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Deposited version:
Accepted Version

Peer-review status of attached file:
Peer-reviewed

Citation for published item:
Vicente, J. J. & Cardoso-Grilo, T. (2025). Cost-effective and operationally sustainable supply chain inventory planning: A mathematical programming approach. In José Luís Reis, José Cascalho, Marc K. Peter, Luís Paulo Reis, Maria Teresa Tiago (Ed.), Marketing and smart technologies: Proceedings of ICMarkTech 2024. (pp. 51-53). São Miguel, Açores: Springer.

Further information on publisher's website:
10.1007/978-981-96-3081-3_3

Publisher's copyright statement:
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Cost-effective and operationally sustainable supply chain inventory planning: a mathematical programming approach

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Abstract

The growing concerns about economically and operationally sustainable supply chains are increasingly at the forefront. It is urgent to create innovative models and less costly systems. This can be achieved through using more efficient supply chains and the rational use of resources.

Inventory management has been widely studied by several authors. Nevertheless, there is still a small number of studies exploring its application at the supply chain level, despite being widely recognized that the collaboration between partners in the supply chain is critical to achieve more efficient inventory management decisions. In fact, such a collaboration fosters the global optimization of supply chains.

In this paper, the inventory supply chain planning problem is addressed through the use of the continuous review (r,Q) policy, aiming at optimizing inventories throughout a supply chain while defining the amount of products transported among facilities over time. A mixed integer linear programming (MILP) model is built for that purpose and a multi-periodic/multi-warehouse/multi-retailer supply chain is considered as reference. This model adds to existing supply chain inventory management literature by extending existing models to a multi-periodic and multi-warehouse supply chain. The model, hereafter called *CRP2InvSC* model, allows to determine inventory levels, order quantities *Q* and reorder points *r* for each time period and each facility on a supply chain. Products flows between supply chain entities are also modelled.

A case study based on a distribution supply chain is used to illustrate the applicability of the proposed model.

Keywords: supply chain management, inventory planning, mixed integer linear programming, continuous review policy.

1. Introduction

Supply chain inventory problem is very important and challenging because all members in the supply chain should be planned and controlled simultaneously to obtain good control parameters and low inventory costs to enhance stakeholders' capability of overseeing essential activities and tasks, such as the upkeep of critical assets and infrastructure. The inventory policy used by each member is also an important issue because it affects the inventory replenishment process on supply chain (You and Grossmann, 2010). Based on these ideas, Krautter (1999) presented new perspectives for corporative management on the inventory theory area and stated that inventories appear as the result of mismanagement of the different supply chain processes. The author recognized that having complete control over the processes is essential for optimizing inventory management. Hence, all supply chain members should collaborate and integrate their orders and inventory policies to achieve more efficient inventory control by aiding decision-making that impacts the company's financial outcomes.

In this context inventory planning appears as an important and challenging problem where decisions made by a supply chain member may affect, with different impacts, the remaining supply chain entities. Nevertheless, although recognized as critical for achieving the optimal performance of supply chains, the planning of inventories at the supply chain level started to be addressed only in recent years, with the vast majority of studies limiting the analysis to one-warehouses supply chains (e.g., Abdul-Jalbar et al. (2006), Monthatipkul and Yenradee (2008) and Malmberg and Marklund (2023)).

Within this context, this study contributes to the literature by developing a supply chain inventory planning model that supports decision-makers when making decisions on inventory levels, order quantities, reorder points and product flows within a multi-period/multi-warehouse/multi-retailer supply chain. The proposed model – hereafter called *CRP2InvSC* model – is developed as a mixed-integer linear programming (MILP) model, and it is a generic model that can be adapted to different structures of distribution networks with two echelons. Demand is satisfied at the retailers and the retailers replenish their inventory from the warehouses through a reordering policy. On the other hand, warehouses replenish their inventory from a central warehouse, located outside of the system boundaries. Costs include ordering, holding and transportation.

