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Competing Art Auctions

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Abstract
This paper examines the optimal choices of two competing auction houses, where each one is able to choose its own auction date, so that the auctions run either simultaneously, sequentially or with an overlap. We show that simultaneously running auctions, *per se*, is an optimal choice for each house. However, if we fix the supply of objects for sale, then the effects of this simultaneity disappear, and the houses choose arbitrary dates. If the sellers' supply of objects to the auction house is taken into account, then overlapping auctions will be the equilibrium outcome and the optimizing behaviour of the auction houses.

Keywords: competing auctions, optimal competing auctions, simultaneous auctions, sequential auctions, overlapping auctions, art auctions
JEL classification: D43; D44; D82; L21

1. Introduction

The rising popularity of online auctions has increased the interest in studying competing auctions, where multiple auctions are competing either simultaneously, sequentially, or in an overlapping manner. This is also the case for more traditionally held auctions, such as in the art market. Indeed, the two largest auction houses, Christie's and Sotheby's, have been found to concentrate their auctions of an art category (e.g. 'prints') within the same period of time. Greve (2023) finds that the percentage of overlapping auction dates between the two auction houses for the sale

of 'prints' is between 50% and 91%, where the 50% refers to the case where the two houses held their auctions on the exact same day.[1]

This raises the question of whether overlapping auctions, where the auction houses are competing for buyers, are the optimal strategy choices for the auction houses. Alternatively, for example, they could have chosen auction dates when they would be unchallenged for potential buyers, a strategy which runs counter to intuition. Hence, this paper examines what would be the optimal competing auction, where an auction house has the option of choosing between running their auction simultaneously with the competing house, sequentially, or with overlapping auction periods.

More specifically, we are presented with a market for art where sellers bring objects to the auction houses with the purpose of selling these objects to potential buyers for the highest possible prices. The accepted objects will, pre-auctioning, be separated into different categories, such as impressionist art, old masters, prints etc., and each category results in an auction catalogue containing different objects to be auctioned at a future pre-determined date and time. Buyers will participate in an auction for a particular object if it corresponds to their tastes or preferences. We formulate these preferences using a description of the objects in terms of several variables, each of which describes a particular aspect of quality. This fits well with practice, where art objects are described by name of the artist, provenance, whether they are signed etc. As is usual, we assume that each of the quality characteristics can be expressed in numerical scales, so that the preferences of buyers can be measured as distance to some ideal object, not necessarily available. Buyers are interested in participating in only one auction at a time. If the auctions take place on different dates, a buyer may decide to refrain from participating in an earlier auction, even when there is an object of interest, because the chance of getting hold of a better object in another auction in the near future is higher. This means that timing matters for the size of the attendance of an auction, and therefore it has an influence upon the revenue of the auction house.

The equilibrium outcome of the game of timing will differ according to the type of objects on sale at the auction. In the first case considered, it is assumed that the auction held by two competing houses is a one-shot affair, in the sense that no similar auctions are expected to take place in the near future. The timing of the auction within the period considered can, however, be chosen by each auction house. In this situation, the risk of losing potential buyers will prevent the auction houses from choosing an early auction date, indeed they will prefer the latest possible date,

[1] We return to this in Section 8.

expecting the other house to do the same.

If the auctions run as recurrent events, so that the decision of the auction house requires it to fix the date of an auction within a given period of time, to be repeated subsequently, then the idea of choosing a 'last' date among the possible choice of dates loses its meaning, as it will be followed immediately by the early dates of the subsequent interval. Indeed, if the number of objects to be auctioned is independent of the timing, there will be no equilibrium choice of auction date to be repeated period after period, rather the auction dates will change in a random way from one period to the next.

This somewhat unrealistic latter pattern will, however, change when the behaviour of sellers is incorporated into the model. So far, sellers have had a passive role, with objects arriving at a fixed rate and are split equally between the auction houses. However, if sellers prefer the auction house which is the first to run its auction, then the supply of objects will vary over the calendar period, unless the auction houses choose essentially the same date for their auctions. A variation in timing will have an impact on the costs for the auction house, which has to maintain a staff with the necessary competence and experience to process and present the objects. In this context, a regular supply of objects will be less costly than a supply with seasonal variations, and as a consequence the equilibrium choice will be that auctions overlap, actually exactly or almost simultaneously. Consequently, the buyers will be exposed to the objects of both houses, before deciding upon where and how much to bid.

The literature on art auctions is quite extensive, see e.g. Ashenfelter (1989), Ashenfelter and Graddy (2003) and Lovo and Spaenjers (2018). The majority of the literature has its focus on independent auctions, but there are contributions to the auction literature that deal with the interdependency across competing auctions, and the impact on the strategies of the buyers and the auctioneer. Here, as for many other sectors, buyers guide themselves with regard to the offers from different auctions, and switch between them, a phenomenon known as cross-bidding, in order to obtain the best alternative (Anwar et al., 2006; Haruvy et al., 2014). The auction houses act accordingly.

Cross-bidding is a significant feature of simultaneous auctions. On one hand, in the context of homogeneous goods, it can reduce the prices of objects, since buyers submit bids in the auction with the lowest price (Anwar et al., 2006; Peters and Severinov, 2006). On the other hand, more buyers will increase competition and therefore increase the prices (Beil and Wein, 2009). Cross-bidding also plays a key role in sequential auctions, where forward-looking behaviour can result in less aggressive bids in a current auction and therefore lower prices (Engelbrecht-Wiggans, 1994).

However, learning from previous auctions can reduce uncertainty about the value of an object, resulting in more aggressive bids and thus higher prices (Kagel and Levin, 1991).

While the cross-bidding literature mostly has its focus on the two extreme cases of running auctions; namely either simultaneously or sequentially, there are few papers which have their focus on cases where the competing auctions overlap. Bapna et al. (2009) study overlapping auctions where identical objects are up for sale, and show that overlaps have a negative impact on the prices, since it incentivizes switching between auctions. Zhuang and Leszczyc (2022) study the optimal strategy of the seller (which in our context would mean a single auction house) in competing auctions, where the auction house has two identical objects for sale, and the buyers have unit demand. If the auction periods of the two objects, which are decided by the seller, are not identical, then buyers may postpone bidding, expecting a lower price, or they may bid right away. The seller may choose between simultaneous, overlapping, and sequential auctioning of the object, and it is shown that the optimal choice depends on buyers' uncertainty about the value of the object and their learning between auction periods, as well as the time discounting.[2]

There are a few contributions to the literature on auctions with competing auction houses, but in general they do not touch upon the choice of auction date or period, and especially where there are non-objects for sale. Landi et al. (2023) study a framework of two competing sellers, offering two heterogeneous objects. The auctions run simultaneously, where second-price (sealed-bid) auctions with reserve prices are used. They compare this situation to one where a single seller is auctioning one object, using a second-price auction with a reserve price. For a large number of buyers, they show that competing auctions generate a greater revenue for the seller than the single seller case, see also Troncoso-Valverde (2014). This literature focuses, however, only on simultaneous auctions, neglecting two other scenarios in real-life auctions - i.e., sequential or overlapping. Greve (2023) is a primary paper of the present paper. He examines a framework of competing auction houses, where a number of heterogeneous objects are for sale. Bidders rank the objects by interest, from the most interesting object to the least. In a two-period model, he shows that the bidders will wait until the second period, where the average number of bidders is larger than in the first period auction. As a consequence, the auction houses will prefer to auction their objects at the later auction. Therefore, their auctions will overlap. The present paper studies the optimal auction format; whether it be simultaneous, sequential or overlapping auctions.

