Normal model's accuracy declined as the significance level dropped. In contrast, SGSt VaR consistently provided more accurate estimates at lower levels, positioning it as a superior alternative in parametric VaR models.

Historical VaR, also known as Historical Simulation VaR, is widely employed by banks. Unlike parametric models that estimate risk using distributions derived from historical data, Historical VaR utilizes the empirical distribution of returns. This method captures the dynamic evolution and dependencies of risk factors based on historical observations (Alexander, 2009), accurately reflecting the empirical skewness and kurtosis of return distributions. However, despite this flexibility, the model has limitations. One major weakness is its difficulty in assessing risk over extended time horizons. To address this, VaR is initially calculated for a single day (1-day VaR) and then scaled to the desired horizon (h-day VaR), requiring h-day historical returns for the extended period. The model's accuracy heavily depends on the chosen sample size, which is both subjective and crucial. Pritsker (2006) points out that while a larger sample size increases diversity in outcomes, it can reduce the model's responsiveness to current market volatility conditions. To improve this, Barone-Adesi et al. (1998) and Boudoukh et al. (1998) suggest refining the traditional approach by assigning more weight to recent observations, thereby making recent data more influential. Additionally, Hull and White (1998) propose a refinement that adjusts past returns to reflect current market volatility levels. Their study, which used nearly nine years of daily data from twelve different exchange rates and five stock indices, found that this volatility adjustment method outperformed both standard Historical VaR and Boudoukh et al. (1998)'s weight adjustment methodology, particularly at the one percent significance level.

In non-parametric methods, VaR can be viewed as a conditional quantile (Xiao et al., 2015), making quantile regressions, introduced by (Koenker and Bassett Jr., 1978), a valuable alternative for estimating VaR. This approach calculates VaR through a quantile regression of a portfolio's returns against selected explanatory variables, bypassing the need to estimate the profit and loss (P&L) distribution. Similar to Historical VaR, quantile regression relies on empirical returns rather than assuming a specific parametric distribution. A key advantage of this method is its flexibility in choosing explanatory variables. However, like Historical VaR, it faces challenges related to sample size and subjective size choice and may not always be suitable for calculating VaR at the risk factor level. In a study by Steen et al. (2015) the performance of Normal, Historical VaR, and Quantile Regression VaR was compared using nearly nineteen years of daily data for futures contracts across nineteen commodities. The results reaffirmed earlier findings on Normal, showing that while it performs adequately at the 5% significance level, its accuracy diminishes at the 1% significance level. Historical VaR generally outperforms Normal model, but Quantile Regression VaR consistently surpasses both models across all significance levels. Despite some subjectivity in configuring the model, such as choosing explanatory variables, Quantile Regression VaR delivers more robust results than Normal model and performs at least as well as, if not better than, Historical VaR. It is especially promising for commodities.

As noted by Alexander (2009), different VaR models can yield varying results for the same portfolio, and using the same model for two distinct portfolios can also produce significantly different outcomes for different portfolios. This highlights the necessity of backtesting, a process that evaluates model performance through statistical tests on historical portfolio returns and corresponding VaR estimates. Backtesting helps identify which model is best suited to a specific portfolio. There is no standard set of statistical tests for backtesting, and the user can choose from various alternatives described in the literature (Zhang and Nadarajah, 2017). A common performance measure is the number of exceedances, where actual losses (P&L) surpass VaR estimates. This project employs the Unconditional Coverage (UC) test (Kupiec, 1995) to assess the number of exceedances across the period under review, as well as in annual sub-periods. Additionally, the BCP test (Berkowitz et al., 2009) is used to examine whether exceedances are autocorrelated, meaning whether they occur independently or cluster together. Identifying such clusters is essential, as it indicates whether the model adapts to rapidly changing market conditions. However, the BCP test is typically more difficult to pass than the UC test.

When VaR exceeds its predefined maximum value, risk management decisions must be made. Adjusting portfolio exposures to comply with EC quotas alters the risk profile of the portfolio, differentiating it from what it would have been without these adjustments. A hedging approach – considered an intermediate strategy (Alexander, 2009) - is commonly used to reduce or mitigate risk by taking an offsetting position in a related asset or market. This strategy involves investing in assets that are expected to move in the opposite direction of the asset or risk being hedged. Essentially, the goal of hedging is to minimize potential losses from adverse market price movements, functioning as a form of insurance (Hopkin, 2018). As Longley-Cook (1998) explains, a monetary gain achieved with substantial risk is less valuable than the same gain achieved with lower risk. Similarly, a return with a high VaR, and therefore a high EC, is less valuable than a return with lower VaR and corresponding lower EC. This underscores the need for a risk-adjusted performance metric. Return on Risk-Adjusted Capital (RORAC) addresses this by providing a ratio between returns and the associated risk (Buch et al., 2011). RORAC formally relates an unadjusted return to a risk-adjusted capital base (Matten, 1996). Therefore, RORAC provides a fair comparison of returns across portfolios with different risk profiles, by relating returns to the corresponding EC (VaR).

Chapter 3

DATA AND PORTFOLIO COMPOSITION

The optimal number of stocks required to minimize risk in a portfolio remains a subject of debate. However, recent findings by De Keizer & De Schaepmeester (2015) suggest that for countries within the PIIGS group (Portugal, Italy, Ireland, Greece and Spain), particularly in Portugal, a portfolio with around nineteen stocks is considered sufficiently diversified. In this study, however, the available Portuguese stocks were not included in the portfolio, and a larger set of assets was employed to explore further risk diversification.

The constructed portfolio is a diversified mix of equities and bonds, with assets sourced from Hong Kong, Japan, European and U.S. markets. The Euro (EUR) serves as the base currency, while the Hong Kong Dollar (HKD), Japanese Yen (JPY) and the U.S. Dollar (USD) are treated as foreign currencies.

The equity part includes sixty stock positions, comprising fifty-two long positions and eight short positions, positioning the portfolio primarily as a long position taker (long-holder). The equities span multiple markets, including Hong Kong, Japan, U.S., and Europe. Daily adjusted closing prices in the respective home currencies were sourced from Yahoo Finance (https://finance.yahoo.com/), along with the corresponding exchange rates (HKD/EUR, JPY/EUR and USD/EUR).

On the fixed-income side, which corresponds to the bonds' part), the portfolio includes eight government bonds, each featuring fixed coupon rates with varying maturities and payment schedules. These bonds were issued in either Euro (EUR) or U.S. Dollar (USD), and originate from markets of Germany, Luxembourg, the Netherlands, and the United States. Bond data, including key characteristics, was obtained from Börse Frankfurt (https://www.boerse-frankfurt.de/bonds), with interest rate data for USD bonds from the Federal Reserve Economic Data (FRED) (https://www.federalreserve.gov/datadownload/Choose.aspx?rel=H15) and for EUR bonds from the European Central Bank (ECB) website for EUR rates (https://sdw.ecb.europa.eu/browseSelection.do?node=9689726.

The data used spans a twenty-year period from January 4, 2007, to February 3, 2024, and daily information was used in relation to the data used. Several constraints were applied during the portfolio construction process over the one-year period: the bond allocation could not exceed twenty percent of the total portfolio value, and the stock positions were equally weighted relative to the overall portfolio, although the long positions were allowed greater exposure than the short positions. Table 1 and Table 2 present the composition of the portfolio as well as characteristics of each position.

Asset	Ticker/ISIN	Currency	Number of Shares/ Face Value	Price (Original Currency)	Value (EUR)	Allocation (%)
Bridgestone Corporation	5108.T	JPY	15,030	4704.87	499,808	1.64%
Canon Inc.	7751.T	JPY	24,890	2841.71	499,920	1.64%
Casio Computer Co., Ltd.	6952.T	JPY	54,600	1295.47	499,939	1.64%
Honda Motor Co., Ltd.	7267.T	JPY	69,200	1022.18	499,956	1.64%
ITOCHU Corporation	8001.T	JPY	17,575	4024.73	499,952	1.64%
Kubota Corporation	6326.T	JPY	-14,990	1887.52	-199,981	-0.66%
Nippon Telegraph and Telephone Corporation	9432.T	JPY	476,035	148.60	499,998	1.64%
Nissan Chemical Corporation	4021.T	JPY	12,245	5776.91	499,978	1.64%
Pacific Metals Co., Ltd.	5541.T	JPY	35,180	2010.71	499,966	1.64%
Sony Group Corporation	6758.T	JPY	6,140	11512.79	499,627	1.64%
Toyota Motor Corporation	7203.T	JPY	38,810	1822.69	499,980	1.64%
Henderson Land Development Company Limited	0012.HK	HKD	153,240	27.83	499,997	1.64%
Sino Biopharmaceutical Limited	1177.HK	HKD	-355,630	4.80	-199,997	-0.66%
Tencent Holdings Limited	0700.HK	HKD	10,435	408.54	499,769	1.64%
BNP Paribas SA	BNP.PA	EUR	9,200	54.33	499,826	1.64%
Carrefour SA	CA.PA	EUR	29,495	16.95	499,926	1.64%
Air France - KLM SA	AF.PA	EUR	-12,375	16.16	-199,980	-0.66%
Kering SA	KER.PA	EUR	950	526.23	499,919	1.64%
L'Oréal S.A.	OR.PA	EUR	1,360	367.55	499,863	1.64%
Vivendi SE	VIV.PA	EUR	53,140	9.41	499,967	1.64%
BE Semiconductor Industries N.V.	BESI.AS	EUR	-3,140	63.63	-199,795	-0.66%
Heineken N.V.	HEIA.AS	EUR	5,680	88.01	499,876	1.64%
Burberry Group plc	BRBY.L	GBP	18,645	23.53	499,958	1.64%
Croda International Plc	CRDA.L	GBP	6,600	66.43	499,637	1.64%
InterContinental Hotels Group PLC	IHG.L	GBP	7,960	55.11	499,878	1.64%
Rio Tinto Group	RIO.L	GBP	7,710	56.88	499,732	1.64%
Schroders plc	SDR.L	GBP	98,185	4.47	499,985	1.64%
St. James's Place plc	STJ.L	GBP	38,085	11.52	499,935	1.64%
Vodafone Group Public Limited Company	VOD.L	GBP	-212,030	0.83	-199,990	-0.66%
Unilever PLC	ULVR.L	GBP	11,515	38.10	499,941	1.64%
American International Group, Inc.	AIG	USD	8,810	61.80	499,732	1.64%
Apple Inc.	AAPL	USD	3,760	144.74	499,533	1.64%
Alphabet Inc.	GOOG	USD	5,405	100.71	499,647	1.64%
Amazon.com, Inc.	AMZN	USD	5,325	102.24	499,730	1.64%
AstraZeneca PLC	AZN	USD	-3,445	63.20	-199,849	-0.66%
AT&T Inc.	T	USD	29,670	18.36	499,992	1.64%
Bank of America Corporation	BAC	USD	15,955	34.14	499,970	1.64%
Brown-Forman Corporation	BF-B	USD	8,290	65.71	499,980	1.64%
Caterpillar Inc.	CAT	USD	2,105	258.33	499,133	1.64%
eBay Inc.	EBAY	USD	11,495	47.38	499,934	1.64%
Exxon Mobil Corporation	XOM	USD	-1,980	109.87	-199,685	-0.66%
FedEx Corporation	FDX	USD	2,930	185.69	499,417	1.64%
Hasbro, Inc.	HAS	USD	10,020	54.34	499,784	1.64%
JPMorgan Chase & Co.	JPM	USD	4,015	135.63	499,856	1.64%
Johnson & Johnson	JNJ	USD	3,385	160.75	499,451	1.64%
Kimberly-Clark Corporation	KMB	USD	-1,770	122.97	-199,792	-0.66%
McDonald's Corporation	MCD	USD	2,065	263.21	498,909	1.64%
Microsoft Corporation	MSFT	USD	2,220	245.08	499,415	1.64%
NIKE, Inc.	NKE	USD	4,355	124.97	499,546	1.64%
Nvidia Corporation	NVDA	USD	2,675	203.55	499,793	1.64%
Simon Property Group, Inc.	SPG	USD	4,565	119.27	499,785	1.64%
UnitedHealth Group Incorporated	UNH	USD	1,140	476.96	499,094	1.64%
Walmart Inc.	WMT	USD	11,650	46.73	499,683	1.64%
Zimmer Biomet Holdings, Inc.	ZBH	USD	4,365	124.68	499,532	1.64%
The Allstate Corporation	ALL	USD	4,440	122.65	499,873	1.64%
The Boeing Company	BA	USD	2,575	211.17	499,120	1.64%
The Coca-Cola Company	KO	USD	9,360	58.17	499,810	1.64%
The Goldman Sachs Group, Inc.	GS	USD	1,605	338.37	498,503	1.64%
The Walt Disney Company	DIS	USD	4,985	109.18	499,597	1.64%
3M Company	MMM	USD	5,010	108.68	499,783	1.64%
Total Equity			-	-	**24,386,867**	**80.18%**
German Bond 2026	DE0001102390	EUR	300,000	95.03%	285,103	0.94%
German Bond 2028	DE000BU25018	EUR	650,000	101.20%	657,821	2.16%
Netherlands Bond 2037	NL0000102234	EUR	800,000	119.81%	958,489	3.15%
Luxembourg Bond 2043	LU2591861021	EUR	1,050,000	117.76%	1,236,487	4.07%
US Treasury Bond 2025	US91282CJD48	USD	450,000	96.65%	434,940	1.43%
US Treasury Bond 2029	US91282CKP58	USD	600,000	98.09%	588,528	1.93%
US Treasury Bond 2032	US91282CEZ05	USD	750,000	71.04%	532,818	1.75%
US Treasury Bond 2041	US912810QN19	USD	1,250,000	106.87%	1,335,914	4.39%
Total Bonds			-	-	**6,030,099**	**19.82%**
Total Portfolio			-	-	**30,416,966**	**100.00%**

Table 1. Portfolio composition at 27 January 2023. This table displays the assets included in the portfolio for this work, along with the corresponding investment amounts. These amounts have been converted to EUR from HKD, JPY, GBP, and USD, where applicable. The exchange rates applied, as of 27 January 2023, are 0.1172, 0.0071, 1.1395, and 0.9179, respectively. Due to rounding, the total may not exactly match the sum of the individual amounts.

