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Uniquely Decodable Signature Sequences Over the Noisy Adder Channel

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Abstract—Signature sequences that are uniquely decodable (UD) can be used to identify active terminals by the observation of their sums. Such sequences provide a solution to the problem of finding which m -out-of- t users are active in grant-free access methods that use physical layer network coding (PLNC). Although the existence of such sequences has been assumed in previous works, their explicit construction is presented here for the first time, along with a performance assessment when the observed sums are disturbed by noise. UD sequences are defined as sets of integers that satisfy the unique-sum property. The construction of the sets is algebraically defined depending on two Galois fields with related dimensions. We use the Bose-Chowla-Lindström approach to computationally find such integer sets that are obtained with reasonable computational complexity. In comparison with orthogonal binary real sequences, these sequences use fewer bits in equivalent setups. The binary signature sequences are mapped to some binary antipodal modulation and assessed in the multiple access adder channel (MAAC) disturbed by noise. Both hard and soft maximum likelihood detectors are used to assess the probability of misidentifying an active user.

Index Terms—Uniquely decodable sequences, non-orthogonal signature sequences, binary multiple access adder channel, noisy sums, grant-free, random access.

I. INTRODUCTION

IN sensor networks and machine-type communications, numerous terminals are deployed but communicate infrequently [1]. Random access methods are appropriate in such scenarios and may be used in communication with massive MIMO [2]. Random access schemes are traditionally grant-based, meaning that one terminal can only start transmitting its message after some access grant is conveyed by the base station (BS). This architecture renders the 1ms latency objective unattainable. Grant-free protocols, in which terminals communicate without waiting for access grants, contribute to achieving ultra-reliable low-latency communication (URLLC) objectives [3]. Grant-free protocols frequently utilize hybrid automatic repeat request (HARQ) retransmissions after collisions, discarding information that may have been obtained from the total of colliding packets. To leverage this, physical layer network coding (PLNC) has been suggested for grant-free random access [4]. In this technique, identifying *how many* and *which* terminals are engaged in a collision is critical and was expected to be known at the BS. When this information is not accessible, successive interference cancellation (SIC)

approaches have been implemented employing uniquely decodable (UD) sequences [1]. In [5], [6], a NOMA-based uplink random access system was suggested, utilizing power-domain separation [7], to allow SIC decoding even when two users collide at the same time.

Terminals in the random access channel can broadcast at BS-defined synchronization times, conveying identification and perhaps data. Multiple active terminals are identified using orthogonal and pseudo-orthogonal sequences (e.g., Hadamard, Zadoff-Chu, Gold, and M-sequences) [8], [9]. Terminal identification may be addressed more efficiently utilizing non-orthogonal sequences that allow recovery of the precise m active terminals from their total, known as the *signature coding problem* [10], [11]. There are two forms of UD sequences: Chang-Weldon (non-optimal) [11], [12] and Lindström (asymptotically optimal) [10], [11], [13]. Lindström sequences, referred to here as Bose-Chowla-Lindström (BCL) sequences, are based on Galois field algebraic structures first presented by Bose and Chowla [14]. The first engineering effort specifically examining such sequences in PLNC-based random access is [1], which assumed the existence of UD sequences (e.g., Lindström's) but did not implement them and rather concentrated their work on techniques for resolving non-decodable collisions.

Signature sequences are associated with uniquely decodable codes (UDCs) via the multiple-access channel (MAC). UDCs are applicable to the *permanent activity case*, where all users are always transmitting, and the total of codewords permits inferring each user's message. In that case, the set of codewords assigned to a user can be associated with different messages, but the zero codeword cannot be assigned to more than one user. UDCs exist for two-user multiple access adder channels (MAACs) [15] and a limited number of T simultaneous users [16], with constructions related to the coin-weighting issue [11], [13]. When just a subset of users is active the *partial activity case* or m -out-of- t model inactive users provide a zero vector, while active users send codewords for identification. The receiver must extract the active subset from the sum. Here, UD sequences are employed, nevertheless, some research refers to them as UDCs or alerting codes [17]. In essence, UD sequences are UDCs with a zero codeword representing user inactivity [10], [13].

In this paper, $\text{BCL}(t, m)$ integer sets are computationally created and listed for feasible (m, t) values, making them available to be tested across several platforms. To the best of our knowledge, such collections were not made available prior to this work. This paper also examines BCL sequences under noisy sums for the first time.

