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Essays on Initial Margin Models

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Essays on Initial Margin Models

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Para a Rosária

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Resumo

Esta tese analisa os modeloes de Margin Inicial (*Initial Margin* – IM) utilizados pelas Contrapartes Centrais (CCPs) e as suas implicações, com foco na estimativa de risco, conformidade regulatória e impacto nos mercados.

Através de três estudos independentes, investiga-se o desempenho dos modelos de IM em períodos de stress, a eficácia das medidas contra a prociclicalidade e a relação entre margens e liquidez de mercado.

O primeiro estudo compara os modelos de IM em conformidade com a regulação durante a crise da COVID-19, concluindo que os modelos de *Value at Risk* (VaR) Paramétrico e de Simulação de Monte Carlo superam a Simulação Histórica na captação de choques de mercado. O segundo estudo avalia as medidas de Anti-Prociclicalidade (APC) previstas no regulamento EMIR, propondo uma nova abordagem baseada na taxa de variação da IM, bem como uma métrica de desempenho baseada na volatilidade da IM. Esta abordagem revela-se mais estável e eficaz do que os métodos regulatórios existentes. O terceiro estudo analisa a relação bidirecional entre IM e liquidez de mercado, demonstrando que os modelos APC *floor* e *stressed* apresentam impactos mais claros na liquidez do mercado, enquanto a influência dos modelos *buffer* é menos evidente devido à sua instabilidade.

Em conjunto, os estudos evidenciam a necessidade de uma calibração mais rigorosa dos modelos e de orientações regulatórias mais claras, de modo a garantir que as CCPs possam mitigar o risco sistémico sem comprometer a liquidez do mercado.

Classificação JEL: C32; C63; G18; G20; G28.

Palavras chave: Mercados Financeiros; Contrapartes Centrais; Margem Inicial; Prociclicidade; Valor em Risco; Liquidez de Mercado; Spread Bid-Ask; Vetores Autorregressivos ; Modelo de Correção de Erros Vetorial

Abstract

This thesis explores the design and implications of Initial Margin (IM) models used by Central Counterparties (CCPs), focusing on risk estimation, regulatory compliance, and market impact. Through three standalone studies, it examines how IM models perform under stress, address procyclicality, and affect market liquidity.

The first study compares regulatory-compliant risk models during the COVID-19 turmoil, finding that Monte Carlo and Parametric Value at Risk (VaR) models outperform Historical Simulation in capturing market shocks. The second study critiques EMIR's Anti-Procyclicality (APC) measures, proposing a novel approach based on IM rate of change and introducing a new performance metric based on margin volatility. This method proves more stable and effective than existing APC tools. The third study analyzes the two-way relationship between IM and market liquidity, showing that while floor and stressed APC models exhibit measurable effects, the buffer model's impact is less clear due to instability.

Together, these papers highlight the need for better model calibration and clearer regulatory guidance to ensure CCPs can manage systemic risk without harming market liquidity.

JEL Classification: C32; C63; G18; G20; G28.

Keywords: Financial Markets; Central Counterparties; Initial Margin; Procyclicality; Value at Risk; Procyclicality; Market Liquidity; Bid-Ask Spread; Vector Autoregression; Vector Error Correction Model

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List of Abbreviations

Abbreviation	Definition
ADF	Augmented Dickey-Fuller
APC	Anti-Procyclicality
AIC	Akaike Information Criterion
ARMA	Autoregressive Moving Average
ARCH	Autoregressive Conditional Heteroskedasticity
BCBS	Basel Committee on Banking Supervision
BIS	Bank for International Settlements
CCP	Central Counterparty
CM	Clearing Member
CPMI	Committee on Payments and Market Infrastructures
EMIR	European Market Infrastructure Regulation
ESCB	European System of Central Banks
ESMA	European Securities and Markets Authority
EWMA	Exponential Weighted Moving Average
FHSVaR	Filtered Historical Simulation Value at Risk
FIA	Futures Industry Association
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GBM	Geometric Brownian Motion
HSVaR	Historical Simulation Value at Risk
IM	Initial Margin
IOSCO	International Organization of Securities Commissions
IRF	Impulse Response Function
KPSS	Kwiatkowski-Phillips-Schmidt-Shin
MLE	Maximum Likelihood Estimation
MCSVaR	Monte Carlo Simulation Value at Risk
MPOR	Margin Period of Risk
MtM	Mark to Market
PFMI	Principles for Financial Market Infrastructures
PP	Phillips-Perron
PVaR	Parametric Value at Risk
RTS	Regulatory Technical Standards
VaR	Value at Risk
VAR	Vector Autoregression
VECM	Vector Error Correction Model
XVA	X Valuation Adjustment

Introduction

In the aftermath of the 2008 global financial crisis, Central Counterparties (CCPs) emerged as critical pillars of financial market infrastructure, designed to enhance market stability and mitigate counterparty risk. By interposing themselves between the buyer and seller of a trade, CCPs assume the counterparty risk and thus play a key role in reducing contagion and promoting trust in cleared markets. A cornerstone of this function is the imposition of Initial Margin (IM) requirements, which serve as an ex-ante risk buffer to protect CCPs against potential Clearing Member (CM) defaults. The calibration and performance of these margin models, particularly under stress conditions, have since become a focal point of regulatory scrutiny and academic research.

This thesis comprises three standalone yet interrelated studies that investigate different dimensions of CCP margin models, each addressing distinct but complementary research questions. Together, these papers contribute to the broader understanding of how margin requirements are designed, how they behave under stress, and how they impact financial markets.

The first paper critically assesses various regulatory-compliant methodologies used by CCPs to estimate IM. It compares the performance of Historical Simulation, Filtered Historical Simulation, Monte Carlo Simulation, and Parametric Value at Risk models in forecasting margin requirements during stressed periods—specifically the COVID-19 turmoil in 2020. The study finds that while Monte Carlo and Parametric VaR models exhibit greater robustness and predictive accuracy, particularly when combined with GARCH-based volatility forecasting. Historical Simulation models are more limited in their responsiveness to market shocks. This paper highlights the trade-offs between model transparency and performance, underscoring the implications for risk management and regulatory guidelines.

The second paper explores the *procyclical nature* of CCP IM models and evaluates the effectiveness of the European Market Infrastructure Regulation's (EMIR) Anti-Procyclicality (APC) requirements. Procyclicality refers to the phenomenon where IM requirements increase sharply during market downturns, potentially exacerbating liquidity stress and triggering destabilizing feedback loops. EMIR mandates CCPs to incorporate at least one of three APC components: a fixed buffer, a floor model, or the inclusion of stressed data. This study not only compares these existing regulatory methods but also proposes a novel APC approach that controls for the *rate of change* of IM requirements. Furthermore, it introduces a new performance measurement framework based on IM volatility, offering a practical tool for both CCPs and regulators. The findings reveal that the proposed APC method outperforms the regulatory alternatives in terms of stability and risk mitigation, while also highlighting inconsistencies and ambiguities in EMIR's APC guidelines.

The third paper shifts focus to the interaction between IM requirements and market liquidity, investigating the bidirectional causal relationship between IM models and liquidity metrics. Using Vector Autoregression (VAR) and Vector Error Correction Models (VECM), the paper analyzes how different APC calibrations affect market functioning. The empirical evidence confirms a two-way causality: not only do worsening market conditions lead to higher IM, but increasing IM requirements also negatively impact liquidity—primarily via widening bid-ask spreads. Importantly, the study finds that the performance of different APC methods varies. While floor and stressed models exhibit more consistent and measurable interactions with market liquidity, the relationship between buffer models and liquidity is less clear, primarily due to the instability and variability of the buffer models themselves. This ambiguity limits their reliability in mitigating procyclicality. These results reinforce the argument for more targeted regulatory reforms and improved model calibration to better balance systemic risk protection with market functioning.

Taken together, the three papers in this thesis offer a comprehensive exploration of the design, calibration, and implications of CCP IM models. By combining risk model analysis, regulatory critique, and market impact evaluation, this work aims to inform academic debate, financial market players and practical policymaking. The findings contribute to the evolving literature on financial risk management, centralized clearing, and regulatory infrastructure, especially at a time when CCPs are increasingly central to financial system resilience.

The remainder of this thesis is structured as follows: each chapter presents one of the standalone papers described above, followed by a concluding section that synthesizes the insights across all three studies and outlines implications for future research and regulatory development.

1. Initial Margin Estimation for E-mini S&P 500 Futures: A Comparative Analysis of Risk Models

Executive Summary: This paper evaluates the predictive performance and resilience of various regulatory-compliant Initial Margin (IM) models used by Central Counterparties (CCPs) during periods of market stress. Using data from the COVID-19 market shock in 2020, the study compares four margining methodologies: Historical Simulation, Filtered Historical Simulation, Monte Carlo Simulation, and Parametric Value at Risk (VaR). The results show that while Monte Carlo and Parametric VaR models are more adaptive and accurate under stress, Historical Simulation models lag due to their lower sensitivity to evolving market conditions. The paper highlights critical trade-offs between transparency, computational complexity, and model performance, providing important implications for both CCPs and regulators.

JEL Classification: G20; G28; C63.

Keywords: Initial Margin; Central Counterparties; Value at Risk

1.1. Introduction

Central Counterparties (CCPs) are crucial pillars of financial markets' stability, as they protect members' contracts against nonperforming counterparties (Gregory, 2014; Vuillemey, 2020). CCPs estimate the Initial Margins (IM) charged to market participants, using risk methodologies to calculate the IM. This ex-ante risk measure is used to protect CCPs against potential adverse market moves and defaults by buyers and sellers. The IM is calculated upon initiating a trade and is complemented by a maintenance margin, which is updated daily until the settlement of the deal.

According to the European Markets Infrastructure Regulation (EMIR) (see EMIR 2012), the IM model should consider a one-tailed 99% confidence interval over an appropriate close-out period in case of default. Also, the lookback period should include at least 1 year of data. The Principles for Financial Market Infrastructure (PFMI) issued by the Committee on Payments and Market Infrastructures (CPMI) and the International Organization of Securities Commissions (IOSCO) (see PFMI 2012) are also aligned with this regulation.

Our research evaluates the backtesting results from different methodologies used by CCPs to estimate IM. The goal is to identify the methodology with the best performance and highlight the weaknesses and strengths of each methodology. This evaluation is relevant given CCPs' role as stable and resilient links between investors in financial markets, and their growing importance, particularly following the market turmoil observed during Q1 2020. Considering this is the first line of defence for CCPs, an IM needs to be well-calibrated to adequately cover market risk. Moreover, as derivatives contract volumes have grown considerably since the subprime crisis, it is important to study the IM behaviour regarding derivatives, especially during times of stress. For this research, we propose to estimate the Initial Margins (IM) required by CCPs for the E-mini S&P 500 futures contracts.

We contribute to the literature on margin models and centralized clearing by assessing CCPs' risk methodologies and current industry practices to estimate the IM. Additionally, our research compares different regulatory-compliant methodologies to estimate the IM, by selecting the most accurate and robust model to estimate the IM and identifying the potential flaws in existing risk methodologies applied by CCPs. This is a critical issue for market participants, Clearing Members, CCPs, and regulators, considering the importance of the X Valuation Adjustment (XVA) framework and counterparty risk modelling. Most current industry practices are based on Value at Risk (VaR) models using the Profit and Loss (P&L) historical distributions to calculate the IM. Accordingly, our analysis tests different approaches to estimate VaR to calculate the IM requirements following EMIR regulation and PFMI guidelines. Specifically, the study uses Historical Simulation, Monte Carlo Simulation, and Parametric Simulation to estimate VaR. Historical Simulation VaR (HSVaR) and Filtered Historical Simulation VaR (FHSVaR) models are widely used by CCPs, with most CCPs using an Exponential Weighted Moving Average (EWMA) model to estimate volatility and filter the returns of the VaR distribution. Similar studies also follow the regulator's guidelines or replicate a CCP's IM model, namely Cotter and Dowd (2006), Lin and Surti (2015), Caspers et al. (2017), and Lee and Seo (2018). Finally, the current research uses 2020 as a backtesting period to capture the COVID-19 market turmoil. IM models are built to handle market stress and, therefore, this seems an appropriate period to test the models (see Foley, Kwan, Philip and Ødegaard, 2022).

The study in this paper confirms that Monte Carlo Simulation and Parametric VaR models achieve better results in estimating IM when compared to Historical Simulation and Filtered Historical Simulation VaR models. Although Monte Carlo Simulation and Parametric models rely on statistical assumptions, these models can achieve more robust results using Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models to estimate volatility and anticipate risk. Conversely, Historical Simulation and Filtered Historical Simulation models heavily depend on historical returns observed during the lookback period, presenting limitations in risk estimation and quickly reacting to volatility. Despite regulators' push for transparent models to compute IM, Monte Carlo and other Parametric models can be powerful tools for market players to anticipate IM behaviour and for the XVA.

The remaining sections of this paper are organized as follows. Section 1.2 presents the literature review. Section 1.3 explains the methodology used. Section 1.4 describes and analyses the empirical results. Section 1.5 concludes.

1.2. Literature Review

Most CCPs use a Historical Simulation VaR model to estimate IM requirements following PFMI guidelines and EMIR regulation (Boudiaf, Scheicher, & Vacirca, 2023). This is a common practice as it defines the worst possible outcome regarding returns of the product cleared, given a confidence level, and the number of days it would take to close out the position on the product, as required by the PFMI guidelines and EMIR regulation. CCP's models have been scrutinized and pushed to evolve over the past decades.

Following the works of Beli and Varadi (2017), Moran and Wilkens (2017), Caspers et al. (2017), and Lee and Seo (2018), our research aims to estimate an IM model. Still, it differs from the previous studies as it compares the most common methodologies CCPs use to estimate the IM. Murphy (2023) also tests different IM models and compares their performance, but does not include Monte Carlo Simulation VaR (MCVaR).

Although our study aims to forecast IM and not volatility, IM models are a function of market volatility.[1] Lee and Seo (2018) propose a Filtered Historical Simulation to compute IM for the Interest Rate Swap in the Korean market. Beli and Varadi (2017) propose a VaR-based model that is EMIR-compliant to compute IM. Hull and White (1998) apply GARCH and EWMA models to incorporate the volatility of returns into a historical simulation method for VaR estimation.

Hence, our paper focuses on VaR IM models. We compare alternative VaR IM models and assess their performance. In addition, we develop VaR IM models that are EMIR compliant and follow PFMI guidelines to estimate IM measures.

1.2.1. VaR

VaR is a standard risk measure that summarizes market risk into a single number representing the maximum expected loss for a given confidence level and over a specific time window. This measure was proposed to a wide audience by JP Morgan Chase through their RiskMetrics approach, leading VaR to become extremely popular in the 90s to measure market and portfolio risk.

VaR literature mostly addresses parametric and nonparametric models. The parametric model approach is based on assumptions about the probability distribution of a security's returns and requires the estimation of its volatility. These models are easy to implement and usually require few parameters to be estimated. Although the most well-known parametric model of VaR relies on the normal distribution of returns, there is evidence that returns are not normally distributed (Sheikh & Qiao, 2009).

[1] See also Bucci (2020), Krauss et al. (2017), and Kristjanpoller et al. (2016).

The Nonparametric approach commonly relies on historical simulation to estimate the VaR. The Historical Simulation bases the VaR estimation on empirical returns. These models are easy to implement, simple, and easy to replicate. More importantly, these models do not rely on assumptions about the probability distribution of returns, nor require the estimation of parameters. On the other hand, these models rely on the historical distribution of returns and assume that past evolution will repeat itself in the future. This research focuses on historical simulation, as it is the most common approach and most used by CCPs.

The Monte Carlo model is a semi-parametric approach that requires parameter estimation and assumes a stochastic process to model and simulate the behaviour of the asset price. Based on this model and its stochastic simulation, it is possible to anticipate risk and draw a Monte Carlo simulated probability distribution of returns. However, these models demand high computational requirements.

The VaR for period t+h is a quantile of the portfolio's returns distribution, given a probability level α∈(0,1). The VaR is defined as the smallest number x such that the probability of the mark-to-market return on the portfolio over t+h not exceeding x is equal to 1-α, the confidence level. Then,

$$VaR_{t+h}^{\alpha} = -\inf\{c \in \mathbb{R}: P(r_{t+h} \leq c|F_t) \geq \alpha\} \quad (1.1)$$

where F_t represents the information available at time t, r represents the returns of the security, and c represents a real number that satisfies the condition for the VaR at the level α.

In the case of the IM, CCPs define the time horizon as the period needed to close the position in the case of default (Basak & Shapiro, 2001; Houllier & Murphy, 2017).

Thus, VaR modelling is about estimating the returns distribution.

1.2.2. Volatility Forecasting

Time Series models are regression models derived from time series data. These models are widely used in finance to estimate, track, and forecast uncertain variables.

First, the Autoregressive Moving Average (ARMA) model is a combination of an Autoregressive (AR) model and a Moving Average (MA) model. The ARMA (ρ, q) process has ρ autoregressive (lagged dependent variable) terms and q-lagged moving-average terms:

$$y_t = u + \emptyset_1 y_{t-1} + \cdots + \emptyset_\rho y_{t-\rho} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \quad (1.2)$$

where y_t denotes the dependent variable in period t, u, $\emptyset_i$, and θ_j are parameters, and ε is an error term. These models assume that the variance of the random variable is constant. Conditional Heteroskedastic models allow for volatility clustering and are more efficient estimators, as per Liu and Morley (2009).

The Autoregressive Conditional Heteroskedasticity (ARCH) model developed by Engle (1982) assumes that the variance of the current error term is related to the previous period's error terms. The conditional variance of u_t can change in time, and it is a linear function of past realizations of the process. A linear regression model with an ARCH effect of order q is usually represented by:

$$y_t = x_t\beta + u_t \tag{1.3}$$

$$u_t = \sigma_t\varepsilon_t \tag{1.4}$$

$$\sigma_t^2 = a_0 + a_1u_{t-1}^2 + a_1u_{t-2}^2 + \cdots + a_1u_{t-q}^2 \tag{1.5}$$

where $u_t$is conditioned by the information set available at the time $t-1$: $u_t|\Phi_{t-1}$, where $\Phi_{t-1} = \{u_{t-1}, u_{t-2}, \dots\}$ and $u_t|\Phi_{t-1} \sim N(0, \sigma_t^2)$. ε_t is a sequence of identically and independently distributed random variables assumed to follow a normal distribution, i.e. $\varepsilon_t \sim N(0,1)$, and β is a vector of parameters whose value is generally unknown.

Finally, the GARCH is a generalized form of the ARCH model (Bollerslev, 1986). The GARCH model is an extension of the ARCH model that uses an ARMA process for variance estimation, allowing it to change over time. A linear regression model with a GARCH effect of order (ρ, q) can be represented by:

$$y_t = x_t\beta + u_t \tag{1.6}$$

$$u_t = \sigma_t\varepsilon_t \tag{1.7}$$

$$\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_iu_{t-i}^2 + \sum_{j=1}^{p} \delta_j\sigma_{t-j}^2 \tag{1.8}$$

where x_t is a vector of explanatory variables, and β is a vector of unknown parameters. The conditions $a_0 > 0$, $a_i > 0$ for $i = 1, 2, \dots, q$, and $\delta_j > 0$ for $j = 1, \dots, p$ ensure that the conditional variance is non-negative. This way the conditional variance depends on the past squared error terms and past variance.

Following the works of Ince (2006), Ječmínek et al. (2020), and Álvarez-Díaz (2020), our study proposes to use GARCH models to estimate the volatility required for the Parametric VaR (PVaR) and MCSVaR. The selection of GARCH is justified by being widely acknowledged in the literature as a robust model to estimate volatility, as well as its use by CCPs.

1.2.3. Parametric Models

The PVaR assumes that the security's returns follow a parametric distribution, such as a Normal distribution. Hence, this method requires estimating the parameters that characterize the assumed distribution. In this research, we use the Normal distribution as it is widely applied in financial theory and used by many CCPs that apply the PVaR to IM.

The PVaR can be estimated as follows:

$$Z_t^a = \frac{VaR_t - u_t}{\sigma_t} \Longrightarrow VaR_t = u_t + \sigma_t Z_t^a \quad (1.9)$$

where *Z* represents the quantile of the normal distribution for probability level a at period t, u_t represents the mean of the parametric distribution in the same period and σ_t represents the volatility of the distribution.

We use the GARCH model to estimate volatility to compare the MCVaR models with the parametric models using the same volatility estimation approaches. Regarding the mean of the distribution, we assume a zero mean, considering the short VaR holding period.

1.2.4. Historical Simulation Models

Most CCPs use the Historical simulation approach. In contrast to the PVaR approach, this method takes into consideration the historical returns and does not depend on any theoretical statistical distribution, nor the accuracy of parameter estimates, nor any linearity assumption of returns.

Historical VaR models rely on empirical returns to draw the historical distribution of returns and compute the VaR. The returns for a defined lookback period are calculated as follows:

$$r_t = \log S_t - log\ S_{t-1} \quad (1.10)$$

where S_t is the price of the security at time t.

Additionally, some CCPs filter the historical return's distribution to allow the model to react to current market conditions. Most CCPs that use this method apply an Exponential Weighted Moving Average (EWMA) model to forecast volatility and adjust the distribution of returns using a scaling function, such as:

$$\sigma_{t_i}^{EWMA} = \sqrt{\lambda \sigma_{t_{i-1}}^2 + (1-\lambda) r_{t_i}^2} \quad (1.11)$$

where λ is the decay factor and r represents the log-daily returns.

The returns are scaled by the following process as suggested by Hull and White (1998):

$$\hat{r_t} = r_t \frac{\sigma_T^{EWMA}}{\sigma_t^{EWMA}} \quad (1.12)$$

where $\hat{r_t}$ represents the adjusted returns, σ_T^{EWMA} is the most recent volatility estimated by the EWMA process and σ_t^{EWMA} is the past volatility estimated by the EWMA process on observation t.

Finally, some CCPs use a smoother scaling function as below:

$$\sigma_T = \frac{\sigma_t^{EWMA} + \sigma_T^{EWMA}}{2} \quad (1.13)$$

1.2.5. Monte Carlo Simulation Models

The Monte Carlo approach uses stochastic simulation to draw random samples of a random variable, with the simulation repeated thousands of times to achieve a large enough number of likely outcomes that build a robust simulated distribution.

In this case, the model aims to approximate the behaviour of the E-mini S&P 500 prices during the VaR holding period. A Geometric Brownian motion (GBM) is assumed to model the futures prices, as this process is widely used in financial literature and banking. Also, this process requires fewer parameters to estimate when compared to others, making it more reliable, less sensitive to parameter estimation risk, and widely accepted. The GBM process is described below:

$$\frac{dS}{S} = udt + \sigma dW_t \quad (1.14)$$

where S represents the security price, u represents the drift of the security, σ represents the volatility and W_t is a Wiener Process that follows a normal distribution as represented below:

$$dW_t \sim N(0, u) \quad (1.15)$$

This stochastic differential equation simulates the price of the security through time. The process requires estimating the drift and volatility. We use the GARCH and ANN methods to estimate the inputs for comparing the MCVaR models with the parametric models using the same volatility estimation approaches. Regarding the drift estimation, we assume a zero-drift considering the short VaR holding period.

This process is applied to simulate the prices of the E MINI contracts through the VaR holding period, building a simulated distribution of returns from which it is possible to estimate the VaR.

1.2.6. Backtesting

Backtesting assesses a model's performance against historical market data. We use the Kupiec and the Conditional Coverage tests to compare the performance of VaR models, given the extensive use of these tests in the VaR literature. The Kupiec test checks the number of breaches of the model. This test is aligned with current CCPs' backtesting practices that analyse whether IM is sufficient to cover potential exposures in the portfolio. The Conditional Coverage test also examines the independence of the breaches observed.

1.2.6.1 Kupiec Test

The models will be tested first by the Kupiec test (Kupiec, 1995), also known as the Unconditional Coverage test. The Kupiec test is a breach frequency test, where a breach is measured as the difference between negative returns whose absolute values exceed the estimated VaR. The test assesses the number of times the returns exceed the forecast VaR. The VaR estimated in t is compared with the

realized returns observed between t and t+h, as the VaR is estimated for the h period. The h period matches the MPOR. The realized returns are the Mark to Market (MtM) valuation. This approach assumes that the VaR (the VaR represents the IM) and MtM estimations are performed at the end of the day.

We represent T as the number of times that the realized returns exceeded the VaR in N observations that form the sample. The rate of realized breaches π is described as:

$$\frac{T}{N} = \pi \tag{1.16}$$

where $\mathrm{T} = \sum I_t$. The indicator variable I_t is defined as below:

$$I_t = \begin{cases} 1 & if \quad |r_t| > |VaR_t| \\ 0 & otherwise \end{cases} \tag{1.17}$$

This test assumes that the breaches follow a Binomial distribution to assign probabilities:

$$\mathrm{T} \sim B(N, \alpha) \tag{1.18}$$

where α is the expected percentage of breaches reflected in the significance level of the VaR model.