Bonds	ISIN	Maturity	Original Currency	Coupon per Year	Coupon (%)	Face Value (EUR)	Fair Value (EUR)
German Bond 2026	DE0001102390	2026-02-15	EUR	1	0.500%	300,000	285,103
German Bond 2028	DE000BU25018	2028-10-19	EUR	1	2.400%	650,000	657,821
Netherlands Bond 2037	NL0000102234	2037-01-15	EUR	1	4.000%	800,000	958,489
Luxembourg Bond 2043	LU2591861021	2043-03-02	EUR	1	3.250%	1,050,000	1,236,487
US Treasury Bond 2025	US91282CJD48	2025-10-31	USD	4	5.465%	450,000	434,940
US Treasury Bond 2029	US91282CKP58	2029-04-30	USD	2	4.625%	600,000	588,528
US Treasury Bond 2032	US91282CEZ05	2032-07-15	USD	2	0.675%	750,000	532,818
US Treasury Bond 2041	US912810QN19	2041-02-15	USD	2	4.750%	1,250,000	1,335,914

Table 2. Bond characteristics. The fair value of the bond is the sum of the present value (PV) of all its future cash flows, discounted to 27 January 2023, and converted to EUR where appropriate. The exchange rate between USD and EUR used on 27 January 2023 is 0.9179. The future cash flows are derived using Equations 2 and 3, and their present value is determined by Equation 4.

Chapter 4
METHODOLOGY

The aim of this work is to assess and manage the Value at Risk (VaR) of a portfolio over a one-year period, beginning January 27, 2023. The objective is to ensure that the portfolio's VaR does not exceed a predefined Economic Capital (EC), which was set as a target based on the VaR values calculated for the upcoming year. A hedging strategy will be employed to maintain the VaR within this limit.

To select the most appropriate VaR model for the upcoming year, a process called backtesting is applied. This process evaluates various VaR models, including different versions within each model class, while considering the current portfolio's characteristics. The most suitable model cannot be determined *a priori*; it requires testing multiple models to assess their accuracy and suitability.

Daily VaR estimates will be calculated for each model using historical data and the current portfolio composition. This simulation spans a ten-year period, from February 11, 2013, to January 27, 2023, allowing the assessment of how VaR would have been estimated in real-time. Four main classes of models will be tested: Normal VaR, SGSt VaR (Skewed Generalized Student-t VaR), Historical VaR and QR VaR (Quantile Regression VaR). Various versions of each model will also be explored to ensure a comprehensive analysis.

During the backtesting process, specific statistical tests will be used to evaluate the performance of each mode and to determine which is best suited for the portfolio's needs moving forward. Key factors influencing model performance include the significance level, the backtesting sample size, and the EWMA (Exponentially Weighted Moving Average) warm-up period. Additionally, the choice of sample size and the EWMA smoothing factor can significantly affect the model's suitability.

One of the chapters of the work will provide a detailed analysis of the different VaR models, their specifications, and how each performed under varying conditions.

4.1. Risk Factor Exposure Mapping

The risk factors, or in other words the sources of risk, are the variables that directly influence the value of the assets in the portfolio. The first step in risk management is to identify these factors and measure the portfolio's exposure to each one through a process known as risk factor mapping. This process involves translating each portfolio position into an equivalent exposure to the relevant risk factors. The specific risk factors that apply to each position vary based on the type of asset, as different assets are associated with different risk factors. To calculate the Value at Risk (VaR) in the local currency, EUR, all the exposures must be expressed in EUR. For that reason, it is necessary to convert the exposures denominated in foreign currencies to EUR, using the appropriate exchange rate as of the Backtest date, which, in this case, is January 27, 2023.

The decomposition by risk factor can be carried out in various ways and in this project the decompositions used were: by asset, by asset class, by currency, by risk factor type and by risk factor group. In each of the decompositions, the Stand-Alone VaR and the Marginal VaR were calculated, and the latter was important to select the strategy to adopt to set the VaR at the predetermined value, the target value, considering the decomposition by risk factor group.

In the following subsections detail the exposure mapping methodology used for each asset class in the portfolio. At the conclusion of this section, Table 3 provides a summary of the exposures to each risk factor resulting from the risk factor mapping process.

4.1.1. Equity

In financial terms, equity can be described as the ownership of the stocks, meaning that the investor has a position in the stock. When evaluating an equity portfolio is important to analyze the correlation between the stock returns that compose the portfolio, and the correlation will be lower the more diversified the portfolio is.

The value of an investment in a stock is determined by the number of shares and the market price of that stock. Consequently, the risk factor for each stock in the portfolio is the change in its market price.

When mapping risk factors, the exposure to price changes for each stock is defined by the amount of capital invested in it. The investment amount is determined by multiplying the number of shares by the market price of the stock:

$$S_{i_t} = N_{i_t} \times P_{i_t} \times e_{i_t} \tag{1}$$

where S_{i_t} is the amount of capital invested, N_{i_t} is the number of shares, P_{i_t} is the market price of the stock and e_{i_t} is the exchange rate verified on 27th January 2023 applied to convert the exposure from foreign currency to local currency (where e is the currency exchange rate in which the asset is denominated against the euro) .

Therefore, the main risk factor when evaluating stocks is the capital amount invested in each stock. However, there is another relevant risk factor the currency risk factor, that will be described below (in subsection 4.1.3).

4.1.2. Bonds

A bond is a type of financial instrument that generates a stream of cash flows over time. The main characteristics of a fixed coupon bond (a bond where the coupon rate is fixed, which was the type of bond used in the portfolio) are the maturity date, the coupon rate, the number of coupons per year, and the face value of the investment, the latter chosen by the investor.

Despite the way used to estimate the future volatility of stock price returns based on the past observations of the stock price return, as it was mentioned above for the equity exposures, the process to achieve the fair value of a bond is obtained by discounting the future cash flows to their present value (PV).

The main risk factor influencing the value of a bond is the interest rate, which is used to determine and discount the cash flow, influencing the bond's value. The bond's value and the interest rate are inversely correlated, which means that if there is an increase (decrease) in the interest rates, the bond's value will decrease (increase). However, it is important to note that interest rates can fluctuate with market conditions, therefore it is necessary to evaluate how sensitive bond positions are to changes in interest rates for risk measurement and management.

In the case of a fixed coupon bond, the future cash flows consist of regular coupon payments made over the bond's duration and a final principal repayment at maturity. The value of each coupon payment is calculated as follows:

$$Coupon = N \times \frac{c_n}{n} \tag{2}$$

In this context, N represents the amount invested in the bond, also referred to as the face value. The variable c_n denotes the annual coupon rate, which is the interest paid yearly by the bond until it matures. Meanwhile, n indicates the coupon frequency, or the number of times per year that coupon payments are made.

The final cash flow, or the last payback payment, happens at the maturity of the bond and comprises the redemption of the face value and the last coupon payment:

$$C_m = N \times \left(1 + \frac{c_n}{n}\right) \tag{3}$$

where C_m is the final payback payment.

The present value (PV) of a cash flow C_T is given by:

$$PV_{C_T, r_T} = C_T \times e^{-r_T \times T} \tag{4}$$

where C_T represents a future cash flow, T indicates the time in years from now until the maturity (also referred as cash flow date), and r_T is the continuously compounding interest rate for the period from the present until time T.

The present value of a basis point, PV01, approximated using a first-order Taylor expansion, measures the sensitivity of a given cash flow's the present in relation to a decrease of a one basis point in the interest rate r_T.

$$\begin{aligned} PV01_{C_T,r_T} &\approx \frac{\partial PV_{C_T,r_T}}{\partial r_T} \times (-0.01\%) \\ &= T \times PV_{C_T,r_T} \times 0.01\% \end{aligned} \tag{5}$$

where T is the maturity of the cash flow.

As a result, the first-order approximation of the change in the PV of a single cash-flow, or its profit and loss (P&L), can be expressed as a function of the $PV01$ as follows:

$$\Delta PV_{C_T,r_T} = -PV01_{C_T} \times \frac{\Delta r_T}{0.01\%} \tag{6}$$

where $\frac{\Delta r_T}{0.01\%}$ denotes the absolute change in interest rate measured in basis points.

Consequently, the profit and loss (P&L) for cash flow C_T is negatively affected by the sensitivity to a one basis point increase in the interest rate ($-PV01_{C_T}$) and the actual change in the interest rate (Δr_T).

For a bond with n cash flows, the total P&L is the sum of the P&L for each cash flow, calculated using Equation 6 for each individual cash flow. However, a bond with n cash flows will have n distinct interest rates as risk factors (one for each cash flow maturity). In a portfolio containing many bonds, this complexity can become overwhelming. Additionally, a challenge arises when a future cash flow is set for a date where no data is available for the corresponding interest rate r_T. Without this interest rate, the cash flow cannot be properly discounted to its present value to compute its $PV01$.

To address these challenges, according to Alexander (2009), the technique suggested includes the adoption of a vertices mapping approach, which can be described by aligning cash flows with non-standard maturities to a series of standard maturity interest rates for which data is available. In this mapping process, a vertex represents a standard maturity that has corresponding interest rate data. Moving forward, the mapping approach that will be used is the $PV + PV01$ invariant mapping method, which aims to maintain the present value (PV) and the $PV01$ of the original cash flows with non-standard maturities.

The preservation of the PV of the original cash flow can be achieved through the following condition:

$$x_{T_1} + x_{T_2} = PV_{C_T} \tag{7}$$

where PV_{C_T} represents the PV of the original cash flow maturing at T, T_1 and T_2 are the standard maturity vertices that lie directly below and above T, respectively, and x_{T_1} and x_{T_2} denote the proportions of the PV_{C_T} allocated to vertices T_1 and T_2, respectively.

In the invariant mapping of $PV01$, the total $PV01$ of the mapped cash flows matches that of the original cash flow. Therefore, the combined P&L of the mapped cash flows aligns with the P&L of the original cash flow when the interest rate curve experiences a parallel shift of one basis point. The maintenance of the $PV01$ of the original cash flow is achieved by applying the following formula:

$$T_1 x_{T_1} + T_2 x_{T_2} = T \times PV_{C_T} \tag{8}$$

The preservation of the PV and $PV01$ simultaneously is achieve by combining Equations 7 and 8, resulting in the values for x_{T_1} and x_{T_2} that satisfy both conditions, expressed as:

$$x_{T_1} = \frac{T_2 - T}{T_2 - T_1} \times PV_{C_T} \tag{9}$$

and

$$x_{T_2} = \left(1 - \frac{T_2 - T}{T_2 - T_1}\right) \times PV_{C_T} \tag{10}$$

The $PV + PV01$ invariant mapping process was applied for each cash flow associated with every bond in the portfolio. Regarding exposure, each cash flow, mapped to a standard maturity, is affected by changes in the corresponding interest rate for that maturity, as determined by its $PV01$.

In the context of risk factor mapping, in view of Equation 6, the considered risk factors are the standard maturity interest rates (vertices) and the respective exposure is calculated as the sum of all $PV01$ values mapped to that maturity, multiplied by -1. Since the portfolio includes bonds from the United States market with cash flows denominated in USD, it was necessary to convert the - $PV01$ exposures to EUR using the exchange rate verified on 27 January 2023, which was 0.9179.

4.1.3. Currency

The portfolio is composed by assets from foreign markets, while the local currency is the EUR. Foreign currency positions are subject to risks arising not only from the specific factors associated with each asset but also from fluctuations in exchange rates between foreign currencies and the EUR. In this portfolio, assets denominated in HKD, JPY, GBP, and USD are exposed to the respective exchange rates of HKD/EUR, JPY/EUR, GBP/EUR, and USD/EUR.

As part of risk factor mapping process, the exposure to the HKD/EUR exchange rate is quantified by converting the total capital invested in HKD-denominated assets, including equities and bonds, into EUR using the exchange rate as of 27 January 2023, which was 0.1172. The same methodology and date are applied to determine the exposure for the JPY/EUR, GBP/EUR, and USD/EUR exchange rates. It is important to highlight that both equities and bonds are impacted by the currency risk factor.

