ISCTE Business School
Instituto Universitário de Lisboa

Carbon Markets and Emission Derivatives – the pricing of Derivatives in the EU ETS

João de Andrade Dias da Costa

Thesis submitted as partial requirement for obtaining the degree of

Master of Science in Business Administration

Supervisor:
Prof. José Carlos Gonçalves Dias, ISCTE Business School, Department of Finance

October 2012

Abstract

The European Union's Emissions Trading Scheme (EU ETS) is the world's largest carbon market operating and an important piece for European environmental policy. Launched in 2005, this market-scheme trades allowances and derivatives' contracts that represent the right to emit a certain amount of pollutant gases.

This work intends to understand the role of carbon markets in general, and the pricing of derivatives' contracts traded in the EU ETS. It was taken as basis the Black-Scholes (1973) model and its further extensions by Merton (1973) and Merton (1976), applying then a model suggested by Daskalakis, Psychoyios and Markellos (2009) to value a call option written on emission allowances. The numerical results suggest that time to maturity and the *moneyness* degree had influence in the options' price, while the jump intensity did not have an influence in the obtained results

Based on the application of the model, it was then derived, using the put-call parity, the value of a put option under the same basic features.

It was also conducted a sensitivity analysis to the call option, in which it was concluded that, under the model specifications, volatility shows a strong influence within the studied call options' value.

Keywords: Black-Scholes-Merton, Derivatives, Carbon Markets, EU ETS.

JEL Classification: G13, Q29

Resumo

O Sistema Europeu de Comércio de Emissões (EU ETS) é o maior mercado de emissões em funcionamento a nível mundial e uma peça chave em termos de política ambiental na Europa. Lançado em 2005, este sistema de mercado é utilizado para a comercialização de direitos e produtos derivados sobre a emissão de uma certa quantidade de gases poluentes.

Este trabalho procura compreender a função dos mercados de carbono em geral e a valorização de produtos derivados em comercialização no EU ETS. Para tal tomou-se por base o modelo de Black-Scholes (1973) e suas extensões por Merton (1973) e Merton (1976), aplicando-se depois o modelo sugerido por Daskalakis, Psychoyios e Markellos (2009) de modo a valorizar uma opção *call* sobre direitos de emissões. Os resultados alcançados sugerem que o tempo até à maturidade e o nível do preço de exercício contribuíram para a alteração no valor da opção, ao passo que a intensidade do "salto" não teve influência nos resultados alcançados.

Com base na aplicação deste modelo, foi igualmente obtido o valor de uma opção *put* com as mesmas características, através da paridade *put-call*.

Foi ainda feita uma análise de sensibilidade à opção *call*, na qual se concluiu que, de acordo com as especificações do modelo, a volatilidade tem uma forte influência no valor das opções *call* estudadas.

Palavras-chave: Black-Scholes-Merton, Derivados Financeiros, Mercados de Carbono, EU ETS

Classificação JEL: G13, Q29

Acknowledgments

I would like to express my gratitude to several people that were very influential and without whom this work couldn't be completed.

First of all, I thank my supervisor, Prof. Dr. José Carlos Gonçalves Dias, for its constant support not only in terms of knowledge and expertise but also in terms of motivation.

Without being able to enumerate all friends I would like to, I mention particularly my friends from "The Ballis Band", António Fernandes, Carlos Silva and Ana Martins, that helped me to keep an important balance between work, academics and leisure.

Finally, I would like to thank my family for providing an important support and all the conditions for me to be able to conduct my studies.

To all, thank you very much.

João Costa

Commonly used notation

BSM – Black-Scholes-Merton

CDM – Clean development mechanism

CER – Certified emission reductions

CO2 – Carbon dioxide

EU ETS – European Union Emissions Trading System

EUA – European Union allowances

ETS – Emission trading system / scheme

ICE – Intercontinental Exchange

S – Asset price

K - Strike price

F - Futures contract price

t – Time period

T - Delivery / ending period

τ - Time to maturity, i.e. $\tau = T - t$

r – Risk-free interest rate

q – Security yield

y – Convenience yield

Index

Figures Index

Table Index

1. Introduction

Over the past three decades financial markets and institutions have faced important changes. Rajan (2005) points out technological development, deregulation and the institutional change as key factors for change within the financial world. Indeed, the development of computer science and telecommunication systems had a major impact, not only in the financial world but also in many other aspects of the society. Derivatives' contracts became widely known and used due to their features which enabled financial institutions to leverage positions and manage risks and exposures with a wide variety of methods and instruments.

Along with these changes within the financial world, environmental issues have faced some important developments as well over the past decades. Climate change was widely related towards human activity, fact that motivated actions intended to reduce the impact of human activity in climate change. Having this in mind, the United Nations promoted the Kyoto Protocol, which comprised mechanisms intended towards greenhouse gases' reduction and among these are Emission-trading schemes, particularly the European Union Emissions Trading System (EU ETS). Being these, markets that allow the trading of financial products whose underlying assets are emissions of pollutant gases, finance academics and professionals developed some work in order to understand how such financial assets could be valued. Daskalakis, Psychoyios and Markellos (2009) conducted one of the most important and complete studies, available up to date, regarding the pricing of carbon emissions in *phase I* of the EU ETS. Taking this study as a guideline it is proposed the adaption of such methodologies to an analysis for *phase II* of the EU ETS.

This thesis is structured as follows:

Chapters 2 and 3 present the theoretical framework under which this thesis is built on. While chapter 2 introduces the Carbon Markets thematic, providing some insights on the main mechanisms established in the Kyoto Protocol and with an obvious highlight to the EU ETS, chapter 3 introduces financial derivatives, focusing on the presentation of financial options. In this chapter, it is also introduced the Black-Scholes model, the most widely known and used method for valuing options contracts, and its extension proposed by Merton (1976), in order to account for jumps in the securities' prices.

Chapter 4 introduces the basic theory to value commodity contracts and the pricing of derivatives contracts within the EU ETS. Moreover, it presents the models that will then be tested.

After presenting some extensive literature and models that allow the valuation of derivatives contracts whose underlying assets are emission allowances, in chapter 5 some of the presented models are applied. Here a call and a put option are valued and a short sensitivity analysis is conducted.

Chapter 6 presents some concluding remarks to the study.

2. Carbon Markets

2.1. Background

Global warming is one of the key issues discussed over the past decades. Many have argued that human activity had an excessive contribution towards climate change, yet, only in the late 1980's there has been made an effort to combat climate change.

Schofield (2007, pp. 246-247) underlines the following actions as the main events against climate change:

- The formation of the Intergovernmental Panel on Climate Change (IPCC) – 1988;
- The Earth Summit, in which it was presented the first report developed by the IPCC – 1992;
- The presentation of the second report by the IPCC and development of the Kyoto Protocol – 1995 and 1997, respectively.

In this last event, the Kyoto Protocol, the presentation of the second report developed by the IPCC concluded that human activity was having a significant effect on the climate, and that this would pose a future threat to human and economic development. Data suggests that from 1960 to 2008 the emissions of carbon dioxide more than tripled worldwide, as shown in Figure 2.1.

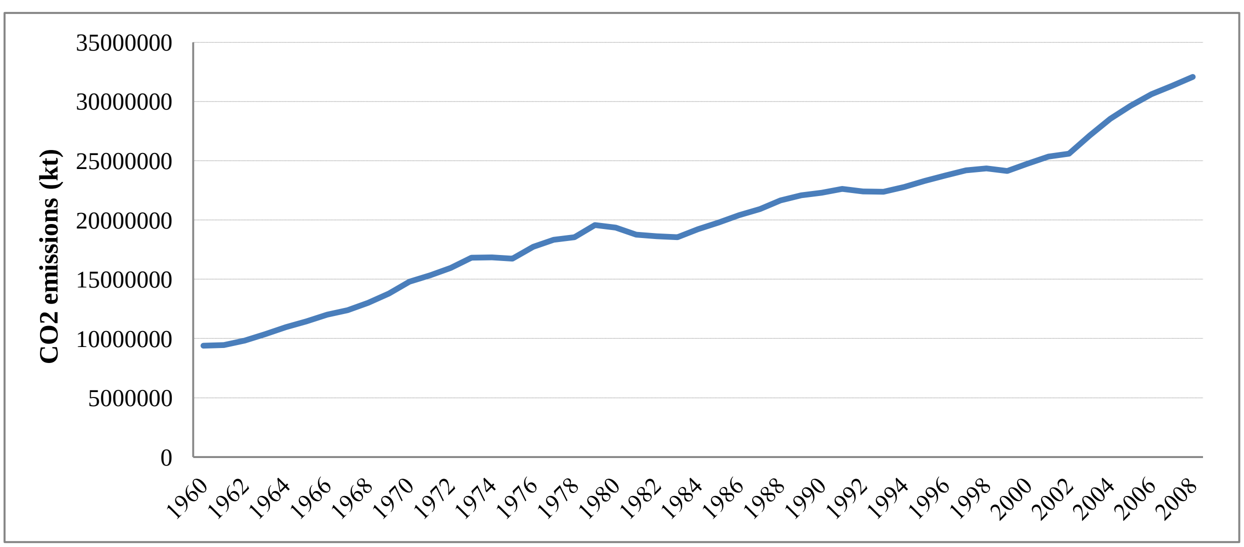


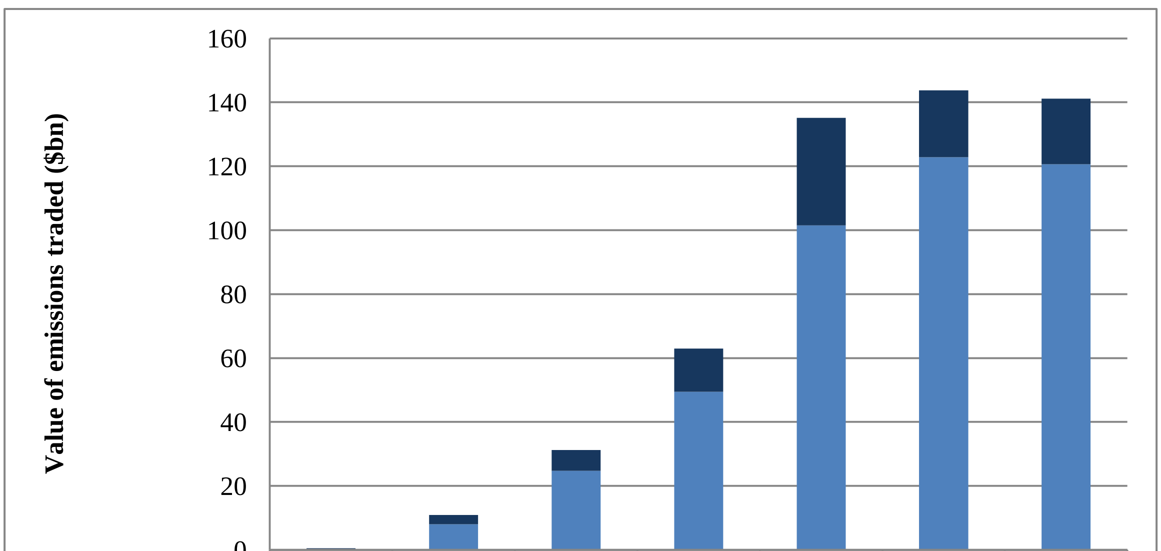
Figure 2.1 World CO2 emissions (kt) **(**The World Bank Group, 2012**)**

In order to combat climate change the protocol set individual targets for industrialized countries willing to reduce their emissions of greenhouse gases. The protocol outlined a variety of mechanisms that would allow countries to meet their environmental commitments.