The Kupiec test assumes that π should be equal to α under the null hypothesis below:

$$H_0: \pi = \alpha$$

The test suggests that the Likelihood Ratio test statistic, as defined below, follows a χ^2 distribution with 1 degree of freedom.

$$LR_{uc} = 2\ln((1-\pi)^{N-T}(\pi)^T) - 2\ln((1-\alpha)^{N-T}(\alpha)^T) \tag{1.19}$$

In line with the previous notation, N and T denote respectively the total number of observations and breaches.

The test result increases as π deviates from α, leading to a rejection of the null hypothesis if the test statistic exceeds 6.635 for a 99% confidence level. Table 1.1 presents the no-rejection intervals for various sample sizes and confidence levels. These intervals mean that the test rejects the model if the observed number of breaches is higher than the upper limit of the interval, suggesting that the model underestimates VaR. The test also rejects the model if the observed breaches are lower than the lower limit of the interval, revealing that the model overestimated VaR.

Table 1.1 – No rejection intervals

Confidence Level	Sample Size		
1-α	N=252	N=500	N=750
95%	7≤T≤19	17≤T≤35	27≤T≤49
99%	0≤T≤6	2≤T≤10	4≤T≤12
99,9%	0≤T≤0	0≤T≤2	0≤T≤3

The Kupiec test is a breach frequency test that does not consider the timing of the breaches in the sample. This means that the test neglects whether the breaches happen in sequence or for short periods. Hence, this test does not detect if the model is suited to deal with volatility clustering, reflecting large market moves followed by additional large market moves.

1.2.6.2 Conditional Coverage Test

We also consider the Conditional Coverage test proposed by Christoffersen (1998). This test aims to solve the problems identified in the Kupiec test by adding a serial independence test. Considering that security returns are assumed to be independent, the VaR breaches should be spread over the period considered in the sample. However, often this is not the case. IM breaches are sequential because the model is unable to deal with volatility clustering.

The Christoffersen Markov independence test assesses the serial independence of the VaR breaches. Let state $i = \{0, 1\}$, where 1 is a breach and 0 is otherwise, and let π_i denote the probability of observing a breach, conditional on the state i the previous day. Accordingly, $\pi = \pi_0 + \pi_1$. $T_{i,j}$ is the number of days, where the state j occurred in one day, while the state i took place the previous day. If the breaches are independent, then the probability of having a breach or not in the next observation should be the same. The null hypothesis is then stated as follows:

$$H_0: \pi_0 = \pi_1$$

The test suggests that the Likelihood Ratio test statistic for independence follows a χ^2 distribution with 1 degree of freedom.

$$LR_{ind} = -2\ln[(1-\pi)^{(n_{0,0}+n_{0,1})}\pi^{(n_{0,1}+n_{1,1})}] + 2\ln[(1-\pi_0)^{n_{0,0}}{\pi_0}^{n_{0,1}}(1-\pi_1)^{n_{1,0}}{\pi_1}^{n_{1,1}}] \quad (1.20)$$

Finally, the Conditional Coverage Test is the combination of the two statistical measures that allows for testing the frequency of breaches and their independence. The respective statistic, defined below, follows a χ^2 distribution with 2 degrees of freedom.

$$LR_{CC} = LR_{uc} + LR_{ind} \quad (1.21)$$

1.2.7. Regulatory Framework

Our research aims to estimate robust IM models, highlight IM models' flaws, and provide adequate models that comply with the regulation. Therefore, it is important to understand the regulations that CCPs need to meet for IM models to develop compliant IM Models that can be used in the industry.

The Regulation (EU) No 648/2012 of the European Parliament and of the Council of 4 July 2012 on OTC derivatives, central counterparties (CCPs), and trade repositories (TRs) was published in the Official Journal of the European Union in 2012. This regulation, also known as the European Markets Infrastructure Regulation (EMIR), addresses the rules for Central Clearing and Central Counterparties, and provides rules for IM models.

Such regulation provides specific details on IM model confidence intervals, lookback period, and time horizon for liquidation (MPOR), among other topics. CCP IM models need to meet these requirements. The regulation also requires that the European Securities and Markets Authority (ESMA) (see ESMA, 2018) provides regulatory technical standards, after consulting with the European Banking Authority (EBA) and the European System of Central Banks (ESCB). These standards should specify appropriate details for CCP Margin Models.

EMIR regulation and ESMA technical standards require that IM models for financial instruments, besides Over the Counter (OTC) derivatives, need to consider at least:

- a 99% confidence interval;
- a 1-year lookback period;
- a 2-day MPOR.

The PFMI guidelines were issued in 2012 by CPMI-IOSCO and BIS and are also aligned with these requirements.

Therefore, the models developed in this research comply with these parameters. All models consider a 99% confidence interval and a 2-day MPOR. Regarding the lookback period, the Historical Simulation models will consider 1, 5, and 10 years of lookback period, whereas Parametric and Monte Carlo simulation models consider volatility models with at least 1 year of data.

1.3. Methodology and Data

We use different VaR models to estimate the IM requirements of the E-mini S&P 500 futures and analyse whether PVaR and MCVaR models outperform historical simulation models. We compare the performance of both models with the most used models in the industry, namely, FHSVaR and HSVaR.

This research focuses on forecasting the IM of the E-mini S&P 500. The E-mini S&P 500 are futures contracts based on the underlying S&P 500. These are financially settled contracts expiring quarterly in March, June, September, and December. We select the E-mini S&P 500 as this is a benchmark for the financial industry in futures contracts. The data is collected from Bloomberg. The dataset comprises price data on E-mini S&P 500 from 05/01/1999 to 29/12/2020.

Future contracts require a long side (buyer) and a short side (seller) of the trade. Hence, CCPs calculate IM for both sides of the trade: the long side and the short side. We estimate and test the VaR models on both sides of the trade.

Additionally, our study focuses on 2 different types of IM models. The forward-looking models use volatility forecasting to anticipate the asset volatility and estimate the VaR. In this study, such models are the PVaR and MCVaR as these use a GARCH model to forecast volatility. The backward-looking models use historical simulation based on a lookback period of historical returns to estimate the VaR.

The IM models are backtested by the Kupiec and Conditional Coverage tests in 2020. Other studies also assess IM performance during the COVID-19 pandemic period, as this was one of the most stressed periods in financial markets in recent years. Foley, Kwan, Philip, and Ødegaard (2022), and Murphy (2023) also test IM performance during the pandemic period.

The VaR models are tested for both sides of the estimated VaR distribution to capture both sides of the trade (long and short sides). Finally, the backtesting considers that there is a lag between the VaR estimation and the realised returns. The VaR estimation considers a 2-day MPOR, and the returns are computed with a 2-day window. Therefore, it is necessary to align the VaR estimations with the returns 2 days ahead to compare the VaR estimation with the realised returns for the MPOR period.

The models are developed and estimated using the Python language. The backtesting results are obtained in R.

1.4. Empirical Results

This section presents the results of the VaR models and the tests performed to identify the most robust VaR model. It also identifies weaknesses in the different methodologies.

The results highlight important features in the VaR models. Specifically, the evidence suggests that the historical simulation is a weak methodology to estimate VaR, while the Monte Carlo simulation seems a robust option. These results are aligned with the previous literature (Pritsker, 1997; Danielsson & De Vries, 2000; & and Seio, 2018). Additionally, all IM models tested show an increase in margin requirements during the COVID-19 pandemic, which is aligned with the findings from Foley, Kwan, Philip, and Ødegaard (2022).

The results section is divided into 3 subsections. First, we present the E-mini S&P 500 series. Second, we describe the GARCH model. Third, we describe and analyse the VaR models and backtesting results.

1.4.1. E-mini S&P 500 Returns

Figure 1.1 shows the logarithmic returns of the E-mini S&P 500 futures using a 2-day window during the backtesting period. The figure shows considerable volatility in March 2020 due to COVID-19 market turmoil.

Figure 1.1 - E-mini S&P500 futures 2-day returns in 2020

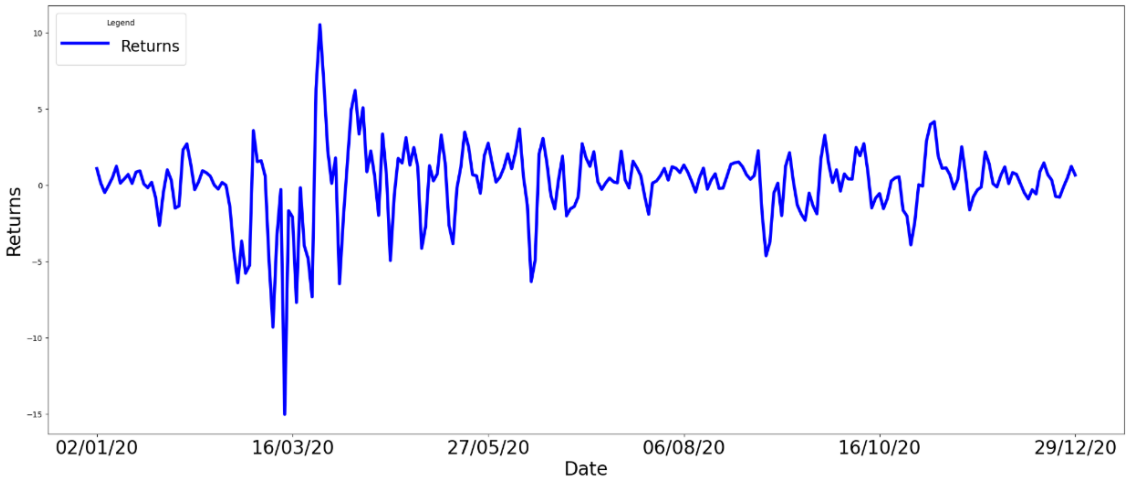


Figure 1.2 displays the histogram of the returns for the backtesting period. It is possible to understand that the returns were positively skewed in 2020 and that there were some large negative returns represented on the left side of the distribution.

Figure 1.42 - E-mini S&P 500 futures 2-day returns histogram in 2020

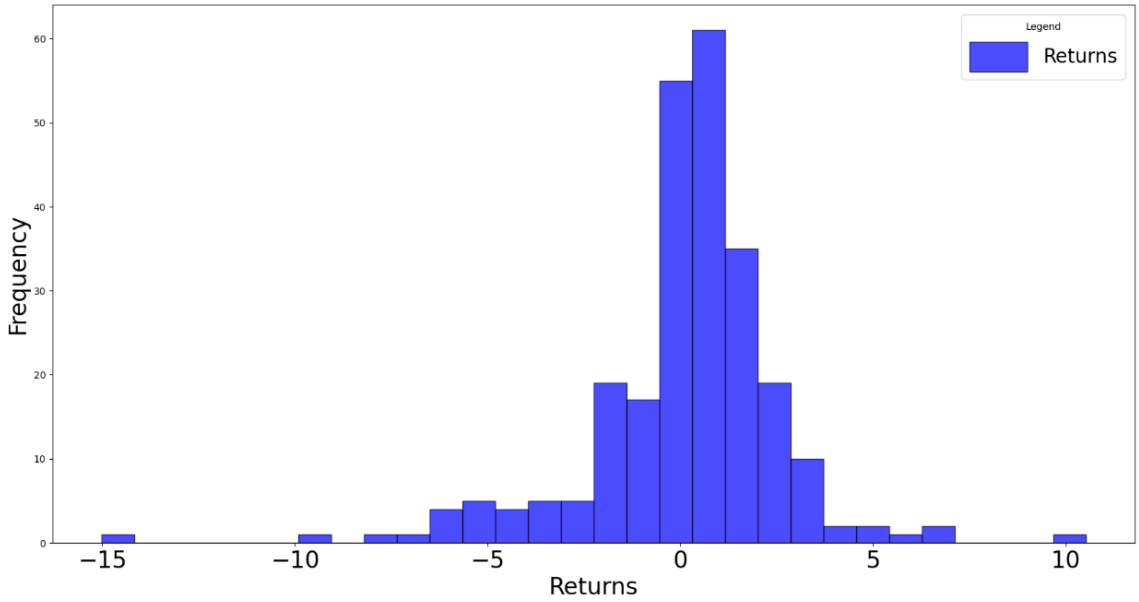


Figure 1.3 describes the logarithmic returns of the E-mini S&P 500 futures using a 2-day window from 1999 to 2020. The graph shows that there are several periods with high volatility. The beginning of the century was marked by high volatility linked with the dot.com bubble and the 2001 terrorist attacks. It is also possible to identify shocks related to the 2008 subprime crisis, the eurozone crisis, and the COVID-19 crisis.

Figure 1.3- E-mini S&P 500 futures 2-day returns 1999 - 2020

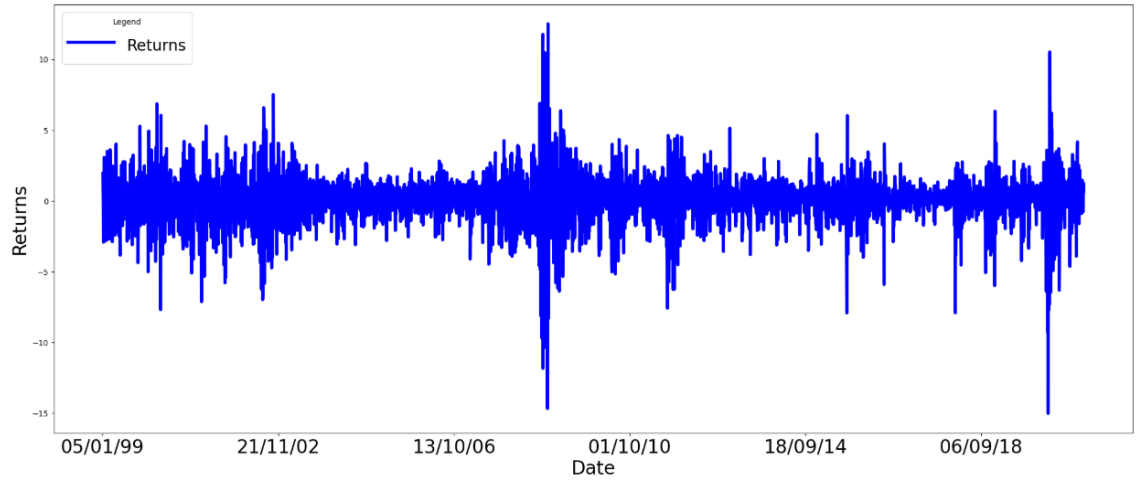
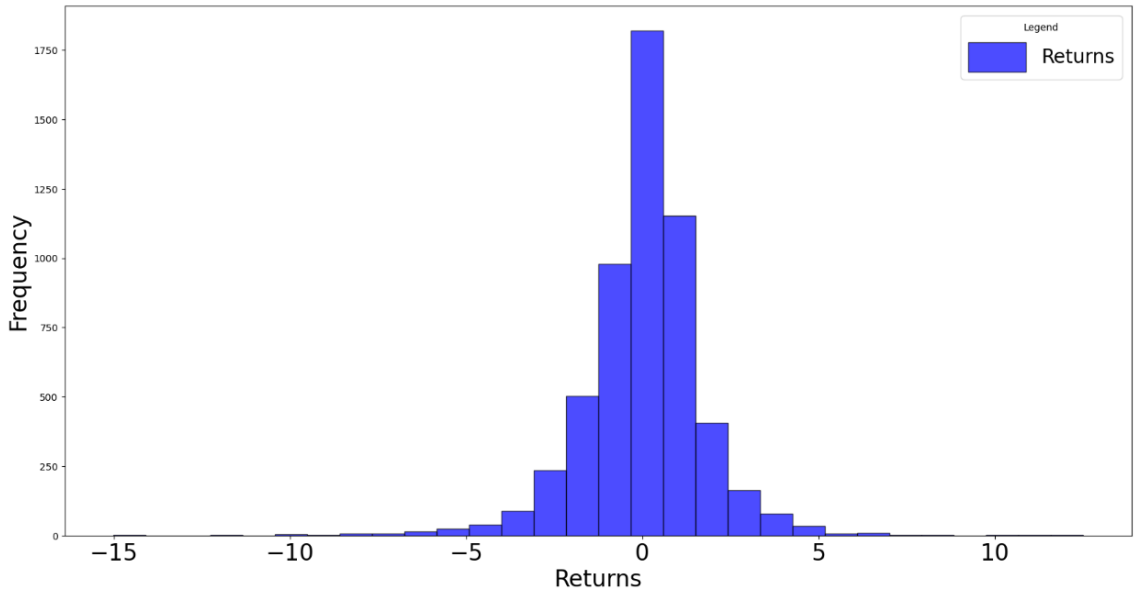


Figure 1.4 displays the histogram of the returns from 1999 to 2020. This graph covers 21 years, where the distribution is closer to a normal distribution when compared to the Figure 2 histogram. Still, there is a left-heavy tail related to several major events that occurred during the sample period.

Figure 1.4 - E-mini S&P 500 futures 2-day returns histogram 1999 – 2020



1.4.2. GARCH Model

In this subsection, the GARCH models are estimated and discussed. The objective is to identify the most accurate GARCH for the PVaR and MCSVaR models.

First, several tests were performed to test if the E-mini S&P 500 series is stationary and does not have unit roots that require additional treatment to apply the GARCH methodology (see results in Appendix I).

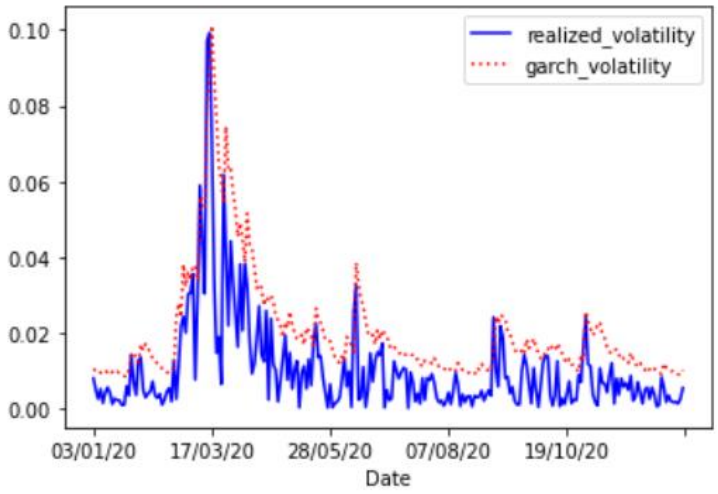
The Augmented Dickey-Fuller (ADF) test assesses if a time series has a unit root, meaning it is non-stationary. The ADF test result rejects the null hypothesis of non-stationarity, implying the time series is stationary. The Phillips–Perron (1988) test checks if a variable has a unit root. The null hypothesis is that the variable contains a unit root. The results show that the nul/ hypothesis is rejected, implying that the series does not have a unit root and was generated by a stationary process. The KPSS test assesses the null hypothesis that the series is stationary. The KPSS results do not reject the null hypothesis.

Second, the autocorrelation and partial autocorrelation graphs were analysed to understand how the time series lags behave and how many lags the GARCH model should consider. The results suggest that the correlation across lags decreases quickly (see results in Appendix I).

Finally, the GARCH model can be estimated. This work tested several GARCH models, considering a constant mean and assuming a Normal distribution for the residuals. The models were estimated using 10 years of time series data until the end of 2019 to avoid using data from the backtesting period. Appendix I shows results for GARCH(1,1) and GARCH(2,2). The GARCH(1,1) presents lower AIC and BIC results and, therefore, it fits better to the historical data. Hence, this is the model used for the PVaR and MCSVaR estimation.

Figure 1.5 presents the results of the GARCH(1,1) model against the realized volatility during 2020 (the backtesting period for the VaR models). The GARCH model estimated volatility is adjusted to match the MPOR period by multiplying the GARCH estimation by the square root of MPOR. The realized volatility considers a 2-day returns window.

Figure 1.5 - GARCH Volatility vs Realized Volatility



1.4.3. Backtesting Results

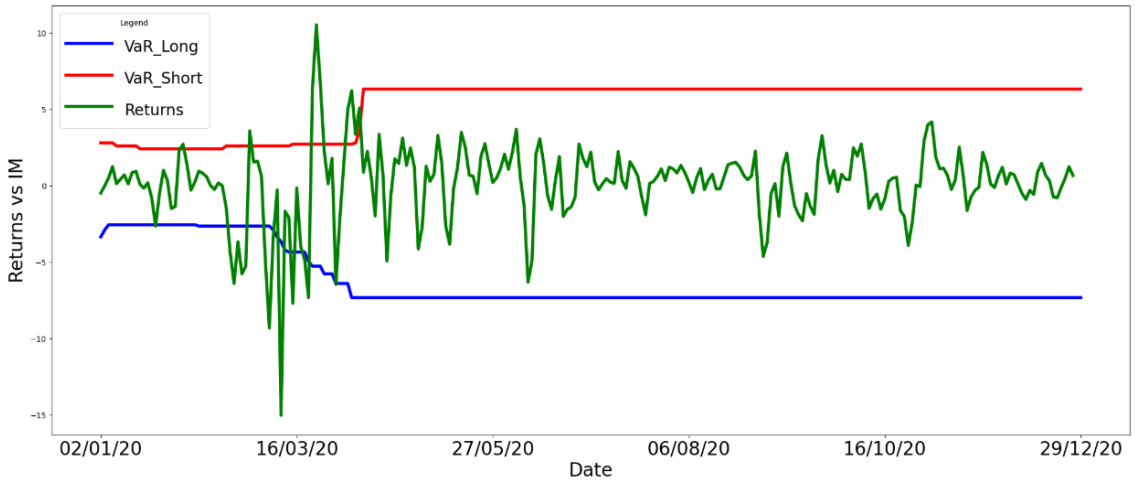
In this subsection, we report the backtesting results for VaR models. First, the VaR models are discussed. Then the backtesting results are analysed.

1.4.3.1 VaR Models

Regarding the HSVaR, we observe that the lookback period has a significant impact on the results. As the lookback period increases, the VaR estimations become more stable and less reactive. The HSVaR with a 1-year lookback period is more reactive, considering that the distribution of returns of the lookback period is smaller. Hence, the stressed returns observed during March 2020 due to the COVID-19 market turmoil had an immediate impact on the VaR estimation in the following observations. This reaction, which translates into higher VaR, allows the model to avoid further breaches in 2020.

On the other hand, the HSVaR with a 1-year lookback period was less conservative when approaching the March 2020 period, as it only includes observations from the past 250 trading days. Therefore, this model presents the biggest breaches during the market turmoil in March 2020. By contrast, the HSVaR with a 10-year lookback period reports the smallest breaches during the COVID-19 market turmoil and shows the slowest reaction. Figure 1.6 presents the performance of the HSVaR model using a 1-year lookback period during the backtesting period. These results are aligned with Danielsson and De Vries (2000), who explain that HSVaR is highly dependent on the historical sample of returns. Lee and Seo (2018) also state that this model is not reliable during times of stress.

Figure 1.6 - HSVaR 1-Year Backtesting Results



The FHSVaR models show that the results depend heavily on the lookback period considered and decay factor parameter calibration. This is aligned with Murphy (2023) that also defends the FHSVaR model performance depends on model calibration. The smaller the decay factor, the more reactive the model becomes. Additionally, the longer the lookback period, the more stable and less reactive the model becomes. Hence, models with a 0.95 decay factor are more reactive and can increase margins faster during the COVID-19 market turmoil in Q1 2020. The 0.95 decay factor allows the model to assign more weight to recent volatility estimates following the EWMA process. This means that the EWMA process will build on volatility and filter the distribution of returns with a higher volatility measure. However, as volatility decreases, the EWMA process using an aggressive decay factor will also reduce the volatility estimation quickly, and, as a result, the same will happen to the VaR estimation. This is evident with the FHSVaR using a 5-year lookback period and a 0.95 decay factor. The model reduces margins too much after the market turmoil and volatility reduction, and cannot avoid later breaches in Q4 2020.

Concerning the lookback period, the relationship is similar to the HSVaR models. A shorter lookback period allows the model to be reactive. On the other hand, the lookback period may not cover significant stressed periods and therefore underestimate risk and VaR. Models with a short lookback period react quicker during the March market turmoil, but these models also face higher breaches.

The figures below illustrate two FHSVaR models using a one-year lookback period and different decay factors, highlighting the crucial role of the decay factor in model calibration. The first model, with a decay factor of 0.95, responds swiftly to market turmoil in March 2020. However, this rapid adjustment also leads to a quick reduction in margins, resulting in breaches later in the year. In contrast, the second model, which applies a 0.99 decay factor, reacts more slowly to the initial volatility in March but maintains greater stability and conservatism throughout the remainder of 2020.

