ISCTE Business School
Instituto Universitário de Lisboa

A COMPARISON ABOUT THE PREDICTIVE ABILITY OF FCGARCH, FACING EGARCH AND GJR

Ricardo Miguel Borges Matias

Project submitted as partial requirement for the conferral of Master in Finance

Supervisor:
Prof. Dr. José Dias Curto, Associate Professor, ISCTE-IUL Business School, Quantitative Methods Department

Co-Supervisor
Dr. Renato Costa, Lecturer, Western Institute of Technology at Taranaki (WITT), Engineering Department

October 2012

Abstract

In order to study the volatility of a stock market, several volatility models have been created, studied and improved throughout the time. Due to the extreme and actual situation in international stock market's volatility, the main objective of this thesis is to focus on the FCGARCH model created by Medeiros and Veiga (2009), and compare it with some of the most popular asymmetric autoregressive conditional heteroskedasticity models, such as EGARCH and GJR.

Using the daily returns of 5 most important international stock market indexes, such as S&P500 (USA), FTSE100 (UK), Nikkei225 (Japan), DAX30 (Germany) and PSI20 (Portugal), and using the Harvey-Newbold test, we are going to check which of these models is the best one to fit the conditional heteroskedastic volatilities of the returns of the indexes under study.

In order to make the thesis possible, I have created the FCGARCH, EGARCH and GJR models' codes in Matlab, with the help of my co-supervisor, Doctor Renato Costa, as well as used the Harvey-Newbold test in E-views, created by my supervisor, Professor José Dias Curto.

According to the estimation results, in the in-sample analysis, when looking at the Quasi-Maximum-Log likelihood goodness-of-fit measure, the FCGARCH fits most of the indexes' returns under study, where, in the out-of-sample analysis, according to the Harvey-Newbold test for multiple forecasts encompassing, the results show that the GJR seems to encompass the other two models in most of the indexes, thus concluding that GJR seems to be the best model to forecast the volatility.

Keywords: Forecasting volatility, EGARCH, GJR, FCGARCH

JEL: C52, C53

Resumo

Para que possamos estudar a volatilidade de uma ação, muitos foram os modelos criados, estudados e melhorados ao longo do tempo. Devido à extrema e atual situação da volatilidade nos mercados acionistas internacionais, o principal objetivo desta tese é focar no modelo FCGARCH, criado por Medeiros e Veiga (2009), e compará-lo com alguns dos mais importantes modelos heterocedásticos, autorregressivos e assimétricos, como o EGARCH e o GJR.

Utilizando os retornos diários de 5 dos índices mais importantes a nível internacional, tais como S&P500 (EUA), FTSE100 (RU), Nikkei225 (Japão), DAX30 (Alemanha) e PSI20 (Portugal), e usando o teste de Harvey-Newbold, vamos descobrir qual dos modelos apresentados é o que melhor descreve o comportamento das variâncias condicionais heterocedásticas dos retornos dos índices sob estudo.

Para que a criação desta tese fosse possível, tive de criar os códigos dos modelos do FCGARCH, EGARCH e GJR no Matlab, com a ajuda do meu co-orientador, o Doutor Renato Costa, assim como usar o teste de Harvey-Newbold no E-views, criado pelo meu orientador, o Professor José Dias Curto.

De acordo com os resultados estimados, na análise in-sample, ao olharmos para a medida de quase-máxima-verosimilhança, o FCGARCH descreve bem a maioria dos retornos sob estudo, enquanto, na análise out-of-sample, de acordo com o teste de Harvey-Newbold para a abrangência de previsões, os resultados demonstram que o GJR parece abranger os outros dois modelos na maioria dos índices, desta forma concluindo que o GJR parece ser o melhor modelo para prever a volatilidade.

Palavras-chave: Previsão de volatilidade, EGARCH, GJR, FCGARCH

JEL: C52, C53

Acknowledgements

Since, with this thesis, I am going to reach a new goal, not only professionally but also in life as well, I would like to thank all the people who helped me throughout this process.

First of all, I would like to thank my friends and the Tuna Académica do ISCTE, for the company, partying and understanding given in these years, which helped me developing my skills, not only in a professional way, but also in a personal way.

I would like to thank also Doctor João Ribeiro and his assistant, Elisabete Faria, for the time and help given in order to finish this thesis in time, as well as the professional and personal development I accomplished through this months since I started working in Mercal.

Furthermore, I would like to thank my co-supervisor, Doctor Renato Costa for all the collaboration, not only for the creation of the Matlab codes, but also for the help and support he had given me through the elaboration of this thesis and the codes related to FCGARCH. Without his insights, this thesis wouldn’t have the same quality as it has now.

I also would like to thank my supervisor, Professor José Dias Curto, for the kind support, guidance and insight in helping me to structure this thesis, as well as to improve the quality of the thesis with the introduction of the Harvey-Newbold test and its codes created by him.

Last but definitely not least, I would like to thank my parents and sister who supported me during all of these years of studies, not only financially, but also emotionally. Having the patience and understanding needed to a person like me. I hope and will try that all of their work with me and my life will not be in vain.

This thesis is dedicated to you all. Thank you so much.

Index

Executive Summary

Due to the financial instability faced nowadays throughout the world, the need of the existence of a plausible study of the volatility in the international stock market's volatility has been the main point of interest among the academy, the industries, and the markets themselves.

In this thesis, we are going to focus on the models which have reached a consensus on being the most important ones for the study of the volatility, starting with the ARCH model by Engle (1982), and continuing with other models such as GARCH by Bollerslev (1986), EGARCH by Nelson (1991), GJR by Glosten, Jagannathan and Runkle (1993), and STGARCH by Hagerud (1997).

However, the focus for this thesis will be on the model proposed by Medeiros and Veiga (2009), the FCGARCH model, which, among others, has as a main advantage the fact that the model can deal with more than 2 regimes, while others cannot.

Thus, this thesis will be focused on the comparison of the FCGARCH model, with other two main important models, the EGARCH and the GJR models. Creating the codes in Matlab for the estimation of the parameters for these models, and using the Harvey-Newbold test code on E-views, we will try to conclude which of these three models seems to be the best one to predict the volatilities of the assets.

Using the S&P500 (USA), FTSE100 (UK), Nikkei225 (Japan), DAX30 (Germany) and PSI20 (Portugal) indexes' returns from January 2001 to December 2011, I divided the sample in two main periods, being the first 2/3 of the observations used for the parameters estimation (In-sample analysis), where, using these results, the final 1/3 are considered as the forecast period (Out-of-sample Analysis).

According to the estimation in-sample results, the Quasi-Maximum-Likelihood shows that the FCGARCH model seems to be the best model in most of the indexes, fitting the dynamics of the returns of the indexes in study. If we consider other goodness-of-fit measures, such as Akaike and Schwarz criterions, although the FCGARCH model has a significant number of parameters, the results confirm the QML conclusions, favoring not only the FCGARCH model, but also the GJR model.

The out-of-sample results, based on the Harvey-Newbold encompassing test, show that the best model to predict the volatility forecasts for the majority of the indexes under study, seems to be, not the FCGARCH model, but the GJR model.

This thesis and its empirical result tends to be useful for those who want, not only to get deeper in the FCGARCH model, but also to be closer to find the best model, regarding the forecast of volatility in financial markets, mainly during periods of high volatility.

Sumário Executivo

Devido à instabilidade económica enfrentada a nível mundial ao longo dos tempos, a necessidade na existência de um estudo plausível da volatilidade nos mercados internacionais acionistas tem vindo a ser o maior ponto de interesse entre a academia, as empresas, e os próprios mercados.

Nesta tese, iremos focar-nos nos modelos aos quais chegou-se a um consenso relativamente a serem os mais importantes a serem estudados para a volatilidade, começando com o modelo ARCH de Engle (1982), e continuando com outros modelos, tais como GARCH de Bollerslev (1986), EGARCH de Nelson (1991), GJR de Glosten, Jagannathan e Runkle (1993), e STGARCH de Hagerud (1997).

No entanto, o foco nesta tese será no modelo proposto por Medeiros e Veiga (2009), o modelo FCGARCH, sendo que, entre outros, tem a principal vantagem de poder lidar com mais do que dois regimes, algo que os outros modelos não conseguem.

Sendo assim, esta tese focar-se-á na comparação entre o modelo FCGARCH, com outros dois principais modelos, o EGARCH e o GJR. Criando os códigos em Matlab para a estimação dos parâmetros para estes modelos, e usando o código do teste de Harvey-Newbold no E-views, vamos tentar concluir quais destes três modelos parece ser o melhor para prever as volatilidades das ações.

Usando os retornos dos índices S&P500 (EUA), FTSE100 (RU), Nikkei225 (Japão), DAX30 (Alemanha) e PSI20 (Portugal), desde Janeiro de 2001 até Dezembro de 2011, dividi a amostra em dois períodos distintos, sendo os primeiros 2/3 das observações usados para a estimação dos parâmetros (análise in-sample), donde, usando esses retornos, o 1/3 final será considerado o período de previsão (análise out-of-sample).