Among these mechanisms some were concerned with the reduction of emissions:

- Clean development mechanism;
- Joint implementation;
- Emission-trading schemes.

Clean development mechanism (CDM) and joint implementation are both project-based mechanisms, while emission-trading schemes (ETS) are market based mechanisms, also known as carbon markets. While market-based mechanisms work through the trade of rights to emit greenhouse gases and compliance with limits, the presented project based mechanisms work through the development and investment in projects that intend to reimburse or reduce carbon emissions in a cost-effective form.



	2004	2005	2006	2007	2008	2009	2010
Project-based	0,5	2,9	6,5	13,6	33,6	20,9	20,5
Allowance-based	0	8	24,7	49,4	101,5	122,8	120,6

Figure 2.2 Global emissions trading (The City UK, 2011)

Figure 2.2 provides some data on the evolution of emissions trading. It is possible to observe that since 2004 there was a significant increase in terms of volume of trading (in billions of dollars), especially from 2007 to 2008 when the market grew more than 114%. Despite the significant growth from 2004 to 2009, 2010 marked a decrease in terms of value of emissions traded, in part due to the global economics conditions (The City UK (2011)).

2.2. Clean development mechanism

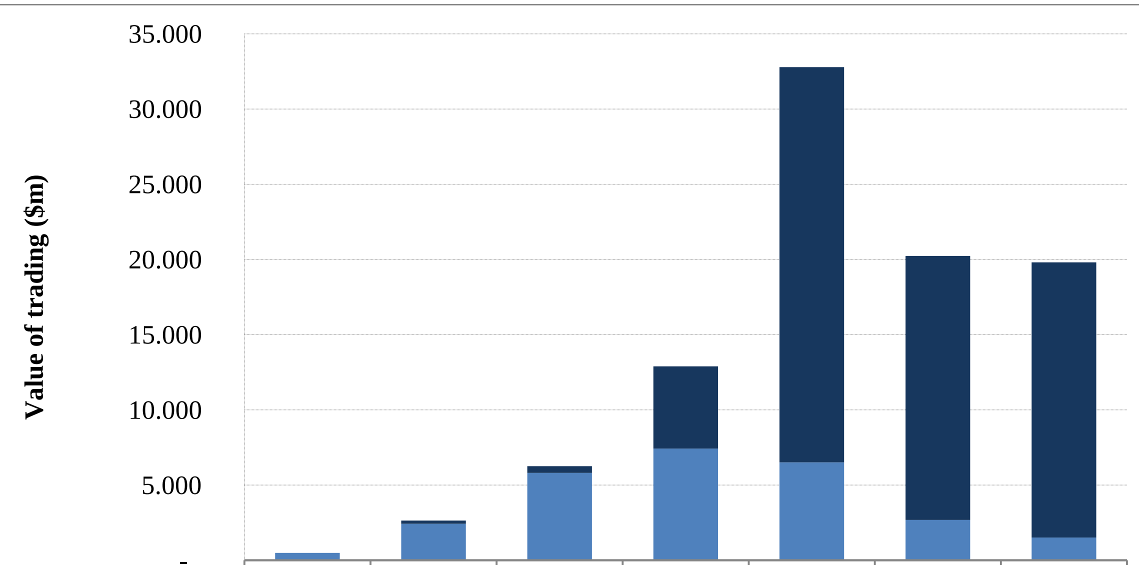
A clean development mechanism (CDM) is an investment made by an Annex I country in a Non-Annex I country[1]. The investment is supposed to contribute towards an improvement of climate conditions in the long run.

This type of project is regulated by the United Nations, which concedes credits known as Certified Emission Reductions (CER). The CER's are conceded according to the amount of emissions saved and will then be available to the investor countries to use them in order to meet their emissions' levels or trade them in the open market. Some examples of CDM's can include: energy efficiency schemes, fuel switching processes, or the capture and destruction of industrial gases. For instance, one of the examples found was conducted in Fiji by the Asian Development Bank. The project aimed towards the expansion of the water supply and sewage systems' grid, and generation of biogas trough the capture of methane from a sewage treatment plant.

From Figure 2.3 it can be assessed that CDM's trading grew until it had a peak in 2008 followed by a significant downturn that is explained by an international recession, which contributed for the reduction of the need for allowances, thus affecting the investment in this project-based mechanism. Until June 2011, the main investors in CDM projects were European, with nearly 70% of the total investments, and Japanese, which represented around 13% of the investments in CDM projects. Regarding European countries' investment it is important to highlight the UK, since it represents more than one quarter of all investments in CDM projects until June 2011. On the other hand, the countries that had a greater inflow of investments in CDM projects are mainly emergent markets such as Brazil, India, China, Mexico, and Indonesia. Brazil, India and China combined, hosted more than 103.249 $m until June 2011, accounting for more than 85% of the investments made. It is also important to refer the investments made in China, which is a receiver of nearly 70% of all the CDM investments until June 2011. In terms of sector of development of

[1] Annex I countries represent the most developed and industrialized countries worldwide, while Non-Annex I countries represent mostly, economies that are undergoing social and economic development processes. Annex A – List of Annex I parties to the Kyoto Protocol presents the list of these countries.

this type of projects until June 2011, hydro electric power and wind were the two main sectors of investment in CDM projects, accounting respectively for 30% and 21% of the overall investments (see, for example, The City UK (2011), United Nations (2012), and Schofield (2007)).



	2004	2005	2006	2007	2008	2009	2010
CDM Secondary	-	221	445	5.451	26.277	17.543	18.300
CDM Primary	485	2.417	5.804	7.433	6.511	2.678	1.500

Figure 2.3 CDM transactions (The City UK, 2011)

2.3. Joint implementation

A joint implementation is an investment project, made by an Annex I country in another Annex I country, with the intention of reducing emissions. This type of project is usually conducted if the costs of emissions' reductions in the investing country are higher when comparing with the invested country. The investing country will then receive allowances, Emission Reduction Units (ERU), according to the amount of emissions saved in the invested country. To exemplify a joint implementation project it can be referred a project led in the Russian Federation (in fact, the first joint implementation project in the Russian Federation) by the Germans' utilities company E.ON, whose goal was to build a gas-fuelled electricity plant with energy efficient systems, that allows to save more than a million tons of carbon by the end of 2012.

Joint implementation transactions peaked in 2007 having a decreasing trend since then. The international recession is also pointed as a cause for the decrease in terms of these transactions (see for example Schofield (2007, p. 249), United Nations (2012) and Bloomberg (2010)).

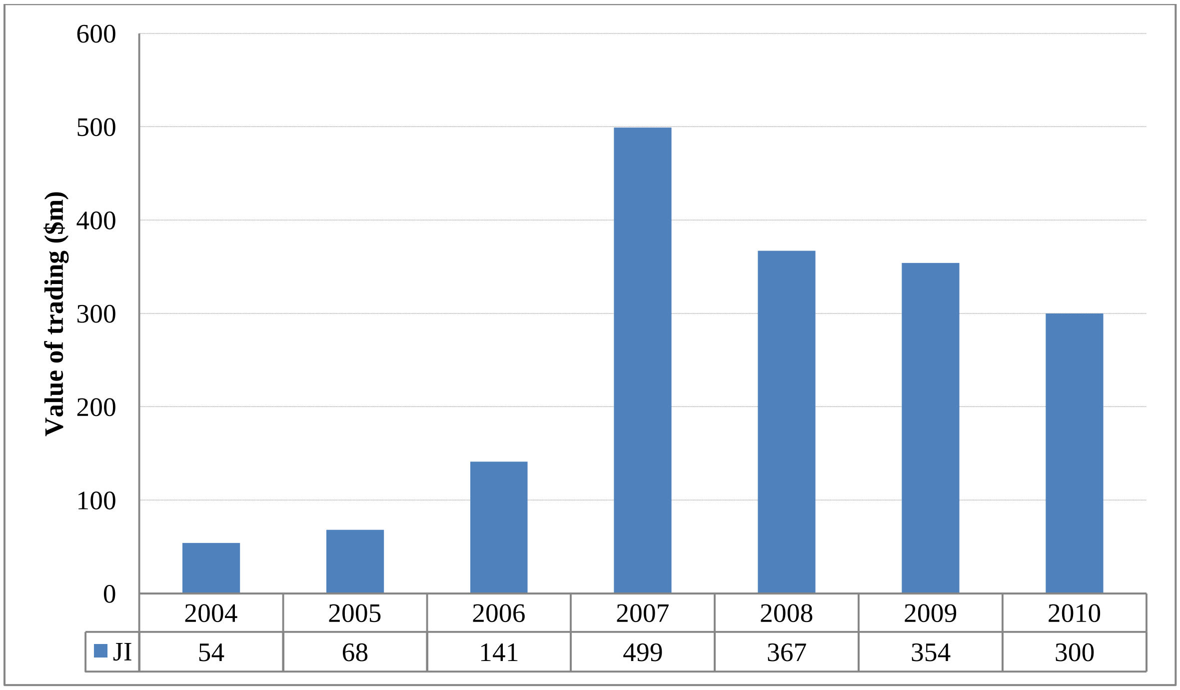


Figure 2.4 Joint implementation transactions (The City UK, 2011)

2.4. Emissions trading

The UN defines emissions trading in Article 17 of the Kyoto Protocol. The document defines emissions/carbon as a new commodity that can be traded, just like any other commonly tradable commodity such as corn, oil or gold. The fundamental idea, as it was previously referred, is a settlement of limits, named allowances, to comply in terms of emissions that can be traded according to the needs of countries and companies. This means that countries or companies with exceeding permits can trade those allowances with others that have a deficit in terms of emissions' allowances. Benz and Trück (2009, p. 5) refer that emissions can then be either an asset or a liability depending on the amount that is produced, as agents must manage their allocated permits and the pollutant gas emissions.

Being in the market as any other common commodity implies that these allowances will have attached a price that represents the value per metric unit of the substance emitted. Thus, it is important to consider factors that can influence the behaviour of prices in carbon markets and that can also be observed in other commodities' markets.[2] Economic growth dynamics as well as the supply and demand for allowances are likely to have an impact on prices of emissions. Weather conditions, fuel prices and the existence of alternative sources of energy production are other important issues to consider when studying prices in carbon markets due to direct relations between these variables. For example, extreme weather conditions would trigger the demand for power generation thus having an impact in fuel prices and in the demand for allowances. Regulation and penalties can also have influence in terms of prices of emissions' allowances (Schofield, 2007, pp. 249-252).

There are several ETS's worldwide, being the EU ETS the one that acquired greater scale and which is the object of this study. Yet, there are other carbon markets that are important to consider since many of these are prior to the EU ETS. Table 2.1 provides an insight on the amounts traded in these carbon markets in terms of volume and value.

The Chicago Climate Exchange (CCX) was a voluntary emissions' trading program established in 2003 and launched within the US and Canada. The program was an effort of US's environmental policy to join the EU in creating a carbon market with the final objective of reducing greenhouse gases' emissions. Despite being a voluntary program, it comprised major corporations, utilities and financial institutions around all US, in 8 Canadian provinces and in 16 other countries. In 2010 though, the CCX's program ended and the business units were acquired or restructured within the holding company. This North American based experiment in carbon markets failed due to the incorrect allocation of allowances, since there was a large imbalance in terms of supply of allowances (see, for example, The City UK (2011), Gronewold (2011), Barnes (2010) and the Chicago Climate Exchange (2011)).