II. SYSTEM MODEL

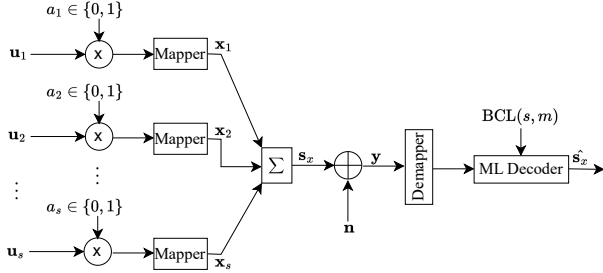


Fig. 1. Adder Channel Model with Uniquely Decodable Signature Sequences.

We examine a system in which t terminals access the channel at random using (non-orthogonal) BCL UD sequences. These sequences demand system design for $s \geq t$ users, where s is a prime number. Without loss of generality, let us suppose $t = s$. In each time slot, m -out-of- s terminals try to access to signal an alarm or initiate message transmission, where $m \leq t \leq s$. If $t < s$, $s - t$ additional users are deemed to be inactive.

A system with m -out-of- s active users is connected with a $\text{BCL}(s, m)$ set of s integers formed using the BCL construct. The system operates at on three levels: (i) the *Galois field level*, over $\text{GF}(s^m)$, where the integer sets $\text{BCL}(s, m)$ have the property that any sums of m , $d_i \in \text{BCL}(s, m)$ are different and the sum $S(\mathcal{A})$ uniquely identifies the terms d_i and therefore their indexes $i \in \mathcal{A}$, defining $\mathcal{A}$ as the set containing the indexes of the active users; (ii) the *logic-level* with binary UD sequences $\mathbf{u}_i$ representing the d_i by $1 + n$ symbols that arise in the adder channel. The n symbols also define UD binary sequences $\mathbf{u}_i$, and their sums can be converted to the unique $S(\mathcal{A})$. The extra bit serves to allow the BS to know the number of active users; (iii) the *physical-layer level*, where the signature sequences are mapped onto the $\{-1, +1\}$ alphabet, resulting in UD sum-vectors where the $1 + n$ channel symbols after the adder channel are isomorphic to the sums of the $\mathbf{u}_i$ and can also be converted back to the BCL integer sums $S(\mathcal{A})$.

Fig. 1 represents the system model for sending BCL signature sequences over MAAC, considering a slotted time system, following [4], [18]. Each terminal is active or inactive ($a \in \{0, 1\}$), with a low activation probability, $P(a = 1)$, guaranteeing that the number of collision terminals m is small compared to the total t . The model for wireless access channels assumes perfect channel state information (CSI) and power management through time division duplex (TDD) with channel reciprocity, calculated from the BS reference signals as in [1], [5], [6]. Before transmission, signature sequences are mapped onto binary phase shift keying (BPSK) or, equivalently, to

amplitude shift keying (ASK). Each user is allocated a unique integer $d_i \in \text{BCL}(s, m)$, transformed to a binary signature sequence $\mathbf{u}_i$ (an n -bit vector), and mapped to the physical layer through a mapper $\mathcal{M}$, producing the n -symbols vector $\mathbf{x}_i = \mathcal{M}(\mathbf{u}_i)$. The unique decodability property of the integers d_i is maintained in both $\mathbf{u}_i$ and $\mathbf{x}_i$ sequences, allowing the sum $S(\mathcal{A}) = \sum_{i \in \mathcal{A}} d_i$ to be obtained from $\mathbf{s}_u = \sum_{i \in \mathcal{A}} \mathbf{u}_i$ or $\mathbf{s}_x = \sum_{i \in \mathcal{A}} \mathbf{x}_i$. Without losing generality, we assume that the m active users are the first m terminals ($a_i = 1$, for $i = 1, 2, \dots, m$, and $a_i = 0$, for $i = m + 1, m + 2, \dots, t$), holding the respective signature sequences $\mathbf{u}_i$, with $i = 1, 2, \dots, m$. At the receiver, the sum signal is distorted by additive white Gaussian noise (AWGN) with zero mean and noise power σ_n^2 , written as:

$$\mathbf{y} = \sum_{i=1}^m \mathbf{x}_i + \mathbf{n} = \mathbf{s}_x + \mathbf{n}. \quad (1)$$