[2]See also the literature on competing mechanisms, e.g. Attar et al. (2020).

From an industrial organization perspective, frequent simultaneous or overlapping auctions between two competing sellers, for a given category of object, could be seen as an indicia of collusive behaviour. The decision on the timing of an auction event is a strategic variable that is expected to be taken by each firm independently (though aware of each other's actions). The present paper shows that simultaneous, overlapping or sequential auctions between two competing firms can occur in a non-cooperative oligopolistic framework. This, however, does not rule out the fact that highly frequent simultaneous or overlapping auctions (in an oligopolistic market) might not be a sign of collusive behaviour between firms. Abstracting from collusion, focusing on the competition between two (art) auction houses and the choice of a buyer when bidding on an object, the model of Chen (2024) has features similar to our model, where a buyer searches for an object with specific characteristics.

A different branch of the industrial organization literature involves the study of endogenous timing in oligopoly markets. Hamilton and Slutsky (1990) study a game with a preplay stage to a basic game, where the players/firms simultaneously decide whether to move early or late. The players will move simultaneously if they both decide to move at the same time (early or late); sequentially, including the order of move, otherwise. Van Damme and Hurkens (1999) extend the paper of Hamilton and Slutsky (1990) to study an environment, where each player can commit to a certain quantity in a stage 1 or wait until stage 2. They show that only the firm with the lowest cost will choose to commit and become the Stackelberg leader. For similar literature on endogenous timing see, for example, Gal-Or (1985; 1987); Mailath (1993); Amir and Stepanova (2006); Mideksa (2025). This literature has, in general, its interest in the effect of strategic commitment to an individual production capacity and therefore, while it shares many of the same themes as the present paper, the models/contexts are different.

This paper is organized as follows. Section 2 presents the model, where the buyers are examined in Section 3, and the auction houses in Section 4. Then follows the analysis of strategic choices of auction date; first in Section 5 for a once-and-for-all type of auction, and then in Sections 6 and 7 in the context of recurrent auctions, either with a fixed supply of objects, or with a supply which depends on the timing of the auctions. Section 8 discusses the model seen in relation to real-world art auctions, while Section 9 concludes. The proofs of the propositions can be found in the appendix.

2. A model of the market for art

We consider a situation where a number N of mutually different objects is set for sale with one of two auction houses, A and B, as intermediaries. The objects are brought to the auction houses by *sellers* and made available for interested individuals, in the following referred to as *buyers*, who may acquire the objects at the auction.

The objects, which here are assumed to be works of art, but might also be antiquities, wine, or other objects, are characterized by their quality. We assume here that all aspects of quality relevant to the buyer are subsumed in k distinct properties. The choice of a many-dimensional quality space has been made for its greater realism as representing the different qualities of a piece of art. Choosing $k = 1$ does not change the reasoning in any fundamental way, but also gives only a slight simplification of the formalism. Technically, this means that the quality of an object can be stated as a k-dimensional vector q, so we introduce an abstract *quality space* taken as a subset Q of $\mathbb{R}^k$, which takes the shape of a k-dimensional cube (in the following referred to as a cube) with sides of length $2L > 0$. This side length can be thought of as a measure of the extension of the quality range of objects which may occur at the auctions. We identify objects with points q in the quality space, and we measure distance between objects $q, q' \in Q$ as

$$d(q, q') = \| q - q' \|_\infty = \max_{h=1,\dots,k} | q_h - q'_h |.$$

For $\delta > 0$ and $q \in Q$, let

$$B_\delta(q_i^0) = \{ q' \in Q \,\big|\, d(q', q) \leq \delta \}$$

be the closed δ-neighbourhood of q, which is a closed k-dimensional cube with sides of length 2δ and volume $(2\delta)^k$.

There is a number M of *buyers* in the art market. We assume that buyers' preferences over objects, that is over elements of Q, are determined by the distance to an ideal object. Thus, for each buyer $i \in \{1, \dots, M\}$, there is a particular object q_i^0 such that i prefers object q to q' if and only if $d(q, q_i^0) < d(q', q_i^0)$. The object q_i^0, which may be considered as the ideal object for buyer i, is not necessarily available, and then i will settle for objects which come close to it.

We assume that each buyer will bid for, at most, one object at an auction. This assumption is clearly restrictive, but it allows us to avoid considerations of budget constraints and selection of optimal multiple bids for each buyer. It may be acceptable if we consider auctions for high value art. It may also be justified by the possible existence of entry fees, or by the presence of reservation prices, so that winning an auction, even with a small additional bid, will have a considerable impact on the

buyer's budget. For a justification of the assumptions made in this paper, including unit demand, see Section 8.

The preferences of a buyer i with ideal object q_i^0 can be described by a utility function v_i defined on objects $q \in Q$,

$$v_i(q, q_i^0) = \alpha - d(q, q_i^0),$$

so that the utility of obtaining the ideal object is α. Assuming quasilinear utility, we have that the net payoff of a buyer i with ideal q_i^0 winning the auction for the object q at a payment b is

$$u(q, b; q_i^0) = v(q, q_i^0) - b. \tag{1}$$

The buyer will accept the purchase of an object q only if $u(q, b; q_i^0) \geq 0$ or equivalently $d(q, q_i^0) \leq \alpha - b$, and they will engage in trade at an auction only if there is an object in the set $B_\alpha(q_i^0)$. The side length of $B_\alpha(q_i^0)$, which is 2α, is a measure of the extension of the sphere of interest of the buyer with ideal q_i^0, and L/α gives an expression for the degree to which buyers are narrowly or broadly interested in the objects for sale. For simplicity, we assume that L/α is an integer.