The New South Wales Greenhouse Gas Abatement Scheme (GGAS) was launched in 2003 in Australia, being one of the first compulsory emission trading schemes. The scheme focuses mainly on the reduction of greenhouse gases' emissions originated from the production and use of electricity. The program is defined as a 'baseline and credit', which means that there are attributed credits according to the improvements made in greenhouse emissions. GGAS comprises electricity retailers and other parties from New South Wales and Australia, that need to meet the established targets for emissions of greenhouse gases. These targets need to be achieved

[2] The price drivers for carbon markets are also considered for the particular case of the EU ETS, see section 2.5.4.

collectively, thus forming a benchmark for each year (see, for example, The City UK (2011) and the Greenhouse Gas Reduction Scheme / Independent Pricing and Regulatory Tribunal (2011)).

The Regional Greenhouse Gas Initiative (RGGI) is a program launched in 2008 in 9 states of the Northeast and Mid-Atlantic regions of the US. It consists in the first mandatory carbon market in the US covering more than 200 facilities in the energy production sector. The RGGI intends to reduce emissions of greenhouse gases through the establishment of a regional limit for the amount permitted for power plants to emit. Like in the CCX, RGGI suffers from over allocation with an exceeding 50% of limits over the emissions (see, for example, The City UK (2011) and the Regional Greenhouse Gas Initiative, Inc (2010)).

Allowance-based transactions						
Volume of trading (mtCO2)						
	EU ETS	GGAS	CCX	RGGI	AAU	Total
2005	321	6	1	0	0	328
2006	1.104	20	10	0	0	1.134
2007	2.060	25	23	0	0	2.108
2008	2.956	31	69	62	48	3.166
2009	5.504	33	41	809	136	6.523
2010	5.712	29	63	178	19	6.001
Value of trading (Million Dollars)						
	EU ETS	NSW	CCX	RGGI	AAU	Total
2005	7.908	59	3	0	0	7.970
2006	24.436	225	38	0	0	24.699
2007	49.065	224	72	0	0	49.361
2008	100.526	183	309	198	276	101.492
2009	118.474	117	54	2.179	2.003	122.827
2010	119.800	100	2	400	260	120.562

Table 2.1 Main emission trading systems (The City UK, 2011)

2.5. European Union Emission Trading System (EU ETS)

The European Union Emission Trading System (EU ETS), launched in 2005, was developed with the objective of cutting greenhouse gases within European Union's member states, and complies with the objectives stated in the Kyoto Protocol. The EU ETS is the largest carbon market operating both in volume and value of transactions as it can be observed in Table 2.1. Being a compulsory program enlarges the scope of operations of the system that covers 30 countries and over 11,000 industrial facilities and power plants, and accounts for nearly half of all EU CO2 emissions, thus making the EU ETS a central instrument in terms of environmental policy (see, for example, the European Commission (2011) and Grubb and Neuhoff (2006, pp. 8-10)).

2.5.1. Creation and development of the EU ETS

As mentioned above, the EU ETS was launched in 2005. However, its creation and development date from several years back.

Ellerman, Convery and de Perthuis (2010, p. 13) mention that the theoretical foundations for the creation of such program date back from the 1960's, especially with the theoretical approach purposed by Coase (1960). This author argued that under a market failure situation[3], such as the pollution and depletion of natural resources, the allocation of environmental property rights can be a solution towards the achievement of an economically efficient solution, through the negotiation of social arrangements between the parties involved. Ellerman, Convery and de Perthuis (2010, p. 13) mention as well the work of Crocker (1966), Dales (1968) and Montgomery (1972), which developed studies in environmental economics, using experimental cases and modelling the socio-economic behaviour of economic agents under the existence of markets for emissions in which a fixed number of emissions are capped and allocated in a system by quotas (see, for example, the work of Montgomery (1972) on the economic analysis and framework for an emissions' market). Price movements obtained from some of these empirical studies implied the incentive to innovation and emissions' reduction.

Concerning political triggers for the development of the EU ETS, the Single European Act of 1986 and the Treaty on European Union of 1992 are important foundations not only in terms of environmental policy, but especially in terms of economic policy development as it created the idea of a single market with more integration and exchange of resources between the member countries. Despite this broader scope of integration, in 1992 the European Commission proposed the creation of a carbon energy tax within the European Union, proposition that failed mainly due to the loss of autonomy in terms of fiscal policy for some EU countries and also due to the strong industry lobbies that opposed the proposition. In 1998, following the Kyoto Protocol negotiations

[3] A common example used to express a market failure from an environmental nature is an Externality. Externalities can be understood using the definition of Friedman (2003): "*an externality is the effect of a transaction between two individuals on a third party who is not consented to or played any role in the carrying out of that transaction*".

and the abandonment of the EU carbon energy tax proposal, fifteen EU countries agreed to adopt measures that would enable the achievement of 8 per cent less emissions than those from 1990, by the 2008 to 2012 period. It was also in 1998 that the European Commission released the strategy document that would define the post-Kyoto actions to adopt in order to meet the goals defined in that Protocol. In this document, *Climate Change: Towards an EU Post-Kyoto Strategy*, it was also determined that an emissions trading scheme would be implemented from 2005 on, and it would be a decisive element in terms of European environmental policy.

Even though the creation of the EU ETS is still quite recent, several countries and organizations within the EU have had experiences with emissions trading before the development and implementation of the EU ETS. The United Kingdom is one of the countries with greater history in terms of carbon taxes and emissions trading, since during the early 1990's it started to discuss the implementation of such taxes and other environmental protection projects which ultimately led to the creation of the UK Emissions Trading Scheme in 2002. Denmark is another example of implementation of emissions trading programs at a local level. This Nordic country has an important background in terms of usage of environmental taxes to achieve environmental goals. Prior to the implementation of the EU ETS the Danish government adopted measures to limit a cap on emissions from the energy sector, particularly the electrical facilities, in a descending trend. Regarding private company programs, it can be mentioned the experiment made by the company British Petroleum (BP) from 1998 to 2001, that introduced an internal trading scheme with successful results as the company managed to reduce the emissions of greenhouse gases by more than 10% (see, for example, Ellerman, Convery and de Perthuis (2010, pp. 16-21) and Victor and House (2006)).

2.5.2. Characteristics of the EU ETS

Several characteristics contribute to the uniqueness of the EU ETS. First of all, and as mentioned previously, there is an important linkage between the EU ETS and the Kyoto protocol, as this serves as a guidance for the application of this European system.

The program is a "cap and trade" system, which means that it works through the creation and distribution of limits in terms of greenhouse gases that can be emitted by the installations covered and companies and countries receive permits for their allocated emission limits. The cap on limits to emit such pollutant gases creates scarcity, thus giving value to the permits.

On a yearly basis, installations/countries must report their emissions and cover them with the corresponding allowances. In the event of a shortage of allowances, the regulator will charge fines to the company or country.

Another characteristic to consider is the progressive reduction of allowances over time, and the multi-period allocation. Hence, it can be said that the intention of the program is to reduce emissions over time and therefore the amounts of carbon allowed will decrease in future allocations.

In terms of its periodicity, the program allocates and implements new regulations (e.g. covered facilities) periodically in 3 different phases:

- *Phase I* marked the beginning of the program; covering the period from 2005 to 2007, it was outside the Kyoto protocol frame and it was important to provide experience in emissions trading and to develop organizations involved in carbon markets. *Phase I* was marked by several challenges for a new market, being the over allocation of permits which drove prices near to zero the most discussed one.
- *Phase II* covers the 2008-2012 schedules and, despite previous knowledge and a reduction of the emissions cap, is marked by a global economic recession that caused a supply of allowances greater than the emissions, and for the inclusion of the aviation sector within the scope of the EU ETS in 2012.
- *Phase III* will cover the 2013 to 2020 periods and presents several important elements, among which are the reduction of the cap on emissions by 1.74% per year, the stronger focus on reductions within the EU and the enlargement of the scope of operations to other sectors and gases, such as chemical producers or nitrous oxide gas.

Regarding this multi-period feature, it is important to consider the possibility of banking and borrowing within any trading period. This means that allowances are issued annually and once they are issued they can be used to cover emissions' limits in any year. The exception on this rule was *phase I* of the EU ETS, in which there was a restriction on banking permits.

Up to certain limits and conditions, installations under the EU ETS can also use credits accomplished outside the European Union to comply with their emissions targets. These credits are known as Certified Emission Reductions (CER) or Emission Reduction Units (ERU) and result from the project-based mechanisms, joint implementation and CDM. The limits mentioned must be accordant to the criteria of the Kyoto Protocol that intends to ensure that a relevant part of the emission reduction is achieved within the territory. In the EU ETS, the limit is a specified percentage of the allocation to each member state that is presented in its National Allocation Plan (NAP).

These National Allocation Plans are in fact a central part in each member state emissions trading policy since these are policy documents that outline the degree coverage and justified allocation of permits in the territory. Each member state is also responsible for monitoring, reporting and compliance of the EU Monitoring and Reporting Guidelines (see, for example, Ellerman, Convery and de Perthuis. (2010), the European Commission (2011) and The City UK (2011)).

2.5.3. Tradable securities

Within the EU ETS there are two main types of possibilities to trade through: European Union Allowances (EUA) or via Certified Emission Reduction (CER). EUA's can be traded, as many other securities in the spot market, in which the underlying is physically delivered between 24 and 48 hours after the negotiation, as forward or futures contracts, in which the underlying security is traded in a future date, or even in the form of swaps and options.

In terms of the CER market, this is a bit more complex. There are some limitations towards the usage of these allowances to comply with the EU ETS' limits, factor that may influence its value. The market for CER is divided into two different segments. The first segment consists in the trade of forward contracts for CER's. These are usually sold by developers and the CER's represent the amount of emissions that will be saved by the underlying project in the future. These projects have risks associated since there are no guarantees of its success. In other words, the reductions purposed may not be achieved and therefore the credits from the project may not be issued. Such market represents a primary market for CER's, as it comprises the development and issuance of the projects and respective credits. The other segment is a secondary market as it comprises the trade of CER credits. In this case, the credits were already issued or its issuance is guaranteed, and the only activity is trading. This secondary market comprises fewer risks when compared to the primary market since the delivery of the security, the CER, is guaranteed. Figure 2.3, previously discussed, presents the evolution of the transactions for CDM projects, in which it can be seen that in the first years of activity, the primary market had a greater weight in terms of transactions. However the secondary market transactions grew significantly over from 2008 on, representing more than 90% of these transactions in 2010 (see, for example, Ellerman, Convery and de Perthuis (2010) and The City UK (2011)).