Given the (logic) binary sequences $\mathbf{u}_i$ as input, the adder channel leads as the non-binary vector $\mathbf{s}_u = \sum_{i=1}^m \mathbf{u}_i$, whose elements $s_u(i) \in \mathbb{S}_u = \{0, 1, 2, \dots, m-2, m-1, m\}$. It is easy to see that $\mathbb{S}_u$ is isomorphic to the set $\mathbb{S}_x$, consisting of the possible values for the elements of the sum vector $\mathbf{s}_x = \sum_{i=1}^m \mathbf{x}_i$, such that $s_x(i) \in \mathbb{S}_x = \{-m, -m+2, -m+4, \dots, m-4, m-2, m\}$. Hence, each $s_u(j)$ can be acquired from the individual symbols in the adder channel, $s_x(i)$, using the one-to-one mapping $s_u(i) \leftrightarrow s_x(j)$, for $j = 1, 2, \dots, n$. Finally, the sum-signal vector $\mathbf{s}_u$ at the logic-level can be converted into the integer sum by applying

$$S(\mathcal{A}) = \sum_{j=1}^n 2^{n-j} s_u(j). \quad (2)$$

A maximum-likelihood (ML) decoder is applied to detect the sum-signal $\mathbf{s}_x$ at the receiver. The ML detector performs an exhaustive search over all possible sums of codewords, selecting the one with the smallest Euclidean distance to the received signal:

$$\hat{\mathbf{s}}_x = \arg \min_{\mathbf{s}_x \in \mathcal{C}} \|\mathcal{Q}(\mathbf{y}) - \mathbf{s}_x\|^2, \quad (3)$$

where $\mathcal{C}$ is the set of all possible sum-vectors $\mathbf{s}_x$ associated to the unique sums generated by the BCL set $\text{BCL}(s, m)$. The operator $\mathcal{Q}(\cdot)$ denotes the quantization to the set $\mathbb{S}_x$, applied in a hard-decision ML decoder. By removing $\mathcal{Q}(\cdot)$ in (3), one obtains a soft-decision ML decoder.

III. BOSE-CHOWLA-LINDSTRÖM SIGNATURE SEQUENCES

The BCL sequences are the most efficient family of UD sequences [10], [11]. For a system with t users and m active users, the sequence length scales is $n(t, m) \sim m \log t$ bits [10], [13], [14], given that:

$$\lim_{m \rightarrow \infty} \lim_{t \rightarrow \infty} \frac{n(t, m)}{m \log t} \leq 1.$$

BCL sequences are created from integers and may be translated onto various Galois fields; therefore, they are not intrinsically binary [10]. The BCL construction generates a set of $s \geq t$ integers, denoted as $\text{BCL}(t, m)$, where each m -out-of- s

sum is uniquely decodable. It requires that s be a prime power (p^i , for a prime p and an integer i). To construct a code for t users, one should first produce a code for $s \geq t$ users, and then discard $s - t$ integers (i.e., keep $s - t$ terminals muted). According to the prime number theory, s satisfies $s \leq 2^{\lceil \log_2 t \rceil}$.

For the construction, let α be a primitive element of $\text{GF}(s^m)$, that is, $\text{GF}(s^m) = \{0, \alpha^0, \alpha^1, \dots, \alpha^{s^m-1}\}$, meaning that all elements in $\text{GF}(s^m)$ can be generated by α^i . Note that any primitive element of any Galois field verifies $\alpha > 1$. Let $\beta_1, \beta_2, \dots, \beta_s$ be elements of $\text{GF}(s)$. Define integers $d_1, d_2, \dots, d_s$ such that:

$$\alpha^{d_i} = \alpha + \beta_i \quad \text{for } i = 1, 2, \dots, s. \quad (4)$$

Since α is one of the primitive elements of $\text{GF}(s^m)$ and $\alpha + \beta_i$ is a nonzero element, such d_i exists for every i . Given that $\alpha > 1$, the right side of (4) is > 1 , and therefore $d_i \neq 0$. Moreover, recalling the definition of primitive power, one gets that $0 < d_i \leq s^m - 1$.