We assume that the ideals of buyers are distributed independently and uniformly over Q. We are considering auctions with *private values,* which seems realistic if objects are acquired for personal use rather than for resale. Buyers know their own values of the objects, but only the distributions of the other buyers' values.

For an object q^0 such that $B_\delta(q^0)$ is contained in Q, the probability that a single draw from a uniform distribution of objects will result in a placement in $B_\delta(q^0)$ is

$$p_\delta = (\delta/L)^k,$$

which is the volume of $B_\delta(q^0)$ divided by the volume of Q. We use the notation $p_\delta^{(S)}$ for the probability that at least one out of S independent draws will be in $B_\delta(q^0)$,

$$p_\delta^{(S)} = 1 - (1 - p_\delta)^S. \tag{2}$$

This probability does not depend on the choice of the point q^0 as long as its distance from the boundary of Q exceeds δ, as will be the case in the following.

Since time will enter into our analysis, we need an assumption about time preferences of buyers, formulated as a discount factor $\gamma(\tau)$ giving the reduction of the utility obtained if getting the object now, as compared to the utility of getting it in an

auction occurring after a time interval of length τ.[3]

Turning now to the role of the *auction houses*, we assume that the objects for sale at the auction, which in their turn are offered by the sellers, are distributed independently and uniformly over Q. Seen from the auction house's side, the probability that an object q will be sold in an auction with M buyers is at most $p_\alpha^{(M)}$, the probability that there is at least one buyer in $B_\alpha(q)$. But even if this happens, the object may remain unsold, as the buyers may choose other objects or refrain from participation in the auction. Thus, the probability that the object remains unsold is at least $1 - p_\alpha^{(M)}$. We assume that unsold objects are returned to the sellers and are not offered at the next auction conducted by this house.

We assume that the auctions are carried out as second-price auctions. This does not quite correspond to what is usual in art auctions, where the auctions would be either open, taking the form of an ascending bid or English auction, or closed but with payment following the bid, that is as a first-price auction. As we shall be mainly concerned with problems of expected payoffs, we may however invoke the revenue equivalence theorem, which holds in our present context of private values, to assert that our results would be the same with these more realistic but less tractable auction formats.

The auction house collects a fee from the buyers in the form of a fixed percentage f of the payment. We assume this fee to be the same for both houses. This does not quite correspond to reality, but is assumed, so that we may concentrate on the competition with respect to the timing of the auctions.

3. Optimal decisions of buyers

We now look more closely at the optimal choice for a buyer. A buyer i with ideal q_i^0 will participate in the auction only if $B_\alpha(q_i^0)$ contains one of the N objects, and in a second-price auction it is optimal to bid the true value for the object $\hat{q}$ closest to q_i^0, adjusted for the payment of a fee to the auction house, we have

$$v_i(\hat{q}, q_i^0)(1+f)^{-1} = \left(\alpha - d(\hat{q}, q_i^0)\right)(1+f)^{-1}.$$

From the point of view of the auction house, the winning bid for $\hat{q}$ will be that of the buyer whose ideal point q_i^0 is closest to $\hat{q}$. If $r_{(1)}^S(\hat{q})$ denotes the smallest distance from $\hat{q}$ to any of S randomly selected points in Q, then the winning bid for $\hat{q}$ in the

[3]The discounting does not purport to the value of possessing the object, which presumably does not change over time: *A thing of beauty is a joy forever* (Keats).

second-price auction with M buyers can be written as

$$\max\left\{0, \left(\alpha - r_{(1)}^{(M)}(\hat{q})\right)(1+f)^{-1}\right\}.$$

The buyer winning $\hat{q}$ at the auction pays the second-highest bid. If there is no other object in $B_\alpha(\hat{q})$, then the winning buyer i is the one situated closest to $\hat{q}$, and the second-highest bid would come from the buyer which is second-closest to $\hat{q}$. If $B_\alpha(\hat{q})$ contains more than one object, then this may not be the case. We shall assume, however, that the buyers will disregard this possible complication, so that the subjectively expected payment $\hat{b}$ of the buyer winning the object $\hat{q}$ will be the bid of the second-closest buyer, if there is one, and otherwise 0, plus the fee to the auction house, amounting to

$$b(\hat{q}) = \max\left\{0, \alpha - r_{(2)}^{M}(\hat{q})\right\},$$

where $r_{(2)}^{S}(\hat{q})$ denotes the second-smallest distance to $\hat{q}$ of S randomly chosen objects in Q.

In the following, we shall repeatedly use versions of the expected payment for an object. For a given object $\hat{q}$ with $B_\alpha(\hat{q}) \subset Q$, the payment for this object, which may or may not be sold, is

$$R^M = \mathbb{E}\left[b(\hat{q})\right] = \alpha - \mathbb{E}\left[\min\left\{\alpha, r_{(2)}^{M}(\hat{q})\right\}\right]. \tag{3}$$

This expected value is independent of $\hat{q}$ and of whether or not there are interested buyers, so we shall refer to it as the *ex ante* expected payment, showing what the auction house may reasonably expect to get from putting object $\hat{q}$ into an auction with M buyers.

For the description of the buyers' bid strategies, we shall need other versions of the expected payment. Suppose that a buyer i with ideal q_i^0 is interested in the object $\hat{q}$, so that $d(q_i^0, \hat{q}) = r < \alpha$. If i wins the auction, then the expected payment will be determined by the value of the buyer being second-closest to $\hat{q}$, who will also be the closest of the buyers with distance to $\hat{q}$ greater than r,

$$\hat{R}^M(r) = \alpha - \mathbb{E}\left[\min\left\{\alpha, r_{(1)}^{M-1}\right\} \,\middle|\, r(\hat{q}) > r\right].$$

The winning buyer gets the utility $\alpha - r$ and pays $\hat{R}^M(r)$. Taking expectations of the payoff with respect to r in the interval $[0, \alpha]$, $\mathbb{E}\left[\alpha - \hat{R}^M(r)\right]$, we get the expected payoff, given that there is an object in $B_\alpha(q_i^0)$, an event which has probability $p_\alpha^{(N)}$, so that the *ex ante* expected payoff to the buyer with ideal q_i^0 at an auction with N objects for sale

is

$$U^{N,M} = p_\alpha^{(N)} \mathbb{E}\left[\alpha - \hat{R}^M(r)\right].$$

We have introduced the main concepts to be used for determining the bidding strategies. In the appendix, we give in the form of Propositions A1-A3, expressions of R^M and $\hat{R}^M(r)$ in terms of the underlying probability distribution on Q. It remains to be seen how the choices of buyers will be influenced by the auction dates. A buyer observing that an object $\hat{q}$ is within their sphere of interest, that is $\hat{q} \in B_\alpha(q_i^0)$, may bid on the object, the optimal bid is the true value (adjusted for the fee), and if they win, the payment will be the adjusted second-largest bid. Alternatively, the bidder may wait for the next upcoming auction, where the chances that an object with a higher subjective value may be presented for sale, meaning that the buyer will abstain from bidding at the present auction. We formulate the above considerations as a proposition.