This study is organized as follows. Section 2 includes a literature review on supply chain inventory planning. Section 3 describes the inventory planning mathematical model for

continuous review policy. In section 4 an inventory planning case study is present and solved. Section 5 wraps up with the final conclusions.

2. Literature review

Existing literature shows that a few models exist representing real world supply chains, since most are focused on single-echelon systems. Research with this system can be found in Shen et al. (2003), where assumptions are made, such as no limit to the storage capacity and all equal lead times from supplier to distribution centers.

Recently, investigations have begun to appear in the literature where inventory management in the supply chain has been studied and where various inventory management policies have been addressed. Inventory planning in supply chains is extremely important – inventory allows flexibility, but also represents costs (You and Grossmann, 2010).

When dealing with the classic deterministic one-warehouse multi-retailer inventory/distribution system, Abdul-Jalbar et al. (2006) considered customer demand rates to be constant. The authors also considered that there was no backlog or lost sales, and no lead time was considered. Retailers submitted orders to meet customer needs, which in turn created demand at the warehouse. Each facility incurred costs that included a fixed fee per order and storage expenses.

Yu-Cheng (2008) studied the traditional deterministic one-warehouse multi-retailer network inventory management. In this research, it was assumed that customer demand rates were both known and unchanging. Shortages were not permitted, and lead times were assumed negligible.

Monthatipkul and Yenradee (2008) developed an inventory control system for a one-product, one-warehouse multiple-retailers distribution supply chain. A MILP model was developed to determine the optimal inventory and distribution plan that minimized the total related costs. The experimental results showed that the system outperformed the periodic review policy when faced with non-stationary demands.

Later on, Li and Hai (2019) studied inventory management for a single-warehouse multi-retailer system while considering carbon emissions. They establish the reorder intervals for both the warehouse and the retailers, aiming to minimize the costs associated with inventory ordering, storage, and carbon emissions.

Recently, Malmberg and Marklund (2023) highlighted the importance of inventory and shipment decisions on distribution systems. These authors studied this topic related to one-

warehouse-multiple-retailer network. The replenishments at the warehouse are made by an outside supplier/manufacturer according to a continuous review (r, Q) policy.

However, few works have been presented in the literature accounting for multi-echelon systems where an integrated view of the inventory along the chain is considered. Most of previous studies rely on a one-warehouse supply chain, and according to the authors review no study was identified considering the extension to a multi-periodic and multi-warehouse supply chain.

Based on this analysis it is clear that some space still exists to generalize the inventory planning models, exploring systems' structures and operations, where the treatment of inventory management should also be considered. Thus, a closer description of the real problems should be explored, namely, considering inventory systems parameters, such as demands and lead times, acting over multi-echelon and multi-period systems.

The research objectives of the paper are presented as follows. The above identified opportunities are explored along the present paper where a model implements the continuous review (r,Q) policy and determines both the inventory levels and order quantities plans that minimize the distribution supply chain total costs.

3. Continuous review (r,Q) policy under supply chain inventory planning

The supply chain inventory planning problem is formulated as MILP model – the CRP2InvSC model. For the purpose of this study, two distribution echelons are considered, and several nodes may exist per echelon. Potential flows may be verified between any two nodes on the two echelons. The overall system is supplied by a central warehouse. To fulfil an order, one or more flows will be planned from one or more origin nodes.

The model is based on a discrete time formulation, where time periods are assumed to have a fixed duration, and a continuous review policy is used as reference. In continuous review (r,Q) policy there are two important decisions to be made: how much to order (Q) and when to order. Due to the lead time between placing and receiving an order, the decision of when to order is typically based on a reorder point (r). The r is given as: r = Demand per time period × Lead time for a new order in time periods (r = D × L). It is assumed that demand during the lead time and lead time itself are constant.