In our implementation, the Galois field arithmetic is handled using the Galois library in Python. It begins by defining the Galois field $\text{GF}(s)$, where s is a prime integer. This field includes elements $\beta_1, \beta_2, \dots, \beta_s$, which are stored in a list for further processing. Next, the procedure defines the extended Galois field $\text{GF}(s^m)$ and identifies its primitive elements α . These primitive elements are critical, as they form the basis for calculating the d_i values for the BCL sets. For each primitive element α , the library iteratively searches for integers d_i such that $(\alpha^{d_i} = \alpha + \beta_i)$ holds for each element β_i in $\text{GF}(s)$. This solution is stored in a table, mapping each primitive element to its corresponding list of d_i values. The crucial characteristic of the integers $d_1, d_2, \dots, d_s$ is that different two sums of exactly m of them cannot coincide. Defining $S(\mathcal{A}) = \sum_{i=1}^m d_{a_i}$ and $S(\mathcal{B}) = \sum_{i=1}^m d_{b_i}$, then

$$S(\mathcal{A}) = S(\mathcal{B}) = S \iff \{a_1, a_2, \dots, a_m\} = \{b_1, b_2, \dots, b_m\}. \quad (5)$$

This means that if the sums of two different subsets of m integers are the same, these sets must consist of the same elements, and given the construction in (4), they also appear in the same order. In short, the sets must be identical.

Let the binary representations of the integers d_i be denoted by $\mathbf{u}_i$. They have an extra digit 1 to allow the BS to count the number of users colliding, as mentioned in Section II. It requires $\lceil \log_2(\ell+1) \rceil$ bits to express integers up to ℓ in binary. Given $d_i < s^m - 1$ and $s \leq 2^{\lceil \log_2 t \rceil}$, consequently:

$$n_{\text{BCL}}(t, m) = \lceil \log_2(s^m - 1) \rceil + 1. \quad (6)$$

An upper bound for the length of BCL sequences can be derived: $n_{\text{BCL}}(t, m) = m \log_2 s + 2 \leq m \lceil \log_2 t \rceil + 2 \leq (1 + \log_2 t)m + 2$. The property in (5) carries over to the binary sequences, so that any sum of two binary sequences corresponds uniquely to the sum in (5) [14], [10]. This is easily confirmed: each bit $\mathbf{x}_i$ corresponds to a weight 2^i according to its position. When vectors are added element-wise, as defined in (1), the weighted sums are preserved, allowing the total sum of the integers, $S(\mathcal{A})$, to be obtained by (2).

TABLE I
LISTS OF d_i INTEGERS FOR VARIOUS SETS OF $\text{BCL}(s, m)$

s	m	α	d_i
5	2	5	{1, 22, 15, 2, 17}
5	3	5	{1, 96, 76, 119, 103}
7	5	7	{1, 12071, 12380, 9244, 8408, 13714, 4038}
11	6	11	{1, 101410, 703772, 279323, 1053498, 1565140, 1622352, 633225, 820891, 1765613, 1042680}
13	3	13	{1, 1713, 1560, 1688, 1064, 1811, 908, 651, 802, 1887, 207, 1516, 918}
17	3	17	{1, 4597, 4660, 4120, 1345, 1385, 4284, 644, 836, 4067, 3475, 1278, 2962, 3873, 2459, 3203, 2800}
31	5	31	{1, 10342133, 9604646, 15264964, 16223190, 20012610, 18106802, 7238195, 11511369, 3708129, 18108920, 14120023, 4186988, 26216728, 26092301, 14986702, 24669807, 18097153, 1939695, 13891837, 4963721, 25706100, 5121522, 2907072, 15948053, 354646, 2686880, 7697960, 19164236, 20302054, 21358055}
67	4	67	{1, 5157084, 540552, 1575080, 2130098, 3868256, 597656, 10183419, 2122602, 15801420, 7487495, 2843820, 2605983, 7463940, 1204340, 12511143, 11869423, 13442311, 2796618, 2174047, 5760790, 10028153, 16363831, 5352422, 2573368, 16441438, 12936522, 5004529, 3911067, 17356877, 6057327, 4394639, 5770568, 12270345, 6526180, 11680883, 15705025, 7884744, 7705300, 2967533, 4664805, 8258716, 3991615, 14462977, 17409065, 2075047, 2097608, 19543030, 8535596, 12077407, 17035334, 3716205, 11052286, 13727578, 7002238, 11442704, 14763519, 9885160, 10496742, 14054764, 15447672, 7860863, 16407366, 972411, 5627734, 171642, 7536112}

Table I lists the d_i integers for various relevant $\text{BCL}(s, m)$ sequences constructed according to (4). The time taken in searching the list of d_i for a given $\text{BCL}(s, m)$ is mostly determined by the value of m . For the set with $(s = 67, m = 4)$, the search exceed 24 hours of computation time. To preserve the order of the calculation of the d_i resulting from the algebraic equation (4) in $\text{GF}(s^m)$, the integers d_i are not sorted in Table I, but rather listed preserving the order dictated by the β_i .