Proposition 1 *Let i be a buyer having ideal q_i^0 with the options to participate either in an auction at date t or alternatively at an auction date $t + \tau$, and assume the closest object $\hat{q}$ to q_i^0 satisfies $d(\hat{q}, q_i^0) \leq \alpha$. Then,*

(a) Expected net payoff from participating is $(1 - p_r^{(M)})\hat{R}^M(r)$,

(b) Ex ante expected payoff from participating in the auction at $t + \tau$ is $U^{N,M}$, so that

(c) Buyer i will participate in the auction at t only if $(1 - p_r^{(M)})\hat{R}^M(r) \leq \gamma(\tau)U^{N,M}$.

The decisions of buyers on participation in the current auction will matter for the auction houses since a reduced participation means that objects may fall outside the sphere of interest of the remaining buyers, and even when this does not happen, the revenue from the auction may be smaller when the second-closest buyers decide to stay away.

For later use, we notice that the particular value of r, for which (c) in the proposition is satisfied with equality, depends on the time span τ to the alternative auction, so that a function r^* can be defined implicitly by the equation

$$(1 - p_{r^*(\tau)}^{(M)})\hat{R}^M(r^*(\tau)) = \gamma(\tau)U^{N,M}. \tag{4}$$

Thus, $r^*(\tau)$ is the upper limit for distances to the ideal for a buyer wanting to participate in the auction of a given object. It is easily seen that r^* is a decreasing function of τ.

In the next section, we shall consider how the choice of participation will influence the profits of the auction houses.

4. Decisions of the auction house

We now consider the situation of the auction house, having put a given number N of objects for sale at date t. The house earns its profits through the percentage fee f on buyers' payments, and the payments in their turn depend on whether the auction house is alone in conducting an auction at date t, or whether the other auction house also conducts an auction at t. We consider the two cases separately.

If only one house is conducting an auction at t, the other auction house will have its auction at $t + \tau$. The first house will then face all the M buyers, and the possibility that there will be interested buyers is determined by $r^*(\tau)$, the critical distance from the ideal introduced in the previous section. If all buyers who bid for the object $\hat{q}$ have ideals at distances no more than $r^*(\tau)$ from the object, then there are no bids lower than $\alpha - r^*(\tau)$ and the payment $b(r^*(\tau))$ of the buyer with the winning bid for this object will be the second-largest bid, if there is one, and otherwise 0, so that expected payment (inclusive fee) for the object is

$$\mathbb{E}[b(r^*(\tau))] = \alpha - \mathbb{E}\left[\min\left\{\alpha, r^M_{(2)} \,\middle|\, r^M_{(2)} \leq r^*(\tau)\right\}\right],$$

The expected revenue of the auction house for this object is therefore

$$\pi_\tau = \mathbb{E}\left[b(r^*(\tau))\right] \frac{f}{1+f}. \tag{5}$$

In the special case where τ is so large that $\gamma(\tau) = 0$, the limiting distance $r^*(\tau)$ becomes equal to α, and we get that

$$\pi_0 = R^M \frac{f}{1+f} \tag{6}$$

with R^M as defined in (3).

If both auction houses decide to choose the same date t for their auctions, then the time to the next auction of the competitor τ is very large (formally so large that $\gamma(\tau) = 0$), and there are $2N$ objects for sale at the auctions. This means that there may be cases where the objects of the auction houses are so close (i.e. closely identical) that they will attract the same buyers. While it may be reasonable to assume that each auction house arranges its objects so as to avoid that the neighborhoods $B_\alpha(q^h_j)$ of their objects are overlapping, they cannot avoid the fact that objects from different houses may be close.

We add some structure to the model so as to deal with this feature, and for this

we return to our basic representation of objects as k-tuples of quality characteristics. The set Q of possible objects can be subdivided into $\mathcal{L} = (L/\alpha)^k$ cubes of the form $B_\alpha(\hat{q})$ with non-intersecting interiors, and the number $\mathcal{L}$ then indicates the maximal number of objects which could be put on auction with no buyer wanting more than one of them. One may see $\mathcal{L}$ as measuring the extension or coverage of the particular type of auction. It is assumed throughout that $\mathcal{L}$ is large compared to the number of buyers in the auction.

Assume now that the objects of a given auction house are arranged so as to be at the center of some of the $\mathcal{L}$ cubes. Strictly speaking, we cannot expect the auction houses to change the positions of the objects delivered by the sellers, but we may accept it as an approximation if houses can postpone the sale of too close objects to a later auction, and indeed what matters is how buyers perceive the objects, something which is influenced by the presentation by the houses of their objects. We then have that each house chooses one of the $\binom{\mathcal{L}}{N}$ possible assignments of its N objects to the $\mathcal{L}$ cubes. If both houses conduct their auction at the same time, there may be cases where the same cube is used by both houses, meaning that the objects proposed are very close.[4] For such objects available in two copies, we assume that buyers choose the version on which to bid with probability 1/2. For an object which has no copy, the expected payment will be

$$\pi^1 = R^M \frac{f}{1+f}, \tag{7}$$

where R^M is given in (3), but for objects with a duplicate it will be

$$\pi^2 = R^{M/2} \frac{f}{1+f}. \tag{8}$$

The expected profit of the auction house will be a weighted average of the two, where the weights are determined by the probabilities that neighborhoods occupied by one auction house are selected by the other house as well.

Proposition 2 *Suppose that at date t only one auction house conducts an auction with N objects, while the auction of the other house is scheduled at $t + \tau$. Then, the revenue of the auction house is*

$$\Pi_\tau = N\pi_\tau = N\mathbb{E}\left[b(r^*(\tau))\right] \frac{f}{1+f} \tag{9}$$

for $\tau \in [0, \infty]$. If both auction houses conduct an auction at t, and each house has N objects

[4]Under our simplifying assumptions they are actually identical, something which does not fit well with objects of art, but it should be kept in mind that our model is a simplification, and even in art it may well happen that several objects are almost indistinguishable, for more about this see Section 8.

for sale, then the expected profit of each auction house is

$$\Pi_c = N\left[\left(1 - \frac{N}{\mathcal{L}}\right)\pi^1 + \frac{N}{\mathcal{L}}\pi^2\right], \tag{10}$$

where π^1 *and* π^2 *are given in (7) and (8), and* $\mathcal{L} = \left(\frac{L}{\alpha}\right)^k \geq N$.