The mathematical formulation is provided below. The following indices, constants, sets, parameters and variables (non-negative continuous and binary) are defined:

Indices

i, j, k	entity node
t	time period

Constants

NW number of warehouses

NR number of retailers

NT number of time periods

Sets

$i, j, k \in I = \{0, 1, 2, \dots, NW, NW + 1, NW + 2, \dots, NW + NR\}$ supply chain entities

$t \in T = \{1, 2, \dots, NT\}$ time periods

$W_o = \{0\}, \quad W_o \subset I$ central warehouse

$W = \{1, 2, \dots, NW\}, \quad W \subset I$ warehouses

$R = \{1,2, \dots, NR\}, \quad R \subset I$ retailers

Parameters

D_{kt}	demand at entity *k* in time period *t* (note that customer demand occurs only at the retailers, but not at the warehouses)
HC_j	unitary holding cost at entity *j* per time period
Ito_j	initial inventory level at entity *j*
L_{jk}	order lead time from entity *j* to entity *k*
OC_j	ordering cost at entity *j* (note that ordering cost is independent of product quantity)
STC_{jt}	storage capacity at entity *j* in time period *t*
TC_{jk}	unitary transportation cost from entity *j* to entity *k*

Non-Negative Continuous Variables

INV_{jt}	inventory at entity *j* at the end of the time period *t*
r_j	reorder point of entity *j*
QP_{jkt}	quantity of product from entity *j* to entity *k* during time period *t*
Q_j	order quantity of entity *j*

Binary Variables

X_{jt}	= 1 if an order is already placed by entity *j* in time period *t*, 0 otherwise
Y_{jt}	= 1 if an order is placed by entity *j* in time period *t*, 0 otherwise

The model aim is to minimize the value of the total cost. This leads to the objective function (1).

$$\text{Minimize total cost} = \sum_{j\in W}\sum_{t\in T} OC_j \times Y_{jt} + \sum_{k\in R}\sum_{t\in T} OC_k \times Y_{kt} + \sum_{j\in W}\sum_{t\in T} HC_j \times INV_{jt}$$

$$\sum_{k\in R}\sum_{t\in T} HC_k \times INV_{kt} + \sum_{i\in W_0}\sum_{j\in W}\sum_{t\in T} TC_{ij} \times QP_{ijt} + \sum_{j\in W}\sum_{k\in R}\sum_{t\in T} TC_{jk} \times QP_{jkt} \quad (1)$$

The minimization of system costs is subject to the following constraints:

$$INV_{jt} = Ito_j + QP_{i,j,t-L_{ij}|L_{ij}=0} - \sum_{k\in R} QP_{jkt}, \;\; i \in W_0, j \in W, t = 1 \quad (2)$$

$$INV_{jt} = INV_{j,t-1} + QP_{i,j,t-L_{ij}|L_{ij}<t} - \sum_{k\in R} QP_{jkt}, \;\; i \in W_0, j \in W, t \in T\backslash\{1\} \quad (3)$$

$$INV_{jt} = Ito_j, \qquad j \in W, t = NT \quad (4)$$

$$INV_{kt} = Ito_k + \sum_{j \in W} QP_{j,k,t-L_{jk}|L_{jk}=0} - D_{kt}, \quad k \in R, t = 1 \tag{5}$$

$$INV_{kt} = INV_{k,t-1} + \sum_{j \in W} QP_{j,k,t-L_{jk}|L_{jk}<t} - D_{kt}, \qquad k \in R, t \in T \backslash \{1\} \tag{6}$$

$$INV_{kt} = Ito_k, \qquad k \in R, t = NT \tag{7}$$

$$INV_{jt} - r_j \leq STC_{jt} \times (1 - X_{jt}), \;\; j \in W, t \in T \tag{8}$$

$$\sum_{u=t-L_{ij}+1}^{t} Y_{ju} = X_{jt}, \qquad j \in W, t \in T \tag{9}$$

$$-STC_{jt} \times X_{jt} + 0{,}0001 \leq INV_{jt} - r_j, \;\; j \in W, t \in T \tag{10}$$

$$QP_{ijt} - Q_j \leq STC_{jt} \times (1 - Y_{jt}), \quad i \in W_0, j \in W, t \in T \tag{11}$$