2.5.4. Carbon price drivers

Within the EU ETS, like in any securities market, price movements are influenced by supply and demand dynamics as well as by a set of other factors. Bataller, Tornero and i Micó (2007) and Chevallier (2011) developed studies on the topic, using the EU ETS as basis. These studies suggest the influence of energy prices, weather conditions, technological development and political and regulatory decisions in the prices of EUA's. Energy prices, more particularly fossil fuels, have a significant impact in the price of carbon allowances. Lowrey (2006) builds up the following rationale describing the influence of energy commodities in the price of carbon allowances: "*if the price of gas increases relatively to the price of coal, then the cost of switching from gas to coal increases and – other things being equal – the demand for coal will increase. Therefore, the demand for carbon allowances to cover that generation will also rise, leading to a resultant increase in emission allowance prices*". Weather conditions are a proxy for the demand and supply of energy, thus affecting carbon allowances' prices. The studies conducted by the aforementioned authors point out that normal weather conditions may not have a very relevant impact. However extreme or unanticipated weather conditions do have a significant impact in carbon prices. Technology is referred by Bataller, Tornero and i Micó (2007) as an important factor to consider when explaining the dynamics of carbon prices, despite being difficult to quantify its influence. One can, however, consider technological development in order to obtain more fuel efficient systems. Finally, Chevallier (2011) provides some views in terms of political and regulatory decisions that influence the prices of carbon allowances. The allocation of allowances, and eventual imbalances that arise from it, are a political decision that influences the pricing of carbon allowances, as it was experienced in *phase I* of the EU ETS, the over allocation of allowances had a significant impact in the price of EUA's. Banking restriction is another example of a political and regulatory measure that can have an impact in carbon prices within the EU ETS. In the first phase of the EU ETS, the existence of this measure implied that allowances "stored" at the end of the period had no commercial value afterwards, thus leading the spot and future prices towards zero as this first period ended.

3. Financial derivatives

Financial derivatives (or just derivatives) are financial instruments whose price depends on the price of another asset, denominated the underlying asset. Derivative instruments can depend on almost any variable, from common financial products such as stocks, interest rates or commodities, to some more elaborate products such as weather conditions.

There are 4 basic types of derivatives' contracts: forward, futures, options and swaps. These will be briefly described bellow.

3.1. Forward contracts

A forward contract is an agreement to trade an asset in a future date for a certain price that is fixed when the contract is celebrated. Such contract is traded over-the-counter, which means that there is no platform connecting buyer and seller, which negotiate the contract among themselves. Figure 3.1 presents the profit/loss behaviour for traders holding long or short positions in a forward contract.

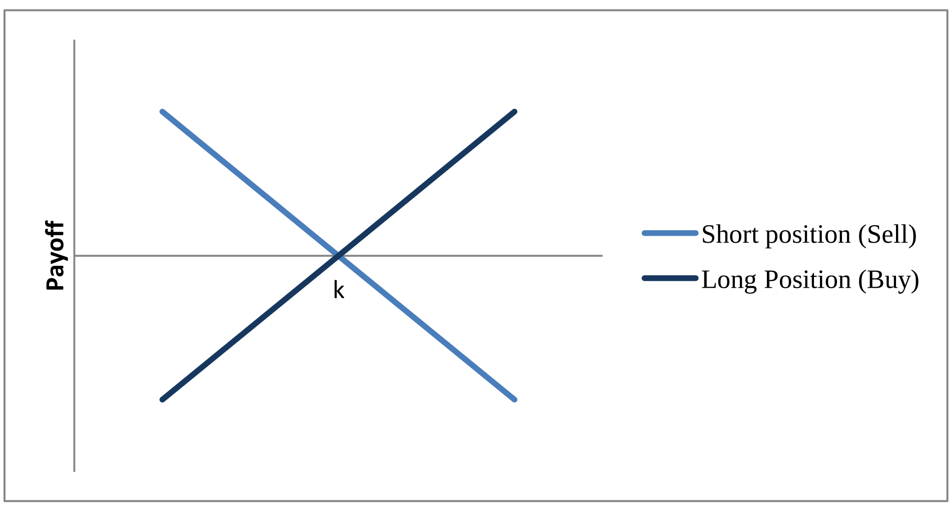


Figure 3.1 Profit/loss of a forward contract

3.2. Futures contracts

A futures contract is an agreement between two agents to trade an asset in a future date at a certain price that is fixed in the present. Despite being quite similar in terms of definition, there are some very important differences between futures and forwards' contracts. Table 3.1 presents the main differences between futures and forwards.

Contract	Futures	Forwards
Form	Standardized, only the price is negotiable	All elements are agreed between the parts
Liquidity	High level of liquidity	Level of liquidity varies according to the elements established between the parts
Margins system	Contracts comprise margins requirements	No margins required, only at maturity there is a financial flow
Closure of the contract	Contracts are closed by offsetting positions	Extinction of contracts with the delivery of the underlying asset
Intermediation	Contracts are agreed within a Board of Exchange/ trade	Contracts are established directly between the parts

Table 3.1 Futures vs. forwards' contracts

3.3. Options

In an options' contract there is a trade of rights and obligations between the parts. First of all, it is important to understand that in an options' contract there are two types of contracts and two types of positions within those contracts. An option can be either a call or a put option. Within a call/put option, the holder has the right but not the obligation to buy/sell an underlying asset at a certain future date, called maturity date, for a specified price, called exercise price. As mentioned there are two positions within a contract, long and short positions. The holder of a contract is defined as having a long position, as it owns a right, while the other part, which sold the contract, has a short position since it assumes an obligation, though it has the right to receive the premium paid at the beginning of the contract.

A call option's payoff can be defined as:

$$c_t = \max(S_t - K\,;\ 0\,) \tag{3.1}$$

while a put option's payoff is defined by:

$$p_t = \max(K - S_t\,; 0\,) \tag{3.2}$$

with S_T being the price of the underlying security at the maturity date T and K the strike price or exercise price.

Figure 3.2 presents the general outcomes from an options' contract according to the position assumed.

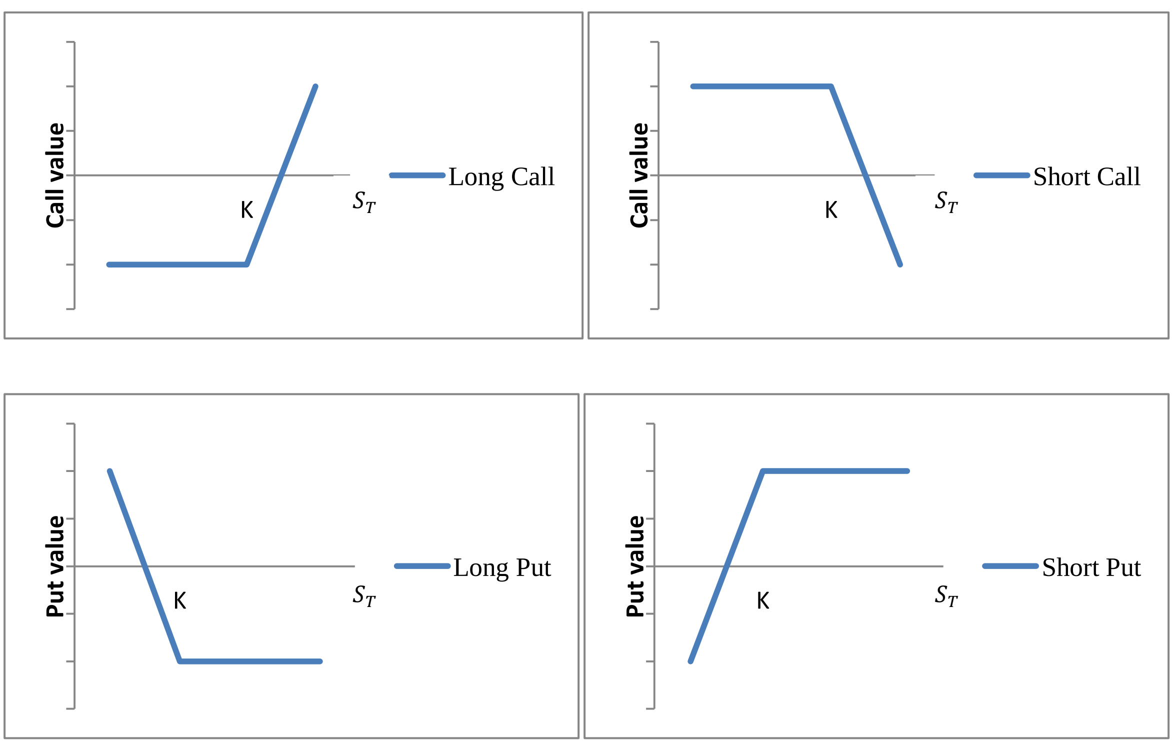


Figure 3.2 Profit/loss on an options' contract

Table 3.2 presents a summary of the types of options contracts, positions and respective rights and obligations.

	Call Option	**Put Option**
Long Position	Long Call (right to buy)	Long Put (right to sell)
Short Position	Short Call (obligation to sell)	Short Put (obligation to buy)

Table 3.2 Options' contracts positions

Options' can also be classified according to its *moneyness* degree. An option is said to be (a) *at-the-money* if its strike price equals the price of the underlying asset, (b) *out-of-the-money* if it delivers a negative payoff or it is not exercised, being then a call option whose exercise price is higher than the underlying asset's price or a put option whose strike price is lower than the underlying assets' price, or (c) *in-the-money* if it is to be exercised, thus being a call option with strike price lower than the underlying assets' price or a put option whose strike price is higher than the underlying asset's price. An option can also be classified as European, if it can only be exercised at maturity, or American, if it is allowed to be exercised prior or at maturity.

It is also important to consider the relationship known as put-call parity, relating the prices of European call and European put options. In practical terms, the expression allows to deduct the value of a European call option from the value of a European put option with the same characteristics (exercise price and exercise date), and vice-versa. If the put-call parity does not hold, there are arbitrage opportunities in the market. The put-call parity is expressed by:

$$c_0 + Ke^{-r\tau} = p_0 + S_0 \tag{3.3}$$

where c_0 is the value for a European call option, p_0 is the value for a European put option, K is the exercise price for the option, r is the risk-free interest rate, τ represents the time to maturity, and S_0 gives the value for the stock at the present date.

3.4. Swaps

In a swap contract two parties agree to exchange cash flows in a future date. Usually, these agreements represent an agreed fixed rate versus a floating rate. For example, a company could manage its currency exchange risk using a swap contract; in this case it could agree a fixed exchange rate in order to avoid fluctuations in the currency markets.

3.5. Valuation of derivatives' instruments

3.5.1. Black-Scholes-Merton model

The Black and Scholes (1973) and Merton (1973) model (hereafter for, BSM model), was a major accomplishment in terms of options' valuation, resulting also in the award of a Nobel Price for their living authors, Myron Scholes and Robert Merton. The BSM model represents a mathematical expression that allows to value European call options on stocks that do not pay dividends.

Introduced by Fisher Black and Myron Scholes (1973), the model was later generalized by Merton (1973), in order to value options on dividend paying stocks and with stochastic interest rates.

The original valuation of the BSM model is built under the idea that in the market, if options were correctly valued, a trader should not be able to make sure profits combining long and short positions in options and their underlying stocks. The mathematical expression derived is build upon some assumptions for ideal market conditions. It is assumed that there are no transactions costs, securities are traded continuously and the underlying stock does not pay dividends. It is also assumed that the risk free interest rate is known and is constant during the contract's period and that there is unlimited borrowing and lending with this rate. There are no penalties to short selling and the option is European that is it can only be exercised at maturity. Finally, it is assumed that the stock price follows a geometric Brownian motion with both constant mean and variance[4].