A. BCL vs Orthogonal signatures

Hadamard sequences (which are orthogonal) and Gold sequences (which are quasi-orthogonal) exist only for lengths 2^k and $2^k - 1$, respectively, for $k \in \mathbb{Z}^+$. BCL sequences exist for any prime s and positive integer $m \leq s$, and their number of bits, n_{BCL} , is given in (6). Hadamard sequences must be designed always considering a number of terminals rounded to the next power of two, $s = \lceil 2^k \rceil$, and in the case of Gold sequences, $s = \lceil 2^k + 1 \rceil$. Hence, BCL sequences can be chosen in a more flexible manner to fit better systems with different numbers of users, as long as $m \ll s$.

IV. SIMULATION RESULTS

We assessed the identification error rate (IDER), defined as the probability that at least one active user is incorrectly, as a function of the signal-to-noise ratio (SNR). The SNR is defined as $\text{SNR} = \mathbb{E} [|s_x(i)|^2] / \sigma^2$, where σ^2 is the variance of the additive white Gaussian noise. We consider two BCL sequences: one with $s = 5, m = 3, n_{\text{BCL}} = 8$, and another with $s = 11, m = 6, n_{\text{BCL}} = 22$. The performance was assessed using both hard and soft ML detection techniques, as explained in Section II. In the hard ML situation, the received signal is first quantized before detection, but in the soft ML case, the

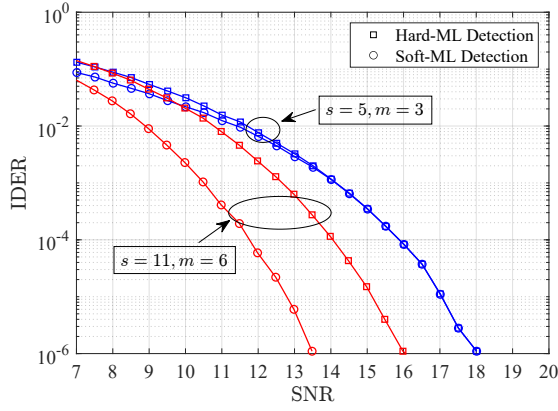


Fig. 2. Identification error rate for BCL sequences using binary antipodal modulation using hard and soft detection in a noisy MAAC, as defined in (3).

unquantized received values are utilized directly in the distance calculation. Fig. 2 shows the performance of the proposed BCL signature sequences under different detection schemes over a noisy MAAC. As expected, soft-ML detection attains a rather better performance compared to the hard-detection, particularly in the low-SNR regime, because it does not make individual decisions on the symbols but rather compares the whole unquantized received sequence with all the possible sum-sequences. Note that the $s = 11$ system will have more competing terminals, and therefore the number of possible values that each $s_x(i)$ can take belongs to a larger set. This leads to a larger symbol error rate (which we measured for each system). However, the IDER in the $s = 11$ system is significantly lower than the one with $s = 5$ terminals. This is a consequence of having longer sequences ($n_{\text{BCL}} = 22$ versus $n_{\text{BCL}} = 8$ bits), which allow for a higher Euclidean distance between the sum sequences, therefore increasing the probability of correctly identifying the sum-sequences.

V. CONCLUSIONS

In this paper, the BCL uniquely decodable sequences are generated for the first time and assessed over the noisy multiple-access adder channel. Our work shows that BCL sequences provide considerable benefits for systems with partial user activity (m -out-of- t model), which is common in massive machine-type communications. Compared to standard orthogonal codes, BCL signatures have two advantages: they have shorter lengths and, most importantly, they provide a much larger flexibility when designing systems with different numbers of users. We have assessed for the first time a grant-free random access scheme using antipodal modulation and applying both hard- and soft detection. The results show that such systems have a reasonable probability of failure in identifying the active terminals, opening doors for more efficient and more flexible non-orthogonal random access schemes, better adapted to the number of users in a system.