$$-QP_{ijt} + Q_j \leq STC_{jt} \times (1 - Y_{jt}), \quad i \in W_0, j \in W, t \in T \tag{12}$$

$$QP_{ijt} \leq STC_{jt} \times Y_{jt}, \quad i \in W_0, j \in W, t \in T \tag{13}$$

$$INV_{kt} - r_k \leq STC_{kt} \times (1 - X_{kt}), \;\; k \in R, t \in T \tag{14}$$

$$\sum_{u=t-L_{jk}+1}^{t} Y_{ku} = X_{kt}, \qquad k \in R, t \in T \tag{15}$$

$$-STC_{kt} \times X_{kt} + 0{,}0001 \leq INV_{kt} - r_k, \;\; k \in R, t \in T \tag{16}$$

$$\sum_{j \in W} QP_{jkt} - Q_k \leq STC_{kt} \times (1 - Y_{kt}), \quad k \in R, t \in T \tag{17}$$

$$-\sum_{j\in W} QP_{jkt} + Q_k \leq STC_{kt} \times (1 - Y_{kt}), \quad k \in R, t \in T \tag{18}$$

$$\sum_{j\in W} QP_{jkt} \leq STC_{kt} \times Y_{kt}, \qquad k \in R, t \in T \tag{19}$$

$$INV_{jt} \leq STC_{jt}\,, \qquad j \in I, t \in T \tag{20}$$

$$INV_{jt}, QP_{jkt}, Q_j \geq 0\,, \qquad j \in I, k \in I, t \in T \tag{21}$$

$$X_{jt}, Y_{jt} \in \{0,1\} \quad j \in I, t \in T \tag{22}$$

The objective function (1), considers the total cost that involve: warehouse ordering cost (first term) and retailer ordering cost (second term); holding cost at both stages of the distribution supply chain, warehouses and retailers (third and fourth terms); and, finally, the transportation cost from central warehouse to warehouses and from warehouses to retailers (fourth and fifth terms).

Apart from the objective function, different constraints constitute the final inventory planning model. Constraints (2) and (3) state that the total incoming quantity at each warehouse *j* is equal to the product quantity from the central warehouse at time period *t*, considering the order lead time through the introduction of a time lag. The total outgoing quantity at each warehouse *j* is equal to the sum of product quantities to the retailers *k* at time period *t*. For *t = 1* the inventory at the end of this time period at warehouses *j* is given by constraint (2), which takes into account the initial inventory level at each warehouse *j* (Ito_j). For the remaining time periods the inventory at the end of these time periods at warehouses is given by constraint (3). Both initial inventory (for time period 1) and final inventory (for time period NT) have been set equal to reduce the impact of the initial inventory for the next planning horizon, as shown in constraint (4).

At each retailer *k*, the incoming quantity is equal to the sum of the product quantity from the warehouses *j* at time period *t*, considering the order lead time. At each retailer *k*, the outgoing quantity is equal to the customer demand at time period *t*. For *t = 1* the inventory at the end

of this time period at the retailers k is given by constraint (5), which accounts for the initial inventory level at retailer k (Ito_k) whereas constraint (6) is applicable for the remaining time periods. Both initial inventory (for time period 1) and final inventory (for time period NT) have been set equal to reduce the impact of the initial inventory for the next planning horizon, as show in constraint (7).

The continuous review (r,Q) policy can be formulated using the following constraints. For warehouses, constraints (8), (9) and (10) state that when the inventory is less than or equal to level r at a given period an order is placed, unless an order has been already placed. Constraint (11), (12) guarantee that an order is placed for Q units of product. Given that transportation occurs after placing an order, it is assumed that the fixed ordering cost is always incurred during this process. Therefore, if the transportation amount is greater than zero, the binary variable Y_{jt} is set to 1, as indicated by constraint (13). The left-hand side of this constraint represents the quantity received by a warehouse j, which comes from the central warehouse.

Equivalent constraints are defined for retailers in constraints (14)-(19). The STC value will have a value that is valid as an upper bound of any quantity that can be ordered by a warehouse or retailer.