The proposition states that if an auction house is without competition at a given date, then the expected revenue reflects the fact that buyers may postpone participation and wait for the auction of the competing house. If they conduct auctions on the same date, then the expected revenue depends on the degree to which the objects of the houses are in direct competition for the individual buyers. This is measured by the fraction $N/\mathcal{L}$ of objects actually for sale, over theoretically possible saleable objects. If $N/\mathcal{L}$ is small, then buyers can choose between objects of both houses only in a few cases, and expected revenue is high, whereas for large values of $N/\mathcal{L}$ competition is more intense and expected revenue will be correspondingly lower.

We have now delineated the outcomes for the auction house. Before we consider the optimal choices, we compare these outcomes under suitable parameter assumptions.

Corollary 1 *Under the assumptions of Proposition 2, the following holds:*
(i) If $\tau' > \tau$ *for* $\tau', \tau \in [0, \infty]$, *then* $\Pi_{\tau'} > \Pi_\tau$.
(ii) There is a $\tau_0 > 0$ *such that for each* $\tau \leq \tau_0$ *there is an* $A_\tau > 0$ *such that*

$$\Pi_\tau \leq \Pi_c \quad \textit{for} \quad N/\mathcal{L} \leq A_\tau$$
$$\Pi_\tau > \Pi_c \quad \textit{for} \quad N/\mathcal{L} > A_\tau.$$

The statements of the corollary are straightforward consequences of the definitions: when another auction is scheduled in the near future, some buyers may prefer to postpone their participation, and this will have a negative effect on the earnings of the auction house. This is reflected in (i), which shows that the negative effect decreases when τ gets larger. In (ii), what is compared is the revenue in latent (that is after a time interval of length τ) competition with that of actual competition. For given τ, the size of $N/\mathcal{L}$, the number of objects relative to the extent of buyers' interests, will according to (10) determine whether contemporary auctions is better than auction with delayed competition. When $N/\mathcal{L}$ is large, both auction houses will propose similar objects in contemporary auctions rather often, with negative consequences for the expected profit of the auction house.

So far, we have considered only the revenues of the auction houses, without touching upon cost. The cost of conducting auctions, arising from the need for organizing and verifying the objects before the sale, will be introduced later when the role of the sellers will also be considered in more detail. In the next section, we consider an intuitive first result regarding the timing of auctions in a simple setting.

5. Equilibrium choices of timing in a two-period model

We now turn to a consideration of the equilibrium actions of the two auction houses with regard to the timing of their auctions. We begin with a case which is simplified as far as possible, where the choices of each auction house are being restricted to one out of two possible dates for the auction. This is clearly an over-simplification, and it does not capture the idea of our auctions as recurrent events, but it serves as an introduction to the more realistic setting, to be treated subsequently. The two consecutive dates, resulting in an early or late auction, are denoted by E or L, and we assume that the interval of time between E and L has length $1 \leq \tau_0$ and that $\gamma(1) > 0$.

Since we consider a case where the two auction houses have equal size, the choice of auction date reduces to a symmetric two person game Γ, the payoff matrix of which is shown in Table 1. Here, the payoffs of the auction houses are as defined above, with $\Pi_1 = \Pi_\tau$ for $\tau = 1$.

Table 1: Payoff matrix for the game Γ

	E	L
E	(Π_c, Π_c)	(Π_1, Π_0)
L	(Π_0, Π_1)	(Π_c, Π_c)

Source: Authors' calculations

The strategies of the auction houses are the dates at which the auctions are conducted, and the payoffs to the auction houses of any pair of strategies are the resulting profits as found in the previous section. The Nash equilibria of Γ will depend on the value of Π in the three different versions as considered in the previous section.

PROPOSITION 3 *Let Γ be the symmetric game between auction houses choosing the date of auction, and let $A = A_1 > 0$ be the constant given in Corollary 1.*

(i) For $N/\mathcal{L} \leq A$, the strategy combination (L, L) is a Nash equilibrium in Γ.

(ii) For $N/\mathcal{L} > A$ there is a symmetric Nash equilibrium in mixed strategies (p_E, p_L) with $p_L > 1/2$, where p_E (p_L) denotes the probability of choosing the strategy E (L).

The proposition shows that simultaneous auctions are an equilibrium outcome of the game under very plausible outcome assumptions of its payoff structure. If $N/\mathcal{L}$ is not too large, that is if the selection of objects for sale in each house is small compared to what could conceivably have been offered under the general theme of the auction, then expected revenue is restricted by competition only to a tolerable degree, and then both houses will select a late auction. If competition is more widespread, the auction house will lose when choosing a fixed decision, either E or L, so mixed strategies must be involved. Even when the Nash equilibrium is one of mixed strategies, the actual play will result in simultaneous actions quite often, indeed with probability close to 1 when p_L is large.

The results so far are reassuring, in the sense that the auction houses typically will select the same date for their auctions, but the model is unsatisfactory, since it deals with a one-time-only event where a possible repetition of the auctions in the future plays no role. This is quite different from the situation with the type of auctions considered, which are recurrent events repeated at rather regular intervals.

6. Equilibrium choices in continuous time with fixed supply

In order to deal with the timing of auctions as recurrent events, to be repeated at more or less regular time intervals, we need to introduce time into the model in a way which fits with our purpose. For this, we shall measure time on a circle with perimeter of length T. Moving ahead in time means moving clockwise on the circle, and the time to the next auction is then the length of the arc from the current point to the first auction occurring using the chosen orientation. We shall use the notation $\leq$ when referring to points on the circle, where $t_1 \geq t_0$ means that t_1 is obtained by moving clockwise from t_0 along an arc of length $a \geq 0$. Adding and subtracting numbers and points on the circle is defined similarly. We will return to a discussion of T, which determines the rate of recurrence of the auction, when we introduce the details of supply of objects and of costs for the auction house.

In order to keep the model simple, we retain the assumption that the number M of potential buyers is constant over time, so that the buyers winning an auction at some date t are replaced by the same number of buyers at the next auction date.

Moreover, we shall simplify the discounting of future auction gains, specifying it as

$$\gamma(\tau) = \begin{cases} 1 & \tau \leq \tau_0 \\ 0 & \tau > \tau_0 \end{cases}$$

where $\tau < T$. Getting an object now, or within a time interval of length up to τ_0, is considered as equivalent, but after τ it has no interest, at least as considered at the given date. Assuming that $\tau_0 > 1$, we may keep the notation Π_1 for the expected profit of an auction house when the auction of the competing house is close by date.