Figure 1.7 - FHSVaR 1-Year 0.95 Decay Factor Backtesting Results

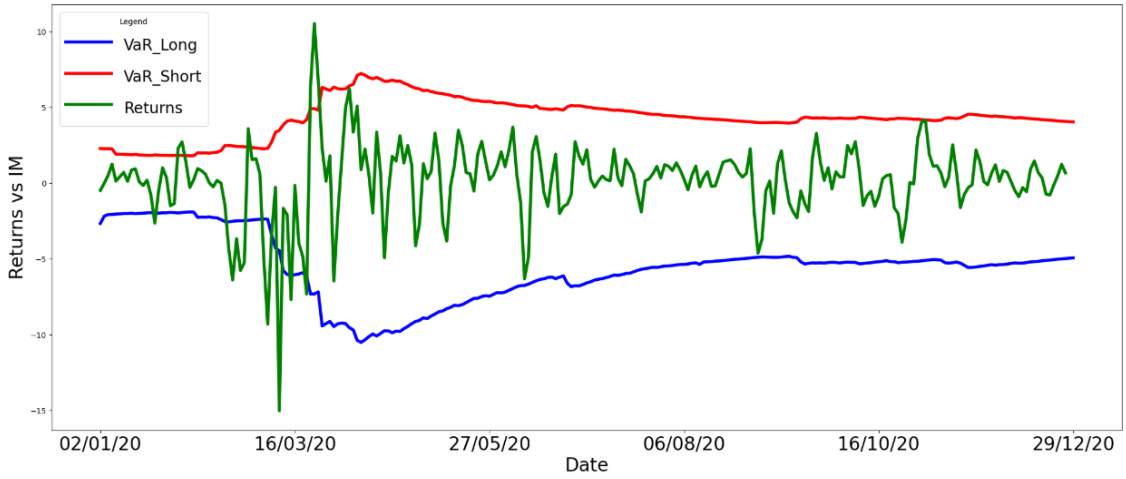
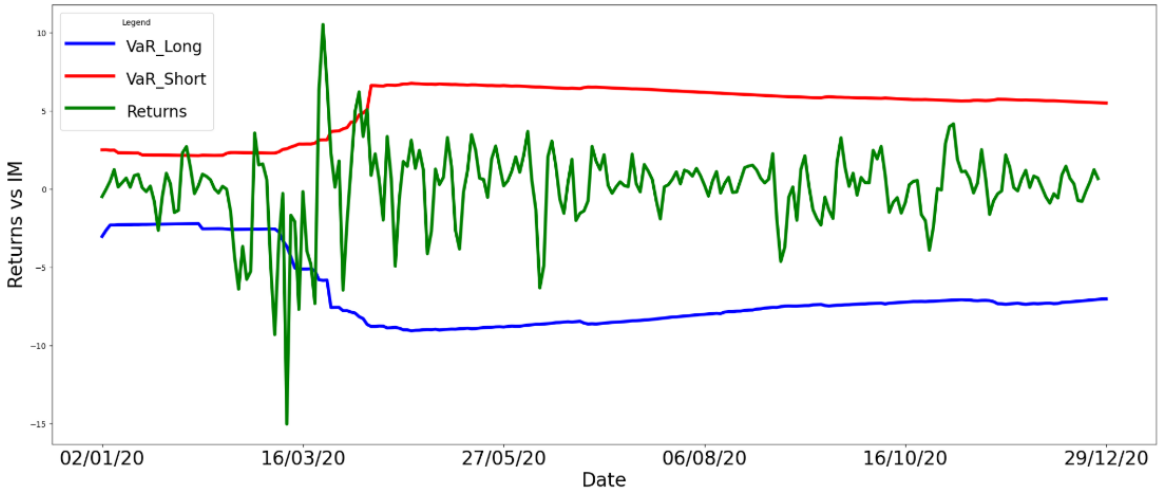
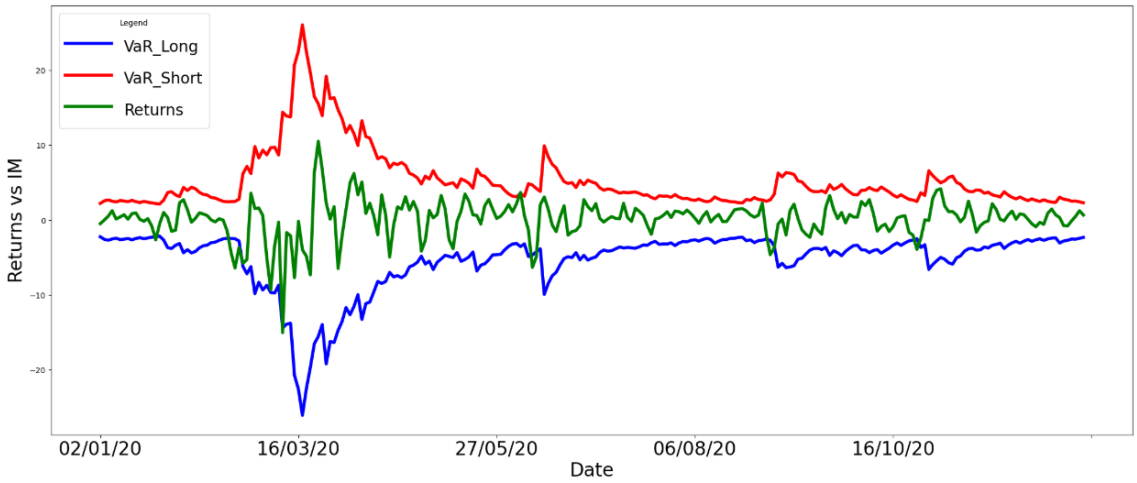


Figure 1.8 - FHSVaR 1-Year 0.99 Decay Factor Backtesting Results



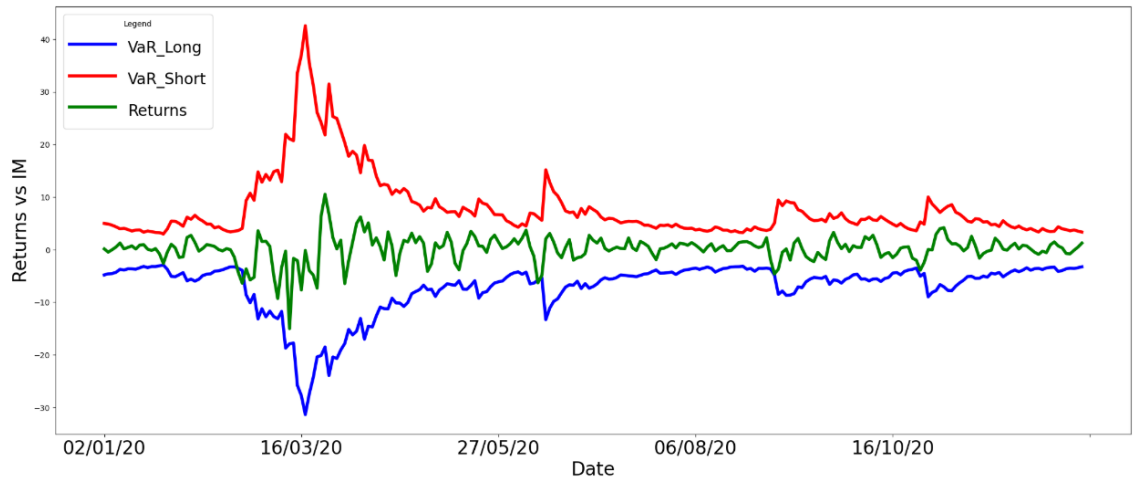
The Parametric models' results are similar regardless of the lookback period considered to estimate the GARCH(1,1) model. Therefore, for the backtesting analysis, we focus on 1 single PVaR model that uses a 10-year lookback period to estimate the GARCH(1,1), considering that PFMI guidelines and EMIR regulation suggest that the lookback period should consider large periods to capture stressed observations. The PVaR models face fewer breaches when compared with HSVaR and FHSVaR models. The models are conservative enough across the backtesting period and can react due to the GARCH(1,1) estimations. The figure below shows the performance of the PVaR model using a GARCH(1,1) and a 10-year lookback period. The model presents some breaches during the backtesting period, but the breach size is small. The PVaR model outperforms the HSVaR and FHSVaR models.

Figure 1.9 - PVaR (10-year lookback period, GARCH(1,1))



The MCSVaR model results are similar to the PVaR models. The lookback period considered for the GARCH(1,1) does not have meaningful implications in the model results. Therefore, for the backtesting analysis, we focus on 1 single PVaR model with a 10-year lookback period to estimate the GARCH(1,1), since PFMI guidelines and EMIR regulation suggest that the lookback period should consider large periods to capture stressed observations. The MCSVaR models leverage a Geometric Brownian Motion to simulate the path of the E-mini S&P 500 returns and a GARCH (1,1) process to estimate volatility. The models are built with 10,000 simulation paths at each VaR estimation. This enables the model to react and set conservative estimations based on the Geometric Brownian Motion simulations. Figure 1.10 presents the backtesting results of the MCSVaR model using a GARCH(1,1) and a 10-year lookback period and 10,000 simulations at each model estimation. The model presents almost no breaches for both sides of the distribution of returns, and the breach size is also small.

Figure 1.10 - MCSVaR (10-year lookback period, GARCH(1,1))



1.4.3.2 Backtesting Results

Regarding the backtesting results, the model's accuracy is measured by two different tests, as explained in section 2.6.

The Kupiec test, or unconditional coverage test, is a breach frequency test. In case the likelihood ratio statistic is higher than the test critical value for a given confidence level, the null hypothesis is rejected, and the model is not fit to estimate the VaR. This means that the model underestimates the risk. The conditional coverage considers the ability of the model to deal with volatility clustering. This test rejects the null hypothesis when the likelihood ratio statistic exceeds the test critical value. This means that the model underestimates risk and that the observations are not independent. The backtesting results are detailed in Appendix II.

Concerning the HSVaR, the backtesting results show that these models are not fit to estimate IM, as both tests reject the null hypothesis for all confidence levels. The models present high breach rates regardless of the lookback period considered for both the long and short sides. All the HSVaR models have the Kupiec test hypothesis rejected for all confidence levels tested. This means that they underestimate risk and perform worse than what the test expected. The number of breaches reported for all the long and short HSVaR models falls outside the interval the test would expect for 249 observations and a 99% confidence level. In addition, the Conditional Coverage test is also rejected for all models. Thus, the model is not able to deal with volatility clustering, as the probability of having a breach after a previous breach is higher than having a non-breach. These results are aligned with Murphy (2023), who refers that the HSVaR models fail to estimate tail risk.

The FHSVaR backtesting results show that the FHSVaR models are also weak IM estimators. Still, these models have a better performance than the HSVaR, despite presenting high breach rates. In this case, the lookback period and decay factor considered have an impact on the model's performance.

The results highlight that all long-side models have the Unconditional Coverage and Conditional Coverage hypotheses rejected for all confidence levels, regardless of the lookback period and decay factor calibration. The models are also not able to deal with volatility clustering. This is aligned with Berkowitz, Christoffersen, and Pelletier (2011), who find that historical simulation VaR models fail to manage volatility clustering. Still, the 1-year lookback period with a 0.95 decay factor FHSVaR short side model and the 1-year lookback period with a 0.97 decay factor FHSVaR short side model do not have the unconditional coverage rejected at the 99% confidence level. The same happens for the 10-year lookback period with a 0.95 decay factor FHSVaR short side model, and the 10-year lookback period with a 0.99 decay factor FHSVaR short side model. These results are aligned with Lee and Seo (2018), who also find that FHSVaR models perform better when compared to HSVaR during times of market stress.

Regarding the PVaR models, the backtesting results show that the Parametric GARCH model is a strong IM estimator compared to the HSVaR and FHSVaR models. The model presents lower breach rates when compared to the previous alternatives for both long and short positions. The PVaR long-side model has the Kupiec test hypothesis rejected for all confidence levels tested. This means the models underestimate risk and perform worse than the test expected. The number of breaches reported falls outside the interval the test would expect for 249 observations and a 99% confidence level. Also, the conditional Coverage test is rejected for all PVaR long-side models, suggesting that the model does not deal with volatility clustering. On the other hand, the PVaR short-side model has a very low breach rate. Hence, the unconditional coverage null hypothesis fails to be rejected for all confidence levels, indicating that the model does not underestimate risk. Additionally, the Conditional Coverage null hypothesis is not rejected for all confidence levels, implying that the model can deal with volatility clustering. These results are not aligned with Murphy (2023), who finds that the PVaR model does not provide accurate estimates of IM. Still, the results differences might be explained by the model calibration and backtesting methodology.

The MCSVaR backtesting results show that the Monte Carlo simulation GARCH model is the strongest IM estimator when compared to the PVaR, HSVaR, and FHSVaR models. That model presents lower breach rates when compared to the previous models for both long and short positions. The MCSVaR long side and short side models have the Kupiec test hypothesis not rejected for all confidence levels tested. This means the models did not underestimate risk and performed in line with expectations. The long side is important during the 2020 backtesting period as the March 2020 market turmoil led to big negative returns (Figure 2). This is the only model where the Kupiec Test Null hypothesis is not rejected for the long side. Additionally, the Conditional Coverage null hypothesis is not rejected for all confidence levels tested, except the 90% confidence level for both the long and short sides. This means the model can properly account for volatility clustering.

IM models play a critical role in safeguarding CCPs by ensuring they maintain sufficient funds to cover potential risks. These results highlight the limitations of the HSVaR and FHSVaR approaches as IM models. During periods of market volatility, these models often fail to accurately anticipate stress conditions, resulting in under-collateralized CCPs. This shortfall leaves CCPS and CMs more vulnerable, with fewer resources than required to effectively manage risk, posing a significant threat to financial stability.

1.5. Conclusion

The IM is the first line of defence for CCPs and sets the initial level of collateral for the portfolios trading through CCPs. This means that the IM should be able to cover the portfolio losses of the clearing members. Otherwise, CCPs trigger additional margin requirements in terms of variation margin to clearing members to cover the MtM portfolio differences. Still, if the IM level is not enough to cover the portfolio risk, there is insufficient collateral to cover losses in case of default, and the CCP is exposed to market risk. Nowadays, CCPs are crucial links in financial markets as most portfolios are traded through them. Hence, they need to be properly collateralised as the volumes trading through CCPs are very high and the risk is concentrated at the CCP level. On the other hand, the IM level should not be over-conservative, as this would mean that the cost of trading through CCPs would be higher, which could push clearing members' trading portfolios over-the-counter.

This study tested the most common VaR models used by CCPs to estimate IM, namely the PVaR, HSVaR, FHSVaR, and MCSVaR. The results are aligned with academic literature and show that the VaR models have critical flaws that must be addressed by CCPs (Lee & Seo, 2018; Berkowitz, Christoffersen, & Pelletier, 2011). Specifically, this study identifies the MCSVaR as the most robust model to estimate IM for the E-mini S&P 500 futures as opposed to the historical simulation models. The MCSVaR model outperforms the historical models (HSVaR and FHSVaR) and the PVaR for both the long and short sides of the distribution of returns. The results also demonstrate that the PVaR model outperforms the HSVaR and FHSVaR models, challenging the finding from Murphy (2023). We conclude that the HSVaR and FHSVaR models are the weakest estimators for IM.

Additionally, the results show that longer lookback periods for historical VaR models do not necessarily improve the robustness of the models. Long lookback periods increase the scenarios of the historical simulation, which may lead to more conservative VaR estimations. However, long lookback periods also decrease the model's ability to react.

Hence, this research highlights that HSVaR and FHSVaR models are weak IM estimators as these underestimate risk and lead to inappropriate collateral levels for the portfolios trading in the CCPs. The models are not able to anticipate volatility and market stress. Our results indicate that MCSVaR and PVaR as more robust models to estimate IM for CCPs. Moreover, our study suggests that the lookback period decision should be asset-specific and that regulatory institutions should not limit CCPs' ability to adjust the models to the portfolio.

These findings are particularly significant given that most CCPs currently rely on HSVaR and FHSVaR models for IM estimation (Boudiaf, Scheicher, & Vacirca, 2023). As institutions tasked with safeguarding the stability and resilience of financial markets, CCPs depend on accurate IM models to ensure they are adequately collateralized against potential losses. However, the observed breaches in these models indicate that CCPs may be undercollateralized, raising serious concerns about their ability to absorb market stress.

This has far-reaching implications: flawed IM models undermine the effectiveness of CCPs in maintaining market stability, potentially eroding market confidence and triggering additional margin calls that affect all clearing members and, by extension, all market participants. The systemic risk introduced by these vulnerabilities challenges the foundational premise of CCPs as bulwarks against financial contagion.

Critically, these results highlight that the performance of IM models is highly sensitive to their calibration. To enhance the robustness of these models—and, by extension, the resilience of financial markets—regulators should consider issuing more detailed and prescriptive guidelines for IM model calibration. Strengthening regulatory oversight in this area is essential to ensure that CCPs can fulfil their mission of preserving financial market stability.

Finally, we acknowledge a limitation in the research, as it does not consider the volatility of IM and its procyclical effects. Procyclicality means that as IM increases during market stress periods, it may overstress the underlying market and lead to more stress. As margins increase, the search for liquid assets intensifies to meet margin requirements, which in turn raises the scarcity of liquid assets and aggravates the market turmoil. EMIR and ESMA guidelines require that IM models consider anti-procyclical components to limit their procyclical effects. Thus, future research should address IM models' procyclical components to study their effectiveness.

2. Mitigating Procyclicality in Initial Margin Models: An Innovative Approach

Executive Summary: This paper investigates the procyclicality of Initial Margin (IM) models and critically assesses the effectiveness of the Anti-Procyclicality (APC) requirements under the European Market Infrastructure Regulation (EMIR). It examines three regulatory tools—fixed buffers, floors, and stressed period data inclusion—and introduces a novel method designed to control the rate of IM change over time. The study also proposes a new evaluation framework based on IM volatility. Empirical results demonstrate that the proposed method outperforms existing APC tools in terms of stability and responsiveness, especially during periods of market stress. The paper also reveals gaps in EMIR's guidance that contribute to inconsistent implementation across CCPs, calling for clearer and more robust regulatory standards.

JEL Classification: C63; G18; G20; G28.

Keywords: Initial Margin; Central Counterparties; Procyclicality

2.1. Introduction

Central Counterparties (CCPs) are institutions that act as intermediaries in transactions of financial instruments to protect clearing members against nonperforming counterparties. CCPs allow members to trade on margin and, in that regard, charge Initial Margins (IM) to market participants. The IM is an ex-ante risk measure that is used to protect CCPs against potential adverse market moves and counterparties' defaults.

The contribution of CCPs to financial market stability is an important topic that has been under the spotlight of research (e.g., Heath et al., 2016; Weber, 2016; Markose et al., 2017). The influence of CCPs' IM on market conditions stems from being defined as a function of market conditions. When stressed market conditions arise, IM requirements increase as trades bear more risk. Hence, the literature suggests that IM requirement increases can lead to a run for cash or cash equivalents and amplify the stressed conditions (Brunnermeier & Pedersen, 2009; Park & Abruzzo, 2016; Glasserman & Wu, 2018). The Futures Industry Association (FIA) (2020) describes "procyclicality" as a negative feedback loop that results from sudden increases in margin requirements. As margins increase, the search for liquid assets increases, which in turn increases the scarcity of liquid assets and intensifies the market turmoil.

Thus, the European Market Infrastructure Regulation (EMIR) requires CCPs' IM models to incorporate Anti-Procyclicality (APC) factors to tackle procyclicality. EMIR proposes three different APC methods and requires CCPs to use at least one in the IM model. Specifically, EMIR proposes a buffer APC component to act as a margin buffer of 25%, a floor component to set a margin floor model, and a stressed component to incorporate stressed observations in the IM model. However, there are questions on the effectiveness of EMIR APC methods, and there is no conventional method to measure the performance of APC methods. As FIA highlighted in October 2020, margin floors and stressed observations are not enough to tackle procyclicality. During the COVID-19 crisis, it was possible to understand that these measures were not effective in avoiding steep increases in margin requirements (FIA, 2020).

Our study compares the different methods currently used by Central Counterparties to include anti-procyclicality in the Initial Margin models. Additionally, we introduce a new approach to tackle IM Procyclical behaviour focused on the rate of change of the margin model. Finally, we propose an approach to measure APC performance based on IM volatility in order to allow CCPs and Clearing Members to analyze IM procyclicality. This innovative approach enables CCPs and Clearing Members to assess APC performance, filling the gap left by the absence of a current measurement framework.

The research in this paper extends the financial literature on margin models and centralized clearing, with its contributions subdivided into three perspectives, following Park and Abruzzo (2016), Glasserman and Wu (2018), Berlinger et al. (2019), Bakoush et al. (2020), Bhanu (2024), and Murphy and Vause (2021).

First, the study compares current EMIR APC methods included in IM models by CCPs. The goal is to test the different EMIR APC methods and identify regulatory weaknesses.

Second, it introduces a new approach to include APC in IM models. This proposal focuses on the rate of change of the IM model rather than the absolute levels of the IM. By focusing on the rate of change of the IM, the model can be calibrated to get the desired level of collateral without over-accelerating IM changes and, consequently, the stressed market conditions. Current industry practices imposed by EMIR for APC are focused on the level of the IM model during non-stressed periods to decrease the absolute change of IM requirements when stressed periods occur. Such an approach is only focused on the absolute change of IM when the market regime changes to tackle IM's procyclical behaviour.

Finally, we propose a framework to assess the performance of the different APC models, as EMIR does not provide any methodologies to measure the APC methods' effectiveness. This framework compares the IM APC models' volatility during the backtesting period. The goal of APC methods is to decrease margin volatility and changes through time to get stable IM requirements to not accelerate stressed market conditions. The volatility of each IM model series will also be compared with the underlying asset volatility for the given period, as suggested by Park and Abruzzo (2016). In addition, it is important to assess the impact of the APC component in the IM model. Hence, the squared differences between the IM APC models and the non-APC IM model are measured across the backtesting period. These squared differences are denoted as X^2 throughout the paper. Then, the average X^2 is compared across the different methods.

Our study shows that the APC method proposed in this study, focused on the rate of change of the IM, outperforms the EMIR APC methods. This is evidenced by both the VaR backtesting results and IM volatility tests. The results show that the EMIR APC methods are flawed when it comes to limiting IM procyclicality. Additionally, this research highlights that EMIR APC guidelines are not clear regarding APC methods in IM models. This means that the EMIR APC methods are highly dependent on the calibration and, thus, their implementation is different across CCPs. The unclear guidelines for implementing APC methods limit CCPs' ability to promote market stability via standardization and transparency. Hence, this paper highlights the importance for regulators to clarify EMIR guidelines regarding APC and improve the APC methods' ability to limit IM volatility and procyclicality.

The remaining sections of this paper are organized as follows. Section 2.2 presents the literature review. Section 2.3 explains the methodology used. Section 2.4 describes and analyses the empirical results. Section 2.5 concludes.

2.2. Literature Review

Park and Abruzo (2016) study the CME's (Chicago Mercantile Exchange) margin methodologies and evaluate their impact on the financial markets. Specifically, they analyse the procyclical behavior of the CME's margin models by comparing the CME model's volatility with market volatility.

Bhanu (2024) studies the effect of APC components in the Indian markets and concludes that strong APC components limit market volatility.

Murphy and Vause (2021) also study different APC tools and compare their performance using a FHSVaR model.

We follow these works and propose an empirical approach to test the different EMIR methods for APC by comparing the different volatility models. Additionally, our study proposes a new method to tackle margin procyclicality.

2.2.1. VaR

The VaR is a standard risk measure that quantifies market risk into a single number that represents the maximum expected loss for a given confidence level and over a specific time window. This risk measure was introduced by JPMorgan Chase through their RiskMetrics approach, leading VaR to become extremely popular in the 1990s to measure market and portfolio risk.

The VaR for the period t+h is a quantile of the portfolio's returns distribution. given a significance level $\alpha \in (0.1)$. The VaR is defined as the smallest number x such that the probability of the mark-to-market return on the portfolio over $t + h$ does not exceed x is $1 - \alpha$, the confidence level. That is.

$$VaR_{t+h}^{\alpha} = -\inf\{c \in \mathbb{R}: P(r_{t+h} \leq c|F_t) \geq \alpha\} \quad (2.1)$$

where F_t represents the information available at time t, r represents the returns of the security, and c represents a real number that satisfies the condition for the VaR at level α.

In the case of IM, the CCPs define the time horizon period as the time that it would take to close the position in case of default (Basak & Shapiro, 2001; Houllier & Murphy, 2017).

2.2.1.1 Filtered Historical Simulation VaR

We focus on the filtered historical simulation VaR (FHSVaR) methods, a non-parametric approach to model VaR, as they are widely used by CCPs and are also compatible with the APC proposed measures. The Historical Simulation takes into consideration the historical returns and does not rely on any theoretical statistical distribution, the accuracy of the parameter estimates, or any linearity assumption of returns. This type of approach is easy to implement, and simple and easy to replicate. On the other hand, it relies on the historical distribution of returns and assumes that the past will repeat itself in the future.