The total inventory stored at any node must respect the storage capacity in each demand node j at any time period t which is enforced by constraint (20). Note that the central warehouse has storage capacity unlimited.

As defined above, the model uses both non-negative continuous variables (21) and binary variables (22).

The above model formed by constraints (2) to (22) and objective function (1) describes the proposed supply chain inventory policy model.

4. Case study

A case study is analyzed using the proposed supply chain inventory planning model. A deterministic set is assumed. The GAMS 41.0 modelling language was used to solve the problem in hand. An Intel CORE i7 CPU 3.34 GHz and 16GB RAM was utilized. The stopping criteria were either a computational time limit of 3 600 seconds or the determination

of the optimal solution (0\% of relative gap). The aim of this case study is to find out the inventory level and order quantity profiles over the time horizon and the reorder point for the retailers.

4.1 Data and parameters

The supply chain considered involves one central warehouse, two warehouses and four retailers. The maximum storage capacity of warehouses and retailers is 5 000 units. A 15-days planning horizon was assumed, which uses a continuous review (r,Q) policy. The ordering cost for warehouses and retailers is 30 euros. The unitary holding costs are 0.2 euros and 0.6 euros per time period, respectively for warehouses and retailers. Tables 1-4 present more parameters' values considered for the case study. Table 1 shows the transportation costs per unit between central warehouse and warehouses as well as between warehouses and retailers. The initial inventory level in each supply chain facility is shown in Table 2. The demand at retailers is presented in Table 3 and, finally, Table 4 shows the order lead times among all supply chain facilities.

Table 1 Unitary transportation (TC) costs (euro).

		Warehouse1	Warehouse2	Retailer1	Retailer2	Retailer3	Retailer4
TC	Warehouse0	0.55	0.22	-	-	-	-
	Warehouse1	-	-	0.22	0.20	0.32	0.38
	Warehouse2	-	-	0.68	0.52	0.34	0.1

Table 2 Initial inventory level (Ito) (unit).

	Warehouse1	Warehouse2	Retailer1	Retailer2	Retailer3	Retailer4
Ito	1200	1100	350	450	500	600

Table 3 Demand (D) at the retailers in each period of time horizon (unit).

	Retailer1	Retailer2	Retailer3	Retailer4
D	50	60	70	80

Table 4 Order lead time (L) (time period).

		Warehouse1	Warehouse2	Retailer1	Retailer2	Retailer3	Retailer4
L	Warehouse0	3	3	-	-	-	-
	Warehouse1	-	-	3	3	3	3
	Warehouse2	-	-	3	3	3	3

The results for the case study under deterministic demand are presented in the subsequent section.

4.2 Experimental results

Results show that costs are minimized when warehouse 1 supplies retailers 1, 2 and 3, and warehouse 2 supplies retailers 3 and 4 (see Appendix). The figures below detail the results obtained for the particular case of warehouse 2 and retailers 3 and 4.

Figure 1 shows the inventory level and order quantity profiles for retailer 3. As can be seen, retailer 3 places an order quantity of 525 units (Q) when inventory level drops of 185 units (r) – 500 units are ordered from warehouse 2 and the remaining 25 are ordered from warehouse 1. The replenishment takes three days (lead time), from day 5 to day 8 and this cycle repeats again from day 12 to day 15. A similar analysis can be done to figure 2 for retailer 4.

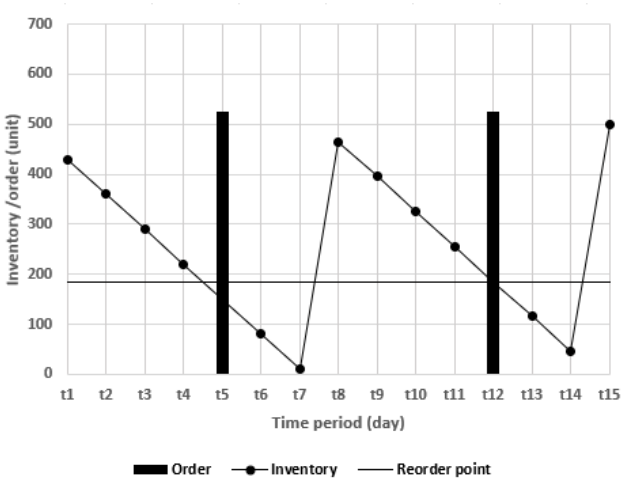


Figure 1 Inventory and order quantity for retailer 3 for 15-days time horizon.