De acordo com os resultados da estimação in-sample, o indicador de Quase-Máxima-Verosimilhança demonstra que o modelo FCGARCH parece ser o melhor, na maioria dos índices, enquadrando-se nas dinâmicas dos retornos dos índices estudados. Se considerarmos outras medidas como os critérios de Akaike and Schwarz, apesar de o modelo FCGARCH apresentar um número significativo de parâmetros, os resultados confirmam as conclusões do QMV, favorecendo não só o FCGARCH como também o GJR.

Os resultados out-of-sample, baseados no teste de Harvey-Newbold, demonstram que o melhor modelo para prever a volatilidade na maioria dos índices, parece ser, não o modelo FCGARCH, mas sim o GJR.

Esta tese e seu resultado empírico tornam-se necessários para aqueles que querem não só aprofundar o modelo FCGARCH, mas também para estar mais próximo de encontrar o melhor modelo, relativamente à previsão da volatilidade nos mercados financeiros, principalmente durante períodos de elevada volatilidade.

1. Introduction

When we talk about stock markets, one of the first things that comes in our mind is volatility. Starting with Bachelier (1990), who identified an impossibility to predict stock returns, due to the huge number of factors that should be included in their forecast, this definition has been developed throughout the years, even more when the concept of market risk (uncertainty about the future market price of a financial asset) has been the main point of interest among the academy, the industries, and the markets themselves.

Nowadays, due to the development of technology and the fastness of how the information is passed throughout the world, we have an infinite number of arriving news which has a huge impact on the prices of stocks in the capital markets. Consequently, we are facing financial markets whose prices and returns change by the second, thus making even more difficult to predict how these markets are going to act in the recent future, and how these acts will affect their prices and returns in the future.

This uncertainty changed some of the minds and behaviors of the investors, who are considered more risk averse (want to invest and have the highest return at the minimum risk possible), and thus are more concerned about the variability of the returns of these markets.

Many authors tried to identify some regularity in the asset returns. Mandelbrot (1963: 418) concluded that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes", thus describing the "volatility clustering effect", in which we can conclude that the returns cannot be independent and identically distributed (i.i.d.). Black (1976) discovered the "Leverage effect", a tendency for negative correlations between changes in the stock prices and in the volatilities of those prices, that is, unexpected bad news or negative innovations have a higher impact on the risk and the prices of a stock than positive news with the same magnitude.

On the same article, Black also discovered that there may be a co-movement among volatilities, saying that high volatility stocks are somewhat more correlated to market volatility changes rather than low volatility changes. Finally, Schwert (1989) found that during financial crisis and recessions, the volatility of the stock rises, but its relation with macroeconomic uncertainty is very weak, which means that maybe stock values are not that closely tied to the health of the economy as we should expect.

All these factors lead to one conclusion: We have to find a model which can predict the risk of certain stocks, always having in mind their regularities. Although this is difficult to find, some authors have reached some helpful models. However, each model has its own disadvantage: there is not a perfect model, so there is a continuous research for the perfect model.

In the last years several models have been proposed to deal with the stock markets' volatility. Starting with the ARCH Model by Engle (1982), and continuing with other models such as GARCH by Bollerslev (1986), EGARCH by Nelson (1991), GJR by Glosten, Jagannathan and Runkle (1993), STGARCH by Hagerud (1997), among many others, we finally reach the FCGARCH model, proposed by Medeiros and Veiga (2009). It is to note the importance of FCGARCH, due to the fact that the model can deal with more than 2 regimes, while EGARCH and GJR can only deal with no more than 2 regimes, thus being FCGARCH a possible solution for the predictability of certain financial assets' volatilities.

There are some researches and studies using FCGARCH which have already been made, such as the one made by Hillebrand and Medeiros (2008), which studied the statistical consequences of neglecting structural breaks and regime switches in autoregressive and GARCH models, proposing the FCGARCH model as a solution for the problem, since the regime-switches are governed by an observable variable, thus concluding that the FCGARCH is a variation of the GARCH model which contains and identifies the different volatility regimes. Another paper worth of mentioning is the one made by Costa (2010). He investigated the pricing of options according to the use of the FCGARCH model, making simulations according to the results estimated in Medeiros and Veiga's paper. Finally, Salgado (2011) tested the predictive accuracy of some GARCH models, including the FCGARCH, concluding that the model has the best performance on realized volatility.

Based on this, the main purpose of this paper is to confirm the importance of the FCGARCH model, computing the predictability of three GARCH type models, mainly EGARCH, GJR and FCGARCH, according to the daily returns of 5 most important international stock market indexes, such as S&P 500 (USA), FTSE 100 (UK), DAX 30 (Germany), Nikkei 225 (Japan), and PSI 20 (Portugal).

In order to analyze the models and their characteristics according to the indexes in study, and with the help of my co-supervisor, Professor Renato Costa, I had to create the code for the programs for the estimation of the models in Matlab, not only for FCGARCH, but also for EGARCH and GJR, so that each model can be compared in-sample according to the Quasi-Maximum-Log-likelihood. It is to note the complexity of these models, specially the FCGARCH, even more due to the fact that this model is very recent, and so there are very few studies talking about this model, as well as there are no codes shared to the public, thus obligating to create the codes from scratch, based on the paper from the creators Medeiros and Veiga (2009). These codes turn out to be crucial for the analysis in this thesis.

Although it was very challenging, I have been successful in this task, even more since the parameters estimated in this thesis follow all the constraints specified in the model and present in the codes. Consequently, for the out-of-sample analysis, I had to use the Harvey-Newbold test. In order to make some conclusions for this test, I used the code in E-views created by my supervisor, Professor José Dias Curto, which allowed me to make a better conclusion regarding the predicting capability of these three models.

The structure of this thesis is based on the following: first, we are going to make a small introduction and recap of the models in study, indicating their importance in the financial market and their main advantages and disadvantages, with a main focus on the FCGARCH model. Then, we are going to describe and analyze the data which we will use in this thesis, followed by the estimation results of the models' parameters based on the same data, and make a comparison of the out-of-sample evaluation results, with the consequent discussion of the results. Finally, some conclusion remarks will be made.

2. Models under Comparison – EGARCH, GJR and FCGARCH

According to Majmudar and Benerjee (2004), financial volatility can be divided in three main groups: realized volatility, which is based on the standard deviation of the returns of the asset; implied/market volatility, representing the market prediction about future price fluctuations; and model volatility, based on theoretical models such as GARCH (Generalized Autoregressive Conditional Heteroskedasticity) and stochastic volatility. Since in this thesis we are focusing on the FCGARCH model, our main concern will be concentrated on this last category.

Let $\{S_t\}$ represent the price of one specific asset at time t. Using the logarithm return,

$$y_t = ln\left(\frac{S_t}{S_{t-1}}\right). \quad \textbf{(1)}$$

Let F_{t-1}represent all the available information at time t-1, and $\varepsilon_t \sim N\ (0,1)$. If we consider a constant volatility, then the returns at time t should be viewed as:

$$y_t = E(y_t|F_{t-1}) + w_t = \mu_{t|t-1} + \sigma\varepsilon_t\ , w_t \sim N\ (0, \sigma^2) \quad \textbf{(2)}$$

What happens with this formula is that although we are using a conditional mean, a constant volatility is not the best assumption to make. There is a volatility clustering effect studied by Mandelbrot (1963) and Fama (1965), which means that we have to assume since then that the volatility does change through time, and consequently, the variance must be heteroskedastic. This means that we have to assume that σ^2 is now a stochastic process, and the conditional variance is denoted by $\sigma^2_{t|t-1}$.

In order to solve this problem, Engle (1982) tried an ARCH (Autoregressive Conditional Heteroskedasticity) model. In this model, we assume that the conditional variance is a linear function of the past q squared innovations, this is:

$$\sigma^2_{t|t-1} = \omega + \sum_{i=1}^{q} \alpha_i\, w^2_{t-i} \quad \textbf{(3)}$$

It is to note that in this model, $\omega > 0$, and $\alpha_i \geq 0$, $i \in [1, q]$, so that the conditional mean and variance can be positive, and $\sum_{i=1}^{q} \alpha_i < 1$, to ensure that this process is covariance stationary. This model has the advantage of being easy to use, due to its simplicity of formulation and estimation, allowing also the impact of volatility clustering.

However, the ARCH model has many drawbacks, being the main one the fact that only w_{t-i}^2 affects the current volatility, which may be unrealistic, since the latter may respond differently to good or bad news ($w_t > 0$ or $w_t < 0$), and the impact of a large shock only lasts for q periods, with a long length implying a large number of parameters, thus being more difficult to estimate.