As we are dealing with continuous time, we must also take into account that auctions themselves take time. The auction houses make the objects available for inspection during a time interval of length η before the date of the auction. The objects of the auction will be known to the public only at the start of this inspection period. If the inspection periods of the two auction houses overlap, buyers are aware of the objects of both houses before submitting their bids, so the auctions will appear as contemporaneous. We assume throughout that $0 < \eta < \tau_0 < T$.

Let t_A, t_B be the starting dates of the auctions of houses A and B, initiating D time periods, during which buyers may inspect the objects and submit their bids. If each auction house has N objects at its disposal at the auction date, then the resulting payoffs of auction house A are

$$\pi_A(t_A, t_B) = \begin{cases} \Pi_1 & t_A + \eta < t_B \leq t_A + \tau_0, \\ \Pi_c & t_A \leq t_B \leq t_A + \eta,\ t_B < t_A \leq t_B + \eta, \\ \Pi_0 & \text{otherwise.} \end{cases} \tag{11}$$

The payoff $\pi_B(t_A, t_B)$ of house B is found using symmetry. This payoff structure is essentially the same as the one displayed in Table 1, now transplanted into the context of recurrent auctions.

The change from the context of a singular event to that of repeated auctions has, however, important consequences for the equilibrium choices, basically due to the fact that there is no longer a 'latest' auction date. Indeed, given any pure strategy combination (t_A, t_B), one of the auction houses can increase its profit by placing its auction at a date at most τ_0 later than that of the other house. This means that mixed strategies must be used, and a particular choice of mixed strategies almost suggests itself.

Proposition 4 *Let Γ^* be the symmetric game for choice of auction dates in the context of*

recurrent auctions with a fixed number N of objects for each auction house. Then, there is a symmetric Nash equilibrium in mixed strategies in Γ^* where both houses choose the uniform distribution over T.

In the mixed equilibrium of the proposition, overlapping auctions may occur, but only as a random event, given that the two auction dates chosen independently are within a distance η of each other, and if η is small relative to T the probability of this event is small. That our model in its present version does not exhibit the phenomenon that we set out to explain may have several explanations, and one of the most obvious is that the supply side of the auctions has not been given any attention so far.

7. Equilibrium choices of timing with time-dependent supply of objects

So far, we have not paid attention to the objects sold at the auctions, assuming only that they are available at each auction house in a given number N. The objects may have their origins from several different sources. They may be newly created or taken from a presumably large stock of older objects, of which a certain fraction will change ownership in each period. We shall not go into details here, but assume that objects are put up for sale at a given rate of 2λ objects per period. If the sellers of these objects select the auction house at random, with probability $1/2$ for each house, then the number N of objects for sale at each house must satisfy $N = \lambda T$. A sufficient number of objects must be collected by the auction house before an auction is initiated. We denote this necessary lot size by N, and assume that it fits with the recurrent period length in the sense $T = N/\lambda$ is the number of time periods that must pass from the start of one auction to the possible initiation of the following one.

It is, however, not to be expected that sellers should choose the auction house completely at random. Indeed, it seems reasonable that timing should matter not only for buyers, but also for the sellers supplying the objects, so that sellers prefer the auction house which holds the earliest auction. For $t \in \mathcal{T}$ and (present or future) auction dates $t_A \geq t$, and $t_B \geq t$ of the two houses, let

$$e(t) = \min\{\tau \geq 0 \mid t_h - \eta = \tau, h \in \{A, B\}\}$$

be the time remaining before the earliest (found by moving clockwise from t in the circle) auction initiates its inspection period. Similarly, we let

$$s(t) = \max\{\tau \geq 0 \mid t_h - \eta = \tau, h \in \{A, B\}\}$$

be the time to the second-earliest, and consequently latest, of the two auctions. Clearly, both $e(t)$ and $s(t)$ depend on the choices of t_A and t_B. We omit explicit reference, since the relevant values will be clear from the context.

It seems intuitive that the seller is attracted towards the earliest auction, but that this attraction will depend on the distance between the auctions of the two houses. We define a time-dependent supply rate $v_1(t)$ at the date t of the earliest auction house as

$$v_1(t) = \begin{cases} 2\lambda & e(t) \leq \tau_1,\ s(t) - e(t) > \eta + \tau_1 \\ \lambda & \text{otherwise.} \end{cases} \tag{12}$$

According to this rule, the auction house with the first auction to be held will get a larger inflow of objects, actually the whole flow, unless the auction dates are too far into the future to matter, here determined by the horizon τ_1. Also, this redirection of everything to the earliest auction house will be the case only if the auction of the other house is far enough away in time to influence the seller, here formulated as a distance in time exceeding $\eta + \tau_1$.

Presumably not all the objects presented to the auction house will be eligible for a particular auction, but we assume that the flows are determined net of discarded or otherwise unwanted objects. However, the process of accepting objects and preparing them for the auction constitutes a cost to the auction house. This examination and subsequent presentation to the public of the objects requires a permanent staff with a long-term relationship to the auction house, so that the cost depends on the inflow, not on any given date, but rather on its maximal value $\nu_h = \max\{v_h(t) \mid 0 \leq t \leq T\}$ in the course of the calendar period, for $h = A, B$. We assume that the cost function is linear, so that this capacity cost is linear, with $K\nu$ being the cost of having a staff able to cope with an inflow of size ν.

Since cost depends on the size of the staff rather than on its current services, it is important for the auction house that the seasonal variations in the arrival of new objects are small. A smaller than average inflow means idle capacity, and a larger than average inflow causes rejection of objects due to lack of qualified staff, resulting in an unacceptable loss of reputation. If there is a large time interval between the scheduled dates of auction of the two houses, then each of them will appear as the first to conduct an auction over a considerable stretch of time, and therefore the capacity must be adapted correspondingly. This cost of extra capacity must be weighed against the advantages of being the only house conducting an auction at the given time.

Under suitable assumptions on supply and cost, we obtain that plans with overlapping auction periods are an equilibrium choice in this model.

Proposition 5 *Let Γ^{**} be the symmetric game for choice of auction dates in the context of recurrent auctions with a variable inflow of object as determined in (12). Assume that $\eta + \tau_1 < T/2$ and that $K\lambda > \Pi_0 - \Pi_c$. Then, any pair (t_A^0, t_B^0) with t_A^0 differing from t_B^0 by at most D periods is a Nash equilibrium for any $t \in T$.*

As seen from the proposition, the equilibria are such that auctions overlap, either exactly, so that the dates of the two houses are identical, or at some interval which is so small that all objects will be disclosed to the buyers before any of the auctions closes. The auction houses will accept this simultaneity, even if auctions at disparate dates would give them a larger revenue, since the cost of processing objects, which come with varying intensity, is greater than if they arrive at a steady rate per period. The specific structure of cost, whereby variation over time gives rise to greater cost, can be expected to occur when objects need a careful scrutiny by highly experienced specialists before being accepted for the auction, and while this may seem reasonable for art auctions, it may be less so for auctions of other objects.

