ISCTE Business School
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THE NONLINEAR FISCAL MULTIPLIER: EVIDENCE FROM PORTUGAL

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"The best way to learn is to teach"
by Frank Oppenheimer

Abstract

The Nonlinear Fiscal Multiplier

by Telma B. Gonçalves

Since the monetarist-Keynesian fiscal policy debate of the late 1960's, researchers have faced an endless discussion about the role that this policy should play in the economy. Recent evidence points to a heterogeneous fiscal multiplier rather than a homogeneous one, that is, a country-specific and state-dependent multiplier. Moreover, a series of recent papers defend that the fiscal multiplier is higher in "recessionary periods" when compared to "normal times". Previously used models, such as *DSGE* or *VAR*, are not capable of capturing the kind of dynamics that we observe in the implementation of modern fiscal policy. This thesis aims to analyse and model nonlinearities for the Portuguese fiscal multiplier. Results suggest that symmetric government spending shocks have asymmetric effects on real GDP depending on the business cycle position, sign and the magnitude of the shock. In order to pursue this analysis, a *Logistic STVAR* model was applied to the Portuguese data and the transition dynamics were reported with Generalized Impulse Response Functions.

Key words: Nonlinear Time Series, Smooth Transition Vector Autoregressive Model - *STVAR*, Fiscal Multiplier.

JEL - Codes: C32, C51, E62

Resumo

Multiplicadores Fiscais Não-lineares
por Telma B. Gonçalves

Desde o debate monetaristas-Keynesianos no final dos anos 60 sobre política orçamental, os investigadores têm enfrentado uma discussão interminável sobre o papel que esta política deve desenvolver na economia. Recentes evidências direcionam para um multiplicador orçamental heterogéneo, em vez de um homogéneo, ou seja, um multiplicador que seja específico ao país e dependente do regime económico. Além disso, um grupo de recentes artigos defendem que o multiplicador orçamental é maior em "períodos de recessão" quando comparado com "períodos normais". Modelos utilizados antigamente, tais como modelos *DSGE* ou *VAR*, não estão construídos para capturar as dinâmicas que observamos na implementação da política fiscal moderna. Assim, esta tese tem por finalidade analisar a existência de não-linearidades no multiplicador orçamental português. Os resultados obtidos sugerem que um choque simétrico nas despesas orçamentais do governo tem efeitos assimétricos no PIB real dependentes da posição do ciclo económico, sinal e magnitude do choque. De modo a prosseguir esta análise, foi aplicado um modelo Logístico *STVAR* a uma base de dados portuguesa e as transições dinâmicas foram reportadas com Funções de Impulso Resposta Generalizadas.

Palavras-chave: Séries Temporais Não-lineares, Modelo Vector Auto-regressivo de Transição Suave - *STVAR*, Multiplicador Orçamental.

JEL - Códigos: C32, C51, E62

To my family and friends

Accomplishing the Master degree in Economics is the first step of a life project I aim to accomplish in order to follow my dream of working as a professional economist. As the life project that it is, this first accomplishment brings a wonderful feeling of achievement which I owe to my dear family and friends.

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Dealing with research, for the first time in my life, made me realize all the challenges and successes that the process implies. Understanding some crucial points of papers was one of the most frustrating exercises, yet rewarding experiences, due to the lack of transparency. Reproducing results and studying routines to be able to understand the robustness of these papers results was a challenge. Therefore I want to remark my appreciation to Saki Bigio who made available a *STVAR* toolbox. I also deeply appreciate the help and guidance of Enrique Moral-Benito who supported me by making his results highly transparent and to be willing to discuss details and economic intuition. In addition, I would like to thank Julia Schmidt, Tolga Omay, Cátia Sousa and Ricardo Nunes for responding to my requests.

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Above all, I wish to express my deep gratitude to my family. To my daughter and husband - Helena and Rafael Romana - who supported me unconditionally, in good and bad times, despite their own sacrifice. Isn't it the best having a psychologist in the family, Rafael?

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Contents

1 Introduction

Times change, so do academic debates. When I think of my undergraduate studies back in 2009, I was amazed with the financial crisis and the recent attempts of policy makers to stabilize the economy.

In most newspapers one could read headlines suggesting the return of the Keynesian theory and the most recent understanding-the-crisis books quickly flew off the shelves. [1]

Although it might seem surprising, I believe it is not wrong to say that the academic world paid little attention to fiscal policy. Despite the nature of the work – abstract, concrete, theoretical or empirical – it seemed that academics were amazed by the monetary policy research, leaving the responsibility of implementing fiscal policy entirely to politicians.

It was only after 2010, and after the fiscal stimulus packages applied by the US government, that the debate around fiscal policy and the efficiency of the fiscal multiplier gained a renewed attention. [2]

The monetarist-Keynesian debate around the efficiency of fiscal policy and the crowding-out effect of the fiscal multiplier is, perhaps, one of the oldest macroeconomic debates. Therefore, why should this topic be revisited in 2014?

In the first place despite the oldness of the fiscal multiplier concept, it seems that new empirical evidence have emerged, highlighting the concept of nonlinear fiscal multipliers. This new research trend, began in 2011 with

[1] The Economist has an entry from October 2009 with the upcoming books about Keyne's comeback: "Keynes: The Twentieth Century's Most Influential Economist", "Keynes: The Return of the Master", "The Keynes Solution: The Path to Global Economic Prosperity".

[2] For a literature with more details about the stimulus packages, see Taylor [2011].

Auerbach and Gorodnichenko's [2012a] paper (henceforth AG12) where the authors provide evidence of a heterogeneous fiscal multiplier. By using a nonlinear regime switching model, the authors showed how symmetric fiscal shocks have asymmetric effects on the real log gdp for the U.S. economy, being larger during recession periods when compared to normal times.

In the second place, the recent European debt crisis and the austerity programs which followed, have shown how important it is for the academic research community to be updated in order to face these new challenges.

In the third place, in my opinion, standard macroeconomic models have three main problems. Firstly, models assume that the economy is represented by only one representative agent, thus everyone takes exactly the same decisions; secondly, it is currently assumed that all agents have rational expectations because they are fully informed agents; and finally, most macroeconomic models describe the economic structure through linear methods, when researchers should, at least, be testing for nonlinear dynamics. This new research line gives an opportunity to consider nonlinearities through regime switching models, by testing for heterogeneous fiscal multipliers.

This thesis aims to contribute to the Portuguese fiscal multiplier literature by exploring this new heterogeneous approach, together with its nonlinear methodology. Given the extensive list of alternatives to deal with nonlinearities, it was chosen the $LSTVAR$ model which is the most commonly method found in this literature.

It is also important to highlight that this thesis is not based on only one paper's contribution, quite the opposite it presents a deep study of two complementary areas, economics and econometrics. In sum, this thesis was economically motivated by AG12's work, but it is based on a diverse source of econometric procedures. The MatLab packages are based on Bigio and Salas' [2006] paper, the model estimation process follows Weise's [1999] paper, while the specification and evaluation stage plus the generalized impulse response functions (henceforth GIRF) are strongly based on Teräsvirta and Yang's [2014a, 2014b] work.

Briefly this thesis is presented in four main steps. Firstly, it presents a short literature review of the twentieth century macroeconomic debate, homogeneous and more recently heterogeneous fiscal multipliers. Secondly, after collecting a data sample from 1960 to 2012 of macroeconomic and fiscal variables for the Portuguese economy, the series are tested for nonlinear evidence and an exploratory analysis is performed. Thirdly, it presents the statistical framework of the $LSTVAR$ model through two stages specification and evaluation, concluding with the estimation process. Finally, results are reported by using several multipliers' concepts while the transition dynamics are reported by using GIRF for three main models, a benchmark plus two robustness check models, which together with the standard linear versions completes a set of 48 models' estimations.

Results seem to suggest that the Portuguese government spending shocks might have stronger effects on output during recession times when compared to expansion periods. Multipliers higher than 1 were found during recession periods while during expansions the multipliers were never higher than 0.4. Results also seem to suggest a counter-cyclical nature of the Portuguese fiscal policy as positive shocks seem to have a stronger effect during recession periods, while negative shocks have a stronger effect during expansion periods. Finally, asymmetries were also found on the shocks' magnitude since doubling the shock does not double the effect because the shifting probabilities are not fixed.

This thesis is structured in 4 sections. Section 2 presents a literature review of the topic. Section 3 presents the model, its econometric specification, estimation and evaluation as well as the exploratory analysis of the empirical data. Section 4 presents the main empirical results and a robustness analysis. Finally, section 5 concludes.

2 The Fiscal Multiplier: A Short Review

The discussion around the magnitude of the fiscal multiplier lead us to the origins of the twentieth century's debate, between the two main schools of economic thought, Neoclassical and Keynesian. These two main schools discussed the role of fiscal policy mainly based on one important assumption: the existence, or lack, of Ricardian families.

In the Keynesian framework, a higher level of government spending or a decrease in the level of taxes should boost aggregate demand. According to the traditional IS-LM model, aggregate demand increases by shifting off the IS curve, making the magnitude of the fiscal multiplier be dependent on how much the IS shifts and, also, on the slope of the LM curve. If we are in the presence of a liquidity trap, which happens when the nominal interest rates are zero or close to zero, the LM curve will be relatively flat which produces a positive fiscal multiplier since it offsets the crowding-out effects.[3] In terms of the "Keynesian Cross Diagram", if the interest rates are held constant, the multiplier for government spending is given by expression (1) and the taxes multiplier by expression (2):

$$\frac{1}{(1-mpc)} \tag{1}$$

$$\frac{-mpc}{(1-mpc)}. \tag{2}$$

where mpc stands for the marginal propensity to consume.

In the light of the counter-cyclical and temporary fiscal policy, if the

[3]This can also be the case when the Central Bank performs an aggressive monetary policy to keep the interest rate fixed, independently of the demand level.

economy suffered from a decline in the level of aggregate demand, caused, for example, by a decline in investment, this decline could be offset by a temporary increase in government purchases, a taxes cut or an increase in transfer payments. Thus the increase in government spending, augmented by possible multiplier effects, fills the "gap" left by the decline in investment. A tax or transfer policy would affect directly disposable personal income and thereby stimulate consumption through a wealth effect. Naturally, the temporary nature of this transmission effect raises doubts about the efficiency of the fiscal stimulus (Hillier [1991]).

Finally, according to the *Mundell-Fleming* ([1960, 1963, 1962]) model, under fixed exchange rate regime, an expansionary fiscal policy would increase the interest rates above the international level, which would cause a pressure on the national currency. To maintain the exchange rate and eliminate the pressure, the Central Bank would increase the money supply through purchasing foreign currency with local currency, until the exchange rate is back to its equilibrium level. Under a flexible exchange rate regime the magnitude of the fiscal multiplier is traditionally lower than no-flexible exchange rate. Thus, in the Keynesian framework the effectiveness of the fiscal policy is determined by the role of the monetary policy. If monetary policy is ineffective (liquidity traps) or if it is dedicated to goals such as keeping the interest rates or exchange rates fixed, an expansionary fiscal policy will produce positive fiscal multipliers (Cos and Moral-Benito [2013]).

Notwithstanding, in the General Theory, Keynes [1936] defended a temporarily counter-cyclical fiscal policy. In order to minimize the crowding-out effect, the government should only put in practice an expansionary fiscal policy in deep recessions, that is, when the economy faces idle resources, when the level of unemployment is persistently high (above what is now termed the "NAIRU" level), when the economy is working with less than full capacity or when there are market failures. The fiscal stimulus would act through two main channels: in the short run would stimulate wealth and expectations, or optimism; in the long run gross domestic product would rise, raising the amount of savings, thus helping the level of investment. Finally, the idea of a more persistent fiscal stimulus was only defended in order to provide public goods due to market failures, since Keynes supported that public and private spending should be seen as complementary instead of substitutes. In sum, one can observe, already in 1936, the defence of a positive and heterogeneous fiscal multiplier that would mainly be effective under deep recessions rather than expansions.

Naturally, from a purely theoretical point of view, the existence and magnitude of the fiscal multiplier is directly dependent on the model's assumptions. The Keynesian framework assumes that the government expenditure is able to increase the capital level, and if one considers investment in public goods as a response to market failures, instead of state failures, the fiscal multiplier is necessarily positive and probably significant, that is, higher than one.

On the other hand, the Neoclassical school defends a different approach since it is based on two main assumptions that lead to a lower level for fiscal multipliers. One of the most influential authors in this field is Barro [1974], who questioned the assumption that people in the economy would treat government bonds as net wealth, pointing out, instead, that if agents are forward-looking they know that the government will have to use future tax revenues to make principal and interest payments on these bonds. If agents have this knowledge they will expect an increase in future taxes, lowering their assessments of their wealth. In sum, the wealth effect associated with government bonds will be offset. The author showed analytically that, under the assumptions of the Ramsey growth model (infinitely lived households and consumption smoothing), consumers will respond to a change in government spending in the same way whether it is financed by an increase in current taxes or by borrowing and running a deficit (i.e., by future taxes). Today, we know this result as the Ricardian Equivalence Theorem.

Nevertheless, these conclusions, either theoretical or empirical, are strictly dependent on the researcher's approach. Differences in the assumptions of the model about the nature of the government spending, the source of financing expenditure and price frictions, different parameters calibrations and persistence of shocks, lead to a diverse set of conclusions about the nature and magnitude of the fiscal multiplier.[4] For instance, both Keynesian and Neoclassical models imply a positive output multiplier in response to an increase in government spending, what differentiates these models is the magnitude of that response, and the response of the GDP components, such as the controversial effect on consumption multiplier, or even the disagreement over the transmission channels. Traditionally, models under the Neoclassical approach will produce a negative wealth effect which is translated into a negative response of consumption, while models under a Keynesian vision will produce a positive consumption multiplier.

2.1 The Homogenous Multiplier

So far, many researchers from both schools of thoughts contributed to this literature. Until very recently, it was defended one homogeneous fiscal multiplier that would be true, independently of the state of the business cycle. The following short review will be presented in separate contributions: theoretical and empirical. The papers and results presented in the theoretical sections emphasize the economic modelization, while the papers presented in the empirical sections emphasize the econometric estimation.

2.1.1 Theoretical Contributions

One of the main contributions to this discussion is Baxter and King's [1993] paper, where the authors used a standard one-sector growth model with variable labor and endogenous capital accumulation to answer several questions in four topics. In each topic, the authors performed two types of analyses, a comparative steady-state analysis for long-run effects and a transition dynamics analysis near the initial steady state for short-run effects.

In a first step, they studied if permanent changes in the government spending would have macroeconomic effects. The authors found that if this

[4] More on this topic can be found on Leeper *et al.* [2012] paper. Through a Bayesian approach these authors analyzed a wide range of DSGE models under several assumptions and proved that the prior distributions of the models impose a very tight range for the multiplier, even before the models are even taken to data.

increase in the government expenditure is financed by lump-sum taxation, the long-run output multiplier will be greater than 1, and moreover, if labor supply is highly elastic, the same conclusion is true for the short-run effect.

In the second place, the temporary output effects of changes in government purchases are lower when compared to permanent effects.

In the third place, the authors found that the financing decision is determinant on the effect of the changes in government purchases and much more important than the direct resource cost of government purchases. When government purchases are financed by general income taxes, a negative effect on output occurs. When they are financed by lump-sum taxation, resources are removed from the economy without altering the equilibrium prices. However, government investment produces a resource benefit through the increase in private factors' productivity, high enough to offset the implicit short-term consumption cost, thus producing a much stronger impact on the economy than a pure rise in government purchases.

Finally, two new assumptions have been introduced to check the robustness of the results. Considering that government purchases augment the productivity of private capital and labor, thus altering private marginal products, public investment policies can have "dramatic effects on output and private investment".

In sum, one can conclude that through the dynamic interaction of labor and capital supplies, the economy might have large multiplier effects of permanent government purchases on output in both the short and long run. The former effect can be explained as a result of a version of the investment accelerator, in which the higher demand for capital stock has an immediate effect on the labor input. The latter effect can be explained as a result of the effects of higher long-run labor input on the steady-state capital stock.

Fiscal policy received little empirical attention compared to monetary policy during the following years, nevertheless the work of Fatás and Mihov [2001] regained some of the most important lines of research during this period. Taking into consideration the Real Business Cycles' results, which imply a negative correlation between consumption and hours worked, the authors studied the empirical effects on consumption and employment as response to a positive government spending shock. The authors' main goal was to confront the mainstream theoretical view against their own empirical results. The authors showed that with lump-sum taxes or when the government expenditures were debt-financed, that the spending stimulus would be expansionary which implies a positive (output) multiplier. The level of consumption decreased regardless the assumptions made.

What differs from the theoretical model are the empirical results concerning the composition of this expansionary effect on output, especially the prediction of a negative response of consumption to government expenditure shocks, due to negative wealth effects. Another example concerns the theoretical classic negative correlation between consumption and employment, which empirically presents a positive conditional correlation in response to an increase in government expenditures. After an increase in government spending, all components of consumption show a persistent rise, while investment also increases during the first 6 quarters. The same trend is registered for employment, which increases for the whole economy, put forth by manufacturing employment. In sum, the authors defend that the consumption dynamics produced by Real Business Cycle model remains a research challenge, considering the empirical results, and one possible way to increase the quality of the model's predictions could be the approximation to reality through, for example, the "inclusion of liquidity constraints, finite horizons or some sort of myopic behavior by consumers".

The fact, dynamic stochastic general equilibrium models (henceforth *DSGE)* show limitations in the reproduction of realistic consumption dynamics as a response to a government spending shock motivated the work of Coenen and Straub [2005], who revisited the effects of government spending shocks on private consumption with a New-Keynesian *DSGE* model with non-Ricardian families. These authors introduced two main features to the model. In the first place, they allow for a passive fiscal policy rule, which stabilizes the intertemporal budget by using distortionary taxes as well as lump-sum taxes/transfers, therefore the model features four types of economic agents: the fiscal and monetary authorities, households and firms. Secondly, they introduced heterogeneous households which can be fully optimizing or liquidity constrained. The latter are assumed to be restricted, so they cannot participate in the asset markets and thereby consume all their disposable income (after tax). The former ones, since they have access to asset markets are able to smooth consumption.

Coenen and Straub found that the existence of non-Ricardian households, above a 35% percentage share, raised the level of consumption response to government spending shocks, when compared to the alternative standard specifications without heterogeneous agents. Nevertheless, due to the highly persistent nature of government spending shocks, the chances that the government spending shocks do actually crowd in consumption were very low and thereby did not generate a sufficient positive fiscal output multiplier. This main result lead the authors to conclude that in order to augment the crowding-in effects, the model needs to be improved in order to reduce the negative wealth effect produced by this highly persistent nature of government spending shocks.

To overcome this limitation Galí *et al.* [2007] build a New Keynesian model with sticky prices, were half percent of the households followed a "rule-of-thumb". This condition provides the major result of the paper: the authors were able to generate a positive response of consumption to a rise in government spending which overcomes the empirical gap seen in the literature up to that point. As Coenen and Straub [2005] argued, in order to achieve a better model, researchers needed to create a mechanism which would contribute to decrease the negative wealth effects. By introducing "rule-of-thumb" consumers in the model, Galí *et al.* were able to partially decrease the negative wealth effect since these agents are more sensitive to the current disposable income than to its present value. However, sticky prices are the necessary condition that allows to offset the negative wealth effect, which makes possible for real wages to increase even in the presence of a decrease in the marginal product of labor. Therefore, with higher real wages and higher employment, current labor income increases just enough to stimulate the consumption of "rule-of-thumb" households. In sum, the authors were able to solve a theoretical conflict regarding the response of consumption to a positive government spending shock, thus producing a response of output systematically above the one generated by the Neoclassical model.

With the main theoretical gap filled up, recent research focused mainly on the topics related to the financial crisis. One recent example is the work of Christiano *et al.* [2011]. These authors developed a *DSGE* model where the monetary policy follows the standard Taylor rule. In this model, when the economy is hit by a government spending shock, the nominal interest rate increases as a response to the expansionary policy, thus reducing the size and magnitude of the fiscal multiplier, which is generally less than one. Their main contribution concerns the results of the zero-lower-bound modelling case, for which the authors found a much higher fiscal multipliers when the nominal interest rate does not respond to an increase in government spending. The timing is also important, large effects on current output will only happen as long as the government spending shock occurs under a zero nominal interest rate scenario, otherwise the current effect of government spending is small. The introduction of capital accumulation into the model, despite decreasing the probability that the zero bound becomes binding, produces a much stronger effect, thus increasing the size of the government-spending multiplier.

Using a different approach, Corsetti *et al.* [2012] focused their work on the recently observed "spending reversals", that is, after a positive spending shock that triggers the level of public debt, over time government spending tend to decline below its long-term trend. These authors confront their empirical results, obtained through a *VAR* model, with the theoretical results from a two-country New-Keynesian *DSGE* model. Under the former model, Corsetti *et al.* achieve standard empirical results: the output response to a government spending shock is big on impact; although private consumption presents a similar pattern as the output response, the crowding-in effect is low; the interest rate increases substantially on impact but its mid-term behavior rapidly drops after two quarters, finally, the public debt shows a persistent and increasing dynamics reaching its peak after 5 years. The latter model, confronts two regimes, the reversal and the non-reversal case. As for the reversal case, it produces a positive initial response both in consumption and output. Moreover, the authors demonstrate that the intertemporal substitution through expectations is the most important driver of the transmission channel. Considering equation (3) from Corsetti's paper

$$\frac{C}{C_t} = \lim_{T\to\infty} \Pi_{s=0}^{T} E_t[\beta^s \frac{1+i_{t+s}}{\Pi_{t+1+s}}] \tag{3}$$

if one observes a significant fall in inflation (Π_{t+1+s}) after a negative spending shock, this will decrease the short-term interest rates (i_{t+s}) and if agents have rational expectations (E_t) they will anticipate a decline in future short-term interest rates which increases the value of present consumption (C_t), thus leading to an expansionary effect on consumption and output in the reversal scenario.[5] In sum, it is important to highlight that the effects of a short-term fiscal stimulus depend on the particular policy measure, expectations about future fiscal policy and the role of monetary policy. Finally, the authors defend that a crowding-in effect on consumption is mainly driven by intertemporal substitution if, and only if, the expansionary shock is strong enough to raise expectations of future interest rate cuts.

To finish, is important to mention that despite the effort to improve theoretical models in order to embrace empirical results, it seems that there is little consensus regarding policy advising. The main results, as mentioned before, depend on the nature of assumptions, calibrations and policy specifications. As Leeper *et al.* [2012] have stated, the model specification imposes "*a ve*ry tight range for the multiplier before the models are taken to data". Nevertheless, it seems that the effort to build models closer to reality marked the crucial role that assumptions, such as nominal rigidities (sticky prices and wages), consumers following a "rule of thumb" or even open-economy specifications, have play in recent modelling strategies. While open-economy specifications contribute to reduce the magnitude of the fiscal multipliers, due to the substitution effect between internal and external consumption, assumptions regarding rigidities imply that, in order to have positive long-run multipliers, a New Keynesian approach is essential. As Hall [2009] demonstrated in his survey, through the analysis of government purchases alone, a

[5] β^s stands for the discount factor while C stands for steady-state value.

Neoclassical general-equilibrium generates positive output multipliers at the cost of negative consumption responses. Thus, to achieve efficacy from the government stimulus one needs to incorporate traditional Keynesian features such as price and wage stickiness, where the former can be interpreted as a negative response of the mark-up after an increase in output, and the latter can be viewed as the necessary level of elasticity of labor supply.

2.1.2 Empirical Contributions

In the light of the theoretical results above described, many authors extended their contributions to the literature through empirical applications reaching the most diverse results. One of the most quoted papers belongs to Blanchard and Perotti [2002] who analysed a structural characterization of the dynamic effects of government spending and taxes' shocks on the US economy and did so, using institutional information about the tax and transfer systems and the timing of tax collections to identify endogenous responses to exogenous fiscal shocks. The authors' main goal focused on the analysis of the output and its components response to a tax and spending shocks. Two specifications were used, a deterministic trend and a stochastic trend approach notwithstanding, the results are very similar and can be summarized in the following way. In the first place, the authors presented a standard result, after an increase in taxes all private components of GDP decrease, while a spending shock produces the opposite result. Secondly, private consumption is consistently crowded out by an increase in taxes, and crowded in by a government spending shock, while investment presents a less standard dynamics, were both fiscal shocks contribute to its crowding-out effect. This result is less standard if we consider the Keynesian approach where the magnitude of the investment response depends, for instance, on the response of the interest rate.

Perotti [2005] extended Blanchard and Perotti's structural model to study the effects of the fiscal multiplier for both spending and taxes shocks, for a group of five countries under two different sample periods, pre and post-1980 periods. His main result concerns a small government spending multiplier effect on output for most countries, sufficiently positive multipliers were only achieved for the US and Germany in the pre-1980 period. The author also expanded the analysis with simulation of the effects on the interest rate and inflation. Reported results, under plausible values of price elasticity of government spending, pointed to small government spending effects on inflation and positive effects on the long-run interest rates in the post-1980 periods. Another important result concerns the difference between the two period samples. The government spending effects have strong impacts in the pre-1980 period where GDP and private consumption presents a positive response, while investment reports a low response and the real interest rate falls or presents a small increase. Considering the post-1980 period the author finds evidence of a negative response from private consumption and investment followed by an increase in the real interest rate. Finally, the author defends that both effects on GDP and its components as a response to both fiscal shocks decreased over time, mainly due to a change in the transmission mechanism, which also contributed to the observed decline of GDP variance in the post-1980 period.

Further empirical developments focused on alternatives for the identification method such as the narrative approach as in Romer and Romer [2010] who studied the effects of tax multipliers on the economic activity and defended that an exogenous tax increase of one percent of GDP contributes to a decrease of the real GDP by approximately three percent. Further extensions also include panel data analyse e.g. Ilzetzki's *et al.* [2010] work, which state that future fiscal multipliers will tend to zero or even negative values due to country's high debt ratios. Among the authors' main conclusions, their most robust result concerns the fact that government consumption (or investment) multipliers are country dependent. The multipliers not only differ with country characteristics but those characteristics are emphasized along with time: short-run multipliers reported small values or close to zero while medium-run and long-run vary. For instance, closed economies, fixed exchange rates, lower level of debts contributed to big fiscal multipliers, while open economies, flexible exchange rate's regimes (closely related to the degree of response to shocks of monetary policy) or indebtedness states provided small multipliers and crowding-out effects.

Similar results, concerning the heterogeneity and country dependent multipliers, can be found in Favero *et al.* [2011] who criticized the role of empirical research upon homogeneous fiscal multipliers and defended that the effect of fiscal multipliers differs across debt dynamics, degree of openness and fiscal reaction functions. Moreover, Hall [2009, :29] highlighted the importance of the literature concerning the zero lower bound as a main source of nonlinearities, and we quote*:*

> *"[multipliers] are not fundamental structural parameters of the economy, invariant to the state of the economy. Quite the contrary, the multipliers are themselves endogenous. The state of the economy in 2009 is a perfect example. With extreme slack in the economy and the federal funds rate at essentially zero, there are good reasons to believe that the government purchases multipliers are higher than in normal times ... [rising] to 1.7 when monetary policy becomes passive with a zero nominal interest rate".*

Their arguments will make the crossing point to our next discussion concerning the research evolution towards heterogeneous multipliers.

Table 1: Homogeneous Contributions - part I

Author	Data	Model	Consumption Effect	Output Effect
Baxter and King (1993)	Postwar U.S data	DSGE	(-)0.2 Long-run	1.16 Long-run
			Considering permanent changes in government purchases, financed by lump-sum taxes	
Fatás and Mihov (2001)	US data	DSGE and VAR	is negative according to RBC	positive if debt or lump-sum financed
			positive according to VAR	negative in presence of distortionary taxes
Coenen and Straub (2005)	Euro Area (1980-1999)	DSGE	Low probability that government spending shocks crowd in consumption due to: (i) the small share of non-Ricardian households and (ii) the highly persistent nature of government spending shocks.	Positive but not significant (smaller than 1)
Galí, López-Salido and Vallés (2007)	U.S. quarterly data (1954:I–2003:IV)	DSGE and SVAR	Goes from 0.17 (on impact) to 0.95 (end of the second year)	Goes from 0.78 (on impact) to 1.74 (end of the second year)
			requires 'rule of thumb' consumers and sticky prices	
Christiano, Eichenbaum and Rebelo (2011)	U.S. data (1960:I-2010:I)	DSGE (ZLB)	The multiplier is larger in economies where the interest rate does not respond. Multipliers between 1.6 with a peak of 2.3 (after 5 periods) were found.	

Table 2: Homogeneous Contributions - part II

<table>
<tr><th>Author</th><th>Data</th><th>Model</th><th>Consumption Effect</th><th>Output Effect</th></tr>
<tr><td>Corsetti, Meier and Müller (2012)</td><td>U.S. data (1983–2007)</td><td>DSGE and VAR</td><td>positive</td><td>positive and significant</td></tr>
<tr><td>Leeper, Richter and Walker (2012)</td><td>Bayesian priors</td><td>Bayesian approach of several DSGE models</td><td colspan="2">Prior distributions influence the probabilities of results before the models are taken to data. Lon-run positive multipliers are achieved with nominal rigidities and non-savers agents. The largest multipliers were found for a scenario of a closed economy with non-savers households with a range from 0.84 to 1.75.</td></tr>
<tr><td rowspan="3">Blanchard and Perotti (2002)</td><td rowspan="3">US data (1947:I-1997:IV)</td><td rowspan="3">SVAR</td><td>Tax Shock: from -0.4 to -0.18</td><td>Tax Shock: from -1.33 to -0.42</td></tr>
<tr><td>Spending Shock: from 0.33 to 1.26</td><td>Spending Shock: from 0.55 to 1.29</td></tr>
<tr><td colspan="2">Results depend on the assumptions about trends</td></tr>
<tr><td>Perotti (2005)</td><td>OECD countries (US, West Germany, UK, Canada, and Australia)</td><td>SVAR</td><td>Positive for all countries: less significant in the post-1980 period</td><td>In general higher than 1 for most countries but less significant in the post-1980 period</td></tr>
<tr><td>Romer and Romer (2010)</td><td>Postwar US data (1945:I-2007:IV)</td><td>Narrative approach</td><td colspan="2">The baseline specification shows that an exogenous tax increase of 1% of GDP decreases real GDP by 3%. Consumption showed a similar contractionary response.</td></tr>
</table>

2.2 The Heterogeneous Multiplier

In alignment with the recent idea of a heterogeneous state-dependent fiscal multiplier, a new approach started to emerge from the criticisms of the limited role that a homogeneous multiplier would play in policy advising activities. Moreover a new question started to be seen as a major ingredient in a new research trend in macroeconomics: is the fiscal multiplier, despite being heterogeneous, dependent upon the business cycle position of the economic activity?

This idea has a Keynesian inspiration related to the counter-cyclical effects of fiscal policy and follows the suggestion that one should study the effects of an expansionary fiscal policy mainly during recessionary periods. Naturally this has a disadvantage, not only because researchers have less available data about deep recessions, but also because most of the works in economics are done under a linear approach, which is much less expensive in terms of computational and numerical methods. Parker [2011] was one of the first authors who highlighted this discussion, where he presents the reasons for the lack of knowledge and a possible methodology for improving the estimate of the efficacy of fiscal policy in recessions. He pointed out two main drawbacks of the methodologies that are currently used to estimate the fiscal multiplier. In the first place, a *methodological* problem, in which both the *DSGE* and *VAR* models do not highlight the importance of the fiscal multiplier in recessions. Under this procedure, the fiscal multiplier is as effective in recessions as in expansions, because these methods do not allow for state dependence. The second problem concerns the *measurement error*, because even a researcher who seeks to calculate the fiscal multiplier in recessions is, in fact, calculating the marginal and not the total multiplier, which will lead to biased results about the magnitude of the fiscal multiplier in response to a shock. In sum, Parker [2011] defends that researchers should invest their time on estimating the multiplier in a recessionary regime:

> *"We do not have a good measure of the effects of fiscal policy in a recession because the methods that we use to estimate the effects of fiscal policy (...) almost entirely ignore the state of*

the economy and estimate 'the' government multiplier, which is presumably a weighted average of the one we care about — the multiplier in a recession — and one we care less about — the multiplier in an expansion."

This section will also present papers which emphasize the economic modelization in the theoretical contributions, while papers which emphasize the econometric estimation will be presented in the empirical contributions.

2.2.1 Theoretical Contributions

As presented, this new line of research rests upon (old) Keynesian views, when it defends that during a recession period the market fails and these failures can be (partially or fully) corrected by a temporary and expansionary fiscal policy, which will bring idle resources back to work. Usually, this would not happen during expansion periods, since fiscal policy would mainly raise price levels and interest rates, which would in turn crowd out private consumption and investment, as the Neoclassical models have systematically pointed out. Parker [2011] also explained how difficult it is to believe that a positive fiscal shock produces the same symmetric response on the economy as a negative fiscal shock, when the economy presents completely different characteristics affecting employment rates and capacity of utilization, respectively, low in recessions and high in expansions.[6] As Ramey [2011] stated, the key question in recent research is not how big fiscal multipliers are when the economy is in the steady state, but how big they are when it starts with idle resources. Nevertheless, to the best of our knowledge, few theoretical examples of this recent literature can be found yet.

Castro *et al.* [2013] used a *DSGE* model in a small open European economy, obtaining different results for "normal times" versus "crisis times" after

[6] For instance, Parker [2011] identifies the literature of fiscal policy at the zero lower bound as a nonlinear example, which it is not emphasize here because although the multipliers are dependent, they depend upon the response of the interest rate and not on the global business cycle position. Nevertheless, besides the partial nonlinearity present in these models, it is important to say that multipliers larger than 3 were found under these specific circumstances.

a negative fiscal shock. This work gains relative importance since it studies specifically the Portuguese fiscal multiplier during a period of crisis and budget consolidation, where it is assumed that the government implements a credible fiscal consolidation, implying a permanent decrease in the debt-to-gdp ratio in the long run. The authors' "crisis time" definition concerns a significant set of calibrations which are representative of the "Great Recession Period". Their main result suggests that the fiscal multipliers in the short run are state dependent and that during the crisis times they equal approximately 2 for a consumption-based fiscal consolidation.

In order to defend a state dependent fiscal multiplier the authors explain how during expansions the effect of an expansionary policy is more likely to be absorbed in nominal terms, while in recessions its effects are of a real nature. The main argument is based on the existence of frictions in wages and prices in general. There are less frictions in a positive adjustment of wages, so in expansions the fiscal multiplier is observed by the nominal increase in the price of the labor market. On the other hand, wages present some frictions when they have to adjust downwards, so there is a real effect on the adjustment process upon the level of employment.

The authors generalized this argument for the whole economy, considering four main assumptions in their model (i) non-Ricardian agents (ii) strong nominal rigidities (iii) hand-to-mouth households and (iv) financial frictions. Thereby, suggesting not only a state-dependent fiscal multiplier but also a sufficiently positive one, since it nearly doubles in crisis times for expenditure-based fiscal consolidation and increases between 30 to 60 percent for revenue-based fiscal consolidations. In sum, the effects are large under an expenditure-based multiplier because it affects directly the level of GDP, while the revenue-based affects, in the first place, the available income and wealth and only later on it will indirectly affect the GDP level. Finally, the effects are also more persistent during crisis times since the nominal rigidities are high enough to create real rigidities. While their work is focused on negative short run effects to achieve a credible fiscal consolidation, Almeida *et al.* [2011] highlighted the trade off between the short-run costs — with losses in output, consumption and welfare — and the long-run benefits with significant increases in the level of output, consumption and welfare improvements.

More recently, Michaillat [2014] adapted a search and matching framework to a New Keynesian model and defended that the government multiplier doubles when the unemployment rate rises from 5 to 8 percentage points. However, this government multiplier only concerns the government spending on public-sector workforce, naming it by "public-employment multiplier", which will provide the information about how many workers are employed per one worker employed in the public sector. His main argument concerns the defense of a higher efficiency of government hiring during recession times, since the crowding-out effect is lower than in expansions. The author explains how public employment, by the process of increasing the tightness in the labor market and so raising the cost of hiring for the private sector, in order to be efficient it needs the presence of a sufficiently high unemployment rate. In sum, the multiplier is 0.45 at a normal unemployment rate of 5.8 percent and reaches 0.71 when the unemployment rate reaches 8 percent. The results highlight two important conclusions. In the first place, the author reinforces how state-dependent the government multipliers seem to be, which implies that studying an average multiplier seems of little relevance in terms of policy implications. In the second place, the author also finds a significant heterogeneity in different types of spending multipliers, which suggests the need for a closer look at this issue when it comes to policy making.

2.2.2 Empirical Contributions

One of the first authors who addressed this topic with an empirical approach was Auerbach and Gorodnichenko [2012a]. These authors, by defending the importance of studying the magnitude of the fiscal multiplier in recessions, provided two main results which started a new research trend. In the first place, they implemented a regime-switching model ($STVAR$) to study the magnitude of the fiscal multiplier, concluding that although the multipliers are similar in terms of impact (either in recessions and expansions), the impulse responses imply different cumulative multipliers that go from 0 to 0.5 in expansions and from 1 to 1.5 in recessions for the US economy, over a significant sample period between 1950 and 2010.

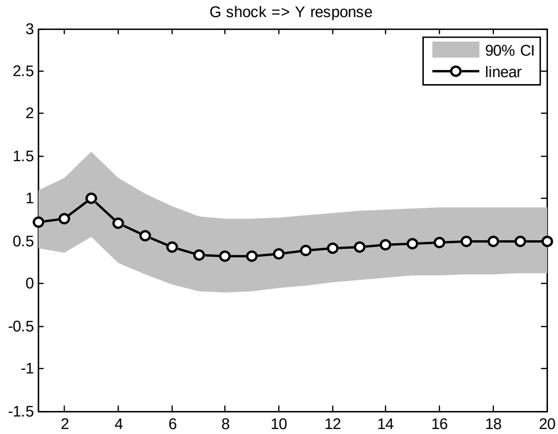

Figure 1: Linear output response after a positive government spending shock. AG12's results.

In order to understand the importance of their results, see Figures (1) and (2) which represent, respectively, the linear and nonlinear response of output to a positive expenditure government shock. Looking at Figure (2), we observe a set of rich and complex dynamics that was not observable in the previous (linear) case. The response of output is state-dependent, similar in the expansionary regime (red line) but it converges to 3 in the recessionary regime (blue line). Thus, one may conclude that fiscal policy has stronger effects during recessionary regimes when compared to expansion periods.[7]

Secondly, the authors introduced expectations to their model, showing that by controlling for predictable components, the fiscal shocks tend to have a higher magnitude in recessions than in expansions. In sum, this paper by showing how different the dynamics of macroeconomic variables are under a linear and nonlinear models, showed the importance of this line of

[7]Figure 1 and 2 were simulated by using the codes of AG12 available on the site of American Economic Association. These results are stronger than the ones presented on their conclusions because it was considered exogenous shifting probabilities, instead of endogenous, to highlight the differences between regimes. This assumption implies that a government shock is not able to change the regime of the economy.

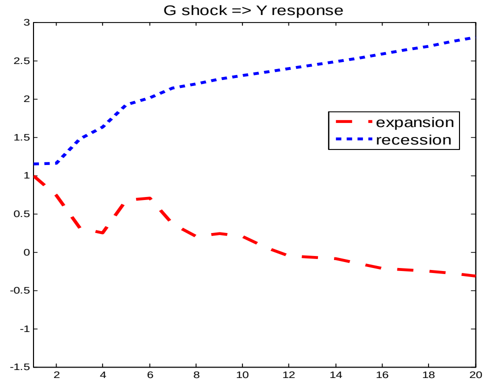

Figure 2: Nonlinear output response after a positive government spending shock. AG12's results.

research for short run analysis and policy making. Thereby, together with the discussion paper of Parker [2011] and these new results of AG12, a literature that studies the heterogeneity, state dependency and country specific fiscal multipliers were reborn.

In another work [2012b] the same two authors used a direct projection method to estimate the multipliers for a panel of countries concluding that government spending shocks crowd out private consumption and private investment in expansions but stimulate them in recessions. Also, they reported a negative response of exports and a positive response of imports which are marginally positive in the recessionary regime. Along with these results they also estimated an "employment multiplier", results of which are comparable to the ones presented previously by Michaillat [2014]. In this case, the authors estimated how much government spending is able to enlarge the level of employment, concluding that it takes about 23 thousand dollars to create a job in a recession. Naturally, the authors found that the effects under an expansionary regime were not significant. The results reported for the labor market wages are also consistent with Castro *et al.*'s [2013] assumptions about real and nominal rigidities, since the authors defend that a positive expenditure shock is absorbed by a higher employment level in recessions and by a higher labor wage in expansions. Another important issue to highlight, specifically for the case of Portugal, concerns the output response to the government debt: higher government debt leads to a smaller response in output. In sum, this paper strengthens the recent conclusions that spending multipliers vary across the business cycle, thus in order to achieve a higher efficiency, policy makers should take into account that the fiscal multiplier seems to be much larger in recessions than in expansions. Moreover, the trade-off effects of this policy, like the crowding-out effect of private investment or the inflationary pressure, are also less likely to happen under deep recessionary regimes than in expansion times.