Under the BSM model it is assumed that the stock price behaviour follows a geometric Brownian motion that is:

$$dS_t = \mu S_t dt + \sigma S_t dW_t^{\mathbb{P}}, \qquad \textbf{(3.4)}$$

with μ and σ representing the mean and standard deviation of stock returns, and W representing a standard Gauss-Wiener process.

The lognormal process for the stock price behaviour in the previous equation has the following transition probability density function:

$$f\ (S_t\ , T\ ;\ S_t\ , t) = n\left(\ln S_t + \left(\mu - \frac{\sigma^2}{2}\right)\tau\ , \sigma\sqrt{\tau}\ \right), \qquad \textbf{(3.5)}$$

with $n($ μ , σ^2 $)$ being a normal density function with mean μ and variance σ^2.

The BSM model option pricing formulae is a function of the stock price St and the option's time to maturity τ. The remaining parameters in the equation are constant and observable, with exception for the standard deviation of the returns for the asset, parameter that accounts for the volatility on the security.

Thus, the formula for pricing European-style call options is given by:

$$C_t(S_t, \tau\ ; K, \sigma^2, r) = \ S_t e^{-q(\tau)} N(d_1) - \ K e^{-r(\tau)} N(d_2), \qquad \textbf{(3.6)}$$

and for pricing an European style put option, the respective formulae is given by:

$$P_t(S_t, \tau\ ; K, \sigma^2, r) = \ K\ e^{-r(\tau)} N(-d_2) - \ S_t e^{-q(\tau)} N(-d_1), \qquad \textbf{(3.7)}$$

---

[4] From stochastic processes it can be understood that these are statistical approaches used to describe the behavior of variables, named stochastic variables, whose value changes over time in an uncertain way. The Brownian motion process, used in the BSM model, is a particular type of stochastic process that derives from the Wiener Process which is a stochastic process that has normal distribution with a mean change of zero, a variance rate of 1 per year and in which the values of for any 2 different short intervals of time are independent. (Hull, 2005, pp. 216-223)

with the volatility parameters $d1$ and $d2$ given by:

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + (r - q + 0.5\sigma^2)(\tau)}{\sigma\sqrt{\tau}} \tag{3.8}$$

and

$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + (r - q - 0.5\sigma^2)(\tau)}{\sigma\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau}, \tag{3.9}$$

with c_t representing the call option's price or its premium at time t, p_t representing the put option's price or its premium at time t, S_t the stock price at t, K the exercise price of the option, $\tau = T - t$ being the time until the expiration of the option, r the risk free interest rate, σ the standard deviation of returns on the security until the expiration date, q is the continuous dividend yield on the security, and $N(d_i)$ the cumulative normal density function.

3.5.2. Merton jump-diffusion model

Following the general BSM model to value options, Merton (1976) proposed an approach that accounts to the possibility of the underlying asset's value taking any level or jump, in a continuous time framework.

A model such as the BSM, in which the price of the underlying asset follows a Brownian motion, is feasible if price changes over a short period of time are not expected to be very large. This means that, unless volatility is extremely high, in the short term prices will never jump.

The jump-diffusion model presented here enables to account for this kind of behaviour in terms of asset's prices. In order to account for extraordinary changes in terms of price dynamics, Merton (1976) describes changes in securities' prices as having two different components, normal and abnormal price vibrations. Normal price vibrations can be described as day-to-day economic movements that are almost certain to occur. Merton exemplifies this type of vibrations referring to supply and demand disequilibrium's or changes in the economic outlook. Normal price vibrations are modelled using a standard geometric Brownian motion with constant variance. Abnormal price vibrations result from new information about the security that has a great impact in the price, such as relevant firm or industry information. These price vibrations are modelled by a jump process. In practical terms, this is implemented using a Poisson distribution for assessing the number of jumps, which is given by:

$$P[N(\tau) = n] = e^{-\lambda\tau}\frac{(\lambda\tau)^n}{n!}, \qquad \textbf{(3.10)}$$

with λ being the jump intensity, τ the time to maturity and n the number of jumps during the options' life.

Merton then defines a random variable X_n to be conditional towards the number of jumps up to time τ, and a variable ε_i to be the expectation operator for the distribution of X_n. Considering the independence among variables it is proposed the following formula for the Merton's jump diffusion option pricing model:

$$M\,(S_t\,,\tau) = \sum_{n=0}^{\infty}\frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!}f_i(S_t,\tau\,,K,\sigma^2,r) \qquad \textbf{(3.11)}$$

with $i \in \{c,p\}$.

4. The pricing of Emissions' Allowances

4.1. Commodities' general theory

Commodities are important instruments not only within the financial industry, but also in terms of economic policy of a country or in terms of the economic and financial strategy of corporations.

First of all, in order to present some of the main theoretical models regarding commodities, it is important to introduce the concepts of contango and backwardation, which are not more than price behaviours within futures markets. A market in contango represents a situation in which the spot price is bellow the forward price, while a market in backwardation represents an opposite situation, the spot price exceeds the forward price. Keynes (1971) refers that markets should, most of the times, be in backwardation. A situation described by this author that is easily understandable is the inexistence of a supply of the commodity in the short run, thus leading the spot price to be higher. Keynes (1971) also mentions that the spot price should exceed the forward price by an amount that allows the producer to hedge himself during the production period. An opposite position was taken by Telser (1958) which argued that there was no reason for the existence of a consistent difference between spot and futures' prices before the expiration of the contract. This author argued as well that arbitrage opportunities would be eliminated immediately in competitive markets.

On this issue, Geman (2005, p. 34) refers that futures contracts' prices reflect a mix between the expectation existent in the market regarding the future spot price for a commodity and the risk premium and risk aversion of market participants, i.e., their behaviour regarding the possibility to pay a fixed price in the present for the future delivery of a commodity or their willingness to risk commodity price movements.

Some other concepts, among the most important in the literature regarding commodities are the theory of storage and convenience yield. The theory of storage was developed by Kaldor (1939) and Working (1949) and focuses on the benefits of ownership of a commodity, defined as convenience yield. As mentioned, the convenience yield is the benefit or cost of having the physical commodity available in the present moment. Geman (2005, p. 24) exemplifies the situation of the unanticipated demand for a commodity and the benefit for holding inventory, thus having attached a convenience yield. One can also refer an opposite situation, the cost of storage attached with the ownership of a commodity.

Geman (2005) mentions as well several issues regarding scarcity and volatility in commodities' markets. Commodities are usually more volatile than other financial assets, and one can argue that those price dynamics of commodities can be explained in part by weather conditions and seasonal patterns of growth (especially for agricultural commodities), demand and supply fluctuations, or geopolitical issues[5]. Under this thematic of volatility in commodities' prices, Samuelson (1965) presented a stochastic model that proofs the randomness of prices within commodity markets.

4.2. Commodities pricing

Concerning the subject of commodity contracts' pricing, some of the most well known works can be traced back to Merton (1973), that provided a derivation *formula* to price European options on commodity contracts, or to Black (1976), which derived a formula to price options on futures on commodities. Geman (2005) also presents a spot-forward price relationship in commodities' markets assuming the absence of arbitrage opportunities, allowing the derivation of the current futures price for a future date T which is related with the spot price at time t. In mathematical terms, this relationship is given by:

$$F_t^T = S_t e^{(r-y)\tau}, \qquad \textbf{(4.1)}$$

with S_t representing the spot price at time t, r representing the interest rate at t for maturity T, y the convenience yield for the commodity, and τ the time to maturity for the contract (or $T - t$).

Concerning the referred approach by Merton (1973), it can be derived a formula to value European-style options on commodity contracts which lays down on the assumptions of the BSM model (see section 3.5.1) with exception of the dividend payments that are substituted by a variable y, representing the aforementioned convenience yield. Merton's equation to derive the price of European call options on commodity contracts is given by:

$$c_t = S_t e^{-y\tau} N(d_1) - K e^{-r\tau} N(d_2), \qquad \textbf{(4.2)}$$

[5] See for example Schofield (2007) for more detailed information regarding specific commodity price drivers, and section 2.5.4 for price drivers on carbon markets.

and to derive European put options, the formula is given by:

$$p_t = Ke^{-r\tau}N(-d_2) - S_t e^{-y\tau}N(-d_1), \tag{4.3}$$

with the Gaussian distribution parameters d_1 and d_2 equal to:

$$d_1 = \frac{\ln\left(\frac{S_t e^{-y\tau}}{Ke^{-r\tau}}\right) + 0.5\sigma^2\tau}{\sigma\sqrt{\tau}} \tag{4.4}$$

and

$$d_2 = d_1 - \sigma\sqrt{\tau}, \tag{4.5}$$

being c_t the call option price for the given commodity at time t, p_t the put option's price for the commodity at time t, S_t the spot price at t for the underlying commodity, K the strike price of the option, $\tau = T - t$ the time frame until the expiration of the option, r the risk free interest rate, σ the standard deviation of returns on the underlying commodity and, as mentioned and y represents the convenience yield for the commodity, keeping in mind that for storable commodities this can be presented as $y = y_1 - c$, being y_1 the benefit inherent from the ownership of the commodity and c the storage cost of the commodity.

The valuation of options on futures has been proposed by Black (1976). The valuation of such contracts is quite important since most commodity contracts traded in financial markets, occur under futures or forward contracts, as mentioned by Geman (2005, p. 93). The model itself builds up under the original BSM model (see section 3.5.1) and its proposed formula to value a European style call option on a futures contract is:

$$c_t = e^{(-r\tau)}[F_t^{T_f} N(d_1) - KN(d_2)], \tag{4.6}$$

while the formula for the respective European put option is given by:

$$p_t = e^{(-r\tau)}\left[KN(-d_2) - F_t^{T_f} N(-d_1)\right], \tag{4.7}$$

Just like in the BSM model and in Merton (1973) model, we have:

$$d_1 = \frac{\ln\left(\frac{F_t^{T_f}}{K}\right) + 0.5\sigma^2\tau}{\sigma\sqrt{\tau}} \quad \textbf{(4.8)}$$

and

$$d_2 = d_1 - \sigma\sqrt{\tau}. \quad \textbf{(4.9)}$$

The notation used is the same when relating to the model presented by Merton (1973), though having in mind that $F_t^{T_f}$ represents the value of a futures' contract expiring at T_f.

Geman (2005, p. 94) points out a particular situation concerning the relationship of the referred models which, if $T_f = T$ (recall that $\tau = T - t$, where T is the maturity date for the option), or in other words, the maturity dates for the futures contract and for the option on that contract are the same, and if the commodity's spot-forward relationship is applied to derive the futures' price then Black (1976) formula will equal Merton (1973) its formula for options on commodity contracts.

4.3. The pricing of emissions' allowances within the EU ETS

As mentioned earlier in section 2.4, carbon market's derivatives are considered commodities. Therefore, in many ways, it is useful to consider some of their properties when modelling carbon prices. Being the EU ETS the largest carbon market in activity worldwide, it is important to assess the pricing of its tradable commodities and which models better describe the behaviour of such assets. Among the extensive literature available related with commodities, specific papers on carbon pricing and emission derivatives pricing are not so common. As referred by Benz and Trück (2006), the lack of historical data is an issue when authors attempt to model price behaviours in the EU ETS. Despite the referred lack of applied literature on carbon markets, several authors contributed with some fundamental theoretical works and/or empirical studies on the subject.

Paolella and Taschini (2006) applied GARCH models in order to conduct an empirical analysis on the returns for the spot prices from the CCX. Fehr, Hinz and Carmona (2009) propose a model that values the strategy followed by market participants involved in carbon emissions' trading.