The Historical VaR methods rely on the empirical returns to draw the historical distribution of returns and compute the VaR. The returns are calculated for a defined lookback period, as per below:

$$r_t = \log S_t - \ log\ S_{t-1} \tag{2.2}$$

where S_t is the price of the security at time t.

Additionally, some CCPs filter the historical return's distribution to allow the model to be more sensitive to current market conditions. Most CCPs that use this method apply an Exponential Weighted Moving Average (EWMA) model to forecast volatility and adjust the distribution of returns using a scaling function, such as:

$$\sigma_{t_i}^{EWMA} = \sqrt{\lambda \sigma_{t_{i-1}}^2 + (1-\lambda) r_{t_i}^2} \tag{2.3}$$

where λ is the decay factor and r represents the log-daily returns.

The returns are scaled by the following process as suggested by Hull and White (1998):

$$\hat{r}_t = \frac{r_t}{\sigma_t^{EWMA}} \sigma_T^{EWMA} \tag{2.4}$$

where $\hat{r}_t$ represents the adjusted returns, σ_T^{EWMA} is the most recent volatility estimated by the EWMA process and σ_t^{EWMA} is the past volatility estimated by the EWMA process on observation t.

Finally, some CCPs use the below scaling function. This approach leads to a smoother adjustment to the returns $\hat{r}_t$, as the alternative approach uses the latest EWMA volatility σ_T^{EWMA} leading to a more aggressive adjustment process.

$$\sigma_T = \frac{\sigma_t^{EWMA} + \sigma_T^{EWMA}}{2} \tag{2.5}$$

2.2.2. Initial Margin Models

CCPs collect the Initial Margin to protect against a potential Member's default when adverse market moves occur. The IM is calculated upon initiating a trade and is updated daily until the settlement date of the deal. It serves as an ex-ante risk measure. This research aims to estimate robust IM models and test the different regulatory APC components.

The Regulation (EU) No 648/2012 of the European Parliament and of the Council of 4 July 2012 on OTC derivatives, central counterparties, and trade repositories (TRs) was published in the Official Journal of the European Union in 2012. This regulation, known as the EMIR (EMIR, 2012), addresses the rules for Central Clearing and Central Counterparties and provides rules for IM models.

EMIR provides specific details on IM model confidence intervals, lookback period, and time horizon for liquidation (MPOR), among other topics. CCP IM models need to meet these requirements. This regulation also requires that the European Securities and Markets Authority (ESMA) (ESMA, 2018) provides regulatory technical standards, after consulting with the European Banking Authority (EBA) and the European System of Central Banks (ESCB). These standards should specify appropriate details for CCP Margin Models.

EMIR regulation and ESMA technical standards require that IM models for financial instruments, besides Over-the-Counter (OTC) derivatives, need to at least consider:

- a 99% Confidence Interval;
- a 1-year lookback period;
- a 2-day MPOR.

The Principles for Financial Market Infrastructure (PFMI) were issued in 2012 by the Committee on Payments and Market Infrastructures (CPMI), the International Organization of Securities Commissions (IOSCO), and the Bank for International Settlements (BIS), and are also aligned with these requirements.

Additionally, FIA (2020) describes procyclicality as a negative feedback loop that results from sudden increases in margin requirements. The literature agrees that an IM increase can lead to a run for cash or cash equivalents and amplify the stressed conditions (Brunnermeier & Pedersen, 2009; Park & Abruzzo, 2016; Glasserman & Wu, 2018).

Hence, the EMIR Regulatory Technical Standards (RTS) Article 28 requires CCPs to include APC components in the IM Model, with most CCPs using at least one of the above methods in their IM models to comply with the APC methods. EMIR defines three possible approaches to include APC in the IM model:

- a 25% buffer of the calculated margins;
- a 25% weight to stressed observations in the lookback period;
- a margin floor model based on volatility estimated over an extended lookback period.

2.3. Methodology

Our study tests different VaR models using APC components to estimate the IM requirements of the E-mini S&P 500 futures. We hypothesize that APC components, based on the model's rate of change, can outperform the regulatory components focused on the absolute change of the IM and lead to less volatile margins, as well as less procyclical behavior.

We focus on forecasting the IM of the E-mini S&P 500 using FHSVaR models with the APC components. The E-mini S&P 500 are futures contracts based on the underlying S&P 500. These are financially settled contracts that expire quarterly in March, June, September, and December. This study analyses the E-mini S&P 500, as it is a benchmark for the financial industry in futures contracts. The data is collected from Bloomberg, with the dataset comprising price data on E-mini S&P 500 from 05/01/1999 to 29/12/2020.

2.3.1. APC Components

This section introduces the different APC components implemented and tested in the VaR Models. We use the FHSVaR method to test the different APC components, as that is the most common method employed by CCPs (Boudiaf, Scheicher & Vacirca, 2023), and the regulatory APC components are compatible with the historical simulation models.

Our work designs a VaR model for each APC component, with the FHSVaR model adjusted to integrate the different APC components imposed by EMIR and to integrate the limit rates component introduced in this paper. The models developed in this research comply with these parameters. All models consider a 99% confidence interval and 2-day MPOR. Regarding the lookback period, the models consider 1, 5, and 10 years.

The table below displays the different parameters used for model estimation.

Table 2.1 - Model Parameters

Method	FHSVaR
APC Component	No APC
	Buffer
	Stressed
	Floor
	Limit Rates
Lookback	1 Year
	5 Years
	10 Years
Decay Factor	0.95
	0.97
	0.99

2.3.1.1 Floor

The Floor component sets a lower barrier to limit the FHSVaR. This component prevents the IM from decreasing too much during periods of low volatility.

The regulatory guidelines state that the floor model should be estimated over an extended lookback period. Hence, we consider a floor model estimated using an HSVaR over a 10-year lookback period. The floor model does not use a filtering method, as the goal is to allow stable and conservative results. This is also aligned with current industry practices.

2.3.1.2 Stressed

The stressed component introduces 25% of stressed observations in the historical scenarios of the VaR. The aim is to ensure that the historical scenarios used to estimate the VaR are stressed enough for a conservative VaR model and the consequent IM.

We consider the 2008 Lehman Brothers failure event as the peak of a crisis period to draw stressed scenarios for the historical scenarios, which is aligned with current industry practices among CCPs. The period selected for the stressed observations is adjusted considering the lookback period of the model, as the stressed observations should represent 25% of the historical simulation distribution.

The following table summarizes the stressed periods selected per the VaR model lookback period.

Table 2.2 - Stressed Periods

Lookback Period	Start Date	End Date
1-year lookback	12/09/08	10/12/08
5-year lookback	01/07/08	25/09/09
10-year lookback	04/05/07	23/10/09

2.3.1.3 Buffer

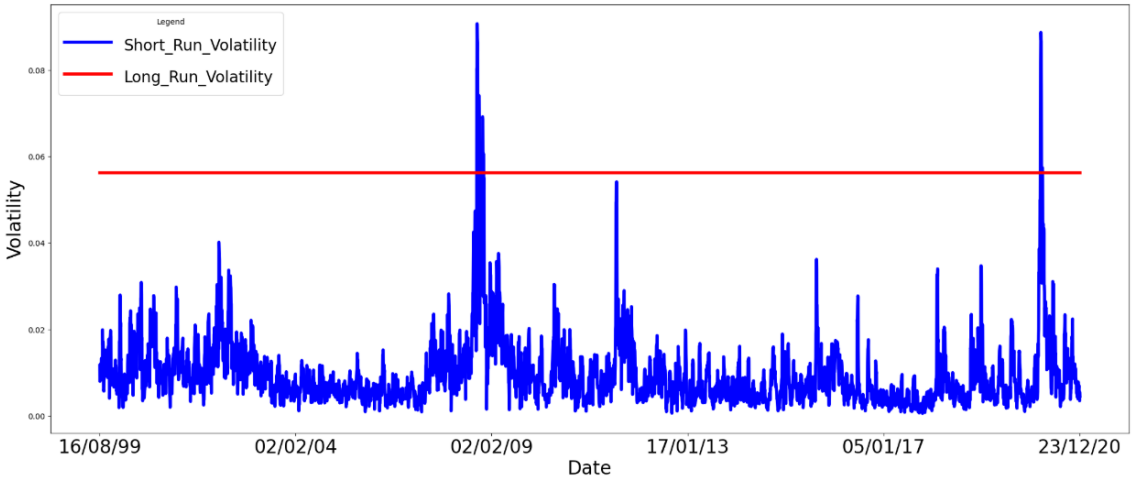
The APC Buffer component is a 25% margin buffer added to the IM. The IM Model should activate the Buffer component during low volatility periods to prevent the IM from decreasing too much during these periods. The goal is to minimize IM jumps when there is a volatility peak following a low-volatility period. On the other hand, the IM Model should deactivate the buffer during high volatility periods.

Hence, the calibration of the buffer component depends on volatility measures for a successful implementation. However, there are no regulatory guidelines for the calibration of the buffer.

We consider different volatility measures to calibrate the buffer component for the VaR models. The buffer calibration uses long-run volatility and short-run volatility measures to identify volatility peaks and activate or deactivate the buffer accordingly. The IM buffer component is activated when the long-run volatility is higher than the short-run volatility, and it is deactivated when the short-run volatility is higher than the long-run volatility measure. The goal is that the buffer is activated during times of low volatility and is deactivated during stressed periods. Several volatility measures were tested to assess an adequate calibration of the buffer component. The short-run volatility measure is the standard deviation of a 5-day returns rolling window to enable the component to capture the most recent volatility based on the last 5 trading days. For the long-run volatility, several measures were tested.

Figure 2.1 shows the short-run volatility based on a 5-day returns rolling window. The long-run volatility shows the buffer threshold that is computed based on the monthly volatility of the total E-mini S&P 500 return series. This threshold was disregarded as it is static and too conservative.

Figure 2.1 - Monthly volatility threshold



The graph below shows the annualized long-run volatility of the E-mini S&P 500 returns computed using an 80-trading-day rolling window against the short-run volatility based on a 5-day rolling window. The graph shows that the long-run volatility measure is higher than the short-run volatility measure in almost every observation. This long-run volatility measure was disregarded as it is consistently higher than the short-run volatility. This leads to a too conservative APC component, which means that the buffer activation trigger is not able to switch on or off according to market volatility.

Figure 2.2 - Annualized volatility threshold

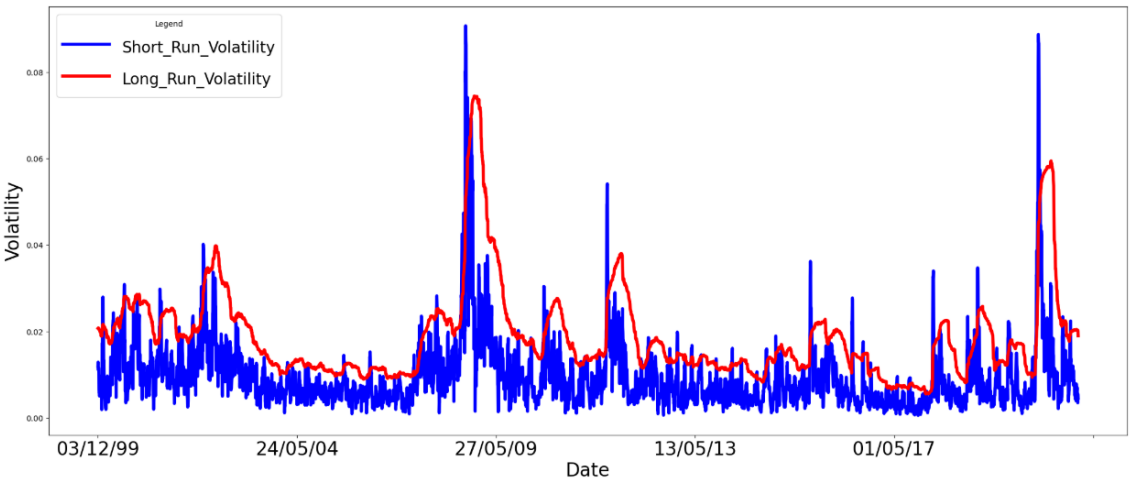
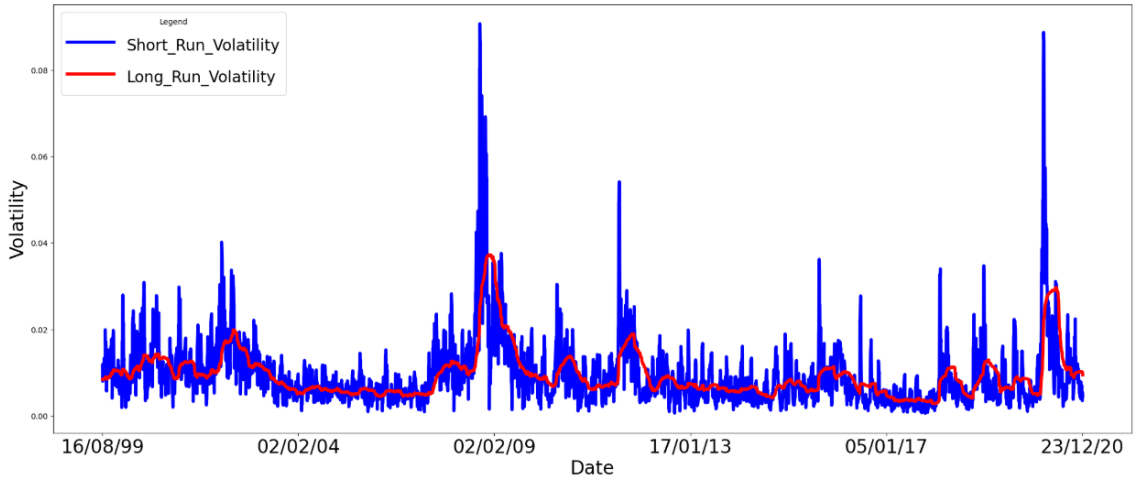


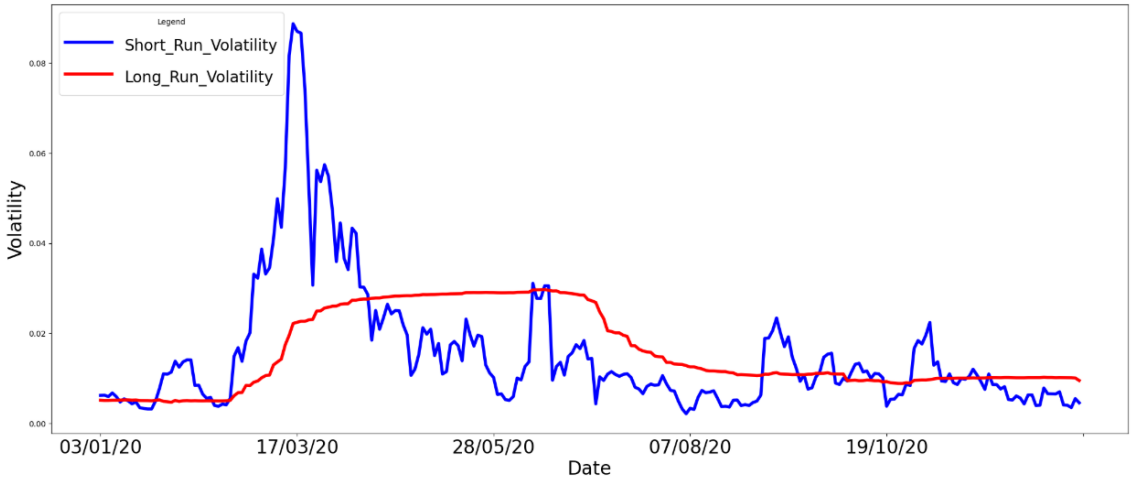
Figure 2.3 shows the quarterly long-run volatility computed using an 80-day rolling window against the short-run volatility based on the 5-day rolling window. This volatility measure is aligned with the short-run volatility series and is not as conservative when compared with the previous volatility measures. This long-run volatility is conservative enough to be higher than the short-run volatility measure in most periods except during times of stress.

Figure 2.3 - Quarterly volatility threshold



Finally, the graph below presents the quarterly long-run volatility measure during the 2020 period. The graph shows that the short-run volatility breaches the long-run volatility measure during the COVID-19 March events and, to a lesser extent, other specific periods. The quarterly long-run volatility measure is not too reactive due to the 80-day rolling window. This leads to a stable buffer component that is only deactivated during high volatility periods.

Figure 2.4 - Buffer calibration during the backtesting period



In conclusion, the long-run volatility measure selected to calibrate the buffer uses quarterly long-run volatility based on an 80-day window. Hence, the buffer calibration compares a weekly short-run volatility measure based on a 5-day rolling period against a quarterly long-run volatility measure based on an 80-day rolling period.

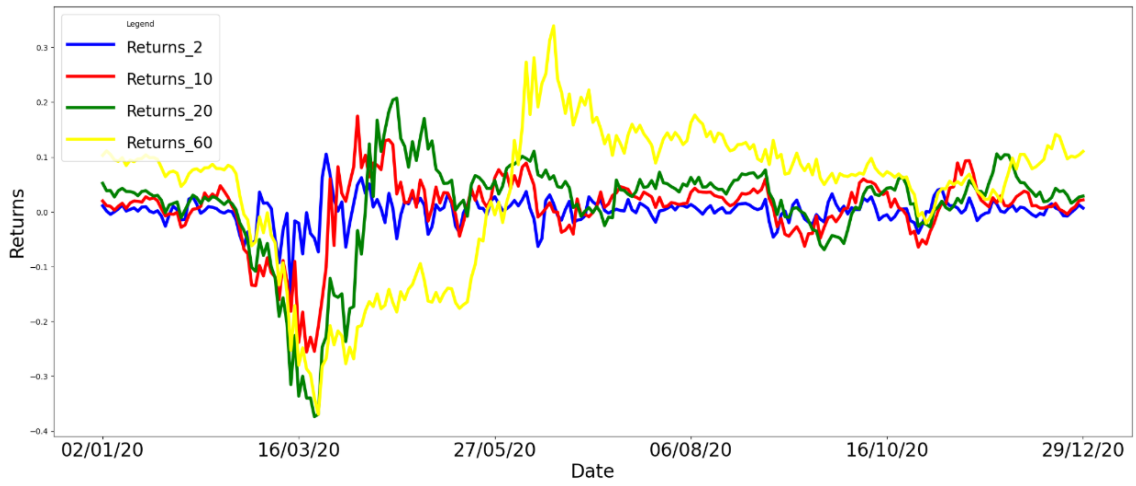
2.3.1.4 Limit Rates

The limit rates APC component focuses on limiting the rate of change of the IM VaR estimations. The goal of these rates is to constrain the model's volatility during times of stress and, at the same time, allow the model to react to volatility shifts.

The limit rates component uses the historical lookback returns to limit the rate of change of the IM Model. For each observation in the backtesting period, the limit rates are compared against the VaR model's daily differences. When the daily difference of the VaR breaches the limit rates, the model adjusts the VaR estimation using the limit rates so that the adjusted daily difference of the VaR does not breach the limit rates.

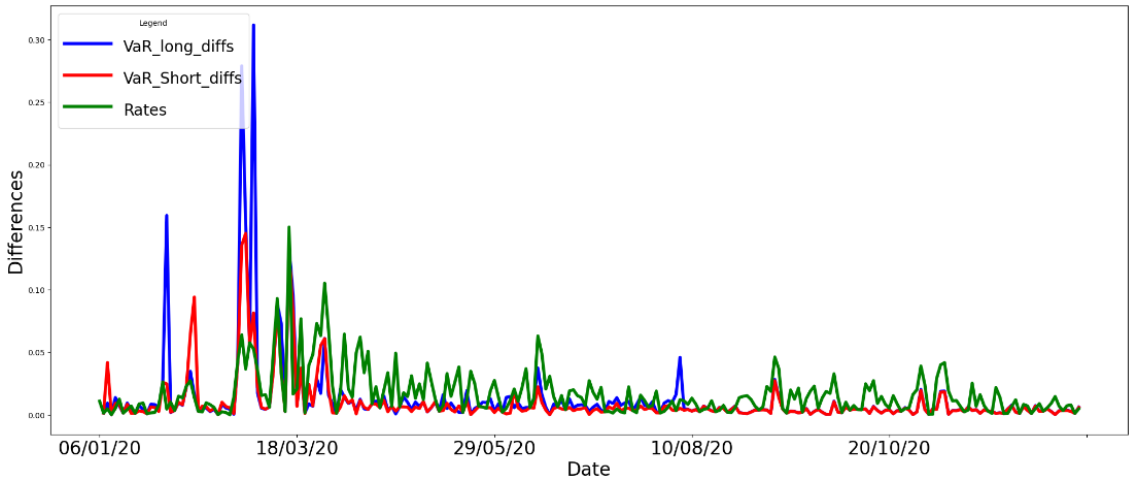
Our study considers several windows of returns for the component. The graph below shows the absolute returns of the E-mini S&P 500 based on 2-, 10-, 20- and 60-day periods. The evidence shows that, the longer the historical window to compute the returns, the greater the absolute returns can get as the differences in the E-mini S&P 500 prices get bigger.

Figure 2.5 - Return windows analysis



Hence, the limit rates APC component selected considers a 2-day return window. Figure 2.6 presents the limit rates computed using a 2-day window return period and the absolute rates of change of the VaR estimation for long and short positions where the VaR is estimated using an FHSVaR model with a 1-year lookback period and a 0.97 decay factor.

Figure 2.6 - Limit Rates vs absolute VaR differences



2.3.2. Backtesting

This section details how the FHSVaR APC model's performance is tested. First, the IM models are backtested by the Kupiec (Kupiec, 1995) and Conditional Coverage test (Christoffersen, 1998), and the models are backtested with observations from 2020. Future contracts require a long side (buyer) and a short side (seller) of the trade. Hence, the VaR models are tested for both sides of the estimated VaR distribution to capture both sides of the trade (long and short sides). Finally, the backtesting considers that there is a lag between the VaR estimation and the realized returns. The VaR estimation considers a 2-day MPOR, and the returns are computed with a 2-day window. Therefore, it is necessary to align the VaR estimations with the 2-day ahead returns to compare the VaR estimation with the realized returns for the MPOR period.

Second, the volatility of the FHSVaR APC models is compared to assess the performance of each method concerning procyclicality. The model's performance depends on the standard deviation of the VaR estimations, as the goal of the APC components is to decrease margin volatility and margin jumps.

Finally, the APC method's impact on the IM model is tested. We assess the impact of the APC component by comparing the APC models with a non-APC model. This study calculates the squared differences between the IM APC model and a non-APC IM model over the backtesting period. Then, the average X^2 of the backtesting period is compared for the different APC methods. The lower the X^2, the lesser will be the distance between the APC and non-APC models, and the lower will be the impact of the APC component. Hence, the better is the model APC performance.

2.4. Results

This section is divided into 2 subsections. The first presents the different VaR models, and the backtesting results are described and analysed. The second presents and analyses the different APC test results.

2.4.1. VaR Models Performance

This subsection presents the backtesting results of the different estimated FHSVaR models. The results are documented in Appendix A. Regarding 1-year lookback models, the results first show that the models with APC components outperform the non-APC model, with the latter presenting higher fail rates. Also, the buffer model presents the lowest breach rates. Finally, it is important to highlight that the conditional coverage and unconditional coverage test hypotheses are rejected for most models, and different decay factors are tested. These results are aligned with chapter 1 - Initial Margin Estimation for E-mini S&P 500 Futures: A Comparative Analysis of Risk Models, which shows that FHSVaR had a poor performance during 2020.

Regarding the 5-year lookback period models, the results show that the performance increases in general. The buffer model presents the lowest breach rates, and the remaining APC model results are very much aligned with similar breach rates and test results. It is important to highlight that the APC models outperform the non-APC models. The non-APC model reports breaches during low volatility periods, as the VaR decreases too much, whereas the APC models can avoid breaches during low volatility periods, maintaining the VaR high.

Regarding the 10-year lookback models, the results indicate that the performance tends to decrease for the stressed model and the no APC model, compared to the other lookback periods. The limit rates, buffer, and floor models tend to present similar results.

Additionally, the results show that the decay factor and lookback period impact on the model performance tend to be the opposite. The models with a 1-year lookback tend to benefit from higher decay factors that allow the model to be more stable and less reactive. By contrast, models with a 10-year lookback tend to benefit from low decay factors, as the extended lookback period tends to make the model more stable and less reactive, and the low decay factor pushes for additional reactiveness.

In summary, the buffer models tend to present the best results followed by the limit rates models. The stressed and no APC models tend to present the worst performance. The 1-year lookback period models and 5-year lookback periods tend to present better results overall.

2.4.2. APC Components Performance

This subsection presents the APC results of the different estimated FHSVaR models. The APC results are documented in Appendix B. Regarding 1-year lookback period models, the results signal that the limit rates model presents the lowest volatility results, whereas the buffer model presents the highest

volatility. Additionally, the limit rates exhibit the lowest X^2 measure except when compared with the floor model using a 0.95 decay factor (short side). By contrast, the buffer presents the highest X^2 for all decay factors tested meaning that the buffer is the APC method that has a higher cost to the CCP clearing members.