Further evidence for the US economy can be found, for example, in the work of Mittnik and Semmler [2012], who estimated a *MRVAR* with two regimes. The authors found clear evidence that the "timing" is crucial for policy makers, since there are significant differences in the responses of output and employment to demand and labor supply shocks. A positive demand shock, in a below average growth regime, produces a multiplier effect on output and on employment growth that is higher when compared to the situation of a high-growth regime.

Because the recent financial crisis is a complex phenomenon, Cos and Moral-Benito [2013] disaggregated the concept of crisis to match the problems of the Spanish economy. By using a *STVAR* analysis similar to AG12, they began by analyzing, in the first place, the traditional expansion/recession dichotomy, reaching standard conclusions — that is the spending multiplier is larger during recessions but irrelevant during expansions — as well as a low crowding-out effect for private consumption and investment during recessions. Secondly, as part of the disaggregate analysis, they analysed two different fiscal scenarios: "good" and "bad". One important debate concerns the role of expectations when there is a weak public finance situation, which is also a characterization of the Portuguese financial case.

Is it possible that an expansionary fiscal multiplier during bad fiscal times contributes to change the sign and magnitude of the fiscal multiplier? The answer is yes. The authors found out, that under a bad financial situation, the effect of government spending shock becomes close to zero, which is in line with the results previously defended by Corsetti *et al.* [2012]. Moreover, the authors also provided results for the dichotomy of banking stress versus no banking stress, where the spending multipliers are found to be large during periods of banking stress but smaller than 1 for government spending. Finally, after studying all three situations separately, the authors combine them to create a proxy for turbulent times, concluding that despite the finance situation contribution to decreasing the cumulative spending multiplier, it is still sufficiently positive in turbulent times.

As a result of the possible benefits for policy advising, several researchers are now studying their own country specific fiscal multipliers and obtaining standard results, with the output response to a government spending shock being higher during recessions than in expansions, as well as the crowding-out effect of private consumption or investment being less likely to happen during recession periods. Rafiq and Zeufack [2012] provided empirical evidence for Malaysia state-dependent fiscal multiplier. Monokroussos and Thomakos [2012] provided evidence for Greece in order to understand the potential output losses due to the austerity program the Greek economy is under since 2010. Herbert [2014], by following the AG12 approach, reproduced their results for the US but also reported results for Italy, Germany and France. This former author found a strong evidence that the spending multiplier was strong in downturns for the US and France, the results had a low magnitude for Germany and were found inconclusive for Italy.[8]

These new results raised some criticisms from, for instance, Ramey [2013a]. In these papers Ramey constructs two main criticism regarding the way how AG12 reported their results, which also applies to the majority of papers previously presented. In the first place, she considers that the traditional way of measuring the fiscal multiplier presents a bias since the ratio of Y/G varies greatly over the sample period. Thus, she proposes a method to convert the variables of interest to the same unit by applying instead:

$$ln(Y_{t+h}) - ln(Y_{t-1})$$

[8] To better understand the Italian case and the reasons behind the inconclusive state-dependent multipliers, see the work of Caprioli and Momigliano [2013].

and

$$\frac{ln(G_{t+h}) - ln(G_{t-1})}{Y_{t-1}}$$

which can be transformed into:

$$ln(G_{t+h}) - ln(G_{t-1})\frac{G_{t-1}}{Y_{t-1}}$$

This method, developed by Ramey, gives directly the dollar changes measure, presenting the ratio of G/Y in each point of time, rather than an average of the entire sample.

Secondly, Ramey criticizes the method used to report the impulse response functions suggesting the Jòrda Method or the Generalized IRF instead.[9] Her main critique relies upon the fact that AG12 assumed fixed transition probabilities, which means that neither a positive nor a negative shock can alter the state under which the economy is in, at moment t. Naturally, such an assumption not only creates an inadequate duration for the average recession periods but also implies that a positive shock in a recessionary period, despite all the economic positive effects defended in their paper, would not be enough to get the economy out of a recession, which is inconsistent with the paper's main policy advise goal. In sum, Ramey [2013b] by using a threshold model with Jordà's local projection technique, found that the US multipliers, contrary to what was defended by AG12, were not state dependent, not even considering a zero-lower-bound hypothesis. Notwithstanding, state-dependent multipliers were reported for the Canadian economy.

Some extensions to more specific topics can already be seen, for instance in the work of Bachmann and Sims [2012]. The authors tried to understand the impact of confidence on the magnitude of the fiscal multiplier and measured the importance of a systematic response of confidence to spending shocks for the spending multiplier. In order to do so, they used three types of models, *DSGE*, *VAR* and *STVAR* to confront the results and perform a robustness analysis. They incorporated "confidence" into the model through an univariate $AR(1)$ process with innovation ($conf_t$) equal to its lag value

[9] More detailed information about Generalized Impulse Response Functions see Koop *et al.* [1996] and Pesaran and Shin [1998].

($conf_{t-1}$), a linear combination of the two structural disturbances ($e_{a,t}$ and $e_{g,t}$) of the model and its own noise term ($e_{c,t}$):

$$conf_t = \rho_t conf_{t-1} + \phi_1 e_{a,t} + \phi_2 e_{g,t} + e_{c,t}$$

Considering the nonlinear model (*STVAR*), the authors found stronger responses of output and confidence to spending shocks in recessionary periods than in normal times. These differences, though, are higher in the long run than in the short run. Notwithstanding, the results suggest that the transmission of spending shocks during recessions might be related to an increase in future productivity rather than to confidence *per se.* This link is explained by the increase of the investment-consumption-expenditure ratio after a spending shock, which remains permanently higher during a recession, hence suggesting long-term consequences for the composition of government spending. In sum, their confidence concept does not seem to be the main driver of the transmission of spending shocks in a recession, rather it seems only to reflect information about future productivity gains that will be a result of the spending multiplier side effect.

Due to the importance of the previous findings, several researchers tried to test and model nonlinearities for their own country hence producing evidence for several countries such as US, Canada, France, Germany, Italy, Malaysia, Greece and recently Japan (Auerbach and Gorodnichenko [2014]), nevertheless empirical evidence for the Portuguese economy seems to be lacking in the literature. To the best of our knowledge, the closest approach can be found on the work of Almeida *et al. [2011]* and Castro *et al.* [2013]. These two papers are focused on a negative shock alone, which is called "expansionary fiscal consolidation", and defend state-dependent effects on the Portuguese output and its components. They also marked a new improvement in theoretical models on the path to approximate the modelling procedure to recent empirical results. However, and here it lies the limitation of *DSGE* models for regime shifting analysis, if it had been considered a positive shock instead of a negative one, their model would produce symmetric results, higher multipliers in "crisis times" than in "normal times" because, as Parker [2011, :6] states:

> *"A linearized solution to a DSGE model implies that the partial derivative of output or consumption (or their present discounted values) with respect to government spending or taxes is the same following a large positive shock to the economy as following a large negative shock to the economy".*

In sum, it seems that there is no empirical evidence for the nonlinear Portuguese fiscal multiplier, therefore no asymmetric effects were tested so far. By testing how does the Portuguese economy reacts to positive and negative fiscal shocks under recessions and expansions, we aim to contribute to decrease this gap in the literature and provide, within our own limitations, a deeper knowledge about the Portuguese economy.

Table 3: Heterogeneous Contributions - part I

Author	Data	Model	Consumption Effect	Output Effect
Castro, Félix, Júlio and Maria (2013)	Portuguese calibration	PESSOA: Normal times	(1) -1.3, (2) -2.3, (3) -1.0 and (4) -1.2	(1) -1.2, (2) -0.7, (3) -0.5 and (4) -0.5
		Crisis Times	(1) -1.7, (2) -2.6, (3) -1.2 and (4) -1.5	(1) -1.7, (2) -1.1, (3) -0.6 and (4) -0.7
			Macroeconomic Impacts on the first year with Financial Frictions: (1) Decrease in government consumption (2) Decrease in lump-sum transfers (3) Increase in labor taxes (4) Increase in consumption tax rate. Results from Table A.2 and B.2	
Almeida, Castro, Félix and Maria (2013)	Portuguese calibration	PESSOA	Negative short term effects after a negative fiscal shock. Positive long-run benefits which implies an "expansionary fiscal consolidation".	
Michaillat (2014)	US calibration	DSGE: normal times and recessions	The public-employment multiplier depends on the position of the business cycle, specifically on the unemployment rate: (i) with a normal unemployment rate of 5.8% the multiplier is 0.45 but (ii) in recessions, with a 8% unemployment rate, the multiplier reaches 0.71.	
Auerbach and Gorodnichenko (2012a)	US data	LSTVAR: normal times and recessions	In both regimes, after a positive fiscal shock, the multipliers are similar on impact but very different in dynamics: the cumulative output multiplier goes from 0 to 0.5 in expansions and from 1 to 1.5 in recessions.	
Auerbach and Gorodnichenko (2012b)	OECD panel data	LSTVAR: expansion	-0.07	0.04
		recession	0.8	0.68
			Maximum response to an unanticipated one percent government spending shock	
Pablo de Cos and Moral-Benito (2013)	Spanish data	STVAR		
		Tranquil Times vs Turbulent		0.79 vs 4.21
		Expansion vs Recession		1.68 vs 2.41
		Good Finance situation vs Bad Finance situation		2.29 vs 0.04 (-0.26 on impact)
		No banking stress vs Banking Stress		0.16 vs 2.27
			Reported results for a government spending shock. Peak multiplier for Output Gap, Deficit-to-gdp ratio, private credit flow and PCA Index I as switching variables.	
Rafiq and Zeufack (2012)	Quarterly dataset from Malaysia	STVAR: higher periods of economic growth	1.4	2
		Lower periodos of economic growth	1.2	2.7
			Maximum median multipliers for government investment spending shocks	

Table 4: Heterogeneous Contributions - part II

Author	Data	Model	Consumption Effect	Output Effect
Monokroussos and Thomakos (2012)	Greek data	SVAR and STVAR: expansion recession	-0.88 1.44	-1.42 1.32
			Reported results for avererage multiplier after a government spending shock	
Herbert (2014)	US, France, Italy and Germany	STVAR Expansion Recession	(1) 1.02 (2) 0.5 (3) 0.503 (1) 3.5 (2) -0.01 (3) 0.89	(1) 0.6 (2) 0.39 (3) 0.76 (1) 2.22 (2) 3.27 (3) 2.08
			Countries: (1) US, (2) France, (3) Germany *Inconclusive results for Italy	
Mittnik and Semmler (2012)	US data	MRVAR: high regime for above average growth low regime for below average growth	Long term effects of 1.1% on output and 0.7% on employment growth Long term effects of 1.3% on output growth and 1.8% on employment growth	
			One percent output shock	
Owyang, Ramey and Zubairy (2013)	US and Canadian	Threshold model: low unemployment rate hight unemployment rate	U.S: (1) 0.72 (2) 0.88, Canada: (1) 0.44 (2) 0.46 U.S: (1) 0.76 (2) 0.78, Canada: (1) 1.60 (2) 1.16	
			Output multipliers results for (1) 2 year integral and for (2) 4 year integral	
Sims (2012)	US data	STVAR (recessions versus normal times), VAR and DSGE	The spending multiplier is larger in recessions (in the neighborhood of 3) than in normal times, because confidence rises in response to unexpected increases in government spending during recessions. However what drives the IRF in recessions is not confidence per se but the effect on future productivity.	
Caprioli and Momigliano (2013)	Italian data	SVAR, ETVAR and STVAR (Recessions and expansions)	Empirical evidence showed weak evidence of a clear expenditure multiplier higher in recessions than in expansions. The results of ETVAR and STVAR do not present a robust result.	

2.3 Concluding Remarks

Through all the papers previously analysed, it is possible to highlight five main conclusions. Firstly, the discussed evidence tends to favor the predictions of traditional (old) Keynesian views over the Neoclassical models, since none of the empirical evidence seem to support a negative comovement between the response of output and consumption after a positive government spending shock.

Secondly, evidence seem to point to the hypothesis that linear models might be outdated for policy making, raising space to consider nonlinear analysis and models instead. When the economy is in a recession, it faces lack of resources, low total productivity and low confidence, high rigidities in the labor market since the demand for labor tends to decrease but the wages are not frictions-free in the adjustment process, which in turn increases the unemployment level. This scenario contrasts in every sense from what is considered by the authors as "normal times" and "expansion periods". In sum, as learned from Castro *et al.* [2013], existent rigidities in normal times are transformed into nominal effects, while in recession periods, due to the market frictions, they are transformed into real effects. Thus, it seems to be well grounded that symmetric shocks have asymmetric effects depending on the fundamentals of the economy, for instance the business cycle position for advanced countries and the rate of growth for developing countries.

Thirdly, it seems that there is no consensus as for the size or the magnitude of the fiscal or public-employment multiplier and therefore no consensus on the role of the fiscal (or employment) policy as an efficient instrument. Even if one considers that researchers accept nonlinear elasticities, assuming that the multiplier is always higher during recessions than in expansions, is also a source of disagreement. The selected model, the way researchers report their results or even the transformation process from elasticities to multipliers constitute points of disagreement among researchers. Nevertheless, results seem to point to higher multipliers in the short-run and medium-run during recessionary periods when compared to normal times.

A more difficult point to formulate, which itself deserves a separate research, concerns the transmission channel of the fiscal shock. What exactly drives the fiscal shock to increasing output during recession periods that does not happen in normal times? As we learned from Bachmann and Sims [2012], one can think about different transmission channels as, for example, the idea that a fiscal shock may increase the perception of commitment to aggregate stability which in turn increases optimism and stimulates demand. Nevertheless, they also defended that although confidence has a main role to play in recessions, which is not present during normal times, it seems that productivity is the main driver for the transmission channel and not confidence per se. Another relevant issue raised by Ramey [2011] concerns the impact that this transmission channel reveals in terms of welfare effects. Such analysis is strictly related to a deeper study of the mechanisms of transmission as well as the possibility of including government consumption into the individual utility function.

Finally, the last point of disagreement concerns the crowding-out effects of private investment and consumption, and the long term effects on welfare and sustainability of debt-to-gdp ratio. This is the point where it seems that most researchers fail to find not only a consensus but also an empirical trend. Most papers, previously discussed, point out to a crowding-out effect on investment and consumption lower during recessionary periods than in expansions. The articles addressing the sustainability of debt-to-gdp ratio usually conclude that countries, which have higher ratios, usually present lower fiscal multipliers when compared to the ones who do not (Cos and Moral-Benito [2013]). These articles also conclude that the size of the output multiplier under recessions is crucial to succeed with an expansionary fiscal consolidation and to avoid a self-destructive austerity (Castro *et al.* [2013]).

3 Econometric Specification

There are several methods to detect, explore and analyse nonlinear features in economic time series, regarding for instance, threshold, smooth transition and Markov switching models.[10] In this thesis the main approach is made by using a *LSTVAR* model which is an intermediate method that balances the efficiency of the model with the complexity of the computer demand.

The choice of this model is suggested by the recent scientific literature, however, it is important to note that our goal does not necessary imply to search for the best nonlinear model but, instead, implies to check if this recent econometric approach, reflects the Portuguese economic data characteristics. In order to pursuit such goal a complex analysis procedure was taken. All computational issues were implemented in MatLab and the main toolbox considered was SSTVAR written by Bigio and Salas [2006] and adapted for the Portuguese data by myself. Nevertheless, throughout several months other toolboxes were collected and analysed. For instance, that is the case of AG12 Bayesian package, LeSage [2009] and Julia Schmidt's [2013] econometric toolboxes. Different software were also considered at an initial stage, but MatLab revealed to have the most flexible options when compared to Eviews, STATA or even R.

In order to capture the nonlinear feature of the time series and to employ the nonlinear dynamic model, several steps were conducted. After collecting the data base, an exploratory analysis of the time series was realized. At the next step, several unit root tests were applied in order to decide about

[10] The former assumes a known threshold caused by an observable variable, while the latter assumes a unknown threshold which depends on shifting probabilities across states that are triggered by unobservable variables - hidden Markov models.

stationarity/nonstationarity of the data. Cointegration tests were also applied for nonstationary data with the purpose to decide between a *VAR* and a *VEC* model. Once the final form of the series of interest were chosen we embraced the main methodology of the *LSTVAR* model, that consist in a three-stage procedure: specification, estimation and evaluation. These stages were build to test and model nonlinearities in the data. Each stage, which will be explained in more detail further in this section, included a battery of tests which were separately collected and adapted to our software language.

After a quite consistent study related to the best way to report our model's results, we chose to report our empirical results by using several multipliers' concepts while the transition dynamics are reported by using GIRF, which is an alternative method to the orthogonalized IRF (henceforth OIRF). Three main models were estimated, a benchmark plus two robustness check models (model II and III), which together with the standard linear versions completes a set of 48 models' estimations.

In sum, it is reasonable to say that this thesis was economically motivated by AG12's work, but it is based on a diverse source of econometric procedures. The software package is based on Bigio and Salas' [2006] paper, the model estimation process follows Weise's [1999] paper, while the specification and evaluation stage plus the GIRF are strongly based on Teräsvirta and Yang's [2014a] work.

3.1 Exploratory Data Analysis

In this section it is presented the primary process that justifies the choice of the model further presented. It will also be discussed some exploratory methods that can be helpful to construct hypothesis about the time series for future tests.

In a first step it was collected quarterly data from, mainly, OECD from 1960 to 2012 with the intention of creating three models, one baseline model plus two disaggregated approaches. The baseline model would produce information regarding the output response to a fiscal shock and would consist on: $y_t = [G_t, T_t, Y_t]'$, with two fiscal variables: Real Government Final

Consumption Expenditure (henceforth G) and Total Tax Revenue (henceforth T), and one macroeconomic variable, the Real Gross Domestic Product (henceforth Y). The first disaggregate approach, would analyse in more detail the difference of the output response to a government consumption shock versus a government investment shock. The model would consist on $y_t = [Cgov_t, Igov_t, T_t, Y_t]'$, with two new fiscal variables and data collected of General Government Final Consumption Expenditure (henceforth $Cgov$) and Gross Fixed Capital Formation (henceforth $Igov$). The second disaggregate approach, would analyse the private consumption and investment multiplier to a fiscal shock: $y_t = [G_t, T_t, C_t, I_t]'$, where C is Private Consumption Expenditure and I is Gross Fixed Capital Formation.

More details about the series can be found in the Appendix A (table 12 and 13).[11]

The initial exploratory analysis of the series is based on basic descriptive statistics and normality tests. The original time series, the associated statistical values and the histograms are presented in figures 12, 14, 16, 18, 20, 22 and 24, Appendix B. We can observe that, with a *p-values* of 0.00 for the Jarque-Bera test on all variables, we reject the null hypothesis of normality for any significance level.[12] The value of the skewness for the variables, G, Y, C, I and $Cgov$, is moderate positive, so the distribution is asymmetric, right skewed, and platykurtic. The variable T presents very high curtose (leptokurtic) and the variable $Igov$ is slightly left skewed.

Weisberg [1985] recommends a logarithmic transformation of any variable, when the ratio between the minimum and the maximum values exceeds 10, thus all the variables were transformed into their logarithmic form henceforth. Moreover, Juselius and Hendry [2000] defended that if a set of series is cointegrated in levels, they will also be cointegrated in log levels.[13]

The value of the skewness for all variables, G, T, Y, C, I, $Cgov$ and $Igov$,

[11] The Total Taxes Revenue, General Government Final Consumption Expenditure and Gross Fixed Capital Formation were extracted in annual levels and adjusted with spline interpolation to simulate quarterly data.

[12] The original series can be found on figures 11, 13, 15, 17, 19, 21, 23.

[13] The logarithmic transformation of the original series can be found on figures 25, 27, 29, 31, 33, 35, 37.

in their logarithmic form is negative so the distribution is asymmetric, left skewed, and platykurtic because the kurtosis presents values lower than 3 (the graphical representation of these time series and the associated descriptive statistics can be observed in Appendix B).

3.2 Stationarity Analysis for Main Variables

In this section, the stationarity hypothesis will be analysed for the considered data. As it is well known, most of statistical inference procedures, in the classical linear regression models, implicitly assume that variables are weakly stationary, which implies constant mean, variance and autocovariances.

In what follows we formally define a weakly stationary process according to Green [2003]:

Definition 1 *A stochastic process x_t is weakly stationary or covariance stationary if it satisfies three main requirements. 1. $E[x_t]$ is independent of t, 2. $Var[x_t]$ is a finite, positive constant and also independent of t, 3 $Cov[x_t, x_s]$ is a finite function of $|t-s|$, but not of t or s, that is, the covariance between observations in the series is a function only of how far apart they are in time, not the time at which they occur.*[14]

For instance, for the $AR(1)$ process (4),

$$x_t = \mu + \gamma_1 x_{t-1} + \varepsilon_t \tag{4}$$

stationarity requires that $|\gamma_1| < 1$. The existence of a unit root, that is, $\gamma_1 = 1$, conduct to a nonstationary process. For $|\gamma_1| > 1$, we are in the presence of an explosive process, which has no economic/econometric meaning.

For the more general case,

$$x_t = \mu + \sum_{i=1}^{p} \gamma_i x_{t-i} + \varepsilon_t$$

[14] "Strong stationarity requires that the joint distribution of all sets of observations $(yt, yt-1, ...)$ be invariant to when the observations are made. For practical purposes in econometrics, this statement is a theoretical fine point."

an autogressive model of order p,it is stationary if the roots of the polynomial

$$z^p - \sum_{i=1}^{p} \gamma_i z^{p-i}$$

lie within the unit circle, that is, each z_i satisfy $|z_i| < 1$. Thus, testing the stationarity hypothesis imply testing the existence of a unit root for the autoregressive process.

Regarding our data, several unit root tests were applied in order to decide about stationarity, namely: the Augmented Dickey Fuller (ADF), the Phillips Perron (PP) and the Kwiatkowski Phillips Schmidt Shin (KPSS) (Greene [2003]). The first two tests consider the hypothesis of unit root under the null, while KPSS considers stationarity (no unit root) under the null hypothesis. Since one of the problems with the ADF test is the lack of valid conclusions when the data presents nonlinearities or structural breaks, it was applied the PP test to overcome this problem, which, according to Popp [2008] is a good test in the presence of a structural break.

Table 5: Stationarity analysis of the main variables in the logarithmic form

Variables	**ADF**	**PP**	**KPSS**
Log G	0.0262**	0***	1.808755***
Log T	0.2438	0.8957	0.395373***
Log Y	0.0054***	0.0004***	1.790703***
Log C	0.0513*	0.0134**	1.765281***
Log I	0.2205	0.1554	1.693525***
Log Cgov	0.9936	1	0.356932***
Log Igov	0.0147**	0.0039***	1.237354***

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

According to: (i) the graphs in Appendix B (figures 25, 27, 29, 31, 33, 35, 37) (ii) the associated t-statistics, (iii) the non-zero values of the means in the histograms (figures 26, 28,30, 32, 34, 36 and 38) and (iv) the values of the information criterion tests Akaike, Schwarz and Hannan-Quinn (not shown in the appendix to save space) it was chosen to run the unit root tests, for series G, Y, C, I, $Igov$ with a constant (the significance of the intercept in the linear trend model was rejected) and for series T and $Cgov$ with a constant and deterministic trend. All variables, with exception of G, Y and $Igov$, were found nonstationary in log levels for a 5% significance level. Table (5) shows the *p-values* for the ADF and PP test and the t-statistic value for the KPSS test.[15]

Table 6: Stationarity analysis of the main variables in the first difference of the logarithmic form

Variables	**ADF**	**PP**	**KPSS**
Dif(2) Log T	0.0001***	0.0008***	0.159069
Dif(1)Log C	0***	0***	0.065896
Dif(1)Log I	0.0004***	0***	0.092684
Dif(1) Log Cgov	0.0641*	0.0641*	0.074863

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

Moreover, Table (6) also shows that C, I and $Cgov$ are integrated of order one ($I(1)$), while T is integrated of order two ($I(2)$).[16]

Since most of the time series presents unit root, the next step will consist on testing the existence of linear combinations between the variables.[17] Such

[15] For more detailed information see Appendix B, from table (16) to table (36)

[16] For more detailed information on stationary tests see Appendix B, from table (37) to table (48).

[17] Recently I have come across with a literature of nonlinear cointegration testing such as Choi and Saikkonen [2010]. Unfortunately, due to time constraints, this type of cointegration is not tested here but it will be an important step for future research.

Table 7: Cointegration Analysis

Variables	ADF	PP	KPSS
Log T and *Log C*	0.073*	0.3846	0.1372
Log T and *Log I*	0.1384	0.3846	0.331266
Log C and *Log I*	0.0278**	0.037**	0.215113
Log Cgov and *Log T*	0.032**	0.0851*	0.391756*

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

a linear combination defines a cointegrating equation which characterizes a long-run relationship between the variables. It was chosen to apply the Engle-Granger cointegration methodology, under the hypothesis of stochastic trend, whose results are summarized in Table (7).[18]

Once again, the results shown in the table concern the *p-values* associated with ADF and PP unit root tests and the t-statistics for the KPSS test. Not all the test results were consistent, so the residual's graphs were determinant on the conclusions.[19] In sum, only the residuals resulting from the linear combination between Private Consumption Expenditure and Gross Fixed Capital Formation were found to be stationary, thus representing a cointegration relationship on the long run.[20]

3.3 Stationarity Analysis for Transition Variables

This section deals with the exploratory analysis of the transition variable z_t . The choice of Z is based on Cos and Moral-Benito's paper [2013], therefore the data was collected from the real GDP Growth Rate (henceforth $z1$), the Output Gap (henceforth $z2$) and the Change of the Unemployment Rate (henceforth $z3$).[21] According to Hubrich and Teräsvirta [2013] these variables

[18]For more detailed information see Appendix B, from table (58) to table (69).

[19]For more detailed information see Appendix B from fig.(45) to fig.(52).

[20]All outputs that are not shown on the appendix, to save space, are available upon request for tfbgs@iscte.pt.

[21]All transition variables are transformations from the original data (OECD and AMECO). For more detailed information see Appendix A (table 13).

should be selected from a set of stationary variables to ensure the choice of a consistent transition variable.

The initial exploratory analysis it is mainly based on the information presented in the histograms illustrated in Figures 40, 42 and 44 (Appendix B). The *p-values* of 0.17($z1$), 0.00($z2$) and 0.00($z3$) for the Jarque-Bera test, indicates a no-rejection of the null hypothesis of normality for a 5% significance level for $z1$. The variables $z2$ and $z3$ are leptokurtic.

In the next step, the stationarity hypothesis is analysed. To test the unit root hypothesis the same battery of tests was run (similarly to the analysis in the above section). According to (i) the graphs showed in Appendix B (figures 39, 41 and 43), (ii) the values of the means in the histograms (figures 40, 42 and 44) and (iii) the values of the information criteria (Akaike, Schwarz and Hannan-Quinn - not shown here to save space), it was chosen to run the unit root tests without intercept or linear trend.

Table 8: Stationarity analysis of the transition variables

Variables	**ADF**	**PP**	**KPSS**
Z1	0***	0***	0.062184
Z2	0***	0***	0.019927
Z3	0.0063***	0.0895*	0.173504**

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

From the information presented in the Table (8) all variables were found to be stationary in levels for at least a 10% significance level.[22]

In sum, since a $LSTVAR$ model requires all variables to be stationary and integrated of the same order, a significant set of information will be lost in the modelling process since it is not possible to build the initial planned

[22] For more detailed information see Appendix B, from table (49) to table (53).

disaggregated models (were we have variables with different order of integration). Only the baseline model, with one fiscal and two macro variables, $y_t = [G_t, Y_t, Z_t]'$, will be considered in what follows.

3.4 The Statistical Framework

One of the most important questions concern the choice of a nonlinear model over the traditional linear vector autoregressive model with k lags ($VAR(k)$) which is defined as:

$$y_t = A_1' y_{t-1} + A_2' y_{t-2} + ... + A_k' y_{t-k} + \Phi' d_t + \varepsilon_t \tag{5}$$

where p is the number of endogenous variables and q the number of exogenous variables, that is, y_t is a $(p \times 1)$ vector of dependent variables and each A_i, for $i = 1, ..., k$ is a $(p \times \ p)$ matrix. Furthermore, d_t is a $(q \times 1)$ vector consisting of deterministic components such as intercepts, trends, seasonal dummies, and exogenous variables. Finally, Φ is a $(q \times p)$ matrix containing the coefficients of the elements of d_t. ε_t is a white noise $(p \times 1)$ vector $\sim i.i.i.N(0, \Omega)$, with mean zero and positive definite covariance matrix Ω (Teräsvirta and Yang [2014a]).

A nonlinear model reclaims its importance when one believes that the macroeconomic variables may react differently to shocks, depending, for example, on the state of the business cycle. In another words, when one believes that symmetric shocks may produce asymmetric effects. As Hubrich and Teräsvirta [2013] defended on their recent article, these type of models may have a role to play in studies of effectiveness of monetary and fiscal policy, because the effects of the policy may be dependent on the phase of the business cycle.

For a thesis about nonlinear models it might seem important to start with a clear definition of nonlinearity. Here we will follow the linearity definition used in Lee, White and Granger [1993], where everything else will be nonlinearity. Consider the model (5) and assume that the variable y_t depends on an explanatory vector ν_t, including lagged y. If,

$$E\{y_t|\nu_t\} = \alpha^{'}\nu_t + g(\nu_t)$$

then the model for the conditional mean is said to be linear if $g(\nu_t) = 0$, so that no nonlinear terms are required (Teräsvirta *et al.* [2010]). If y_t is a function of some other variable, in order to be linear, the same cannot happen to any element of ν_t.

The *STVAR* model is the vectorized form of the smooth transition autoregressive (*STAR*) model developed by Granger and Teräsvirta, [1993] and extended by Weise [1999], which also had its origins on Bacon and Watts [1971]. Some algebraic transformations applied to (5) conduct to the *LST-VAR* model, which has the following representation:

$$y_t = \{\Sigma_{i=1}^{m}(G_t^{i-1} - G_t^{i})F_i'\}x_t + \varepsilon_t \tag{6}$$

with

$$G_t^i = g(z_{jt}|\gamma_{ij}, c_{ij}) = (1 + \exp\{-\gamma_{ij}(z_{jt} - c_{ij})\})^{-1}, \gamma_{ij} > 0 \tag{7}$$

and where $i = 1, ..., m$ represents the number of regimes, $j = 1, ..., k$ the number of lags, $F_i = (A_{i1}', ..., A_{ik}^{'}, \Phi_i')'$ is a $(kp+q) \times p$ matrix of coefficients, x_t is the matrix of lag variables $x_t = (y_{t-1}^{'}, ..., y_{t-k}^{'}, d_t^{'})$, and each A_{ij} is a matrix of type $(p \times \ p)$. Furthermore, d_t is a $(q \times 1)$ vector consisting of deterministic components such as intercepts, trends, seasonal dummies, and exogenous variables. Finally, Φ_i is a $(q \times p)$ matrix containing the coefficients of the elements of d_t.

The element G_t^i in (7) is a diagonal matrix of bounded logistic transition functions, where, z_{jt} is the transition variable, γ_{ij} is the shape of the transition function, c_{ij} is the threshold parameter for $i = 1, ..., m-1$ and $j = 1, ..., p$. While the slope parameter γ controls the slope, c controls the location of the transition function. According to our definition of nonlinearity, because $G_t^i \neq 0$ this model is a nonlinear model in the sense that this G function allows to capture the existent asymmetry.

This model (6) has three main advantages. In the first place, it allows a smooth transition between states, rather than an abrupt one. Since we are dealing with the transition along phases of the business cycles, the empirical knowledge lead us to assume that the transition period between a deep recession and a strong expansion is a continuous path rather than an abrupt change.[23] Secondly, it allows not only differential dynamic responses but also differential contemporaneous responses to structural shocks. Finally, the last advantage overcomes one problem presented previously by Parker [2011] where he defended that one constraint of studying the fiscal multipliers in recessions is due to the lack of data. *LSTVAR* model uses all available information to complete the estimation process, since it computes the probability of being in each regime, and, therefore, the estimation process is based on a larger set of observations.

In what follows we are going to present the three common stages of nonlinear models, namely: specification, estimation and evaluation.

3.5 Specification

The specification stage consists on two phases according to Teräsvirta and Yang [2014b]. First, the linear model (5) will be tested against the *LSTVAR* (6). Secondly, if the linear specification is rejected, the *LSTVAR* model will be selected together with the optimal number of lags and the transition variable.

This specification stage consists on testing the data for nonlinearities, and for this purpose, a Lagrange Multiplier (*LM*) and a Likelihood-Ratio (*LR)* test were applied. The *LM* test confronts a standard linear model against a third-order Taylor series expansion of the logistic function in the Logistic *STAR* and the specification is made equation-by-equation. In order to avoid a spurious rejection of the null hypothesis, the standard linear version was built upon stationary series and integrated of the same order.

[23] The work of Weise [1999] or Saki and Salas [2006], where the smooth parameter tended to infinity, respectively 67.75 and 100 (when the position of the business cycle is selected as transition variable) suggests a quicker change between regimes. Unlike them, my optimal smooth parameter after the grid search, is 1.015 which suggests a very smooth transition between a deep recession and a strong expansion.

$$LR = [T - (kp + q)](LN|\Omega_{\varepsilon linear}| - LN|\Omega_{\varepsilon nonlinear}|) \sim \chi^2(kp^2) \qquad (8)$$

The LR test was calculated under the specification in (8) which includes modifications proposed by Sims [1980] to take into account small-sample bias.[24]

These tests give three types of information: first, they show if the nonlinearities of the data are statistically significant; secondly, because several transition variables are tested, it will indicate which one has the highest rejection of the null hypothesis and finally, it has power to test $K = 1$ (logistic) against $K = 2$ (exponential). Thus, the test has the power to test 1 versus 2 regimes, which belongs to the specification stage, while testing 2 or more regimes belongs to the evaluation stage.

The null hypothesis for a one-equation-case (that is γ_i, instead of γ_{ij}) of linearity can be written as $H_0 : \gamma_i = 0$ and the alternative as H_1 : *at least one* $\gamma_i > 0$, for $j = 1, ..., p$.[25].

Table 9: Linearity analysis

LR	Z1	Z2	Z3
1	0.08*	0***	0***
2	0.59	0***	0***
3	1	0***	0***
4	0.08*	0***	0***

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

In the first step, since the linearity is rejected (see table (9)) for all transition variables, one can conclude that nonlinearities are statistically significant, hence a $LSTVAR$ ($K = 1$) model will be considered in the next stage.

[24] χ^2 stands for chi-squared distribution.

[25] For more detailed information see the work of Terasvirta, Tjostheim and Granger [2010] and see also the Appendix C.

The second step concerns the choice of the transition variables and its lag specification. According to the same test one can conclude that for 1 lag all transition variables reject linearity, nevertheless $z2$ and $z3$ reject it more strongly according to Table (9).

Since the model considers an endogenous transition variable which is incorporated in the VAR model, $z3$ will be the final choice for two main reasons. Firstly, because $z2$ is a linear transformation of Y it will not be considered to avoid multicollinearity issues. Secondly, $z3$ is a more reliable indicator for policy makers since unlike the output gap, the change in unemployment rate is directly observable. Moreover, the output gap measure has two main drawbacks: firstly, it produces symmetric cycles, and secondly it is extremely sensitive to the filter choice.[26]

The final step concerns the specification of the model's lag structure for which it was used a likelihood ratio test. We determine a lag length of 3 lags.[27]

3.6 The Model

The particular model studied in this thesis is an univariate $LSTVAR$ model, with two regimes and a logistic transition function[28]:

$$y_t = [y_{1t}, y_{2t}, y_{3t}]'$$

for time $t = 1, ..., T$

[26]The LM test was also run and presents the information equation-by-equation which can be seen on the Appendix C in table (74).

The individual LM test equation-by-equation suggest that perhaps a bivariate $LSTVAR$ model could be an alternative to this univariate model, since the G equation might present a different transition variable from Y and Z equations. This is a topic to be considered in future research.

[27]Because the optimal lag length of 4 would be superior to the number of variables in the VAR, and to avoid problems with the *degrees of freedom*, the final chosen lag number will be 3 instead of 4. More detailed information can be seen from table (70) to table (73).

[28]A univariate STVAR model implies that there is only one common nonlinear feature, that is, only one transition variable for each equation.

$$y_t = (1 - F(z_{t-1}))[\phi^E + \sum_{i=1}^{p} \Gamma_k^E y_{t-k}] + F(z_{t-1})[\phi^R + \sum_{i=1}^{p} \Gamma_k^R y_{t-k}] + \varepsilon_t, \quad (9)$$

where $\varepsilon_t \sim N(0, \Omega_t)$ and $\Omega_t = \Omega_E(1 - F(z_{t-1})) + \Omega_R F(z_{t-1})$,

$$g(z_t|\gamma, c) = (1 + \exp\{-\gamma(z_t - c)/\hat{\sigma}_{z_t}\})^{-1}, \gamma > 0, \quad (10)$$

$$var(z_t) = 1, E(z_t) = 0 \quad (11)$$

The transition function (10) is assumed to be responsible for capturing the regime changes. Matrix y_t consists of one fiscal variable and two macroeconomic variables $y_t = [G_t, Y_t, Z_t]'$, where G is for the real government expenditure, Y is for the real gross domestic product (GDP) in national currency at the reference year of 2005 and Z is for the unemployment growth rate.[29]

In order to solve the identification problem, the variables were ordered by applying a Cholesky decomposition which implies that shocks in output and unemployment change have no contemporaneous effect on government spending. In another words, it is assumed that government spending shocks affect the economy immediately, whereas government spending reacts to other shocks with a delay. This assumption is explained in Blanchard and Perotti [2002] as the minimum delay, since it is intuitive to assume that the government is unable to adjust its spending in response to changes in macroeconomic conditions due to bureaucracy's frictions.[30] Moreover, the contemporaneous relationship between the output and the unemployment change is left unrestricted in the tradition of the semi-structural *VAR* liter-

[29] It is important to note that in order to follow the standard notation an unfortunate coincidence regarding the y_t notation occurs. Thereby, it is important to highlight that y and Y are different variables. In our model y_t represents a vector, where its second element y_2 is GDP, a times series commonly denominated by Y_t.

[30] This solution to the identification problem does not solve for the Luca's critique, though. It does not guarantee that the shocks cannot be anticipated which would imply a different macroeconomic dynamic. Other authors such Auerbach and Gorodnichenko [2012a] control for the expectation by using some confidence proxies, nevertheless that is a data base which is not available for the Portuguese case.

ature.[31].

As presented, one of the main advantages of this model is to allow for propagation of the structural shock either contemporaneously, via differences in covariance matrices Ω_E and Ω_R, and dynamically, via differences in lag polynomials Γ_k^E and Γ_k^R. By looking at (9) the dependent variable y_t is explained by its lag variable, weighted by periods of expansion (E) and recession (R).

The transition function allows the coefficients for lagged values of y_t, to change smoothly. Since we are considering the *logistic* form of the transition function, it allows the local dynamics to be different for high and low values of the transition variable. Moreover, the model is assumed to be univariate, which implies that there is only one common nonlinear feature, only one transition variable for each equation $z_t^G = z_t^T = z_t^Y = z_t$.[32] Finally, the value of $F(z_t)$ is bounded between 0 and 1. $F(z_t) = 0$ for the lower regime, which is driven by low values of z_t, and $F(z_t) = 1$ for the higher regime, which is driven by high values of z_t.

3.7 Estimation

There are several available estimation methods, for instance Teräsvirta and Yang [2014a, 2014b] suggested a maximum likelihood technique with complete information, non-linear least squares or Bayesian methods. Apart from the Bayesian, these estimation processes have one possible problem which might compromise the quality of the estimation due to the presence of several local maxima, explained by the highly sensitiveness to the smooth parameter γ. For instance the higher the γ, the largest will be the differences across states, while differences among distinct sign or size shocks will tend to dilute.

$$y_t = [G_t, Y_t, Z_t]' \tag{12}$$

[31]See the work of Bernanke and Blinder [1992] and Bernanke and Mihov [1998] on semi-structural VARs.

[32]Indeed, the empirical evidence supports this assumption. In the specification stage there is a common transition variable which rejects the null hypothesis of linearity.

$$y_t = (1 - F(z_{t-1}))[\phi^E + \sum_{i=1}^{3} \Gamma_3^E y_{t-3}] + F(z_{t-1})[\phi^R + \sum_{i=1}^{3} \Gamma_3^R y_{t-3}] + \varepsilon_t, \quad (13)$$

In our case, the estimation process will follow the same direction as in Weise [1999] with an equation-by-equation OLS for the $LSTVAR$ model ((12) and (13)), where the position of the business cycle is proxied by the change in the unemployment rate.