Bollerslev (1986) tried to circumvent this problem proposing the Generalized ARCH (GARCH) model, in which the conditional variance is a linear combination not only of the past q squared innovations, but also of the past p conditional variances, being represented by:

$$\sigma_{t|t-1}^2 = \omega + \sum_{i=1}^{q} \alpha_i \, w_{t-i}^2 + \sum_{i=1}^{p} \beta_i \, \sigma_{t-i}^2 \qquad \textbf{(4)}$$

It is to note that $\omega > 0$; $\alpha_i \geq 0$, with $i \in [1, q]$; $\beta_i \geq 0$, with $i \in [1, p]$, in order to assure a positive conditional variance and $\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i < 1$, to ensure a covariance stationary process. This model has the advantage of being more flexible than the ARCH model, when parametrizing the conditional variance. The GARCH model not only captures thick tailed returns, but also the volatility clustering effect.

However, with the GARCH model we are concluding that really bad news ($w_t < 0$) have the same impact on future volatility than really good news ($w_t > 0$), both with the same impact. Since we are only considering the square of the errors, and not their sign, we are excluding the "financial leverage effect" determined by Black (1976) and Christie (1982). This means that GARCH model is not appropriate to predict conditional volatility at all.

Nelson (1991) proposed the Exponential GARCH (EGARCH) model to include this effect. In this model, the conditional variance depends on both the size and the sign of the residuals. This is:

$$ln\left(\sigma_{t|t-1}^2\right) = \omega + \sum_{i=1}^{q} \alpha_i \frac{|w_{t-i}|}{\sigma_{t-i}} + \sum_{i=1}^{q} \gamma_i \frac{w_{t-i}}{\sigma_{t-i}} + \sum_{i=1}^{p} \beta_i \, ln(\sigma_{t-i}^2) \qquad \textbf{(7)}$$

With this model, it is to note that, γ_i must be negative. By doing so, the leverage effect is considered, this is, negative news/shocks ($w_t < 0$) will increase volatility more than positive news/shocks ($w_t > 0$). It is also to note that with the use of the log-conditional variance we don't have to impose that all the coefficients must be positive, to ensure a positive conditional variance.

There is also one model created by Glosten, Jagannathan and Runkle (1993) that is very used on the conditional volatility subject: the GJR model. The specificity of this model is the fact that, modifying the GARCH formula, we are now using a dummy variable, which makes it possible to analyze the impact of the negative shocks (news), as we can see below:

$$\sigma_{t|t-1}^2 = \omega + \sum_{i=1}^{q}\left(\alpha_i + \gamma I_{w_t<0}\right)w_{t-i}^2 + \sum_{i=1}^{p}\beta_i\,\sigma_{t-i}^2 \qquad \textbf{(8)}$$

As we can see, the coefficient for the dummy variable is represented by γ, which, if it is positive and statistically significant, $w_t < 0$ (we have bad news), and $I = 1$, which indicates a negative asymmetric volatility response.

Continuing with the idea of the news impact on the volatility, Hagerud (1997) identified the existence of several regimes which have a different impact on the returns and the volatility, thus concluding that the conditional volatility can't always be a linear function. By doing so, he indicated a nonlinear model which allowed a smooth transition among those regimes, depending on the sign of the past returns. He introduced the Smooth Transition GARCH (STGARCH) Model, which is represented by:

$$\sigma_{t|t-1}^2 = \omega + \sum_{i=1}^{q}\left[\alpha_{1j} + \alpha_{2j}F(w_{t-j})\right]w_{t-i}^2 + \sum_{i=1}^{p}\beta_i\,\sigma_{t-i}^2 \qquad \textbf{(9)}$$

It is to recall that α_{1j} and α_{2j} are constants, and $F(w_{t-j})$ is the transition function, which can take the logistic form $\left\{F\left(w_{t-j}\right) = \left[1 + \exp\left(-\theta w_{t-j}\right)\right]^{-1} - 1/2\right\}$, or the exponential form $\left\{F\left(w_{t-j}\right) = 1 - \exp\left(-\theta w_{t-j}^2\right)\right\}$, both forms with $\theta > 0$. As the name of the model suggests, it allows a smooth transition between volatility regimes. The exponential form highlights the differences between the effects of big and small shocks, and the logistic form implies the "financial leverage effect".

This study of nonlinear models leads us finally to the Flexible Coefficient GARCH (FCGARCH) model, presented by Medeiros and Veiga (2009). The main characteristic of this model is that, comparing to other nonlinear GARCH models, it allows more than two limiting regimes. Being $y_t = \sigma_t\varepsilon_t$, with $\varepsilon_t \sim N(0,1)$, $y_t \sim N(0,\sigma^2)$. With $m = H + 1$ limiting regimes, for an FCGARCH(1,1,m), this model has the following expression,

$$\sigma_{t|t-1}^2 = \alpha_0 + \beta_0\sigma_{t-i}^2 + \lambda_0 y_{t-i}^2 + \sum_{i=1}^{H}\left[\alpha_i + \beta_i\sigma_{t-i}^2 + \lambda_i y_{t-i}^2\right]\frac{1}{1+e^{-\gamma_i(s_t-c_i)}} \qquad \textbf{(10)}$$

As we can see, the FCGARCH has the logistic elements of the STGARCH, and by that the stationary condition is satisfied, even with weak restrictions.

The slope parameter:

$$f_i(s_t;\gamma_i;c_i) = \frac{1}{1+e^{-\gamma_i(s_t-c_i)}}, \quad \textbf{(11)}$$

with $i \in [1, H]$, determines the speed of the transition between two regimes. s_t is the transition variable, measurable at time $t-1$, and often interpreted as $s_t = y_{t-1}$.

According to Medeiros and Veiga (2009), and Hillebrand and Medeiros (2008), the main advantages of FCGARCH in comparison to other nonlinear models are the following:

- More than two limiting regimes can be modeled, with the number of regimes being determined by a simple and easy sequence of tests.
- Unlike the STGARCH, there are no fixed values for where the transitions are located;
- It is highly flexible, since it is stationary and ergodic independently of the intensity of the regimes and the strength of the restrictions;
- It describes several stylized facts of financial time series, such as the volatility clustering effect, the financial leverage effect, among others, that other models can't do with the same accuracy;

For the FCGARCH to be successful, some assumptions must be made. According to Medeiros and Veiga (2009), the main ones are:

1. $\psi = \{\alpha_0, \alpha_1, \dots, \alpha_H, \beta_0, \beta_1, \dots, \beta_H, \lambda_0, \lambda_1, \dots, \lambda_{H,} c_1, \dots, c_H\} \in \Psi \subseteq \mathbb{R}^{3+5H}$. This is, the true parameter vector is in the interior of Ψ, a compact and convex parameter space. This is important for the estimation of the model.
2. $\{\varepsilon_t\}$ must be drawn from a continuous, symmetric, unimodal, positive everywhere density and bounded in a neighborhood of zero.
3. For the model to be identifiable, $-\infty < c_1 < \cdots < c_2 < \infty$, and $\gamma_i > 0$.
4. To guarantee strictly positive conditional variances, $f_i(s_t;\gamma_1;c_1) \geq f_i(s_t;\gamma_2;c_2) \geq \cdots \geq f_i(s_t;\gamma_H;c_H)$, $\forall t$.
5. To guarantee strictly positive conditional variances, $\sum_{i=0}^{H} \alpha_i > 0$, $\sum_{i=0}^{H} \beta_i \geq 0$, and $\sum_{i=0}^{H} \lambda_i \geq 0$.

6. $\{\varepsilon_t\}$ must be strictly stationary and ergodic, since if it is nonstationary some regimes may be unidentifiable.
7. To guarantee strict stationary of the model, $\frac{1}{2}(\beta_0 + \lambda_0) + \frac{1}{2}\sum_{i=1}^{H}(\beta_i + \lambda_i) < 1$.
8. For the model to be identifiable, α_i, β_i and λ_i can't vanish jointly for some $i \in [1, H]$, thus not allowing any irrelevant regimes.
9. In order to cause identifiability of the model, $f(s_t; \gamma_i; c_i) \neq 1 - f(s_t; -\gamma_i; c_i)$
10. In order to guarantee the fourth moment of y_t, we must also guarantee that $\beta_0^2 + \beta_0\beta_U + \frac{\beta_U^2}{2} + \mu_4\left(\lambda_0 + \lambda_0\lambda_U + \frac{\lambda_U^2}{2}\right) + 2\lambda_0\beta_0 + \beta_0\lambda_U + \lambda_0\beta_U + \lambda_U\beta_U < 1$.