Regarding the 5-year lookback period models, the results are very similar to the 1-year lookback period. The limit rates model presents the lowest volatility levels and the lowest X^2, whereas the buffer model presents the highest volatility and X^2 measures. It is important to highlight that the floor model performance is very close to the limit rates model. Additionally, the X^2 of the buffer model is lower compared to the 1-year lookback period model, which indicates that the buffer component has a higher impact on short lookback periods.

Regarding the 10-year lookback period models, the results show that the floor model presents the lowest volatility using 0.95 and 0.97 decay factors as the floor APC component is a stable HSVaR using a 10-year lookback period. Still, the limit rates model outperforms the other APC models regarding the X^2 measure across all decay factors tested. The floor, stressed, and limit rates models using a 0.99 decay factor present smaller volatility measures compared to the underlying security returns' volatility.

Finally, it is very important to highlight that the volatility and X^2 measures of the different models tend to decrease as the lookback period increases and the models become more stable, less reactive, and therefore less volatile.

In summary, the results indicate that the buffer component performs the worst in terms of both the volatility of the IM model and its overall impact when compared to a non-APC model. The buffer method exhibits high volatility and contributes to procyclical margin behaviors. This makes it an expensive APC measure, as it raises trading costs during periods of low volatility without effectively preventing procyclical margin fluctuations. Consequently, the increased cost of trading is not offset by a more stable margin behavior.

Moreover, the buffer method requires a complex calibration process to activate or deactivate, relying on volatility measures to trigger or release the buffer. This results in non-transparent margin models that are highly dependent on calibration, making them difficult to replicate and harmonize, contradicting the Principles for Financial Market Infrastructures (PFMI). These findings align with Murphy and Vause (2021), who also identify the buffer as an ineffective APC tool, noting that it increases trading costs more than other APC components while offering limited effectiveness in reducing procyclicality. This has significant economic implications, as higher trading costs can negatively impact financial market liquidity.

Conversely, the limit rates model outperforms other APC methods in terms of model volatility and the X^2 measure. This suggests that the limit rates component effectively reduces model volatility while maintaining a relatively low impact compared to a non-APC model. This is particularly important because a lower APC impact translates to reduced costs for CCP clearing members.

Among the EMIR APC methods, the floor model consistently yields better results in terms of model volatility compared to both the stressed and buffer models. However, when evaluated against the stressed model using the X^2 measure, the comparative performance of the floor model varies depending on the model's calibration.

Our findings align with those of Cominetta, Grill, and Jukonis (2019), who examine IM procyclicality and emphasize the critical role of IM model calibration. The effectiveness of IM models and APC components in mitigating risk and procyclicality depends heavily on proper calibration. Therefore, it is essential for regulators to closely monitor the implementation of these models and provide clear guidance on managing and measuring procyclicality. Finally, these authors also state that the floor outperforms the other methods (not considering the limit rates introduced in this study) which is aligned with the results of this study.

2.5. Conclusion

This paper shows that the EMIR APC methods have limited capabilities to reduce the procyclicality of the IM. The EMIR APC methods focus on the absolute levels of IM and tend to overcharge clearing members during low volatility periods to minimize the IM jump when volatility increases. The results show that the limit rates APC method introduced in this paper outperforms the EMIR APC components. The limit rates model performance is aligned with the other EMIR APC models regarding VaR backtesting results. The models show similar breach rates across the different tested lookback periods and decay factors, and similar conditional coverage and unconditional coverage results, despite the FHSVaR being a deficient VaR method that relies on historical returns.

Regarding APC results, the limit rates VaR models show lower volatility in almost all lookback periods and decay factors combinations tested and a smaller distance to the non-APC Model measured by the X^2. This means that the limit rates component is efficient when it comes to limiting the IM model rates of change and has a smaller impact on the IM model compared to the EMIR APC methods. On the other hand, this paper highlights that the buffer APC method is highly deficient as an APC method, leading to highly volatile IM levels and a significant cost to CCP clearing members. This is aligned with Murphy and Vause (2021), who suggest the buffer is the worst-performing APC tool. Additionally, the results demonstrate that the non-APC model leads to much higher model volatility when compared to the APC models. This highlights how important it is to establish robust APC models to promote IM and market stability.

This research is particularly relevant in the context of EMIR and CCPs, which seek to promote financial market stability and mitigate credit risk through robust risk management standards, including requirements for IM. The findings of this study highlight a critical issue: the anti-procyclical margining methods currently recommended under EMIR are flawed. The results demonstrate that these methods, while well-intentioned, lack sufficient guidance for practical calibration and are not supported by a standardized performance testing framework. This lack of clarity and rigor undermines their effectiveness in achieving the regulatory goals.

Moreover, the procyclicality of IM has a direct and significant impact on market liquidity (Foley, Kwan, Philip & Ødegaard, 2022). In times of stress, excessively procyclical margin requirements can amplify market volatility, exacerbate liquidity pressures, and contribute to systemic risk—precisely the outcomes EMIR is designed to prevent. Therefore, policymakers and regulatory bodies must revisit and refine the current guidelines to ensure that margining practices support financial stability. A more nuanced and empirically validated approach to IM calibration and performance assessment is essential to align regulatory intent with actual market outcomes.

Finally, this paper highlights that the EMIR guidelines on APC calibration are not clear, and the EMIR APC methods' performance is highly dependent on calibration. The EMIR APC guidelines lead to different APC implementations across CCPs, which are not aligned with CCPs principles of IM stability, replicability, and standardization. Additionally, the EMIR APC methods are designed for HSVaR IM models rather than PVaR and MCSVaR IM models. More concretely, the stressed and floor APC methods are designed for HSVaR models. On the contrary, the limit rates APC method and the Buffer methods are compatible with any VaR model.

Lastly, our study tests the IM APC methods for the E-mini S&P 500 futures. Additional research is required to test the IM APC methods across other asset classes, considering the APC methods' sensitivity. Additionally, the study does not consider the impact of the IM in the underlying market. The results of our study encourage future research to study the relationship between IM models and the behaviour of the underlying security to understand how the IM impacts the underlying market.

3. Reassessing Anti-Procyclicality: The Impact of Initial Margin Models on Market Liquidity

Executive Summary: This study examines the two-way causal relationship between Initial Margin (IM) requirements and market liquidity, with a particular focus on the impact of various APC model calibrations. Using Vector Autoregression (VAR) and Vector Error Correction Models (VECM), it explores how changes in IM not only respond to market stress but also influence key liquidity indicators such as bid-ask spreads. The findings confirm a bidirectional dynamic where rising IM contributes to liquidity deterioration, and worsening liquidity conditions drive further increases in margin requirements. Among APC tools, floor and stressed models show clearer and more consistent effects, while buffer models produce ambiguous outcomes due to their unstable calibration. The study underscores the need for more refined regulatory tools that consider both systemic risk containment and market functioning.

JEL Classification: C32; C63; G18; G20; G28.

Keywords: Initial Margin; Central Counterparties; Market Liquidity; bid-ask spread; Vector Autoregressive; Vector Error Correction Model

3.1. Introduction

Central Counterparties (CCPs) are financial market infrastructures that allow members to trade on margin and charge margin requirements to market participants. These margin requirements are used to protect CCPs against potential adverse market moves and counterparties' defaults.

The contribution of CCPs to financial market stability is an important topic for academic research (Weber, 2016; Berndsen, 2021). CCPs' margin requirements depend on the underlying market conditions. Specifically, the Initial Margin (IM) is charged to members when opening a trade and is maintained until the trade is closed. The literature suggests that margin requirements in turn affect market conditions (Brunnermeier & Pedersen, 2009; Park & Abruzzo, 2016; Glasserman & Wu, 2018; Bakoush, Gerding & Wolfe, 2019). The Futures Industry Association (FIA) (2020) describes "procyclicality" as a negative feedback loop that results from sudden increases in margin requirements. As margins increase in more stringent market conditions, the search for liquid assets expands, which increases the scarcity of liquid assets and intensifies market turmoil.

Therefore, the European Market Infrastructure Regulation (EMIR) requires CCPs to use IM models that incorporate Anti Procyclicality (APC) factors. The EMIR guidelines introduce three APC methods, and CCPs are mandated to apply at least one APC component in the IM model. Specifically, the EMIR proposes a buffer APC component to act as a margin buffer of 25%, a floor component to set a margin floor model, and a stressed component to incorporate stressed observations in the IM model.

We analyse the relationship between CCPs´ Initial Margin (IM) requirements and market liquidity using a Vector Autoregressive model (VAR) and a Vector Error Correction model (VECM). Our study compares the IM requirements behaviour with the market liquidity performance to assess potential causality.

The research in this paper expands the financial literature on margin models and centralized clearing from two perspectives. First, it tests the relationship between IM and market liquidity. Knowing the extent of such relationship is particularly important for understanding the procyclical impact of the IM Models on market liquidity (Chou, Wang, & Wang, 2015; Foley, Kwan, Philip & Ødegaard, 2022).

Second, the study examines how different calibrations of Initial Margin (IM) models and Anti-Procyclicality (APC) measures influence their relationship with market liquidity. By comparing a range of IM model and APC measure combinations, the study aims to uncover how variations in model calibration—both in terms of margin sensitivity and procyclicality mitigation—can lead to differing impacts on market liquidity dynamics. This is particularly important as FIA highlighted in October 2020 that the COVID-19 market crisis showed that APC methods are not enough to prevent sharp increases in margin requirements (FIA, 2020). This component of the study complements academic literature as there are no studies yet on how different IM APC measures might impact market liquidity differently.

Based on causality test results, our study shows that IM models and market liquidity have a causal relationship that works both ways. This means the IM can be explained by market conditions, but market conditions can also be explained by the IM. Additionally, the results show that the EMIR APC measures have different performances regarding market impact. Specifically, the study confirms that the floor and stressed APC IM models have a meaningful relationship with market liquidity, while the buffer IM Models do not exhibit a relevant relationship with market liquidity. Furthermore, the study shows that the VAR model fails to capture the relationship between the IM Model series and the bid-ask spread series, as well as that the VECM is the appropriate method to model the relationship as the series are cointegrated. Finally, our results confirm that IM models affect market liquidity conditions and that the model calibration parameters have different performances regarding the impact on market liquidity. The study highlights the importance of regulators improving EMIR guidelines regarding APC and model calibration.

The remaining sections of this paper are organized as follows. Section 3.2 presents the literature review. Section 3.3 explains the methodology used. Section 3.4 describes and analyses the empirical results. Section 3.5 concludes.

3.2. Literature Review

Our study follows the work of Brunnermeier and Pedersen (2009) to study the relationship between IM and market liquidity using VAR models. The next subsections summarize the fundamentals and relevant literature on IM models, VAR Models, and the impulse response function (IRF).

3.2.1. Initial Margin Models

CCPs collect the Initial Margin to protect against a potential Member's default when adverse market moves occur. The IM is calculated upon initiating a trade and daily until the settlement date of the deal. It serves as an ex-ante risk measure. Our research aims to estimate robust IM models and test the regulatory APC components.

The Regulation (EU) No 648/2012 of the European Parliament and of the Council of 4 July 2012 on OTC derivatives, central counterparties, and trade repositories (TRs) was published in the Official Journal of the European Union in 2012. This regulation, known as the EMIR, addresses the rules for Central Clearing and Central Counterparties and provides rules for IM models.

The EMIR provides specific details on IM model confidence intervals, lookback period, and time horizon for liquidation (MPOR), among other topics. CCP IM models need to meet these requirements. The regulation also requires that the European Securities and Markets Authority (ESMA) provides regulatory technical standards, after consulting with the European Banking Authority (EBA) and the European System of Central Banks (ESCB). These standards should specify appropriate details for CCP Margin Models.

EMIR regulation and ESMA technical standards require that IM models for financial instruments, besides Over the Counter (OTC) derivatives, need to consider at least:

- a 99% Confidence Interval;
- a 1-year of lookback period;
- a 2-day MPOR.

The Principles for Financial Market Infrastructure (PFMI) are aligned with these requirements. Such principles were issued in 2012 by the Committee on Payments and Market Infrastructures (CPMI), the International Organization of Securities Commissions (IOSCO), and the Bank for International Settlements (BIS).

Additionally, the FIA (2020) describes procyclicality as a negative feedback loop that results from sudden increases in margin requirements. The literature agrees that an IM increase can lead to a run for cash or cash equivalents and amplify the stressed conditions (Brunnermeier & Pedersen, 2009; Park & Abruzzo, 2016; Glasserman & Wu, 2018).

Hence, Article 28 of the EMIR Regulatory Technical Standards (RTS) requires CCPs to include APC components in the IM Model, with most CCPs using at least one of the above methods in their IM models to comply with APC methods. EMIR defines three possible approaches to include APC in the IM model:

- a 25% buffer of the calculated margins;
- a 25% weight to stressed observations in the lookback period;
- a margin floor model based on volatility estimated over an extended lookback period.

3.2.2. Market Liquidity

The bid-ask spread is the gap between the bid price and the ask price of an asset. The bid price is the highest price a buyer is willing to pay for that asset, reflecting demand and the price at which investors can sell the asset. The ask price is the lowest price at which a seller is willing to trade the asset, and so it is the price investors on the demand side will have to pay to buy the asset.

The bid-ask spread may therefore be expressed by:

$$\text{Bid-Ask Spread} = Ask\ Price - Bid\ Price \tag{3.1}$$

Following the works of Chordia, Roll, and Subrahmanyam (2008), and Amihud and Mendelson (1986), we use the bid-ask spread to measure market liquidity. The bid-ask spread is widely used in financial literature as a good indicator of market liquidity, and it is an indicator that can be tracked daily. In this case, the underlying market is the E Mini S&P500 future contracts

3.2.3. VAR and IRF

The Vector Autoregressive (VAR) model, introduced by Sims (1980), is used to model multivariate time series data. It captures the interdependencies and dynamic relationships between multiple time series without requiring prior theoretical assumptions about the underlying structural relationships among variables.

The VAR model is a system of equations in which each variable is modeled as a linear function of its past values, along with the past values of the other variables in the system. The general form of a VAR model with p lags for a system of k variables can be written as:

$$y_t = \emptyset_0 + \sum_{i=1}^{p} \emptyset_i y_{t-i} + \varepsilon_t \tag{3.2}$$

where,

- y_t = $(y_{1t}, y_{2t}, \dots, y_{kt})$ is a vector of k stationary variables
- $\emptyset_0$ is a k-constant vector
- $\emptyset_i$ is a k × k matrix of coefficients to be estimated
- ε_t is a k vector of error terms

Additionally, the VAR model can be transformed into a Moving Average (MA) process.

$$y_t = \mu + \sum_{i=0}^{\infty} \Phi_i \varepsilon_{t-i} \quad (3.3)$$

where,

- y_t = $(y_{1t}, y_{2t}, \dots, y_{kt})$ is a vector of k stationary variables
- μ is a vector of constants
- Φ_i is a matrix of impulse response coefficients at time i
- ε_{t-i} is a vector of residual shocks at time $t - i$

The IRF is used to explain how market liquidity and IM interact dynamically over time. The IRF describes how a system responds to an external input, known as an "impulse", in a VAR model. The IRF uses the VAR model MA matrix of impulse response functions to describe how a shock to one variable affects the future values of that variable and potentially other variables in the system over time. For example, the IRF shows the evolution of y_t after a shock to ε_t at time t=0.

The VAR and IRF are used to study structural analysis by testing how financial variables relate to each other. They are also utilized for policy analysis to assess how policy shocks affect financial variables. For example, Murphy and Murphy (2012) study the relationship between interest rate swaps and market liquidity using a VAR model. Haan and End (2013) and Wickens and Motto (2001) also use VAR models and IRF to study market liquidity and shock responses.

We use the VAR to model the relationship between market liquidity and IM; therefore, the VAR will be bivariate. The IRF is used to explain how IM and market liquidity interact. Specifically, a change in IM represents a change in market circumstances that might affect market liquidity and vice versa.

3.2.4. VECM

The VECM is an extension of the VAR for non-stationary time series that are cointegrated. It is used to model the short-term and long-term relationships among multiple time series that share a common stochastic trend. The VECM formulation includes:

- Cointegrating vectors that describe the equilibrium relationships between the variables.
- Error correction terms that represent the adjustment towards equilibrium after a short-term shock.
- Short-term dynamics that reflect how the differenced variables interact in the short run.

The VECM is used in academic literature when the time series of the variables are cointegrated and non-stationary. For example, Rehman, Mahdzan, and Zainudin (2016) use the VECM model to study macroeconomic and market liquidity variables. Chikwira and Mohammed (2023) study the impact of the stock market on liquidity and economic growth also using a VECM approach.

3.3. Methodology and Data

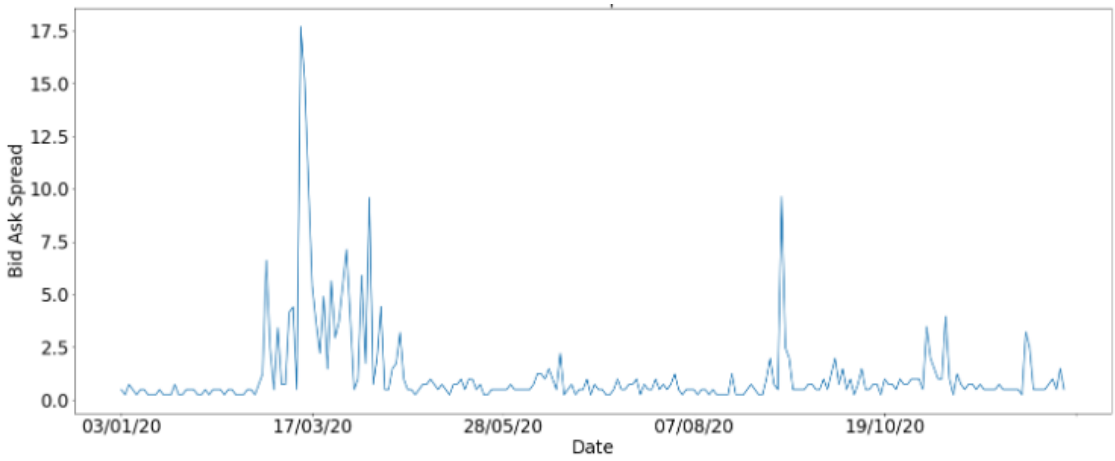
This section details the methodology used in this research and describes the data, correlation analysis, and tests executed to utilise the models.

3.3.1. Data

We analyse the E Mini S&P 500 future series liquidity and the IM for these securities. The E Mini S&P 500 are futures contracts based on the underlying S&P500. These contracts are financially settled and expire quarterly in March, June, September, and December.

We measure the liquidity of the E Mini S&P 500 contracts by the end of day bid-ask spread. The graph below shows the evolution of the spread during 2020, which seems a relevant period due to the negative and abnormal economic effects caused by the COVID-19 pandemic. For example, the graph confirms abnormally high values of the bid-ask spread during the market turmoil, in March 2020.

Figure 3.1 - E Mini S&P500 bid-ask spread series



The IM series are estimated using a Filtered Historical Simulation Value at Risk (FHSVaR) method and a 99% confidence level with a 2-day margin period of risk (MPOR), in line with EMIR. These models include the anti-procyclical components required by EMIR: floor, buffer, and stressed components. The FHSVaR is used in this study as it is one of the most used methods by CCPs (Boudiaf, Scheicher & Vacirca, 2023), and it is simpler to adapt to integrate the anti-procyclical components.

This study focuses on the analysis in 2020 considering it was a challenging period for CCPs and IM due to the COVID-19 market turmoil.

3.3.2. Model Data Analysis

This section describes the correlation analysis and Granger causality tests performed to understand the relationship between the IM model series and the bid-ask spread series. Following the results of both approaches, we focus on the relationship between the IM floor and stressed models with the bid-ask spread series and disregard the buffer IM models, as these do not have a significant relationship with the bid-ask spread series.

3.3.2.1 Correlation Analysis

The VAR and VECM models study the relationship between IM and the bid-ask spread. Hence, it is important to study the correlation between both.

Figures 3.2, 3.3, and 3.4 below plot the results from three IM Models (using the buffer, the stressed, and the floor APC components) and the E Mini S&P500 bid-ask spread series. The three models selected for this analysis relate to three different APC components, with the models using a 0.95 decay factor and a 1-year lookback period. The plots suggest a positive relationship between IM models and the bid-ask spread, which is aligned across models.

Figure 3.2 - IM with a FHSVaR (95 decay - stressed – 1 year lookback) and bid-ask spread series

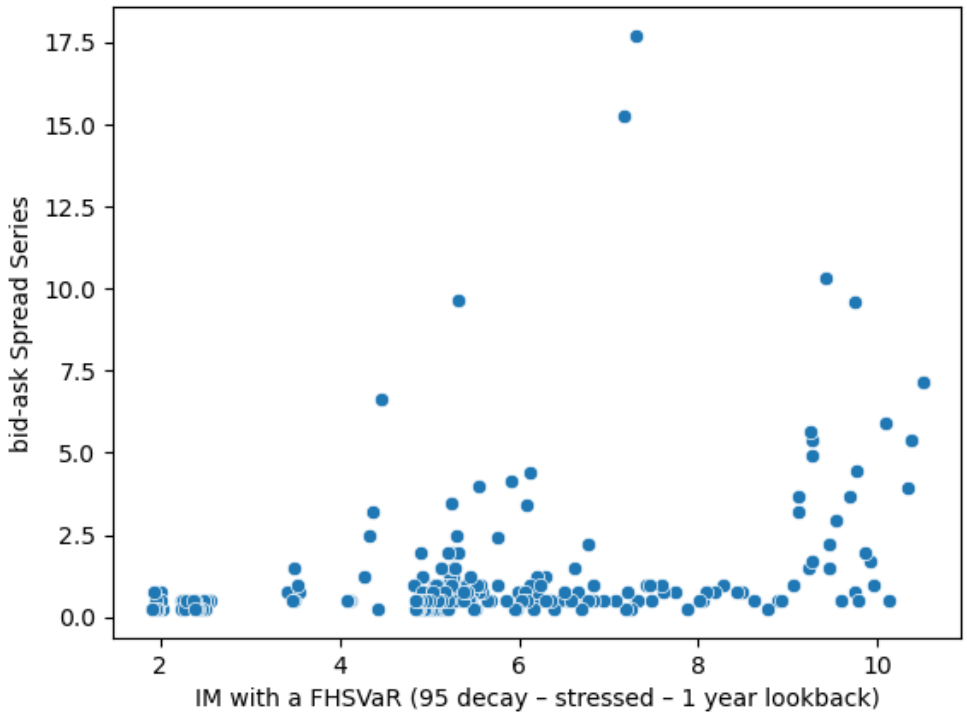


Figure 3.3 - IM with a FHSVaR (95 decay - floor – 1 year lookback) and bid-ask spread series

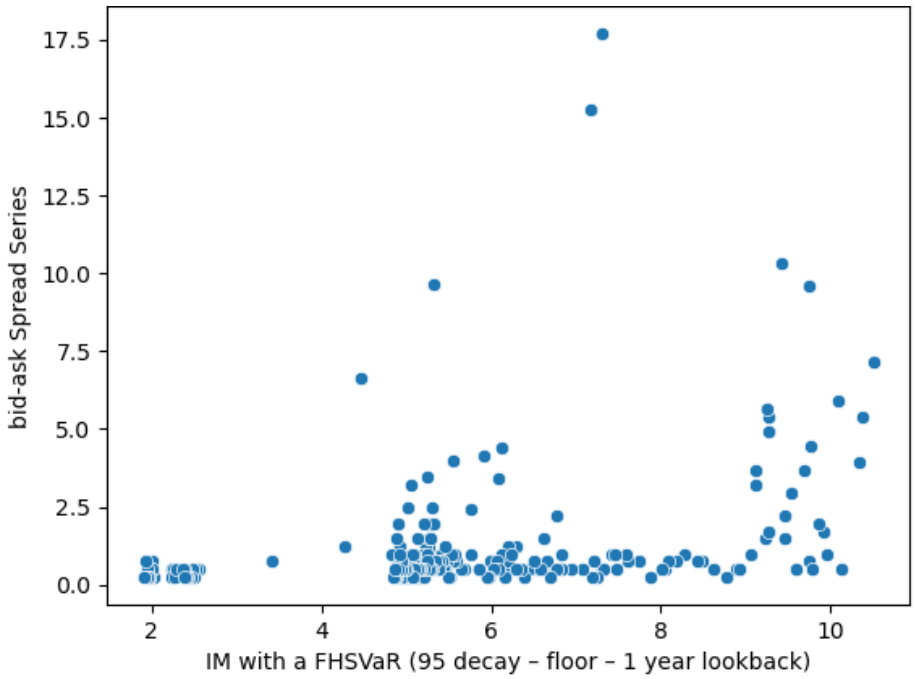
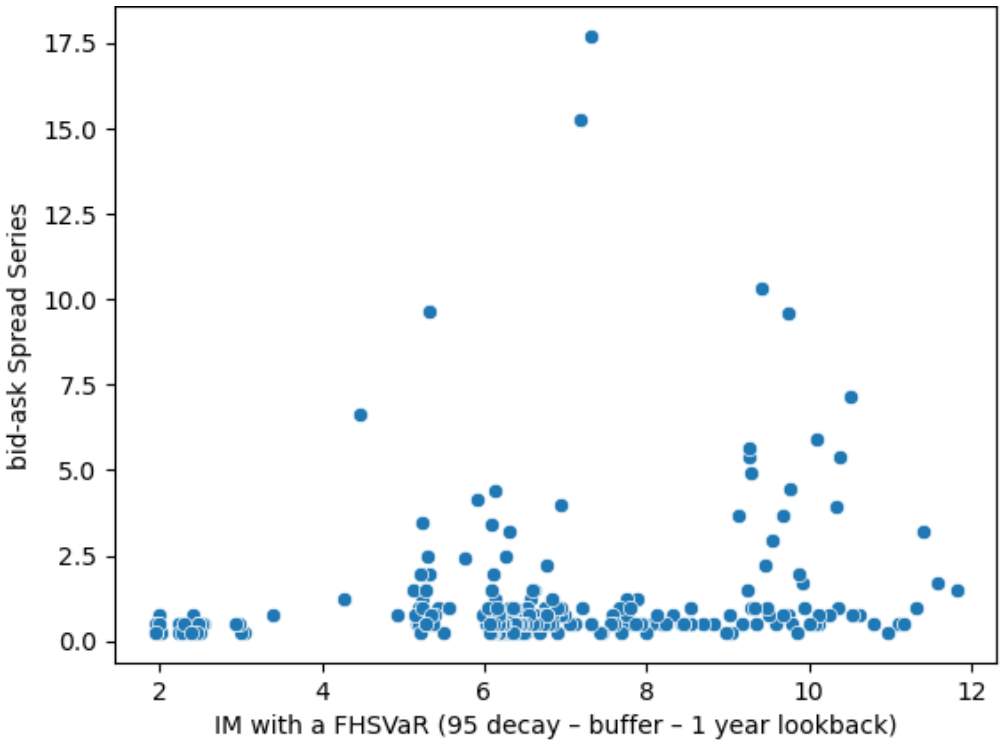


Figure 3.4 - IM with a FHSVaR (95 decay – buffer – 1 year lookback) and bid-ask spread series



Additionally, the correlation coefficients between the IM Models' first differences and the bid-ask spread first differences are detailed in Appendix A. The results show that most IM models differences have a positive correlation with the bid-ask spread differences series. Two key observations emerge from the analysis. First, the Buffer IM models exhibit weaker correlation coefficients with the bid-ask spread series, which can be attributed to the inherent volatility and instability of these models. Second, higher decay factors and longer lookback periods are associated with stronger correlations with the bid-ask spread. In contrast, lower decay factors and shorter lookback windows make the IM models more reactive to short-term fluctuations, thereby reducing its stability and predictive power.