In order to find suitable values for the optimization process a grid search was conducted over parameters γ and c. Nevertheless, due to the sensitiveness to the smooth parameter two types of approaches will be reported. The first consists on a calibrated value to match the recession probabilities $\gamma = 5$, and the second consists on a grid search. Both choices regarding the transition function and the calibration of the smooth parameter follow Cos and Moral-Benito's paper [2013]. Nevertheless, the necessary calculations were made to assure the fitness of this calibration.[33]

A general model is considered:

$$y_t = \Psi_t' B' x_t + \varepsilon_t \quad (14)$$

The model has the following parameters:

$$\begin{aligned} \theta &= \{B, \Omega, \Gamma, C\} \\ \Psi &= (I_p, G_t^1, ..., G_t^{m-1})' \\ B &= (B_1 + B_2 + ... + B_m) \\ \Gamma &= \{\gamma_{ij}\} \\ C &= \{c_{ij}\} \\ i &= 1, ..., m-1 \\ j &= 1, ..., p \end{aligned}$$

Then, least squares estimators are obtained by solving a minimization problem of the sum of squared residuals:

[33]The grid search graphs can be seen in fig.53, fig.66 and fig.84.

$$\hat{\theta} = \arg\min_{\theta} Q_T(\theta)$$

$$Q_T(\theta) = \sum_{t=1}^{T} (y_t - \Psi_t' B' x_t)'(y_t - \Psi_t' B' x_t)$$

The benchmark model will be estimated for the entire sample's observations, nevertheless in order to assure the robustness of the results, a model excluding the financial crisis from the sample (henceforth model II) and a model with rates of growth as variables (henceforth model III) will also be estimated. As it is a standard procedure in the associated scientific literature, the results from the LSTVAR model will be presented in opposition to a standard OLS estimated linear *VAR*. Thus, a total of 48 models were estimated, thereby, for space reasons, it is not possible to present all outputs. We will focus only on one example which is representative of the overall quality of the model, which the caracteristics are: 1 standard error deviation (SD) government spending shock in recessions; 500000 Monte Carlo simulations were considered in order to assure stability; a sample period of 1960Q2:2012Q4, for a smooth parameter of $\gamma = 5$.

$$\begin{aligned} y_{1t} = {} & \underset{0.00}{2.14} y_{1,t-1} \underset{0.00}{-1.51} y_{1,t-2} + \underset{0.00}{0.36} y_{1,t-3} + \underset{0.92}{0.00} y_{2,t-1} + \underset{0.59}{0.04} y_{2,t-2} + \underset{0.91}{0.00} y_{2,t-3} \\ & + \underset{0.66}{0.01} y_{3,t-1} + \underset{0.89}{0.00} y_{3,t-2} + \underset{0.91}{0.00} y_{3,t-3} + [(1 + exp5(z_{t-1}))^{-1}] \times (\underset{0.00}{-1.63} y_{1,t-1} \\ & + \underset{0.00}{2.23} y_{1,t-2} \underset{0.00}{-0.62} y_{1,t-3} + \underset{0.18}{0.21} y_{2,t-1} \underset{0.66}{-0.11} y_{2,t-2} + \underset{0.61}{0.00} y_{2,t-3} \\ & \underset{0.00}{-0.17} y_{3,t-1} + \underset{0.00}{0.29} y_{3,t-2} \underset{0.00}{-0.16} y_{3,t-3}) \underset{0.02}{-0.13} \end{aligned} \tag{15}$$

$$
\begin{aligned}
y_{2t} = {} & \underset{0.07}{0.27} y_{1,t-1} - \underset{0.07}{0.52} y_{1,t-2} + \underset{0.10}{0.24} y_{1,t-3} + \underset{0.00}{0.81} y_{2,t-1} + \underset{0.00}{0.34} y_{2,t-2} - \underset{0.09}{0.15} y_{2,t-3} \\
& \underset{0.02}{-0.08}\, y_{3,t-1} + \underset{0.05}{0.13} y_{3,t-2} - \underset{0.04}{0.08} y_{3,t-3} + [(1 + exp5(z_{t-1}))^{-1}] \times (\underset{0.09}{-0.33} y_{1,t-1} \\
& + \underset{0.09}{0.51} y_{1,t-2} - \underset{0.37}{0.17} y_{1,t-3} + \underset{0.00}{0.72} y_{2,t-1} - \underset{0.00}{0.99} y_{2,t-2} + \underset{0.23}{0.26} y_{2,t-3} \\
& + \underset{0.55}{0.04} y_{3,t-1} \underset{0.31}{-0.12}\, y_{3,t-2} + \underset{0.17}{0.09} y_{3,t-3}) + \underset{0.48}{0.06} \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
y_{3t} = {} & \underset{0.03}{-0.51} y_{1,t-1} + \underset{0.03}{0.96} y_{1,t-2} \underset{0.02}{-0.49}\, y_{1,t-3} + \underset{0.67}{0.06} y_{2,t-1} \underset{0.40}{-0.15}\, y_{2,t-2} + \underset{0.25}{0.15} y_{2,t-3} \\
& + \underset{0.00}{2.57} y_{3,t-1} \underset{0.00}{-2.54}\, y_{3,t-2} + \underset{0.00}{0.94} y_{3,t-3} + [(1 + exp5(z_{t-1}))^{-1}] \times (\underset{0.03}{+0.64} y_{1,t-1} \\
& \underset{0.09}{-0.78}\, y_{1,t-2} + \underset{0.59}{0.15} y_{1,t-3} + \underset{0.06}{0.62} y_{2,t-1} - \underset{0.11}{0.83} y_{2,t-2} + \underset{0.54}{0.20} y_{2,t-3} \\
& \underset{0.17}{-0.14}\, y_{3,t-1} + \underset{0.02}{0.42} y_{3,t-2} \underset{0.01}{-0.24}\, y_{3,t-3}) \underset{0.00}{-0.37} \qquad (17)
\end{aligned}
$$

Traditionally in nonlinear models, such as $LSTVAR$, the coefficients are not interpreted. Instead, the analysis is conducted with nonlinear impulse response functions (GIRF) which will be explained in a following section. Moreover, the traditional analysis of the *p-values* is also questionable because these *p-values* rely on critical values derived from linear distributions. In the presence of nonlinear models it is difficult to conclude about the significance of these values, therefore we suggest that they are not informative about the statistical significance of each individual explanatory variable.

3.8 Evaluation

This last phase is usually ignored in the literature.[34] Nevertheless it seems important to check if the final model satisfies the assumptions under which it was estimated. In order to do so, it will be employed three multivariate misspecification tests suggested by Teräsvirta and Yang [2014a]. They are:

[34] The evaluation of the nonlinear models are commonly omitted in the economic papers, such as the ones mentioned in the literature review. To the best of our knowledge the exception concerns econometric papers of authors such as Teräsvirta or Yang.

(i) the no serial correlation test which is derived from the autocorrelation VARMA form, (ii) the test of no additive nonlinearity, to test $m = 2$ against $m > 2$ and (iii) the heteroskedasticity-robust test.[35]

Table (75) suggests that for a 5% significance level, and with the exception of $G(lag1)$, the residuals associated to all equations strongly reject additive nonlinearity. Thus, one can conclude that the 2 regime specification fits well the model.

Tables (76), (78) and (80) present the correlation among the lags of the residuals associated to each equation. Since none of the values are higher than 0.3, in absolute value, the hypothesis of residuals autocorrelation is rejected.

Moreover, in order to understand how much the past (lags) explain the dependent variable at time t, autogressions models were also run. From Table (77) one can conclude that the coefficients associated to the lag of the residuals associated to G are statistical significant. However according to R-squared value of this $AR_G(3)$ regression, the past values of the residuals associated to G only contribute to explain the dependent variable in 8%, which suggests no significant autocorrelation.

Table (79) of the $AR_Y(3)$ regression suggests no autocorrelation for the residuals associated to Y equation because the coefficients of the lag terms are statistically insignificant.

Table (81) of the $AR_Z(3)$ regression suggests, in overall, no autocorrelation for the residuals associated to Z equation with the exception or the first lag. However, and once again, according to R-squared value of this $AR_Z(3)$ regression, the first past value of the residuals associated to Z only contribute to explain the dependent variable in 8%, which suggests no significant autocorrelation.

Finally, Table (82) allows to conclude that, in overall, the coefficients associated to the residuals of each equation reject conditional heteroskedasticity, with the exception of the first two lags of the G equation. Thereby,

[35] In Teräsvirta and Yang's paper [2014a] the authors suggested a new battery of tests to test the adequacy of the model. Unfortunately due to the lack of routines to run some of the new tests and also due to time constraints only two tests were selected to evaluate the model.

evidence suggest no need to report the bootstrap *p-values* of the residuals test as it is suggested by Teräsvirta and Yang [2014b] in the case of conditional heteroskedasticity.

In sum, with this battery of tests one may conclude that it seems that there is no evidence of misspecification or omitted variables in the model.

3.9 Generalized Impulse Response Functions

There is a big controversy regarding the best way to report the transition dynamics in nonlinear models. Here, only two methods will be considered: generalized impulse response functions (GIRF) and orthogonalized impulse response functions (OIRF). Since we are dealing with a nonlinear model which aims to test the existence of asymmetric effects from symmetrical shocks, the GIRF method presented by Koop, Pesaran and Potter [1996] will be applied.[36]

The main advantage of GIRF rely on the fact that, unlike OIRF, they allow the regime to switch after a structural shock, being more suitable for policy making. Another main reason that explains why this method is considered superior to OIRF is the fact that instead of controlling the impact of correlation among residuals, computes the mean by integrating all other shocks, i.e., when we compute a shock to one variable, all other variables will also vary. Moreover, it is not affected by the ordering of the variables. On the contrary to the OIRF, the GIRF method does not require that we identify any structural shock, thus it cannot explain exactly how output reacts to a fiscal policy shock. Instead, provides a tool for describing the dynamics in a time series model by mapping out the reaction of output to one standard deviation shock of the residual in the fiscal spending equation.

The main limitations of the OIRF, which is overcome by GIRF, concerns the restrictive property of symmetry. A shock of -1 has exactly the opposite effect of a shock of $+1$, and a shock of size 2 has exactly twice the effect of a shock of size 1. The GIRF method offers a solution to this problem by introducing a history dependence property, where the response constructed

[36] The GIRFs were also developed by Pesaran and Shin [1998]

is an average of what might happen in the future given the present and the past.

Three important details to understand in this method: (i) the type of the shock at time t; (ii) the state (or regime) in which the economy was at time $t-1$, before being shocked and (iii) the type of shock that is going to hit the economy from period $t+1$ to $t+n$. Therefore, following the notation of Teräsvirta *et al.* [2010]:

$$GIRF(h, \varepsilon_t, \Omega_{t-1}) = E\{y_{t+h}|\varepsilon_t, \Omega_{t-1}\} - E\{y_{t+h}|\Omega_{t-1}\} \tag{18}$$

where $\Omega_{t-1} = \{\omega_{t-j} : j \geq 1\}$is the set of possible histories and ε_t and Ω_{t-1} are random condition variables, h goes for h_{th} impulse response, where the response ultimately vanishes as $h \to \infty$.[37]

Although we consider the GIRF method a better alternative to the OIRF, Hyeongwoo [2009] seems to have analysed some possible limitations concerning the identifying assumptions, which he considers that can be misleading if the covariance matrix fails to be diagonal. A possible way to overcome both limitations from both methods in a future research, could be the Jordà method (a projection method) presented by Jordà [2005].

[37]These GIRF were computed with a Monte Carlo technique for 500000 simulations. The details concerning the algorithm can be seen in Koop *et al.* [1996] from page 135 to 137. The Gaussianity method was not assumed, instead, in order to pick a history, it was used the bootstrap method.

4 Results

This section aims to present the main results regarding the output multiplier after a fiscal spending shock by analyzing the transition dynamics with the GIRF method. Results will be reported by using the multiplier concept, for different types of standard-error-deviation shocks (henceforth SD, 1SD, -1SD, 2SD and -2SD).[38] It will be considered the calibrated smooth parameter ($\gamma = 5$) and 500000 Monte Carlo simulations, which are the necessary simulations to achieve stability in the model.[39]

A smooth parameter of $\gamma = 5$ is the calibration that fits better the Portuguese recession dates. According to OECD Recession Indicators for Portugal, the Portuguese economy spends about 43 percent of time in a recessionary regime, which corresponds to a probability of $P[F(z)] > 0.57$.[40]

Moreover, this time series composed by OECD reflects a dummy variable which represents periods of expansion and recession (shaded area). According to OECD definition, which follows NBER, the recession begins the first day of the period following a peak and ends on the last day of the period of the trough.

[38] A 1SD shock, corresponds to 1.56 percent increase of the government spending, while 2SD corresponds to a 3.12 percent increase. For instance, the graphs can be therefore interpreted as the response of the log level of output to a permanent 1.56/3.12 percent increase, or decrease, in the log level of government spending.

[39] The outputs for the other specifications are available upon request to *tfbgs@iscte.pt.*

[40] Composite Leading Indicators: Reference Turning Points and Component Series", www.oecd.org/std/cli (Accessed 27/01/2014)

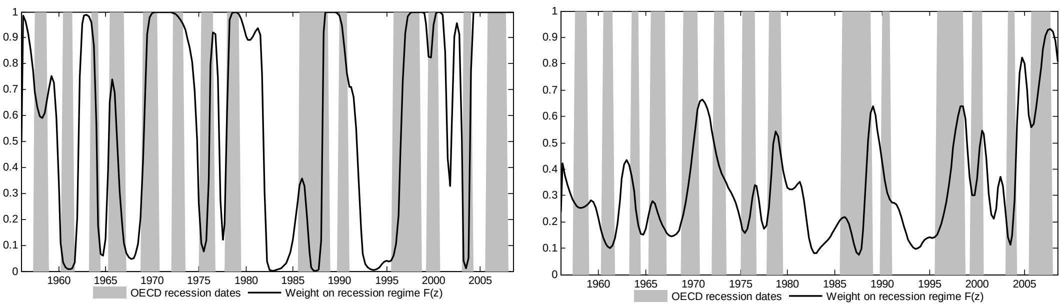

Figure 3: These figures compare the dynamic of the weight in a recession regime with recessions identified by OECD for: a $P[F(z)] > 0.57$, for $\gamma = 5$,(right panel) and $\gamma = 1.015$ (left panel) all sample period, and 500000 Monte Carlo simulations.

Table 10: Multipliers values for the benchmark model

Benchmark model		Impact Multiplier		Peak Multiplier		Cumulative Multiplier	
		Recession	Expansion	Recession	Expansion	Recession	Expansion
Shock 1sd	smooth=5	0.23	0.15	0.99	0.50	1.03	0.29
	smooth=1.015	0.99	-0.06	1.37	0.35	0.99	0.11
Shock 2sd	smooth=5	-0.13	0.97	1.97	0.97	0.95	0.39
	smooth=1.015	1.00	-0.11	1.80	0.71	0.53	0.11
Shock -1sd	smooth=5	0.06	0.06	0.06	0.06	-0.95	-1.39
	smooth=1.015	-0.37	0.06	0.00	0.06	-0.60	-0.11
Shock -2sd	smooth=5	0.13	-2.34	0.13	0.00	-0.95	-1.17
	smooth=1.015	-1.30	0.11	0.00	0.11	-0.48	-0.11

Figure (3) compares the dynamics of the weight in a recession regime with recessions identified by OECD. Results seem to suggest that the properties of the left panel have a better fit when compared to the right panel, which will constitute the robustness check of our results.

Table (10) reports the output multipliers that are direct transformations, calibrated by the following ratio $\frac{Y}{G}$, of the elasticities reported by the GIRF. For instance, 500000 Monte Carlo simulations were considered, the identification of government shocks follows Cholesky ordering, with G ordered first,

Y second, and Z third, for the entire sample period.[41]

The *fiscal multiplier* term refers to the ratio of a change in output, ΔY, to an exogenous change in the fiscal balance ΔG or ΔT. The former relates to a change in government spending, while the latter relates to a change in government revenue. The *impact multiplier* (henceforth m_i) is defined by the ratio of a contemporaneous change to an exogenous change in the fiscal balance: $\frac{\Delta Y_{t_0}}{\Delta G_{t_0}}$. More commonly, most authors are interested in presenting results for the *peak multiplier* (henceforth m_p) or *cumulative multiplier* (henceforth m_c). The former (second column) is defined as the ratio of the largest change in output over any time horizon N periods to an exogenous change in the fiscal balance at time t_0 (max $\frac{\Delta Y_{t_0+i}}{\Delta G_{t_0}}$). The latter (third column) is defined as the ratio of the cumulative change in output over an exogenous change in the fiscal balance over a time horizon of N periods: $\frac{\Sigma_{i=0}^{N}(\Delta Y_{t_0+i})}{{}_{i=0}^{N}(\Delta G_{t_0+i})}$, with $i = 0, 1, ...N$ (Monokroussos and Thomakos [2012]).

Another challenge concerns the identification of the exogenous shock. There are several methods, as discussed in Cos and Moral-Benito [2013], here, as explained before, it is adopted the Cholesky decomposition method. Therefore if the government spending is in the first position of the system in equation (12), it is assumed that it is contemporaneously exogenous to the remaining variables in the *VAR*, that is, fiscal policy does not react to output or to unemployment changes in the current quarter, hence the implicit shocks can only be exogenous.

4.1 Main Results

There are three main questions to be addressed, and for which, impulse response functions are the most convenient way to answer.

- Firstly, do fiscal shocks have different effects at different points in the business cycle, for instance, recessions and expansions?
- Secondly, do positive fiscal shocks have different effects from negative shocks?

[41] This is the same procedure followed by Auerbach and Gorodnichenko [2012a].

- Thirdly, is the asymmetry affected by the magnitude of the shock?

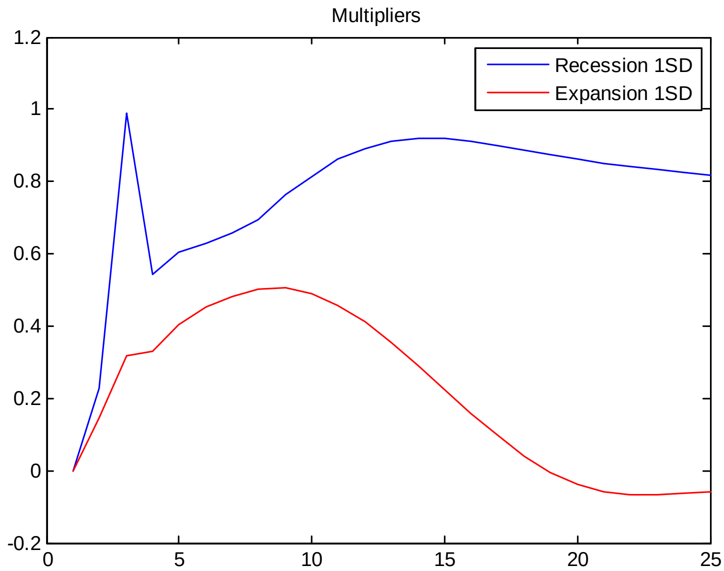

Figure 4: Accumulated response of output (Y) to a 1SD government spending shock for both regimes, recession (blue line) and expansion (red line).

Figure (4) allows to provide an answer to the first question. According to this figure the major difference is not on the impact output multiplier of both regimes (0.23 for recession and 0.15 for expansion) but on the dynamics. A positive 1SD shock has a higher effect on output during recession periods (blue line) when compared to expansion periods (red line), where it converges to zero after approximately 20 quarters.[42] We may also conclude that the persistence of the output response after a positive government shock is higher in recessions when compared to expansions. Moreover, according to Table (10) the values of m_i, m_p and m_c are higher in recessions than in expansions. For instance, the value of m_c after 24 quarters is 1.03 for the recessionary

[42]Note that, as Auerbach and Gorodnichenko [2012a] pointed out, the difference across states is not on the contemporaneous response to a government spending shock but on the dynamics. This suggests that the differences in the magnitudes of the multipliers are explained via the polynomials (Γ_k^E and Γ_k^R.) rather than the covariance of error terms (Ω_E and Ω_R).

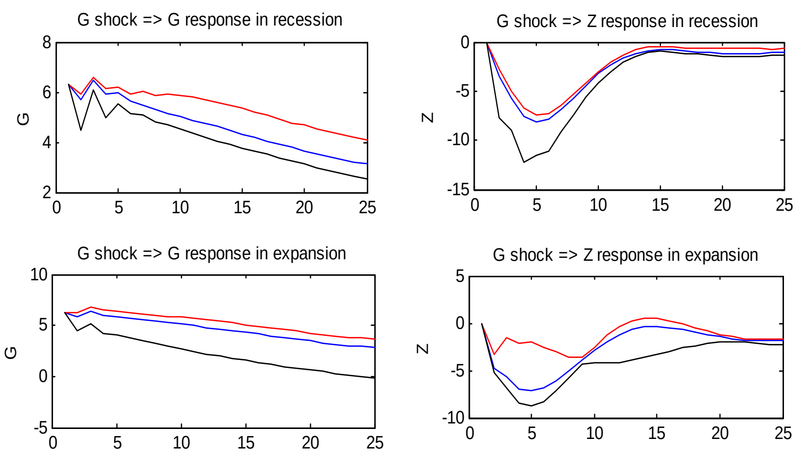

Figure 5: Accumulated response of the government spending G (left panels) and the unemployment rate change Z (right panels) to a 1SD government spending shock in both regimes: recession (top panels) and expansion (bottom panels).

regime, and 0.29 for the expansionary regime which is consistent with the Keynesian view of business cycle. [43]

Moreover, by looking at Figure (5) both dynamics of G seem to suggest that the persistence of the government shock is slightly lower in the expansionary regime (bottom-left panel) when compared to the recessionary regime (top-left panel). In what respects the unemployment rate change (Z) both regimes present a similar response to a positive 1SD shock (right panels). The unemployment decreasing rate is fast during the first 5 quarters, but slows down converging to its original equilibrium value after approximately 13 quarters. These dynamics, allow us to conjecture that the economy might be converging from a recession (top-right panel) to an expansion during 15 quarters.[44]

[43] These results are consistent with the qualitative conclusions of Poirier [2014], who estimated a $TVAR$ for the Portuguese fiscal multiplier.

[44] More detailed information can be found on fig. (54) and fig. (55) in the Appendix D. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

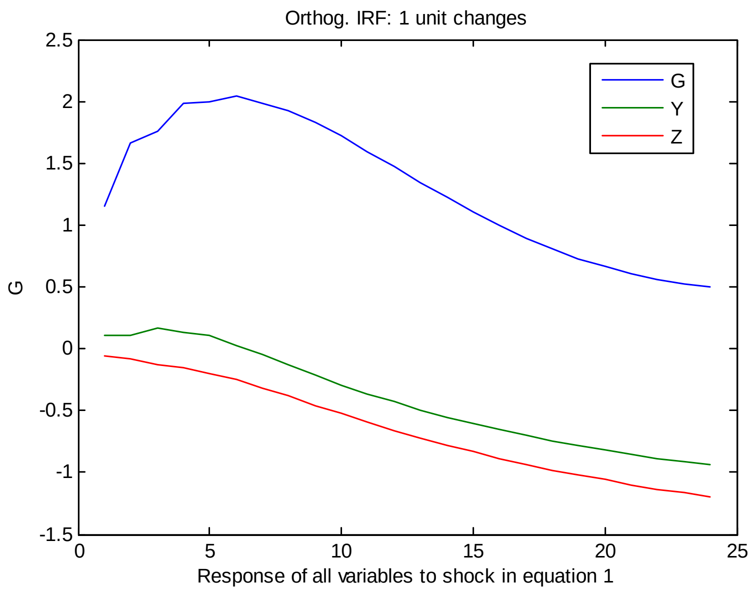

Figure 6: Accumulated response of government spending (G), output (Y) and unemployment change (Z) to a 1 unit change in government spending shock for a linear VAR model.

Another major conclusion regards the output response in the linear model, Figure (6), where the output response is negative after a positive shock. Therefore, we can conclude that, as defended, linear models might be outdated for policy advising because they are not capable of replicating the kind of dynamics that we expect to find in a modern market economy. Symmetric government spending shocks have asymmetric effects depending on the position of the business cycle.

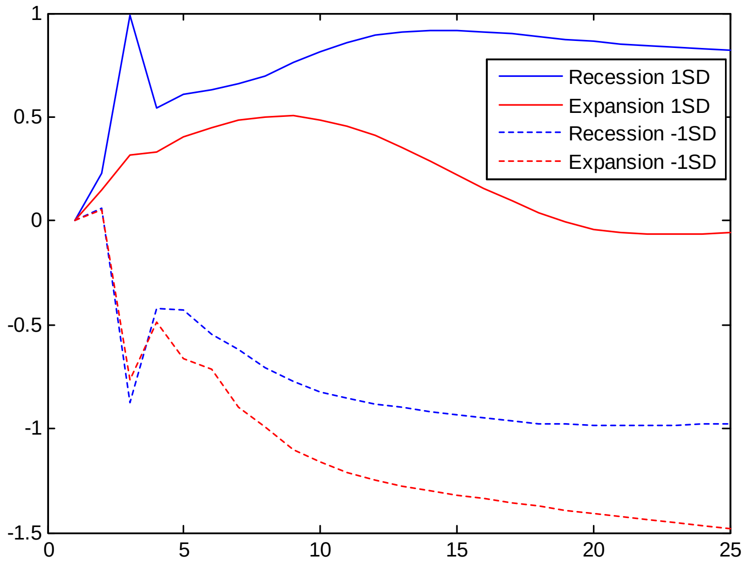

Figure 7: Accumulated response of output (Y) to a 1SD government spending shock for positive shocks and -1SD for negative shocks.

Figure (7) allows to conclude that positive fiscal shocks have different effects from negative shocks. While an expansionary fiscal policy boosts the economy, a consolidating fiscal policy decreases the output in both regimes. Considering a negative 1SD shock, the recessionary regime presents a value of $m_c = -0.95$, while the expansionary regime presents a stronger negative value of $m_c = -1.39$.The model predicts that a consolidation fiscal policy in times of recession depresses the economy, which is consistent with the short-run results of Castro *et al.* [2013] but leave some doubts about positive long-run results from an expansionary fiscal consolidation.

One major conjecture from this Figure (7) regards the apparent counter-cyclical nature of fiscal spending policy. While a positive shock has a higher effect on output under a recessionary regime than in expansions, a negative shock seems to have a stronger effect during expansions than in recessions, both contributing to reduce the amplitude of the business cycles.

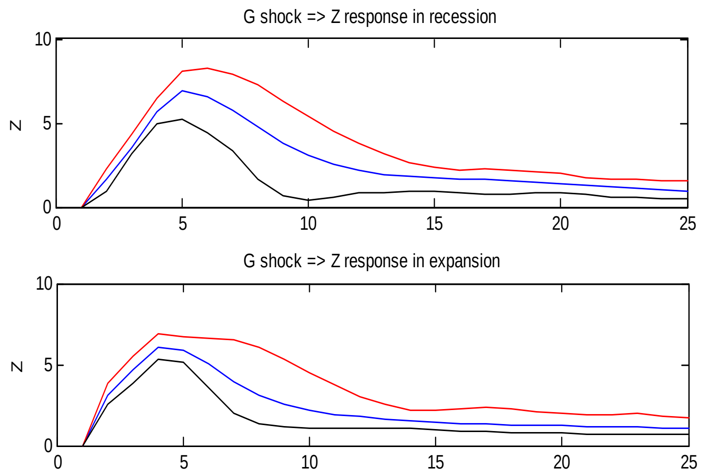

Figure 8: Accumulated response of the unemployment rate change (Z) to a -1SD government spending shock in both regimes: recession (top panel) and expansion (bottom panel).

Intuitively, a negative fiscal shock increases the unemployment rate change in a similar way for both regimes (figure (8)).[45] Nevertheless, the response is more accentuated in the recessionary regime (top panel), probably due to the sensitiveness of firms. Traditionally, Portuguese firms, react to a change in the output by adjusting firstly productivity, then salaries and only as last resource, the employment level. It is expected a stronger response of unemployment in recessions because there is less room to adjust when compared to expansionary regimes.

[45] For more detailed information see figures (56) and (57).

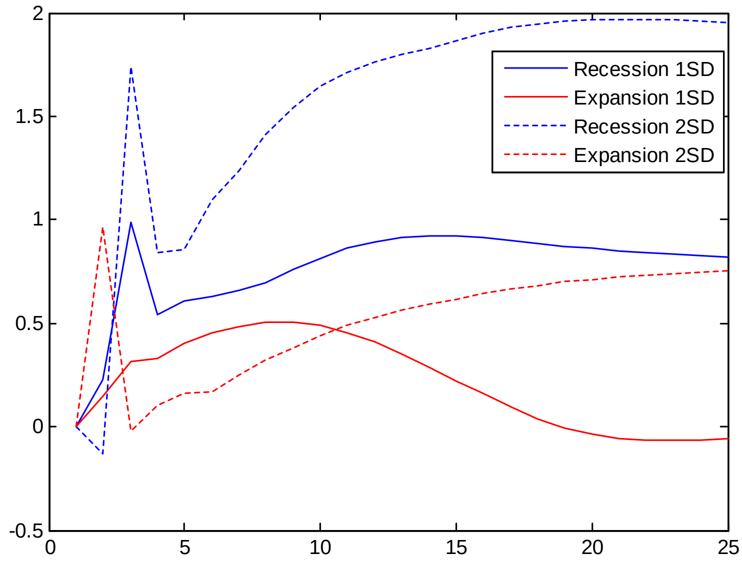

Figure 9: Accumulated response of output (Y) to a 1SD and 2SD government spending shock.

Finally, Figure (9) answers the last question by showing that the dynamic after a 1SD shock differs from the dynamic after a 2SD shock, mainly in expansions where the effect no longer converges to zero but to a positive value. There are two main effects that a 2SD shock may instigate. In the first place, a 2SD shock has a stronger effect when compared to a 1SD shock, which is visible in the higher values of m_p. In the second place, the value for m_c in the recessionary regime is lower after a 2SD shock than it is after a 1SD shock. This fact might be explained by the non-constant shifting probabilities. If the multipliers are lower in expansions than they are in recessions and if a stronger shock produces a stronger effect, this shock might increase the probability of shifting regime from recession to expansion, which will produce a smaller value for m_c. On the other hand, one could argue that a stronger effect under an expansionary regime would trigger a crowding-out effect but, on the contrary, the expansionary values of m_c seem to report a similar response derived from a stronger shock.

In sum, these findings suggest that the three sources of asymmetries - different effects at different business cycle positions, sign and magnitude of the shock - all have important effects on the resulting estimates, leaving linear models with underestimated results on three major grounds.[46]

Firstly, government spending shocks have a stronger effect during recession times, compared to expansion periods. Multipliers higher than 1 were found during recession periods against multipliers lower than 0.4 in expansions. In short, the estimates for the output multipliers seem to be largely consistent with the theoretical arguments of both old and modern Keynesian approaches.

Secondly, asymmetries are also present depending on the sign of the shock. Results suggest that expansionary government spending shocks tend to have a stronger effect during recessions, while contractionary government spending shocks tend to have stronger effects during expansions, highlighting the counter-cyclical nature of the fiscal policy.

Finally, asymmetries are also present depending on the magnitude of the shock, since doubling the shock does not double the effect. Because the model allows for non-constant probabilities of shifting regimes, the stronger the shock is the higher will be the probability of changing regime. For instance, considering a high positive government spending shock during recession times, increases the probability of shifting to a recovery period, which produces lower multipliers when compared to small positive shocks.

4.2 Robustness Analysis

As explained, 48 models were already estimated in order to construct a robustness analysis of the model's quality, making not possible, due to space and time constraints, to report all the results and outputs.

The results of the benchmark model were analysed under the assumption of a calibrated parameter ($\gamma = 5$). Now we will check the robustness of these results by looking at an optimal smooth parameter of 1.015. The higher the

[46] These results are qualitatively similar to the ones found by Poirier [2014]. Nevertheless, because the author uses a $TVAR$ with only two small samples as representativeness of an expansion and recession regime, it lacks some richness in terms of endogenous dynamics.

smooth parameter is, the quicker will be the convergence between states. Under the smooth parameter of 5 we have learned that the multipliers are higher in recession periods when compared to expansion periods.

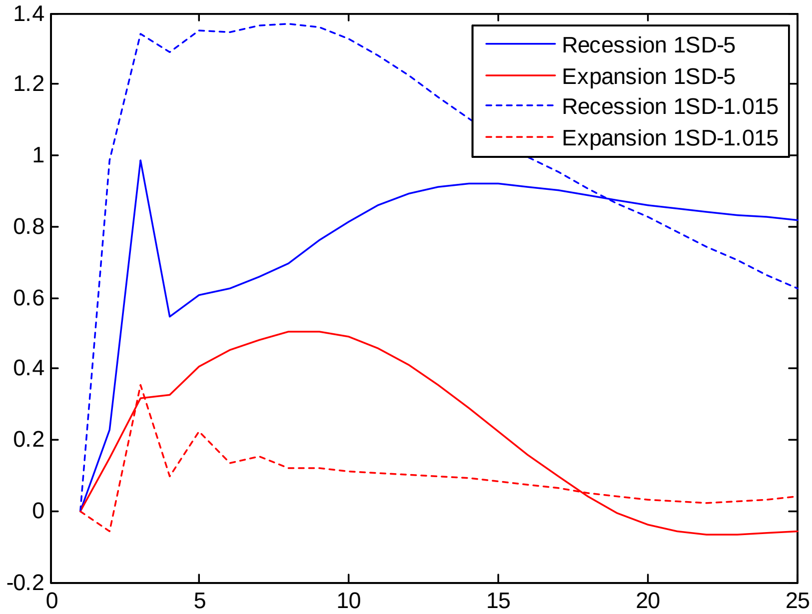

Figure 10: Accumulated response of output (Y) to a 1SD government spending shock for both regimes, recession and expansion and both smooth parameters. 500000 Monte Carlo simulations were considered in order to assure stability. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4.

Table 11: Multipliers values for the model II and model III

Sample periodo 1960Q2 to 2007Q4		Impact Multiplier		Peak Multiplier		Cumulative Multiplier	
Model II		Recession	Expansion	Recession	Expansion	Recession	Expansion
Shock 1sd	smooth=5	-0.02	-0.07	1.64	0.80	1.35	0.34
	smooth=1.015	1.99	-0.59	1.99	0.26	0.57	-0.57
Shock 2sd	smooth=5	0.41	0.12	3.81	0.12	1.54	-0.29
	smooth=1.015	0.29	0.40	0.89	1.79	0.26	0.55
Shock -1sd	smooth=5	-0.78	-0.96	0.00	0.00	-0.85	-1.36
	smooth=1.015	-0.15	-0.20	0.29	0.00	-0.26	-0.55
Shock -2sd	smooth=5	-1.88	-1.93	0.00	0.00	-1.25	-1.36
	smooth=1.015	-1.44	-0.40	0.00	0.00	-0.87	-0.55
Growth Rates 1960Q2:2012Q4							
Model III							
Shock 1sd	smooth=5	0.22	0.45	0.77	0.45	1.24	0.18
	smooth=1.015	0.26	0.94	0.87	0.94	1.06	0.93
Shock 2sd	smooth=5	3.21	1.14	3.21	1.14	1.24	0.37
	smooth=1.015	1.80	0.03	2.11	0.37	1.39	0.15
Shock -1sd	smooth=5	0.29	-0.45	0.29	0.51	-1.09	-0.45
	smooth=1.015	-1.01	-0.62	0.01	0.09	-1.53	-0.64
Shock -2sd	smooth=5	-0.44	-1.14	0.00	0.26	-1.24	-0.37
	smooth=1.015	-0.51	-1.58	0.09	0.22	-1.06	-2.58

.

A closer look to the values of 1SD with a smooth parameter of 1.015, Figure (10), reveals that the differences across states are even bigger. The value of m_i is close to one for recessions (0.99) but negative for expansions (-0.06), the value of m_p is also higher in recessions (1.37) than in expansions and the differences between states are more accentuated for the values of m_c, 0.99 and 0.11 for recessions and expansions respectively. The conclusion regarding higher multipliers in recessions than in expansions periods for a $\gamma = 5$ specification, is robust for model II, as well, for model III presented in Table (11).[47]

Moreover, it is worth noticing that the multipliers for model II (1SD and 2SD), without the financial crisis are higher when compared to the benchmark model for a $\gamma = 5$ specification. This suggests that the weak financial situation over the public finances might have compromised the credibility of economic agents, thus contributing to low output responses after government spending shocks, which is consistent with the literature that proved weak values for multipliers under periods of weak public finances.[48] For instance, Cos and Moral-Benito [2013] found evidence suggesting lower multipliers during periods of weak public finances.

The conclusions regarding the asymmetries of the shock sign seem to be robust independently of the model, indeed negative shocks have different dynamic effects when compared to positive shocks. The benchmark model suggested that negative government spending shocks have stronger effects during expansionary periods, while positive shocks have stronger effects during recessionary periods. These results seem to be, in overall, robust with two exceptions. In the first place, for a -2SD shock and a smooth parameter of $\gamma = 1.015$, all models present the opposite conclusion, which is also the case for 2SD in model II.

Secondly, while the models which have logarithmic variables suggest that a negative shock has stronger effects during expansion times than in recessions, when the growth rates of those variables are considered, model III

[47] See also figures: fig.(73), fig.(74), fig.(87) and fig.(88).

[48] This conclusion can also be seen in the linear model of the subsample, see fig.(70), which presents a higher output response when compared to the linear model of the benchmark model (fig.(6)).

suggests that both positive and negative shocks have stronger effects during recession times than in expansions. Notwithstanding, in all cases, the negative shock is never a negative symmetric response of the positive shock, both GIRF and the values of m_c present significant differences. Regarding the unemployment rate change, it was concluded that a negative fiscal shock increased the unemployment rate change in a similar way for both regimes, but with a stronger magnitude during recession times. The robustness of this result is dependent on the γ parameter. All models achieved the same conclusion for a smooth parameter of $\gamma = 5$ (a negative shock affects more the unemployment rate change in recessions), but once the $\gamma = 1.015$ is considered the conclusions are reverted and the effects are stronger during expansions.

We may conjecture then, that due to the intrinsic nature of these parameters (the higher the value of the parameter, the quicker will be the convergency rate between states) is expected to have a stronger response in recessions than in expansions if the rate of convergence is high, because a negative shock during expansion times has a quicker transition from expansion to recession. We believe the result under $\gamma = 5$ specification is more economically intuitive, because empirically, negative variations of unemployment are biding, in expansions, by the NAIRU limit. However, during recessions, positive variations of unemployment are not biding and can achieve huge rates. For instance consider the Portuguese case, where during almost 30 years the unemployment rate ranged between 4% and 8%, and just in the last 4 years it increased up to, almost, 18%.

Finally, different magnitudes also contribute to asymmetric effects after a symmetric shock. From Table (11) we may conclude that some results from a 2SD government shock under the $\gamma = 1.015$ are not robust. Nevertheless, we may conjecture that under the benchmark model a 2SD shock takes the economy out of recession quicker because the multipliers are slightly lower after a 2SD shock when compared to 1SD shock. This result is robust for model II under parameter $\gamma = 5$ specification and for model III under all specifications.

In sum, we may conclude that this specification and estimation process of the *LSTVAR* model, despite being very sensitive to the values of the smooth parameter, it seems to do a good job to test asymmetries and to conclude about state-dependent dynamics. Despite the fact that there is still room for possible improvements, if the results are robust and consistent is expected that these results have a practical guidance for fiscal policy, especially when this conclusions highlight a counter-cyclical view.

5 Concluding Remarks

A recent trend in the literature highlighted the *LSTVAR model* as a potential nonlinear approach for the decision-making process regarding fiscal policy. It also presented evidence suggesting that linear models might be outdated for the same process. For instance, several researchers produced their own nonlinear-country-specific multiplier. To the best of our knowledge, evidence of nonlinear fiscal multipliers were produced for several countries such as US, Canada, France, Germany, Italy, Malaysia, Greece and Japan but none for the Portuguese economy. This thesis aims to fill this gap in the literature by providing empirical evidence about the Portuguese nonlinear government spending multiplier.

Results suggest that symmetric shocks have asymmetric effects depending on the position of the business cycle, multipliers higher than 1 were found during recessions against multipliers lower than 0.4 during expansions. The analysis was also extended to incorporate the analysis of asymmetric effects depending on differences on the sign and magnitude of the shocks. Results suggest that expansionary government spending shocks tend to have a stronger effect during recessions, while contractionary government spending shocks tend to have stronger effects during expansions, highlighting the counter-cyclical nature of the fiscal policy. Because the model allows for non-constant probabilities of shifting regimes the magnitude also plays a role. The stronger the shock is, the higher will be the probability of changing regime.

These results seem consistent with the recent literature and with the empirical results suggested by several authors in several countries.

Regarding future research, we believe there are two main paths to follow. Firstly, the complexity of the nonlinear nature of the data clearly requires a

Bayesian analysis that allows the creation of nonlinear models with more efficiency, specifically in terms of superior optimization algorithms and superior estimation processes. Secondly, many important variables were not analysed in this thesis but they are of great importance to understand the economic dynamics, such as, the output components and the debt-to-gdp ratio.