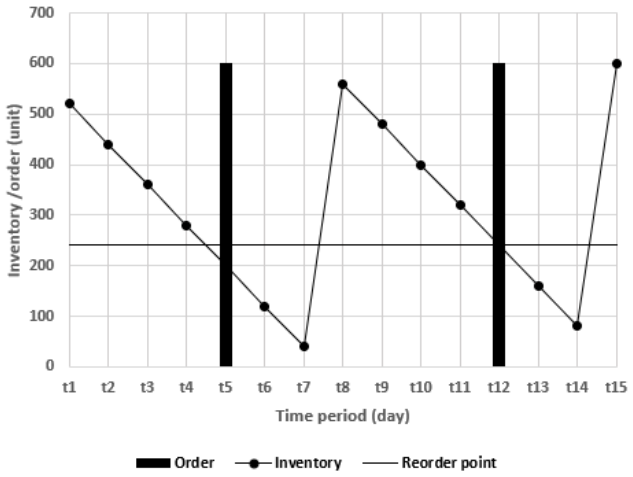


Figure 2 Inventory and order quantity for retailer 4 for 15-days time horizon.

Figure 3 shows the inventory level and order quantity profiles for warehouse 2. As can be seen, warehouse 2 places an order quantity of 1 100 units (Q) when inventory level drops below 0 (r). The replenishment takes three days (lead time), from day 8 to day 11 and this cycle repeats again from day 12 to day 15.

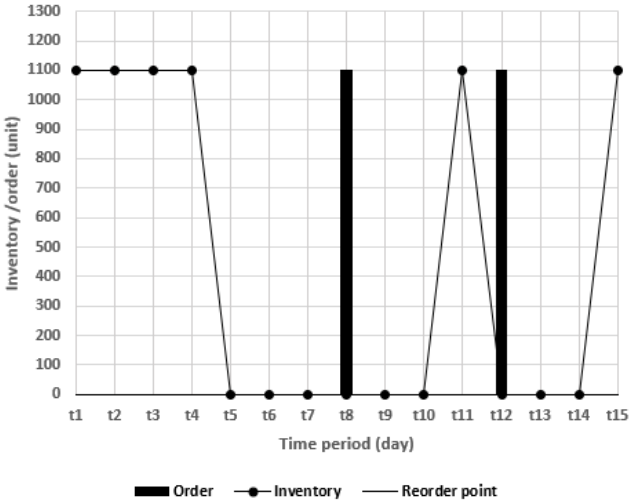


Figure 3 Inventory and order quantity for warehouse 2 for 15-days time horizon.

Table 5 presents computational statistics related to the performance of the model. The value of the objective function is 14 730 euros. It was achieved an optimal solution (0% of relative gap) in less than 10 seconds.

Table 5 Computational statistics.

MIP solution (euro)	14 730
Best possible (euro)	14 730
Relative gap (%)	0
Single equations	907
Single variables	448
Discrete variables	180
Computational time (second)	9.98

4.3 Discussion of the results

The suggested model (CRP2InvSC model) aims to enhance the synchronization between tactical planning and supply chain inventory management. A unified inventory policy strategy was set for the whole time horizon and for all supply chain facilities. To evaluate how mathematical formulation can be applied, a case study is optimized using an inventory policy.