This strategy builds up on models from other commodities, particularly energy commodity pricing models, and assumes that market participants follow an optimization strategy of carbon allowances in order to manage risks related with the trade of such emissions. Seifert, Uhrig-Homburg and Wagner (2007) present a stochastic equilibrium model and analyze the spot price dynamics for the EU ETS. Unlike the previous model, in this case it is taken the role of a central planner that intends to minimize the costs and maximize the profits. There is a cap on emissions to be released in the beginning of each trading period and during the trading period the market participants decide on the costless alternative to approach emission reductions, that is the cost efficiency between abatement strategy and/or the payment of a penalty. The authors found the inexistence of any deterministic seasonal component related with the emissions rate in the spot price of allowances, and that volatility increases when reaching the end of a trading period. Benz and Trück (2009) conduct an analysis of the spot price behaviour of carbon allowances within the EU ETS in the short-run. Their study applies a stochastic process and econometric models to account for heteroskedasticity in order to study the spot price dynamics and volatility in this market. The study concluded that regime-switching models with auto-regressive processes better fit the data, when comparing the log returns from the EU ETS with the models' forecast.

Daskalakis, Psychoyios and Markellos (2009) compared several models to value allowance prices within the EU ETS and conducted an empirical analysis comparing the main markets trading spot and futures contracts. The study was conducted during *phase I* of the EU ETS, in which there was a banking prohibition (see section 2.5.2), fact that led the authors to conclude that the standard approach to price commodities (Black's formula, see section 4.2) leads to significant pricing errors. The econometric analysis conducted, which compared the French Powernext and the Nordic Nord Pool, concluded that the spot price behaviour for carbon allowances between these markets is quite similar and there is a strong correlation coefficient between these markets. Regarding the analysis for the futures markets, conducted for the Dutch European Climate Exchange and the Nordic Nord Pool, it was concluded that futures on carbon allowances were negatively correlated with equity market returns. It was also concluded that the nature of the contracts traded had a different behaviour, since the analysis comprised contracts ending within *phase I* (inter-phase contracts) of compliance and contracts with maturity date in *phase II* (inter-phase contracts) of compliance of the EU ETS. The price of intra-phase contracts evolved following the behaviour of the spot prices, while the price of inter-phase contracts deviated from the spot price. The authors provide some insights on this difference in terms of behaviour, as it appears to exist convenience yields in inter-phase contracts. In order to model the price behaviour for such contracts the authors applied six different continuous-time stochastic models.

In order to price futures contracts the authors use the aforementioned approach (see section 4.2) defined in equation (4.1). The convenience yield, in this situation, would be equal to zero since the storage of permits is costless.

Regarding the pricing of options on futures written on carbon allowances, the authors propose an approximation scheme using the basic models discussed previously. Under this model the price at time t of a call option expiring at T_1, written on a future contract having maturity date T_1 would be:

$$C(t;T_1;T) = P(t;T_1)\sum_{n=0}^{\infty}\left[\frac{e^{\left(-\lambda(T_1-t)\right)}\left(\lambda(T_1-t)\right)^n}{n!}\right]\left[F(t;T)N(d_1) - KN(d_2)\right], \qquad \textbf{(4.10)}$$

where $P(t;T_1)$ is the price at time t of a zero-coupon bond with maturity at time T_1, K is the strike price for the option, while λ represents the jump intensity.

5. Model simulation

The model presented by Daskalakis, Psychoyios and Markellos (2009) was applied by considering the BSM model as a basis and adapting it to account for jumps, which leads to the price of a European call option written on a futures contract.

All the obtained results in this work were performed in Matlab. The respective programming code is presented in Annex B.

5.1. Model specification and assumptions

To model the presented derivatives' products it is assumed as the initial date the 1st of June of 2012. This will allow us to conduct a simulation of the price of the aforementioned call option, using EUA's as the underlying asset, whose maturity is in the last Monday of the contract month, in this situation, the 31st of December 2012. Each contract entitles to one lot of 1000 CO2 EU Allowances which represents the right to emit one tonne of a carbon dioxide gas. The contracts are settled and delivered physically. Trading is conducted in Euros and is continuous throughout the trading hours.

In order to apply the presented model to market data it was necessary to consider some assumptions regarding the elements to derive the values, as presented in equation (4.10). The variables to account for in the model are expressed bellow:

Variable	Value
$\boldsymbol{F(t;T_1)}$	6,34€
$\boldsymbol{\lambda}$	0 ; 0,5 ; 1
σ	3,57%
$\boldsymbol{K}$	5,706€ ; 6,34€, 6.974€
$\boldsymbol{\tau = T_1 - t}$	1 ; 0.5 ; 0,0096
$\boldsymbol{r}$	0.943% ; 0,665% ; 0,357%
Nmax	10

Table 5.1 Summary of values used for modelling derivatives' contracts

The futures contract price was considered from the Intercontinental Exchange [6](ICE) with maturity in December 2012. Regarding the lambda parameter (λ), used to account for the jump intensity, three possibilities were considered, having a jump intensity of 0, 0.5 or 1, in order to access on the sensibility of this parameter, considering that, according to Merton (1976), $\lambda = 0$ provides no specification error and $\lambda = 1$ provides maximum misspecification. Concerning the strike price for the option it was considered several options, taking as basis that the option is *at-the-money* and using then a 10% coefficient being then the option either *in-the-money*, with $F(t;T_1) \times (1 - 10\%)$, or *out-of-the-money*, with $F(t;T_1) \times (1 + 10\%)$.

The parameter tau ($\tau = T_1 - t$), represents the time to maturity. Therefore, at the beginning of the simulation this value would be 1, since the contract is assumed to start at the assumed present date, 1st of June of 2012. In order to assess the time dynamics it was considered three moments to evaluate the options' price, at the beginning, with $\tau = 1$, at the middle, with $\tau = 0{,}5$, and one week until the end of the contract which represents the 24th of December 2012 being $\tau = 0{,}0096$. Regarding the value for the interest rate, it was considered the European Interbank Offered Rate (EURIBOR) according the time to maturity of the contract that is 6 months, 3 months or 1 week. The data collected refers to the 1st of June 2012. The variable nmax is used in order to compute the jump process, as defined by Merton (1976). In this situation nmax would account for the maximum number of jumps during the options' life. For the purpose of this simulation it was assumed that the maximum number of jumps to occur during the options' life would be 10.

5.2. Dataset specification and volatility

In terms of volatility, it has been computed the historical volatility using a dataset from the ICE, considering as underlying a futures contract with maturity in December 2012, and that comprised 258 observations starting at the 1st of June 2011 until the 1st of June 2012. The descriptive statistics for the prices and log-returns are presented in Table 5.2, while the price evolution and log-returns are presented in Figure 5.1 and in Figure 5.2, respectively. Analyzing the data and its representative figures, it is easy to notice the drop in the price of EUA futures from June 2011 until June 2012. There are some sharpest movements. Yet, once the mean and standard deviation

[6] The Intercontinental Exchange (www.theice.com) is an exchange focused on the trade of financial assets as well as commodities, both in the spot and derivatives' markets.

of log returns are observed, it can be seen that the movements, in a global perspective, are not as sharp.

Price		Log Returns	
Mean	10,0643	**Mean**	-0,004
Std. Error	0,1871	**Std. Error**	0,0022
Median	9,13	**Median**	-0,003
Std. Deviation	3,0048	**Std. Deviation**	0,0357
Variance	9,0291	**Variance**	0,0013
Kurtosis	-0,4067	**Kurtosis**	2,4172
Skewness	0,6963	**Skewness**	-0,1415
Minimum	6,15	**Minimum**	-0,1628
Maximum	17,8	**Maximum**	0,1277
Count	258	**Count**	257

Table 5.2 Descriptive statistics of EUA futures' contracts with maturity in December 2012