In sum, despite the error margin that this approach might present we believe that this thesis highlights the counter cyclical view of fiscal policy by suggesting that country and time-specific multipliers should be reconsidered since empirical evidence suggest that symmetric fiscal shocks have asymmetric effects, stronger during recessions when compared to expansions.

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Part I
Appendix

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9 Appendix A - Fiscal and Macroeconomic Series

Table 12: Fiscal Variables

		FISCAL VARIABLES		
Variable	Source	Series	Measure	Definition
G	OECD	General Government Final Consumption Expenditure 1960Q1 to 2012Q4	Millions in National Currency, Volume Estimates OECD Reference Year, 2005 Quarterly Levels Seasonally Adjusted	Final consumption expenditure by general government is equal to the compensation of employees, plus intermediate consumption, plus consumption of fixed capital, plus expenditure on market goods and services by general government on the behalf of households, minus partial payments.
T	OECD	Total Tax Revenue 1965Q1 to 2012Q4	Millions in National Currency Annual Levels Ajusted by the author - spline serie Spline interpolation	Total tax revenue indicates the amount in millions that is collected by the government through taxes. It can be regarded as one measure of the degree to which the government controls the economy's resources.
C gov	Eurostat	General Government Final Consumption Expenditure 1977Q1 to 2012Q4	Millions of national currency (including euro fixed' series for euro area countries) European System of Accounts (ESA 1995) Spline interpolation	Consumption expenditure by general government includes the value of goods and services purchased or produced by general government and directly supplied to
I gov	Eurostat	General Government Gross fixed capital formation 1977Q1 to 2012Q4	Millions of national currency (including euro fixed' series for euro area countries) European System of Accounts (ESA 1995) Spline interpolation	General government gross fixed capital formation (ESA95 code P.51) consists of resident producers' acquisitions, less disposals of fixed assets during a given period plus certain additions to the value of non-produced assets realized by the productive activity of government producer or units. Fixed assets are tangible or intangible assets produced as outputs from processes of production that are themselves used repeatedly, or continuously, in processes of production for more than one year.

Table 13: Macroeconomic Variables

		MACROECONOMIC VARIABLES		
Variable	Source	Series	Measure	Definition
Y	OECD	Gross Domestic Product Expenditure approach 1960Q1 to 2012Q4	Millions in National Currency Volume Estimates OECD Reference Year, 2005 Quarterly Levels Seasonally Adjusted	Gross domestic product is an aggregate measure of production equal to the sum of the gross values added of all resident institutional units engaged in production (plus any taxes, and minus any subsidies, on products not included in the value of their outputs). The sum of the final uses of goods and services (all uses except intermediate consumption) measured in purchasers' prices, less the value of imports of goods and services, or the sum of primary incomes distributed by resident producer units.
C	OECD	Private final consumption expenditure 1960Q1 to 2012Q4	Millions in National Currency Volume Estimates OECD Reference Year, 2005 Quarterly Levels Seasonally Adjusted	Private final consumption expenditure consists of the expenditure, including imputed expenditure, incurred by resident households on individual consumption goods and services, including those sold at prices that are not economically significant.
I	OECD	Gross fixed capital formation 1960Q1 to 2012Q4	Millions in National Currency Volume Estimates OECD Reference Year, 2005 Quarterly Levels Seasonally Adjusted	Gross capital formation is measured by the total value of the gross fixed capital formation, changes in inventories and acquisitions less disposals of valuables for a unit or sector.
Z1	Author	Gross Domestic Product Growth 1960Q1 to 2012Q4	Millions in National Currency Volume Estimates OECD Reference Year, 2005 Quarterly Levels Seasonally Adjusted Percent Change from the previous year	
Z2	Author	Output gap of the total economy 1960Q1 to 2012Q4	Deviation of actual GDP from potential GDP as a per cent of potential GDP. Hodrick-Prescott Filter, ?=1600	Potential gross domestic product (GDP) is defined in the OECD's Economic Outlook publication as the level of output that an economy can produce at a constant inflation rate. Although an economy can temporarily produce more than its potential level of output, that comes at the cost of rising inflation. Potential output depends on the capital stock, the potential labour force (which depends on demographic factors and on participation rates), the non-accelerating inflation rate of unemployment (NAIRU), and the level of labour efficiency.
Z3	AMECO	Unemployment Rate: All Persons for Portugal, 1960Q1 to 2012Q4	Change From the Previous Year, Spline interpolation	Unemployed persons as a share of the total active population (labour force). The labour force is the number of people employed and unemployed (see also ESA 95, paragraph 11.21)

10 Appendix B - Exploratory Data Analysis

10.1 Variables in Levels

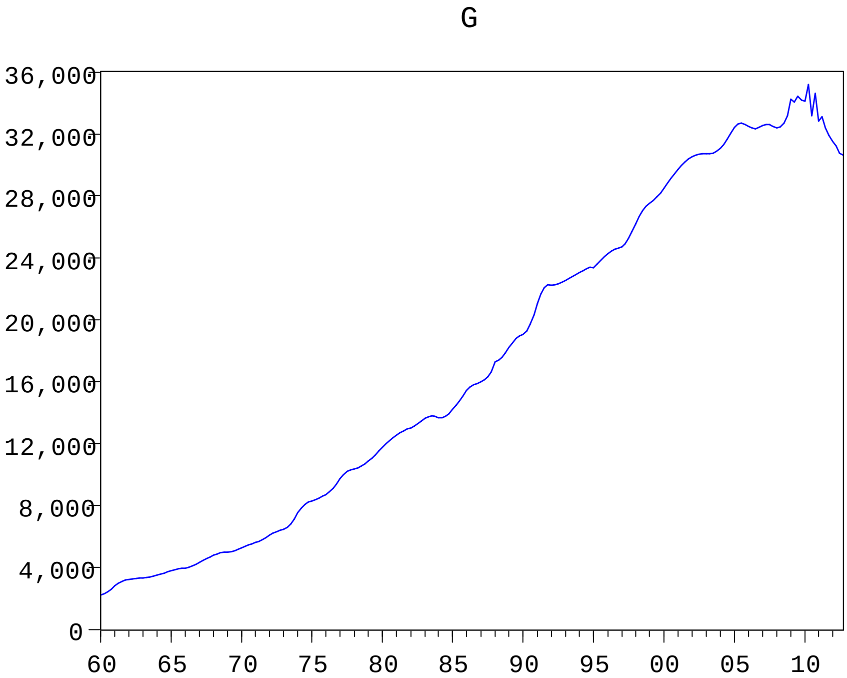

Figure 11: Representation of Real Government Final Consumption Expenditure Time Series

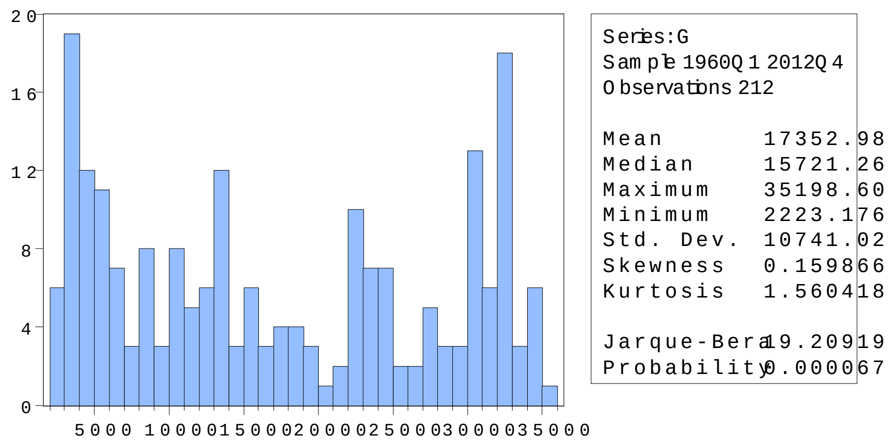

Figure 12: Histogram of Real Government Final Consumption Expenditure

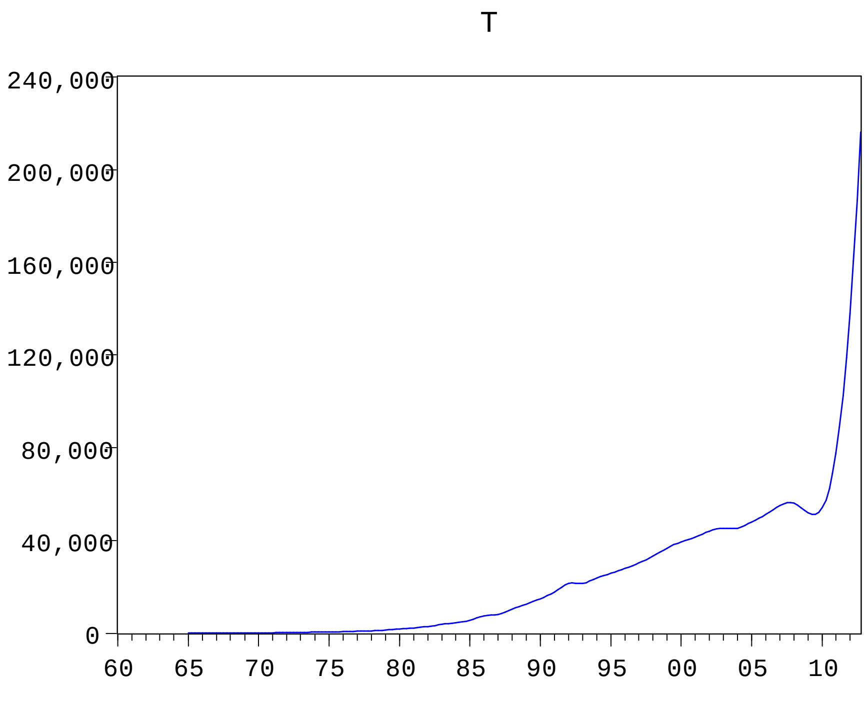

Figure 13: Representation of Total Tax Revenue Time Series

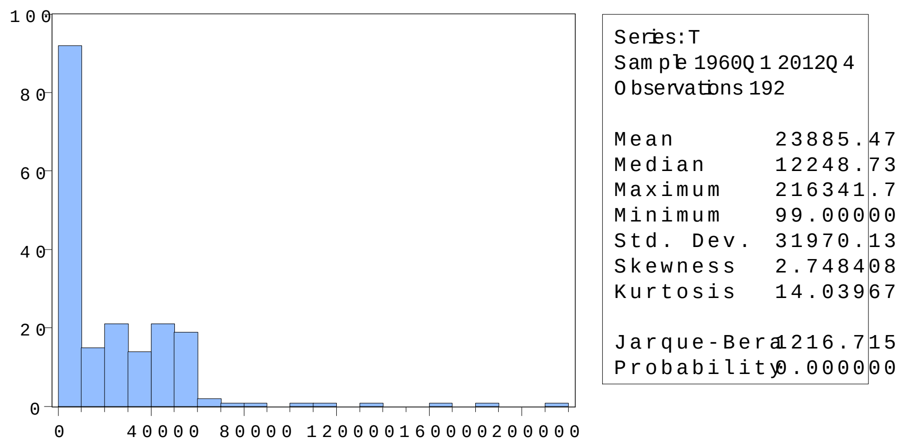

Figure 14: Histogram of Total Tax Revenue

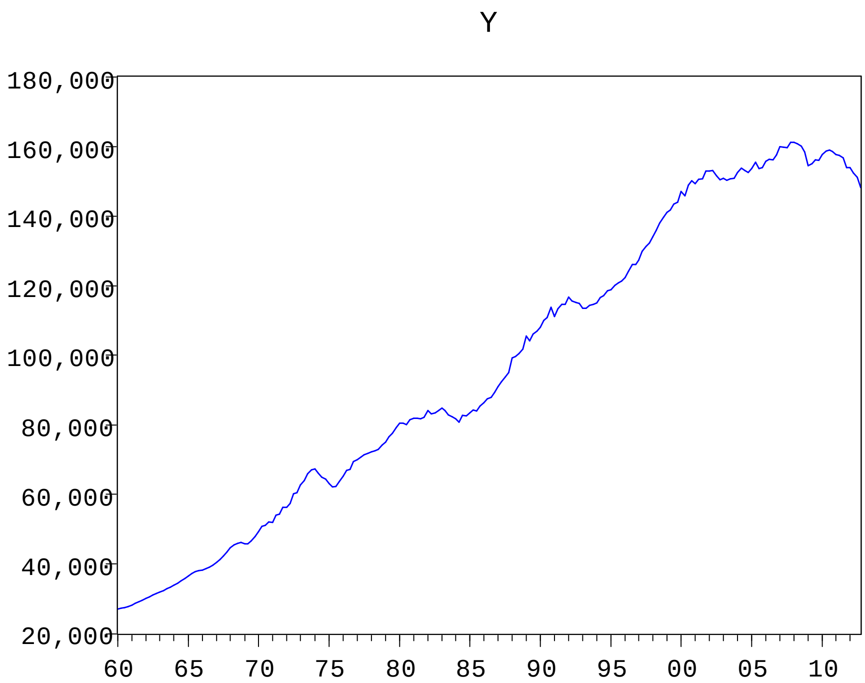

Figure 15: Representation of Real Gross Domestic Product Time Series

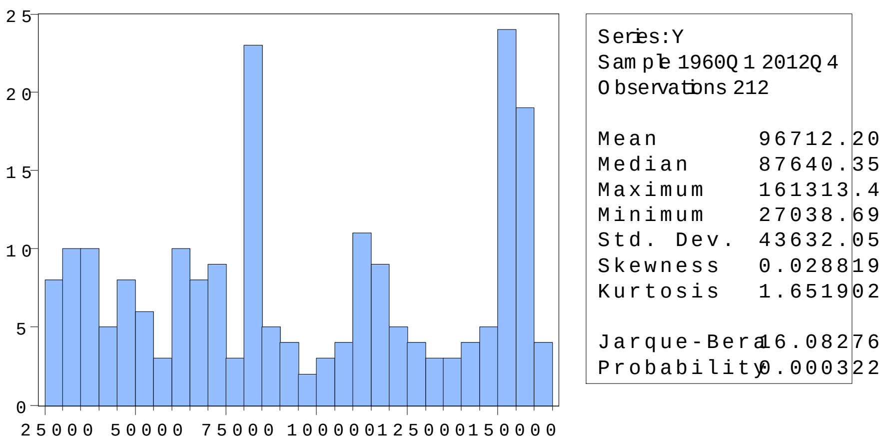

Figure 16: Histogram of Real Gross Domestic Product

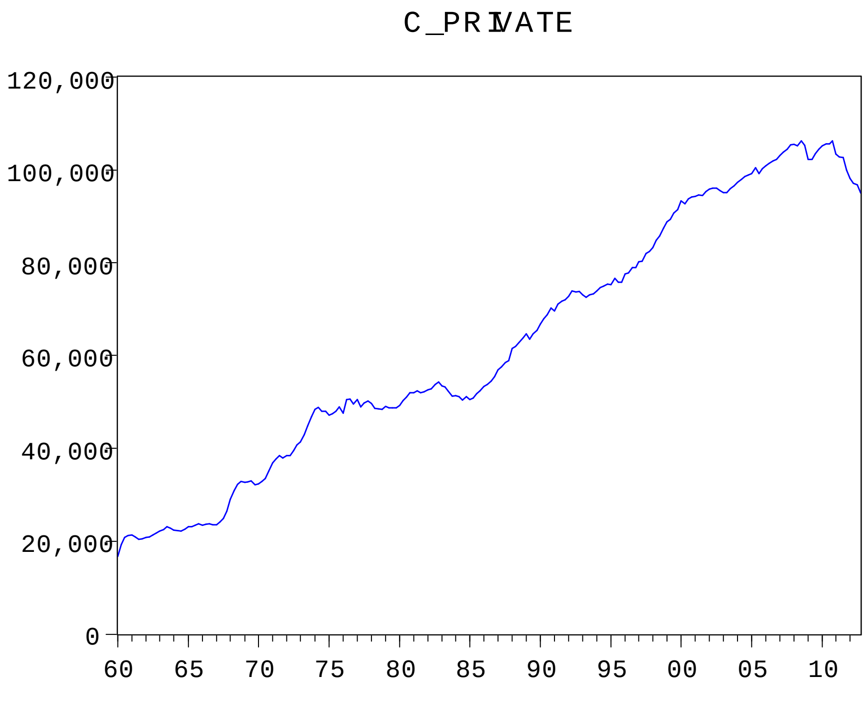

Figure 17: Representation of Private Consumption Expenditure Time Series

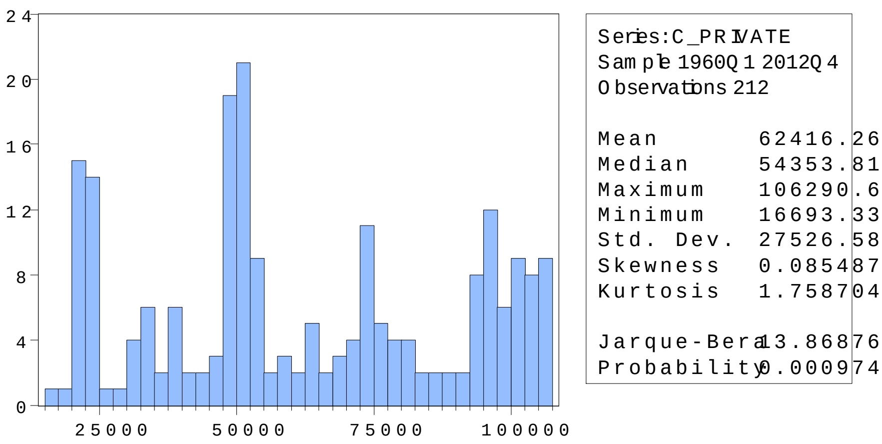

Figure 18: Histogram of Private Consumption Expenditure

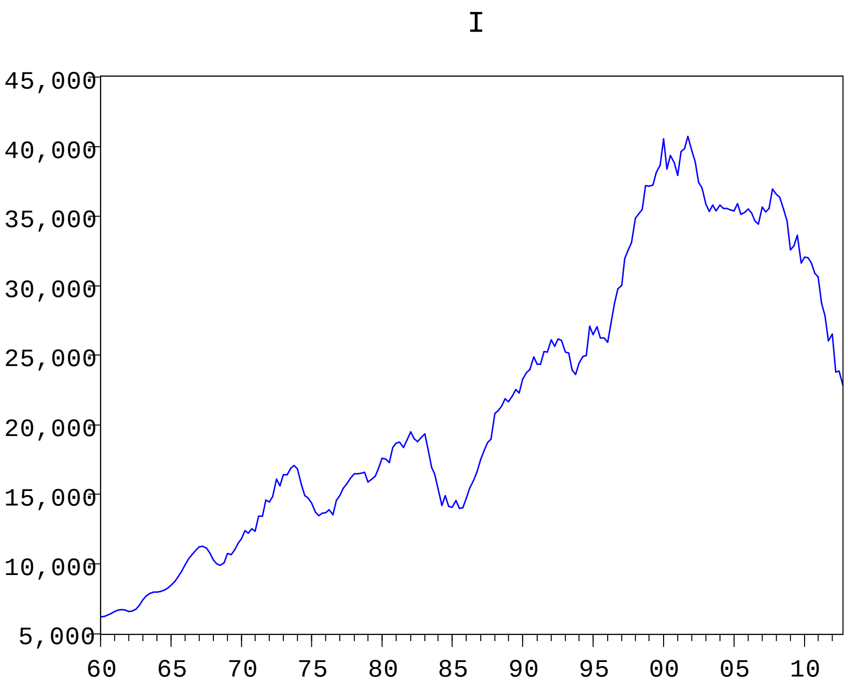

Figure 19: Representation of Gross Fixed Capital Formation Time Series

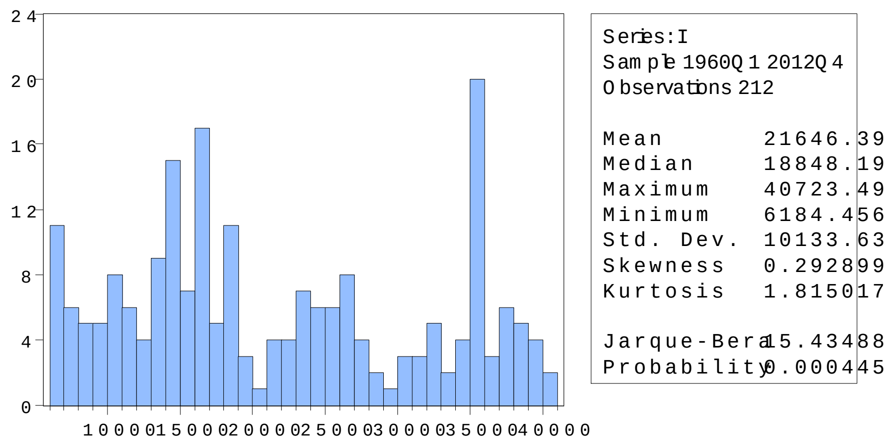

Figure 20: Histogram of Gross Fixed Capital Formation

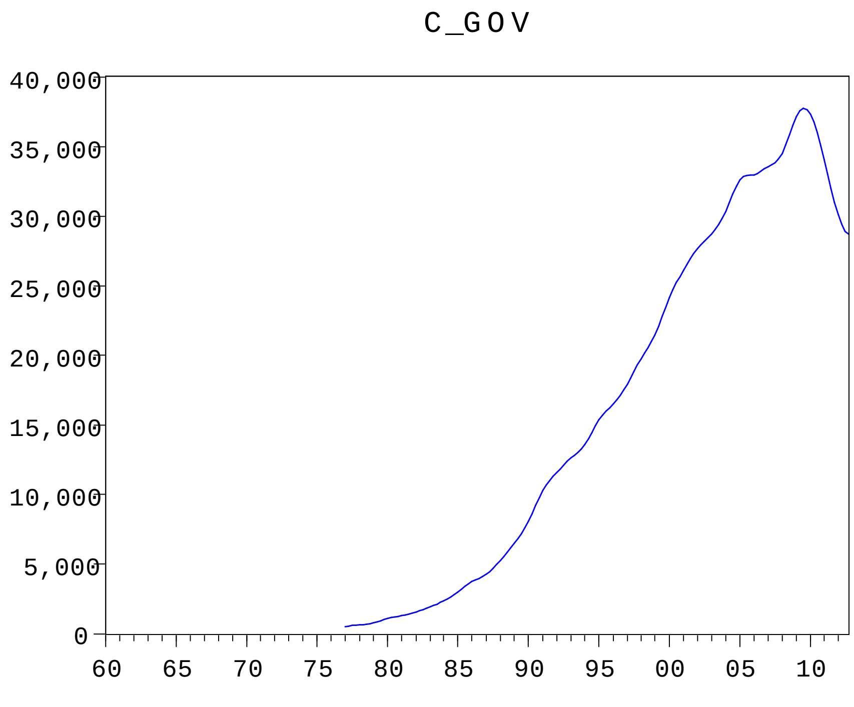

Figure 21: Representation of General Government Final Consumption Expenditure Time Series

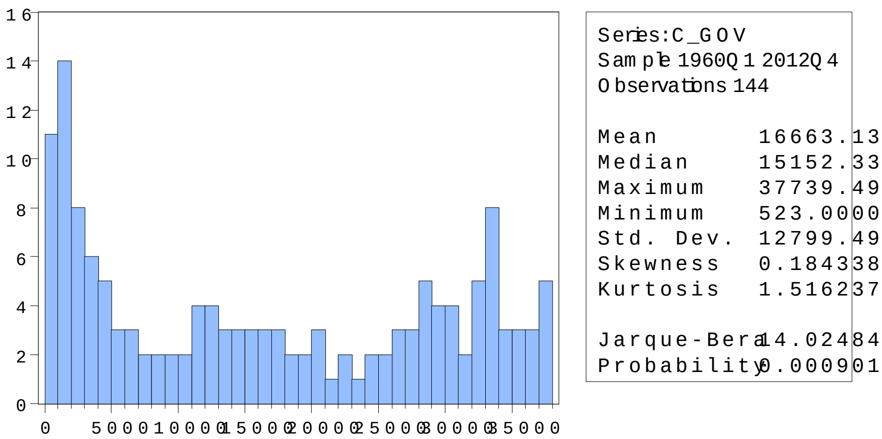

Figure 22: Histogram of General Government Final Consumption Expenditure

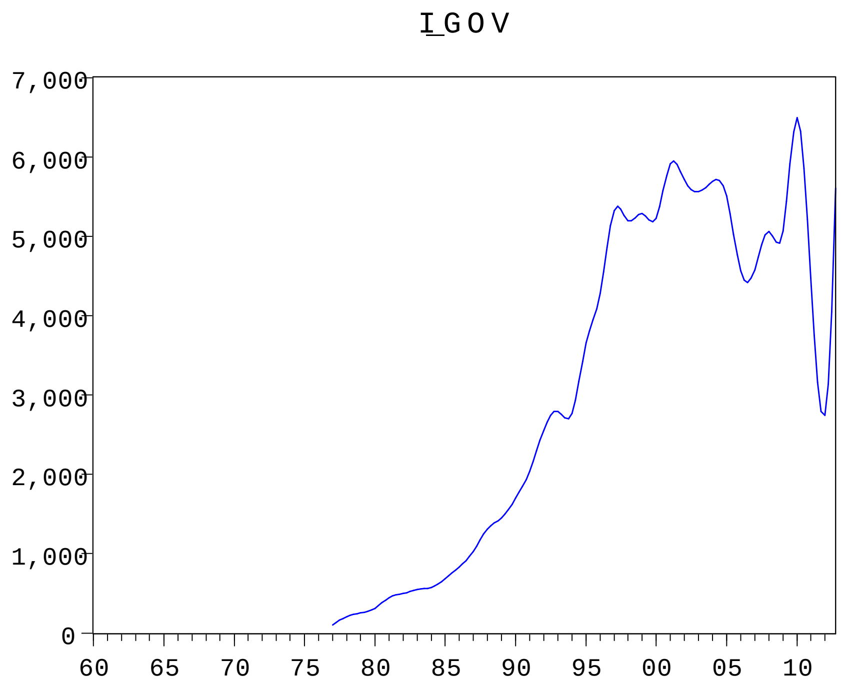

Figure 23: Representation of General Government Gross Fixed Capital Formation Time Series

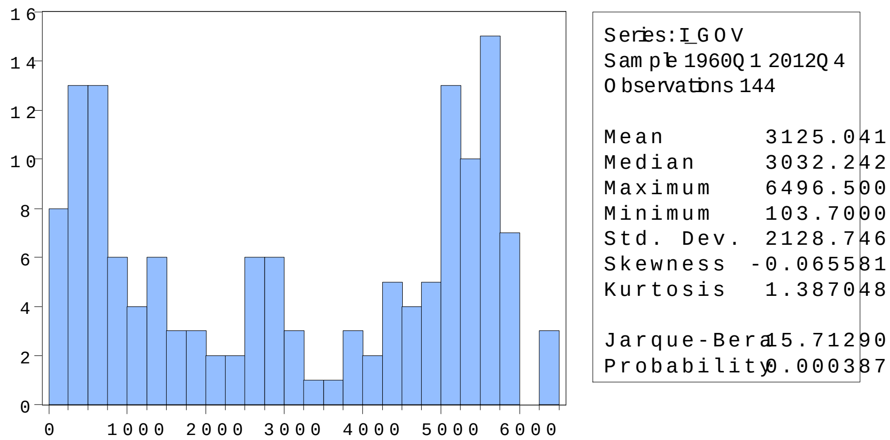

Figure 24: Histogram of General Government Gross Fixed Capital Formation

10.2 Variable in Logarithms

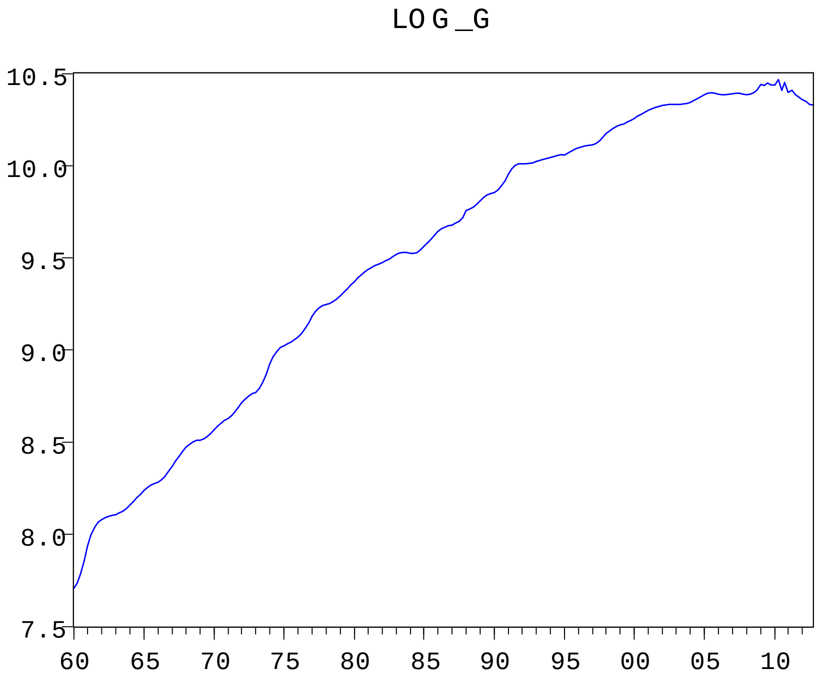

Figure 25: Representation of Log Real Government Final Consumption Expenditure

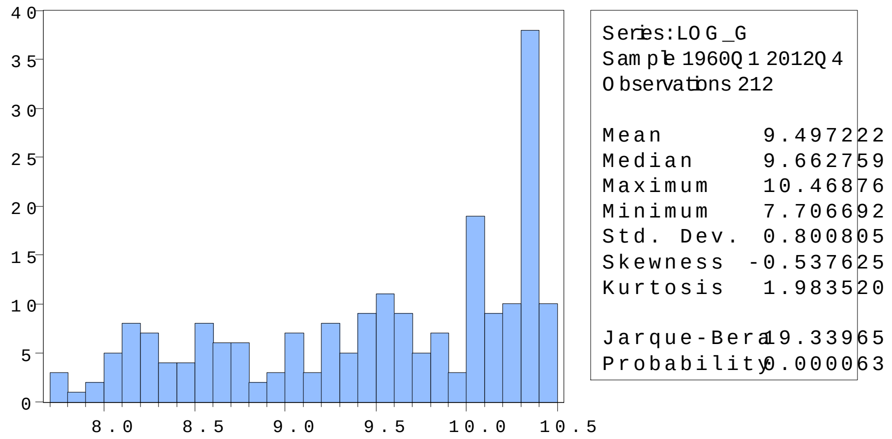

Figure 26: Histogram of Log Real Government Final Consumption Expenditure

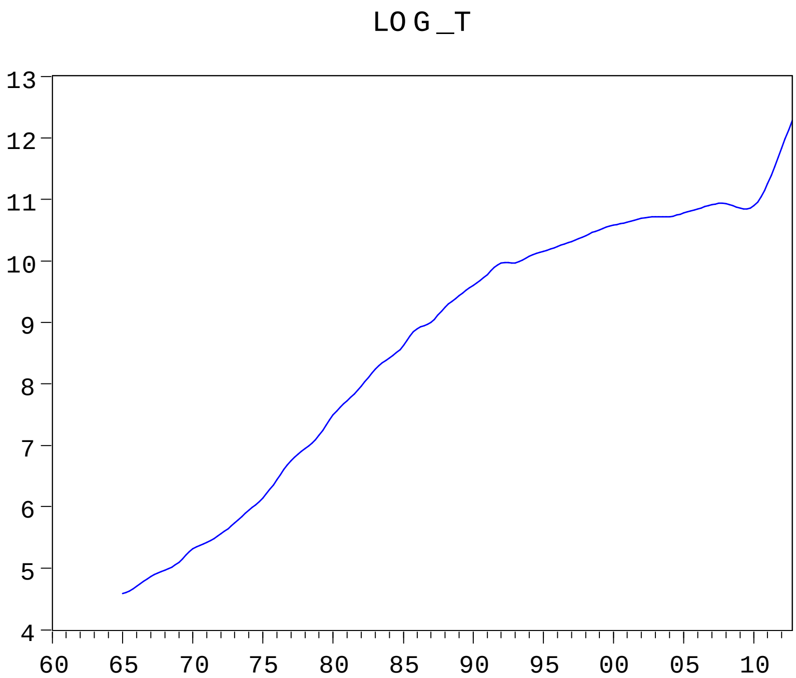

Figure 27: Representation of Log Total Tax Revenue

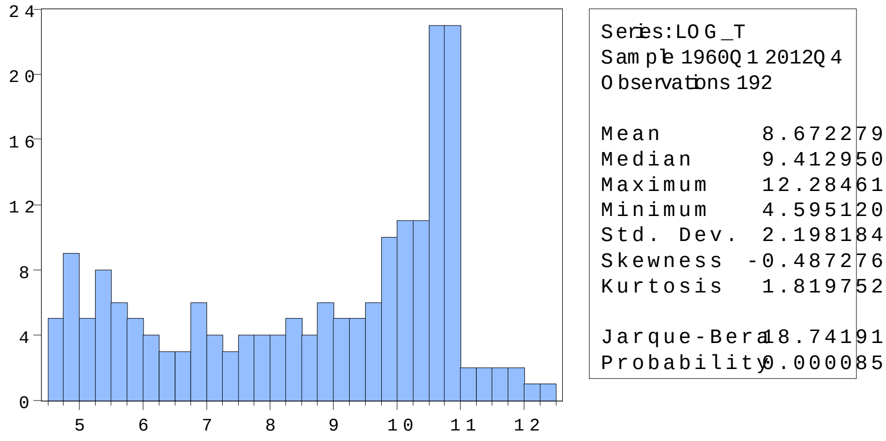

Figure 28: Histogram of Log Total Tax Revenue

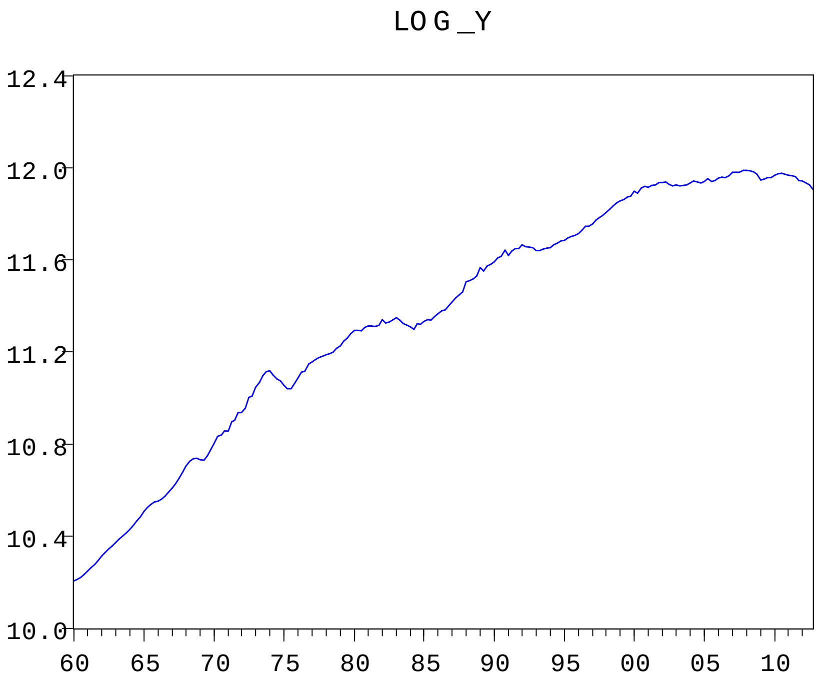

Figure 29: Representation of Log Real Gross Domestic Product

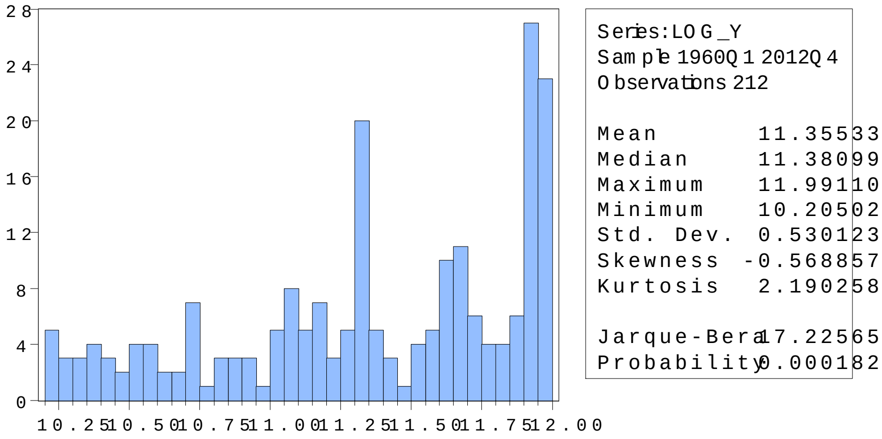

Figure 30: Histogram of Log Real Gross Domestic Product

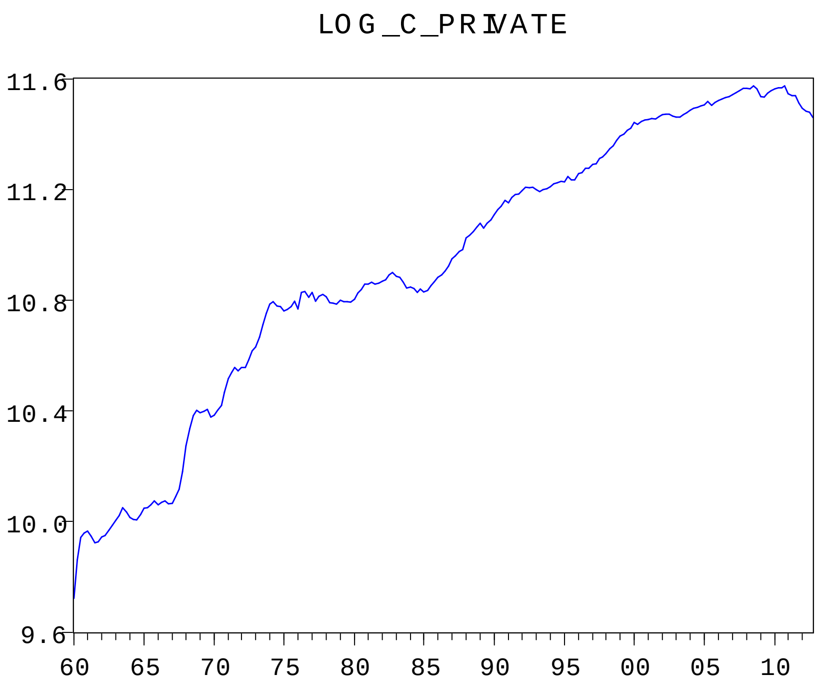

Figure 31: Representation of Log Private Consumption Expenditure

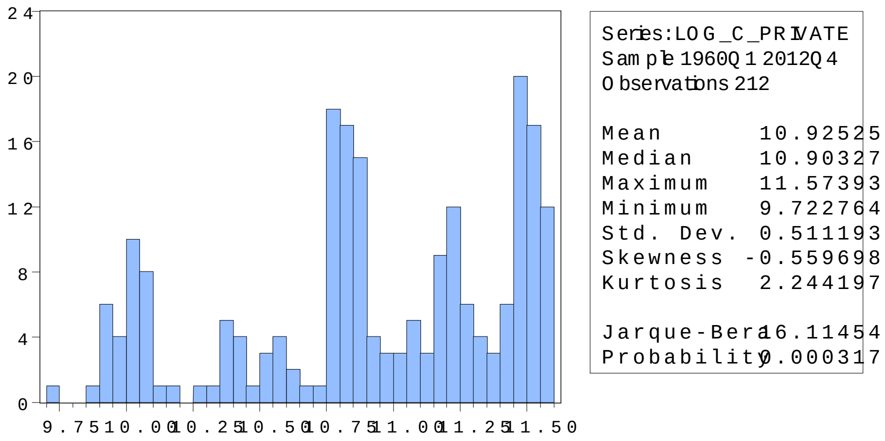

Figure 32: Histogram of Log Private Consumption Expenditure

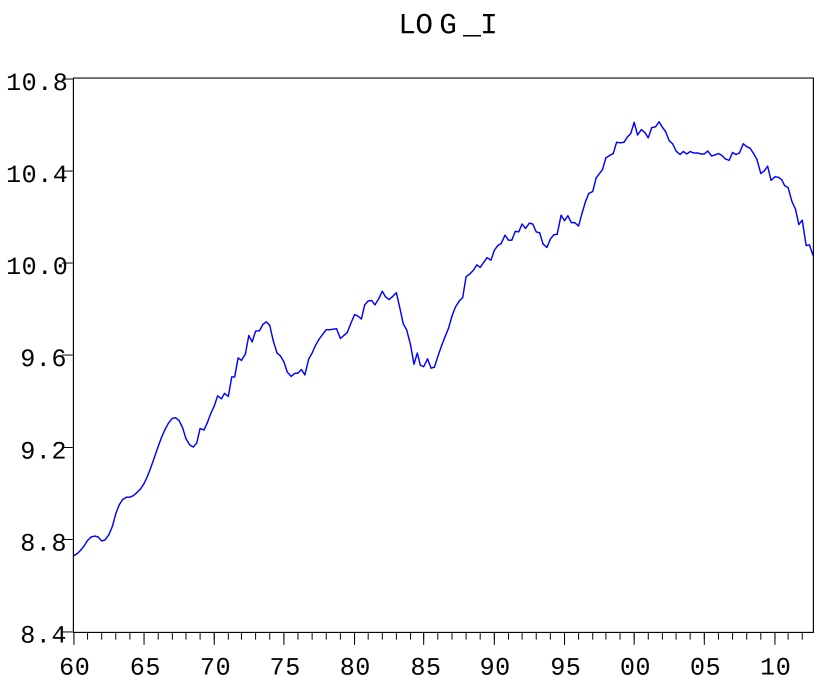

Figure 33: Representation of Log Gross Fixed Capital Formation

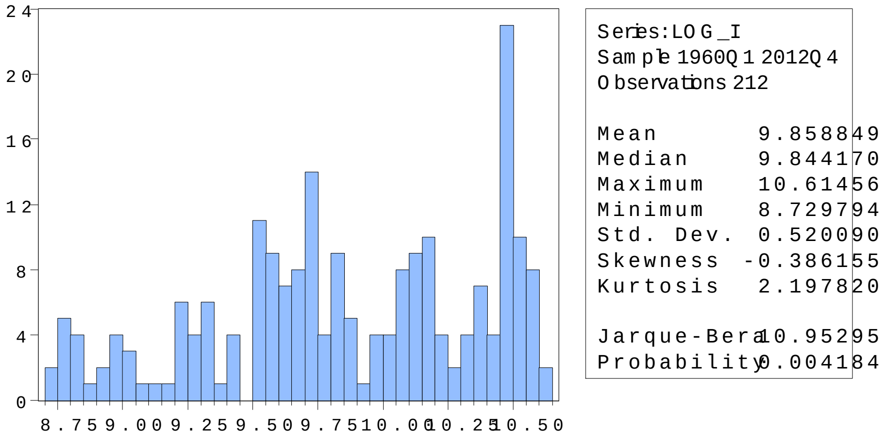

Figure 34: Histogram of Log Gross Fixed Capital Formation

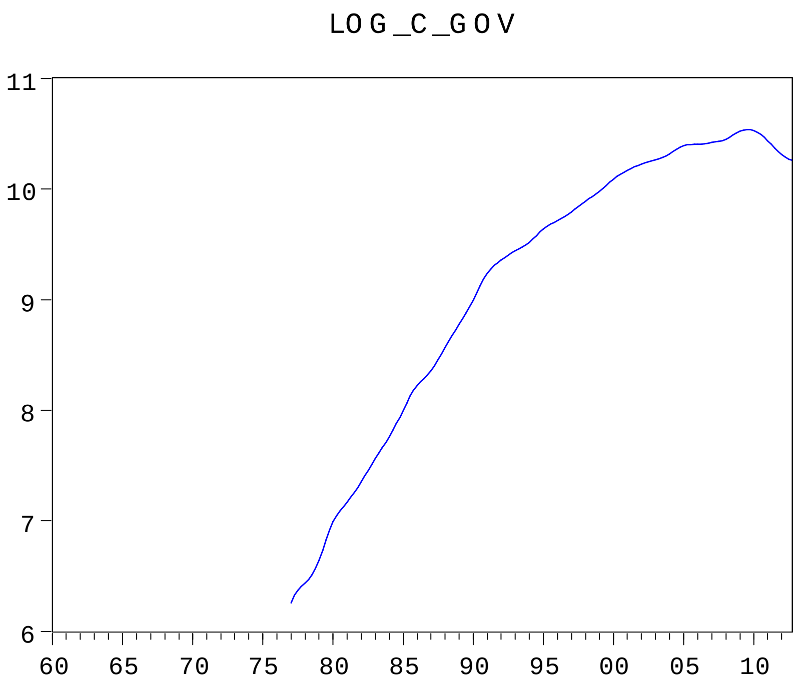

Figure 35: Representation of Log General Government Final Consumption Expenditure