4.1.4. Portfolio Exposures

To conclude this topic, Table 3 summarizes the risk factors and their corresponding mapped exposures on 27 January 2023. Equity exposures are calculated using the methodology detailed in subsection 4.1.1 of this chapter. For bonds, the risk factors are the standard maturity interest rates, and the exposure to each interest rate is the sum of all $-PV01$ values for that maturity. The $-PV01$ values are determined using the approach described in subsection 4.1.2. Currency exposures for the four different currencies – HKD, JPY, GBP, and USD – relative to their respective exchange rates, HKD/EUR, JPY/EUR, GBP/EUR, and USD/EUR, are calculated based on the methodology outlined in subsection 4.1.3.

Equity		Bonds		Currency	
Risk Factor	**Exposure (EUR)**	**Risk Factor**	**Exposure (EUR)**	**Risk Factor**	**Exposure (EUR)**
5108.T	499,808	EUR3M	-1.45	JPYEUR	4,799,143
7751.T	499,920	EUR6M	0.60	HKDEUR	799,769
6952.T	499,939	EUR1Y	-7.26	GBPEUR	3,299,076
7267.T	499,956	EUR2Y	-15.84	USDEUR	15,781,478
8001.T	499,952	EUR3Y	-116.05		
6326.T	-199,981	EUR5Y	-257.54		
9432.T	499,998	EUR7Y	-246.59		
4021.T	499,978	EUR10Y	-326.71		
5541.T	499,966	EUR15Y	-922.27		
6758.T	499,627	EUR20Y	-1488.74		
7203.T	499,980	USD3M	-1.94		
0012.HK	499,997	USD6M	-0.67		
1177.HK	-199,997	USD1Y	-7.63		
0700.HK	499,769	USD2Y	-37.62		
BNP.PA	499,826	USD3Y	-120.42		
CA.PA	499,926	USD5Y	-152.35		
AF.PA	-199,980	USD7Y	-344.32		
KER.PA	499,919	USD10Y	-753.15		
OR.PA	499,863	USD20Y	-1170.82		
VIV.PA	499,967				
BESI.AS	-199,795				
HEIA.AS	499,876				
BRBY.L	499,958				
CRDA.L	499,637				
IHG.L	499,878				
RIO.L	499,732				
SDR.L	499,985				
STJ.L	499,935				
VOD.L	-199,990				
ULVR.L	499,941				
AIG	499,732				
AAPL	499,533				
GOOG	499,647				
AMZN	499,730				
AZN	-199,849				
T	499,992				
BAC	499,970				
BF-B	499,980				
CAT	499,133				
EBAY	499,934				
XOM	-199,685				
FDX	499,417				
HAS	499,784				
JPM	499,856				
JNJ	499,451				
KMB	-199,792				
MCD	498,909				
MSFT	499,415				
NKE	499,546				
NVDA	499,793				
SPG	499,785				
UNH	499,094				
WMT	499,683				
ZBH	499,532				
ALL	499,873				
BA	499,120				
KO	499,810				
GS	498,503				
DIS	499,597				
MMM	499,783				

Table 3. Risk factor exposures map in EUR at 27 January 2023. These exposures are utilized to calculate historical portfolio returns using Equation 14 for the backtesting process.

4.2. Returns

Returns represent the gain or loss on an investment over a specific period, expressed as a percentage of the initial investment cost. It is a key measure of an asset performance, applicable to stocks, bonds, real estate, and other asset classes. Returns includes both incomes generated from the asset, such as dividends or interest, and capital gains (or losses) resulting from changes in the price of the asset.

Risk factor returns, often referred to as profit and loss (P&L) as a percentage of the amount invested, are calculated for each asset class using the same approach applied to determine risk factor exposure.

The P&L of the stocks that composed the portfolio is derived from changes in market price of the stocks risk factor, which is the stock price, and can be measured as:

$$P\&L_{Stock_t} = M_{Stock} \times \left(\frac{P_t}{P_{t-1}} - 1\right) \tag{11}$$

where M_{Stock} denotes the capital invested, which remains constant.

In the case of the investment in a bond, each cash flow is subject to a different interest rate and consequently, to changes in that specific rate. For bonds belonging to the portfolio, while keeping the capital invested and the negative present value of a basis point constant, - $PV01$, constant, the P&L is calculated as the sum of the P&L of all the mapped cash flows, according to Equation 6:

$$P\&L_{Bonds_t} = \sum_{i=1}^{n} - PV01_{T_i} \times \frac{\Delta r_{T_i}}{0.01\%} \tag{12}$$

The investments made in an asset denominated in foreign currency, as mentioned above, produce exposure not only to the risk factor of the specific asset but also to the exchange rate between the foreign currency and the local currency. Meaning that, when investing in foreign-denominated stocks and bonds, it is necessary to evaluate the exposure to the exchange rate between the foreign and local currencies equivalent to the amount invested ($M_{Currency}$). Consequently, the P&L resulting from foreign currency exposure can be calculated using the following expression:

$$P\&L_{Currency_t} = M_{Currency} \times \left(\frac{e_{i_t}}{e_{i_{t-1}}} - 1\right) \tag{13}$$

where i represents the exchange rate between foreign and local currency.

Using the available data, specifically the series of daily stock prices, spot interest rates, and currency exchange rates, along with the mapped exposures to each risk factor as of January 27, 2023 (outlined in Table 3), it is possible to compute the historical time series of daily P&Ls for the portfolio composition at 27 January 2023, which is represented in vector form as:

$$P\&L_{Portfolio_t} = \begin{bmatrix} M_{Stock_i} \\ \vdots \\ -PV01_{T_i} \\ \vdots \\ M_{Currency} \end{bmatrix}^T \times \begin{bmatrix} \left(\frac{P_{i_t}}{P_{i_{t-1}}} - 1\right) \\ \vdots \\ \frac{\Delta r_{T_i}}{0.01\%} \\ \vdots \\ \left(\frac{i_t}{i_{t-1}} - 1\right) \end{bmatrix} \tag{14}$$

where the first vector transposed represents the exposures to each risk factor, which is known as the vector of risk factor loadings, and remains constant, while the second vector corresponds to the exchange rate for each respective risk factor.

Finally, the portfolio P&L can be converted into a percentage return using the formula:

$$R_t(\%) = \frac{P\&L_{Portfolio_t}}{Portfolio\ Value} \tag{15}$$

Considering that the denominator corresponds to the portfolio value at 27 January 2023 (observed in Table 1).

4.3. Volatility

The process of identifying and obtaining the exposures to each risk factor, as well as calculating the portfolio returns values, constitutes the initial steps in completing the backtest. However, to successfully carry out this process, it is necessary to model portfolio volatility to be able to obtain VaR estimates from the VaR models. With the data obtained in the previous subsection, the information available is a series of historical returns for the current portfolio composition.

To compute the previously discussed VaR models, the next step is volatility modeling. Volatility, represented by σ, refers to the standard deviation of returns. The most straightforward method to estimate the volatility is by selecting a historical sample of past returns and calculating their standard deviation. Nonetheless, this approach assumes that all observations in the sample are equally weighted, irrespective of when they occurred. This assumption may be problematic, as it treats older observations, which may be less relevant to current market conditions, the same as more recent data.

As VaR is a forward-looking measure, historical data from the distant past may not be as relevant. To address this, the Exponential Weighted Moving Average (EWMA) volatility model gives greater importance to more recent observations, thus better capturing current market conditions. The parameter λ, which can range from 0 to 1, controls the weighting. A lower λ value gives more emphasis to recent data. Although the selection of λ is subjective, the RiskMetrics technical (a specification of the Normal VaR approach) document by J.P. Morgan and Reuters (1996) recommends a λ value of 0.94 for daily returns. Therefore, 0.94 is the value that will be used in the majority subsequent calculations for the several models analysed.

The estimation of the EWMA variance recursively, using the historical daily returns, is obtained as follows:

$$\hat{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2 \tag{16}$$

where $\hat{\sigma}_t^2$ corresponds to the variance estimated for day t on day t - 1, r_{t-1} refers to the return observed on day $t - 1$ and $\lambda \in (0,1)$ is the smoothing factor.

4.4. Value-at-Risk (VaR) Models

Value-at-Risk (VaR) quantifies the maximum expected loss, due to market movements, over a specified future time horizon h for a given significance level α. In this analysis, the significance level will be equal to α = 1%, which corresponds to a confidence level of 1 - α, 99%, and a time horizon of 1 day (h = 1). This implies that, while maintaining the current portfolio unchanged, there is 99% confidence that the observed loss over the next day will not exceed the VaR estimate.

Formally, the h-day 100α% VaR ($VaR_{h,\alpha}$) is defined as the negative value of the α-quantile of the h-day return distribution. As noted by Alexander (2009), for any 0 < α < 1, the α-quantile of the h-day distribution of a continuous random variable X is a real number x_α such that:

$$P(X < x_\alpha) = \alpha \tag{17}$$

If the distribution function of X is known, the α-quantile (x_α) for any specified value of α can be determined as follows:

$$x_a = F^{-1}(\alpha) \tag{18}$$

where F^{-1} corresponds to the inverse cumulative distribution function of X.

The α-quantile value (x_a) indicates the maximum loss that is expected to be exceeded with a probability of α. Since VaR is described in terms of a loss, its value is presented in absolute terms:

$$VaR_{h,\alpha} = -F^{-1}(\alpha) = -x_a \quad (19)$$

For the various VaR models, the VaR can be decomposed into Total VaR and Systematic VaR. The Systematic VaR, or Risk Factor VaR captures the risk associated with the risk factor mapping. In combination with the Residual VaR, or Specific VaR, which captures the risk that is not accounted for by the mapping, this gives the value of the Total VaR.

Typically, the Total VaR is computed at the portfolio level due to the complexity of modelling all individual risk factors in a multivariate distribution. It can be directly calculated using a univariate series of portfolio returns or P&L's.

The systematic VaR is computed at the risk factor level, considering disaggregation and risk management. It is measured using the indices to which the stock belongs, rather than the stock itself, as is the case in Total VaR. Lastly, the Specific VaR is computed at the portfolio level and reflects the risk not captured during the mapping process.

The VaR can further be disaggregated, for example by asset, by market, among others, using several methods, including the Marginal VaR and the Stand-Alone VaR. Marginal VaR assigns the proportion of the total risk to each component as an additive measure, describing the relative risk contributions from different factors to the systematic risk of a diversified portfolio. Therefore, the sum of the marginal VaR components is the Systematic VaR and represents the sensitivity of VaR to changes in the risk factor models parameters. The Stand-Alone VaR measures the risk of an individual component, such as a stock, an index, a currency or currency, in isolation, as if all other risk factors exposures in the portfolio (all the other components) were removed.

The Marginal VaR for a slice, denoted as θ^s, measures the proportion (percentage) of the overall portfolio's VaR attributed to that particular slice, θ^s. Therefore, let ∇ symbolize the gradient vector, which captures how sensitive the portfolio's VaR is to small changes in exposure to each of the n risk factors from their current values, θ. Meanwhile, let S be the decomposition vector, which lists the current exposures to each risk factor (θ_i), based on the type of the decomposition being performed. The formula for calculating Marginal VaR is:

$$Marginal\ VaR^s = \nabla f(\theta)^T \theta^s = \sum_{j=1}^{n} \frac{\partial VaR}{\partial \theta_j} \times \theta_j^s \quad (20)$$

Here, $\nabla f(\theta)$ represents the gradient vector, which can be expressed as:

$$\nabla f(\theta) \equiv \frac{\partial f(\theta)}{\partial \theta} = \begin{bmatrix} \frac{\partial f(\theta)}{\partial \theta_1} \\ \frac{\partial f(\theta)}{\partial \theta_2} \\ \vdots \\ \frac{\partial f(\theta)}{\partial \theta_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial VaR}{\partial \theta_1} \\ \frac{\partial VaR}{\partial \theta_2} \\ \vdots \\ \frac{\partial VaR}{\partial \theta_n} \end{bmatrix} \tag{21}$$

Alternatively, the gradient vector ∇ can be calculated using the formula:

$$\nabla f(\theta) = \frac{VaR_{\theta_1} - VaR_{\theta}}{\varepsilon} \tag{22}$$

This process is repeated for each risk factor, θ_i, to calculate the respective sensitivities.

Hence, the Marginal VaR is given by:

$$Marginal\, VaR = S^T \nabla \tag{23}$$

In the upcoming subsections, the methodology used to compute the four types of VaR models will be examined.

4.4.1. Normal VaR

The most elemental statement in the Normal VaR model is that it assumes risk factor returns are normally distributed, namely $X_h = N \sim (\mu_h, \sigma_h)$, where μ_h is the estimated mean and σ_h is the standard deviation.

Therefore, assuming a normal distribution, and using Equation 19, results in:

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha) \times \sigma_h - \mu_h \tag{24}$$

where $\Phi^{-1}(\alpha)$ denotes the α-quantile of the standard normal distribution.

According to Alexander (2009), it is expected to assume that the present value of the expected return is zero, $\mu_h = 0$, for short risk horizons. In this case, since it was used daily data and is calculated the daily VaR estimates (h = 1), it is reasonable to adopt this assumption. Therefore, without the drift adjustment, the normal linear formula is simply:

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha) \times \sigma_h \tag{25}$$

where σ_h is estimated using the EWMA volatility model specified in Equation 16.

The RiskMetrics VaR is a specific instance of the Normal VaR model, which assumes a normal distribution and calculates volatility using the EWMA Smoothing Factor with a value of $\lambda = 0.94$.