8. Competing auctions in the art market

In the preceding sections, we have considered a theoretical model of art auctions, where sellers bring objects to an auction house, which will attempt to sell them at the highest possible price. If auction houses compete for sellers as well as buyers, and the timing of their auctions matters for both parties, then their choices of auction dates will be of central importance. In our model, the equilibrium choices will be such that the auctions of the two houses take place essentially on the same date. We shall now relate these results to the actual behaviour of the two largest auction houses.

The objects accepted by the houses will, pre-auctioning, be gathered into different categories, such as impressionist art, old masters, prints etc., and each category becomes an auction catalogue containing different objects to be auctioned at a future pre-determined date and time. In our model, we have considered only a single category.

The auction catalogues are placed online, on an auction house's website, prior to auctioning. The auction houses, Christie's and Sotheby's place their catalogues online on average 18 days and 14 days, respectively, prior to auctioning (cf. Greve, 2023). In this period, the potential buyer cannot submit a bid on an object, but only view the objects for sale (hereafter called *preview*). This corresponds to the inspection

Table 2. The category 'Prints': Auction start, end and turnover of two auction houses.[6]

Date	Catalogue	Auction start	Auction end	Sale ($)
March 2021				
Sotheby's	Prints & Multiples	5-Mar	19-Mar	864,360
Sotheby's	Prints & Multiples	9-Mar	17-Mar	9,046,822
Christie's	Prints & Multiples	16-Mar	1-Apr	4,394,571
April 2021				
Christie's	Prints & Multiples	15-Apr	16-Apr	6,003,250
Sotheby's	Important Prints & Multiples: Part I	15-Apr	22-Apr	4,371,570
Sotheby's	Important Prints & Multiples: Part II	15-Apr	23-Apr	1,870,911
July 2021				
Sotheby's	Contemporary Prints & Multiples	16-Jul	23-Jul	1,024,506
September 2021				
Sotheby's	Prints & Multiples	8-Sep	15-Sep	2,894,332
Sotheby's	Pop Prints & Multiples	9-Sep	16-Sep	804,510
Christie's	Prints & Multiples	16-Sep	28-Sep	4,496,028
October 2021				
Sotheby's	Important Prints & Multiples: Part I	15-Oct	21-Oct	3,931,830
Sotheby's	Important Prints & Multiples: Part II	15-Oct	22-Oct	3,354,120
Christie's	Prints & Multiples	21-Oct	22-Oct	14,651,375
December 2021				
Sotheby's	Prints & Multiples	3-Dec	10-Dec	1,359,540

Source: Greve (2023)

period of length τ in the formal model, cf. Section 6 above. A catalogue can be auctioned as an online or live auction, where an online auction has a preview, a starting and an ending date, while a live auction has a preview and an ending date.

The auction houses issue catalogues for auctioning several times a year. In 2021, Christie's auctioned its main catalogue in the category 'prints' four times - in March, April, September and October (see Table 2), the same is the case for Sotheby's, who, however, added a catalogue in July and December.[5]

In the light of the results obtained by our model, some characteristic features may be noticed:

Overlapping auctions. It seems reasonable to consider an 'overlap' as a situation where a buyer has access to auction catalogues of the same category from two different auction houses at the same time. As seen in Table 2, the starting date of Christie's April 2021 online auction '*Prints & Multiples*' was 15 April, whereas the ending date

[5]All references to catalogues can be found on the websites of Christie's and Sotheby's, under the name 'Auction Results' and the respective date(s).

was 16 April. The starting date of the online auction '*Important Prints & Multiples: Part I* ' from Sotheby's was also 15 April, whereas the ending date was 22 April. Hence, a buyer could assess the objects for sale at each house before deciding which object to bid on, and participation in an auction could not be decided upon after inspection without uncertainty regarding a possibly better future supply of objects. Thus, the configuration of auctions over time corresponds (with a few exceptions to be commented upon below) to what is stated in Proposition 5.

The notion of overlap can be extended to situations where a buyer has access to two (or more) catalogues of different categories. Thus the starting date of Christie's March 2021 online auction '*Prints & Multiples*' was 16 March and the ending date was 1 April. The starting date of Sotheby's 'old master' auction '*Master Paintings and Drawings*' was 16 March (2021), where the ending date was 23 March. Again, a buyer can compare objects from both catalogues before deciding which (object) to bid for.

Simultaneous auctions. It is rare to find two absolutely simultaneous auctions from two different auction houses, where the dates of the 'preview', starting and ending are all the same. However, one might consider two auctions as simultaneous auctions if they end at the same time (so that they run with the same starting and ending hours on that day). For example, Christie's held their live auction '*Old Master and British Works on Paper: Drawings, Watercolours and Prints 1500-1900*' on 5 July 2022, which was the same date as the auction '*Master Sculpture from Four Millennia*' held by Sotheby's. This kind of simultaneity is however of secondary importance in relation to our model, since what matters is whether one of the houses can enjoy a second-mover advantage by hiding its catalogue until the other house has finished its auction.

Sequential auctions. Christie's ran 331 auctions in 2022 and Sotheby's a total of 512 auctions, split over the 12 months of the year, where both houses were offering narrow and similar lines of categories, meaning that auctions (even with the same category of objects) would be running sequentially, with some distance of time to those of the competing house. Thus, Christie's held their online auction '*Prints & Multiples*' in October 2021 (together with other Sotheby's auctions of the same category), but Sotheby's held a separate online auction '*Prints & Multiples*', in December 2021. Another example is furnished by the Banksy catalogues, where all objects are auctioned sequentially, cf. Table 3. The tendency of the auction to be spread out over time, as prescribed by Proposition 4, indicates that the supply of art objects for these auctions may be less regular that for those considered above.

Table 3. Sequential auctions for Banksy art

Date	Catalogue	Auction start	Auction end	Sale ($)
March 2022				
Christie's	Laugh now but one day we'll be in charge: Banksy and 21st Century Editions	1-Mar	15-Mar	3,444,599
April 2022				
Sotheby's	Banksy	20-Apr	26-Apr	4,321,588
August 2022				
Sotheby's	Banksy	11-Aug	17-Aug	2,245,159
September 2022				
Christie's	Laugh now but one day we'll be in charge: Banksy and 21st Century Editions	7-Sep	21-Sep	1,435,448

Source: Chrisitie's /Sotheby's

Observing the auctions run by Christie's and Sotheby's may also be useful in assessing some of our model assumptions.