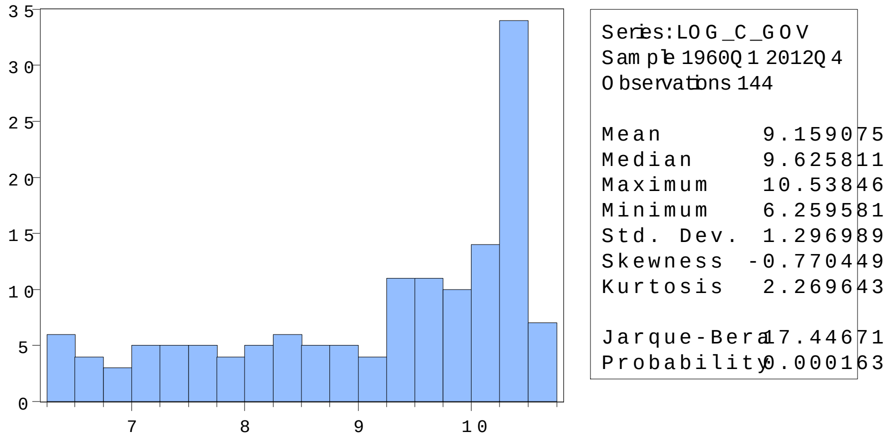

Figure 36: Histogram of Log General Government Final Consumption Expenditure

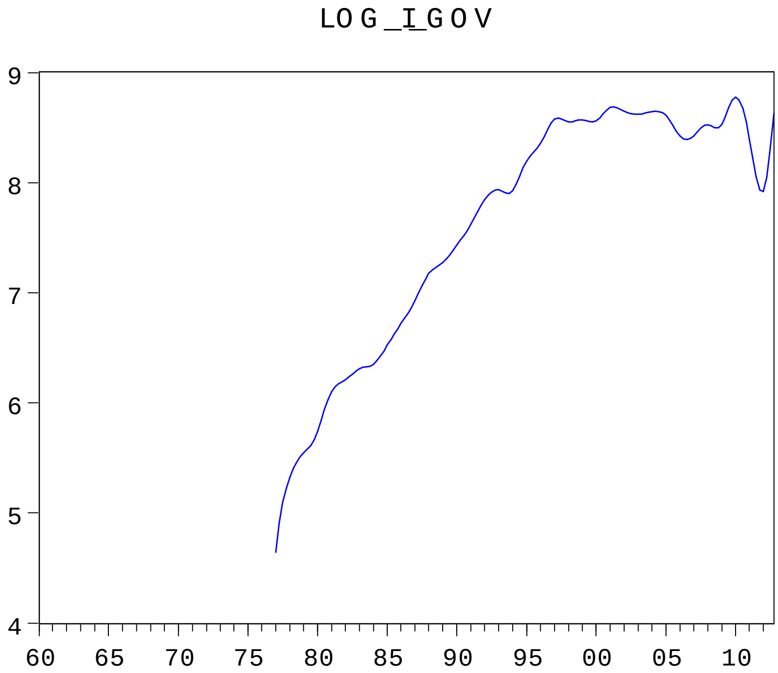

Figure 37: Representation of Log General Government Gross Fixed Capital Formation

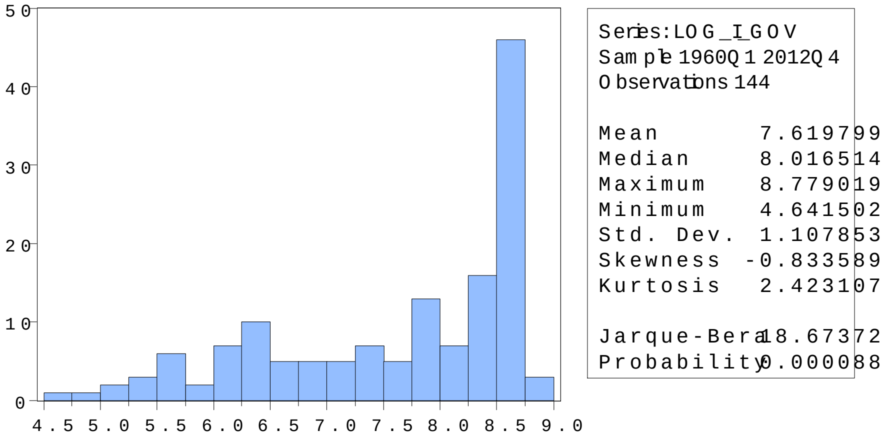

Figure 38: Histogram of Log General Government Gross Fixed Capital Formation

10.2.1 Stationarity Analysis for Main Variables

The choice of the stationarity test specification, especially for the Log Real GDP and for Log Real Government Final Consumption Expenditure is controversial and reflects a complex debate in the literature. There are several authors addressing the question, whether if real GDP has, or not, a unit root, who find very hard to distinguish between a model with a unit root and a stationary model with a root lower but close to one. For instance Campbell and Mankiw [1987] argue that GDP has indeed a unit root, while Stock and Watson [1, page 55] "estimated that the 90% confidence interval for the largest autoregressive root in the time series for log real GDP in the U.S. runs from 0.96 to 1.10." (Blinder, [2004])

This question, more than a statistical question, matters for an economic perspective since it implies different economic dynamics for the business cycle. For instance, as defended by Lucke [2005, :2] for the German economy:

"(...) impulse responses at the five-year horizon imply shock magnification by a factor of 1.6 under difference stationary specification while the same shock dies out rapidly (shrinkage factor of 0.15) under a trend stationary model estimated from the same set of data."

The critical question concerns the correct specification of the underlying trend imposed by the unit root tests in order to conclude about the stationary nature: stationary, trend-stationary or difference stationary. Since this topic deserves a separate research alone, it will be followed the literature where the log of real gdp and the log of real government final consumption expenditure are used directly in stationary VARs.

Since the *VAR* only accepts variables that are stationary and integrated of the same order this lead us to another important assumption: the log of total tax revenue variable is undoubtedly non-stationary and integrated of order 2, therefore will be excluded. Nevertheless, the other two variables are controversial and inconclusive, therefore in order to build the model it will be taken as hypothesis that these variables are also stationary, since both models reject the significance of the intercept parameter under the modelization with a linear trend.

In sum, the VAR will be built with the logarithmic form of G and Y as it is commonly done in the literature (for instance, see Pereira and Wemans [2013] for the Portuguese economy) together with a stationary transition variable, all variables assumed to be integrated of order 0.

Table 14: Stationarity Analysis for Log G

Variable G	**Pvalue**	**AIC**	**SC**	**HC**
ADF with constant	0.0262**	-6.69	-6.56	-6.64
ADF with constant plus linear trend	1.00	-6.71	-6.60	-6.67
PP with constant	0***	-5.88	-5.85	-5.87
PP with constant plus linear trend	0.99	-5.90	-5.85	-5.88
KPSS with constant	1.808755***	2.40	2.41	2.40
KPSS with constant plus linear trend	0.457542***	-0.57	-0.54	-0.56

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

Table 15: Stationarity Analysis for Log G

Variable Y	**Pvalue**	**AIC**	**SC**	**HC**
ADF with constant	0.0054***	-6.20	-6.14	-6.18
ADF with constant plus linear trend	0.99	-6.20	-6.12	-6.16
PP with constant	0.0004***	-6.11	-6.07	-6.09
PP with constant plus linear trend	0.99	-6.11	-6.06	-6.09
KPSS with constant	1.790703***	1.57	1.59	1.58
KPSS with constant plus linear trend	0.386049***	-1.36	-1.33	-1.35

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

Table 16: Unit Root Augmented Dickey-Fuller test statistic for Log Real Government Final Consumption Expenditure

Null Hypothesis: LOG_G has a unit root				
Exogenous: Constant				
Lag Length: 6 (Automatic based on SIC, MAXLAG=14)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-3.12602	0.0262
Test critical values:	1% level		-3.462253	
	5% level		-2.875468	
	10% level		-2.574271	
*MacKinnon (1996) one-sided p-values.				

Table 17: Unit Root Phillips-Perron test statistic for Log Real Government Final Consumption Expenditure

Null Hypothesis: LOG_G has a unit root				
Exogenous: Constant				
Bandwidth: 7 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-5.596303	0
Test critical values:	1% level		-3.461327	
	5% level		-2.875062	
	10% level		-2.574054	
*MacKinnon (1996) one-sided p-values.				

Table 18: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log Real Government Final Consumption Expenditure

Null Hypothesis: LOG_G is stationary				
Exogenous: Constant				
Bandwidth: 11 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				1.808755
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 19: Unit Root Augmented Dickey-Fuller test statistic for Log Total Tax Revenue

Null Hypothesis: LOG_T has a unit root				
Exogenous: Constant, Linear Trend				
Lag Length: 4 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-2.685677	0.2438
Test critical values:	1% level		-4.007882	
	5% level		-3.434036	
	10% level		-3.140923	
*MacKinnon (1996) one-sided p-values.				

Table 20: Unit Root Phillips-Perron test statistic for Log Total Tax Revenue

Null Hypothesis: LOG_T has a unit root				
Exogenous: Constant, Linear Trend				
Bandwidth: 10 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-1.253548	0.8957
Test critical values:	1% level		-4.006824	
	5% level		-3.433525	
	10% level		-3.140623	
*MacKinnon (1996) one-sided p-values.				

Table 21: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log Total Tax Revenue

Null Hypothesis: LOG_T is stationary				
Exogenous: Constant, Linear Trend				
Bandwidth: 11 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.395373
Asymptotic critical values*:		1% level		0.216
		5% level		0.146
		10% level		0.119
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 22: Unit Root Augmented Dickey-Fuller test statistic for Log Real Gross Domestic Product

Null Hypothesis: LOG_Y has a unit root				
Exogenous: Constant				
Lag Length: 2 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-3.657441	0.0054
Test critical values:	1% level		-3.46163	
	5% level		-2.875195	
	10% level		-2.574125	
*MacKinnon (1996) one-sided p-values.				

Table 23: Unit Root Phillips-Perron test statistic for Log Real Gross Domestic Product

Null Hypothesis: LOG_Y has a unit root				
Exogenous: Constant				
Bandwidth: 7 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-4.383725	0.0004
Test critical values:	1% level		-3.461327	
	5% level		-2.875062	
	10% level		-2.574054	
*MacKinnon (1996) one-sided p-values.				

Table 24: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log Real Gross Domestic Product

Null Hypothesis: LOG_Y is stationary				
Exogenous: Constant				
Bandwidth: 11 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				1.790703
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 25: Unit Root Augmented Dickey-Fuller test statistic for Log Private Consumption Expenditure

Null Hypothesis: LOG_C_PRIVATE has a unit root				
Exogenous: Constant				
Lag Length: 4 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-2.864488	0.0513
Test critical values:	1% level		-3.461938	
	5% level		-2.87533	
	10% level		-2.574198	
*MacKinnon (1996) one-sided p-values.				

Table 26: Unit Root Phillips-Perron test statistic for Log Private Consumption Expenditure

Null Hypothesis: LOG_C_PRIVATE has a unit root				
Exogenous: Constant				
Bandwidth: 4 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-3.363502	0.0134
Test critical values:	1% level		-3.461327	
	5% level		-2.875062	
	10% level		-2.574054	
*MacKinnon (1996) one-sided p-values.				

Table 27: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log Private Consumption Expenditure

Null Hypothesis: LOG_C_PRIVATE is stationary				
Exogenous: Constant				
Bandwidth: 11 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				1.765281
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 28: Unit Root Augmented Dickey-Fuller test statistic for Log Gross Fixed Capital Formation

Null Hypothesis: LOG_I has a unit root				
Exogenous: Constant				
Lag Length: 3 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-2.16342	0.2205
Test critical values:	1% level		-3.461783	
	5% level		-2.875262	
	10% level		-2.574161	
*MacKinnon (1996) one-sided p-values.				

Table 29: Unit Root Phillips-Perron test statistic for Log Gross Fixed Capital Formation

Null Hypothesis: LOG_I has a unit root				
Exogenous: Constant				
Bandwidth: 8 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-2.356834	0.1554
Test critical values:	1% level		-3.461327	
	5% level		-2.875062	
	10% level		-2.574054	
*MacKinnon (1996) one-sided p-values.				

Table 30: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log Gross Fixed Capital Formation

Null Hypothesis: LOG_I is stationary				
Exogenous: Constant				
Bandwidth: 11 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				1.693525
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 31: Unit Root Augmented Dickey-Fuller test statistic for Log Gross Fixed Capital Formation

Null Hypothesis: LOG_C_GOV has a unit root				
Exogenous: Constant, Linear Trend				
Lag Length: 4 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-0.148954	0.9936
Test critical values:	1% level		-4.025426	
	5% level		-3.442474	
	10% level		-3.145882	
*MacKinnon (1996) one-sided p-values.				

Table 32: Unit Root Phillips-Perron test statistic for Log General Government Final Consumption Expenditure

Null Hypothesis: LOG_C_GOV has a unit root				
Exogenous: Constant, Linear Trend				
Bandwidth: 7 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			1.301537	1
Test critical values:	1% level		-4.023506	
	5% level		-3.441552	
	10% level		-3.145341	
*MacKinnon (1996) one-sided p-values.				

Table 33: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log General Government Final Consumption Expenditure

Null Hypothesis: LOG_C_GOV is stationary				
Exogenous: Constant, Linear Trend				
Bandwidth: 10 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.356932
Asymptotic critical values*:		1% level		0.216
		5% level		0.146
		10% level		0.119
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 34: Unit Root Augmented Dickey-Fuller test statistic for Log General Government Gross Fixed Capital Formation

Null Hypothesis: LOG_I_GOV has a unit root				
Exogenous: Constant				
Lag Length: 4 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-3.344558	0.0147
Test critical values:	1% level		-3.477835	
	5% level		-2.882279	
	10% level		-2.577908	
*MacKinnon (1996) one-sided p-values.				

Table 35: Unit Root Phillips-Perron test statistic for Log General Government Gross Fixed Capital Formation

Null Hypothesis: LOG_I_GOV has a unit root				
Exogenous: Constant				
Bandwidth: 4 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-3.783379	0.0039
Test critical values:	1% level		-3.476472	
	5% level		-2.881685	
	10% level		-2.577591	
*MacKinnon (1996) one-sided p-values.				

Table 36: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log General Government Gross Fixed Capital Formation

Null Hypothesis: LOG_I_GOV is stationary				
Exogenous: Constant				
Bandwidth: 10 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				1.237354
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 37: Unit Root Augmented Dickey-Fuller test statistic for Log Total Tax Revenue I(2)

Null Hypothesis: D(LOG_T,2) has a unit root				
Exogenous: None				
Lag Length: 13 (Automatic based on SIC, MAXLAG=14)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-3.98864	0.0001
Test critical values:	1% level		-2.578167	
	5% level		-1.942645	
	10% level		-1.615502	
*MacKinnon (1996) one-sided p-values.				

Table 38: Unit Root Phillips-Perron test statistic for Log Total Tax Revenue I(2)

Null Hypothesis: D(LOG_T,2) has a unit root				
Exogenous: None				
Bandwidth: 14 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-3.383665	0.0008
Test critical values:	1% level		-2.577255	
	5% level		-1.942517	
	10% level		-1.615583	
*MacKinnon (1996) one-sided p-values.				

Table 39: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log Total Tax Revenue I(2)

Null Hypothesis: D(LOG_T,2) is stationary				
Exogenous: Constant				
Bandwidth: 5 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.159069
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 40: Unit Root Augmented Dickey-Fuller test statistic for Log Private Consumption Expenditure

Null Hypothesis: D(LOG_C) has a unit root				
Exogenous: Constant, Linear Trend				
Lag Length: 3 (Automatic based on SIC, MAXLAG=14)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-7.646081	0
Test critical values:	1% level		-4.003005	
	5% level		-3.431682	
	10% level		-3.139538	
*MacKinnon (1996) one-sided p-values.				

Table 41: Unit Root Phillips-Perron test statistic for Log Private Consumption Expenditure

Null Hypothesis: D(LOG_C) has a unit root				
Exogenous: Constant, Linear Trend				
Bandwidth: 0 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-10.72046	0
Test critical values:	1% level		-4.002354	
	5% level		-3.431368	
	10% level		-3.139353	
*MacKinnon (1996) one-sided p-values.				

Table 42: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log Private Consumption Expenditure

Null Hypothesis: D(LOG_C) is stationary				
Exogenous: Constant, Linear Trend				
Bandwidth: 3 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.065896
Asymptotic critical values*:		1% level		0.216
		5% level		0.146
		10% level		0.119
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 43: Unit Root Augmented Dickey-Fuller test statistic for Log Gross Fixed Capital Formation

Null Hypothesis: D(LOG_I) has a unit root				
Exogenous: Constant, Linear Trend				
Lag Length: 2 (Automatic based on SIC, MAXLAG=14)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-4.934254	0.0004
Test critical values:	1% level		-4.002786	
	5% level		-3.431576	
	10% level		-3.139475	
*MacKinnon (1996) one-sided p-values.				

Table 44: Unit Root Phillips-Perron test statistic for Log Gross Fixed Capital Formation

Null Hypothesis: D(LOG_I) has a unit root				
Exogenous: Constant, Linear Trend				
Bandwidth: 8 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-13.27631	0
Test critical values:	1% level		-4.002354	
	5% level		-3.431368	
	10% level		-3.139353	
*MacKinnon (1996) one-sided p-values.				

Table 45: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log Gross Fixed Capital Formation

Null Hypothesis: D(LOG_I) is stationary				
Exogenous: Constant, Linear Trend				
Bandwidth: 8 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.092684
Asymptotic critical values*:		1% level		0.216
		5% level		0.146
		10% level		0.119
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 46: Unit Root Augmented Dickey-Fuller test statistic for Log General Government Final Consumption Expenditure

Null Hypothesis: D(LOG_C_GOV) has a unit root				
Exogenous: None				
Bandwidth: 0 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-1.830251	0.0641
Test critical values:	1% level		-2.581349	
	5% level		-1.94309	
	10% level		-1.61522	
*MacKinnon (1996) one-sided p-values.				

Table 47: Unit Root Phillips-Perron test statistic for Log General Government Final Consumption Expenditure

Null Hypothesis: D(LOG_C_GOV) has a unit root				
Exogenous: None				
Bandwidth: 0 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-1.830251	0.0641
Test critical values:	1% level		-2.581349	
	5% level		-1.94309	
	10% level		-1.61522	
*MacKinnon (1996) one-sided p-values.				

Table 48: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Log General Government Final Consumption Expenditure

Null Hypothesis: D(LOG_C_GOV) is stationary				
Exogenous: Constant, Linear Trend				
Bandwidth: 7 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.074863
Asymptotic critical values*:		1% level		0.216
		5% level		0.146
		10% level		0.119
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

10.3 Transition Variables

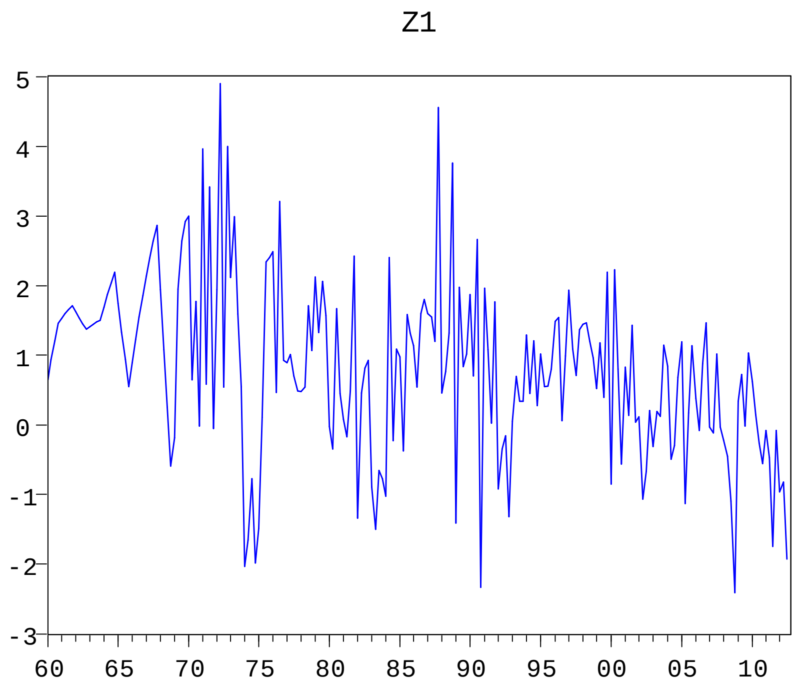

Figure 39: Representation of GDP Growth

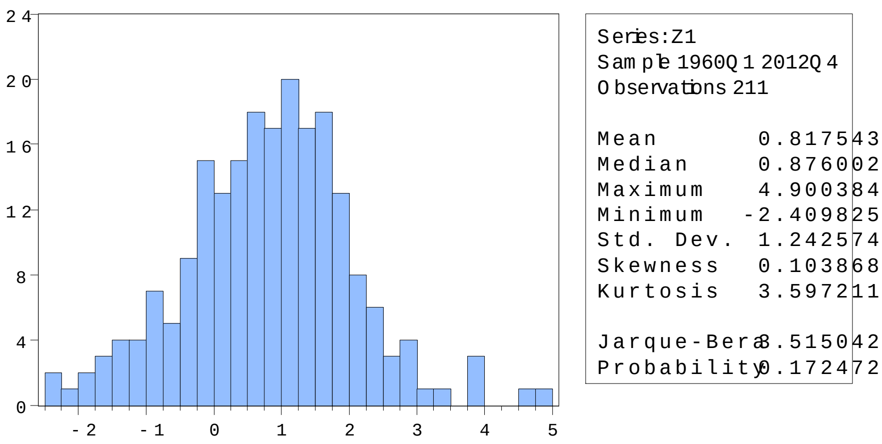

Figure 40: Histogram of GDP Growth

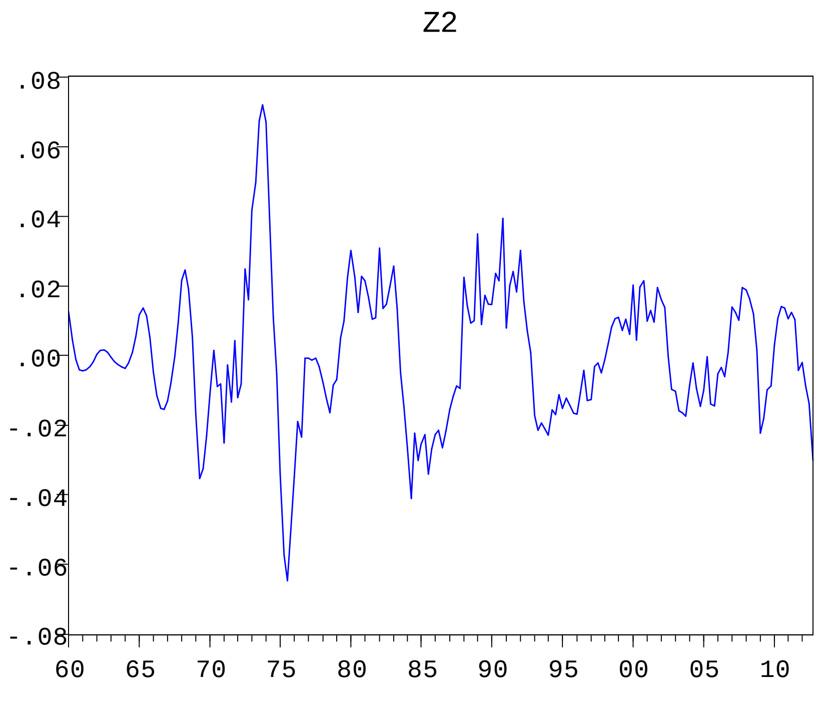

Figure 41: Representation of Output Gap

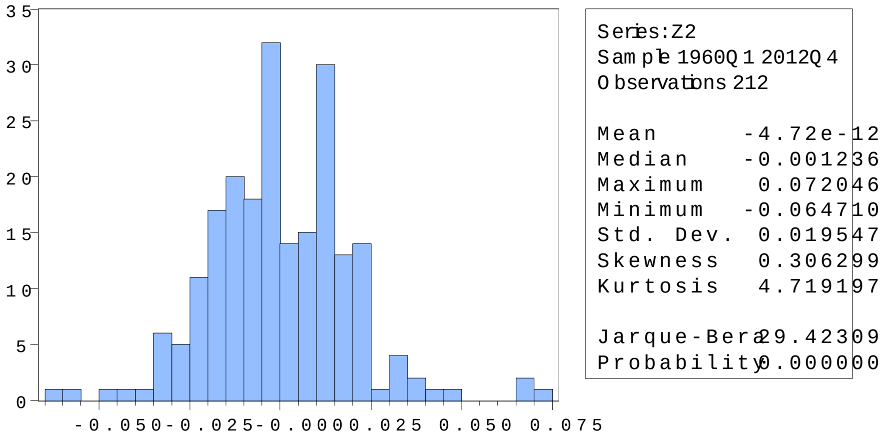

Figure 42: Histogram of Output Gap

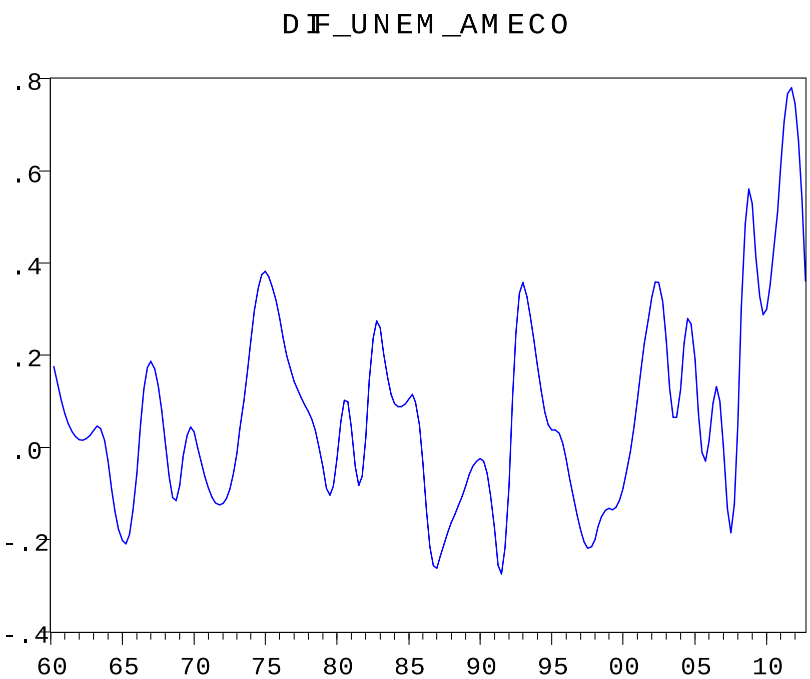

Figure 43: Representation of Change in the Unemployment Rate

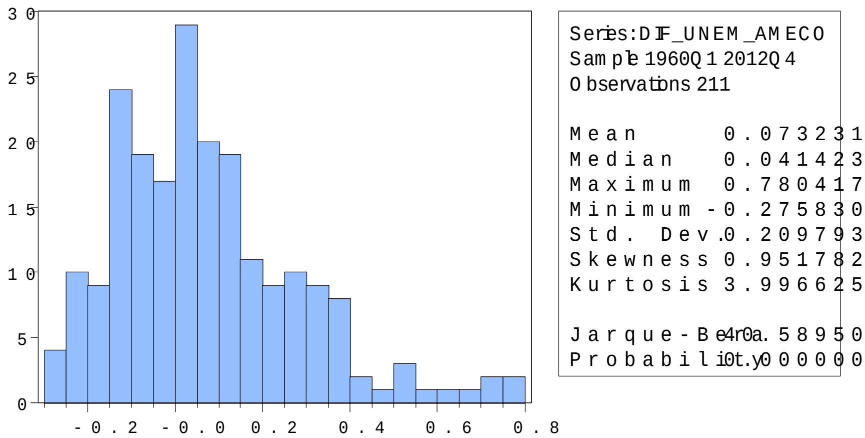

Figure 44: Histogram of Change in the Unemployment Rate

10.3.1 Stationarity Analysis for Transition Variables

Table 49: Unit Root Augmented Dickey-Fuller test statistic for GDP Growth

Null Hypothesis: Z1 has a unit root				
Exogenous: None				
Lag Length: 1 (Automatic based on SIC, MAXLAG=14)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-4.339668	0
Test critical values:	1% level		-2.576073	
	5% level		-1.942353	
	10% level		-1.615688	
*MacKinnon (1996) one-sided p-values.				

Table 50: Unit Root Phillips-Perron test statistic for GDP Growth

Null Hypothesis: Z1 has a unit root				
Exogenous: None				
Bandwidth: 9 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-9.522973	0
Test critical values:	1% level		-2.57602	
	5% level		-1.942346	
	10% level		-1.615693	
*MacKinnon (1996) one-sided p-values.				

Table 51: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for GDP Growth

Null Hypothesis: Z1 is stationary				
Exogenous: Constant, Linear Trend				
Bandwidth: 7 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.062184
Asymptotic critical values*:		1% level		0.216
		5% level		0.146
		10% level		0.119
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 52: Unit Root Augmented Dickey-Fuller test statistic for Output Gap

Null Hypothesis: Z2 has a unit root				
Exogenous: None				
Lag Length: 2 (Automatic based on SIC, MAXLAG=14)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-5.492008	0
Test critical values:	1% level		-2.576073	
	5% level		-1.942353	
	10% level		-1.615688	
*MacKinnon (1996) one-sided p-values.				

Table 53: Unit Root Phillips-Perron test statistic for Output Gap

Null Hypothesis: Z2 has a unit root				
Exogenous: None				
Bandwidth: 5 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-4.701678	0
Test critical values:	1% level		-2.575968	
	5% level		-1.942338	
	10% level		-1.615698	
*MacKinnon (1996) one-sided p-values.				

Table 54: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Output Gap

Null Hypothesis: Z2 is stationary				
Exogenous: Constant				
Bandwidth: 9 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.019927
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 55: Unit Root Augmented Dickey-Fuller test statistic for Change in the Unemployment Rate

Null Hypothesis: DIF_UNEM_AMECO has a unit root				
Exogenous: None				
Lag Length: 3 (Automatic based on SIC, MAXLAG=3)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-2.734097	0.0063
Test critical values:	1% level		-2.576181	
	5% level		-1.942368	
	10% level		-1.615679	
*MacKinnon (1996) one-sided p-values.				

Table 56: Unit Root Phillips-Perron test statistic for Change in the Unemployment Rate

Null Hypothesis: DIF_UNEM_AMECO has a unit root				
Exogenous: None				
Bandwidth: 0 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-1.671429	0.0895
Test critical values:	1% level		-2.57602	
	5% level		-1.942346	
	10% level		-1.615693	

Table 57: Unit Root Kwiatkowski-Phillips-Schmidt-Shin test statistic for Change in the Unemployment Rate

Null Hypothesis: DIF_UNEM_AMECO is stationary				
Exogenous: Constant, Linear Trend				
Bandwidth: 10 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.173504
Asymptotic critical values*:		1% level		0.216
		5% level		0.146
		10% level		0.119
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

10.3.2 Cointegration Analysis

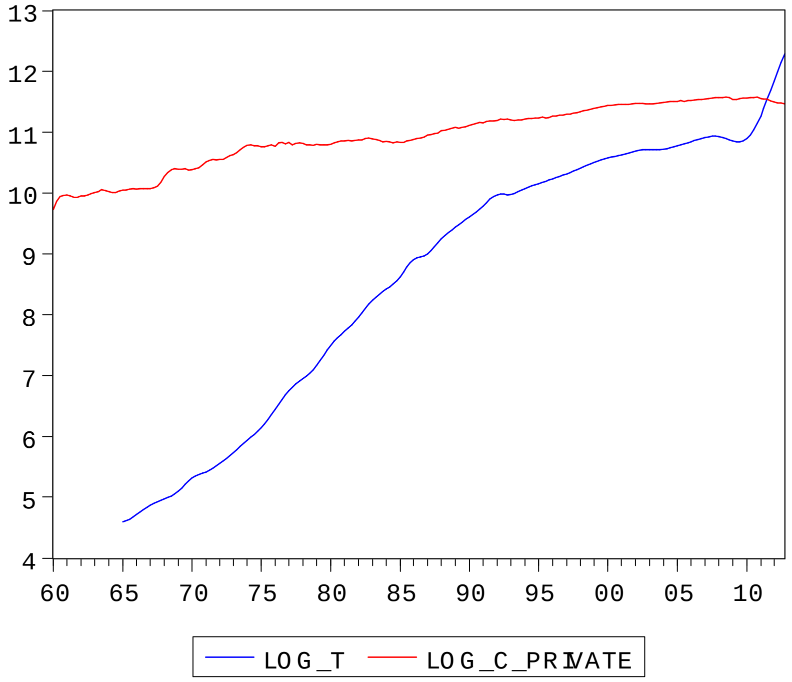

Figure 45: Representation of Log Total Tax Revenue and Log Private Consumption Expenditure Long Run Relationship

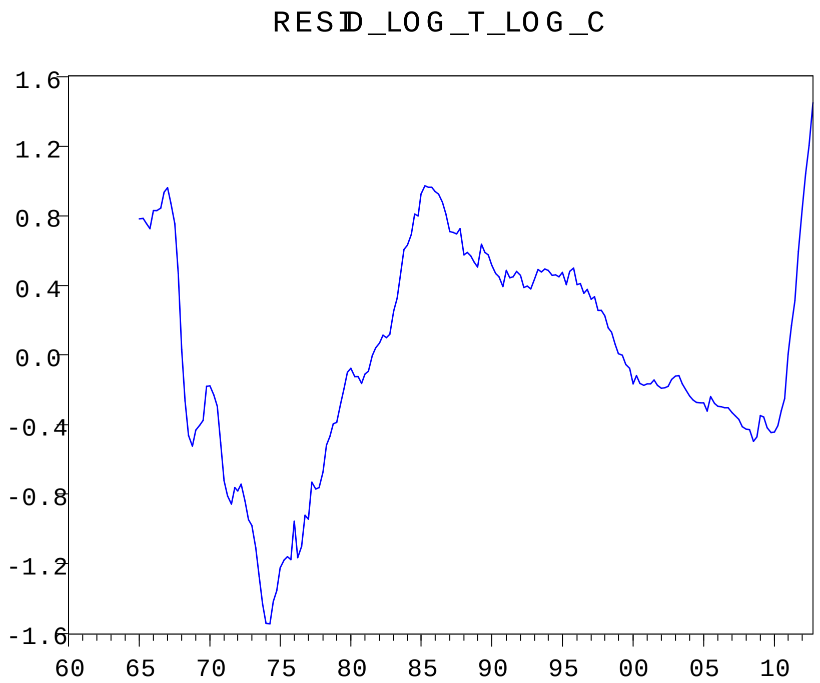

Figure 46: Residuals of Log Total Tax Revenue and Log Private Consumption Expenditure

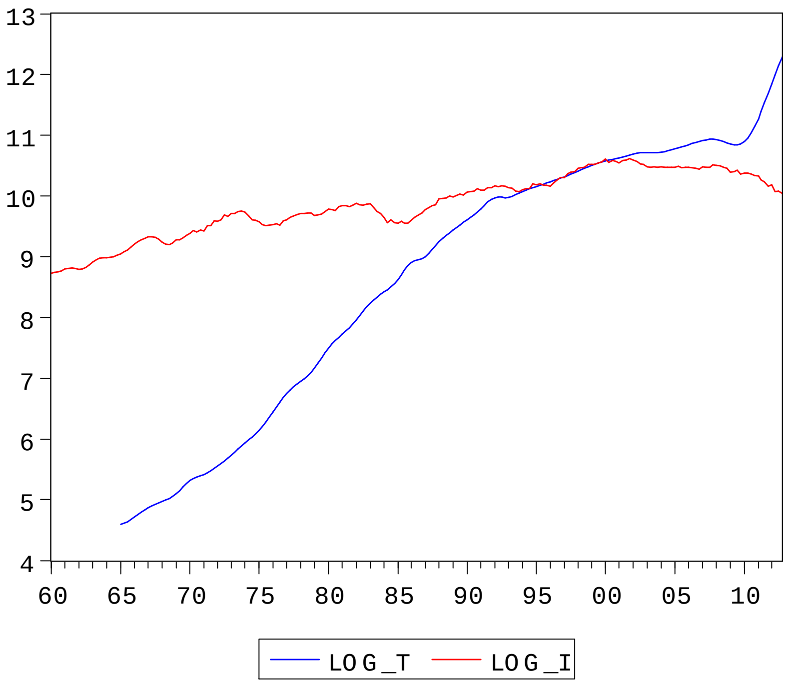

Figure 47: Representation of Log Total Tax Revenue and Log Gross Fixed Capital Formation Long Run Relationship