For the purposes of the project, two different VaR series were tested. The primary difference between these models lies in the value of the EWMA Smoothing Factor, which is set to 0.94 and 0.965, respectively.

4.4.2. Skewed Generalized Student-t VaR

Financial asset returns often deviate from a normal distribution, typically exhibiting fatter tails. As a result, the normal distribution tends to underestimate the likelihood of large negative returns. Consequently, relying on the normal distribution can lead to an underestimation of the VaR at low significance level, which is the case at the $\alpha = 1\%$ considered in this thesis.

The Skewed Generalized Student-t (SGSt) is an extension of the classical Student-t distribution, designed to accommodate asymmetry and offer greater flexibility in modeling the tail and central regions of a distribution.

The standardized SGSt distribution (Theodossiou, 1998) is specifically formulated to capture deviations from normality in asset returns. Its density function, denoted as $T_{0,1,\lambda,p,q}$,, is dependent on three parameters: λ, p and q. The parameter λ, which lies within the range $(-1, 1)$, determines the skewness of the distribution. When $\lambda = 0$, the distribution is symmetric, when $\lambda > 0$ or $\lambda < 0$, the distribution is positively or negatively skewed, respectively. The parameter $p > 0$ controls the shape of the central region of the distribution, while $q > 0$ influences the shape of the tail region.

To estimate these parameters, the maximum likelihood method is used, aiming to align the SGSt distribution closely with the actual return distribution of the portfolio. To ensure the model reflects up-to-date market conditions, the parameters are re-estimated each trading month. Hence, this results in the computation of five different SGSt VaR series, differing in the sample size of portfolio returns used for parameter estimation: variants using 300, 500 and 800 daily observations, along with two different EWMA smoothing factors, $\lambda = 0.93$ and $\lambda = 0.94$.

The formula used to compute the h-day 100α% SGSt VaR is:

$$VaR_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) \times \sigma_h - \mu_h \tag{26}$$

where $T^{-1}_{0,1,\lambda,p,q}(\alpha)$ represents the α-quantile of the standard SGSt distribution. Similarly to the Normal VaR, it was used $\mu_h = 0$ and estimate σ_h using the EWMA volatility model, simplifying the equation to:

$$VaR_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) \times \sigma_h \tag{27}$$

4.4.3. Volatility-Adjusted Historical VaR

Despite the assumptions made for the parametric VaR models, mainly the Normal and SGSt VaR models, which assume that the portfolio returns follow a specific distribution, the Historical VaR directly uses the empirical distribution of returns. It relies on the α-quantile of this empirical distribution to estimate the VaR. Therefore, the 100α% h-day VaR is defined as the α-quantile of the empirical h-day discounted P&L (or return) distribution.

To estimate the Historical VaR the first step is to select the sample size, n. Next, empirical h-day returns are computed while maintaining the current portfolio exposure to risk factors constant throughout the sample period. This approach simulates the returns that the current portfolio would have experienced under past market conditions. The returns are then sorted from the worst to best, and probabilities are assigned starting from the worst return (with each observation given a probability $\frac{1}{n}$). Finally, the VaR is determined as the negative of the return at the cumulative probability level α.

The sample size chosen for this model is critical since a larger sample size increases the diversity of returns but may reduce the model's responsiveness to current market conditions. This limitation arises from the simple Historical VaR method, where each observation is equally weighted, meaning that current and older return volatilities have the same. To address this, Hull and White (1998) proposed a refinement known as the volatility-adjusted Historical VaR. This adjustment modifies the volatility of the entire return series while still assigning equal weight to each observation, ensuring the sample more accurately reflects current market conditions. The implementation of this modification involves first calculating a series of volatility estimates ($\hat{\sigma}_t$) and then adjusting the series of returns accordingly:

$$\hat{r}_t = \frac{r_t}{\hat{\sigma}_t} \hat{\sigma}_T \tag{28}$$

where $\hat{r}_t$ denotes the adjusted return, T represents the VaR measurement date, and $t \leq T$. This model is referred to as the T volatility-adjusted Historical VaR.

Due to the importance of the sample size for this VaR model, six different VaR series were computed, with sample sizes of 800 and 1000 daily observations. Additionally, the VaR series vary based on the EWMA Smoothing Factor, so the variants used for the analysis are 0.94, 0.95, and 0.96.

Formally, the 100α% h-day volatility-adjusted Historical VaR is defined as the negative α-quantile of the sample of volatility-adjusted returns.

4.4.4. Quantile Regression VaR

Similarly to the Historical VaR model, the Quantile Regression VaR model is also a non-parametric approach, as the VaR is calculated as minus the α-quantile of a series of portfolio returns. In this model, VaR is estimated through quantile regression, where the dependent variable is the portfolio return, and explanatory variables are used, which can vary depending on the purpose of the model.

The α-quantile (q_α) regression VaR can be estimated using the following formula:

$$VaR_\alpha = -q_{\alpha,y} = -\left(\hat{a} + \hat{b}x_i\right) \tag{29}$$

where y indicates the portfolio return, and $\hat{a}$ and $\hat{b}$ are the estimated parameters of the α-quantile regression of y onto x. These parameters can be determined through the following minimization problem (Koenker and Bassett Jr, 1978):

$$\left(\hat{a}, \hat{b}\right) = arg\ \min_{a,b} \sum_{t=1}^{n} [y_i - (a + bx_i)]\left(\alpha - I_{y_i - (a + bx_i) < 0}\right) \tag{30}$$

where $I_{y_i - (a + bx_i) < 0}$ indicates whether the event occurs, specifically:

$$I_{y_i - (a + bx_i) < 0} = \begin{cases} 1, & y_i - (a + bx_i) < 0 \\ 0, & otherwise \end{cases} \tag{31}$$

A sample size with an appropriate number of daily observations is required to estimate the parameters, which are re-estimated every trading month, similar to the SGSt VaR. Six series of daily VaR estimates are generated for the different Quantile Regression VaR specifications. In all models, only one independent variable, $volatility$, is used. The dependent variable, y, represents the portfolio return and is defined as:

$$y_t = b \times \sigma_t + \varepsilon_t \tag{32}$$

where the explanatory variable σ_t represents the volatility estimate for day t, obtained using the EWMA volatility model.

In summary, the following table, Table 4, provides the specifications for each model computed, with each model assigned a unique number for identification.

Model Number	Description
1	Normal, $\lambda = 0.965$
2	Normal, $\lambda = 0.94$
3	SGSt, sampe size of 300 observations, $\lambda = 0.93$
4	SGSt, sampe size of 300 observations, $\lambda = 0.94$
5	SGSt, sampe size of 500 observations, $\lambda = 0.93$
6	SGSt, sample size of 800 observations, $\lambda = 0.93$
7	SGSt, sample size of 1000 observations, $\lambda = 0.93$
8	T Volatility-Adjusted Historical, sample size of 800, $\lambda = 0.94$
9	T Volatility-Adjusted Historical, sample size of 800, $\lambda = 0.95$
10	T Volatility-Adjusted Historical, sample size of 800, $\lambda = 0.96$
11	T Volatility-Adjusted Historical, sample size of 1000, $\lambda = 0.94$
12	T Volatility-Adjusted Historical, sample size of 1000, $\lambda = 0.95$
13	T Volatility-Adjusted Historical, sample size of 1000, $\lambda = 0.96$
14	Quantile Regression, with EWMA volatility as the explanatory variable, sample size of 300, $\lambda = 0.94$
15	Quantile Regression, with EWMA volatility as the explanatory variable, sample size of 400, $\lambda = 0.94$
16	Quantile Regression, with EWMA volatility as the explanatory variable, sample size of 500, $\lambda = 0.94$
17	Quantile Regression with EWMA volatility as the explanatory variable, sample size of 600, $\lambda = 0.94$
18	Quantile Regression, with EWMA volatility as the explanatory variable, sample size of 700, $\lambda = 0.94$
19	Quantile Regression, with EWMA volatility as the explanatory variable, sample size of 800, $\lambda = 0.94$

Table 4. Model number and respective description. The EWMA Smoothing Factor is denoted by λ. All models are evaluated based on the Total VaR.

Chapter 5

BACKTEST AND MODEL SELECTION

In the previous chapter, the processes required to calculate the VaR for the four different models were explored. In total, there were estimated 20 different models, generating daily historical VaR estimates over a ten-year period, from 11 February, 2013, to 27 January, 2023, referred to as the global period. After this, it is necessary to evaluate the performance of each model and select the best one for application to the upcoming one-year period.

The primary performance metric used is the number of exceedances. An exceedance occurs when the actual return for the day is worse than the VaR estimate for that same day. To assess these exceedances, two inference tests were employed: the Unconditional Coverage (UC) test (Kupiec, 1995) and the Berkowitz Conditional Coverage (BCP) test (Berkowitz et al., 2011). The UC test evaluates the number of exceedances, while the BCP test examines the autocorrelation between exceedances.

Although both tests assess model performance from different perspectives, the first criterion for evaluation is based on the UC test results. The BCP test will be used to distinguish between models with similar UC test performance. This approach is necessary because a model with a low number of exceedances may still fail the BCP test if those exceedances are clustered together. Conversely, a model with a higher number of exceedances may pass the BCP test if these exceedances are spread out over time.

The UC and the BCP tests will first be applied to the global period and, where relevant, to each individual year within this period. Evaluating model performance across specific time periods can provide insights into how models perform under different market conditions, which may recur in the future. However, the final decision will primarily rely on the results from the global period, as this provides a more consistent and comprehensive dataset.

The following sections will detail the methodology for each test, along with the backtest results, ultimately identifying the selected model for the upcoming one-year period.

5.1. Unconditional Coverage Test

The Unconditional Coverage (UC) test, introduced by Kupiec (1995), is based on a null hypothesis that is suitable when the number of exceedances aligns with the significance level α of the VaR model (Alexander, 2009). Recall that the VaR is defined as the worst loss that will not be exceeded with a confidence level of $1 - \alpha$, implying a α chance that the actual loss will indeed be worse than the VaR. For instance, in a sample of 1000 daily VaR estimates at a 99% confidence level (where the significance level α = 1%), it is expected that the worst observed loss will exceed the VaR 1,000 x α = 10 times, resulting in 10 exceedances. This corresponds to an exceedance rate of $\frac{10}{1,000} = 1\% = \alpha$.

Formally, for a sample of n observations, an exceedance for each observation is detected using an indicator function, defined as:

$$I_{\alpha,t} = \begin{cases} 1, & if\ r_t < -VaR_{1,\alpha,t} \\ 0, & otherwise \end{cases} \tag{33}$$

where r_t denotes the return observed for day t, and $VaR_{1,\alpha,t}$ represents the VaR estimated for day t. This results in a series of n observations, where each observation is either 1 or 0, depending on the indicator function.

The next step is to test the null hypothesis that the indicator function follows an independent and identically distributed (i.i.d.) Bernoulli process with a probability equal to the significance level α of the VaR model (Alexander, 2009). Therefore, the null and alternative hypothesis for the UC test can be expressed as:

$$H_o : \pi_{obs} = \pi_{exp} = \alpha$$
$$H_1 : \pi_{obs} \neq \pi_{exp}$$

where π_{obs} and π_{exp} are the observed and expected exceedance rates, respectively.

Formally, the test statistic is:

$$LR_{UC} = \left(\frac{\pi_{exp}}{\pi_{obs}}\right)^{n_1} \left(\frac{1 - \pi_{exp}}{1 - \pi_{obs}}\right)^{n_0} \tag{34}$$

where n_1 is the number of exceedances and $n_0 = n - n_1$ is the number of non-exceedances.

Under the null hypothesis ($\pi_{obs} = \pi_{exp}$), the distribution of the test statistic follows a chi-squared distribution with one degree of freedom: $-2\ ln\ (LR_{UC}) \sim \chi_1^2$.

As mentioned earlier, the model can be considered well-specified if the null hypothesis is not rejected at the 95% confidence level. This indicates that, according to the test, the exceedance rate is consistent with the expected value of α.

5.2. BCP Test

In the case of the BCP test, a model is considered well-specified when the exceedances are independent of each other (Berkowitz et al., 2011). This means that the occurrence of one exceedance does not provide information about when the next exceedance might occur, implying that the autocorrelation in exceedances is zero across all lags. For instance, if VaR is estimated daily, a model that is slower to adapt to changes in volatility may experience several exceedances during a week when volatility spikes suddenly and remains elevated. Even if such an outcome was not anticipated, these exceedances can persist and be observed on subsequent trading days, either consecutively or intermittently. This clustering of exceedances suggests that the model may not be adjusting quickly enough to recent increases in volatility. The BCP test is designed to detect such clustering events.

The null and alternative hypothesis for the BCP test can be expressed as follows:

$$H_0 : \hat{\rho}_k = 0, \forall k \in \{1, \ldots, K\}$$
$$H_1 : \exists k \in \{1, \ldots, K\}\, s.t. \hat{\rho}_k \neq 0$$

where $\hat{\rho}_k$ is the lag k autocorrelation of the series of n observations where each observation is either 1 or 0, given by the indicator function described in the UC test (Equation X) and K is the maximum autocorrelation lag considered in the test.

