

The Fisheries Dissipative Effect Modelling Through Dynamical Systems and Chaos Theory

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Abstract

It is presented a very simple mathematical model to study the dissipative effect in fisheries, based in dynamical systems and in its approach through chaos theory. Actually, due to constant misconducts, forgetting their consequences and only prosecuting easy profits, the risk of extinction is very real for fisheries resources. From it results the importance of this kind of models.

Keywords: Chaos, dynamical systems, fisheries.

1 Introduction

Many studies have been performed on fisheries in many environments worldwide, using various frameworks. In this work, chaos theory is used to analyse how fisheries can be organized, trying to avoid its dissipative effect. It is intended to consider chaos theory and complexity to build a model to explain the catches' dissipative effect in fisheries systems. Another work on this subject is[6].

2 Equations and Chaos

Chaos is present in many mathematical computer problems and in laboratory research. As soon as the idea of nonlinearity is introduced into theoretical models, chaos becomes obvious. The chaos theory and the complexity theory reflect that phenomena in many activities, such as fisheries.

Williams (1997), see [1], refers that phenomena happen over time either at discrete intervals or as continuously. Discrete intervals can be spaced evenly in time or irregularly in time. Continuous phenomena in principle might be measured continuously. But generally they are measured at discrete interval, being possible to obtain good approximations.

Differential equations are often the most adequate mathematical way to describe a smooth continuous evolution. But, frequently, to solve them is not an easy task. Difference equations can be solved right away if they are linear. And, in this form, they are often differential equations acceptable approximations. Olsen and Degn (1985), see [3], even risk to say that difference equations are the most powerful vehicle to the understanding of chaos. Others difference equations, than the linear, present difficulties even greater than the differential equations.

3 The Model

The fisheries mathematical model to be presented follows the model presented in Berliner (1992), see [4], related to dissipative systems in the presence of chaos. That author shows that non-invertibility is required to observe chaos for one-dimensional dynamic systems. He refers also that “everywhere invertible maps in two or more dimensions can exhibit chaotic behavior”. The strange attractors

study shows that in the long term, as time proceeds, the systems trajectories may become trapped in certain system state space bounded regions.

The model presented in Berliner (1992), see again [4], is an example, in two dimensions, of the Hénon map, exhibiting the property of having a strange attractor.

The Hénon map appears represented by the equations: $x_{t+1} = 1 + y_t - ax_t^2$ and $y_{t+1} = bx_t$, for fixed values of a and b , $t = 0, 1, \dots$. This invertible map has strange attractors and simultaneously strong sensitivity to initial conditions.

The Hénon map, representing a transformation from \mathbb{M}^2 to \mathbb{M}^2 , has Jacobian equal to $-b$. If $0 < b < 1$, the Hénon map contracts the domains to which it is applied. These maps are said to be dissipative.

It is evidently possible to suggest a model on this basis for fisheries. If a general situation is considered, the following equations may represent a system in which fish stocks, at time t , are given by x_t and catches by y_t . The model is as follows:

$$x_{t+1} = F(x_t) - y_t \text{ and } y_{t+1} = bx_t.$$

It is a Hénon model generalization. The Jacobian is equal to b . As y_{t+1} is a portion of x_t , $0 < b < 1$. So, it is a dissipative model and the values of x_t are restricted to a bounded domain. Considering the particular case below:

$$x_{t+1} = x_t - y_t, \text{ and } y_{t+1} = bx_t,$$

so, $x_{t+2} = x_{t+1} - y_{t+1}$ and $x_{t+2} - x_{t+1} + bx_t = 0$. Now, after solving the characteristic equation associated to the difference equation, see [5], it is obtained:

$$k = \frac{1 + \sqrt{1 - 4b}}{2} \text{ or } k = \frac{1 - \sqrt{1 - 4b}}{2}; \text{ calling } \Delta = 1 - 4b \text{ and being } 0 < b < 1,$$

$-3 < \Delta < 1$. So, $0 < \Delta < 1$ if $0 < b < \frac{1}{4}$ and $-3 < \Delta < 0$ if $\frac{1}{4} < b < 1$, being $\Delta = 0$ when $b = \frac{1}{4}$.

Consequently, for $0 < b < \frac{1}{4}$,

$$x_t = A_1 \left(\frac{1 + \sqrt{1 - 4b}}{2} \right)^t + A_2 \left(\frac{1 - \sqrt{1 - 4b}}{2} \right)^t.$$

And for $b = \frac{1}{4}$,

$$x_t = (A_1 + A_2 t) \left(\frac{1}{2} \right)^t.$$

Finally, for $\frac{1}{4} < b < 1$

$$x_t = (\sqrt{b})^t \left[A_1 \cos \left(\left(\arccos \frac{1}{2\sqrt{b}} \right) t \right) + A_2 \operatorname{sen} \left(\left(\arccos \frac{1}{2\sqrt{b}} \right) t \right) \right].$$

In these solutions, A_1 and A_2 are real constants.

Note that the bases of t powers are always between 0 and 1. So, $\lim_{t \rightarrow \infty} x_t = 0$ and whatever the value of b , the dissipative effect is real, even leading to the extinction of the specie. Of course, this is evident according to the premises of this particular situation of the model.

Concluding this approach, the general model does not allow obtaining in general such explicit solutions. But, of course, with simple computational tools it is possible to obtain recursively concrete time series solutions after establishing the initial value x_0 and to check the dissipative effect.

4 Conclusions

Chaos theory has become important in many contexts of dynamic systems explanation. Fish stocks modeling may be considered on the basis of an approach associated with chaos theory, instead considering the usual prospect based on classical models.

In fisheries analysis it is quite important to understand that overfishing may cause a problem of irreversibility in the recovering of several species. Anyway, to analyze this case specific situation, it is mandatory to obtain enough data to estimate the function which fits for that particular case.

In this work, a model evidencing the dissipative effect of catches on fisheries was presented. Additionally a particular model showed how stocks are dissipated and may tend to the extinction.

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Received: December 8, 2013