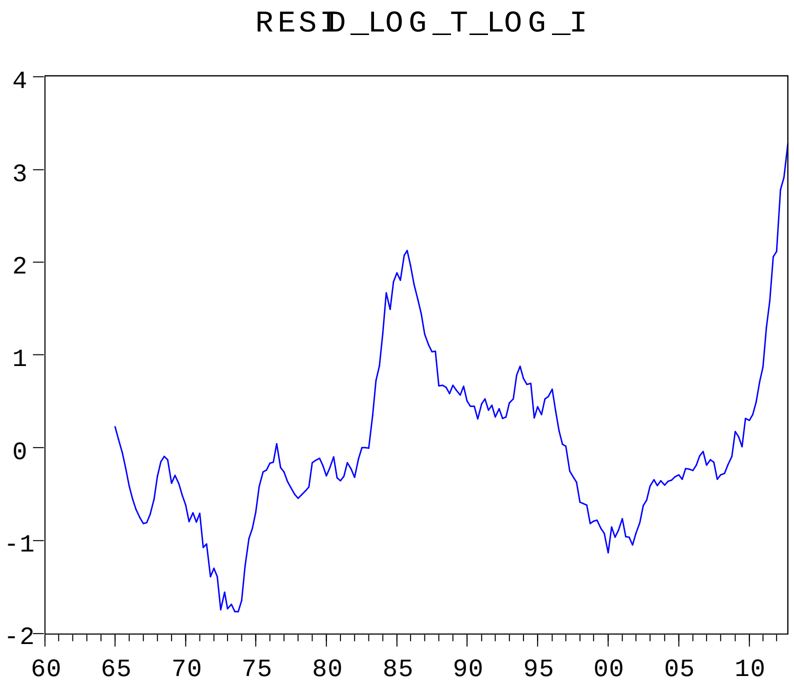

Figure 48: Residuals resulting from the linear combination of Log Total Tax Revenue and Log Gross Fixed Capital Formation

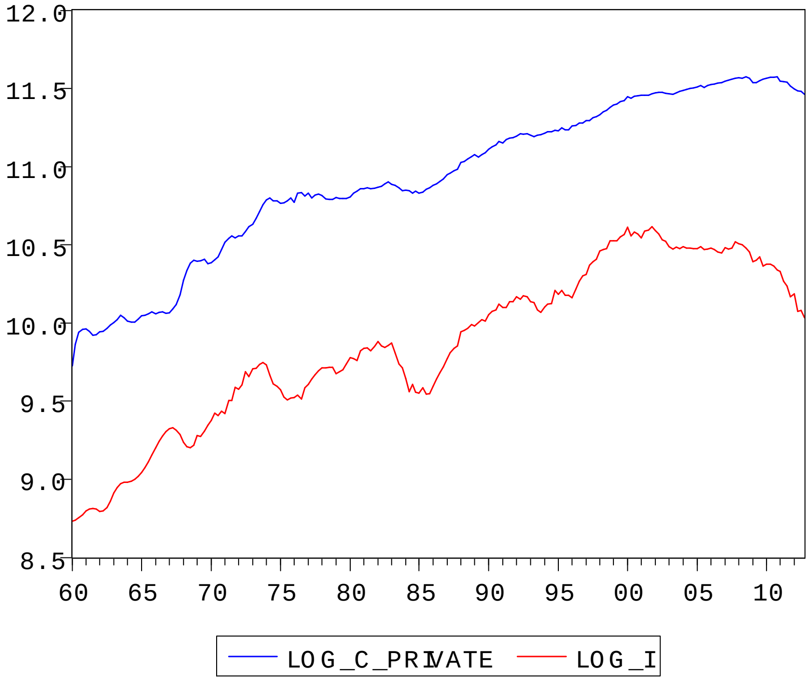

Figure 49: Representation of Log Private Consumption Expenditure and Log Gross Fixed Capital Formation Long Run Relationship

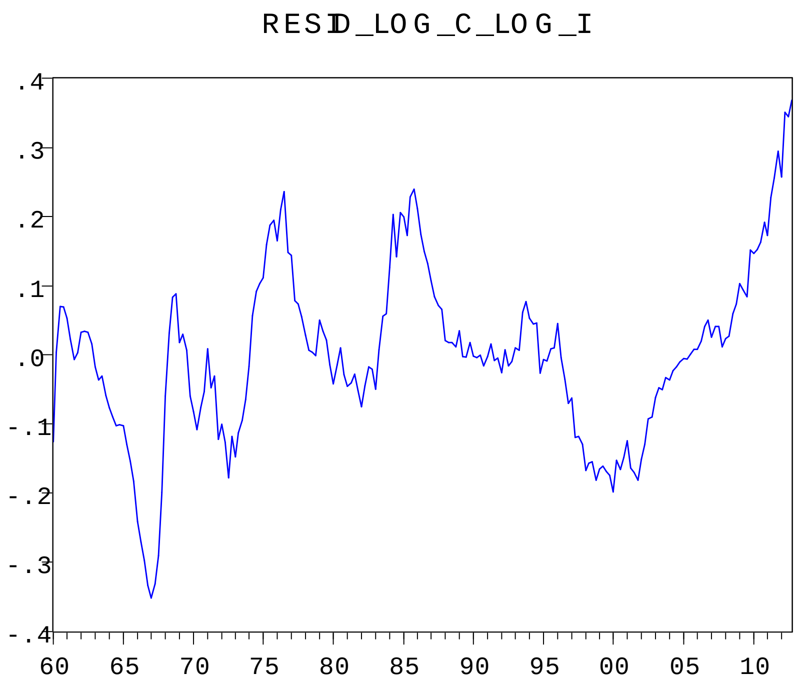

Figure 50: Residuals resulting from the linear combination of Log Private Consumption Expenditure and Log Gross Fixed Capital Formation

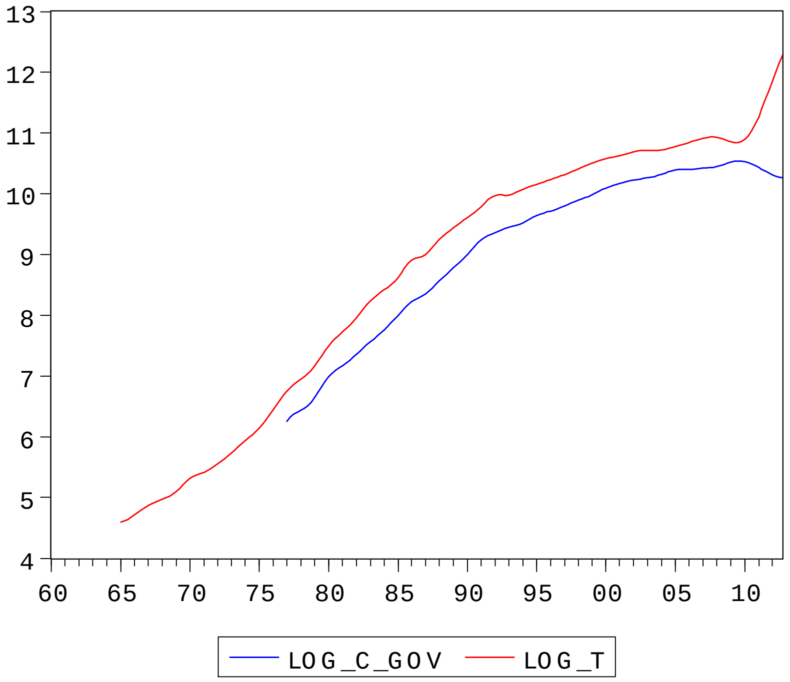

Figure 51: Representation of Log General Government Final Consumption Expenditure and Log Total Tax Revenue Long Run Relationship

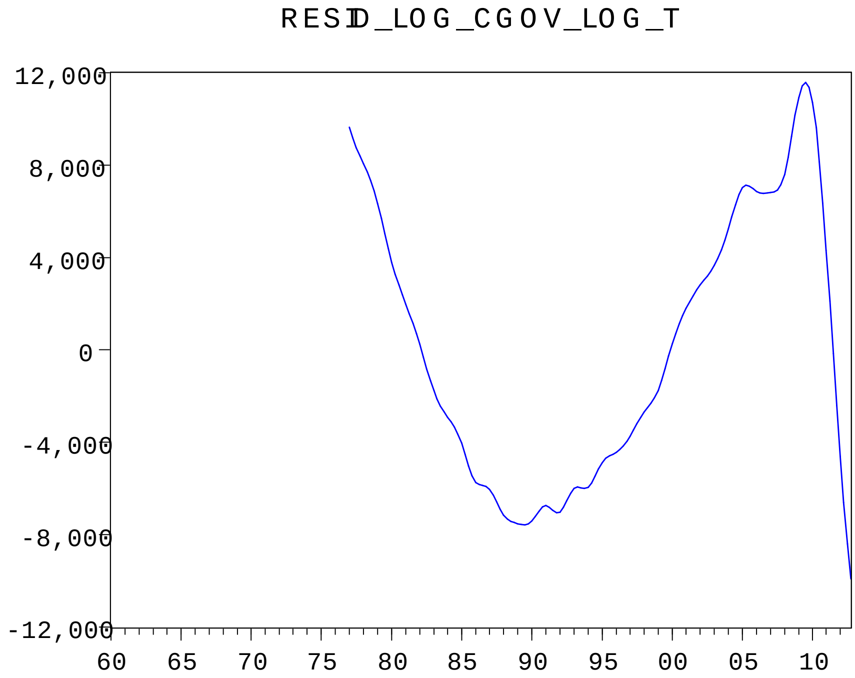

Figure 52: Residuals resulting from the linear combination of Log General Government Final Consumption Expenditure and Log Total Tax Revenue

Table 58: Engle Granger methodology to test the residuals with Augmented Dickey-Fuller test for Total Tax Revenue and Private Consumption Expenditure

Null Hypothesis: RESID_LOG_T_LOG_C has a unit root				
Exogenous: None				
Lag Length: 2 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-1.769292	0.073
Test critical values:	1% level		-2.577255	
	5% level		-1.942517	
	10% level		-1.615583	
*MacKinnon (1996) one-sided p-values.				

Table 59: Engle Granger methodology to test the residuals with Augmented Phillips-Perron test for Total Tax Revenue and Private Consumption Expenditure

Null Hypothesis: RESID_LOG_T_LOG_C has a unit root				
Exogenous: None				
Bandwidth: 8 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-1.449545	0.1372
Test critical values:	1% level		-2.577125	
	5% level		-1.942499	
	10% level		-1.615594	
*MacKinnon (1996) one-sided p-values.				

Table 60: Engle Granger methodology to test the residuals with Kwiatkowski-Phillips-Schmidt-Shin test for Total Tax Revenue and Private Consumption Expenditure

Null Hypothesis: RESID_LOG_T_LOG_C is stationary				
Exogenous: Constant				
Bandwidth: 10 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.230913
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 61: Engle Granger methodology to test the residuals with Augmented Dickey-Fuller test for Total Tax Revenue and Gross Fixed Capital Formation

Null Hypothesis: RESID_LOG_T_LOG_I has a unit root				
Exogenous: None				
Lag Length: 3 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-1.444538	0.1384
Test critical values:	1% level		-2.57732	
	5% level		-1.942527	
	10% level		-1.615577	
*MacKinnon (1996) one-sided p-values.				

Table 62: Engle Granger methodology to test the residuals with Augmented Phillips-Perron test for Total Tax Revenue and Gross Fixed Capital Formation

Null Hypothesis: RESID_LOG_T_LOG_I has a unit root				
Exogenous: None				
Bandwidth: 9 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-0.76305	0.3846
Test critical values:	1% level		-2.577125	
	5% level		-1.942499	
	10% level		-1.615594	
*MacKinnon (1996) one-sided p-values.				

Table 63: Engle Granger methodology to test the residuals with Kwiatkowski-Phillips-Schmidt-Shin test for Total Tax Revenue and Gross Fixed Capital Formation

Null Hypothesis: RESID_LOG_T_LOG_I is stationary				
Exogenous: Constant				
Bandwidth: 10 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.331266
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 64: Engle Granger methodology to test the residuals with Augmented Dickey-Fuller test for Private Consumption Expenditure and Gross Fixed Capital Formation

Null Hypothesis: RESID_LOG_C_LOG_I has a unit root				
Exogenous: None				
Lag Length: 2 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-2.189674	0.0278
Test critical values:	1% level		-2.576073	
	5% level		-1.942353	
	10% level		-1.615688	
*MacKinnon (1996) one-sided p-values.				

Table 65: Engle Granger methodology to test the residuals with Augmented Phillips-Perron test for Private Consumption Expenditure and Gross Fixed Capital Formation

Null Hypothesis: RESID_LOG_C_LOG_I has a unit root				
Exogenous: None				
Bandwidth: 6 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-2.071867	0.037
Test critical values:	1% level		-2.575968	
	5% level		-1.942338	
	10% level		-1.615698	
*MacKinnon (1996) one-sided p-values.				

Table 66: Engle Granger methodology to test the residuals with Kwiatkowski-Phillips-Schmidt-Shin test for Private Consumption Expenditure and Gross Fixed Capital Formation

Null Hypothesis: RESID_LOG_C_LOG_I is stationary				
Exogenous: Constant				
Bandwidth: 11 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.215113
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

Table 67: Engle Granger methodology to test the residuals with Augmented Dickey-Fuller test for General Government Final Consumption Expenditure and Total Tax Revenue

Null Hypothesis: RESID_LOG_CGOV_LOG_T has a unit root				
Exogenous: None				
Lag Length: 4 (Automatic based on SIC, MAXLAG=4)				
			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-2.135045	0.032
Test critical values:	1% level		-2.581705	
	5% level		-1.94314	
	10% level		-1.615189	
*MacKinnon (1996) one-sided p-values.				

Table 68: Engle Granger methodology to test the residuals with Augmented Phillips-Perron test for General Government Final Consumption Expenditure and Total Tax Revenue

Null Hypothesis: RESID_LOG_CGOV_LOG_T has a unit root				
Exogenous: None				
Bandwidth: 9 (Newey-West using Bartlett kernel)				
			Adj. t-Stat	Prob.*
Phillips-Perron test statistic			-1.695425	0.0851
Test critical values:	1% level		-2.581233	
	5% level		-1.943074	
	10% level		-1.615231	
*MacKinnon (1996) one-sided p-values.				

Table 69: Engle Granger methodology to test the residuals with Kwiatkowski-Phillips-Schmidt-Shin test for General Government Final Consumption Expenditure and Total Tax Revenue

Null Hypothesis: RESID_LOG_CGOV_LOG_T is stationary				
Exogenous: Constant				
Bandwidth: 10 (Newey-West using Bartlett kernel)				
				LM-Stat.
Kwiatkowski-Phillips-Schmidt-Shin test statistic				0.391756
Asymptotic critical values*:		1% level		0.739
		5% level		0.463
		10% level		0.347
*Kwiatkowski-Phillips-Schmidt-Shin (1992, Table 1)				

11 Appendix C - Model Estimation

11.1 Specification Tests

11.1.1 Algorithm: Lagrange Multiplier Test

According to Teräsvirta, Tjostheim and Granger [2010] the Lagrange multiplier test can be carried out by two regressions:

Algorithm 2 *1 - Estimate a linear model under H_0: regress Y on X. Compute the residuals $\tilde{\varepsilon}_t, t = 1, ..., T$, and the matrix residual sum of squares SSR_0.*

2. Run an auxiliary regression of $\tilde{\varepsilon}_t$ on $(X,\ Z)$. Collect the residuals $\tilde{\epsilon}$, and compute the matrix residual sum of squares SSR_1.

3. Compute the asymptotic test statistic with the F-version to consider the heteroskedasticity-robust version:

$$LM = \frac{(SSR_0 - SSR_1)/n}{SSR_1/\{T - (k+p+1) - n\}}$$

Table 70: LR test for the entire system

X=[G,Y,Z]	LR statistic	probability
H1 H0		
nlag=5 4	17.2904	0.04436**
nlag= 4 3	61.4036	0***
nlag= 3 2	262.2174	0***
nlag=2 1	495.8812	0***

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

Table 71: LR test for G equation

G		LR statistic	probability
	H1 H0		
	nlag=5 4	2.8418	0.09184*
	nlag= 4 3	28.2324	0***
	nlag= 3 2	59.0333	0***
	nlag=2 1	226.1198	0***

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

Table 72: LR test for Y equation

GDP		LR statistic	probability
	H1 H0		
	nlag= 4 3	-0.3093	1.00
	nlag= 3 2	13.8741	0***
	nlag=2 1	4.1831	0.04**

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

Table 73: LR test for Z equation

Z		LR statistic	probability
	H1 H0		
	nlag=9 8	2.0508	0.15
	nlag=8 7	32.6993	0***
	nlag=7 6	80.3364	0***
	nlga=6 5	11.9778	0***
	nlag=5 4	12.1845	0***
	nlag= 4 3	31.3841	0***
	nlag= 3 2	191.9059	0***
	nlag=2 1	267.0575	0***

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

Table 74: Lagrange Multiplier test

LM for lag 1	Z1	Z2	Z3
G	0***	0.01**	0***
Y	0.04**	0***	0***
Z	0.05*	0***	0.07*

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

11.2 Evaluation Tests

Table 75: Pvalue of a no additive nonlinearity test.

Residuals	Lags	Pvalue
G	1.00	0.00***
	2.00	0.68
	3.00	0.64
	4.00	0.78
Y	1.00	0.43
	2.00	0.94
	3.00	0.10
	4.00	0.62
Z	1.00	0.0533*
	2.00	0.18
	3.00	0.10
	4.00	0.77

*rejection of null hypothesis for 10% significance level

** rejection of null hypothesis for 5% significance level

*** rejection of null hypothesis for 1% significance level

Table 76: Correlation analysis of G up to 3 lags.

Correlation	RESID_G	RESID_G1	RESID_G2	RESID_G3
RESID_G	1.00	-0.16	0.22	0.07
RESID_G1	-0.16	1.00	-0.16	0.22
RESID_G2	0.22	-0.16	1.00	-0.18
RESID_G3	0.07	0.22	-0.18	1.00

Table 77: Test of no autocorrelation in the residuals. The dependent variable is the residuals associated to G equation. Method: Least Squares. Sample (adjusted): 1960Q4 2011Q4 (included observations: 205 after adjustments).

	Coefficient	Std. Error	t-Statistic	Prob.
RESID_G1	-0.158253	0.069684	-2.271024	0.0242
RESID_G2	0.224303	0.068784	3.260988	0.0013
RESID_G3	0.144962	0.067995	2.131959	0.0342
C	-5.72E-05	0.000457	-0.12514	0.9005
R-squared	0.087172	Mean dependent var		-6.77E-05
Adjusted R-squared	0.073548	S.D. dependent var		0.006804
S.E. of regression	0.006549	Akaike info criterion		-7.19969
Sum squared resid	0.008621	Schwarz criterion		-7.13485
Log likelihood	741.9681	Hannan-Quinn criter.		-7.17346
F-statistic	6.398308	Durbin-Watson stat		1.950921
Prob(F-statistic)	0.000368			

Table 78: Correlation analysis of Y up to 3 lags.

Correlation	RESID_GDP	RESID_GDP1	RESID_GDP2	RESID_GDP3
RESID_GDP	1.00	-0.01	0.02	0.02
RESID_GDP1	-0.01	1.00	-0.01	0.01
RESID_GDP2	0.02	-0.01	1.00	-0.01
RESID_GDP3	0.02	0.01	-0.01	1.00

Table 79: Test of no autocorrelation in the residuals. The dependent variable is the residuals associated to Y equation. Method: Least Squares. Sample (adjusted): 1960Q4 2011Q4 (included observations: 205 after adjustments)

	Coefficient	Std. Error	t-Statistic	Prob.
RESID_GDP1	-0.008971	0.0712	-0.126	0.8999
RESID_GDP2	0.021695	0.071159	0.304881	0.7608
RESID_GDP3	1.65E-02	0.07122	0.231047	0.8175
C	2.71E-05	0.00068	0.039779	0.9683
R-squared	0.000799	Mean dependent var		3.09E-05
Adjusted R-squared	-0.014114	S.D. dependent var		0.009671
S.E. of regression	0.009739	Akaike info criterion		-6.406089
Sum squared resid	0.019063	Schwarz criterion		-6.34125
Log likelihood	660.6241	Hannan-Quinn criter.		-6.379863
F-statistic	0.0536	Durbin-Watson stat		1.980926
Prob(F-statistic)	0.983605			

Table 80: Test of no autocorrelation in the residuals. The dependent variable is the residuals associated to Y equation. Method: Least Squares. Sample (adjusted): 1960Q4 2011Q4 (included observations: 205 after adjustments)

Correlation	RESID_Z	RESID_Z1	RESID_Z2	RESID_Z3
RESID_Z	1.00	0.26	0.17	0.14
RESID_Z1	0.26	1.00	0.25	0.15
RESID_Z2	0.17	0.25	1.00	0.24
RESID_Z3	0.14	0.15	0.24	1.00

Table 81: Test of no autocorrelation in the residuals. The dependent variable is the residuals associated to Z equation. Method: Least Squares. Sample (adjusted): 1960Q4 2011Q4 (included observations: 205 after adjustments)

	Coefficient	Std. Error	t-Statistic	Prob.
RESID_Z1	0.23053	0.071056	3.24433	0.0014
RESID_Z2	0.092067	0.072741	1.265669	0.2071
RESID_Z3	8.68E-02	0.071667	1.211482	0.2271
C	-8.80E-05	0.000994	-0.088564	0.9295
R-squared	0.08619	Mean dependent var		9.08E-06
Adjusted R-squared	0.072551	S.D. dependent var		0.014768
S.E. of regression	0.014222	Akaike info criterion		-5.648726
Sum squared resid	0.040656	Schwarz criterion		-5.583887
Log likelihood	582.9945	Hannan-Quinn criter.		-5.622501
F-statistic	6.319412	Durbin-Watson stat		1.88341
Prob(F-statistic)	0.000407			

Table 82: Arch Test: pvalues higher than the significance level indicate the acceptance of the null hypothesis, suggesting no ARCH effects, that is, homoscedasticity at the corresponding element of lags; pvalues lower than the significance level reject the null hypothesis, suggesting ARCH effects, that is, heteroskedasticity at the corresponding element of lags

Heteroskedasticity Test	Pvalue
G (lag 1)	0.0022***
G (lag 2)	0.0138**
G (lag 3)	0.0815*
Y (lag 1)	0.4253
Y (lag 2)	0.409
Y (lag 3)	0.3905
Z (lag 1)	0.8726
Z (lag 2)	0.9891
Z (lag 3)	0.9993

*** rejection of null hypothesis for 1% significance leve

** rejection of null hypothesis for 5% significance level

*rejection of null hypothesis for 10% significance level

12 Appendix D - Results

In this appendix it will only be presented the main figures, charts and tables. There are several outputs that will not be presented due to space-saving reasons, for instance the model's coefficients (betas, standard errors and *p-values*), but they are available upon request for *tfbgs@iscte.pt.*

12.1 Main Results: Benchmark Model

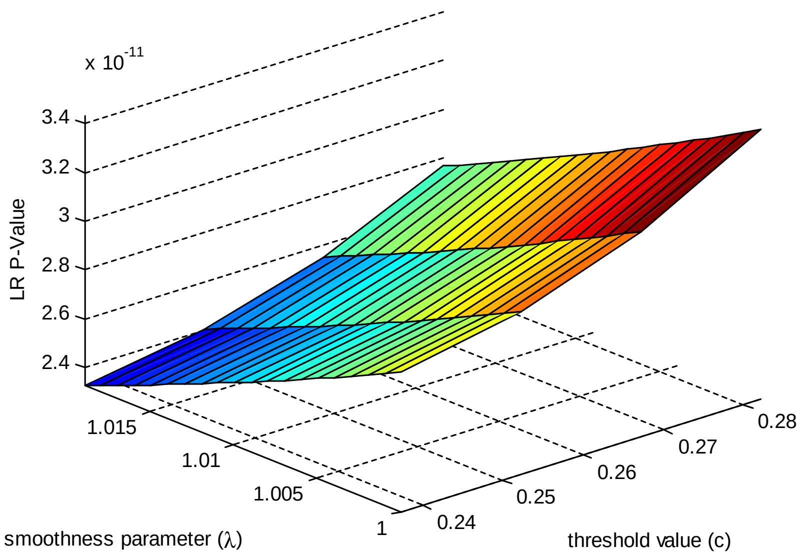

Figure 53: Grid search: LR pvalues for different thresholds and smoothness parameters given the unemployment change (one lag) as the transition variable. Sample period: 1960Q2:2012Q4.

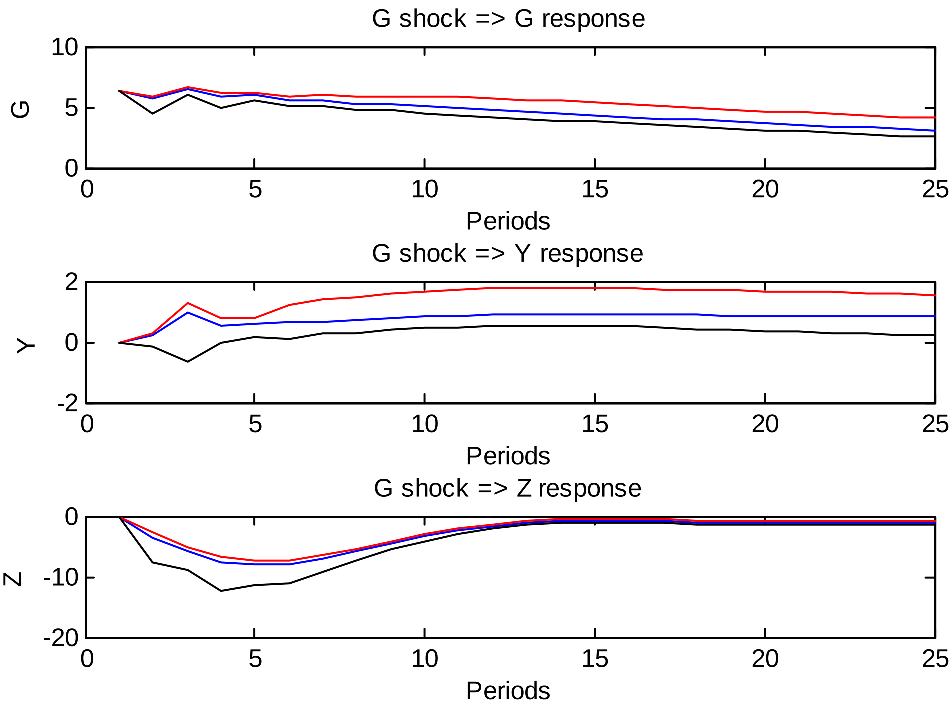

Figure 54: Accumulated response of all variables to a 1SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

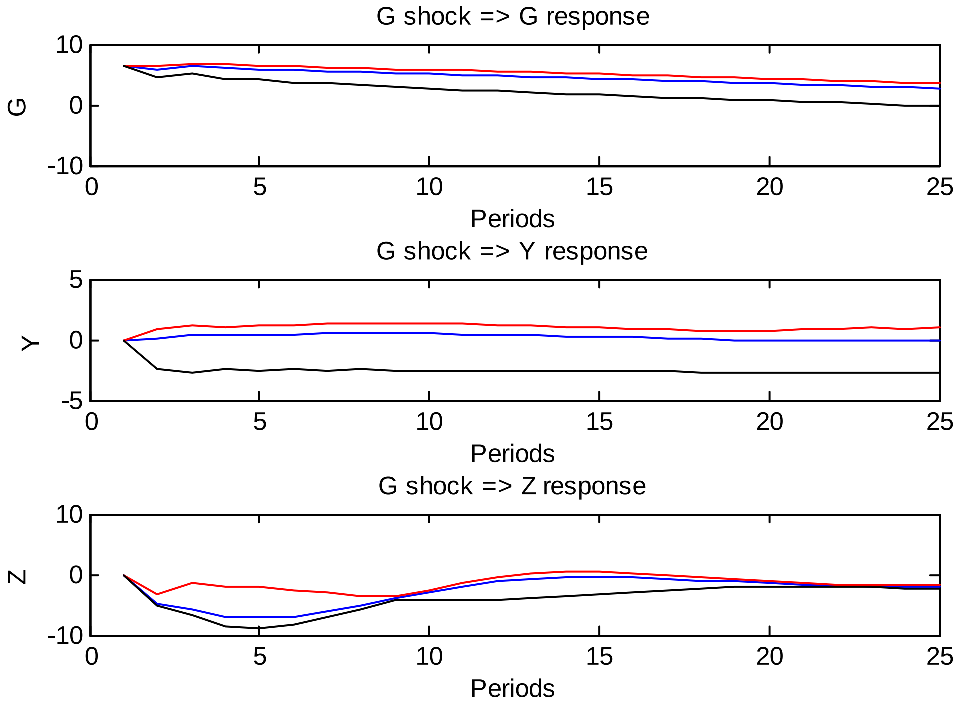

Figure 55: Accumulated response of all variables to a 1SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

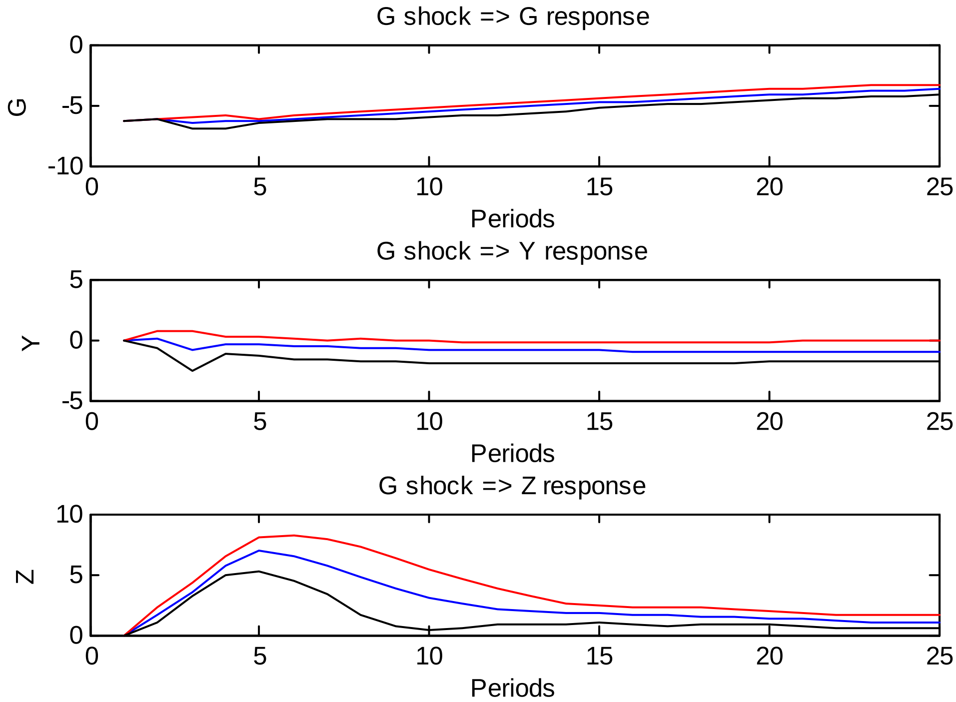

Figure 56: Accumulated response of all variables to a -1SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

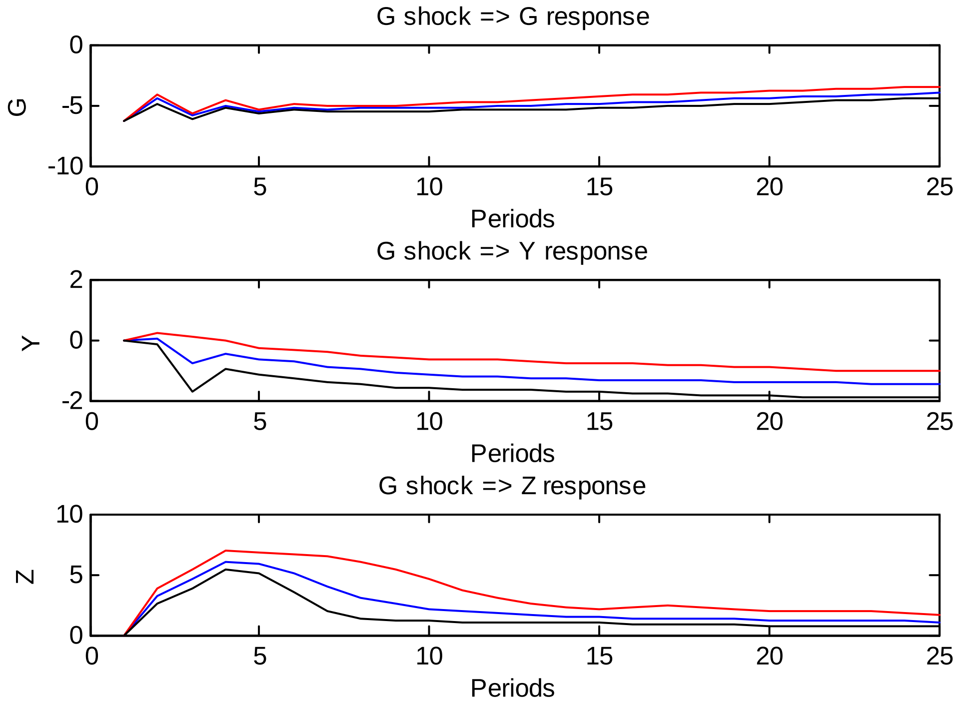

Figure 57: Accumulated response of all variables to a -1SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

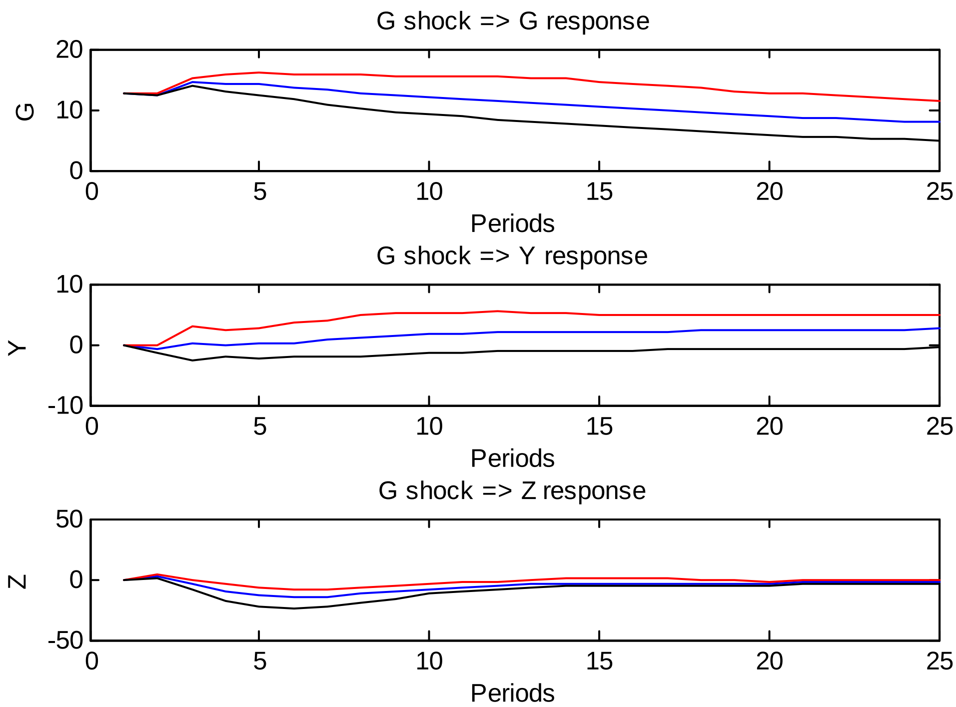

Figure 58: Accumulated response of all variables to a 2SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

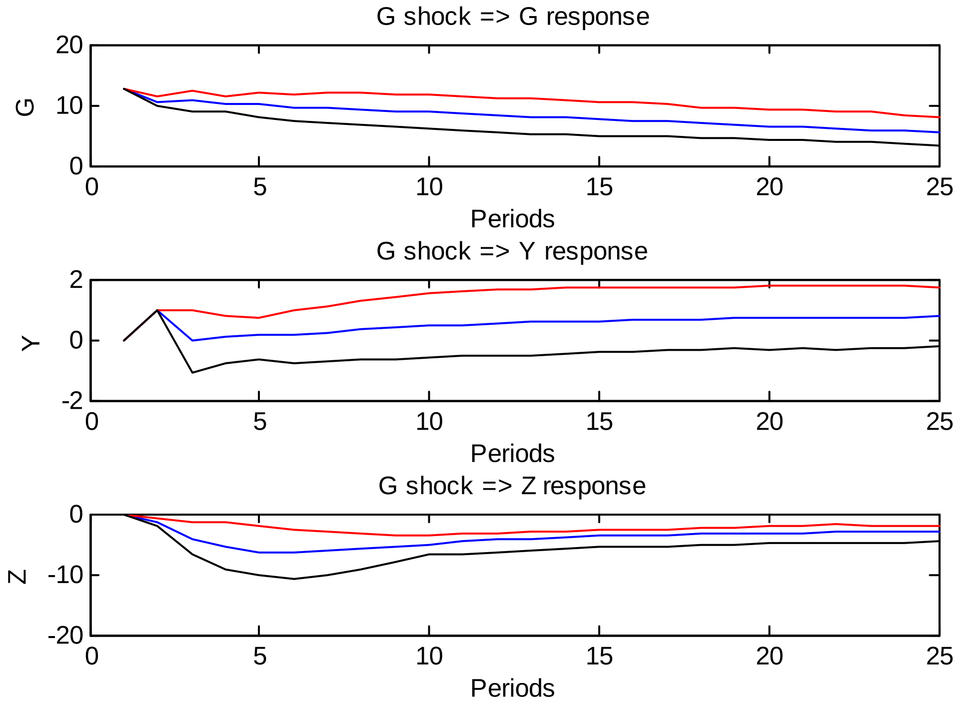

Figure 59: Accumulated response of all variables to a 2SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

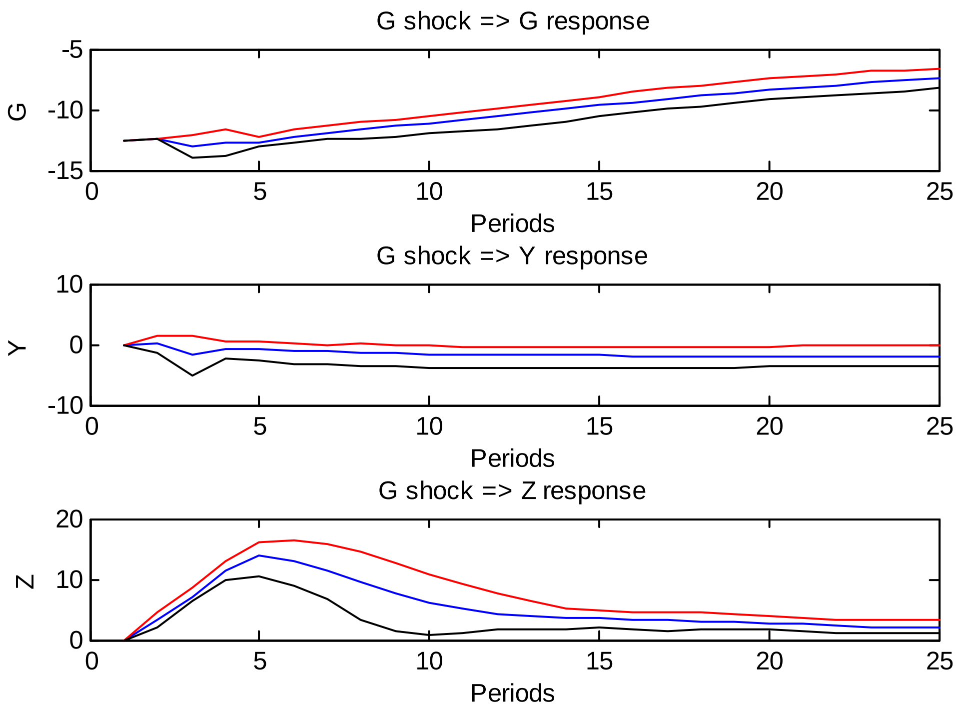

Figure 60: Accumulated response of all variables to a -2SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

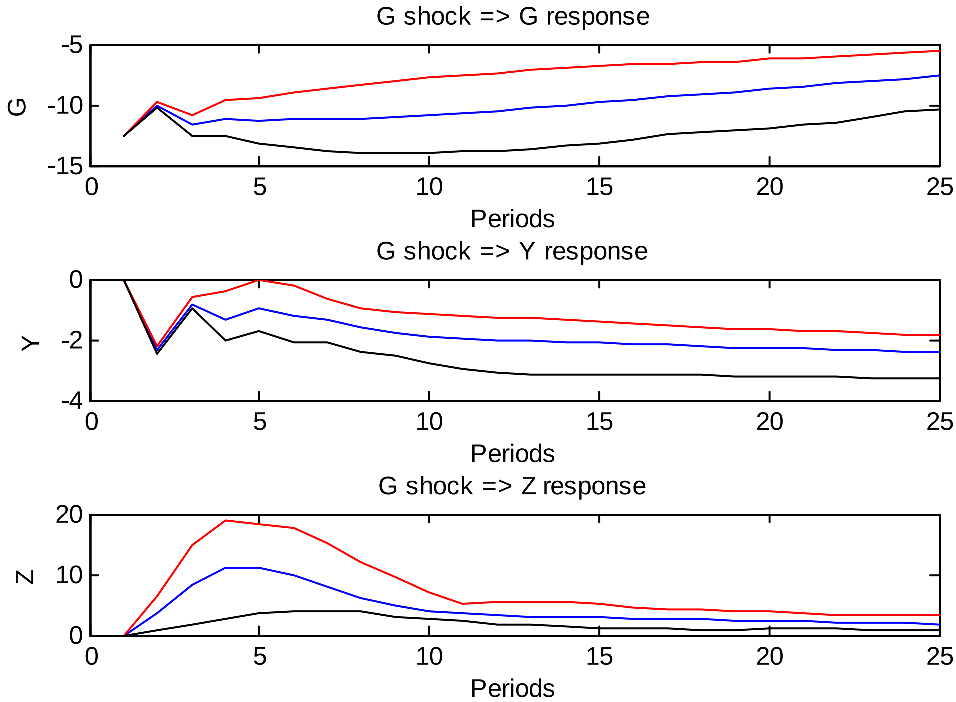

Figure 61: Accumulated response of all variables to a -2SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

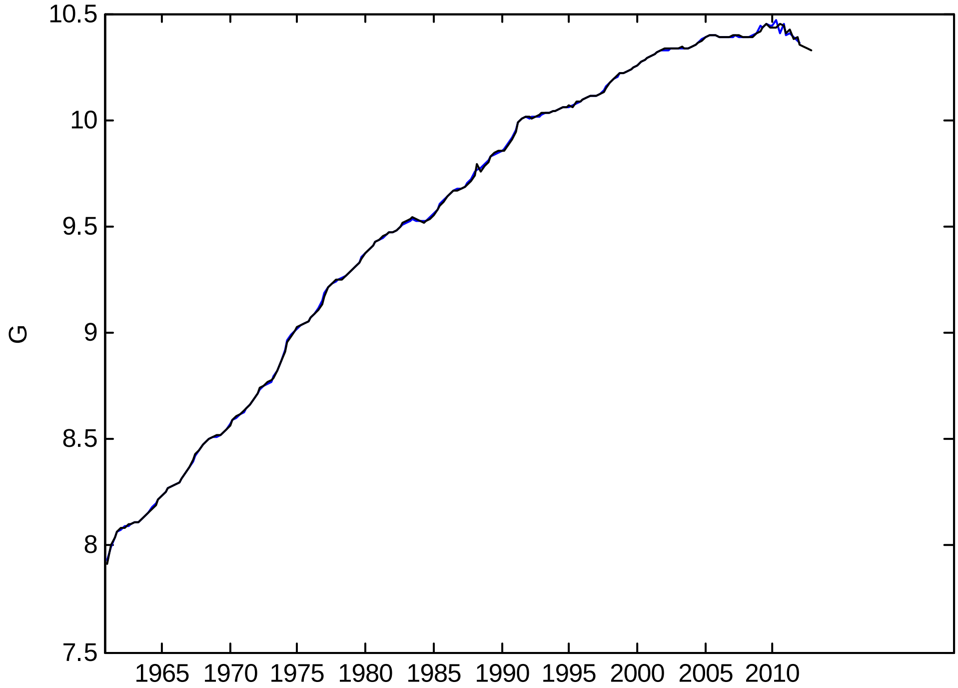

Figure 62: This figure confronts the actual values of G (blue line) with the predicted values by the model (black line). Sample 1960Q2:2012Q4, $\gamma = 5$, and 500000 Monte Carlo simulations.