Formally, the test statistic for the BCP test is:

$$BCP\,(K) = n\,(n + 2) \sum_{k=1}^{K} \frac{\hat{\rho}_k^2}{n - k} \tag{35}$$

where n is the sample size (number of observations) of the test.

Under the null hypothesis, $\hat{\rho}_k = 0$, the distribution of the test statistic follows a chi-squared distribution with K degrees of freedom: $BCP\,(K) \sim \chi_K^2$. The lag K can be chosen, but it is important to consider the implications of selecting a larger or smaller value for K. A larger K provides insight into higher-order autocorrelations, but since the test statistic follows a chi-squared distribution with K degrees of freedom, increasing K raises the critical value, making it more difficult to reject the null hypothesis. Conversely, a smaller K increases the test sensitivity but ignores autocorrelations order greater than K.

Given these considerations, several BCP tests will be computed with K values ranging from 1 to 10. This approach allows for the detection of autocorrelations up to the 10^{th} lag while maintaining an acceptable level of test sensitivity.

The first criterion for selecting the most suitable model is the UC test for the global period. Models that pass the UC test for the global period will then be evaluated using the BCP test. The final analysis will assess the models 'performance for the annual sub-periods. However, it is important to note that it is possible that none of the models may pass the BCP test. In such cases, the analysis will focus solely on the UC test, both for the global period and for the annual sub-periods.

5.3. Backtest Results and Model Selection

The daily VaR estimates for all the models are computed using the global period spanning ten years starting from 11 February, 2013, to 27 January, 2023. Based on the tests described in the previous subsections, the UC test and the BCP test, the performance of each model will be evaluated not only for the global period but also for the annual sub-periods, ultimately selecting the most suitable model for the current portfolio composition.

During the analysis, only models based on Total VaR are evaluated. This approach is chosen because it provides a more reliable expectation of the portfolio's performance and viability, considering its dimension. Although the portfolio consists of sixty different stocks, this does not guarantee sufficient diversification to support an analysis based on Systematic VaR, which could lead to unreliable results.

The global period includes a total of n = 2600 observations. Given that the VaR models are calculated using a significance level of α = 1%, a well-specified model would be expected to produce approximately 26 exceedances ($2600 \times 1\% \approx 26$) from the perspective of the UC test.

Conventionally, the null hypothesis is rejected if the p-value of the test statistic is below 5%. Therefore, a model passes the UC or BCP tests if the null hypothesis is not rejected, meaning the p-value is above 5%.

The following table, Table 5, displays the number of exceedances, the exceedance rate, and the p-value of the UC test for all models over the global period.

Model Class	Model Number	Global Period		
		Number of Exceedances	Exceedance Rate (%)	p-value (%)
Normal	1	51	1.96%	0.00%
	2	50	1.92%	0.00%
SGSt	3	52	2.00%	0.00%
	4	51	1.96%	0.00%
	5	38	1.46%	2.69%
	6	**34**	**1.31%**	**13.22%**
	7	**36**	**1.38%**	**6.25%**
Historical	8	48	1.85%	0.01%
	9	45	1.73%	0.07%
	10	38	1.46%	2.69%
	11	48	1.85%	0.01%
	12	43	1.65%	0.22%
	13	39	1.50%	1.70%
QR	**14**	**36**	**1.38%**	**6.25%**
	15	**34**	**1.31%**	**13.22%**
	16	**30**	**1.15%**	**44.15%**
	17	**27**	**1.04%**	**84.47%**
	18	**26**	**1.00%**	**100.00%**
	19	**29**	**1.12%**	**56.15%**

Table 5. UC test results for the global period. The rows in bold indicate the models that pass the UC test. A model passes the UC test when its corresponding p-value is greater than 5%. For a description of each model, refer to Table 4.

Based on the number of exceedances, both Normal models, models 1 and 2, show a high number of exceedances, which points to a significant flaw: the assumption of a normal distribution for returns, which is not valid for the portfolio during the global period. The portfolio returns exhibit negative skewness, indicating an asymmetric distribution skewed to the left, along excess kurtosis (kurtosis > 3), which results in fatter tails compared to a normal distribution. The skewness and the kurtosis can be confirmed by the descriptive statistics present in Appendix A.1.. These statistics indicate that the portfolio may not be suitable for applying the Normal VaR, approach, as it exhibits a negative skewness of approximately -0,6920 and a kurtosis of 13,7173 over global period.

For the SGSt VaR models, only the models with a higher sample size pass the UC test, specifically models 6 and 7. These models, both with the same EWMA Smoothing Factor $\lambda = 0.93$, use 800 and 1000 observations, respectively. The first three SGSt VaR models display a much higher number of exceedances than expected (approximately 26 exceedances) and have exceedance rates exceeding the expected 1%.

Regarding Historical VaR models (models 8 to 13), none pass the UC test at the 95% confidence level. These models exhibit a higher number of exceedances and exceedance rates than expected.

Model Class	Model Number	p-value (%)		
		Global p-value	**Lowest p-value**	**Lag**
Normal	1	0.00%	0.00%	All
	2	0.00%	0.00%	2 and 3
SGSt	6	0.01%	0.01%	1 and 3
	7	0.01%	0.01%	3 and 4
QR	14	0.02%	0.02%	2
	15	0.01%	0.01%	1
	16	0.00%	0.00%	All
	17	0.00%	0.00%	All
	18	0.00%	0.00%	3, 4, 7, 8, 9 and 10
	19	0.00%	0.00%	All

Table 6. BCP test results for the global period. Since none of the models passed the BCP test for the global period and all have p-values below 5%, the table presents the p-value for the global period, along with the lowest p-value and the corresponding lag where it occurs.

Finally, for the Quantile Regression VaR (models 14 to 19), all pass the UC test at the 95% confidence level, with the number of exceedances closely aligning with the expected value ($2{,}600 \times 1\% \approx 26$). Detailing, despite all the models passing the UC test, the model that has the most suitable performance is model 18, because it is the model with whose number of exceedances is closest to 26 and an exceedance rate approximately equal to the significance level (α = 1%). However, it is important to note that model 17 also demonstrates a great performance for the current portfolio.

Table 6 presents the p-values of the BCP test for the global period, focusing on the lowest p-values and the corresponding lag where it occurs. Appendix B.1. provides the BCP test results for the global period for all the models.

As mentioned in the previous subsection, it is possible that none of the models pass the BCP test. In such a case, all models would have a p-value below 5%, which leads a premise to rejecting the null hypothesis. This suggests that none of the models perform well enough to pass the BCP test. The reasons for this could vary, including factors such as the analysis period or the portfolio composition.

Therefore, to draw a conclusion on the most suitable model, it is essential to further evaluate the UC test for the annual sub-periods. Table 7 shows the exceedance rate (%) and p-value (%) of the UC test across the annual sub-periods within the global period for the models that pass the UC test for the global period. The full UC test results for the annual sub-periods for all models are provided in Appendix B.2..

Model Class	Model Nr.	2023-2013		2023-2022		2022-2021		2021-2020		2020-2019		2019-2018	
		Exc.	p-value	Exc.	p-value	Exc.	p-value	Exc.	p-value	Exc.	p-value	Exc.	p-value
Normal	1	1.96%	0.00%	1.92%	**18.44%**	2.31%	**7.01%**	3.85%	0.04%	1.92%	**18.44%**	2.31%	**7.01%**
	2	1.92%	0.00%	1.92%	**18.44%**	2.31%	**7.01%**	3.46%	0.18%	1.92%	**18.44%**	1.92%	**18.44%**
SGSt	6	1.31%	**13.22%**	0.77%	**69.67%**	1.54%	**41.87%**	2.69%	2.34%	1.54%	**41.87%**	0.77%	**69.67%**
	7	1.38%	**6.25%**	0.77%	**69.67%**	1.92%	**18.44%**	3.08%	0.69%	1.54%	**41.87%**	0.77%	**69.67%**
QR	14	1.38%	**6.25%**	1.15%	**80.77%**	1.15%	**80.77%**	1.54%	**41.87%**	1.92%	**18.44%**	0.77%	**69.67%**
	15	1.31%	**13.22%**	0.77%	**69.67%**	0.77%	**69.67%**	1.54%	**41.87%**	1.92%	**18.44%**	0.77%	**69.67%**
	16	1.15%	**44.15%**	0.38%	**25.44%**	0.77%	**69.67%**	1.54%	**41.87%**	1.54%	**41.87%**	0.77%	**69.67%**
	17	1.04%	**84.47%**	0.38%	**25.44%**	0.77%	**69.67%**	1.54%	**41.87%**	1.54%	**41.87%**	0.77%	**69.67%**
	18	1.00%	**100.00%**	0.00%	2.22%	0.77%	**69.67%**	1.92%	**18.44%**	1.54%	**41.87%**	0.38%	**25.44%**
	19	1.12%	**56.15%**	0.00%	2.22%	0.77%	**69.67%**	1.92%	**18.44%**	1.54%	**41.87%**	0.77%	**69.67%**

Model Class	Model Nr.	2023-2013		2018-2017		2017-2016		2016-2015		2015-2014		2014-2013	
		Exc.	p-value	Exc.	p-value	Exc.	p-value	Exc.	p-value	Exc.	p-value	Exc.	p-value
Normal	1	1.96%	0.00%	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	1.54%	**41.87%**	1.15%	**80.77%**
	2	1.92%	0.00%	1.92%	**18.44%**	1.15%	**80.77%**	2.31%	**7.01%**	1.15%	**80.77%**	1.15%	**80.77%**
SGSt	6	1.31%	**13.22%**	1.54%	**41.87%**	0.77%	**69.67%**	1.92%	**18.44%**	0.77%	**69.67%**	0.77%	**69.67%**
	7	1.38%	**6.25%**	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	0.77%	**69.67%**	0.38%	**25.44%**
QR	14	1.38%	**6.25%**	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	1.54%	**41.87%**	1.15%	**80.77%**
	15	1.31%	**13.22%**	1.92%	**18.44%**	0.77%	**69.67%**	1.92%	**18.44%**	1.15%	**80.77%**	1.54%	**41.87%**
	16	1.15%	**44.15%**	1.54%	**41.87%**	0.77%	**69.67%**	1.92%	**18.44%**	1.15%	**80.77%**	0.77%	**69.67%**
	17	1.04%	**84.47%**	0.77%	**69.67%**	0.77%	**69.67%**	1.92%	**18.44%**	1.15%	**80.77%**	0.77%	**69.67%**
	18	1.00%	**100.00%**	0.77%	**69.67%**	0.77%	**69.67%**	1.92%	**18.44%**	1.15%	**80.77%**	0.77%	**69.67%**
	19	1.12%	**56.15%**	1.15%	**80.77%**	0.77%	**69.67%**	1.92%	**18.44%**	1.54%	**41.87%**	0.77%	**69.67%**

Table 7. UC test results for the annual sub-periods. A model passes the UC test when its corresponding p-value is greater than 5%.

Notably, only model 17 pass the UC test for all the annual sub-periods. It is important to note that model 17 includes a sample size of 600 observations (n = 600 observations), an EWMA smoothing factor of $\lambda = 0.94$, and the use of EWMA volatility as the explanatory variable.

Taking into account the UC test for the global period, model 17 record 27 exceedances, which is close to the expected number ($2{,}600 \times 1\% \approx 26$). Additionally, the exceedance rate is 1.04%, which is closer to the significance level ($\alpha = 1\%$).

A final evaluation involves reviewing the BCP test results for the annual sub-periods. However, since the BCP test results are unfavorable for all the models evaluated, they will be disregarded in selecting the most suitable model, with the focus instead placed on the results of the UC test for both the overall period and the annual sub-periods.

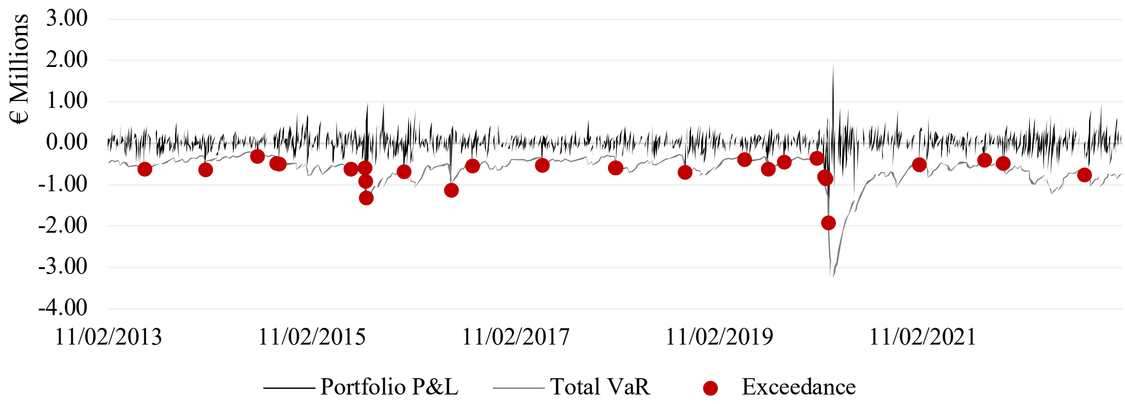


Figure 1. Quantile Regression model 17. The red dots represent the P&L of the days in which the VaR was exceeded.