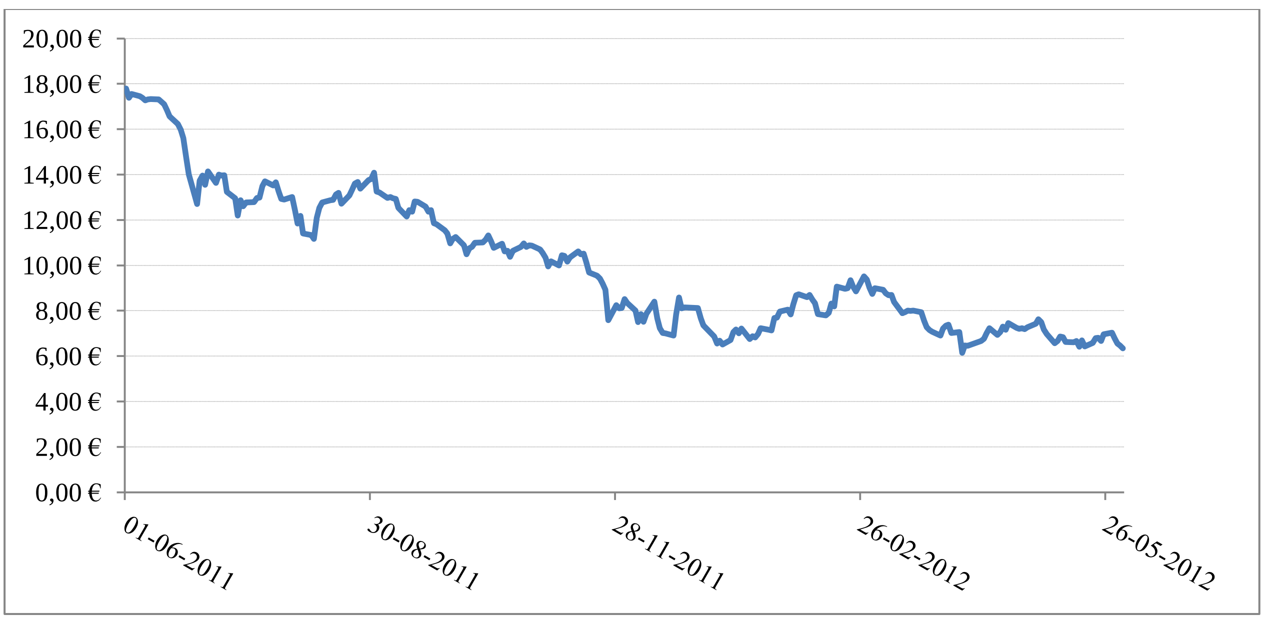


Figure 5.1 EUA Futures price

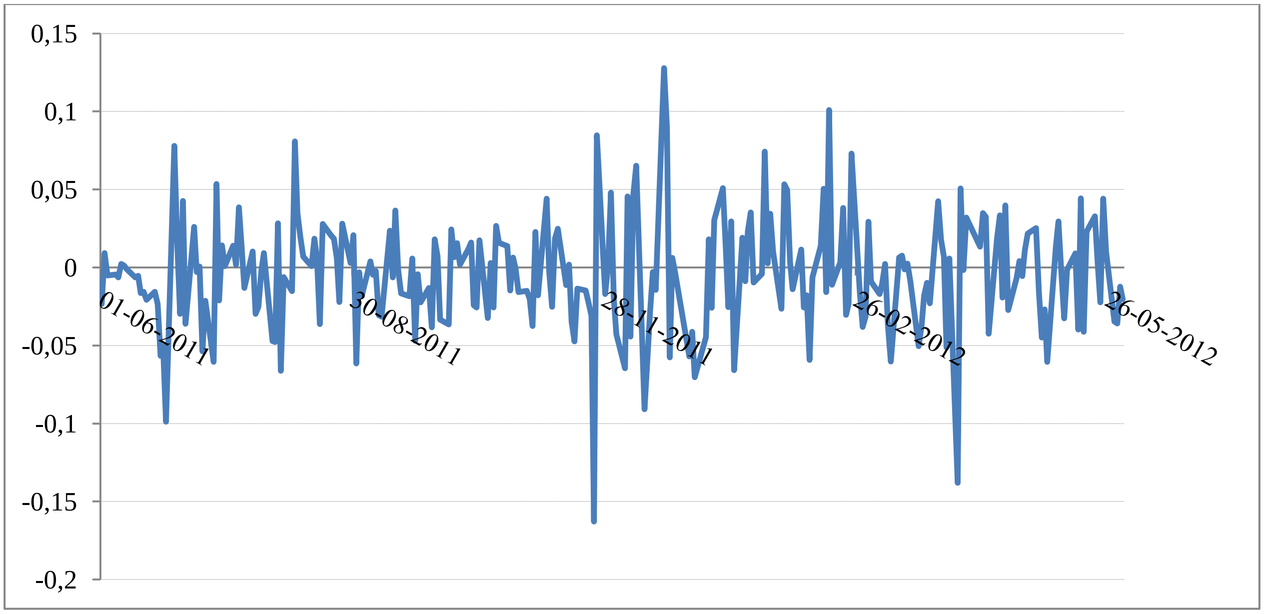


Figure 5.2 Log-returns for EUA Futures price

5.3. Simulations' results and analysis

After listing the assumptions and methods that provide theoretical and technical basis for the computation of the call options' price, the following simulation was conducted. The results are provided in Table 5.3. This table presents not only the effective value for each European call option calculated, but also the setup used for it, i.e., the assumptions underlying each calculation, which, as it was mentioned, can vary according to the strike price, lambda or tau, as the remaining parameters are assumed to be constant.

SPOT(€)	STRIKE(€)	LAMBDA	TAU	VOLATILITY	RATE	NMAX	VALUE(€)
6,34	6,34	0	1	0,0357	0,00943	10	0,0895
6,34	5,706	0	1	0,0357	0,00943	10	0,6282
6,34	6,974	0	1	0,0357	0,00943	10	0,0003
6,34	6,34	0,5	1	0,0357	0,00943	10	0,0895
6,34	5,706	0,5	1	0,0357	0,00943	10	0,6282
6,34	6,974	0,5	1	0,0357	0,00943	10	0,0003
6,34	6,34	1	1	0,0357	0,00943	10	0,0895
6,34	5,706	1	1	0,0357	0,00943	10	0,6282
6,34	6,974	1	1	0,0357	0,00943	10	0,0003
6,34	6,34	0	0,5	0,0357	0,00665	10	0,0635
6,34	5,706	0	0,5	0,0357	0,00665	10	0,631
6,34	6,974	0	0,5	0,0357	0,00665	10	0
6,34	6,34	0,5	0,5	0,0357	0,00665	10	0,0635
6,34	5,706	0,5	0,5	0,0357	0,00665	10	0,631
6,34	6,974	0,5	0,5	0,0357	0,00665	10	0
6,34	6,34	1	0,5	0,0357	0,00665	10	0,0635
6,34	5,706	1	0,5	0,0357	0,00665	10	0,631
6,34	6,974	1	0,5	0,0357	0,00665	10	0
6,34	6,34	0	0,0096	0,0357	0,00357	10	0,0089
6,34	5,706	0	0,0096	0,0357	0,00357	10	0,6339
6,34	6,974	0	0,0096	0,0357	0,00357	10	0
6,34	6,34	0,5	0,0096	0,0357	0,00357	10	0,0089
6,34	5,706	0,5	0,0096	0,0357	0,00357	10	0,6339
6,34	6,974	0,5	0,0096	0,0357	0,00357	10	0
6,34	6,34	1	0,0096	0,0357	0,00357	10	0,0089
6,34	5,706	1	0,0096	0,0357	0,00357	10	0,6339
6,34	6,974	1	0,0096	0,0357	0,00357	10	0

Table 5.3 Results' table from call options' simulation

The following tables present the results obtained from the simulation, divided in terms of options' *moneyness* degree, being *at-the-money*, *in-the-money* or *out-of-the-money*, respectively. It is then possible to compare options' values according to the values of tau and lambda.

It is possible to observe that in the simulation the parameter lambda does not have any influence in the results, since for each value of the jump intensity assumed (lambda) the obtained price for the call option is the same, being every other assumption equal. Considering this, the major changes in the price that were obtained are a result of the evolution in terms of maturity of the option and the respective *moneyness* degree. In the case of *at-the-money* and *out-of-the-money* options, it is then possible to observe a decrease in the options' value when approximating maturity, with values close to 0 in the case of *at-the-money* options and 0 in the case of *out-of-the-money* options. *In-the-money* options have a different signal dynamics since they present an increase throughout the contracts' life. The nature in which such contracts are established, in line with *phase II* of the EU ETS, and being an intra-phase contract, can be reflected in the values obtained. Recall that the contracts analyzed here give the holder the right to buy 1 EUA at the 31st of December 2012 for the predefined price. These features can enable investment opportunities at the end of the period since banking restrictions no longer apply in the EU ETS, therefore, EUA's acquired until the 31st of December 2012 (end of *phase II*) can be transferrable and used thereafter.

		Strike		
		ITM	**ATM**	**OTM**
		5.706	**6.34**	**6.974**
Tau	**1**	0,6282	0,0895	0,0003
	0,5	0,631	0,0635	0
	0,0096	0,6339	0,0089	0

Table 5.4 Simulation results according to *moneyness* and time to maturity

5.4. Put-option value estimation

After computing the value for the call option, it is possible to estimate the value for a put option under the assumptions that were previously used.

In short, one can easily apply the put-call parity to estimate this value. Another option is to use Matlab, given that it modelling the code is fairly straight forward taking the call options' code as a basis. Recall equation (3.3) for the put-call parity, considering that in this situation the underlying asset would be the EUA's spot price instead of a stock price, the put option's value would be given by:

$$p = c + Ke^{-rT} - S_0$$

The values obtained are displayed in Table 5.5.

		Strike		
		OTM	**ATM**	**ITM**
		5.706	**6.34**	**6.974**
Tau	**1**	0	0,0301	0,5691
	0,5	0	0,0338	0,6013
	0,0096	0	0,0083	0,6334

Table 5.5 Put option value estimation using put-call parity

Considering the theoretical statement underlying the put-call parity, the obtained values would reflect the absence of arbitrage opportunities, i.e. if options trading in the market, under the stated conditions, are priced differently then there are opportunities for sure profits.

5.5. Sensitivity analysis

Considering now the initial situation, that is pricing a call option using the programming code previously stated, one can access which variables other can have a greater impact in the options' price.

For this experimental purpose, let us consider separately some variations in terms of volatility and interest rate for the underlying asset[7]. Considering for the purpose of this experiment, upward and downward variation of 10% in terms of volatility and interest rate, the new underlying assumptions for the model would be those stated in Table 5.6.

[7] Usually traders use measures called Greeks, which represent measures of sensitivity for options according to the underlying assumptions. Generalizing when computing Greeks, the volatility sensitivity is called Vega, while the interest rate sensitivity is called Rho.

Variable	Value
$\boldsymbol{F(t;T_1)}$	6,34€
$\boldsymbol{\lambda}$	0 ; 0,5 ; 1
σ	3,213% ; 3,927%
$\boldsymbol{K}$	5,706€ ; 6,34€, 6.974€
$\boldsymbol{\tau = T_1 - t}$	1 ; 0.5 ; 0,0096
$\boldsymbol{r}$ **(6 months)**	0,854% ; 1,044%
$\boldsymbol{r}$ **(3 months)**	0,599% ; 0,732%
$\boldsymbol{r}$ **(1 week)**	0,321% ; 0,393%
nmax	10

Table 5.6 Summary of values used for modeling derivatives' contracts - sensitivity analysis

The results from the new simulations are displayed in Annex C – Results for call options' price on the sensitivity analysis, while the results for the respective sensitivities are displayed from Table 5.7 up toTable 5.10.

As it can be observed in Table 5.7 and in Table 5.8, there is some sensitivity for *at-the-money* options as these appear to have positive response to changes in the value of volatility. In this situation, the 10% increase/decrease in the volatility contributed for a 10% increase/decrease in the options' value at the beginning or in the middle of the maturity time, and for a 9% increase/decrease of the options' value, one week until the end of the contract. There is also some sensitivity observable in *out-of-the-money* options in the beginning of the contract ($\tau = 1$) with a variation of 100% with a 10% upward volatility, and -67% of variation with a 10% downward volatility. Despite that, it must be taken into account that these values, for *out-of-the-money* options, can be easily ignored due to the absolute value of such options, which is very close to 0.

In terms of interest rate sensitivity, it wasn't found any sensitivity to changes in this variable, fact that can be an indication of a greater influence of volatility changes for the price of the call options here valued.

		Strike		
		ITM	ATM	OTM
	1	0%	10%	100%
Tau	0,5	0%	10%	0%
	0,0096	0%	9%	0%

Table 5.7 Sensitivity with upward volatility

		Strike		
		ITM	ATM	OTM
	1	0%	-10%	-67%
Tau	0,5	0%	-10%	0%
	0,0096	0%	-10%	0%

Table 5.8 Sensitivity with downward volatility

		Strike		
		ITM	ATM	OTM
	1	0%	0%	0%
Tau	0,5	0%	0%	0%
	0,0096	0%	0%	0%

Table 5.9 Sensitivity with upward interest rate

		Strike		
		ITM	ATM	OTM
	1	0%	0%	0%
Tau	0,5	0%	0%	0%
	0,0096	0%	0%	0%

Table 5.10 Sensitivity with downward interest rate

6. Conclusion

This work intended to provide an insight on carbon markets, particularly the EU ETS, give some background, and test a model that can be used to value call options traded in this market.

Carbon markets and the EU ETS grew over the past few years becoming important both for managers and investors. Options and futures contracts traded in the EU ETS are relatively new products and their features and regulatory framework are, yet, being subject of study and development. Banking prohibition and allowance over allocation are policy issues that provide researchers a variety of economic and financial methods to study and model.

The EU ETS's contracts allow companies and countries to acquire allowances that represent an amount of carbon gases that can be released. Following the study of theoretical background, related both with carbon markets and financial derivatives, it was conducted a simulation for the price of a call option written on a futures' contract trading in the EU ETS. The used model was based on the BSM model with a jump process. Analysing the obtained results it was possible to conclude that variations in the options' price occurred mostly due to the *moneyness* degree and time to maturity.

Moreover it was derived the price for a put option under the same basic assumptions, using the put-call parity relationship.

Considering then the initial simulation, it was conducted a sensitivity analysis for the volatility and the interest rate, used to compute the first simulation. Results express a sensitivity of *at-the-money* call options' price to volatility, that is, with increases/decreases in volatility the value of *at-the-money* options' increases/decreases.