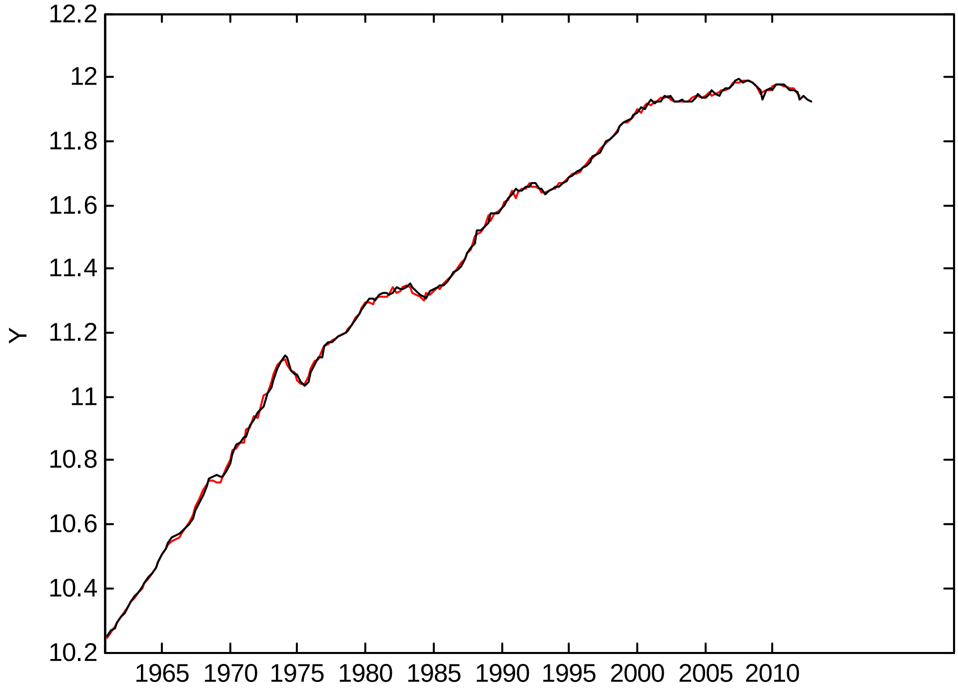

Figure 63: This figure confronts the actual values of Y (red line) with the predicted values by the model (black line). Sample 1960Q2:2012Q4, $\gamma = 5$, and 500000 Monte Carlo simulations.

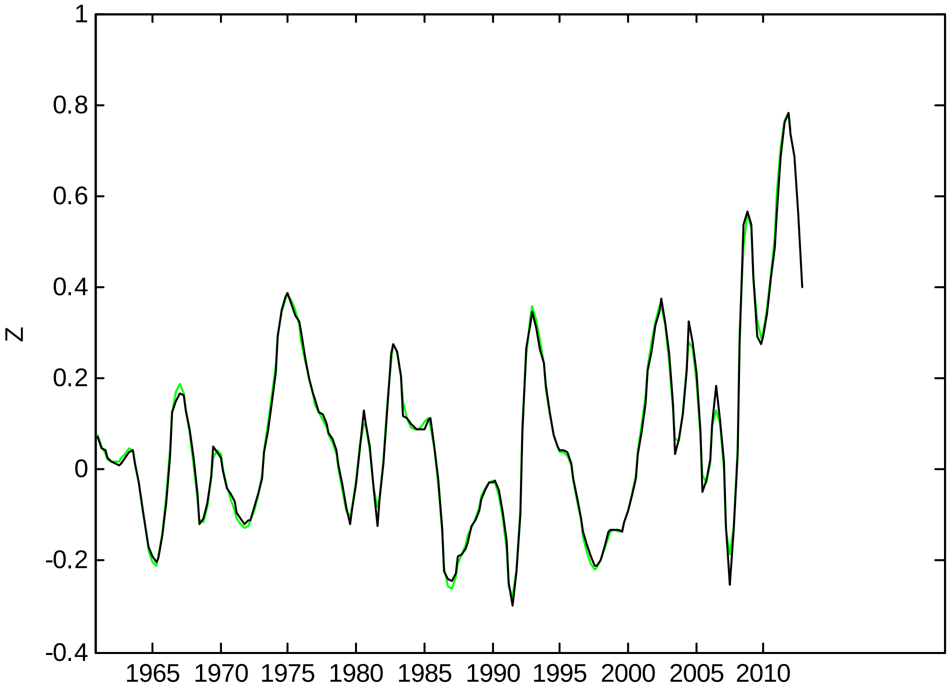

Figure 64: This figure confronts the actual values of Z (green line) with the predicted values by the model (black line). Sample 1960Q2:2012Q4, $\gamma = 5$, and 500000 Monte Carlo simulations.

12.2 Robustness Analysis

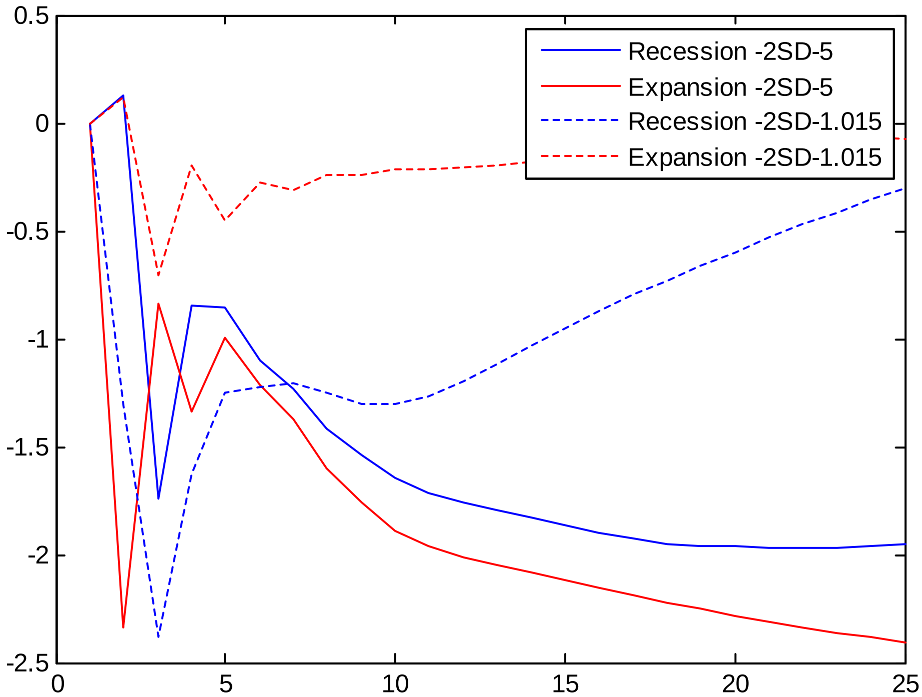

Figure 65: Accumulated response of output (Y) to a 2SD government spending shock for both regimes, recession and expansion and both smooth parameters. 500000 Monte Carlo simulations were considered in order to assure stability. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2012Q4.

12.2.1 Model II

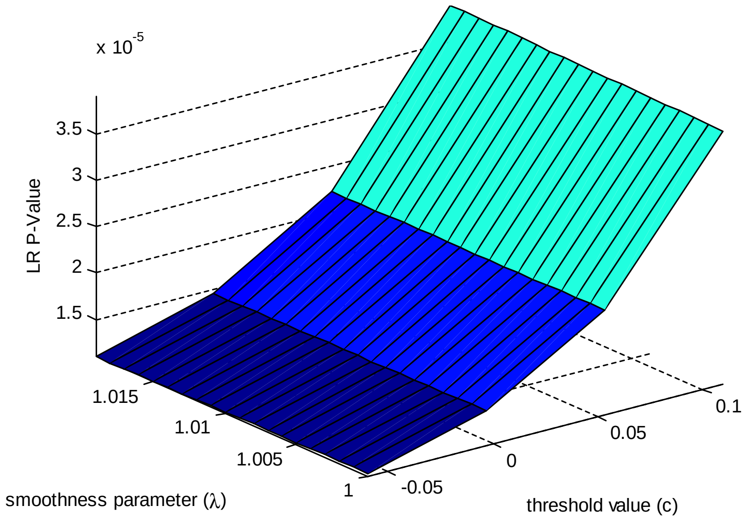

Figure 66: Grid search: LR pvalues for different thresholds and smoothness parameters given the unemployment change (one lag) as the transition variable for the subsample model. Sample period: 1960Q2:2007Q4.

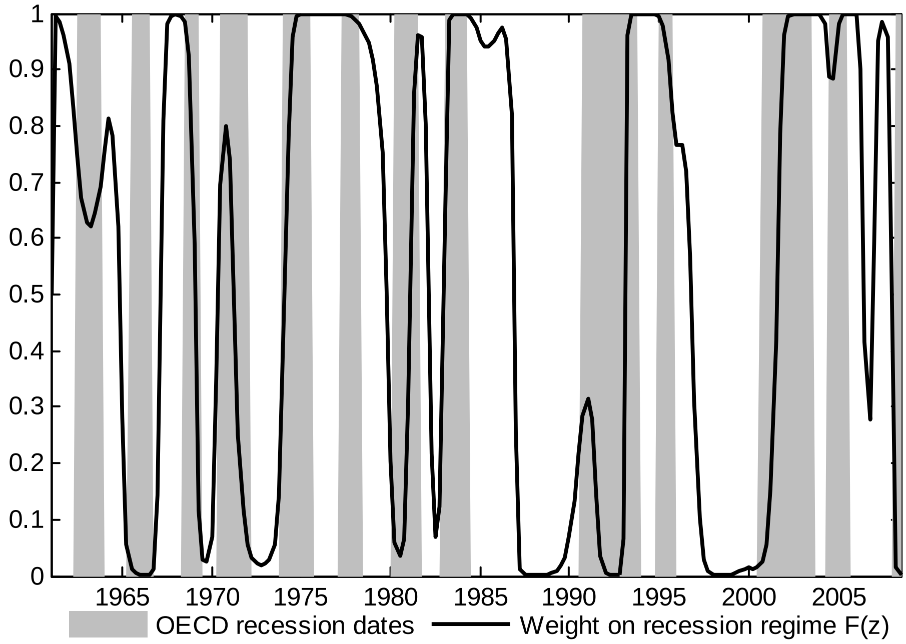

Figure 67: This figure compares the dynamic of the weight in a recession regime with recessions identified by OECD for: a $P[F(z)] > 0.57$, $\gamma = 5$, sample period of 1960Q2:2007Q4, and 500000 Monte Carlo simulations.

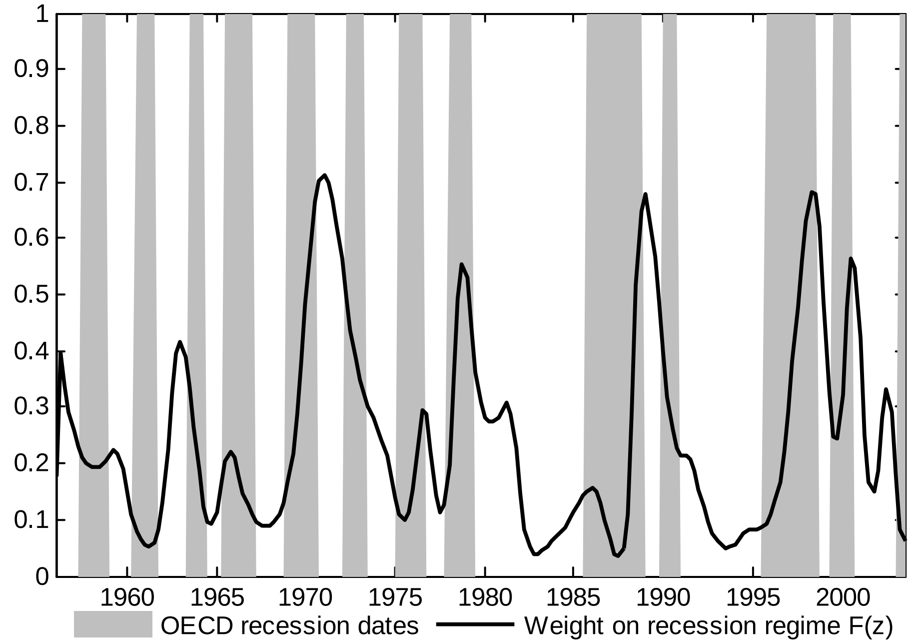

Figure 68: This figure compares the dynamic of the weight in a recession regime with recessions identified by OECD for: a $P[F(z)] > 0.57$, $\gamma = 1.0155$, sample period of 1960Q2:2007Q4, and 500000 Monte Carlo simulations.

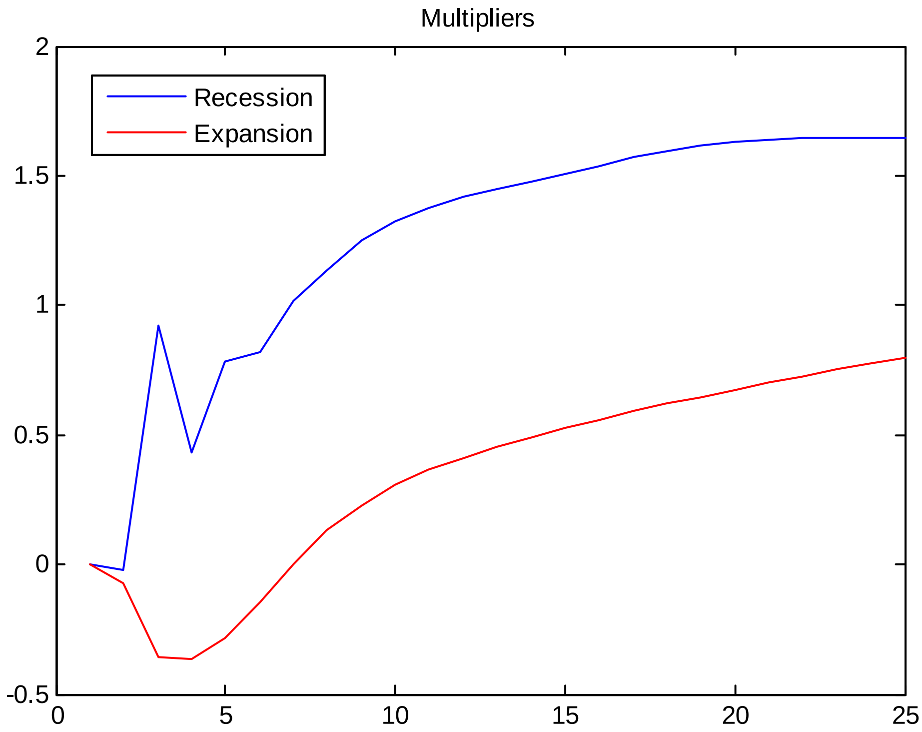

Figure 69: Accumulated response of output (Y) to a 1SD government spending shock for both regimes, recession and expansion. 500000 Monte Carlo simulations were considered in order to assure stability. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$.

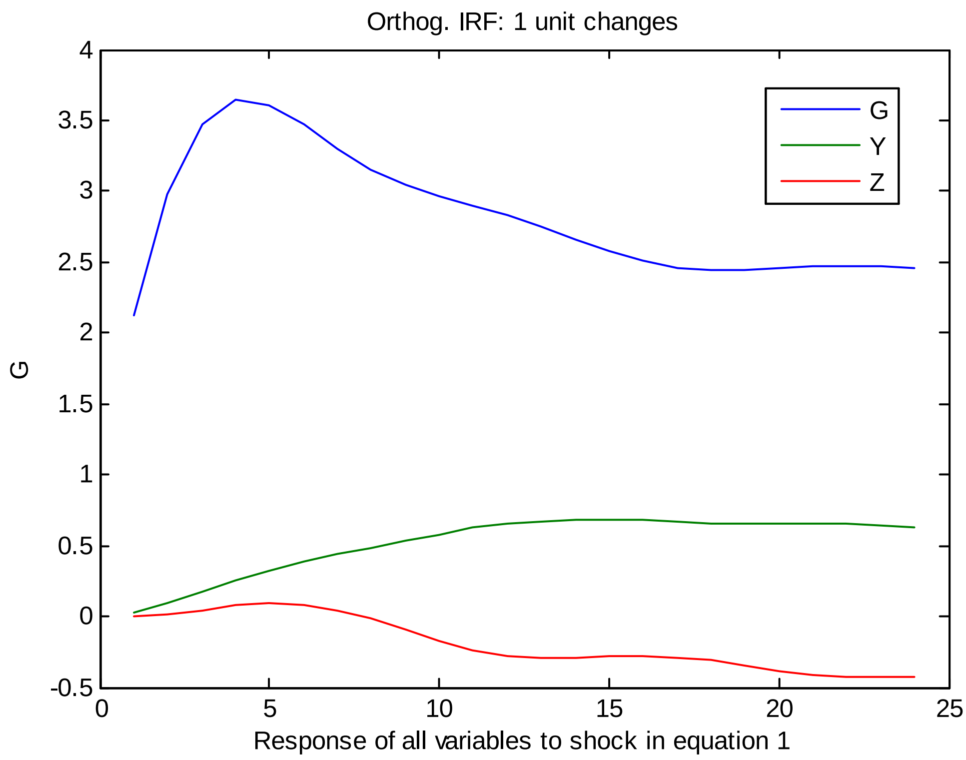

Figure 70: Accumulated response of government spending (G), output (Y) and unemployment change (Z) to a 1 unit change in government spending shock for a linear VAR model. Sample period is 1960Q2:2007Q4.

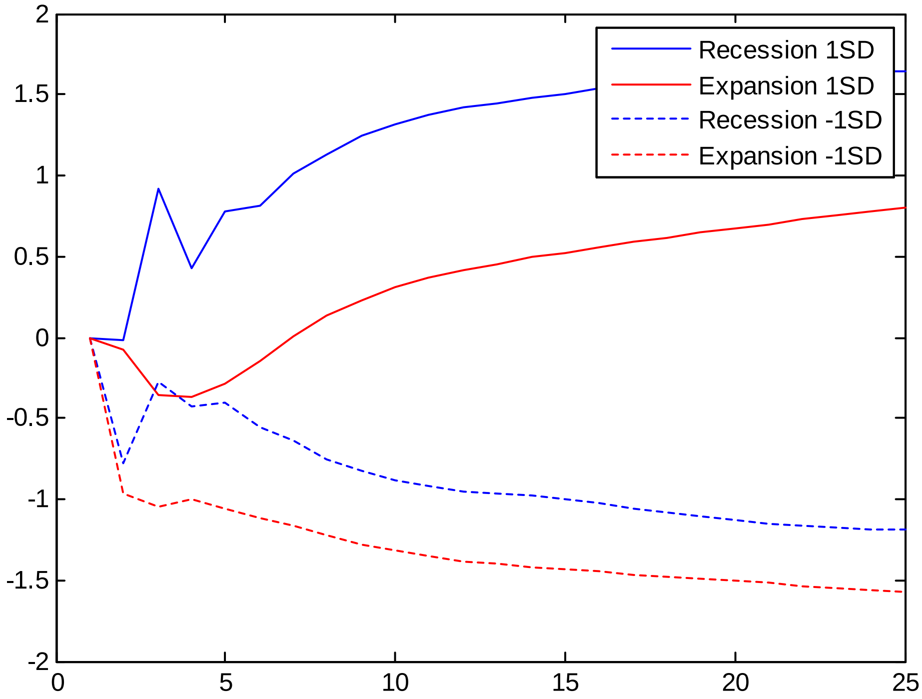

Figure 71: Accumulated response of output (Y) to a 1SD government spending shock for positive shocks and -1SD for negative shocks. 500000 Monte Carlo simulations were considered in order to assure stability. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$.

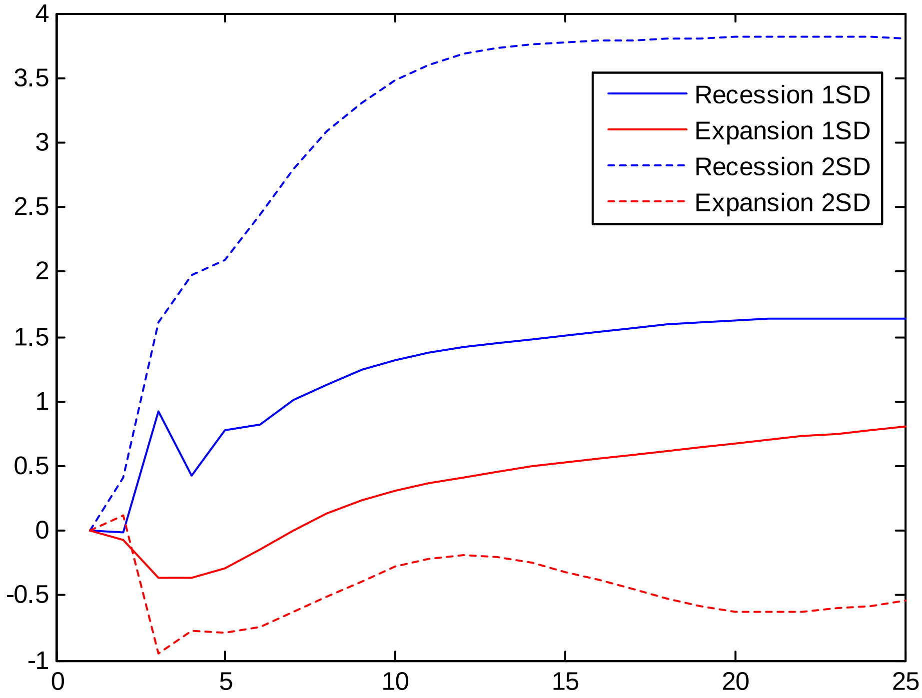

Figure 72: Accumulated response of output (Y) to a 1SD and 2SD government spending shocks. 500000 Monte Carlo simulations were considered in order to assure stability. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$.

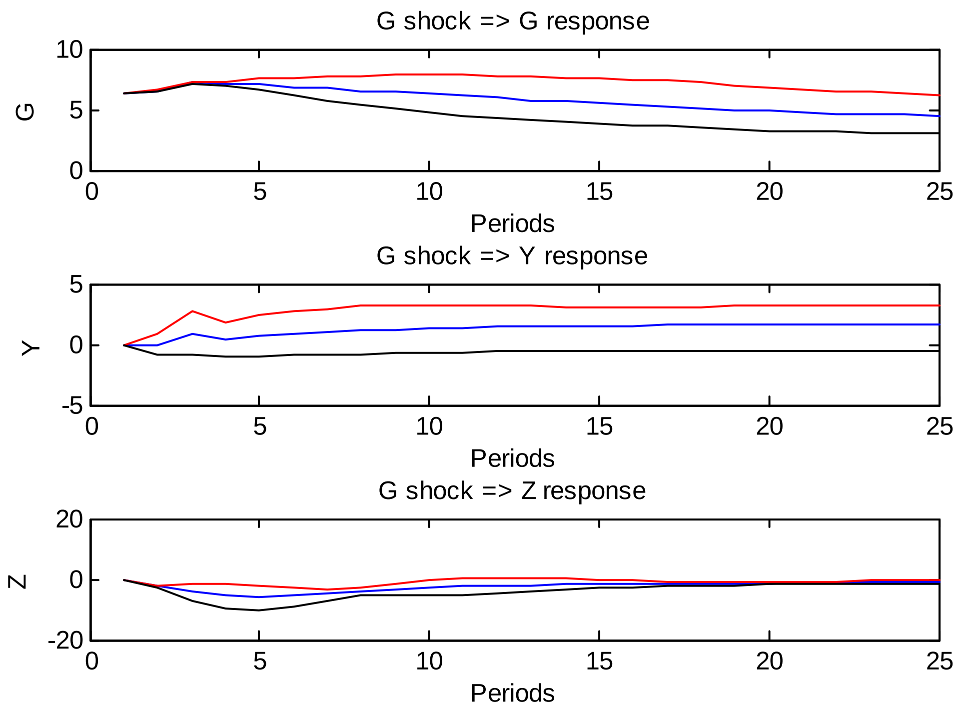

Figure 73: Accumulated response of all variables to a 1SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

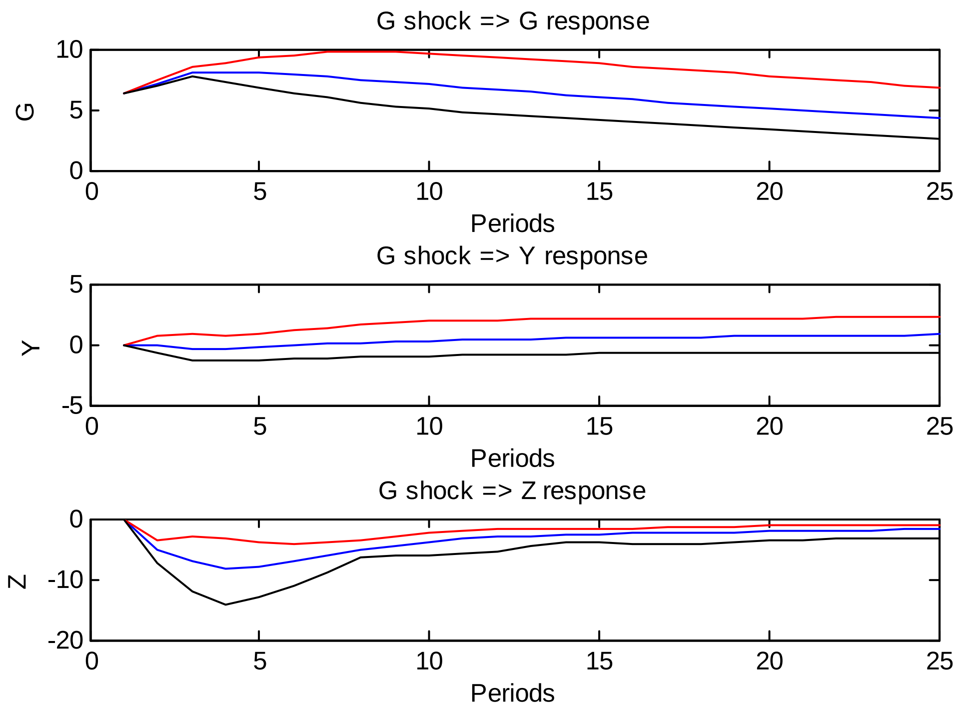

Figure 74: Accumulated response of all variables to a 1SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

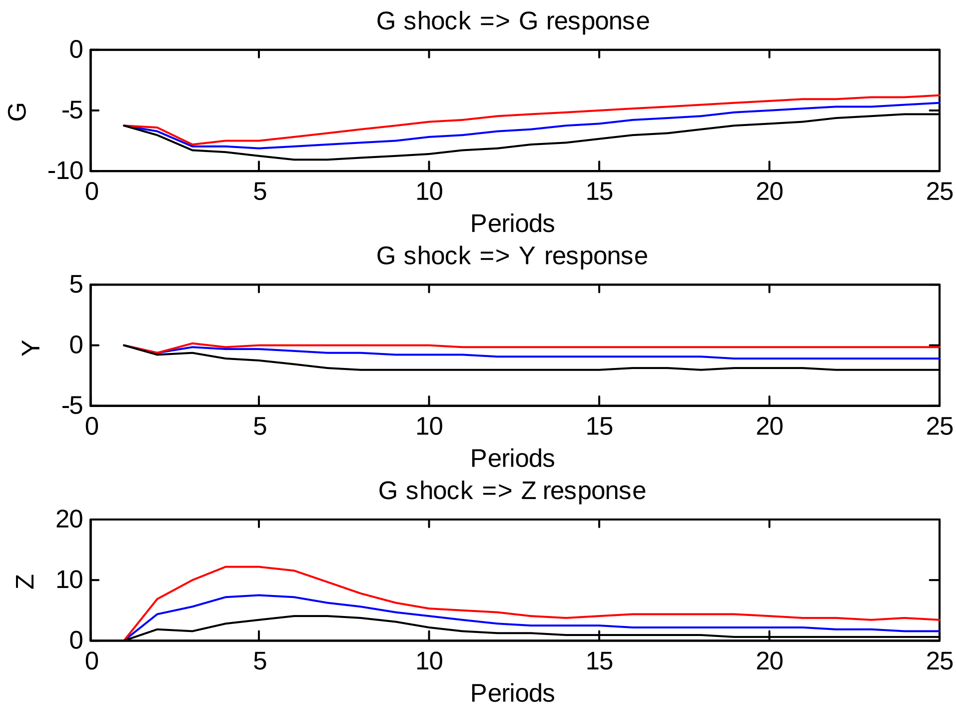

Figure 75: Accumulated response of all variables to a -1SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

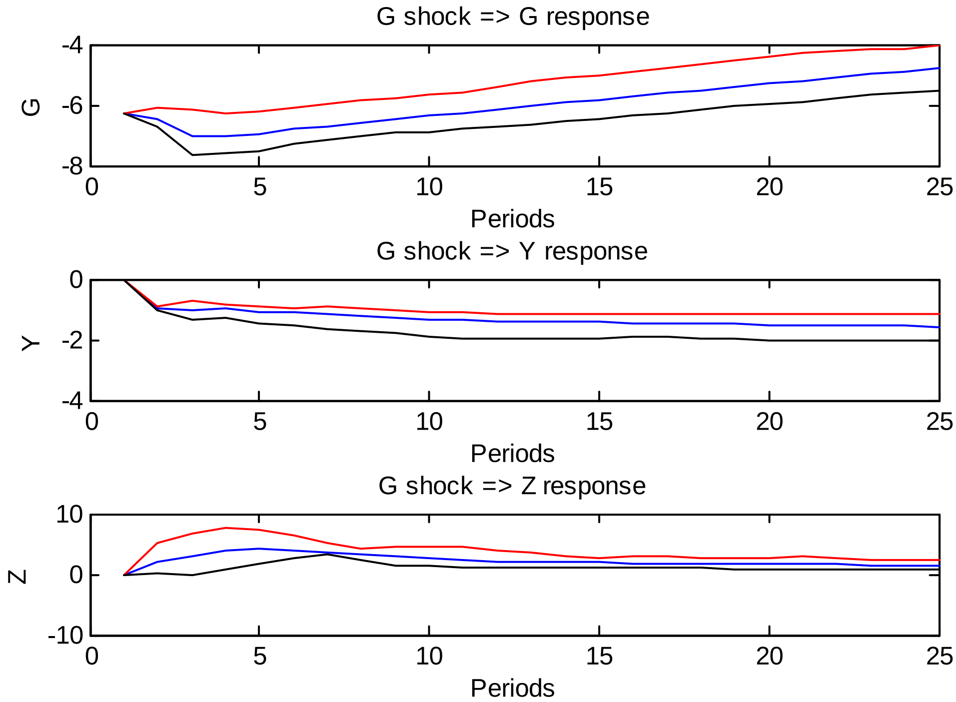

Figure 76: Accumulated response of all variables to a -1SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

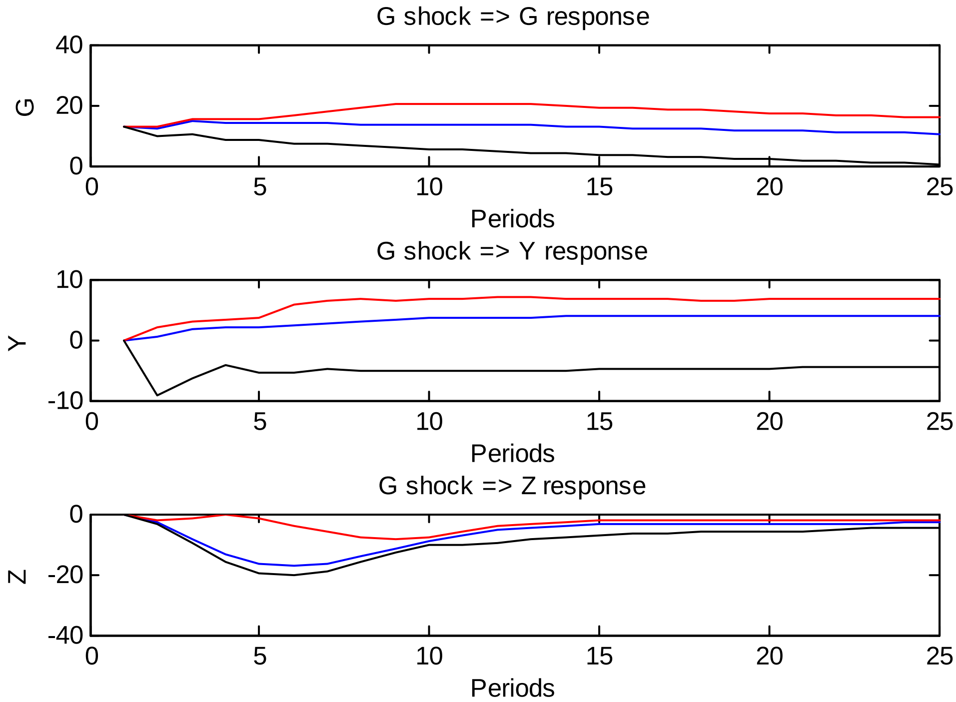

Figure 77: Accumulated response of all variables to a 2SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

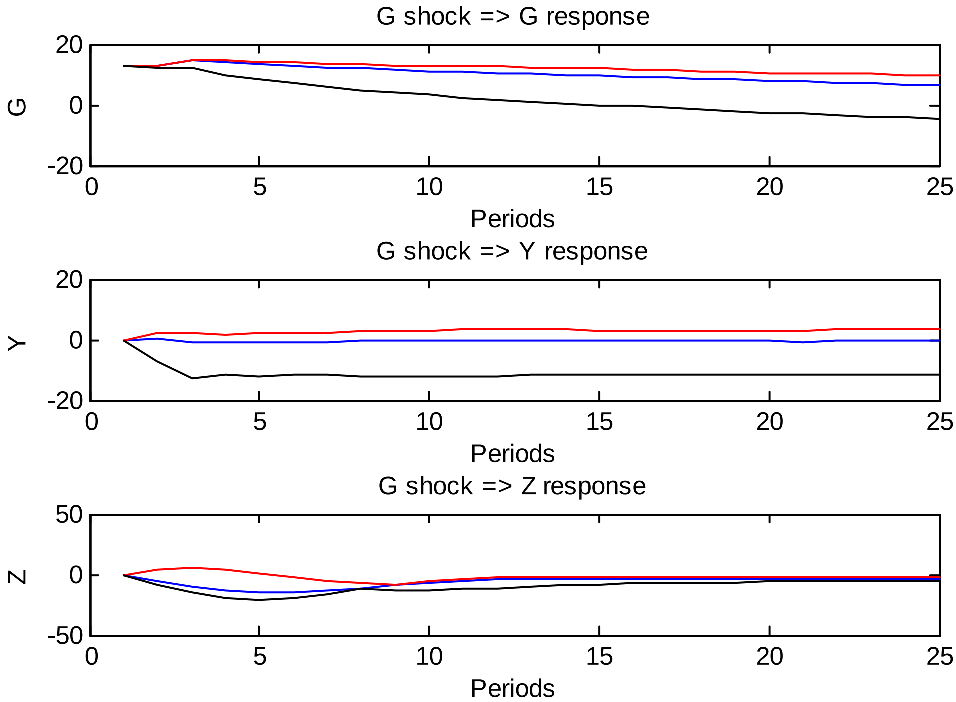

Figure 78: Accumulated response of all variables to a 2SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

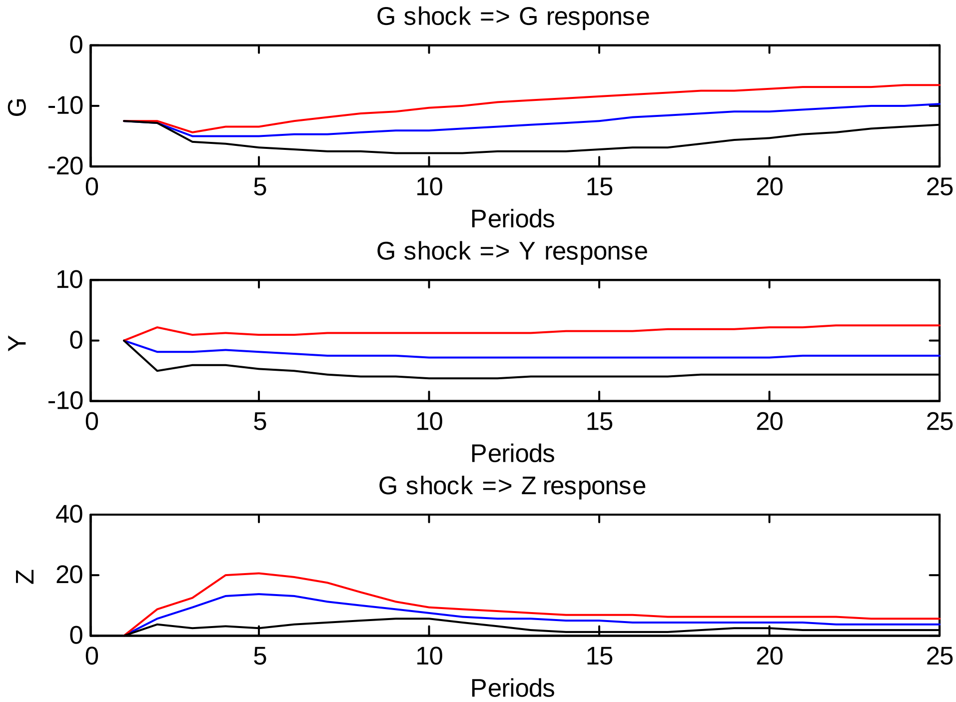

Figure 79: Accumulated response of all variables to a -2SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

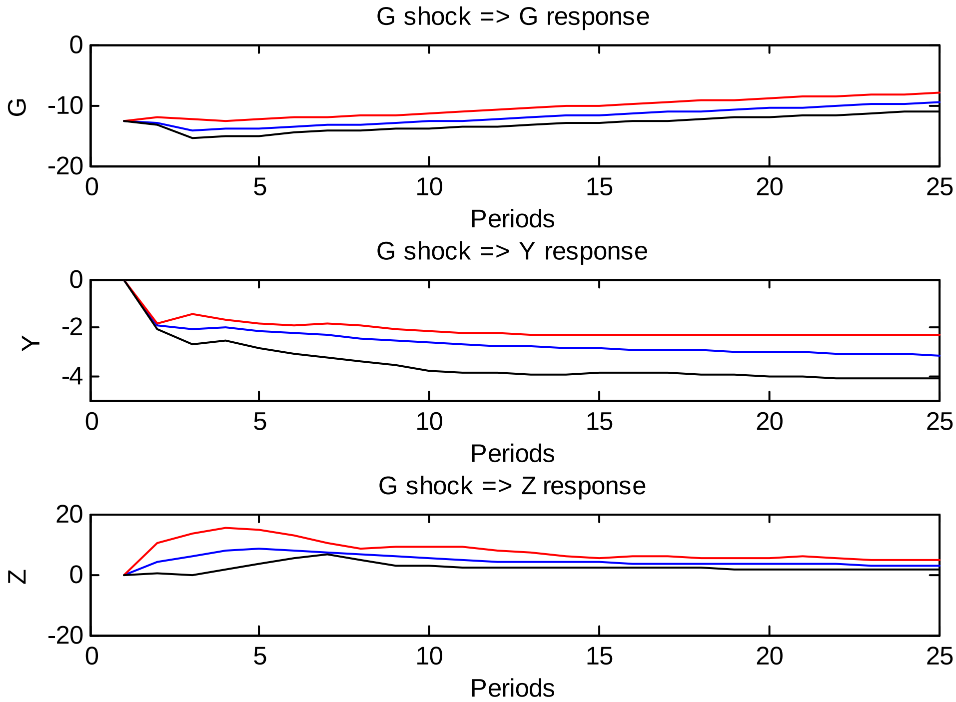

Figure 80: Accumulated response of all variables to a -2SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Sample period is 1960Q2:2007Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

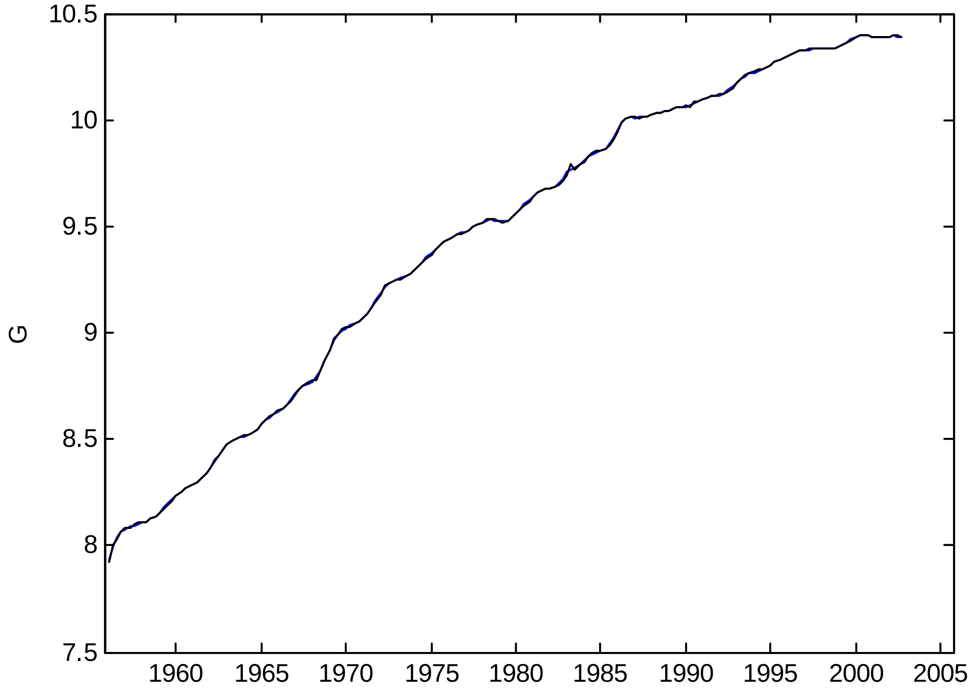

Figure 81: This figure confronts the actual values of G (blue line) with the predicted values by the model (black line). Sample 1960Q2:2007Q4, $\gamma = 5$, and 500000 Monte Carlo simulations.