3.3.2.2 Granger Causality

Our study also analyses the potential causal relationships between the first differences in the bid-ask spread and the first differences in the IM models. For this purpose, we use the Granger causality test, which assesses if one time series can predict or "cause" another time series (Granger, 1969).

The following equation represents the unrestricted model to test whether X_t Granger-causes Y_t (dependent variable).

$$Y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \sum_{j=1}^{p} \beta_j X_{t-j} + \varepsilon_t \quad (3.4)$$

where,

- p is the selected lag length for both time series
- α_0 represents the intercept term
- α_i represents the coefficients on lagged values of Y
- β_j represents the coefficients on lagged values of X
- ε_t is the white noise error term

The null hypothesis in the Granger causality test assumes that none of the lagged values of X are statistically significant to explain Y_t in which case all β_j coefficients are 0. This means that X does not granger-cause Y.

We apply this test to check if the IM Models Granger caused the bid-ask spread series. The results, detailed in Appendix B, show that the IM Models Granger cause the bid-ask spread series except for the Buffer IM Models. The null hypothesis is not rejected for the buffer IM Models, which means that the past lag values of the Buffer IM Models are not statistically significant to explain the bid-ask spread series. Hence, the VAR and VECM models are not estimated for the buffer models. These results are aligned with the correlation results described above.

3.3.3. Model Methodology

This study tests both the VAR and VECM models to study the relationship between the IM and market liquidity.

3.3.3.1 VAR Models Methodology

The Vector Autoregression (VAR) model is estimated using the Ordinary Least Squares (OLS) method, where each variable in the system is regressed on its lags as well as the lags of all other variables. For OLS to yield unbiased and consistent estimates, the regressors must be exogenous—specifically, lagged values of the variables should not be correlated with the model's error terms. Additionally, OLS assumes that residuals are homoscedastic and free from autocorrelation. These assumptions require the time series involved to be stationary.

To validate this requirement, we conduct stationarity tests on the IM models and the bid-ask spread series. We employ three standard tests: the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. The ADF and PP tests reject the null hypothesis of a unit root in the bid-ask spread series, indicating stationarity, while the KPSS test confirms this result by failing to reject the null of stationarity.

However, the IM model series exhibited non-stationarity, even after the first differencing. Only after applying second differences did these series meet the stationarity criteria. As such, the VAR models were estimated using the second-differenced data for both the IM models and the bid-ask spread series to ensure compliance with OLS assumptions and the absence of unit roots.

To determine the appropriate lag structure and prevent overfitting, model selection criteria such as the Akaike Information Criterion (AIC) were used.

3.3.3.2 VECM Models Methodology

The Vector Error Correction Model (VECM) is estimated using a Maximum Likelihood Estimation (MLE) method, which simultaneously determines the cointegration vectors, short-term dynamics, and error correction terms by maximizing the log-likelihood function. This approach allows for the identification of both long-run equilibrium relationships and short-run deviations among non-stationary variables.

A key prerequisite for VECM estimation is that the time series involved are cointegrated—meaning they exhibit a stable, long-term relationship despite being individually non-stationary. To verify this, the Johansen cointegration test was applied to the first-differenced series of the bid-ask spread and the IM models (both floor and stressed versions). The test confirmed the existence of statistically significant cointegrating relationships between the bid-ask spread and both the IM floor model series and the IM stressed model series.

Given these findings, the VECM models were estimated using first-differenced data. While the individual series remain non-stationary, their cointegration justifies the use of the VECM framework. As with the VAR model, the Akaike Information Criterion (AIC) was used to select the optimal lag length.

3.4. Results

In this section, we analyse separately the results from the VAR model and from the VECM model.

3.4.1. VAR Models Results

The results from the VAR model show that it fails to describe the relationship between the IM model series and the bid-ask spread series. The models were analysed in detail by checking the statistical significance of the estimated coefficients and the autocorrelation of the error terms of the model and the IRF plots.

First, the coefficients of the VAR models were analysed. Most VAR models estimated for the IM Model series and the bid-ask spread series show that the IM model lags are not statistically significant to estimate the bid-ask spread. The IM model's series lags show high p values, suggesting the IM model lags are not required to explain the bid-ask spread series.

Second, the VAR models were tested using the Ljung-Box test, which indicates whether there is significant autocorrelation in the residuals of a time series model. The test checks whether the residuals of a time series are independent and identically distributed by testing for autocorrelation at multiple lags. The results show that the VAR models estimated for the bid-ask spread series and the IM models series present autocorrelated errors. This means that the VAR model has not fully captured the dependencies in the data, indicating the need for model refinement.

Hence, the results show that the VAR models are not appropriate to assess the relationship between the IM models and the bid-ask spread.

3.4.2. VECM Models Results

The VECMs were estimated using a cointegration rank of 2, as determined by the results of the cointegration test. The appropriate number of lags was selected based on the AIC, where different lag lengths were tested through VAR models, and the lowest AIC value defined the optimal lag order.
The diagnostic tests performed on the residuals of all estimated VECM models indicate that there is no significant autocorrelation, as confirmed by the Ljung-Box test. This result suggests that the models are well specified and provide a reliable fit to the data.

The analysis of the VECM models reveals that the p-values of the IM model lag terms are not statistically significant in explaining the bid-ask spread, when the decay factor is low and the lookback period is shorter. However, as the decay factor increases and the lookback period lengthens, the p-values of the IM model's lag terms become statistically significant, indicating a stronger relationship with the bid-ask spread series. A similar pattern exists for the bid-ask spread lags in explaining the IM model series. Specifically, when the decay factor is higher and the lookback period is longer, the bid-ask spread lags exhibit greater statistical significance in explaining the IM model series.

Regarding the cointegration relations of the models, the results show that, in general, the IM model series and the bid-ask spread series share a long-run equilibrium. Regarding the adjustment coefficients, the VECM model results show that the spread reacts strongly to long-run equilibrium forces, suggesting that deviations in the spread are highly unstable and adjust rapidly. The IM model adjusts more gradually, suggesting that financial risk measures are more stable and correct deviations at a slower rate. In general, the higher the decay factor and the longer the lookback period, the lower the adjustment coefficients of the IM model.

Furthermore, the AIC results demonstrate that lower AIC values are associated with models where the IM model exhibits a higher decay factor and a longer lookback period. This finding suggests that the relationship between the IM models and the bid-ask spread is better captured by the VECM models when the IM model is less reactive.

The IRF plots consistently exhibit a predictable response pattern across different impulses. In general, positive shocks in the bid-ask spread lead to an immediate short-term increase in the IM model, while positive shocks in the IM model similarly cause an immediate short-term rise in the bid-ask spread. This is aligned with expectations that an increase in margin requirements leads to less liquidity translated into a wider bid-ask spread. However, these shock responses are not persistent over time and tend to return to equilibrium following the initial reaction. In the short term, both the IM model series and the bid-ask spread series exhibit a positive response to shocks, but this effect is later reversed, indicating a counterbalancing dynamic. These findings imply that the IM models and bid-ask spread series reinforce each other in the short term, but a consistent long-term increase in response to a unit positive shock in either series is not expected. This is aligned with the cointegration relations results of the VECM models.

These results are aligned with Chou, Wang, and Wang (2015), who study the relationship between margins and liquidity, and also find that market liquidity is negatively related to margin requirements. Our simulation results confirm that the bid-ask spread is positively related to margin requirements and vice versa. Additionally, this study is aligned with Foley, Kwan, Philip, and Ødegaard (2022), who state that margin requirements increased considerably during the COVID-19 pandemic and that positive changes in margin requirements led to a decrease in market liquidity. Our results show a significant IM increase during the COVID-19 market turmoil and confirm that increases in margin requirements affect the market liquidity measured by the bid-ask spread.

Figures 3.5 and 3.6 detail the IRF for the bid-ask spread series and the FHSVaR Stressed model, using a 0.99 decay factor and a 10-year lookback period, and the IRF for the bid-ask spread series and the FHSVaR Stressed model, using a 0.95 decay factor and a 1-year lookback period. Notably, the magnitude of shock responses is smaller for IM models with low decay factors and short lookback periods when compared to IM models with higher decay factors and longer lookback periods. This suggests that IM models with a slower rate of responsiveness exhibit a stronger interaction with the bid-ask spread over time.

Figure 3.5 - IRF bid-ask spread series and FHSVaR stressed 99decay and 10 years lookback

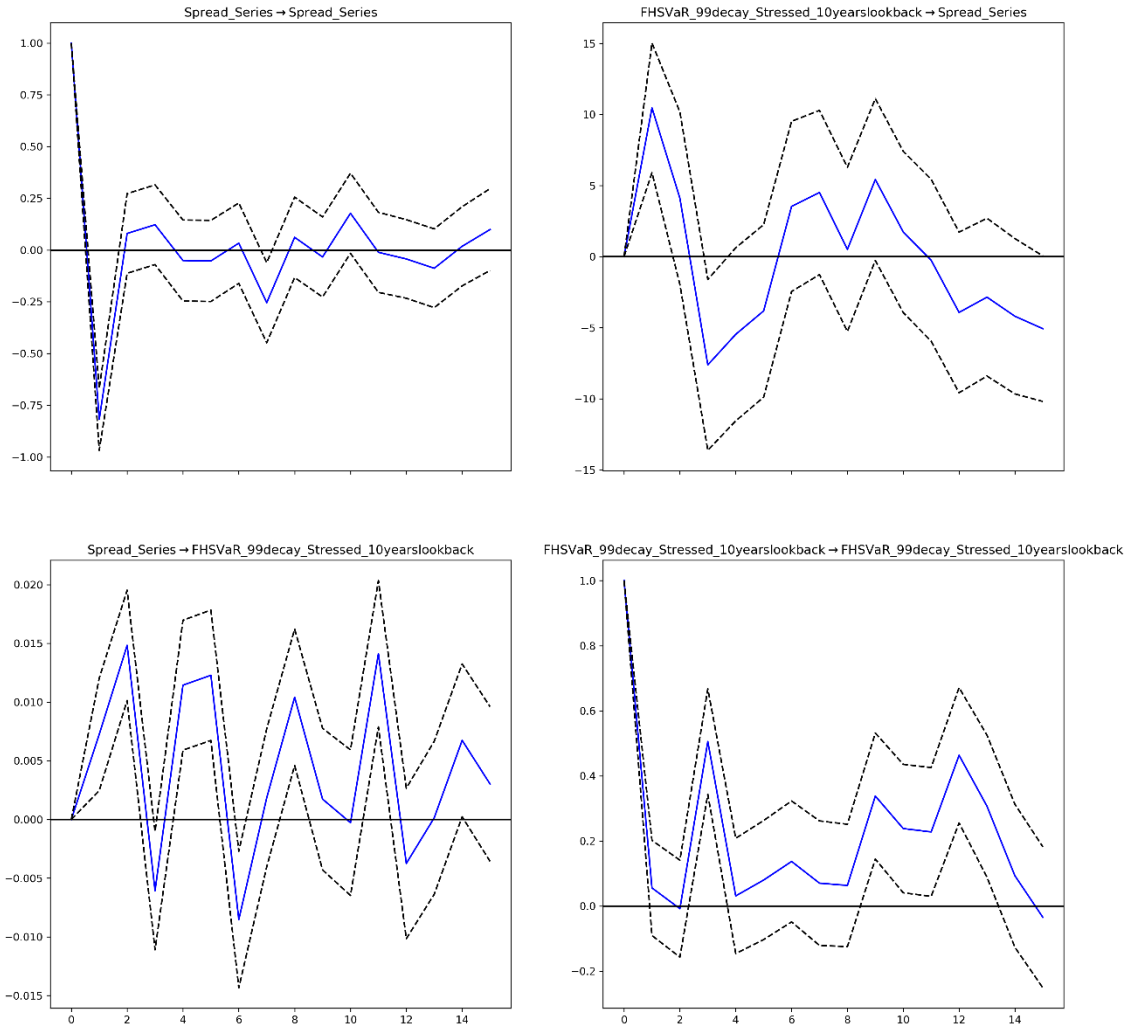
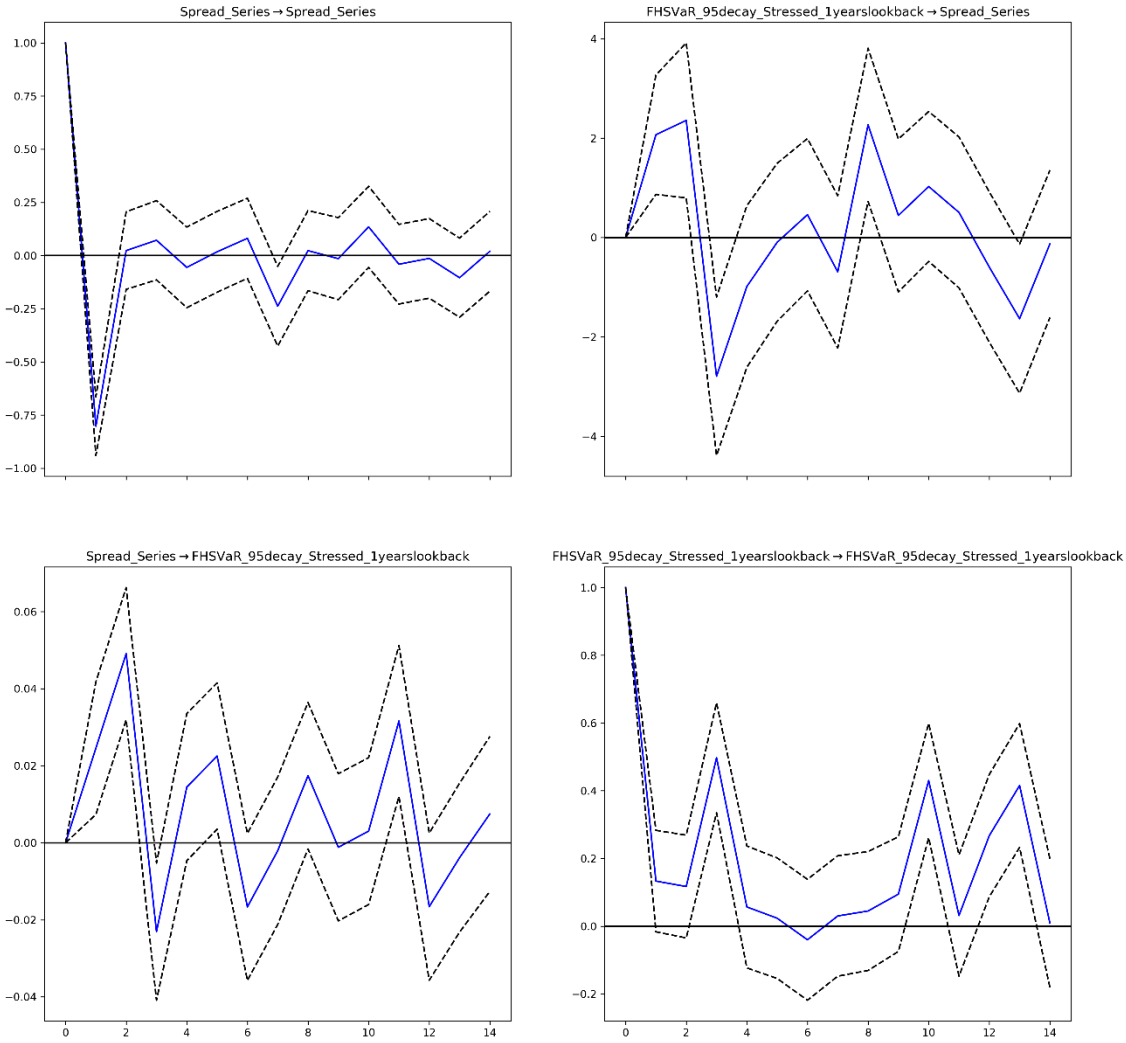


Figure 3.6 - IRF bid-ask spread series and FHSVaR stressed 95decay and 1 year lookback



Finally, it is important to highlight that the results detailed are consistent in general across the IM Floor models and the IM Stressed models.

3.5. Conclusion

This study examined the relationship between the IM Model series and the bid-ask spread series using VAR and VECM models.

The results show that the VAR models fail to explain the relationship between the IM models and the bid-ask spread, as expressed by their inability to statistically capture the influence of the IM model on the bid-ask spread. Both the lack of statistical significance in the coefficient estimates and the presence of autocorrelated errors in the residuals suggest that VAR models are insufficient for capturing the complex dynamics between these variables.

In contrast, the VECM models display a strong performance when explaining these relationships. The results from the VECM show that the relationship between the IM models and the bid-ask spread is highly dependent on the parameters of the IM models, specifically the decay factor and the lookback period. Shorter lookback periods and lower decay factors are associated with weaker, statistically insignificant relationships, while longer lookback periods and higher decay factors result in stronger, more stable relationships. This suggests that IM models with slower response times are more aligned with the bid-ask spread dynamics, making it easier to explain and interpret their interactions.

This study finds that the VECM model performance decreases when applied to more responsive IM models and the bid-ask spread. This happens because more responsive IM models are less predictable and more unstable. Hence, a sensitive IM model relationship with the bid-ask spread is more difficult to capture as these models are less predictable. These results highlight how IM model calibration affects IM model sensitivity and the ability to study their impact on underlying market conditions. This is extremely important as regulators and market players need to understand the impact of IM models on market conditions.

Moreover, the results underline the potential liquidity consequences of IM changes. The positive relationship observed between the IM models and the bid-ask spread—where increases in IM lead to wider bid-ask spreads and vice versa—aligns with existing literature, particularly the findings of Chou, Wang, and Wang (2015), and Foley, Kwan, Philip and Ødegaard (2022), who suggest that market liquidity is negatively related to margin requirements.

The IRF plots further reinforce this positive interaction, showing that shocks to the IM models or the bid-ask spread result in immediate, short-term responses, but these effects dissipate over time, returning to equilibrium. These findings imply that while short-term dynamics may be influenced by margin requirements, the long-term impact on market liquidity is less pronounced, which is consistent with the cointegration relations in the VECM models.

In summary, the study's findings demonstrate that VECM models, especially when applied to less responsive IM models, provide a more robust and meaningful understanding of the relationship between margin requirements and liquidity. This underscores the importance of IM model calibration, where the stability and responsiveness of the IM models can have significant implications for understanding market dynamics.

Concluding Remarks

This thesis explored the design, performance, and market impact of IM models used by CCPs, with a particular focus on their ability to accurately estimate risk, mitigate procyclicality, and preserve market liquidity. Across three standalone yet interconnected studies, the findings consistently highlight that while IM models are a critical component of CCP risk management frameworks, their effectiveness hinges on methodological robustness, calibration choices, and the regulatory guidance underpinning their application.

The first paper demonstrated that many CCPs still rely heavily on Historical Simulation models (HSVaR and FHSVaR) despite their known limitations in capturing market risk during stress periods. More advanced models, such as MCSVaR and PVaR, exhibited superior performance, particularly in anticipating portfolio losses under volatility. The continued use of weaker models raises serious concerns about undercollateralization and systemic vulnerability. These findings underscore the need for regulators to issue clearer, more prescriptive guidelines on IM model selection and calibration to ensure that CCPs are properly collateralized without imposing excessive costs on market participants.

The second paper critically assessed the APC measures mandated under EMIR. It found that current APC tools—especially the buffer model—are insufficient in curbing the procyclical nature of IM requirements. The study introduced a novel “limit rates” approach that more effectively limits margin volatility while maintaining model performance. However, the analysis also revealed that EMIR’s APC guidance lacks clarity, leading to inconsistent implementation across CCPs and poor alignment with best practices in model stability and transparency. The results advocate for a fundamental regulatory reassessment that incorporates performance-based criteria, clearer calibration standards, and compatibility across IM methodologies.

The third paper examined the interaction between IM models and market liquidity, finding strong evidence that IM changes influence bid-ask spreads—particularly when models are highly responsive. Using the VECM, the study showed that more stable IM configurations (i.e., with longer lookback periods and higher decay factors) have a more predictable and interpretable relationship with liquidity measures. Conversely, more reactive IM models exhibit greater sensitivity to market fluctuations, making their relationship with market liquidity harder to capture and model accurately. This reduced predictability complicates the analysis of their market impact and may contribute to more volatile and disruptive outcomes. The analysis also highlighted that buffer-based APC methods, though intended to dampen volatility, exhibited high instability and limited effectiveness, further weakening their role in managing procyclicality and market impact.

Together, the three papers present a cohesive argument for more rigorous and harmonized regulatory oversight of IM models and APC frameworks. Specifically, regulators should:

- Encourage the adoption of more robust IM methodologies, such as MCSVaR and PVaR;
- Provide clearer calibration criteria tailored to asset class and risk profile;
- Standardize APC performance benchmarks across CCPs;
- Reevaluate or replace underperforming APC tools like the buffer model;
- Consider the liquidity implications of margin policies to minimize systemic feedback loops.

Ultimately, if CCPs are to fulfill their role as anchors of financial market stability, they must rely on IM models that are not only technically sound but also guided by well-defined, empirically supported regulatory frameworks. By addressing the gaps identified in this research, policymakers can better align risk management practices with the broader goals of market resilience and systemic protection.

References

Álvarez-Díaz, M. (2020). Is it possible to accurately forecast the evolution of Brent crude oil prices? An answer based on parametric and nonparametric forecasting methods. *Empirical Economics, 59*(3), 1285–1305.

Amihud, Y., & Mendelson, H. (1986). Asset pricing and the bid-ask spread. *Journal of Financial Economics, 17*(2), 223–249. https://doi.org/10.1016/0304-405X(86)90065-6

Anderson, D. T., Bezdek, J. C., Popescu, M., & Keller, J. M. (2010). Comparing fuzzy, probabilistic, and possibilistic partitions. *IEEE Transactions on Fuzzy Systems, 18*(5), 906–918.