Figure 1 displays the daily VaR estimates for model 17 alongside the portfolio's daily P&L over the backtest period. Table 8 illustrates the performance of model 17 during the backtest period, covering approximately 10 years of data, and highlights the magnitude of the 26 exceedances. Following a thorough analysis, Model 17 was identified as the most suitable model, and daily VaR estimates will be calculated accordingly.

Model 17				
Date of Exceedance	-VaR (€)	P&L (€)	Size of Exceedance	
			Value (€)	% of VaR
2022-09-13	-696,089.11 €	-765,621.37 €	-69,532.26 €	9.08%
2021-11-26	-468,248.83 €	-479,272.05 €	-11,023.22 €	2.30%
2021-09-20	-359,245.32 €	-407,071.55 €	-47,826.22 €	11.75%
2021-01-29	-480,511.84 €	-507,804.40 €	-27,292.56 €	5.37%
2020-03-09	-1,360,165.30 €	-1,918,729.99 €	-558,564.69 €	29.11%
2020-02-27	-827,638.55 €	-857,957.78 €	-30,319.23 €	3.53%
2020-02-24	-465,001.04 €	-797,081.96 €	-332,080.92 €	41.66%
2020-01-27	-288,709.14 €	-357,241.02 €	-68,531.88 €	19.18%
2019-10-02	-400,697.64 €	-453,147.34 €	-52,449.70 €	11.57%
2019-08-05	-370,737.37 €	-622,789.91 €	-252,052.54 €	40.47%
2019-05-13	-345,646.21 €	-386,836.21 €	-41,190.00 €	10.65%
2018-10-10	-310,030.82 €	-698,027.51 €	-387,996.69 €	55.58%
2018-02-05	-509,386.86 €	-593,725.14 €	-84,338.28 €	14.20%
2018-02-02	-292,537.77 €	-591,262.53 €	-298,724.76 €	50.52%
2017-05-17	-310,778.80 €	-529,650.55 €	-218,871.75 €	41.32%
2016-09-09	-396,114.51 €	-534,615.50 €	-138,500.99 €	25.91%
2016-06-24	-528,979.42 €	-1,123,432.81 €	-594,453.39 €	52.91%
2016-01-07	-551,719.31 €	-679,170.93 €	-127,451.61 €	18.77%
2015-08-24	-889,412.12 €	-1,309,971.02 €	-420,558.89 €	32.10%
2015-08-21	-623,369.21 €	-912,552.21 €	-289,182.99 €	31.69%
2015-08-20	-481,064.35 €	-590,537.28 €	-109,472.93 €	18.54%
2015-06-30	-449,805.19 €	-624,246.63 €	-174,441.44 €	27.94%
2014-10-16	-463,409.27 €	-500,125.33 €	-36,716.05 €	7.34%
2014-10-07	-344,591.04 €	-486,897.86 €	-142,306.82 €	29.23%
2014-07-31	-238,090.58 €	-306,432.57 €	-68,341.99 €	22.30%
2014-01-24	-287,245.62 €	-632,813.23 €	-345,567.61 €	54.61%
2013-06-20	-566,199.17 €	-620,293.73 €	-54,094.56 €	8.72%
Average			**-184,514.22 €**	**25.05%**

Table 8. Model 17 global period exceedance details. The date of the exceedances and the days between them add additional detail to the BCP test results.

As shown in Figure 1, most of the exceedances throughout the global period, occurred with significant gaps between them. Table 8 provides a detailed breakdown of these exceedances for the global period, for both models.

Chapter 6

VALUE-AT-RISK MANAGEMENT

According to Alexander (2009), Economic Capital (EC) can be described as the capital at risk due to a portfolio's activities over a specified time horizon and at a certain significance level. This risk is typically measured by Value at Risk (VaR), meaning EC and VaR are numerically identical. By establishing a target EC for the portfolio, it is possible to base the risk management strategies on this benchmark.

Moving forward, the VaR will be calculated using model 17. As mentioned in the previous section, the model belongs to the Quantile Regression VaR class with a single independent variable (EWMA volatility), a sample size of 600 observations, and an EWMA smoothing factor of $\lambda = 0.94$. As shown in Figure 1 of the previous chapter, during the backtesting period, VaR ranged between €200,000 and €3,600,000, with most results falling between €400,000 and €1,000,000. On January 27, 2023, the Total VaR was, approximately, €650,990. Notably, during the one-year period analysis, VaR showed a declining trend, indicating improved portfolio performance. To set the EC according to what will happen in the market during the upcoming year, the final capital risk will only be determined after calculating the 1-day VaR.

The process used for this calculation involved estimating the next day's VaR at the close of each trading day based on the current portfolio composition. This process will be repeated daily for one year, from January 27, 2023, to February 1, 2024.

By using the 1-day VaR results from model 17 over 264 days (one year), it is possible to establish the EC. As expected, the Total VaR decreased over the analysis period. Had the EC been fixed earlier, the chosen value might be higher than any of the actual results (since the highest Total VaR for the 1-day VaR was around €650,000). Hence, for the upcoming year, it is expected that VaR will remain around €425,000, meaning the maximum daily EC of €425,000, slightly below the geometric average for the year, which is, approximately, €434,735. This represents the capital at risk for holding the portfolio over the next day, as indicated by the 1-day VaR, should not surpass €425,000.

The key question is which strategy to implement. To reduce risk, it is possible to either adjust the portfolio's positions or hedge current exposures. For this portfolio, a hedging strategy will be used. To do so effectively, first it is necessary to assess the contribution of each risk factor to the portfolio's VaR. This is accomplished through marginal VaR decompositions, which breaks down the portfolio's overall VaR into portions attributed to each risk factor. Essentially, while VaR represents the entire risk, marginal decomposition shows how the risk is distributed across different factors.

The hedging strategy implemented during the VaR estimation process involves investing the difference between the VaR and the EC in the S&P 500 index by taking a short position. This is executed through a short position in a one-day maturity future or forward contract on the relevant index, - in this case, the S&P 500. Doing so also implicitly creates a short exposure to the associated exchange rate (USA/EUR). To neutralize this currency exposure, a corresponding long position in a USD/EUR future or forward is required. However, reducing the S&P 500 exposure alone was insufficient to bring VaR within the required limit. Consequently, across the 264 days analyzed, full hedging was not always achievable to fully neutralize the position. On such occasions, only the short position on the S&P 500 was implemented, resulting in active exposure to two risk factors: the equity index and the corresponding exchange rate.

Section 1 will detail the methodology for the marginal VaR decomposition and the resulting strategy, while Section 2 will present the outcomes of this strategy over the one-year period.

6.1. VaR Decomposition and Management Strategy

The choice of decomposition is made by the user. In this case, the VaR estimate is initially broken down by asset class for 27 January, 2023, the first day when VaR exceeds €425,000. Table 9 illustrates this decomposition by asset class.

Description	Asset Class		Total
	Equity	Bonds	
VaR (%)	90.42%	9.58%	100.00%
VaR (EUR)	571,958.26 €	60,623.57 €	632,581.83 €
VaR Breakdown (%)	90.42%	9.58%	100.00%

Table 9. VaR decomposition by asset class. This tables shows the contribution of each asst class to the VaR estimate for 27 January 2023.

As expected, since the portfolio consists of approximately 80% equities, equity exposure significantly increases the VaR, while bond exposure has a comparatively smaller impact.

Given that equity exposure has a higher contribution to the VaR, the marginal used to guide the hedging strategy is based on risk factor groups. This divides the Total VaR into three parts: exchange rates (HKD/EUR, JPY/EUR, GBP/EUR, USD/EUR), indices (N225, HIS, FCHI, AEX, FTSE, GSPC), and interest rates (IR_EUR, IR_USD). This approach helps identify which factors contribute most to the VaR, and it emphasizes understanding the exposure to the currency in which stocks are denominated relative to EUR through exchange rates.

Table 10 displays the marginal VaR decomposition by risk factor group for the first day it exceeds €425,000: January 27, 2023.

Description		Asset Class		
		Market	VaR (%)	VaR (€)
Equity	Market	N225	5.32%	33,635.09 €
		HSI	1.68%	10,634.19 €
		FCHI	8.92%	56,405.21 €
		AEX	-0.09%	-558.79 €
		FTSE	8.79%	55,576.82 €
		GSPC	48.19%	304,868.97 €
	Currency	JPY/EUR	5.06%	32,020.41 €
		HKD/EUR	0.69%	4,371.73 €
		GBP/EUR	0.26%	1,666.27 €
		USD/EUR	14.19%	89,794.61 €
Bonds	Interest Rate	IR_EUR	3.85%	24,384.50 €
		IR_USD	3.13%	19,782.81 €
Total			**100.00%**	**632,581.83 €**

Table 10. VaR decomposition by risk factor group. This illustrates the contribution of Equity and Bonds to the VaR estimate for 27 January, 2023. Equities are divided into Market and Currency components, while Bonds are categorized by Interest Rate.

Based on this analysis, one potential strategy to reduce the VaR is to hedge the exposure to the GSPC index (S&P 500), as many portfolio stocks belong to this index. This can be achieved by taking a short position in the GSPC index. To maintain exposure to the USD/EUR exchange rate, a long position in the USD/EUR exchange rate should be added in the same proportion. Therefore, the strategy would involve taking a short position in the GSPC and a long position in the USD/EUR exchange rate simultaneously, where possible.

If the VaR estimate exceeds €425,000 on the following day, a short position in the GSPC index will be implemented to ensure that the portfolio's VaR does not exceed this threshold. At the same time, when possible, the exposure to the USD/EUR exchange rate will be preserved.

Since VaR is estimated daily, the same reinvestment strategy (described above in this section) will be applied, and the VaR decomposition will be reviewed each day based on the risk factor group. The hedging strategy will be triggered if the VaR exceeds €425,000 and will not be used if it falls below this level. This process will continue daily until February 1, 2024.

6.2. VaR Management Results

To facilitate comparison, the portfolio with the hedging strategy will be referred as "Hedged Portfolio", and the same portfolio without the hedging strategy as the "Unhedged Portfolio".

Figure 2 shows the daily VaR estimates for both the Hedged and Unhedged Portfolios throughout the one-year period.

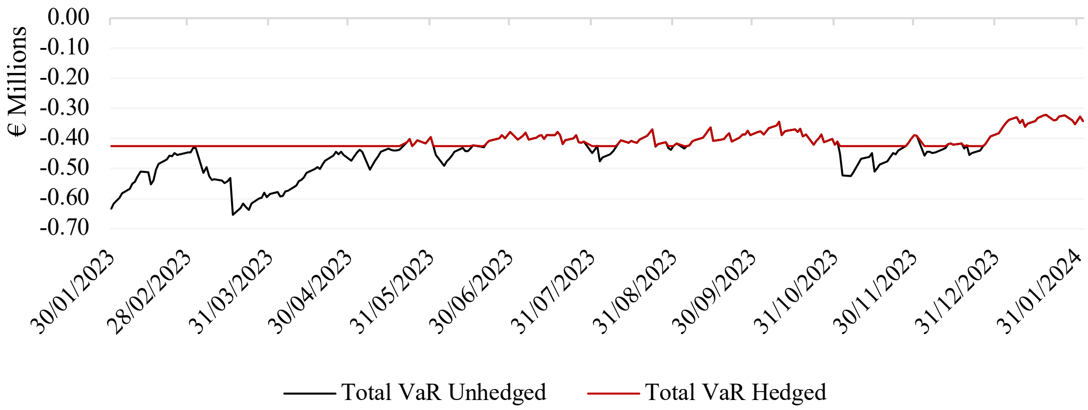


Figure 2. Daily VaR estimates.

Additionally, Figure 3 illustrates how the hedge positions evolved throughout the one-year period.