Regarding the discussed issue, there are several interesting opportunities for future research. It may prove to be interesting to assess on the impacts of new regulatory policies, adopted under *phase II* and *phase III* of the EU ETS, in the price movements of the derivatives traded. Other interesting research approach, already studied for *phase I* of the EU ETS, was banking prohibition effect in the correlation of the EU ETS with other commodities and equity indexes. Despite these suggestions, many other studies under the EU ETS and its derivatives can be object of study. Some authors have made important contributions so far, yet many specific issues in economics and finance are still available for study.

Bibliography

Asian Development Bank. (2011, Setember). *Kinoya Sewerage Treatment Project: first clean development mechanism initiative of its kind in Fiji*. Retrieved October 13, 2012, from Asian Development Bank: http://www.adb.org/publications/kinoya-sewerage-treatment-project-first-clean-development-mechanism-initiative-its-kind

Barnes, E. (2010, November 09). *Collapse of Chicago Climate Exchange Means a Strategy Shift on Global Warming Curbs.* Retrieved March 10, 2012, from Fox News: http://www.foxnews.com/politics/2010/11/09/collapse-chicago-climate-exchange-means-strategy-shift-global-warming-curbs/

Bataller, M. M., Tornero, Á. P., & i Micó, E. V. (2007). CO2 Prices, Energy and Weather. *The Energy Journal 28* , 73-92.

Benz, E., & Trück, S. (2006). CO2 Emission Allowances Trading in Europe – Specifying a New Class of Assets. *Problems and Perspectives in Management 4* , 30-40.

Benz, E., & Trück, S. (2009). Modeling the Price Dynamics of CO2 Emission Allowances. *Energy Economics 31* , 4-15.

Black, F. (1976). The Pricing of Commodity Contracts. *Journal of Financial Economics 3* , 167-179.

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy 81* , 637-654.

Chevallier, J. (2011, April 16). *Carbon Price Drivers: An Updated Literature Review.* Retrieved March 10, 2012, from Hyper Articles en Ligne (HAL): http://halshs.archives-ouvertes.fr/docs/00/58/65/13/PDF/chevallier_carbon_16_04_11.pdf

Chicago Climate Exchange. (2011). *Fact Sheet.* Chicago Climate Exchange.

Coase, R. (1960). The Problem of Social Cost. *Journal of Law and Economics 3* , 1-44.

Cootner, P. (1960). Returns to Speculators: Telser versus Keynes. *Journal of Political Economy* .

Crocker, T. D. (1966). The Structuring of Atmospheric Pollution Control Systems. In H. Wolozin, *The Economics of Air Pollution* (pp. 61-86). W.W. Norton & Co.

Dales, J. H. (1968). *Pollution, Property & Prices: An Essay in Policy-Making and Economics.* Toronto: University of Toronto Press.

Daskalakis, G., Psychoyios, D., & Markellos, R. (2009). Modeling CO2 Emission Allowance Prices and Derivatives: Evidence from the European Trading Scheme. *Journal of Banking & Finance 33* , 1230-1241.

Ellerman, D., Convery, F., & de Perthuis, C. (2010). *Pricing Carbon - The European Union Emissions Trading Scheme.* New York: Cambridge University Press.

European Commission. (1998). *Climate change - Towards an EU Post-Kyoto Strategy*. European Commission.

European Commission. (2011, January 25). *European Commission*. Retrieved December 03, 2011, from European Commission - Climate Action: http://ec.europa.eu/clima/policies/ets/index_en.htm

Fehr, M., Hinz, J., & Carmona, R. (2009). Optimal Stochastic Control and Carbon Price Formation. *Siam Journal on Control and Optimization 48* , 2139-2167.

Friedman, M. (2003). The Corporation. (M. Achbar, Interviewer)

Geman, H. (2005). *Commodities and Commodity Derivatives: Modeling and Pricing for Agriculturals, Metals and Energy*. Chichester, West Sussex: Wiley Finance.

Greenhouse Gas Reduction Scheme / Independent Pricing and Regulatory Tribunal. (2011). *Introduction to the Greenhouse Gas Reduction Scheme (GGAS).* Greenhouse Gas Reduction Scheme.

Gronewold, N. (2011, January 3). *Chicago Climate Exchange Closes Nation's First Cap-And-Trade System but Keeps Eye to the Future*. Retrieved March 10, 2012, from The New York Times: http://www.nytimes.com/cwire/2011/01/03/03climatewire-chicago-climate-exchange-closes-but-keeps-ey-78598.html?pagewanted=all

Grubb, M., & Neuhoff, K. (2006). Allocation and Competitiveness in the EU Emissions Trading Scheme: Policy Overview. *Climate Policy Special Issue 6* , 5-28.

Hull, J. (2005). *Options, Futures and Other Derivatives 5th Edition.* Upper Saddle River, New Jersey: Prentice Hall.

Kaldor, N. (1939). Speculation and Economic Stability. *The Review of Economic Studies 7* , 1-27.

Keynes, J. M. (1971). *A Treatise on Money.* London: Royal Economic Society.

Krukowska, E. (2010, October 18). *UN approves Russian Joint Implementation CO2 reduction project by E.ON*. Retrieved 10 13, 2012, from Bloomberg: http://www.bloomberg.com/news/2010-10-18/un-approves-russian-joint-implementation-co2-reduction-project-by-e-on.html

Lowrey, C. (2006). A Changing Environment. *FOW Energy, Spring* , 24-26.

Merton, R. (1976). Option Pricing when Underlying Stock Returns are Discontinous. *Journal of Financial Economics 3* , 125-144.

Merton, R. (1976). The Impact on Option Pricing of Specification Error in the Underlying Stock Returns. *The Journal of Finance 31* , 333-350.

Merton, R. (1973). Theory of Rational Option Pricing. *The Bell Journal of Economics and Management Science 4* , 141-183.

Montgomery, W. D. (1972). Markets in Licenses and Efficient Pollution Control Programs. *Journal of Economic Theory 5* , 395-418.

Paolella, M., & Taschini, L. (2006). An Econometric Analysis of Emission Trading Allowances. *Journal of Banking and Finance 32* , 2022-2032.

Rajan, R. G. (2005). Has Financial Development Made the World Riskier? *Jackson Hole Conference* (pp. 313 - 314). Kansas City: Federal Reserve Bank of Kansas City.

Regional Greenhouse Gas Initiative, Inc. (2010). *RGGI Fact Sheets.* Regional Greenhouse Gas Initiative (RGGI).

Samuelson, P. (1965). Proof that Properly Anticipated Prices Fluctuate Randomly. *Industrial Management Review 6 Spring* , 41-49.

Schofield, N. C. (2007). *Commodity Derivatives - Markets and Applications.* Chichester, West Sussex: John Wiley & Sons Ltd.

Seifert, J., Uhrig-Homburg, M., & Wagner, M. (2007). Dynamic Behavior of CO2 Spot Prices. *Journal of Environmental Economics and Management 56* , 180-194.

Telser, L. (1958). Futures Trading and the Storage of Cotton and Wheat. *Journal of Political Economy 66* , 233-255.

The City UK. (2011). *Financial Markets Series Carbon Markets - July 2011.* The City UK.

The Intercontinental Exchange. (2012). *ICE ECX EUA Futures Product Specifications*. Retrieved August 30, 2012, from The Intercontinental Exchange: https://www.theice.com/productguide/ProductSpec.shtml?specId=197#

The World Bank Group. (2012). *The World Bank*. Retrieved August 30, 2012, from The World Bank Data: http://data.worldbank.org/topic/environment

Triami Media. (October de 2012). *Euribor Rates* . Obtido em 23 de June de 2012, de Euribor-rates: http://www.euribor-rates.eu/

United Nations. (2012). *Kyoto Protocol to the United Nations Framework Convention on Climate Change.* Retrieved January 08, 2012, from United Nations: http://unfccc.int/essential_background/kyoto_protocol/items/1678.php

United Nations. (2012). *United Nations - Framework Convention on Climate Change*. Retrieved 08 27, 2012, from List of Annex I Parties to the Convention: http://unfccc.int/parties_and_observers/parties/annex_i/items/2774.php

Victor, D., & House, J. (2006). BP's Emissions Trading System. *Energy Policy 34* , 2100-2112.

Working, H. (1949). The Theory of the Price of Storage. *American Economic Review 36* , 1254-1262.

Annex A – List of Annex I parties to the Kyoto Protocol

Australia
Austria
Belarus
Belgium
Bulgaria
Canada
Croatia
Czech Republic
Denmark
Estonia
European Community
Finland
France
Germany
Greece
Hungary
Iceland
Ireland
Italy
Japan
Latvia
Liechtenstein
Lithuania
Luxembourg
Malta
Monaco
Netherlands
New Zealand
Norway
Poland
Portugal
Romania
Russian Federation
Slovakia
Slovenia
Spain
Sweden
Switzerland
Turkey
Ukraine
United Kingdom of Great Britain and Northern Ireland
United States of America

Annex B – Matlab programming code for EU ETS call options

```
function value = Daskalakis2(fut, strike, lambda_star, tau_one, sigma, rate,
nmax)

tic
InputMatrix = [fut, strike, lambda_star, tau_one, sigma, rate, nmax];
N = size(InputMatrix,1);
value = zeros(N,1);

for j = 1:N

  value(j,1) =
callJumpfun(fut(j,1),strike(j,1),lambda_star(j,1),tau_one(j,1),sigma(j,1),rate
(j,1),nmax(j,1));

end

value = [(1:N)' InputMatrix value];

toc

end



function y = NormalSDist(d)

y = 0.5*(1+erf(d/sqrt(2)));

end



function call = callfun(fut, strike, tau_one, sigma)

d_one = (log(fut/strike) + 0.5*sigma^2*tau_one)/(sigma*sqrt(tau_one));
d_two = d_one - sigma*sqrt(tau_one);

call = fut*NormalSDist(d_one) - strike*NormalSDist(d_two);

end
```

```
function callJump = callJumpfun(fut, strike, lambda_star, tau_one, sigma,
rate, nmax)

sum = 0;
n = 0;

sum = sum + (exp(-
lambda_star*tau_one)*(lambda_star*tau_one)^n)/factorial(n)*...
    callfun(fut, strike, tau_one, sigma);

for n = 1:nmax

    sum = sum + (exp(-
lambda_star*tau_one)*(lambda_star*tau_one)^n)/factorial(n)*...
          callfun(fut, strike, tau_one, sigma);

end

callJump = exp(-rate*tau_one)*sum;

end
```

Annex C – Results for call options' price on the sensitivity analysis

Upward volatility

		ITM	ATM	OTM
		5.706	**6.34**	**6.974**
Tau	**1**	0,6283	0,0984	0,0006
	0,5	0,6319	0,0700	0
	0,0096	0,6340	0,0097	0

Downward volatility

		ITM	ATM	OTM
		5.706	**6.34**	**6.974**
Tau	**1**	0,6281	0,0805	0,0001
	0,5	0,6319	0,0573	0
	0,0096	0,6340	0,0080	0

Upward interest rate

		ITM	ATM	OTM
		5.706	**6.34**	**6.974**
Tau	**1**	0,6275	0,0894	0,0003
	0,5	0,6317	0,0636	0
	0,0096	0,6340	0,0089	0

Downward interest rate

		ITM	ATM	OTM
		5.706	**6.34**	**6.974**
Tau	**1**	0,6287	0,0895	0,0003
	0,5	0,6321	0,0636	0
	0,0096	0,6340	0,0089	0