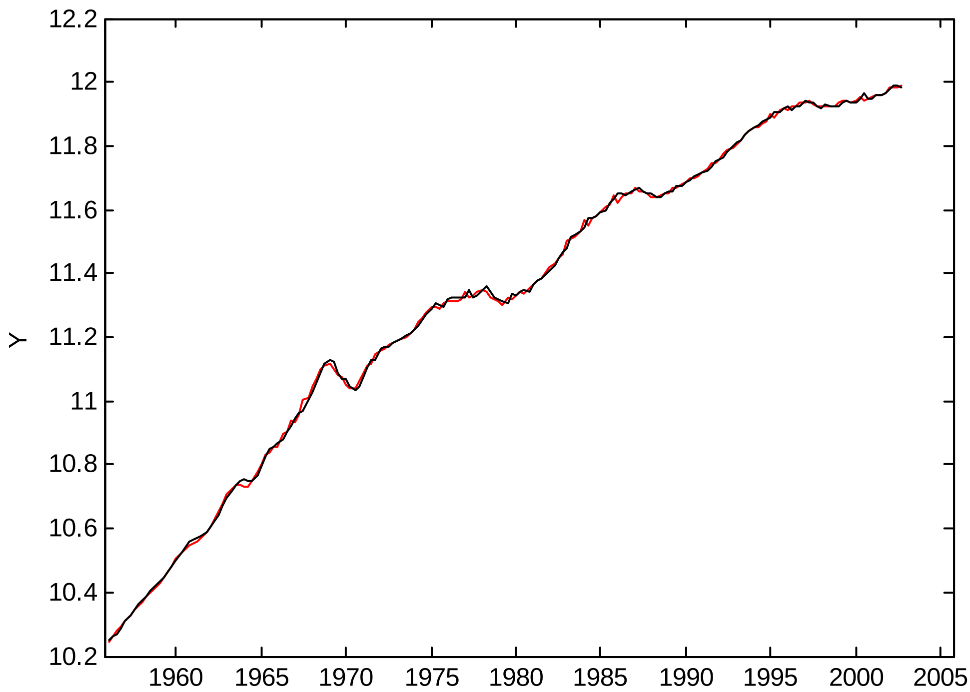

Figure 82: This figure confronts the actual values of Y (red line) with the predicted values by the model (black line). Sample 1960Q2:2007Q4, $\gamma = 5$, and 500000 Monte Carlo simulations.

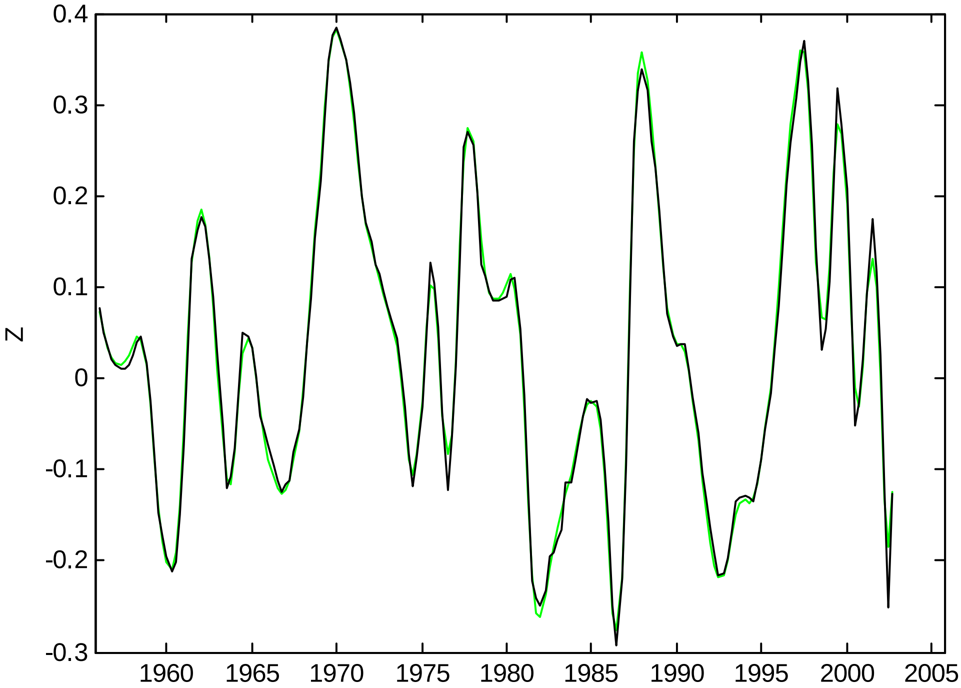

Figure 83: This figure confronts the actual values of Z (green line) with the predicted values by the model (black line). Sample 1960Q2:2007Q4, $\gamma = 5$, and 500000 Monte Carlo simulations.

12.2.2 Model III

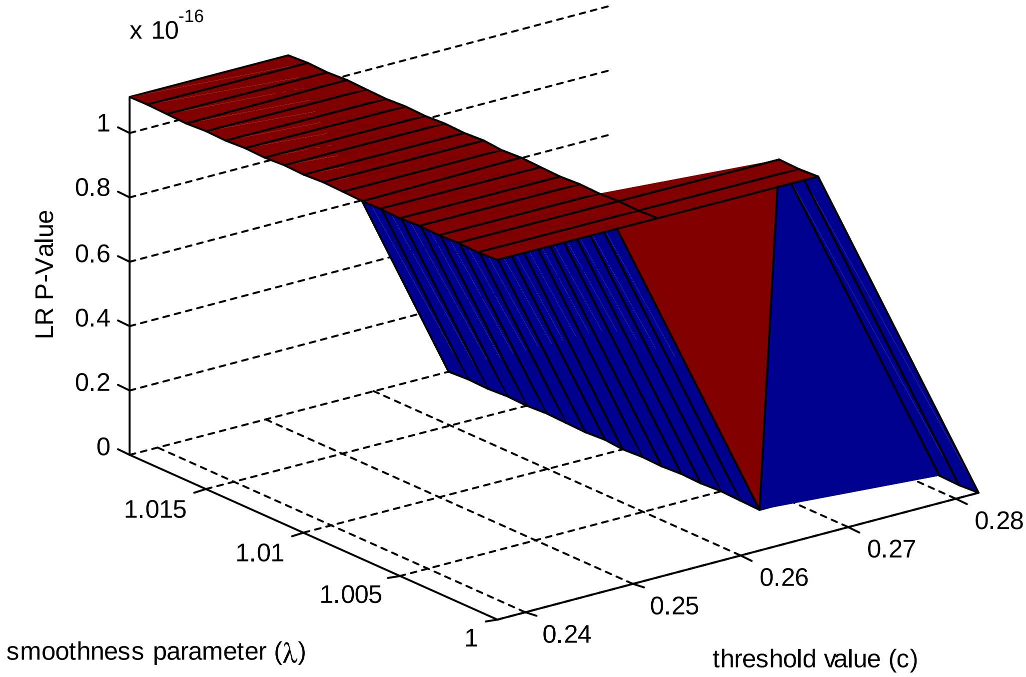

Figure 84: Grid search: LR pvalues for different thresholds and smoothness parameters given the unemployment change (one lag) as the transition variable for the growth rate model. Sample period: 1960Q2:2012Q4.

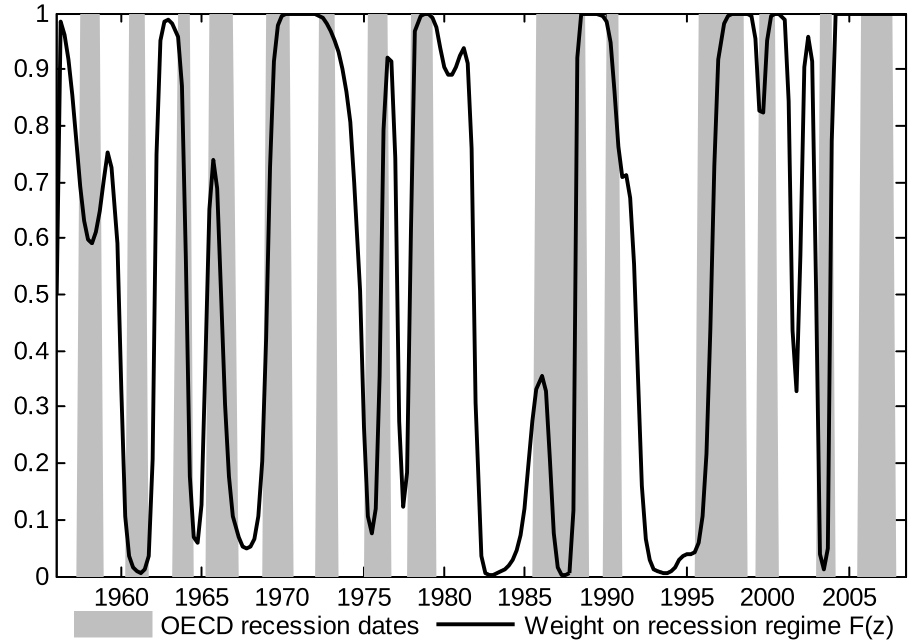

Figure 85: This figure compares the dynamic of the weight in a recession regime with recessions identified by OECD for: a $P[F(z)] > 0.57$, $\gamma = 5$. Growth rate model for a sample period of 1960Q2:2012Q4, and 500000 Monte Carlo simulations.

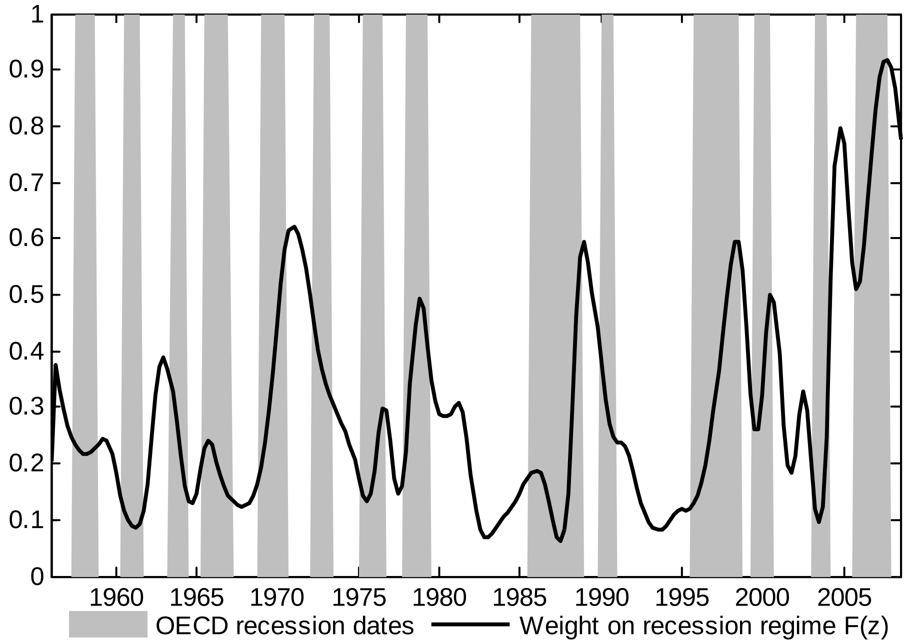

Figure 86: This figure compares the dynamic of the weight in a recession regime with recessions identified by OECD for: a $P[F(z)] > 0.57$, $\gamma = 1.015$. Growth rate model for a sample period of 1960Q2:2012Q4, and 500000 Monte Carlo simulations

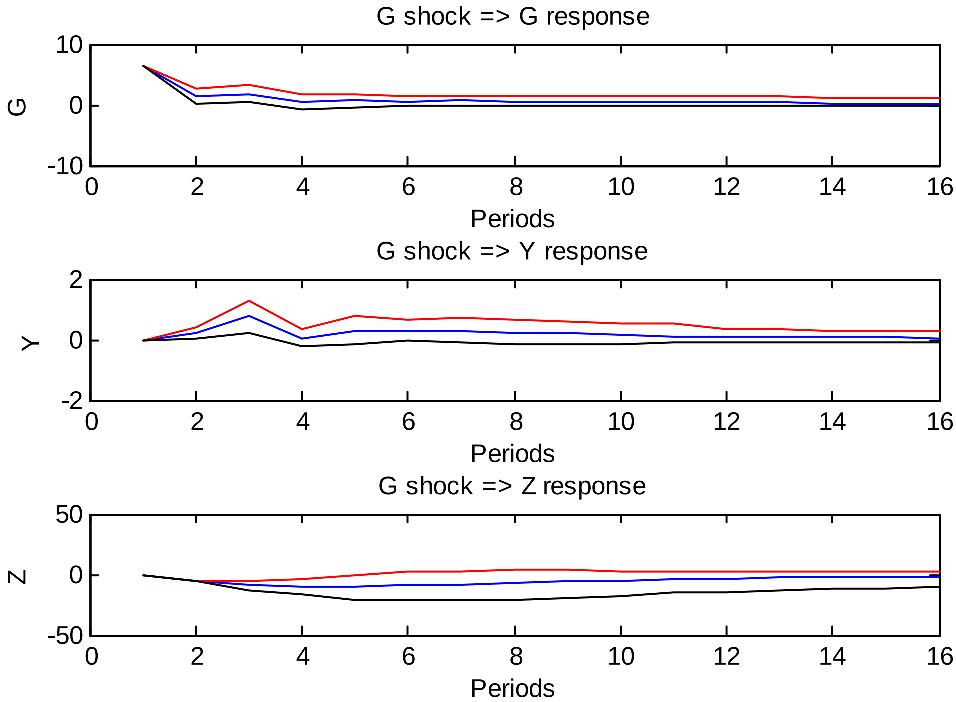

Figure 87: Accumulated response of all variables to a 1SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Growth rate model for sample period of 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

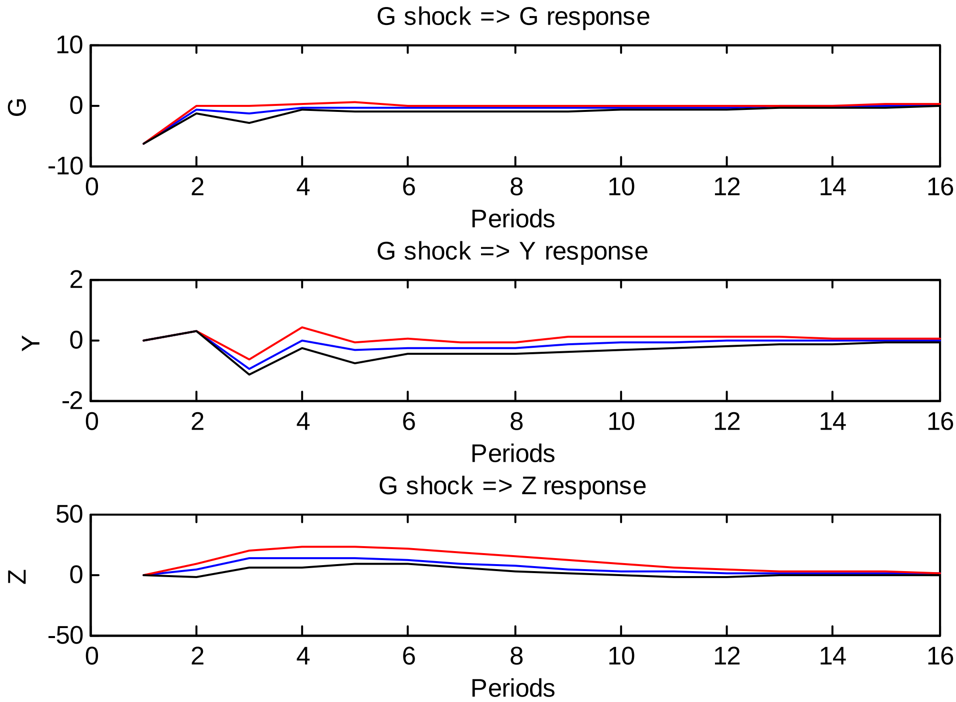

Figure 88: Accumulated response of all variables to a 1SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Growth rate model for sample period of 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

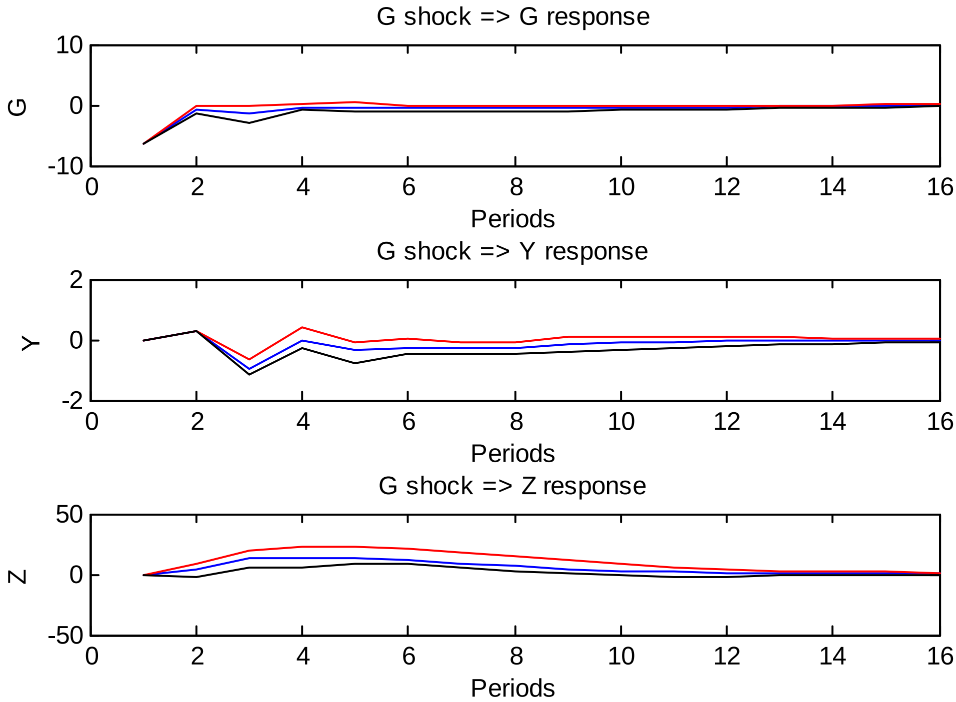

Figure 89: Accumulated response of all variables to a -1SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Growth rate model for sample period of 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

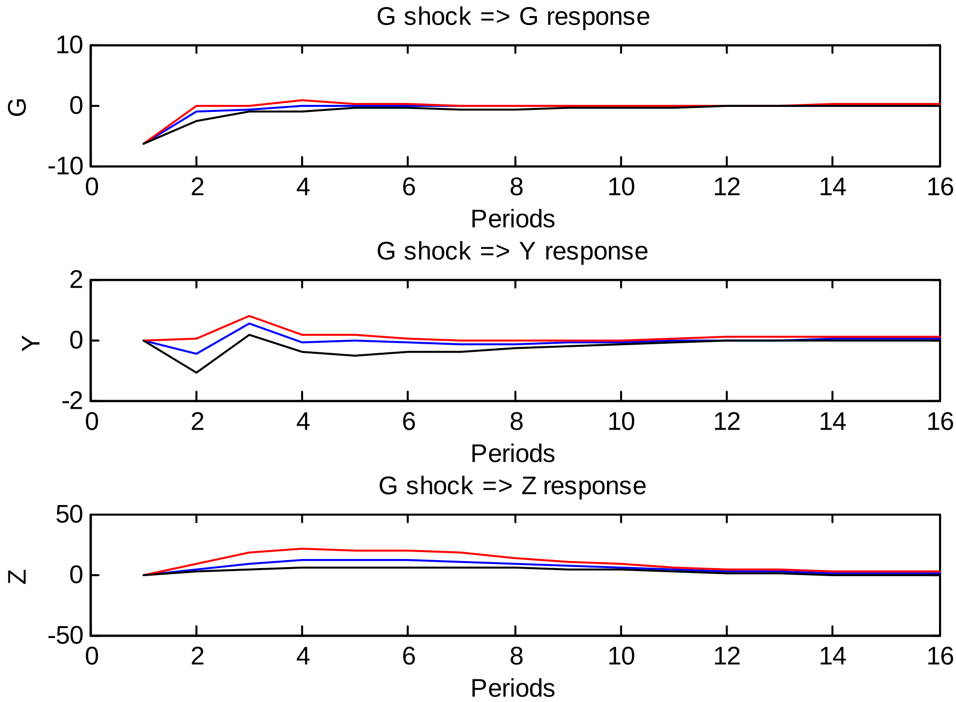

Figure 90: Accumulated response of all variables to a -1SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Growth rate model for sample period of 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

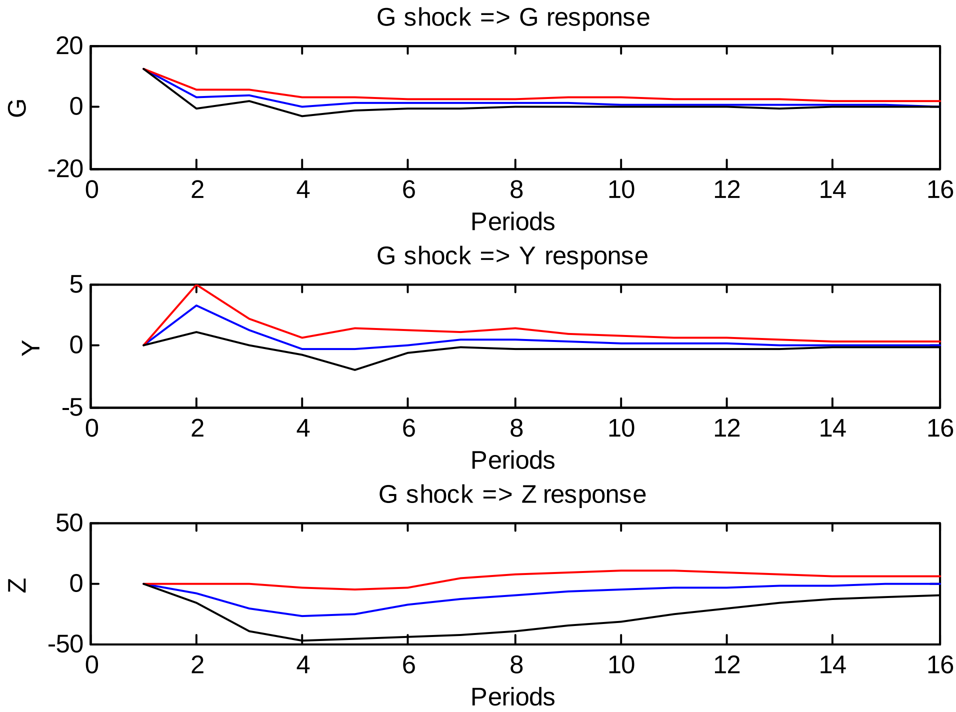

Figure 91: Accumulated response of all variables to a 2SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Growth rate model for sample period of 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

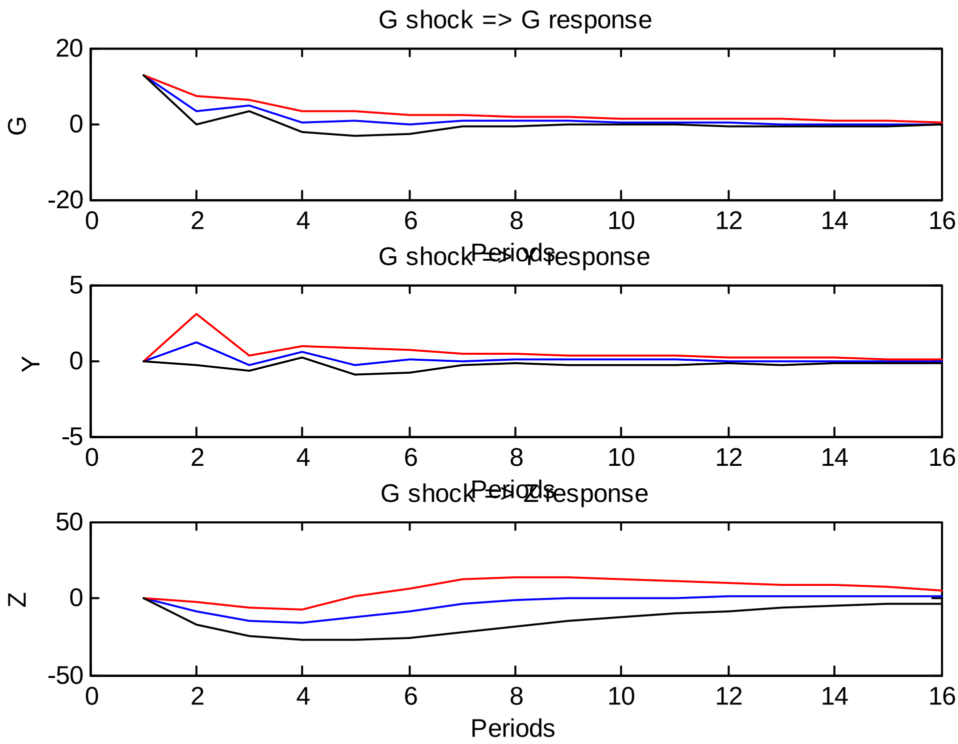

Figure 92: Accumulated response of all variables to a 2SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Growth rate model for sample period of 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

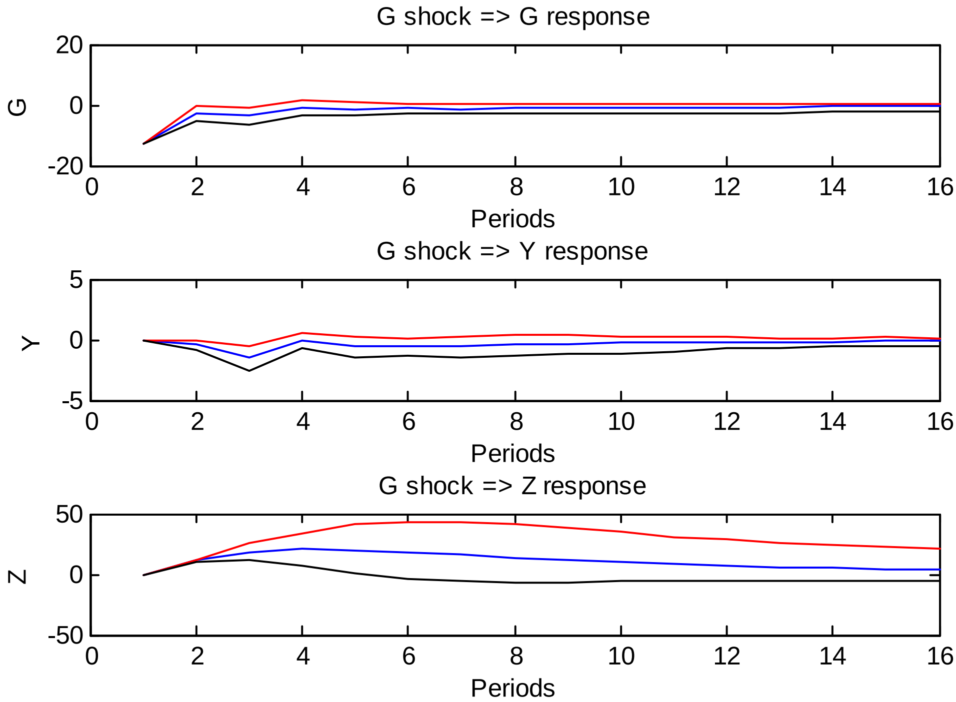

Figure 93: Accumulated response of all variables to a -2SD government spending shock for recessionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Growth rate model for sample period of 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

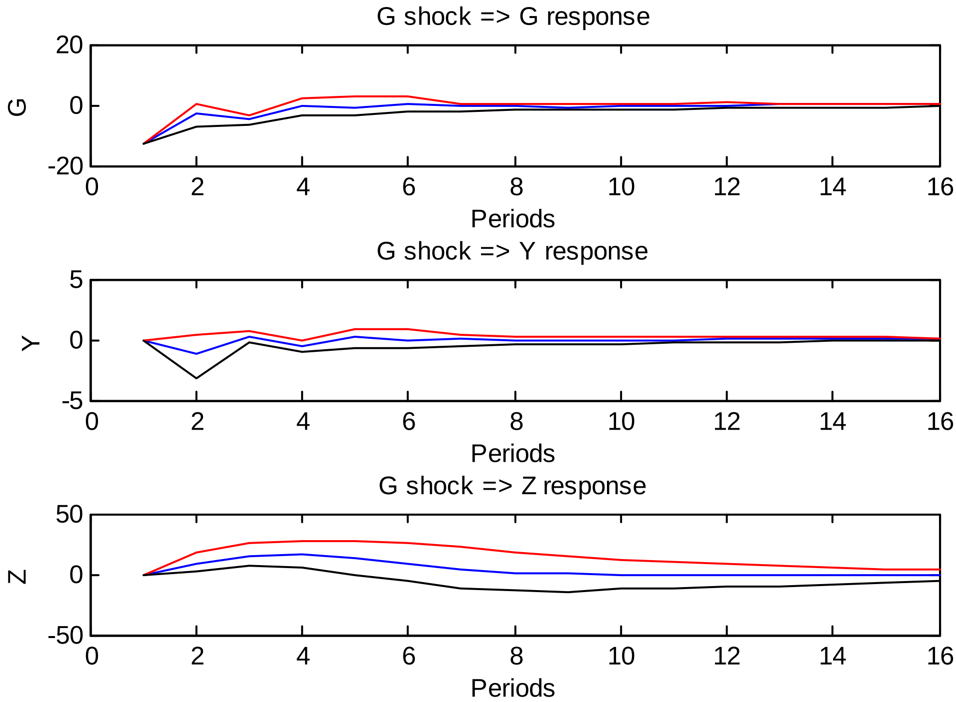

Figure 94: Accumulated response of all variables to a -2SD government spending shock for expansionary regime. 500000 Monte Carlo simulations were considered. Identification of government shocks follows Cholesky ordering, with G ordered first, Y second, and Z third. Growth rate model for sample period of 1960Q2:2012Q4 for $\gamma = 5$. Confidence bands for a 95% level are represented by the superior band (red line) and inferior band (black line).

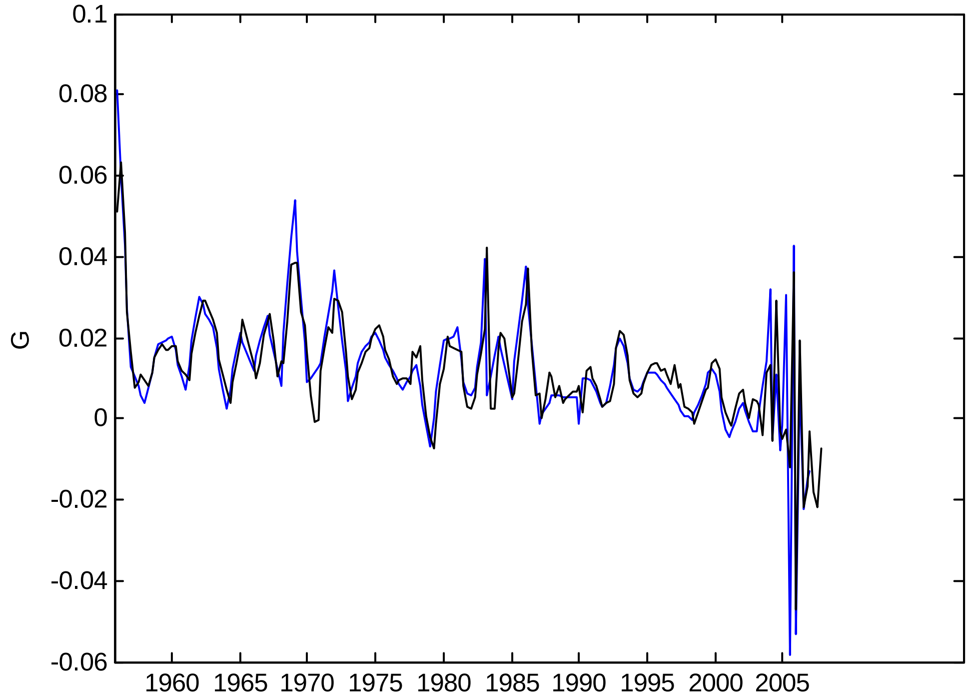

Figure 95: This figure confronts the actual values of G (blue line) with the predicted values by the model (black line). Growth rate model for sample 1960Q2:2012Q4, $\gamma = 5$, and 500000 Monte Carlo simulations.

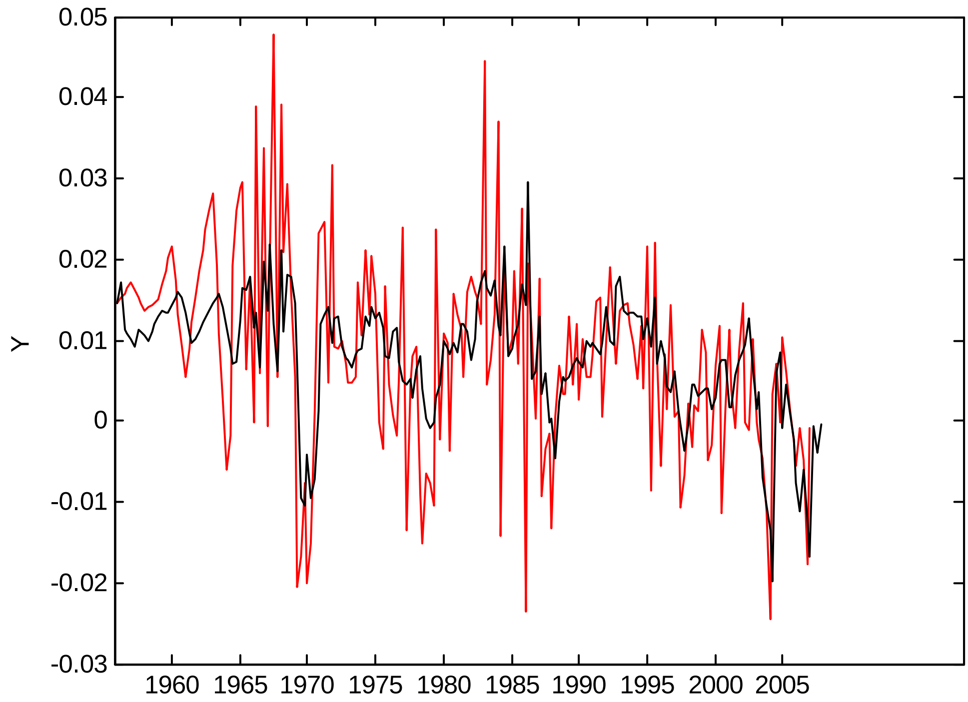

Figure 96: This figure confronts the actual values of Y (red line) with the predicted values by the model (black line). Growth rate model for sample 1960Q2:2012Q4, $\gamma = 5$, and 500000 Monte Carlo simulations.

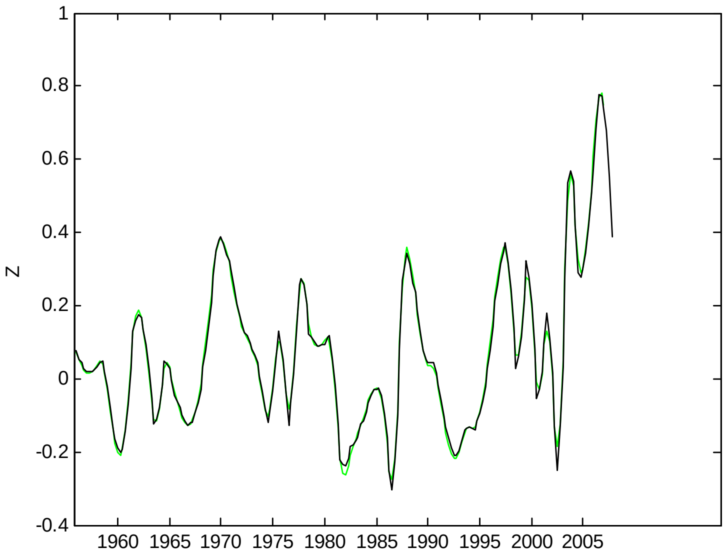

Figure 97: This figure confronts the actual values of Z (green line) with the predicted values by the model (black line). Growth rate model for sample 1960Q2:2012Q4, $\gamma = 5$, and 500000 Monte Carlo simulations.