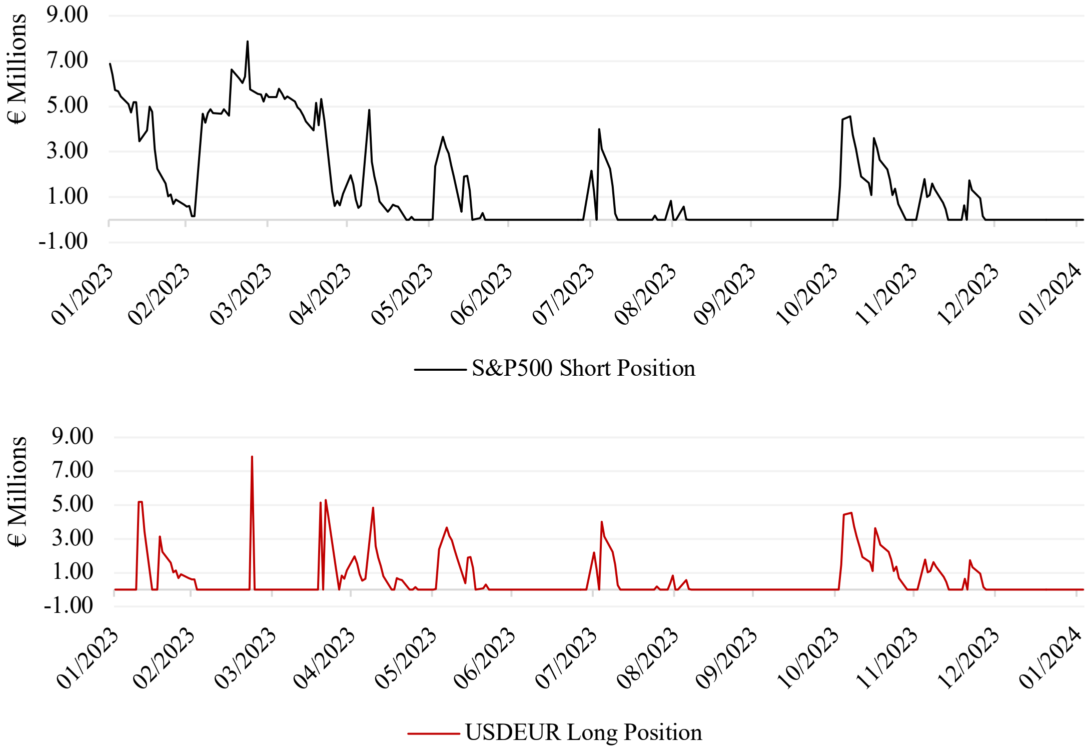
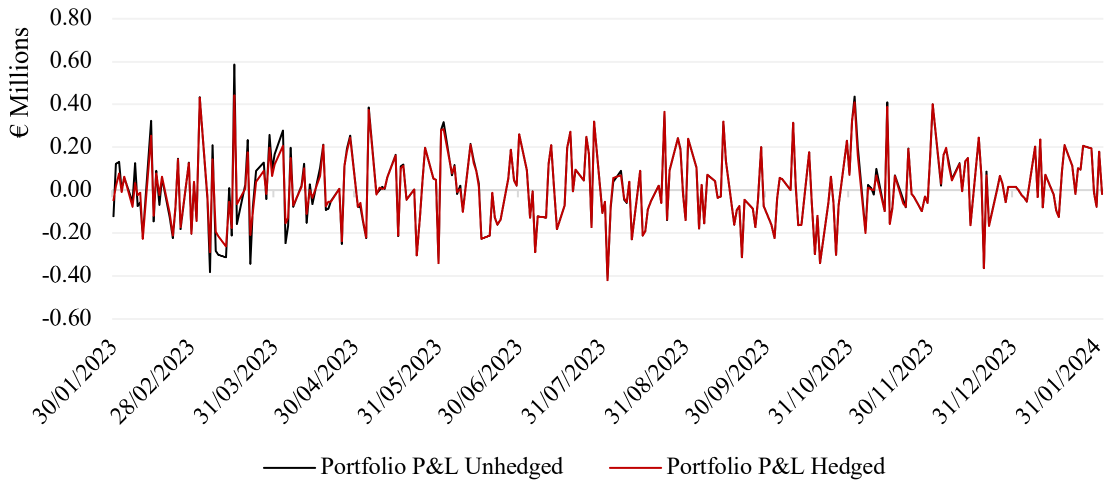


Figure 3. Daily short positions in GSPC and long positions in USD/EUR, respectively.

From Figures 2 and 3, it is possible to conclude that whenever the VaR exceeds the €425,000 target value, a short position in the S&P500 stock index is initiated. Furthermore, when conditions allow, – specifically, when a Target Change of zero can be achieved, with both strategies outlined in the previous section, without causing constant fluctuations between positive and negative values – a long position is also taken in the USD/EUR exchange rate, in equal proportion. This strategy ensures that the VaR of the Hedged portfolio stays consistently at €425,000 throughout the year. In contrast, the VaR of the Unhedged portfolio exceeds this limit, particularly during the first half of the year, reaching a maximum of €653,364.20 for the VaR estimation on March 17, 2023.

The next figure, Figure 4, presents the daily P&L for both the Unhedged and Hedged portfolios over the one-year period. Figure 5 provide a detailed comparison of the VaR performance (shown as a loss) for each portfolio during the period.



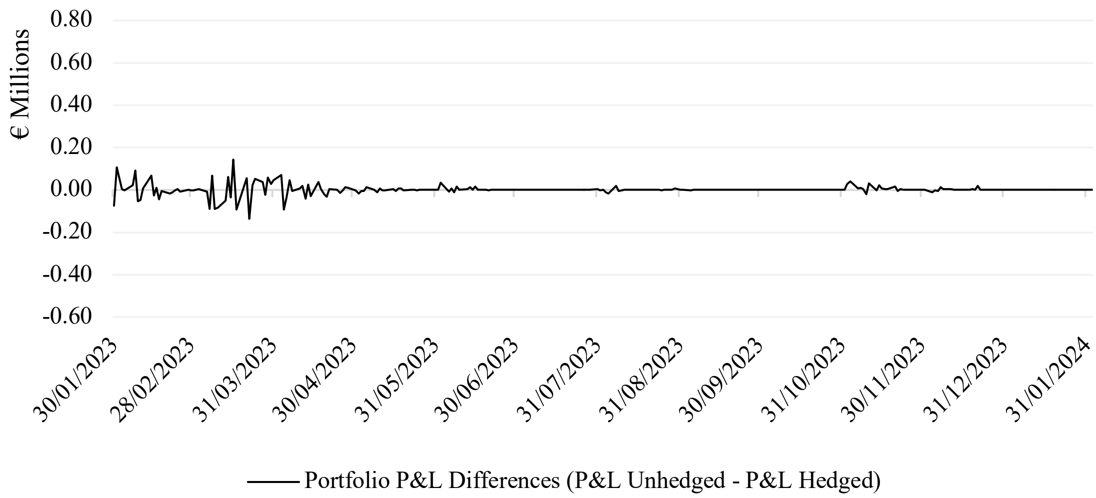


Figure 4. Daily P&L and the difference between the Unhedged and Hedged portfolio.

As expected, Figure 5 illustrates that the Unhedged portfolio exhibits more volatility in its daily P&L compared to the Hedged portfolio. This increased volatility is particularly noticeable during periods with larger short positions, resulting in a significant divergence between the P&Ls of the two portfolios – especially from the end of January 2023 through June 2023, and in smaller periods thereafter.

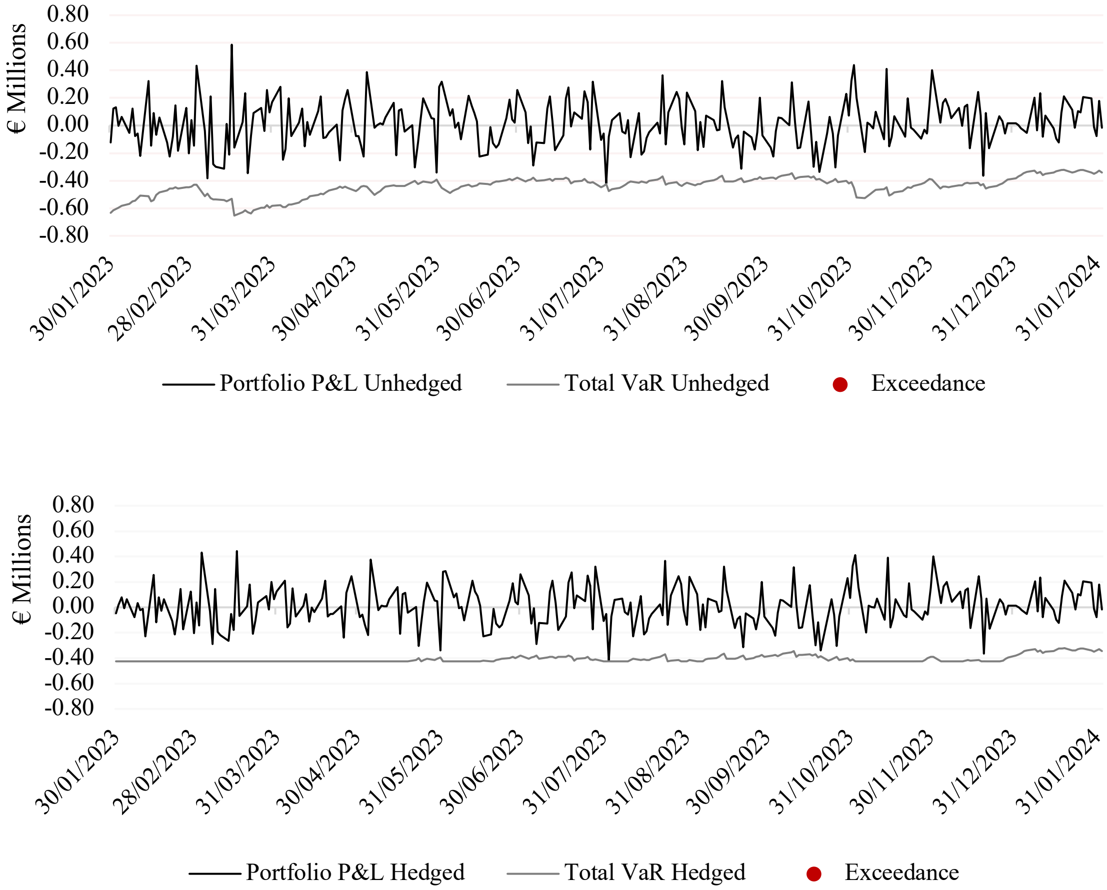


Figure 5. Unhedged VaR performance (Panel A) and Hedged VaR performance (Panel B) for the one-year period.

Furthermore, Figure 5 confirm that neither portfolio exceeded its VaR limits during the year, which means that neither portfolio had exceedances during the one-year period. As expected, since the unhedged portfolio has no exceedances, the hedged portfolio also shows none. This outcome reflects the model's ability to adapt to changing market conditions. In isolation, higher volatility leads to a higher VaR, which raises the threshold for exceedances – making them less likely, particularly in the case of the hedged portfolio.

Figure 6 displays the daily cumulative P&L for both Unhedged and Hedged portfolios over the one-year period. From this, it is possible to conclude that while both portfolios experienced a profit, the Hedged portfolio's profit was smaller, indicating that the hedging strategy was effective in reducing further losses and limiting overall risk, even though it resulted in lower returns compared to the Unhedged portfolio.

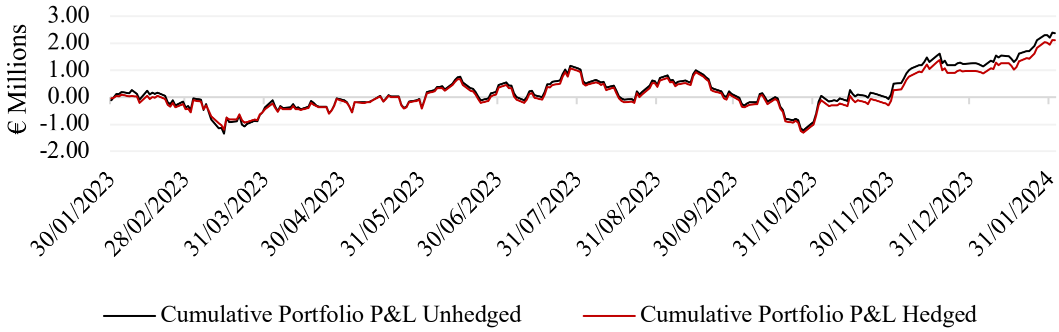


Figure 6. Daily cumulative P&L.

Table 11 displays the returns for the year for both the Unhedged and Hedged portfolios. The conclusion taken by the observation of Table 11 is that the Hedged portfolio delivered a higher return for the year compared to the Unhedged portfolio. However, because the two portfolios have different risk profiles due to the hedging strategy, a direct comparison of their returns may not be entirely fair without considering the level of risk taken by each portfolio. This highlights the importance of using a risk-adjusted performance metric for a more balanced evaluation.

Metric	Portfolio	
	Unhedged	Hedged
P&L	2,375,643.52 €	2,102,345.41 €
Return (%)	7.81%	6.91%

Table 11. One year portfolio returns.

The Return on Risk-Adjusted Capital (RORAC) metric, as defined by Matten (1996), relates the return to the risk taken to achieve it. It is calculated as follows:

$$RORAC = \frac{P\&L}{EC} \tag{36}$$

where $P\&L$ represents the average P&L for the year, and EC denotes the average daily EC (measured by the VaR) over the course of the year.

Table 12 presents the average RORAC for both Unhedged and Hedged portfolios. Based on these results, it is evident that the Hedged portfolio delivered smaller profits compared to the Unhedged portfolio, while simultaneously reducing its risk exposure. While the outcome was not optimal given the market conditions during the year, it indicates potential areas for improvement.

Metric	Portfolio	
	Unhedged	**Hedged**
P&L (€)	8 998.65 €	7 933.38 €
EC (€)	440 472.46 €	406 373.59 €
RORAC (%)	2.04%	1.95%

Table 12. Average RORAC for the one-year period.

Chapter 7

CONCLUSION

As mentioned earlier, the aim of this project is to measure and manage the daily Value-at-Risk (VaR) of the chosen portfolio over the one-year period from January 27, 2023, to February 1, 2024 ensuring it remained below the specified maximum limit, the Economic Capital (EC) target value.

The portfolio includes equities from Hong Kong, Japan, Europe, and U.S., as well as bonds from the U.S. and European markets. Before managing the VaR, it is necessary to measure it. With various types of models available to assess market risk, the goal was to select the most suitable model to proceed with the analysis. Four types of VaR models were evaluated: Normal, Skewed Generalized Student t (SGSt), Historical, and Quantile Regression (QR). In total, nineteen distinct models were analyzed to ensure a comprehensive evaluation. Each model's performance was tested through a Backtest, using the Unconditional Coverage (UC) test and the Berkowitz, Christoffersen, and Pelletier (BCP) test.