For the sake of understanding how this model is used, let us suppose that $H = 2$, c_1 is highly negative, and c_2 is highly positive. We have the following FCGARCH(1,1,3) model:

$$\sigma_{t|t-1}^2 = \alpha_0 + \beta_0\sigma_{t-i}^2 + \lambda_0 y_{t-1}^2 + [\alpha_1 + \beta_1\sigma_{t-i}^2 + \lambda_1 y_{t-1}^2]\frac{1}{1+e^{-\gamma_1(s_t-c_1)}} + [\alpha_2 + \beta_2\sigma_{t-i}^2 + \lambda_2 y_{t-1}^2]\frac{1}{1+e^{-\gamma_2(s_t-c_2)}} \qquad \mathbf{(12)}$$

As we can see, 3 regimes are determined: extremely low negative shocks/"very bad news" ($\alpha_0 + \beta_0\sigma_{t-i}^2 + \lambda_0 y_{t-1}^2$), low absolute returns/"tranquil periods" ($\alpha_0 + \beta_0\sigma_{t-i}^2 + \lambda_0 y_{t-1}^2 + [\alpha_1 + \beta_1\sigma_{t-i}^2 + \lambda_1 y_{t-1}^2]\frac{1}{1+e^{-\gamma_1(s_t-c_1)}}$), and very high positive shocks, or "very good news" ($\alpha_0 + \beta_0\sigma_{t-i}^2 + \lambda_0 y_{t-1}^2 + [\alpha_1 + \beta_1\sigma_{t-i}^2 + \lambda_1 y_{t-1}^2]\frac{1}{1+e^{-\gamma_1(\varepsilon_{t-1}-c_1)}} + [\alpha_2 + \beta_2\sigma_{t-i}^2 + \lambda_2 y_{t-1}^2]\frac{1}{1+e^{-\gamma_2(s_t-c_2)}}$).

Salgado (2011) created a small but resuming table, which explains the engine of a FCGARCH(1,1,2) model. Assuming that $\gamma_1 > 0$:

Table 1: The Engine of a FCGARCH (1,1,2)

	$\alpha_0 + \beta_0\sigma_{t-i}^2 + \lambda_0 y_{t-1}^2$			$[\alpha_1 + \beta_1\sigma_{t-i}^2 + \lambda_1 y_{t-1}^2]\frac{1}{1+e^{-\gamma_1(s_t-c_1)}}$					
c_1	N/A			Highly Negative			Highly Positive		
y_{t-1}	$\to -\infty$	$\to 0$	$\to +\infty$	$\to -\infty$	$\to 0$	$\to +\infty$	$\to -\infty$	$\to 0$	$\to +\infty$
$\frac{1}{1+e^{-\gamma_1(\varepsilon_{t-1}-c_1)}}$	N/A			$\to 0$	≈ 1	$\to 1$	$\to 0$	≈ 0	$\to 1$
Regime is	Active			Inactive	Active	Active	Inactive	Inactive	Active

In order to know how many regimes should be determined, Medeiros and Veiga (2009) created a somehow complex but effective test for the presence of an additional regime. By using the form $[\alpha_H + \beta_H \sigma_{t-i}^2 + \lambda_H y_{t-i}^2] f_{H,t}$, their test consists on the following:

$$\begin{cases} H_0: \lambda_H = 0 \\ H_a: \lambda_H > 0 \end{cases}$$

Assuming a normal distribution, we should:

1. Estimate the FCGARCH model with the new regime, under H_0, and call its variance $\sigma'^2_{0,t}$.
2. Compute the Sum of Squares Residual:

$$SSR_0 = \sum_{t=1}^{T} \left(\frac{y_t^2}{\sigma'^2_{0,t}} - 1 \right)^2. \qquad \textbf{(13)}$$

3. Regress $\left(\frac{y_t^2}{\sigma'^2_{0,t}} - 1 \right)$ on $\hat{z}_t$ and $\hat{u}_t$, and compute its Sum of Squares Residual: SSR_1.
4. Compute the LM Statistic:

$$LM = T \frac{SSR_0 - SSR_1}{SSR_0} \qquad \textbf{(14)}$$

or the F statistic:

$$F = \frac{\frac{SSR_0 - SSR_1}{3}}{\frac{SSR_1}{T - 5H + 2}}, \qquad \textbf{(15)}$$

in order to study the significance of the new regime.

So, as we know how FCGARCH works, according to Medeiros and Veiga (2009), in order to estimate the parameters of the model, since $\{\varepsilon_t\}$ is unknown, there should be a maximization of the following quasi-maximum likelihood (QML) function:

$$\begin{cases} L_T(\psi) = \frac{1}{T} \sum_{t=1}^{T} l_t(\psi) \\ l_t(\psi) = -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln(\sigma_t^2) - \frac{y_t^2}{2\sigma_t^2} \end{cases} \qquad \textbf{(16)}$$

The use of the QML is reliable, since, according to Lee and Hansen (1994), Jeantheau (1998), and Ling and McAleer (2003), this function is consistent and asymptotically normal, even under weaker conditions. By always assuming a normal distribution for the errors, the $L_T(\psi)$ is not conditional on the true $(\varepsilon_t, \sigma_t)$, which makes it easier to be used in practical applications.

3. Empirical Study

3.1. Statistical Properties of Returns

The daily returns of the indexes in which we are going to implement the FCGARCH model, as well as EGARCH and GJR models, are 5 of the most important international stock market indexes: S&P 500 (USA), DAX 30 (Germany), FTSE 100 (UK), PSI 20 (Portugal), and Nikkei 225 (Japan).

The closing prices of these indexes were taken from Yahoo Finance, and can be seen, as well as its returns, in the annexes 1.1.

Being S_t the closing price of the index at time t, using the logarithm return,

$$y_t = ln\left(\frac{S_t}{S_{t-1}}\right) \quad \textbf{(17)}$$

the descriptive statistics of the returns of these indexes are presented in Table 2:

Table 2. Summary Statistics of Returns

	S&P500	**DAX**	**FTSE100**	**PSI20**	**Nikkei 225**
Starting	02-01-2001	01-01-2001	02-01-2001	01-01-2001	04-01-2001
Ending	30-12-2011	30-12-2011	30-12-2011	30-12-2011	30-12-2011
Observations	2766	2839	2777	2672	2695
Mean	-0,0000073	-0,0000306	-0,0000370	-0,0002390	-0,0001788
Median	0,000682206	0,000409408	0,000337363	0	0,000195023
Std. Deviation	0,013845017	0,016282365	0,013346508	0,012044358	0,016181109
Maximum	0,109571959	0,107974655	0,09384244	0,125297903	0,13234585
Minimum	-0,094695145	-0,090084123	-0,092645552	-0,137764669	-0,121110256
Skewness	-0,166292013	0,000400489	-0,114410308	-0,211229535	-0,387071593
Kurtosis	7,611780592	4,447750073	5,878927454	18,73589903	6,742653538
Jarque-Bera	2463,945	247,93708	965,07486	27588,065	1640,2167

As we can see, from January 2001 to December 2011, all the indexes had a negative mean, as well as a negative skewness (except DAX 30, although is very close to 0, thus being close to having a symmetric distribution), which implies an asymmetry, namely a heavier and longer left tail in the sample distribution. The high kurtosis computed in the returns, being leptokurtic, implies that the distribution of these returns have fatter tails, and a more sensitive peak around the mean, when compared to the normal distribution.

According to the Jarque and Bera (1987) Test:

$$JB = \frac{n}{6}\left[S^2 + \frac{1}{4}(K-3)^2\right], \quad \textbf{(18)}$$

being n the number of observations, S the skewness, and K the kurtosis, the results confirm the rejection of the normality assumption for each of the series.

The sample used in this thesis is divided in two main periods. The first 2/3 of the observations are the ones used for the parameters estimation (In-sample analysis), where, using these results, the final 1/3 are considered as the forecast period (Out-of-sample Analysis).

3.2.In-sample Analysis

3.2.1. Descriptive Statistics

The descriptive statistics of the returns of these indexes, based on the in-sample data, are presented in Table 3:

Table 3. Summary Statistics of Returns (In-sample)

	S&P500	DAX	FTSE100	PSI20	Nikkei225
Starting	02-01-2001	01-01-2001	02-01-2001	01-01-2001	04-01-2001
Ending	05-05-2008	16-04-2008	30-04-2008	30-10-2007	25-04-2008
Observations	1843	1892	1850	1781	1797
Mean	0,0000501	0,0000217	-0,0000077	0,0001246	0,0000070
Median	0,000488856	0,000330803	0,000336925	9,03318E-06	0,000196695
Std. Deviation	0,010860767	0,01518862	0,011538219	0,008383281	0,014200594
Maximum	0,055744324	0,073462813	0,059037795	0,03839921	0,072217435
Minimum	-0,05046794	-0,090084123	-0,056374301	-0,046319284	-0,068644566
Skewness	0,099304048	-0,183715002	-0,160760091	-0,530253689	-0,173758728
Kurtosis	2,53974093	3,58378559	3,335268367	3,553110146	1,549037881
Jarque-Bera	386,2752241	731,3999207	331,6989761	2199,080982	3422,102645

As we can see, according to the results presented in the in-sample analysis, contrary to the ones presented in the whole sample, we can see that most of the indexes have a positive mean. One of the reasons for this situation is the fact that in these sample are not considered the last 3/4 years (mid 2008 to 2011), when the world crisis had and has an extreme impact on the assets' returns, thus suffering a decrease, and consequently contributing for the negative mean, if we consider the whole sample.

However, most of the indexes continue to have an asymmetry, with a heavier and longer left tail, except for S&P 500, which has a longer right tail, but the value seems to be significant. The kurtosis implies that all of them are leptokurtic, and the Jarque-Bera test results confirm the rejection of the normality assumption, the same conclusions as if we consider the whole sample.