Dispersed objects. We have assumed that auction houses may arrange that the objects for sale are suitably different from each other, even though two different auction houses may well have similar objects. This may be justified in several ways: according to Newman and Bloom (2012), bidders perceive art objects as unique and therefore dispersed, and since prices decrease with supply, an auction house should search for dispersed objects to minimize the risk of lower revenues. Also the art catalogues provide examples to justify the assumption:

Christie's November 2022 auction, '*20th Century Evening Sale*' had a Monet painting for sale and a couple of paintings by Magritte, hence, two artists from different genres. The Monet was from 1880 and estimated to sell for $4m-$6m. The competing November 2022 auction from Sotheby's, named '*Modern Evening Auction*', also had a Monet for sale. It was from 1886 and also estimated to sell for $4m-$6m.

Compared to paintings, prints are less rare objects, since they are produced in a series of impressions, known as limited edition prints. Andy Warhol is one of the best known modern artists within this field of work. Christie's September 2022 auction '*Prints and Multiples*' had Warhol's Queen Elizabeth II and Elizabeth Taylor prints for sale. A buyer might be interested only in the print of Queen Elizabeth II, and for this buyer the two prints could be considered as dispersed objects. Sotheby's September 2022 auction also had a Queen Elizabeth II for sale.

In their May 2022 auction, '*Modern Day Auction*', Sotheby's had two Jackson Pollock prints for sale, estimated at the same price, but only one of the prints was signed. The signed print was sold for $315,000, while the other was sold for $94,500. Hence, it is likely that buyers considered these two prints as dispersed objects.

Almost identical objects. Even for prints, there are no absolutely identical objects. There will always be a difference between two objects, in terms of, for example, the number of editions made or the ageing of the sheet. However, it is often possible to find almost identical objects for sale. In its September 2022 catalogue '*Prints and Multiples*', Christie's had the print 'Queen Margrethe II of Denmark' from Andy Warhol for sale. The review date was 2 September and the auction was 14-28 September, the estimated price was £15,000-£25,000 and the hammer price was £37,800. Sotheby's had the same print for sale in their September 2022 catalogue ' *Prints & Multiples*', with a review date 9 September and auction 9-14 September. Their estimate was the same as Christie's, but this print was sold for £56,700. The prints had the same subject and colouring, both were signed, but the print from Christie's was numbered '28/40' (the 28th print out of an edition of 40), whereas the print from Sotheby's was numbered '11/30'. It, thus, appears that what could be considered as tiny differences (in this case in the print run) may have importance.

Unit demand. We have assumed throughout the paper that a bidder has unit demand, that is, they want (and potentially bid on) at most one object in each time period. This is a standard assumption in the literature on auctions, see e.g. Engelbrecht-Wiggans (1980), Beggs and Graddy (1997) and Hong et al. (2015). In the present case, acquiring a piece of art can be considered a large investment, as indicated in Table 4.

Table 4. Average price of art in a catalogue

	Catalogue	Auction date	Price realised Total ($)	Price realised Average ($)
Evening sale				
Christie's	20th/21st Century: London Evening Sale	1 Mar 2022	240,574,101	3,758,970
Sotheby's	Contemporary Evening Auction	2 Mar 2022	253,037,802	4,685,885
Impressionist				
Christie's	Impressionist and Modern Art Day Sale	13 Nov 2021	29,330,000	191,699
Sotheby's	Modern Day Auction	17 Nov 2021	53,015,168	180,324
Prints				
Christie's	Prints and Multiples	14-28 Sep 2022	6,335,999	35,006
Sotheby's	Prints & Multiples	8-14 Sep 2022	3,354,419	24,307

Source: Chrisitie's /Sotheby's

As shown in the table, the average unit price of evening sales (high value art) is around $4m. For impressionist art it is $200,000, and $30,000 for prints. It seems reasonable to assume that few buyers have the means to bid for more than one object.

9. Concluding comments

In the previous sections, we have been concerned with the competition between two auction houses as manifested in their choices of auction dates. It was shown that the equilibrium choice of the houses would result in (almost) simultaneous or overlapping auctions. The reason for this somewhat surprising outcome is to be found in the behaviour of the buyers as well as sellers. That it does occur in practice has already been argued in Greve (2023), dealing with art auctions held by Sotheby's and Christie's, as mentioned above in Section 8.

In this paper, we have identified the optimal choices of the auction houses and the resulting equilibria. We have shown that the optimal timing of the auction changes from situation to situation, depending on whether it is a one-time-only or a recurrent events, as well as on the supply of objects and cost of running the auctions. It was shown that simultaneous and overlapping auctions may occur with sufficient frequency to be considered an important feature of competing auctions. The theory seems to be consistent with our empirical findings, in the sense that it is not uncommon to find high percentages of overlapping auction dates at Christie's and Sotheby's.

In our model of competition between auction houses we have used several simplifying assumptions. Relaxing them is a challenging task for further research. First of all, buyers' values may be selected randomly using other distributions over the quality configurations, allowing for some general considerations of more or less valuable art objects. This is something which may provide an opportunity for more realistic decision problems for the auction houses, which would also involve the selection of objects for auctions. Next, the seller side could be elaborated upon, so that their preferences would influence decisions about objects for sale. Moving further, one might introduce some dependence of preferences on observed prices of objects sold at previous auctions. Allowing for greater realism of the model is highly desirable, but keeping the model tractable requires some further considerations. This will be a task for future research.

We have used the art market as a motivation, backed by the observation of real-life phenomena. However, the model might turn out to be useful in many other cases where there are competing (auction) sales. Examples could be two countries selling their 5G services, or the sale of two state-owned airline companies. One could also ignore the use of auctions and consider the competition between supermarkets and the question of when a supermarket should promote a particular product (i.e. object) or group of products. Another example could be a filmmaker who has to decide when to launch a new film.

In the present paper, our focus was upon the choice of time for auctioning objects. However, one could add other parameters in order to explore how one enterprise competes with another, e.g. commission fees, to attract certain sellers or more buyers. Alternatively, one could consider a framework with two competing auction houses of equal size (like Christie's and Sotheby's) and one smaller house seeking to increase its sales. A final suggestion could be to extend the analysis with an empirical study of an optimal auction. We leave these possible extensions for future work.

Supplementary Material

Supplementary material is available at the Oxford Economic Papers Journal online. This includes the online appendix.

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Conflict of Interest

The authors declare that they have no conflicts of interest.

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