Based on the experimental results presented in the previous subsection, it is possible to verify the validity of the model, since the behavior found complies with the expected for a continuous review policy (r,Q). This policy was adjusted to a discrete-time model, loosening the requirement for the inventory level to be exactly at the reorder point to trigger a replenishment. The replenishment orders are consistent in size and appropriately scheduled. The CRP2InvSC model determines inventory levels, order quantities (Q), and reorder points (r). These inventory and order parameters are crucial to the supply chain operations manager's decision making.

The inventory volume increases from retailer 1 to retailer 4. This happens because daily demand also increases from retailer 1 to retailer 4. Furthermore, the reorder point also increases to meet demand satisfaction during the lead time.

The CRP2InvSC model provides optimal planning solutions and define clear management strategies for warehouse and retailer inventory control. Additionally, the computational efficiency of the model is an issue to be highlighted.

5. Conclusions

This paper proposes an inventory planning policy model applied to a multi-periodic/multi-warehouse/multi-retailer distribution supply chain. The model provides an inventory and distribution plan that minimizes the total costs for the global supply chain. The proposed model implements the continuous review (r,Q) policy applied to supply chain inventory planning. The model determines inventory levels, order quantities, reorder points and shipping quantities between supply chain entities.

The proposed model brings a contribute to the supply chain inventory management literature not only because there is still a limited number of studies around this topic, but also because no study was identified extending this planning to a multi-periodic and multi-warehouse supply chain.

The results show that retailers can use this inventory policy integrated in a supply chain, working together to obtain the minimum cost for their operations.

Although the model proposed resulted to be quite flexible and able to solve real inventory systems problems, further research is still required. There are some limitations to be further addressed, in particular the need for more detailed validation of the proposed model should be pursued. Namely, more complex distribution supply chain structures should be studied over larger periods of time and with more products flowing. Aspects within the inventory planning systems should be further studied, such as the definition of the safety stock levels and the comparisons with another inventory policies under uncertain conditions. Alternative research directions can be explored, such as addressing the environmental and social impacts of the supply chain.

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Appendix

Inventory supply chain planning for 15-time period horizon

		t1		t2		t3		t4		t5		t6		t7		t8	
		II	INV	II	INV	II	INV	II	INV	II	INV	II	INV	II	INV	II	INV
	w1	1200	1200	1200	1200	1200	1200	1200	1200	1200	350	350	350	350	350	350	350
	w2	1100	1100	1100	1100	1100	1100	1100	1100	1100	0	0	0	0	0	0	0
D																QP From w0 1100	
50	r1	350	300	300	250	250	200	200	150	150	100	100	50	50	0	0+ 375	325
										QP From w1 375							
60	r2	450	390	390	330	330	270	270	210	210	150	150	90	90	30	30+ 450	420
										QP From w1 450							
70	r3	500	430	430	360	360	290	290	220	220	150	150	80	80	10	10+ 525	465
										QP From w1 25							
										QP From w2 500							
80	r4	600	520	520	440	440	360	360	280	280	200	200	120	120	40	40+ 600	560
										QP From w2 600							

II Initial Inventory INV Final Inventory D Demand QP Quantity of Product

		t9		t10		t11		t12		t13		t14		t15	
		II	INV	II	INV	II	INV	II	INV	II	INV	II	INV	II	INV
	w1	350	350 QP From w0 850	350	350	350	350	350+ 850 QP From w0 850	350	350	350	350	350	350+ 850	1200
	w2	0	0	0	0	0+ 1100	1100	1100 QP From w0 1100	0	0	0	0	0	0+ 1100	1100
D															
50	**r1**	325	275	275	225	225	175	175 QP From w1 375	125	125	75	75	25	25+ 375	350
60	**r2**	420	360	360	300	300	240	240 QP From w1 450	180	180	120	120	60	60+ 450	450
70	**r3**	465	395	395	325	325	255	255 QP From w1 25 QP From w2 500	185	185	115	115	45	45+ 525	500
80	**r4**	560	480	480	400	400	320	320 QP From w2 600	240	240	160	160	80	80+ 600	600

II Initial Inventory INV Final Inventory D Demand QP Quantity of Product