3.2.2. Estimation Results

According to Bollerslev, Chou and Kroner (1992), and more recently, Hansen and Lunde (2005), the use of $(p,q) = (1,1)$ model is satisfactory when estimating the volatility of financial assets. Since most of the empirical papers deal with this specification, Medeiros and Veiga (2009) focused their attention on a first-order FCGARCH specification. Thus, when estimating the FCGARCH model, and in order to compare it with EGARCH and GJR, we are going to use this specification.

Moreover, when estimating the volatility using the FCGARCH model, due to the complexity of the estimation of the regimes in Matlab, and since in Medeiros and Veiga (2009) paper most of their estimated results were based on 3 regimes, we are going to use that assumption in our estimation, that is, H=2.

According to Franses and Van Dijk (2000), as well as Fan and Yao (2003), since we don't know the "true" distribution of the innovations/news, the use of QML can provide a practical way to estimate non-normal GARCH parameters, being much more simple its estimation. So, in order to compare the in-sample results, we are going to use the quasi maximum log-likelihood (QML) on the three models, so we can make a more realistic analysis and comparison.

The use of this estimator assumes that the innovations follow a normal distribution, although it can also be used for non-normal distributions, such as Student t (1925), for example. Medeiros and Veiga (2009) used the QML in their paper, so we are going to do the same. Since the FCGARCH model properties were developed for the normal distribution, we will also assume in this thesis that the innovations follow a Gaussian (Normal) distribution.

It is also to be acknowledged that, although we have seen that the observations for this thesis seem to follow a non-normal distribution, as seen in the results for the Jarque-Bera test, the use of FCGARCH, according to Medeiros and Veiga (2009: 117), has the main advantage of "generating series with high kurtosis", so the assumption for the normality of the innovations is plausible for this thesis.

The codes which were used for the estimation of the three models were made on Matlab program, and are present in annexes 1.2. It is to note the complexity of the FCGARCH model and consequently of its incorporation in Matlab, due to the number of parameters and regimes incorporated in the model. However, in this thesis we have reached a point where all the conditions are satisfied, the parameters seem reasonable, if compared to the ones predicted by Medeiros and Veiga (2009), and it was created a prediction code of FCGARCH, all of them working, with no errors whatsoever, thus contributing for a better interpretation and conclusion of the results presented in this thesis.

It is also to note, since I created the codes for the FCGARCH model, in order to make a plausible comparison between the models in study, I thought it would be better if I created not only the estimation codes for the FCGARCH, but also for EGARCH and GJR models. With the use of these codes, based on the in-sample observations, we reach the following results, presented on tables 4, 5 and 6:

Table 4. Estimation Results

Parameters	S&P 500			DAX 30		
	EGARCH	GJR	FCGARCH	EGARCH	GJR	FCGARCH
ω	-0,2618	1,5508E-06	-	-0,2792	2,85E-06	-
α_0	0,0851	0	1,0000E-06	0,1192	0	3,1822E-06
β_0	0,9792	0,9149	1,2028	0,9786	0,901	1,3006
λ_0	-	-	0,0737	-	-	0,0701
α_1	-	-	2,6376E-06	-	-	-7,3908E-07
β_1	-	-	-0,3242	-	-	-0,4881
λ_1	-0,1222	1,1377	-0,0737	-0,1246	0,1671	0,0381
γ_1	-	-	-0,72	-	-	-0,72
c_1	-	-	2,52	-	-	2,52
α_2	-	-	-2,6376E-06	-	-	3,0621E-05
β_2	-	-	-0,3326	-	-	-0,4882
λ_2	-	-	0,0713	-	-	-0,0759
γ_2	-	-	1,56	-	-	1,56
c_2	-	-	2,85	-	-	2,85
Quasi-Log-Likelihood	6.0149e+03	6.0066e+003	6.0188e+003	5.6624e+03	5.6616e+03	5.6558e+03
Akaike Information Criterion	-1,2030E+08	-1,2013E+08	-1,2038E+08	-1,1325E+08	-1,1323E+08	-1,1312E+08
Schwarz Bayesian Criterion	-1,2030E+08	-1,2013E+08	-1,2038E+08	-1,1325E+08	-1,1323E+08	-1,1312E+08

Table 5. Estimation Results

Parameters	FTSE 100		
	EGARCH	**GJR**	**FCGARCH**
$\boldsymbol{\omega}$	-0,2821	1,83E-06	-
$\boldsymbol{\alpha_0}$	0,1098	0	1,0000E-08
$\boldsymbol{\beta_0}$	0,9795	0,8881	1,2968
$\boldsymbol{\lambda_0}$	-	-	0,0565
$\boldsymbol{\alpha_1}$	-	-	2,4500E-06
$\boldsymbol{\beta_1}$	-	-	-0,4717
$\boldsymbol{\lambda_1}$	-0,151	0,1839	0,0569
$\boldsymbol{\gamma_1}$	-	-	-0,72
$\boldsymbol{c_1}$	-	-	2,52
$\boldsymbol{\alpha_2}$	-	-	-2,4500E-06
$\boldsymbol{\beta_2}$	-	-	-0,4636
$\boldsymbol{\lambda_2}$	-	-	-0,0562
$\boldsymbol{\gamma_2}$	-	-	1,56
$\boldsymbol{c_2}$	-	-	2,85
Quasi-Log-Likelihood	6.0696e+03	6.0627e+003	6.0785e+003
Akaike Information Criterion	-1,2139E+08	-1,2125E+08	-1,2157E+08
Schwarz Bayesian Criterion	-1,2139E+08	-1,2125E+08	-1,2157E+08

Table 6. Estimation Results

Parameters	Nikkei 225			PSI 20		
	EGARCH	GJR	FCGARCH	EGARCH	GJR	FCGARCH
ω	-0,3609	2,98E-06	-	-1,5266	4,49E-06	-
α_0	0,1704	0,0356	9.0685E-07	0,2139	0,048	4,7407E-05
β_0	0,9736	0,9041	1,1994	0,8584	0,8102	0,5225
λ_0	-	-	0,0640	-	-	0,7731
α_1	-	-	-8.9685E-07	-	-	-3,9627E-05
β_1	-	-	-0,2884	-	-	0,0324
λ_1	-0,0785	0,0941	-0,0640	-0,1589	0,1625	-0,7731
γ_1	-	-	-0,72	-	-	-0,72
c_1	-	-	2,52	-	-	2,52
α_2	-	-	4.1674E-05	-	-	1,4257E-05
β_2	-	-	-0,6589	-	-	-0,5549
λ_2	-	-	0,1162	-	-	0,2080
γ_2	-	-	1,56	-	-	1,56
c_2	-	-	2,85	-	-	2,85
Quasi-Log-Likelihood	5.2486e+003	5.2465e+003	5.2634e+003	6.1476e+03	6.1638e+003	6.1319e+003
Akaike Information Criterion	-1,0497E+08	-1,0493E+08	-1,0527E+08	-1,2295E+08	-1,2328E+08	-1,2264E+08
Schwarz Bayesian Criterion	-1,0497E+08	-1,0493E+08	-1,0527E+08	-1,2295E+08	-1,2328E+08	-1,2264E+08

As we can see, the in-sample estimation results confirm the high sensitivity of the market according to the "bad news", since the sign of the parameter λ_1 is negative in the EGARCH and positive in the GJR. The same happens in FCGARCH model, since each regime is related to certain kinds of innovations, as explained before and seen in the tables.

We can also see, in the case of the S&P500, DAX30 and FTSE indexes, we have an α_0 which is exactly 0 in the GJR model, according to the codes created for this thesis. Since the parameter is combined with the dummy related to bad news, we can conclude that future positive news have no impact at all in the future volatilities, thus focusing mainly on the leverage effect explained before.

Another point worth of mentioning is the fact that, in the FCGARCH model, the γ and c parameters are exactly the same in all the indexes estimated. However, these parameters are only related to the transition between regimes, and since the conditions are satisfied, the logistic function is considered a smooth transition function in the FCGARCH model linking the regimes included in the model. So, our main focus will be on the GARCH parameters of the FCGARCH model, that is, the α, β and λ.

As we can also see, the Quasi-Maximum-Log-Likelihood, which its results show how well the data is incorporated in the models, testing how the data used for the estimation is more likely to fit in a certain model than in another, shows positive values for all the models studied in this thesis. Since Matlab cannot maximize, thus can only minimize, the codes present in annex 1.2.1. show a negative function for the QML, thus showing the most negative possible value. So, the sign of the output must be changed, which means that the results shown in Matlab should be changed back to positive values, as seen in tables 4, 5 and 6.

According to the Quasi-Maximum-Log-Likelihood, we see that the FCGARCH model has the highest values, mainly on S&P 500, FTSE 100 and Nikkei 225 indexes, being GJR the best model in PSI 20 and DAX 30.