Bakoush, M., Gerding, E. H., & Wolfe, S. (2019). Margin requirements and systemic liquidity risk. *Journal of International Financial Markets, Institutions and Money, 58*, 78–95. https://doi.org/10.1016/j.intfin.2018.10.008

Bakoush, M., Gerding, E. H., & Wolfe, S. (2020). Interest rate swaps clearing and systemic risk. *Finance Research Letters, 33*, 101218.

Bank for International Settlements & International Organization of Securities Commissions. (2012). *Principles for financial market infrastructures*. https://www.bis.org/cpmi/info_pfmi.htm

Basak, S., & Shapiro, A. (2001). Value-at-risk-based risk management: Optimal policies and asset prices. *The Review of Financial Studies, 14*(2), 371–405.

Basel Committee on Banking Supervision (BCBS), & International Organization of Securities Commissions (IOSCO). (2015). *Margin requirements for non-centrally cleared derivatives*. Bank for International Settlements & OICU-IOSCO.

Béli, M., & Váradi, K. (2017). A possible methodology for determining the initial margin. *Financial and Economic Review, 16*(2), 119–147.

Berlinger, E., Dömötör, B., & Illés, F. (2019). Anti-cyclical versus risk-sensitive margin strategies in central clearing. *Journal of International Financial Markets, Institutions and Money, 62*, 117–131.

Berndsen, R. (2021). Fundamental questions on central counterparties: A review of the literature. *Journal of Futures Markets, 41*(12), 2009–2022. https://doi.org/10.1002/fut.22249

Berkowitz, J., Christoffersen, P., & Pelletier, D. (2011). Evaluating value-at-risk models with desk-level data. *Management Science, 57*(12), 2213–2227.

Bhanu, A. (2024). The effect of anti-procyclical central counterparty margins on trading. *Journal of Futures Markets*.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, 31*, 307–327.

Boudiaf, I. A., Scheicher, M., & Vacirca, F. (2023). CCP initial margin models in Europe (*Occasional Paper Series No. 314*). European Central Bank. https://www.ecb.europa.eu/pub/pdf/scpops/ecb.op314~60bc490fd6.en.pdf

Brunnermeier, M. K., & Pedersen, L. H. (2009). Market liquidity and funding liquidity. *The Review of Financial Studies, 22*(6), 2201–2238. https://doi.org/10.1093/rfs/hhn098

Caspers, P., Giltinan, P., Lichters, R., & Nowaczyk, N. (2017). Forecasting initial margin requirements: A model evaluation. *Journal of Risk Management in Financial Institutions, 10*(4), 365–394.

Chikwira, C., & Mohammed, J. I. (2023). The impact of the stock market on liquidity and economic growth: Evidence of volatile market. *Economies, 11*(6), 155. https://doi.org/10.3390/economies11060155

Chordia, T., Roll, R., & Subrahmanyam, A. (2008). Liquidity and market efficiency. *Journal of Financial Economics, 87*(2), 249–268. https://doi.org/10.1016/j.jfineco.2007.03.005

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International Economic Review, 39*(4), 841–862.

Chou, R. K., Wang, G. H., & Wang, Y. Y. (2015). The effects of margin changes on the composition of traders and market liquidity: Evidence from the Taiwan Futures Exchange. *Journal of Futures Markets, 35*(10), 894–915.

Cominetta, M., Grill, M., & Jukonis, A. (2019). Investigating initial margin procyclicality and corrective tools using EMIR data. *Macroprudential Bulletin, 9*.

Cotter, J., & Dowd, K. (2006). Extreme spectral risk measures: An application to futures clearinghouse margin requirements. *Journal of Banking & Finance, 30*(12), 3469–3485.

Danielsson, J., & De Vries, C. G. (2000). Value-at-risk and extreme returns. *Annales d'Économie et de Statistique, 60*, 239–270.

De Haan, L., & van den End, J. W. (2013). Banks' responses to funding liquidity shocks: Lending adjustment, liquidity hoarding and fire sales. *Journal of International Financial Markets, Institutions and Money, 26*, 152–174.

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association, 74*(366a), 427–431.

European Market Infrastructure Regulation (EMIR). (2012). *Regulation (EU) No 648/2012*. https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex%3A32012R0648

European Securities and Markets Authority (ESMA). (2015). *EMIR review report no. 2: Review on the efficiency of margining requirements to limit procyclicality* (ESMA/2015/1252). https://www.esma.europa.eu/sites/default/files/library/2015/11/esma-2015-1252_-_emir_review_report_no.2_on_procyclicality.pdf

European Securities and Markets Authority (ESMA). (2018). *Guidelines on EMIR anti-procyclicality margin measures for central counterparties* (ESMA70-151-1293). https://www.esma.europa.eu/press-news/esma-news/esma-promotesconsistent-anti-procyclicality-margin-measures-ccps

FIA (Futures Industry Association). (2020). *Revisiting procyclicality: The impact of the COVID crisis on CCP margin requirements*. https://www.fia.org/resources/revisiting-procyclicality-impact-covid-crisis-ccp-margin-requirements

Foley, S., Kwan, A., Philip, R., & Ødegaard, B. A. (2022). Contagious margin calls: How COVID-19 threatened global stock market liquidity. *Journal of Financial Markets, 59*, 100689. https://doi.org/10.1016/j.finmar.2021.100689

Glasserman, P., & Wu, Q. (2018). Persistence and procyclicality in margin requirements. *Management Science, 64*(12), 5705–5724.

Granger, C. W. J. (1969). Investigating causal relations by econometric models and cross-spectral methods. *Econometrica, 37*(3), 424–438.

Gregory, J. (2014). *Central counterparties: Mandatory central clearing and initial margin requirements for OTC derivatives*. John Wiley & Sons.

Heath, A., Kelly, G., Manning, M., Markose, S., & Shaghaghi, A. R. (2016). CCPs and network stability in OTC derivatives markets. *Journal of Financial Stability, 27*, 217–233.

Houllier, M., & Murphy, D. (2017). Initial margin model sensitivity analysis and volatility estimation. *Journal of Financial Market Infrastructures, 6*(4), 29–56.

Hull, J., & White, A. (1998). Incorporating volatility updating into the historical simulation method for value-at-risk. *Journal of Risk, 1*(1), 5–19.

Ince, H. (2006). Non-parametric regression methods. *Computational Management Science, 3*(2), 161–174.

Ječmínek, J., Kukalová, G., & Moravec, L. (2020). Volatility modeling and VaR: The case of Bitcoin, Ether, and Ripple. *DANUBE: Law, Economics and Social Issues Review, 11*(3), 253–269.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control, 12*(2–3), 231–254.

Kupiec, P. H. (1995). Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives, 3*, 73–84.

Lee, K., & Seo, B. K. (2018). Filtered historical simulation for initial margin of interest rate swap under Korean market. *Emerging Markets Finance and Trade, 54*(11), 2516–2532.

Lin, L., & Surti, J. (2015). Capital requirements for over-the-counter derivatives central counterparties. *Journal of Banking & Finance, 52*, 140–155.

Liu, W., & Morley, B. (2009). Volatility forecasting in the Hang Seng Index using the GARCH approach. *Asia-Pacific Financial Markets, 16*(1), 51–63.

Markose, S. M., Giansante, S., & Rais Shaghaghi, A. (2017). A systemic risk assessment of OTC derivatives reforms and skin-in-the-game for CCPs. *Financial Stability Review, 21*, 111–126.

Moran, L., & Wilkens, S. (2017). Capturing initial margin in counterparty risk calculations. *Journal of Risk Management in Financial Institutions, 10*(2), 118–129.

Murphy, D. (2023). What can we expect from a good margin model? Observations from whole-distribution tests of risk-based initial margin models. *Journal of Risk Model Validation, 17*(2).

Murphy, D., & Vause, N. (2021). A cost–benefit analysis of anti-procyclicality: Analyzing approaches to procyclicality reduction in central counterparty initial margin models. *Journal of Financial Market Infrastructures, 9*(4).

Park, Y. H., & Abruzzo, N. (2016). An empirical analysis of futures margin changes: Determinants and policy implications. *Journal of Financial Services Research, 49*(1), 65–100.

Pritsker, M. (1997). Evaluating value at risk methodologies: Accuracy versus computational time. *Journal of Financial Services Research, 12*(2–3), 201–242.

Rehman, I. U., Mahdzan, N. S., & Zainudin, R. (2016). Is the relationship between macroeconomy and stock market liquidity mutually reinforcing? Evidence from an emerging market. *International Journal of Monetary Economics and Finance, 9*(3), 294–316.

Sheikh, A. Z., & Qiao, H. (2010). Non-normality of market returns: A framework for asset allocation decision making. *The Journal of Alternative Investments, 12*(3), 8–21.

Sims, C. A. (1980). Macroeconomics and reality. *Econometrica, 48*(1), 1–48.

Vuillemey, G. (2020). The value of central clearing. *The Journal of Finance, 75*(4), 2021–2053.

Weber, M. (2016). Central counterparties in the OTC derivatives market from the perspective of the legal theory of finance, financial market stability and the public good. *European Business Organization Law Review, 17*(1), 71–103. https://doi.org/10.1007/s40804-015-0023-z

Wickens, M. R., & Motto, R. (2001). Estimating shocks and impulse response functions. *Journal of Applied Econometrics, 16*(3), 371–387.

A. Appendix A

A.1 GARCH Model

The ADF test result rejects the null hypothesis of non-stationarity, implying the time series is stationary.

Table A.1 - ADF test results

ADF test Statistic:	**-13,8648**
p-value:	**0**
Critical Values:	
1%:	-3,431
5%:	-2,862
10%:	-2,567

The Phillips–Perron (1988) test results show that the null hypothesis is rejected, implying that the series does not have a unit root.

Table A.2 - PP test

PP Test Statistic	-83,698
P-value	0
Lags	34
Critical Values:	
1%:	-3,43
5%:	-2,862
10%:	-2,567

The KPSS test results do not reject the null hypothesis that the process is weakly stationary. The value of the test statistic is lower than the critical values, hence, the null hypothesis is not rejected.

Table A.3 - KPSS test

Test Statistic	0,466
P-value	0,049
Lags	26
Critical Values:	
1%:	0,74
5%:	0,46
10%:	0,35

The autocorrelation and partial autocorrelation graphs below suggest that the correlation across lags decreases quickly.

Figure A.1 - ACF E-mini S&P 500 futures daily returns

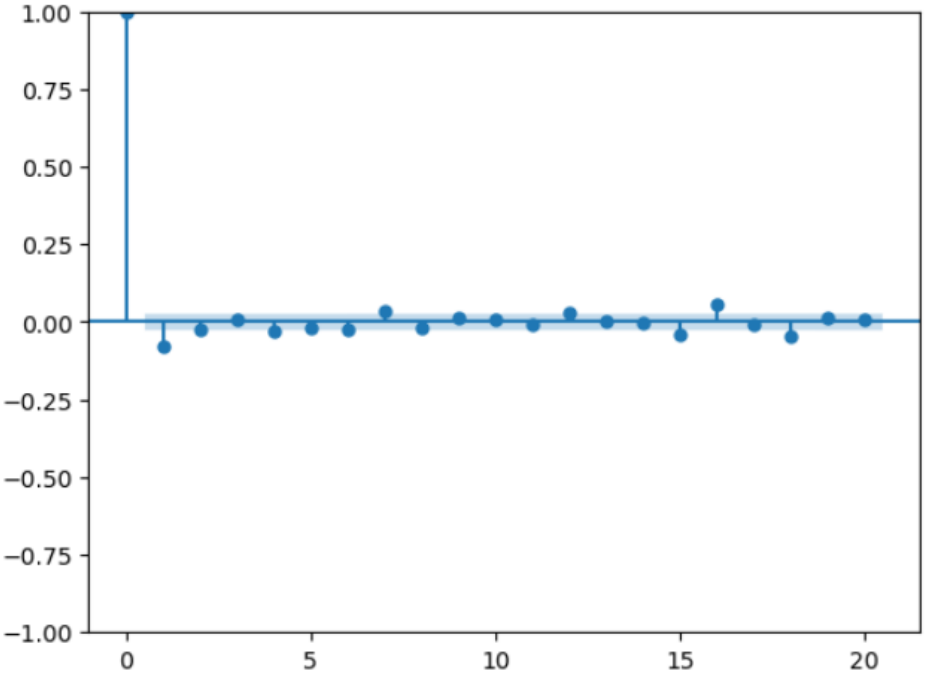
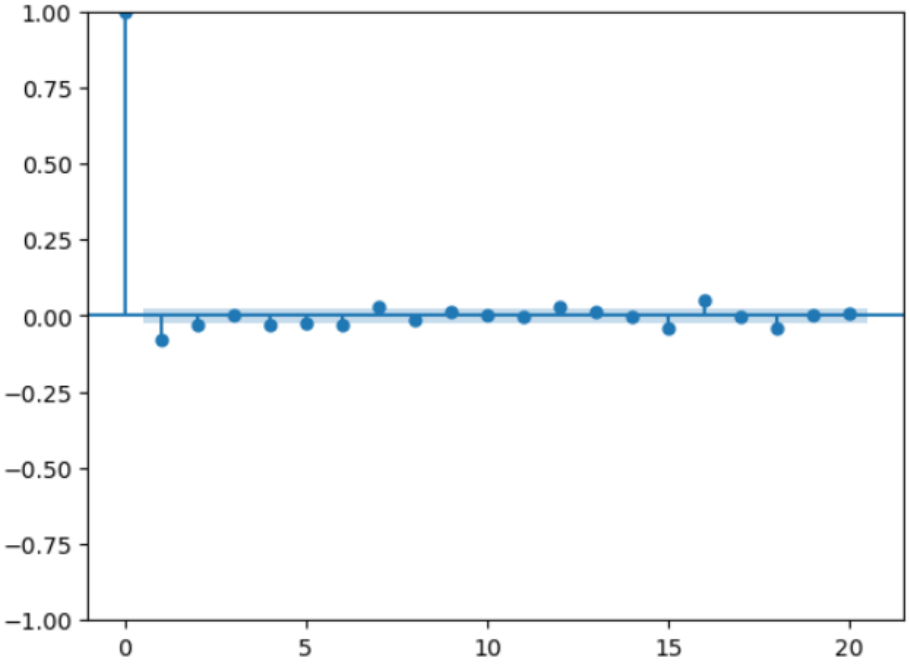


Figure A.2 - PACF E-mini S&P 500 futures daily returns



The GARCH(1,1) and GARCH(2,2) models are presented below.

Table A.4 - GARCH (1,1)

Dep. Variable:		log_rtn	**R-squared:**		0,00
Mean Model:		Constant Mean	**Adj. R-squared:**		0,00
Vol Model:		GARCH	**Log-Likelihood:**		8495,56
Distribution:		Normal	**AIC:**		-16983,10
Method:		Maximum Likelihood	**BIC:**		-16959,80
No. Observations:		2500	**Df Residuals:**		2499
Mean Model	**coef**	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
mu	8.4332e-04	2.725e-08	3.095e+04	0.000	[8.433e-04,8.434e-04]
Volatility Model	**coef**	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
omega	8.8878e-06	2.172e-13	4.092e+07	0.000	[8.888e-06,8.888e-06]
alpha[1]	0.2000	2.188e-02	9.141	6.167e-20	[0.157, 0.243]
beta[1]	0.7000	1.666e-02	42.023	0.000	[0.667, 0.733]

Table A.5 - GARCH (2,2)

Dep. Variable:		log_rtn	**R-squared:**		0
Mean Model:		Constant Mean	**Adj. R-squared:**		0
Vol Model:		GARCH	**Log-Likelihood:**		8496
Distribution:		Normal	**AIC:**		-16980
Method:		Maximum Likelihood	**BIC:**		-16945,1
No. Observations:		2500	**Df Residuals:**		2499
Mean Model	**coef**	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
mu	7.9958e-04	1.379e-06	580.019	0.000	[7.969e-04,8.023e-04]
Volatility Model	**coef**	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
omega	8.8881e-06	1.412e-11	6.296e+05	0.000	[8.888e-06,8.888e-06]
alpha[1]	0.1000	5.752e-02	1.739	8.210e-02	[-1.273e-02, 0.213]
alpha[2]	0.1000	7.432e-02	1.345	0.178	[-4.567e-02, 0.246]
beta[1]	0.3500	0.673	0.520	0.603	[-0.969, 1.669]
beta[2]	0.3500	0.643	0.544	0.586	[-0.911, 1.611]

A.2 VaR Backtesting Results

This section presents the backtesting results of the VaR models. Regarding the PVaR and MCSVaR models, this section only presents the models estimated using a 10-year lookback period GARCH model, as the difference to other lookback periods considered is insignificant.

A.2.1. Historical Simulation VaR Backtesting Results

Table A.6 - HSVaR 1-Year Backtesting Results

		Historical Simulation VaR					
		1 Year Lookback					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	14	14	14	9	9	9
	Breaches Rate	6%	6%	6%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	25,87568	25,87568	25,87568	10,28241	10,28241	10,28241
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	3,64129E-07	3,64129E-07	3,64129E-07	0,001343046	0,001343046	0,001343046
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	51,2241	51,2241	51,2241	34,62411	34,62411	34,62411
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	7,53064E-12	7,53064E-12	7,53064E-12	3,0302E-08	3,0302E-08	3,0302E-08
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.7 - HSVaR 5-Year Backtesting Results

		Historical Simulation VaR					
		5 Years Lookback					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	13	13	13
	Breaches Rate	5%	5%	5%	5%	5%	5%
Unconditional Coverage	Likelihood Ratio statistic	19,09467	19,09467	19,09467	22,40394	22,40394	22,40394
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	1,24392E-05	1,24392E-05	1,24392E-05	2,20921E-06	2,20921E-06	2,20921E-06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	30,03667	30,03667	30,03667	43,44505	43,44505	43,44505
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	3,00344E-07	3,00344E-07	3,00344E-07	3,68152E-10	3,68152E-10	3,68152E-10
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.8 - HSVaR 10-Year Backtesting Results

		Historical Simulation VaR					
		10 Years Lookback					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	15	15	15	10	10	10
	Breaches Rate	6%	6%	6%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	29,49891	29,49891	29,49891	13,0172	13,0172	13,0172
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	5,59482E-08	5,59482E-08	5,59482E-08	0,000308643	0,000308643	0,000308643
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	41,10511	41,10511	41,10511	27,3515	27,3515	27,3515
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	1,18615E-09	1,18615E-09	1,18615E-09	0,00000115	0,00000115	0,00000115
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

A.2.2. Filtered Historical Simulation VaR Backtesting Results

Table A.9 - FHSVaR 1-Year 0.95 Decay Factor Backtesting Results

		Filtered Historical Simulation					
		1 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	7	7	7
	Breaches Rate	4%	4%	4%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	15,9607	15,9607	15,9607	5,533804	5,533804	5,533804
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	6,47E-05	6,47E-05	6,47E-05	0,01865251	0,01865251	0,01865251
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	34,88654	34,88654	34,88654	18,99762	18,99762	18,99762
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	2,66E-08	2,66E-08	2,66E-08	7,49E-05	7,49E-05	7,49E-05
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.10 - FHSVaR 1-Year 0.97 Decay Factor Backtesting Results

		Filtered Historical Simulation					
		1 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	7	7	7
	Breaches Rate	5%	5%	5%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	19,09467	19,09467	19,09467	5,533804	5,533804	5,533804
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	1,24392E-05	1,24392E-05	1,24392E-05	0,01865251	0,01865251	0,01865251
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	42,64241	42,64241	42,64241	18,99762	18,99762	18,99762
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	5,50E-10	5,50E-10	5,50E-10	7,49E-05	7,49E-05	7,49E-05
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.11 - FHSVaR 1-Year 0.99 Decay Factor Backtesting Results

		Filtered Historical Simulation					
		1 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	8	8	8
	Breaches Rate	5%	5%	5%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	19,09467	19,09467	19,09467	7,778629	7,778629	7,778629
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	1,24392E-05	1,24392E-05	1,24392E-05	0,00528679	0,00528679	0,00528679
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	42,64241	42,64241	42,64241	26,68347	26,68347	26,68347
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	5,50E-10	5,50E-10	5,50E-10	1,61E-06	1,61E-06	1,61E-06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.12 - FHSVaR 5-Year 0.95 Decay Factor Backtesting Results

		Filtered Historical Simulation					
		5 Years Lookback					
		0.95 decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	9	9	9
	Breaches Rate	5%	5%	5%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	19,09467	19,09467	19,09467	10,28241	10,28241	10,28241
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	1,24392E-05	1,24392E-05	1,24392E-05	0,001343046	0,001343046	0,001343046
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	35,84712	35,84712	35,84712	26,71322	26,71322	26,71322
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	1,64E-08	1,64E-08	1,64E-08	1,58E-06	1,58E-06	1,58E-06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.13 - FHSVaR 5-Year 0.97 Decay Factor Backtesting Results

		Filtered Historical Simulation					
		5 Years Lookback					
		0.97 decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	9	9	9
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	15,9607	15,9607	15,9607	10,28241	10,28241	10,28241
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	6,46713E-05	6,46713E-05	6,46713E-05	0,001343046	0,001343046	0,001343046
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	42,36896	42,36896	42,36896	26,71322	26,71322	26,71322
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	6,31E-10	6,31E-10	6,31E-10	1,58E-06	1,58E-06	1,58E-06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.14 - FHSVaR 5-Year 0.99 Decay Factor Backtesting Results

		Filtered Historical Simulation					
		5 Years Lookback					
		0.99 decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	10	10	10
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	15,9607	15,9607	15,9607	13,0172	13,0172	13,0172
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	6,46713E-05	6,46713E-05	6,46713E-05	0,000308643	0,000308643	0,000308643
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	42,36896	42,36896	42,36896	27,3515	27,3515	27,3515
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	6,31E-10	6,31E-10	6,31E-10	1,15E-06	1,15E-06	1,15E-06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.15 - FHSVaR 10-Year 0.95 Decay Factor Backtesting Results

		Filtered Historical Simulation					
		10 Years Lookback					
		0.95 decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	13	13	13	7	7	7
	Breaches Rate	5%	5%	5%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	22,40394	22,40394	22,40394	5,533804	5,533804	5,533804
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	2,20921E-06	2,20921E-06	2,20921E-06	0,01865251	0,01865251	0,01865251
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	43,44505	43,44505	43,44505	18,99762	18,99762	18,99762
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	3,68E-10	3,68E-10	3,68E-10	7,49E-05	7,49E-05	7,49E-05
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.16 - FHSVaR 10-Year 0.97 Decay Factor Backtesting Results

		Filtered Historical Simulation					
		10 Years Lookback					
		0.97 decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	8	8	8
	Breaches Rate	4%	4%	4%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	15,9607	15,9607	15,9607	7,778629	7,778629	7,778629
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	6,46713E-05	6,46713E-05	6,46713E-05	0,00528679	0,00528679	0,00528679
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	34,88654	34,88654	34,88654	19,26942	19,26942	19,26942
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	2,66E-08	2,66E-08	2,66E-08	6,54E-05	6,54E-05	6,54E-05
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table A.17 - FHSVaR 10-Year 0.99 Decay Factor Backtesting Results

		Filtered Historical Simulation					
		10 Years Lookback					
		0.99 decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	6	6	6
	Breaches Rate	5%	5%	5%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	19,09467	19,09467	19,09467	3,583938	3,583938	3,583938
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	1,24392E-05	1,24392E-05	1,24392E-05	0,05834069	0,05834069	0,05834069
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	42,64241	42,64241	42,64241	19,47525	19,47525	19,47525
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	5,50E-10	5,50E-10	5,50E-10	5,90E-05	5,90E-05	5,90E-05
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

A.2.3. Parametric GARCH VaR Backtesting Results

Table A.18 - PVaR 10-year lookback GARCH (1,1) Backtesting Results

		Parametric GARCH(1,1) VaR					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	10	10	10	1	1	1
	Breaches Rate	4%	4%	4%	0%	0%	0%
Unconditional Coverage	Likelihood Ratio statistic	13,0172	13,0172	13,0172	1,164423	1,164423	1,164423
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	0,000308643	0,000308643	0,000308643	0,2805502	0,2805502	0,2805502
	H0 Decision	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	27,3515	27,3515	27,3515	1,17252	1,17252	1,17252
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	1,15E-06	1,15E-06	1,15E-06	5,56E-01	5,56E-01	5,56E-01
	H0 Decision	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0	Fail to Reject H0