The Backtest results, based on the portfolio composition as of January 27, 2023, produced both expected and surprising outcomes. As predicted, the Normal VaR model, which assumes normally distributed returns, failed the Backtest, indicating that the portfolio's returns are not normally distributed and making this model unsuitable. In the parametric category, the SGSt VaR model, designed to improve upon Normal, did not fail the Backtest but also did not outperform the others. Among non-parametric models, the Historical VaR showed poor results, with a higher exceedance rate and more frequent exceedances compared to the SGSt VaR. Ultimately, the models from QR VaR class performed the best in the Backtest, with fewer exceedances and a lower exceedance rate than the other models. Further analysis of individual annual sub-periods led to the selection of the Quantile Regression VaR model, using a rolling sample of 600 observations, one explanatory variable (EWMA volatility), and an EWMA smoothing factor of $\lambda = 0.94$, as the most suitable model for the portfolio in analysis.

To manage the VaR effectively, the daily maximum EC limit was set at €425,000, based on the average of the daily estimated Total VaR for the one-year period starting on 27 January, 2023 and ending on 1 February, 2024. After collecting all the estimates for Total VaR for the one-year period, if the estimate for the following day (starting on 30 January, 2023) exceeded this limit, a risk management strategy would be implemented to reduce the VaR back to the €425,000 threshold.

Since the EC was established according to the average of the estimated Total VaR, approximately fifty percent of the days exceeded the target value. Therefore, whenever the VaR surpassed the

EC, a VaR decomposition was conducted, revealing that the primary source of risk was U.S. equities, which made up the largest part of the portfolio. To reduce the VaR to the target level, a hedging strategy was implemented. This involved taking short positions in the S&P 500 and, when possible, a proportional long position in the USD/EUR exchange rate (though this was only applicable on a few occasions due to the large gap between the EC and VaR). This strategy was maintained throughout the year, with adjustments made based on the VaR estimates relative to the maximum limit. For clarity, the portfolio with the hedging strategy is referred to as the Hedged portfolio, while the same portfolio without the hedging strategy is the Unhedged portfolio.

The results indicated that the hedging strategy successfully kept the VaR at or below the €425,000 threshold throughout the year. In contrast, without active management, the VaR would have exceeded this limit for a significant portion of the year, peaking at nearly €690000 in May. When comparing the returns of the Hedged and Unhedged portfolios using the RORAC (Return on Risk-Adjusted Capital) performance metric, both portfolios experienced positive returns for most of the year. However, contrary to the expectation that the hedging strategy would reduce risk without significantly impacting returns, the Hedged portfolio did not have only lower risk but also smaller returns compared to the Unhedged portfolio. This was mainly due to the larger exposure to U.S. equities, a significant part of the portfolio. Although the risk was reduced, the returns did not increase proportionally. In conclusion, the Hedged portfolio offered better protection against potential losses compared to the Unhedged portfolio.

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Appendixes

APPENDIX A

Portfolio Returns Descriptive Statistics

A.1.

Period	Mean	Median	Max	Min	StdDev	Skew	Kurt
Global Period (2013-2023)	**0.0005**	**0.0007**	**0.0638**	**-0.0703**	**0.0078**	**-0.6920**	**13.7173**
2022-2023	-0.0002	-0.0007	0.0316	-0.0264	0.0092	0.2758	3.5672
2021-2022	0.0002	0.0003	0.0316	-0.0264	0.0077	0.0820	4.3004
2020-2021	0.0006	0.0007	0.0638	-0.0703	0.0106	-1.1241	16.0866
2019-2020	0.0007	0.0011	0.0638	-0.0703	0.0105	-1.1894	16.9372
2018-2019	0.0004	0.0010	0.0195	-0.0229	0.0060	-0.5362	4.3877
2017-2018	0.0002	0.0009	0.0195	-0.0229	0.0056	-0.5227	4.6047
2016-2017	0.0005	0.0008	0.0275	-0.0369	0.0060	-0.3928	7.4546
2015-2016	0.0007	0.0007	0.0327	-0.0431	0.0083	-0.3788	6.0203
2014-2015	0.0008	0.0011	0.0327	-0.0431	0.0076	-0.3711	6.6303
2013-2014	0.0007	0.0009	0.0249	-0.0208	0.0059	-0.2791	4.1753

Table 13. Portfolio Return Descriptive Statistics for global period and annual sub-periods. The global Backtest period spans from 11 February, 2013, to 27 January, 2023. Returns are calculated by dividing the portfolio P&L by the portfolio value on the first day of the one-year period, which was € 3,0416,966 as of 27 January, 2023 (used to estimate the VaR for 30 January, 2023).

APPENDIX B

Detailed Backtest Results

B.1.

Model Class	Model Nr.	p-value (%)										
			Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
Normal	1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	2	0.00%	0.16%	0.00%	0.00%	0.01%	0.02%	0.03%	0.04%	0.05%	0.02%	0.02%
SGSt	3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	5	0.03%	0.09%	0.06%	0.03%	0.07%	0.13%	0.22%	0.39%	0.59%	0.86%	0.38%
	6	0.01%	0.01%	0.04%	0.01%	0.02%	0.05%	0.09%	0.14%	0.23%	0.37%	0.09%
	7	0.01%	0.03%	0.02%	0.01%	0.01%	0.03%	0.06%	0.10%	0.17%	0.26%	0.09%
Hist.	8	0.00%	0.07%	0.16%	0.04%	0.12%	0.28%	0.41%	0.46%	0.62%	**1.09%**	**1.13%**
	9	0.01%	0.02%	0.04%	0.01%	0.02%	0.04%	0.07%	0.07%	0.10%	0.19%	0.17%
	10	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	11	0.04%	0.07%	0.16%	0.04%	0.12%	0.19%	0.29%	0.32%	0.44%	0.47%	0.50%
	12	0.01%	0.01%	0.01%	0.01%	0.03%	0.07%	0.11%	0.09%	0.14%	0.10%	0.08%
	13	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
QR	14	0.02%	0.03%	0.02%	0.04%	0.09%	0.18%	0.32%	0.10%	0.17%	0.26%	0.39%
	15	0.01%	0.01%	0.04%	0.09%	0.20%	0.37%	0.65%	0.14%	0.23%	0.37%	0.51%
	16	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	17	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	18	0.00%	0.06%	0.09%	0.00%	0.00%	0.01%	0.02%	0.00%	0.00%	0.00%	0.00%
	19	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 14. BCP test results for the global period. As shown in the table, none of the models pass the BCP test for the global period, as all p-values are below the 1% significance level for the global period. For the ten lags comprising the global period, p-values exceeding 1% are highlighted in bold. The characteristics of each model are detailed in Table 4.

B.2.

Model Class	Model Nr.	2023-2013 Exc.	2023-2013 p-value	2023-2022 Exc.	2023-2022 p-value	2022-2021 Exc.	2022-2021 p-value	2021-2020 Exc.	2021-2020 p-value	2020-2019 Exc.	2020-2019 p-value	2019-2018 Exc.	2019-2018 p-value	2018-2017 Exc.	2018-2017 p-value	2017-2016 Exc.	2017-2016 p-value	2016-2015 Exc.	2016-2015 p-value	2015-2014 Exc.	2015-2014 p-value	2014-2013 Exc.	2014-2013 p-value
Normal	1	1.96%	0.00%	1.92%	**18.44%**	2.31%	**7.01%**	3.85%	0.04%	1.92%	**18.44%**	2.31%	**7.01%**	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	1.54%	**41.87%**	1.15%	**80.77%**
	2	1.92%	0.00%	1.92%	**18.44%**	2.31%	**7.01%**	3.46%	0.18%	1.92%	**18.44%**	1.92%	**18.44%**	1.92%	**18.44%**	1.15%	**80.77%**	2.31%	**7.01%**	1.15%	**80.77%**	1.15%	**80.77%**
SGSt	3	2.00%	0.00%	1.15%	**80.77%**	1.54%	**41.87%**	8.46%	0.00%	1.54%	**41.87%**	0.77%	**69.67%**	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	0.77%	**69.67%**	1.15%	**80.77%**
	4	1.96%	0.00%	1.15%	**80.77%**	1.15%	**80.77%**	8.46%	0.00%	1.54%	**41.87%**	0.77%	**69.67%**	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	0.77%	**69.67%**	1.15%	**80.77%**
	5	1.46%	2.69%	0.77%	**69.67%**	1.92%	**18.44%**	3.46%	0.18%	1.54%	**41.87%**	1.15%	**80.77%**	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	0.77%	**69.67%**	0.38%	**25.44%**
	6	1.31%	**13.22%**	0.77%	**69.67%**	1.54%	**41.87%**	2.69%	2.34%	1.54%	**41.87%**	0.77%	**69.67%**	1.54%	**41.87%**	0.77%	**69.67%**	1.92%	**18.44%**	0.77%	**69.67%**	0.77%	**69.67%**
	7	1.38%	**6.25%**	0.77%	**69.67%**	1.92%	**18.44%**	3.08%	0.69%	1.54%	**41.87%**	0.77%	**69.67%**	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	0.77%	**69.67%**	0.38%	**25.44%**
Hist.	8	1.85%	0.01%	0.77%	**69.67%**	1.92%	**18.44%**	2.69%	2.34%	1.92%	**18.44%**	1.92%	**18.44%**	1.54%	**41.87%**	1.15%	**80.77%**	2.69%	2.34%	1.92%	**18.44%**	1.92%	**18.44%**
	9	1.73%	0.07%	0.77%	**69.67%**	1.15%	**80.77%**	2.69%	2.34%	1.92%	**18.44%**	1.92%	**18.44%**	1.54%	**41.87%**	1.15%	**80.77%**	2.31%	**7.01%**	1.92%	**18.44%**	1.92%	**18.44%**
	10	1.46%	2.69%	0.77%	**69.67%**	1.15%	**80.77%**	2.31%	**7.01%**	1.54%	**41.87%**	1.54%	**41.87%**	1.54%	**41.87%**	0.77%	**69.67%**	1.92%	**18.44%**	1.92%	**18.44%**	1.15%	**80.77%**
	11	1.85%	0.01%	1.15%	**80.77%**	1.92%	**18.44%**	3.08%	0.69%	1.92%	**18.44%**	1.92%	**18.44%**	1.54%	**41.87%**	1.15%	**80.77%**	2.69%	2.34%	1.15%	**80.77%**	1.92%	**18.44%**
	12	1.65%	0.22%	0.77%	**69.67%**	1.15%	**80.77%**	2.69%	2.34%	1.92%	**18.44%**	1.54%	**41.87%**	1.54%	**41.87%**	1.15%	**80.77%**	2.69%	2.34%	1.54%	**41.87%**	1.54%	**41.87%**
	13	1.50%	1.70%	0.77%	**69.67%**	1.15%	**80.77%**	2.69%	2.34%	1.54%	**41.87%**	1.54%	**41.87%**	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	1.54%	**41.87%**	1.15%	**80.77%**
QR	14	1.38%	**6.25%**	1.15%	**80.77%**	1.15%	**80.77%**	1.54%	**41.87%**	1.92%	**18.44%**	0.77%	**69.67%**	1.54%	**41.87%**	1.15%	**80.77%**	1.92%	**18.44%**	1.54%	**41.87%**	1.15%	**80.77%**
	15	1.31%	**13.22%**	0.77%	**69.67%**	0.77%	**69.67%**	1.54%	**41.87%**	1.92%	**18.44%**	0.77%	**69.67%**	1.92%	**18.44%**	0.77%	**69.67%**	1.92%	**18.44%**	1.15%	**80.77%**	1.54%	**41.87%**
	16	1.15%	**44.15%**	0.38%	**25.44%**	0.77%	**69.67%**	1.54%	**41.87%**	1.54%	**41.87%**	0.77%	**69.67%**	1.54%	**41.87%**	0.77%	**69.67%**	1.92%	**18.44%**	1.15%	**80.77%**	0.77%	**69.67%**
	17	1.04%	**84.47%**	0.38%	**25.44%**	0.77%	**69.67%**	1.54%	**41.87%**	1.54%	**41.87%**	0.77%	**69.67%**	0.77%	**69.67%**	0.77%	**69.67%**	1.92%	**18.44%**	1.15%	**80.77%**	0.77%	**69.67%**
	18	1.00%	**100.00%**	0.00%	2.22%	0.77%	**69.67%**	1.92%	**18.44%**	1.54%	**41.87%**	0.38%	**25.44%**	0.77%	**69.67%**	0.77%	**69.67%**	1.92%	**18.44%**	1.15%	**80.77%**	0.77%	**69.67%**
	19	1.12%	**56.15%**	0.00%	2.22%	0.77%	**69.67%**	1.92%	**18.44%**	1.54%	**41.87%**	0.77%	**69.67%**	1.15%	**80.77%**	0.77%	**69.67%**	1.92%	**18.44%**	1.54%	**41.87%**	0.77%	**69.67%**

Table 15. UC test results for the annual sub-periods. A model passes the UC test when its corresponding p-value is greater than 5%.