One thing that it is to note, is the fact that, although the FCGARCH has the main advantage of modeling more than two limiting regimes, and has a higher QML in some of the indexes studied, this model's QML result is very close to the ones from EGARCH and GJR. Since the model deals with a high number of parameters, contrary to EGARCH and GJR, the

FCGARCH may fail in some goodness-of-fit measures, such as Akaike Information Criterion (1978):

$$AIC = 2k - 2LLF, \qquad \textbf{(19)}$$

as well as Schwarz Bayesian Criterion (1978):

$$SBC = -2LLF + k\ln(n), \qquad \textbf{(20)}$$

being k the number of parameters in the model, n the number of observations used for the estimation, and LLF the Log-Likelihood function, which, in this case, will be the Quasi-Maximum-Log-Likellihood.

The main characteristic of these criterions is the fact that they are based mainly on the number of parameters estimated: the less parameters the model uses, the more likely it is the best model. However, as we can see, the results confirm the QML conclusions, showing that the FCGARCH model seems to be the best one in S&P 500, FTSE 100 and Nikkei 225, since it has the lowest values from the three models presented. Regarding DAX 30 and PSI 20, the results favour EGARCH and GJR, respectively.

In conclusion, by looking at the parameters' estimation results, and at the QML, we see that FCGARCH and GJR models seem to be the most prone to generate and predict the future data.

3.3.Out-of-sample Analysis - Results

In order to check which of the models studied in this paper predicts best the volatilities in-sample, we use the quasi-log-likelihood maximum value. However, for the out-of-sample, there is also a volatility forecast comparison test, proposed by Harvey and Newbold (2000), for multiple surrounding forecasts.

Assuming that $(f_{1t}, f_{2t}, \ldots, f_{Kt})$ are the competing forecasts, made one-step ahead with non-autocorrelated errors, of the actual quantity A_t, the test can be written by the following formula:

$$e_{it} = \sum_{j=2}^{K}\left[\lambda_{j-1}(e_{it} - e_{jt})\right] + \varepsilon_t \qquad \textbf{(21)}$$

It is to note that $e_{it} = A_t - f_{it}$, with $i = 1, \dots, K$, $0 \leq \lambda_j \leq 1$ with $j = 2, \dots, K$, and ε_t is the error of the combined forecast. Thus, we can write the regression in its matrix general form, being the following:

$$y_t = X_t^{'}\beta + \varepsilon_t \text{ and } y = X\beta + \varepsilon \quad \textbf{(22)}$$

Where $y_t = e_{it}$, $\beta = [\lambda_1\ \lambda_2\ \dots\ \lambda_{K-1}]$ and $X_t = [(e_{1t} - e_{2t})(e_{1t} - e_{3t}) \dots (e_{1t} - e_{Kt})]$. The F test used for this case is an F test of the joint significance of the parameters of e_{it}, being the null hypothesis the fact that f_1 encompasses $f_2, \dots, f_K$, that is:

$$\begin{cases} H_0: \lambda_1 = \lambda_2 = \cdots = \lambda_{K-1} = 0 \\ H_a: \lambda_1 = \lambda_2 = \cdots = \lambda_{K-1} > 0 \end{cases}$$

The codes which were used for the test of these three models were made on E-views program, by my supervisor, Professor José Dias Curto, and are present in annex 1.2.3. What I have made is comparing one model according to the 2 competing ones, and test their joint significance, applying the out-of-sample observations, according to the hypothesis shown before, thus testing the predictive capability of the models in study.

In order to use the Harvey-Newbold test I have created a volatility prediction code for the FCGARCH model in Matlab, present in annex 1.2.2.1. The predictions for the variances of the other two models were made by hand, using the Microsoft Excel.

Table 6 shows the results for using the Harvey-Newbold test for the forecasts of the 5 indexes, according to the models studied in this paper.

Table 6. Harvey-Newbold Test

Indexes	**EGARCH**		**GJR**		**FCGARCH**	
	Value	**Significance**	**Value**	**Significance**	**Value**	**Significance**
S&P 500	482.0812	0,000000	21914,60	0,000000	431.9928	0,000000
DAX 30	417.4045	0,000000	1,096722	0,295254	363.2588	0,000000
FTSE 100	444.1585	0,000000	0,439739	0,507414	632.3077	0,000000
Nikkei 225	420.3645	0,000000	0,564861	0,452505	342.6930	0,000000
PSI 20	135.8052	0,000000	3,564647	0,059347	129.4079	0,000000

The results seem to favour the GJR model in 4 out of 5 indexes, where we fail to reject the null that the GJR forecasts can't be improved by combination with the other two models, except for S&P 500, where the null is rejected (for a 5% significance level), which means that the FCGARCH and/or EGARCH predictions, with the ones from GJR, would lead to an improvement of the S&P 500 index forecast performance. However, since the FCGARCH presents a much lower value, although the test is not statistically significant, we can conclude that the FCGARCH seems better to encompass EGARCH and GJR in this index, thus seeming to be a better model than EGARCH and GJR.

4. Conclusions

Nowadays, due to the complexity and constant fastness of how the information is passed throughout the world, it is very difficult not only to predict the price of a certain asset, but also to forecast its volatility. Therefore, there is a need to create a model which can be used in order to make the most rightful decision.

Several regularities of these assets have been discovered, and consequently, several models have been determined, with the most popular ones being GARCH models, namely EGARCH, GJR and FCGARCH. The main purpose of this thesis was to discover which one of these models seems better to predict the financial markets' volatility.

The empirical analysis is based on the daily returns of 5 most important international stock market indexes, such as S&P 500 (USA), FTSE 100 (UK), DAX 30 (Germany), PSI 20 (Portugal), and Nikkei 225 (Japan). These returns, from January 2001 to December 2011, were divided in two main periods, being the first 2/3 used for the estimation of the parameters, and the final 1/3 for the forecast period.

According to the estimation in-sample results based on the codes created in Matlab for all three models, based on the Quasi-Maximum-Likelihood, the FCGARCH model seem to be the best in S&P 500, FTSE 100 and Nikkei 225 indexes, fitting the dynamics of the returns of the indexes in study.

If we consider other goodness-of-fit measures, such as Akaike and Schwarz criterions, the results confirm the QML conclusions, showing that the FCGARCH model, although it has a significant number of parameters, seems to be the best one in the same indexes, since it has the lowest values from the three models presented. These criterions also seem to favour the GJR model.

The out-of-sample results, based on the Harvey-Newbold encompassing test, based on the codes created in E-views, show that the best model to predict the volatility forecasts for the majority of the indexes under study, seems to be not FCGARCH, but the GJR model.

Some of the limitations and future research that should be considered in this thesis are the following ones:

- The fact that, first, we assume that the observation's returns follow a normal distribution, when in fact, the descriptive statistics show the opposite (a non-normal distribution). Thus, it could be useful to use other distributions than the Gaussian one in this study, such as Student t (1925), for example.

- According to Andersen and Bollerslev (1998), the use the daily squared returns as a proxy of daily volatility can be considered a noisy estimator, which may lead us to use an alternative proxy for the daily volatility, such as the intraday realized volatility, for example.
- For this thesis, the in-sample observations do not include the financial crisis period started in 2008. It could be a plus if we could separate the in-sample data and the out-of-sample data in a more convenient way, thus considering the extreme variances of the returns of the indexes in these last years.
- In this thesis we used the PSI20 index, a Portuguese one which was not considered in the paper by Medeiros and Veiga (2009). It could be considered for future analysis other indexes than the ones studied, not also in their paper but also in this thesis.
- Lastly, it is to note that the variance estimated coefficients for the three models in this thesis do not have their significance measure for the errors, thus not knowing if they are statistically significant and thus plausible for study. However, the main objective of this thesis was to compare the models, obtaining the maximum value for the QML, thus knowing which of the models seems to be the best one to describe the volatility. For future research, it should be considered a measure for the significance of the estimated coefficients.

This thesis and its empirical result tends to be useful for those who want to get deeper in the FCGARCH model, and study its advantages, disadvantages, parameters and results, thus being closer to find the best model, regarding the forecast of volatility in financial markets, mainly during periods of high volatility.