A.2.4. Monte Carlo Simulation GARCH VaR Backtesting Results

Table A.19 - MCSVaR 10-year lookback GARCH (1,1) Backtesting Results

		Monte Carlo Simulation GARCH(1,1) VaR					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	3	3	3	3	3	3
	Breaches Rate	1%	1%	1%	1%	1%	1%
Unconditional Coverage	Likelihood Ratio statistic	0,09903333	0,09903333	0,09903333	0,09903333	0,09903333	0,09903333
	Test Critical Value	2,705543	3,841459	6,634897	2,705543	3,841459	6,634897
	Test P-Value	0,7529928	0,7529928	0,7529928	0,7529928	0,7529928	0,7529928
	H0 Decision	Fail to Reject H0	Fail to Reject H0	Fail to Reject H0	Fail to Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	5,516337	5,516337	5,516337	5,516337	5,516337	5,516337
	Test Critical Value	4,60517	5,991465	9,21034	4,60517	5,991465	9,21034
	Test P-Value	6,34E-02	6,34E-02	6,34E-02	0,06340781	0,06340781	0,06340781
	H0 Decision	Reject H0	Fail to Reject H0	Fail to Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0

B. Appendix B

B.1 VaR Backtesting Results

B.1.1. 1 Year Lookback Period

Table B.1 - FHSVaR 1 Year 0.95 Decay Factor No APC Backtesting results

		Filtered Historical Simulation No APC					
		1 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	7	7	7
	Breaches Rate	4%	4%	4%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	15.96	15.96	15.96	5.53	5.53	5.53
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.02	0.02	0.02
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	34.89	34.89	34.89	19.00	19.00	19.00
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.2 - FHSVaR 1 Year 0.97 Decay Factor No APC Backtesting results

		Filtered Historical Simulation No APC					
		1 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	7	7	7
	Breaches Rate	5%	5%	5%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	19.09	19.09	19.09	5.53	5.53	5.53
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.02	0.02	0.02
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	42.64	42.64	42.64	19.00	19.00	19.00
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.3 - FHSVaR 1 Year 0.99 Decay Factor No APC Backtesting results

		Filtered Historical Simulation No APC					
		1 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	8	8	8
	Breaches Rate	5%	5%	5%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	19.09	19.09	19.09	7.78	7.78	7.78
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.01	0.01	0.01
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	42.64	42.64	42.64	26.68	26.68	26.68
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.4 - FHSVaR 1 Year 0.95 Decay Factor Limit Rates 2 days Backtesting results

		Filtered Historical Simulation Limit Rates 2 days					
		1 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	10	10	10	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	13.14	13.14	13.14	3.64	3.64	3.64
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	27.41	27.41	27.41	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.5 - FHSVaR 1 Year 0.97 Decay Factor Limit Rates 2 days Backtesting results

		Filtered Historical Simulation Limit Rates 2 days					
		1 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.39	10.39	10.39	3.64	3.64	3.64
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.76	26.76	26.76	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.6 - FHSVaR 1 Year 0.99 Decay Factor Limit Rates 2 days Backtesting results

		Filtered Historical Simulation Limit Rates 2 days					
		1 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.39	10.39	10.39	3.64	3.64	3.64
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.76	26.76	26.76	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.7 - FHSVaR 1 Year 0.95 Decay Factor Floor Backtesting results

		Filtered Historical Simulation Floor					
		1 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	10	10	10	4	4	4
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	13.08	13.08	13.08	0.79	0.79	0.79
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.37	0.37	0.37
	H0 Decision	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	27.38	27.38	27.38	12.99	12.99	12.99
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.8 - FHSVaR 1 Year 0.97 Decay Factor Floor Backtesting results

		Filtered Historical Simulation Floor					
		1 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	4
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	0.79
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.37
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.74	26.74	26.74	19.48	19.48	12.99
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.9 - FHSVaR 1 Year 0.99 Decay Factor Floor Backtesting results

		Filtered Historical Simulation Floor					
		1 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.74	26.74	26.74	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.10 - FHSVaR 1 Year 0.95 Decay Factor Stressed Backtesting results

		Filtered Historical Simulation Stressed					
		1 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	10	10	10	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	13.08	13.08	13.08	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	27.38	27.38	27.38	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.11 - FHSVaR 1 Year 0.97 Decay Factor Stressed Backtesting results

		Filtered Historical Simulation Stressed					
		1 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.74	26.74	26.74	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.12 - FHSVaR 1 Year 0.99 Decay Factor Stressed Backtesting results

		Filtered Historical Simulation Stressed					
		1 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.74	26.74	26.74	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.13 - FHSVaR 1 Year 0.95 Decay Factor Buffer Backtesting results

		Filtered Historical Simulation Buffer					
		1 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.74	26.74	26.74	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.14 - FHSVaR 1 Year 0.97 Decay Factor Buffer Backtesting results

		Filtered Historical Simulation Buffer					
		1 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.74	26.74	26.74	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.15 - FHSVaR 1 Year 0.99 Decay Factor Buffer Backtesting results

		Filtered Historical Simulation Buffer					
		1 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	8	8	8	6	6	6
	Breaches Rate	3%	3%	3%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	7.82	7.82	7.82	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.01	0.01	0.01	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.70	26.70	26.70	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

B.1.2. 5 Years Lookback Period

Table B.16 - FHSVaR 5 Year 0.95 Decay Factor No APC Backtesting results

		Filtered Historical Simulation No APC					
		5 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	9	9	9
	Breaches Rate	5%	5%	5%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	19.09	19.09	19.09	10.28	10.28	10.28
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	35.85	35.85	35.85	26.71	26.71	26.71
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.17 - FHSVaR 5 Year 0.97 Decay Factor No APC Backtesting results

		Filtered Historical Simulation No APC					
		5 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	9	9	9
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	15.96	15.96	15.96	10.28	10.28	10.28
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	42.37	42.37	42.37	26.71	26.71	26.71
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.18 - FHSVaR 5 Year 0.99 Decay Factor No APC Backtesting results

		Filtered Historical Simulation No APC					
		5 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	10	10	10
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	15.96	15.96	15.96	13.02	13.02	13.02
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	42.37	42.37	42.37	27.35	27.35	27.35
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.19 - FHSVaR 5 Year 0.95 Decay Factor Floor Backtesting results

		Filtered Historical Simulation Floor					
		5 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	16.03	16.03	16.03	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	28.52	28.52	28.52	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.20 - FHSVaR 5 Year 0.97 Decay Factor Floor Backtesting results

		Filtered Historical Simulation Floor					
		5 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	8	8	8	6	6	6
	Breaches Rate	3%	3%	3%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	7.82	7.82	7.82	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.01	0.01	0.01	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.70	26.70	26.70	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.21 - FHSVaR 5 Year 0.99 Decay Factor Floor Backtesting results

		Filtered Historical Simulation Floor					
		5 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	10	10	10	9	9	9
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	13.08	13.08	13.08	10.34	10.34	10.34
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	34.46	34.46	34.46	26.74	26.74	26.74
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.22 - FHSVaR 5 Year 0.95 Decay Factor Stressed Backtesting results

		Filtered Historical Simulation Stressed					
		5 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	9	9	9
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	16.03	16.03	16.03	10.34	10.34	10.34
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	28.52	28.52	28.52	26.74	26.74	26.74
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.23 - FHSVaR 5 Year 0.97 Decay Factor Stressed Backtesting results

		Filtered Historical Simulation Stressed					
		5 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	10	10	10
	Breaches Rate	5%	5%	5%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	19.17	19.17	19.17	13.08	13.08	13.08
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	35.89	35.89	35.89	27.38	27.38	27.38
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.24 - FHSVaR 5 Year 0.99 Decay Factor Stressed Backtesting results

		Filtered Historical Simulation Stressed					
		5 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	10	10	10	10	10	10
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	13.08	13.08	13.08	13.08	13.08	13.08
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	34.46	34.46	34.46	27.38	27.38	27.38
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.25 - FHSVaR 5 Year 0.95 Decay Factor Buffer Backtesting results

		Filtered Historical Simulation Buffer					
		5 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	7	7	7	8	8	8
	Breaches Rate	3%	3%	3%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	5.57	5.57	5.57	7.82	7.82	7.82
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.02	0.02	0.02	0.01	0.01	0.01
	H0 Decision	Reject H0	Reject H0	Fail to Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	12.28	12.28	12.28	26.70	26.70	26.70
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.26 - FHSVaR 5 Year 0.97 Decay Factor Buffer Backtesting results

		Filtered Historical Simulation Buffer					
		5 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	7	7	7	8	8	8
	Breaches Rate	3%	3%	3%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	5.57	5.57	5.57	7.82	7.82	7.82
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.02	0.02	0.02	0.01	0.01	0.01
	H0 Decision	Reject H0	Reject H0	Fail to Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	12.28	12.28	12.28	26.70	26.70	26.70
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.27 - FHSVaR 5 Year 0.99 Decay Factor Buffer Backtesting results

		Filtered Historical Simulation Buffer					
		5 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	10	10	10
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	13.08	13.08	13.08
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	20.15	20.15	20.15	27.38	27.38	27.38
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.28 - FHSVaR 5 Year 0.95 Decay Factor Limit Rates 2 days Backtesting results

		Filtered Historical Simulation Limit Rates 2days					
		5 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	9	9	9
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	16.10	16.10	16.10	10.39	10.39	10.39
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	28.56	28.56	28.56	26.76	26.76	26.76
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.29 - FHSVaR 5 Year 0.97 Decay Factor Limit Rates 2 days Backtesting results

		Filtered Historical Simulation Limit Rates 2days					
		5 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	8	8	8	8	8	8
	Breaches Rate	3%	3%	3%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	7.87	7.87	7.87	7.87	7.87	7.87
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.01	0.01	0.01	0.01	0.01	0.01
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	26.71	26.71	26.71	26.71	26.71	26.71
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.30 - FHSVaR 5 Year 0.99 Decay Factor Limit Rates 2 days Backtesting results

		Filtered Historical Simulation Limit Rates 2days					
		5 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	10	10	10	10	10	10
	Breaches Rate	4%	4%	4%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	13.14	13.14	13.14	13.14	13.14	13.14
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	34.48	34.48	34.48	27.41	27.41	27.41
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

B.1.3. 10 Years Lookback Period

Table B.31 - FHSVaR 10 Year 0.95 Decay Factor No APC Backtesting results

		Filtered Historical Simulation No APC					
		10 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	13	13	13	7	7	7
	Breaches Rate	5%	5%	5%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	22.40	22.40	22.40	5.53	5.53	5.53
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.02	0.02	0.02
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	43.45	43.45	43.45	19.00	19.00	19.00
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.32 - FHSVaR 10 Year 0.97 Decay Factor No APC Backtesting results

		Filtered Historical Simulation No APC					
		10 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	8	8	8
	Breaches Rate	4%	4%	4%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	15.96	15.96	15.96	7.78	7.78	7.78
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.01	0.01	0.01
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	34.89	34.89	34.89	19.27	19.27	19.27
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.33 - FHSVaR 10 Year 0.99 Decay Factor No APC Backtesting results

		Filtered Historical Simulation No APC					
		10 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	6	6	6
	Breaches Rate	5%	5%	5%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	19.09	19.09	19.09	3.58	3.58	3.58
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	42.64	42.64	42.64	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.34 - FHSVaR 10 Year 0.95 Decay Factor Stressed Backtesting results

		Filtered Historical Simulation Stressed					
		10 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	14	14	14	9	9	9
	Breaches Rate	6%	6%	6%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	25.97	25.97	25.97	10.34	10.34	10.34
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	44.74	44.74	44.74	26.74	26.74	26.74
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.35 - FHSVaR 10 Year 0.97 Decay Factor Stressed Backtesting results

		Filtered Historical Simulation Stressed					
		10 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	13	13	13	9	9	9
	Breaches Rate	5%	5%	5%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	22.49	22.49	22.49	10.34	10.34	10.34
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	37.29	37.29	37.29	26.74	26.74	26.74
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.36 - FHSVaR 10 Year 0.99 Decay Factor Stressed Backtesting results

		Filtered Historical Simulation Stressed					
		10 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	12	12	12	10	10	10
	Breaches Rate	5%	5%	5%	4%	4%	4%
Unconditional Coverage	Likelihood Ratio statistic	19.17	19.17	19.17	13.08	13.08	13.08
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0
Conditional Coverage	Likelihood Ratio statistic	35.89	35.89	35.89	27.38	27.38	27.38
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.37 - FHSVaR 10 Year 0.95 Decay Factor Floor Backtesting results

		Filtered Historical Simulation Floor					
		10 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	14.93	14.93	14.93	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.38 - FHSVaR 10 Year 0.97 Decay Factor Floor Backtesting results

		Filtered Historical Simulation Floor					
		10 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	14.93	14.93	14.93	19.48	19.48	19.48
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.39 - FHSVaR 10 Year 0.99 Decay Factor Floor Backtesting results

		Filtered Historical Simulation Floor					
		10 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	20.15	20.15	20.15	11.72	11.72	11.72
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.40 - FHSVaR 10 Year 0.95 Decay Factor Buffer Backtesting results

		Filtered Historical Simulation Buffer					
		10 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	8	8	8	7	7	7
	Breaches Rate	3%	3%	3%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	7.82	7.82	7.82	5.57	5.57	5.57
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.01	0.01	0.01	0.02	0.02	0.02
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	13.38	13.38	13.38	19.01	19.01	19.01
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.41 - FHSVaR 10 Year 0.97 Decay Factor Buffer Backtesting results

		Filtered Historical Simulation Buffer					
		10 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	7	7	7	7	7	7
	Breaches Rate	3%	3%	3%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	5.57	5.57	5.57	5.57	5.57	5.57
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.02	0.02	0.02	0.02	0.02	0.02
	H0 Decision	Reject H0	Reject H0	Fail to Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	12.28	12.28	12.28	19.01	19.01	19.01
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.42 - FHSVaR 10 Year 0.99 Decay Factor Buffer Backtesting results

		Filtered Historical Simulation Buffer					
		10 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	9	9	9	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	10.34	10.34	10.34	3.61	3.61	3.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	20.15	20.15	20.15	11.72	11.72	11.72
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.43 - FHSVaR 10 Year 0.95 Decay Factor Limit Rates 2 days Backtesting results

		Filtered Historical Simulation Limit Rates 2days					
		10 Year Lookback					
		0.95 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	11	11	11	7	7	7
	Breaches Rate	4%	4%	4%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	16.10	16.10	16.10	5.61	5.61	5.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.02	0.02	0.02
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	34.95	34.95	34.95	19.02	19.02	19.02
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.44 - FHSVaR 10 Year 0.97 Decay Factor Limit Rates 2 days Backtesting results

		Filtered Historical Simulation Limit Rates 2days					
		10 Year Lookback					
		0.97 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	10	10	10	7	7	7
	Breaches Rate	4%	4%	4%	3%	3%	3%
Unconditional Coverage	Likelihood Ratio statistic	13.14	13.14	13.14	5.61	5.61	5.61
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.02	0.02	0.02
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	27.41	27.41	27.41	19.02	19.02	19.02
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

Table B.45 - FHSVaR 10 Year 0.99 Decay Factor Limit Rates 2 days Backtesting results

		Filtered Historical Simulation Limit Rates 2days					
		10 Year Lookback					
		0.99 Decay Factor					
		Long			Short		
		90%	95%	99%	90%	95%	99%
	Nº Backtesting Observations	249	249	249	249	249	249
	Expected Breaches	2	2	2	2	2	2
	Actual Breaches	10	10	10	6	6	6
	Breaches Rate	4%	4%	4%	2%	2%	2%
Unconditional Coverage	Likelihood Ratio statistic	13.14	13.14	13.14	3.64	3.64	3.64
	Test Critical Value	2.71	3.84	6.63	2.71	3.84	6.63
	Test P-Value	0.00	0.00	0.00	0.06	0.06	0.06
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Fail to Reject H0	Fail to Reject H0
Conditional Coverage	Likelihood Ratio statistic	34.48	34.48	34.48	11.73	11.73	11.73
	Test Critical Value	4.61	5.99	9.21	4.61	5.99	9.21
	Test P-Value	0.00	0.00	0.00	0.00	0.00	0.00
	H0 Decision	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0	Reject H0

B.2 APC Test Results

B.2.1. 1 Year Lookback Period

Table B.46 - APC Results 1 Year 0.95 decay factor

APC VaR Model	Lookback	Decay	VaR Long Volatility	VaR Short Volatility	Daily Returns Volatility	X2 Long	X2 Short
Limit Rates	1 year	0.95	0.0382	0.0274	0.0212	0.6116	0.2208
Floor			0.0420	0.0247		0.6821	0.5223
Stressed			0.0436	0.0362		0.7610	0.1558
Buffer			0.0794	0.0760		1.4867	0.8124
Non APC			2,0981	1,2798		0	0

Table B.47 - APC Results 1 Year 0.97 decay factor

APC VaR Model	Lookback	Decay	VaR Long Volatility	VaR Short Volatility	Daily Returns Volatility	X2 Long	X2 Short
Limit Rates	1 year	0.97	0.0347	0.0219	0.0212	0.5028	0.1627
Stressed			0.0399	0.0282		0.6788	0.1239
Floor			0.0706	0.0692		1.8457	1.0102
Buffer			0.0779	0.0746		1.7401	0.9378
Non APC			2,1756	1,3104		0	0

Table B.48 - APC Results 1 Year 0.99 decay factor

APC VaR Model	Lookback	Decay	VaR Long Volatility	VaR Short Volatility	Daily Returns Volatility	X2 Long	X2 Short
Limit Rates	1 year	0.99	0.0249	0.0198	0.0212	0.3209	0.1748
Floor			0.0291	0.0197		0.3546	0.4413
Stressed			0.0351	0.0283		0.7723	0.2112
Buffer			0.0755	0.0749		2.4548	1.4745
Non APC			2,1651	1,5788		0	0

B.2.2. 5 Year Lookback Period

Table B.49 - APC Results 5 Year 0.95 decay factor

APC VaR Model	Lookback	Decay	VaR Long Volatility	VaR Short Volatility	Daily Returns Volatility	X2 Long	X2 Short
Limit Rates	5 year	0.95	0.0291	0.0281	0.0212	0.4249	0.2192
Floor			0.0343	0.0245		0.4671	0.5110
Stressed			0.0346	0.0348		0.6478	0.2802
Buffer			0.0759	0.0755		1.1230	0.5571
Non APC			1,6853	1,2571		0	0

Table B.50 - APC Results 5 Year 0.97 decay factor

APC VaR Model	Lookback	Decay	VaR Long Volatility	VaR Short Volatility	Daily Returns Volatility	X2 Long	X2 Short
Limit Rates	5 year	0.97	0.0220	0.0220	0.0212	0.2686	0.1537
Floor			0.0257	0.0205		0.2893	0.3762
Stressed			0.0264	0.0260		0.4326	0.1938
Buffer			0.0745	0.0744		1.3053	0.6255
Non APC			1,6096	1,176		0	0

Table B.51 - APC Results 10 Year 0.99 decay factor

APC VaR Model	Lookback	Decay	VaR Long Volatility	VaR Short Volatility	Daily Returns Volatility	X2 Long	X2 Short
Limit Rates	5 year	0.99	0.0172	0.0147	0.0212	0.1728	0.0513
Stressed			0.0186	0.0143		0.3413	0.1023
Floor			0.0197	0.0128		0.1830	0.1880
Buffer			0.0740	0.0731		1.6910	0.7150
Non APC			1,4894	0,8173		0	0

B.2.3. 10 Year Lookback Period

Table B.52 - APC Results 10 Year 0.95 decay factor

APC VaR Model	Lookback	Decay	VaR Long Volatility	VaR Short Volatility	Daily Returns	X2 Long	X2 Short
Floor	10 year	0.95	0.0248	0.0235	0.0212	0.5651	0.3850
Limit Rates			0.0271	0.0261		0.3473	0.2033
Stressed			0.0337	0.0322		0.3968	0.2713
Buffer			0.0751	0.0746		0.9042	0.5410
Non APC			1,5339	1,2		0	0

Table B.53 - APC Results 10 Year 0.97 decay factor

APC VaR Model	Lookback	Decay	VaR Long Volatility	VaR Short Volatility	Daily Returns	X2 Long	X2 Short
Floor	10 year	0.97	0.0201	0.0186	0.0212	0.3342	0.2527
Limit Rates			0.0203	0.0196		0.2271	0.1320
Stressed			0.0272	0.0240		0.3648	0.1730
Buffer			0.0739	0.0734		1.0725	0.6155
Non APC			1,452	1,0923		0	0

Table B.54 - APC Results 10 Year 0.99 decay factor

APC VaR Model	Lookback	Decay	VaR Long Volatility	VaR Short Volatility	Daily Returns	X2 Long	X2 Short
Limit Rates	10 year	0.99	0.0147	0.0131	0.0212	0.1129	0.0476
Stressed			0.0153	0.0140		0.2492	0.1222
Floor			0.0156	0.0124		0.1438	0.1042
Buffer			0.0732	0.0726		1.3712	0.7702
Non APC			1,1524	0,765		0	0

C. Appendix C

C.1 Correlation Matrix Results

The table below describes the correlation coefficients for the IM models 1st differences series tested against the E Mini S&P 500 bid-ask spread 1st differences series.

Table C.1 - Correlation Coefficients

Model	Spread Series Correlation Coefficient
FHSVaR_95decay_Buffer_10yearslookback	0,021
FHSVaR_95decay_Buffer_1yearslookback	0,03847
FHSVaR_95decay_Buffer_5yearslookback	0,01711
FHSVaR_95decay_Floor_10yearslookback	0,15534
FHSVaR_95decay_Floor_1yearslookback	0,14583
FHSVaR_95decay_Floor_1yearslookback_new	0,14583
FHSVaR_95decay_Floor_5yearslookback	0,10809
FHSVaR_95decay_Stressed_10yearslookback	0,10275
FHSVaR_95decay_Stressed_1yearslookback	0,13956
FHSVaR_95decay_Stressed_5yearslookback	0,10953
FHSVaR_97decay_Buffer_10yearslookback	0,00962
FHSVaR_97decay_Buffer_1yearslookback	0,03602
FHSVaR_97decay_Buffer_5yearslookback	0,01499
FHSVaR_97decay_Floor_10yearslookback	0,13791
FHSVaR_97decay_Floor_1yearslookback	0,14176
FHSVaR_97decay_Floor_5yearslookback	0,14084
FHSVaR_97decay_Stressed_10yearslookback	0,11913
FHSVaR_97decay_Stressed_1yearslookback	0,15753
FHSVaR_97decay_Stressed_5yearslookback	0,12964
FHSVaR_99decay_Buffer_10yearslookback	-0,006283
FHSVaR_99decay_Buffer_1yearslookback	0,00763
FHSVaR_99decay_Buffer_5yearslookback	0,00521
FHSVaR_99decay_Floor_10yearslookback	0,14064
FHSVaR_99decay_Floor_1yearslookback	0,1579
FHSVaR_99decay_Floor_5yearslookback	0,16621
FHSVaR_99decay_Stressed_10yearslookback	0,1108
FHSVaR_99decay_Stressed_1yearslookback	0,10318
FHSVaR_99decay_Stressed_5yearslookback	0,14659

C.2 Granger Causality Results

The table below details the results of the Granger causality test. This test was performed on the spread series and model series with second absolute differences computed (2 differences computed to ensure stationarity of all series). The Granger causality null hypothesis assumes that the past lags of the IM models do not cause the bid-ask spread series. The test null hypothesis is rejected for most of the IM Models tested below.

Table C.2 - Granger Causality Results

IM Model	Spread Series
FHSVaR_95decay_Buffer_10yearslookback_y	0,0974
FHSVaR_95decay_Buffer_1yearslookback_y	0,0333
FHSVaR_95decay_Buffer_5yearslookback_y	0,0765
FHSVaR_95decay_Floor_10yearslookback_y	0
FHSVaR_95decay_Floor_1yearslookback_y	0
FHSVaR_95decay_Floor_1yearslookback_new_y	0
FHSVaR_95decay_Floor_5yearslookback_y	0
FHSVaR_95decay_Stressed_10yearslookback_y	0
FHSVaR_95decay_Stressed_1yearslookback_y	0
FHSVaR_95decay_Stressed_5yearslookback_y	0
FHSVaR_97decay_Buffer_10yearslookback_y	0,3637
FHSVaR_97decay_Buffer_1yearslookback_y	0,1072
FHSVaR_97decay_Buffer_5yearslookback_y	0,3107
FHSVaR_97decay_Floor_10yearslookback_y	0
FHSVaR_97decay_Floor_1yearslookback_y	0,1049
FHSVaR_97decay_Floor_5yearslookback_y	0
FHSVaR_97decay_Stressed_10yearslookback_y	0
FHSVaR_97decay_Stressed_1yearslookback_y	0
FHSVaR_97decay_Stressed_5yearslookback_y	0
FHSVaR_99decay_Buffer_10yearslookback_y	0,5083
FHSVaR_99decay_Buffer_1yearslookback_y	0,3933
FHSVaR_99decay_Buffer_5yearslookback_y	0,6307
FHSVaR_99decay_Floor_10yearslookback_y	0
FHSVaR_99decay_Floor_1yearslookback_y	0
FHSVaR_99decay_Floor_5yearslookback_y	0
FHSVaR_99decay_Stressed_10yearslookback_y	0
FHSVaR_99decay_Stressed_1yearslookback_y	0
FHSVaR_99decay_Stressed_5yearslookback_y	0