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REFERENCES

- [1] J. Goseling, Č. Stefanović, and P. Popovski, "Sign-compute-resolve for tree splitting random access," *IEEE Transactions on Information Theory*, vol. 64, no. 7, pp. 5261–5276, Jul. 2018.
- [2] S. Allahkaram, F. A. Monteiro, and I. Chatzigeorgiou, "URLLC with coded massive MIMO via random linear codes and GRAND," in *Proc. of IEEE 96th Vehicular Technology Conference (VTC2022-Fall)*, London, UK, Sep. 2022, pp. 1–5.
- [3] P. Popovski, Č. Stefanović, J. J. Nielsen, E. de Carvalho, M. Angelichinoski, K. F. Trillingsgaard, and A.-S. Bana, "Wireless access in ultra-reliable low-latency communication (URLLC)," *IEEE Transactions on Communications*, vol. 67, no. 8, pp. 5783–5801, May 2019.
- [4] J. Goseling, M. Gastpar, and J. H. Weber, "Random access with physical-layer network coding," *IEEE Transactions on Information Theory*, vol. 61, no. 7, pp. 3670–3681, Jul. 2015.
- [5] J.-B. Seo, B. C. Jung, and H. Jin, "Nonorthogonal random access for 5G mobile communication systems," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 8, pp. 7867–7871, Aug. 2018.
- [6] —, "Performance analysis of NOMA random access," *IEEE Communications Letters*, vol. 22, no. 11, pp. 2242–2245, Nov. 2018.
- [7] R. Alberto and F. A. Monteiro, "Downlink MIMO-NOMA with and without CSI: A short survey and comparison," in *12th Inter. Symp. on Communication Systems, Networks and Digital Signal Processing (CSNDSP)*, Porto, Portugal, 2020, pp. 1–6.
- [8] J. H. Sørensen, E. de Carvalho, Č. Stefanović, and P. Popovski, "Coded pilot random access for massive MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 17, no. 12, pp. 8035–8046, Dec. 2018.
- [9] R.-A. Pitaval, B. M. Popovic, F. Berggren, and P. Wang, "Overcoming 5G PRACH capacity shortfall by combining Zadoff-Chu and M-sequences," in *Proc. of the IEEE International Conference on Communications (ICC)*, Kansas City, MO, USA, May 2018, pp. 1–6.
- [10] D. Danev, Z. Füredi, and M. Ruzinkó, "Multiple access channels: theory and practice," in *Multiple Access Channels*, E. Biglieri and L. Györfi, Eds. Amsterdam, Netherlands: IOS press, 2007, pp. 26–53.
- [11] B. Laczay, "General theory of information transfer and combinatorics," in *General Theory of Information Transfer and Combinatorics*, R. Ahlswede, L. Băumer, N. Cai, H. Aydinian, V. Blinovskiy, C. Deppe, and H. Mashurian, Eds. Berlin, Germany: Springer, 2006, ch. Coding for the Multiple-Access Adder Channel, pp. 165–195.
- [12] D. G. Cantor and W. H. Mills, "Determination of a subset from certain combinatorial properties," *Canadian Journal of Mathematics*, vol. 18, p. 42–48, 1966.
- [13] B. Lindström, "Determining subsets by unramified experiments," in *A Survey of Statistical Design and Linear Models*, J. N. Srivastava, Ed. Amsterdam, Netherlands: North-Holland, 1975, pp. 407–418.
- [14] R. C. Bose and S. Chowla, "Theorems in the additive theory of numbers," *Commentarii Mathematici Helvetici*, vol. 37, no. 1, pp. 141–147, Dec. 1962.
- [15] R. Ahlswede and V. Balakirsky, "Construction of uniquely decodable codes for the two-user binary adder channel," *IEEE Transactions on Information Theory*, vol. 45, no. 1, pp. 326–330, Jan. 1999.
- [16] L. Kiviluoto and P. R. J. Ostergard, "New uniquely decodable codes for the T -user binary adder channel with $3 \leq T \leq 5$," *IEEE Transactions on Information Theory*, vol. 53, no. 3, pp. 1219–1220, Feb. 2007.
- [17] V. Gritsenko, G. Kabatiansky, V. Lebedev, and A. Maevskiy, "Signature codes for noisy multiple access adder channel," *Designs, Codes and Cryptography*, vol. 82, no. 1, pp. 293–299, Jan. 2017.
- [18] S. Lu, W. Hou, and J. Cheng, "A family of $(k+1)$ -ary signature codes for noisy multiple-access adder channel," *IEEE Transactions on Information Theory*, vol. 61, no. 11, pp. 5848–5853, Nov. 2015.