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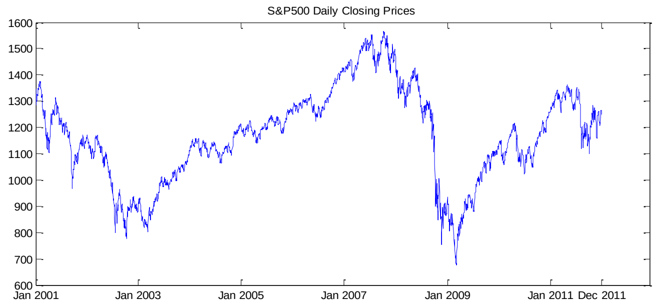
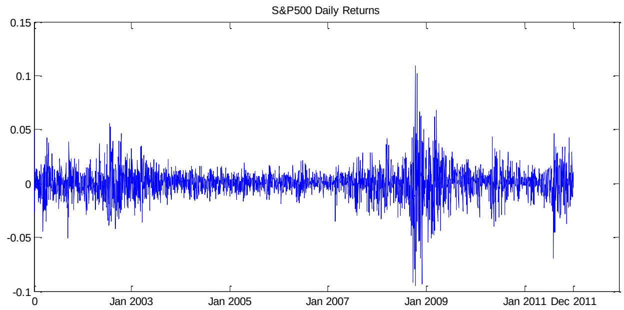
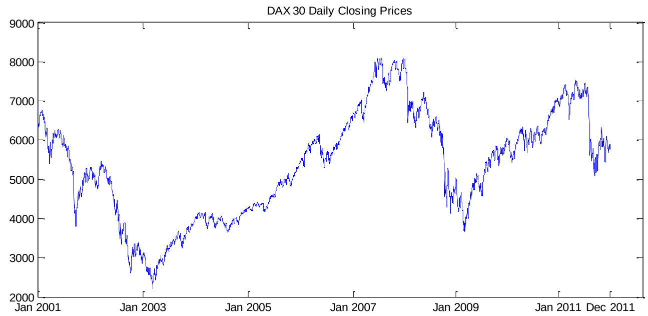
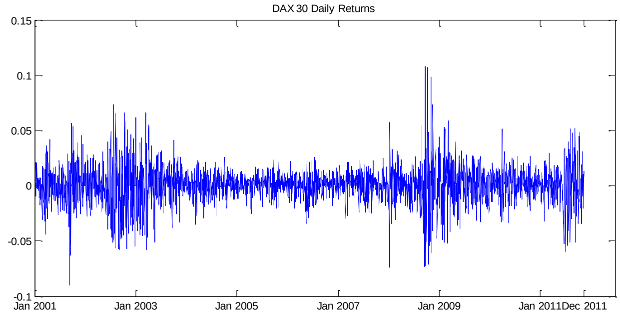
Annexes

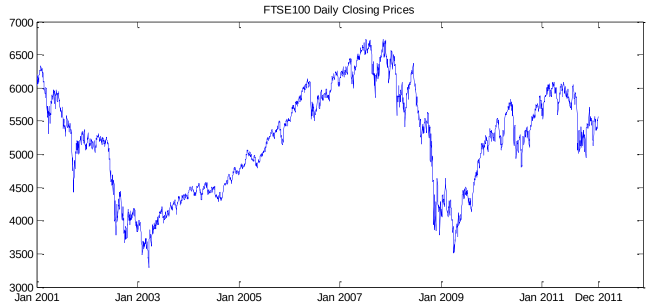
1.1.Daily Closing Prices and Returns of the assets

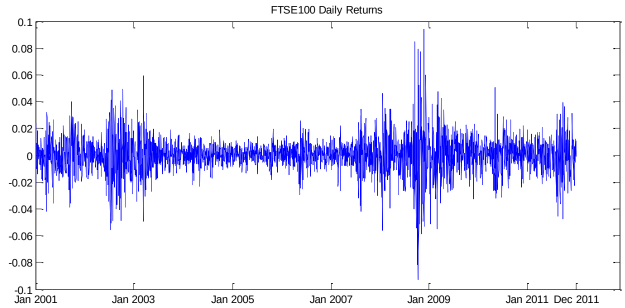


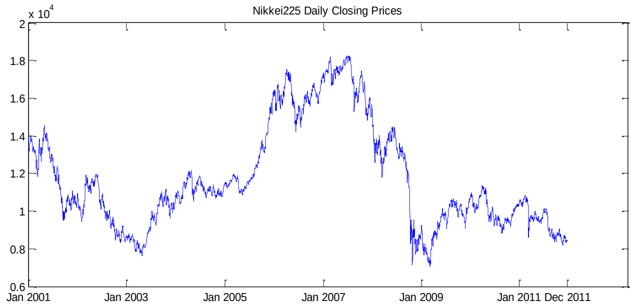


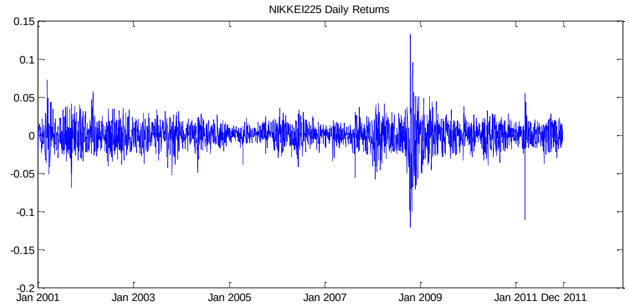


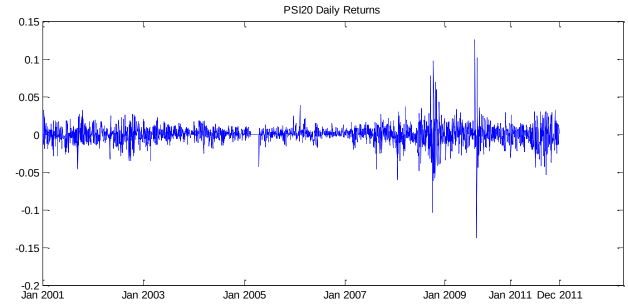












1.2.Matlab Codes

1.2.1. Estimation of Parameters

1.2.1.1.EGARCH

```
clear all;
close all;
clear global;
clc;
global x;
x=xlsread('C:\...\*.xlsx');

T=size(x,1);
h(1)=x(1)^2;

constant = 0.0005;
alpha  = 0.15;
beta = 0.8;
gama = -0.04;

x0=[constant,alpha,beta,gama];
options = optimset('Algorithm','sqp','display', 'iter','TolX', 1e-14,
'TolCon',1e-14, 'TolFun', 1e-14, 'MaxFunEvals', 2000);
[parameters, fval,exitflag,output] =
fmincon('objfunegarch',x0,[],[],[],[],[],[],'confunegarch',options,x)
```

```
function [cons, ceq] = confunegarch(parameters,x)

T=size(x,1);

constant  = parameters(1);
alpha = parameters(2);
beta   = parameters(3);
gama = parameters(4);

% Nonlinear inequality constraints; note: make (...)<0
cons = [gama;
        abs(beta)-0.9999999999999];
% Nonlinear equality constraints
ceq = [] ;
```

```
function likeli = objfunegarch(parameters,x)

constant  = parameters(1);
alpha = parameters(2);
beta   = parameters(3);
gama = parameters(4);

T=size(x,1);
h(1)=(x(1))^2;

for t=2:T;
h(t,1)=exp(constant+alpha*(abs(x(t-1,1))/((h(t-1))^(1/2)))+gama*(x(t-
1,1)/((h(t-1))^(1/2)))+beta*log((h(t-1))^2));
end
```

```
likeli=T*((1/T)*sum((1/2)*log(2*pi)+(1/2)*log(h)+(x.^2)./(2*h))); %note:
used - because matlab can't maximize. There are points after x because x is
a vector.

end
```

1.2.1.2.GJR

```
clear all;
close all;
clear global;
clc;
global x;
x=xlsread('C:\...\*.xlsx');

T=size(x,1);
h(1)=x(1)^2;

constant = 0.0005;
beta  = 0.8;
lambda = 0.15;
gama = 0.04;

x0=[constant,beta,lambda,gama];
options = optimset('Algorithm','sqp','display', 'iter','TolX', 1e-14,
'TolCon',1e-14, 'TolFun', 1e-14, 'MaxFunEvals', 2000);
[parameters, fval,exitflag,output] =
fmincon('objfun2012gjr',x0,[],[],[],[],[],[],'confun2012gjr',options,x)
```

```
function [cons, ceq] = confun2012gjr(parameters,x)

T=size(x,1);

constant  = parameters(1);
beta  = parameters(2);
lambda = parameters(3);
gama = parameters(4);

% Nonlinear inequality constraints; note: make (...)<0
cons = [-constant;
        -beta;
        -lambda;
        -lambda-gama;
        beta+lambda+(1/2)*gama-1];
% Nonlinear equality constraints
ceq = [] ;
```

```
function likeli = objfun2012gjr(parameters,x)

constant  = parameters(1);
beta  = parameters(2);
lambda = parameters(3);
gama = parameters(4);
```

```
T=size(x,1);
h(1)=(x(1))^2;
dummy=0;

for t=2:T;
    if x(t-1)<0;
       dummy=1;
       else dummy=0;
    end
    h(t,1)=constant+(lambda+gama*dummy)*x(t-1)^2+beta*h(t-1);
end

likeli=T*((1/T)*sum((1/2)*log(2*pi)+(1/2)*log(h)+(x.^2)./(2*h))); %note: used - because matlab can't maximize. There are points after x because x is a vector.

end
```

1.2.1.3.FCGARCH

```
clear all;
close all;
clear global;
clc;
global x;
x=xlsread('C:\...\*.txt');

H=2;
T=size(x,1);
h=ones(T,1);
h(1)=x(1)^2;
alphas=  [2.22*(10^-16), 2.55*(10^-5), 3.73*(10^-4) ];
betas=[1.21, -0.32, -0.25];
lambdas=[0.06,-0.01,-0.04];
c=[-0.72,1.56];
gama=[2.52,2.85];

x0=[alphas,betas,lambdas,c,gama];
options = optimset('Algorithm','sqp','display', 'iter','TolX', 1e-14, 'TolCon',1e-14, 'TolFun', 1e-14, 'MaxFunEvals', 1500);
[parameters, fval,exitflag,output] =
fmincon('objfun2012',x0,[],[],[],[],[],[],'confun2012',options,x)
```

```
function [cons, ceq] = confun2012(parameters,x)

T=size(x,1);

alphas  = parameters(1:3);
betas   = parameters(4:6);
lambdas = parameters(7:9);
c = parameters(10:11);
gama = parameters(12:13);

sum_alphas= cumsum(alphas);   %cum sum A=[1,2,3]; cumsum(A) = [1,3,6]
sum_betas= cumsum(betas);
sum_lambdas= cumsum(lambdas);
```

```
% Nonlinear inequality constraints; note: make (...)<0
cons = [-c(2)+c(1)+(10^-8);   %assumption 3 r1; +10^-18 because it's
strictly positive
        -c(1)-realmax;         %assumption 3 r1; realmax = best MATLAB
approximation to infinity
        -c(2)-realmax;         %assumption 3 r1
        -gama(1)+10^-18;       %assumption 3 r2
        -gama(2)+10^-18;       %assumption 3 r2
        -sum_alphas(1)+10^-8;
        -sum_alphas(2)+10^-8; %assumption 5 r1;
        -sum_alphas(3)+10^-8;
        -(1./(1+exp(-gama(1)*(x-c(1)))))+(1./(1+exp(-gama(2)*(x-c(2)))));
%assumption 4; There are points after 1 because 1 is a vector.

        -sum_betas(1);         %assumption 5 r2
        -sum_betas(2);
        -sum_betas(3);
        -sum_lambdas(1);       %assumption 5 r3
        -sum_lambdas(2);
        -sum_lambdas(3);
        (1./(1+exp(-gama(1)*(x-c(1)))))+(1./(1+exp(gama(1)*(x-c(1)))))-1; %
assumption 9
        (1./(1+exp(-gama(2)*(x-c(2)))))+(1./(1+exp(gama(2)*(x-c(2)))))-1;

(1/2)*(2*betas(1)+betas(2)+betas(3)+2*lambdas(1)+lambdas(2)+lambdas(3))-
1+10^-18;%assumption 7

betas(1)^2+betas(1)*(betas(2)+betas(3))+(((betas(2)+betas(3))^2)/2)+(sum(x.
^4)/T)*(lambdas(1)+lambdas(1)*(lambdas(2)+lambdas(3))+((lambdas(2)+lambdas(
3))^2)/2)+2*lambdas(1)*betas(1)+betas(1)*(lambdas(2)+lambdas(3))+lambdas(1)
*(betas(2)+betas(3))+(lambdas(2)+lambdas(3))*(betas(2)+betas(3))-1+10^-18
%assumption 10
         ];
% Nonlinear equality constraints
ceq = [] ;
```

```
function likeli = objfun2012(parameters,x)

alphas  = parameters(1:3);
betas   = parameters(4:6);
lambdas = parameters(7:9);
c = parameters(10:11);
gama = parameters(12:13);

H=2;
T=size(x,1);
h(1)=(x(1))^2;

  for t=2:T;
    regi(1)=alphas(1)+betas(1)*h(t-1)+lambdas(1)*x(t-1)^2;
        for j=1:H;
           if  (1/(1+exp(-gama(j)*(x(t-1)-c(j))))) <0
               disp('AFFLog')
           end
        regi(j+1)=(alphas(j+1)+betas(j+1)*h(t-1)+lambdas(j+1)*x(t-
1)^2)*(1/(1+exp(-gama(j)*((x(t-1)/(h(t-1))^(1/2))-c(j)))));
        regac=sum(regi);
        end
```

```
    h(t,1)=regac;

    if min(h)<0
        disp('AFF')
        h
        pause
    end
 end

 likeli=T*((1/T)*sum((1/2)*log(2*pi)+(1/2)*log(h)+(x.^2)./(2*h)));
%note: used - because matlab can't maximize. There are points after x
because x is a vector.

end
```

1.2.2. Forecasting Returns

1.2.2.1.FCGARCH

```
clear all
clc
H=2;
T=899;
k=1000; %number of simulations
p=13863.47; %closing price at t-1

tic

%using estimated results of the parameters (for each index)
alpha=[1.5152*10^-05,-8.1461*10^-06,-6.9958*10^-06];
beta=[1.1994,-0.2884,-0.6589];
lambda=[0.0640,-0.0640,0.1122];
gama=[-0.72,1.56];
c=[2.52,2.85];

sigma=0.3; %initial value for sigma
sigma2=sigma^2;
diasnoano=360;

h(1:k,1)=((sigma2)/diasnoano);
Nor=randn(k,T); % creates random numbers, which will be the noises. It will
be created a number of vectors throughout the simulations

for i=1:k;
    y(i,1)=h(i,1)^(1/2)*Nor(i,1);
    for t=2:T;
    regi(1)=alpha(1)+beta(1)*h(i,t-1)+lambda(1)*(h(i,t-1)^(1/2)*Nor(i,t-
1))^2;
        for j=1:H;
        regi(j+1)=(alpha(j+1)+beta(j+1)*h(i,t-1)+lambda(j+1)*(h(i,t-
1)^(1/2)*Nor(i,t-1))^2)*(1/(1+exp(-gama(j)*(y(i,t-1)-c(j)))));
        regac=sum(regi);
        end
    h(i,t)=regac;
    if h(i,t)<0;
        h(i,t)=0.0000001;
    end
```

```
    y(i,t)=h(i,t)^(1/2)*Nor(i,t);
    end
end
retornoac=sum(y,2);     sum of y throughout the lines (,2)
ST=p*(exp(retornoac))

toc
```

1.3. E-views Codes

1.3.1. Harvey-Newbold Test

```
series e1t=rt2-sigfcgarch
vector(1000) yt
stomna(e1t, yt)
series e2t=rt2-sigegarch
series e3t=rt2-siggjr
series de12t=e1t-e2t
series de13t=e1t-e3t

equation eq3
eq3.ls e1t de12t de13t
show eq3.output
!obs=@obssmpl
!s2=@se^2
!K=@ncoef
eq3.makeresids res1
vector(1000) resf1
stomna(res1, resf1)
vector(!K) LAMB
for !i=1 to !K
        LAMB(!i)=@coefs(!i)
next

matrix(1000,2) X
group HNg1 de12t de13t
stomna(HNg1, X)
matrix(1000,2) HatM
matrix(1000,2) HatQ
matrix(1000,2) HatD
HatM=(!obs^-1)*@transpose(X)*X
HatQ=!s2*HatM
HatD=@inverse(HatM)*HatQ*@inverse(HatM)

'F standard test
matrix(1,1) F11
F11=@transpose(lamb)*@inverse(HatD)*lamb
scalar F=!obs*(!K-1)^(-1)*@trace(F11)
!ProbF=1-@cfdist(F,!K-1,!obs-!K+1)

'Computing F1
!q=0
!cm=@columns(HatM)
!rm=@rows(HatM)
matrix(!cm, !rm) HatQ1
for !i=1 to !cm
        for !j=1 to !rm
                for !l=1 to !obs
                        !q=!q+X(!l, !i)*X(!l,!j)*resf1(!l)^2
```

```
                next
                HatQ1(!i,!j)=!q*!obs^(-1)
                !q=0
        next
next

matrix(!rm, !cm) HatD1
HatD1=@inverse(HatM)*HatQ1*@inverse(HatM)

matrix(1,1) F12
F12=@transpose(lamb)*@inverse(HatD1)*lamb
scalar F1=!obs*(!K-1)^(-1)*@trace(F12)
!ProbF1=1-@cfdist(F1,!K-1,!obs-!K+1)

'Computing F1
!q=0
!cm=@columns(HatM)
!rm=@rows(HatM)
matrix(!cm, !rm) HatQ2
for !i=1 to !cm
        for !j=1 to !rm
                for !l=1 to !obs
                        !q=!q+X(!l, !i)*X(!l,!j)*yt(!l)^2
                next
                HatQ2(!i,!j)=!q*!obs^(-1)
                !q=0
        next
next

matrix(!rm, !cm) HatD2
HatD2=@inverse(HatM)*HatQ2*@inverse(HatM)

matrix(1,1) F13
F13=@transpose(lamb)*@inverse(HatD2)*lamb
scalar F2=!obs*(!K-1)^(-1)*@trace(F13)
!ProbF2=1-@cfdist(F2,!K-1,!obs-!K+1)

'COMPUTING MS*
scalar MS=(!obs-(!K-1)*F2)^(-1)*(!obs-!K+1)*F2
!ProbMS=1-@cfdist(MS,!K-1,!obs-!K+1)

series dmsv1=(sigfcgarch-rt2)^2
'MAE MSV-EGARCH
series dmsv2
dmsv2=@abs(sigfcgarch-rt2)

show resulta
```