12th CONFERENCE ON APPLIED MATHEMATICS **APLIMAT 2013**
Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava

P R O C E E D I N G S

CHAOS IN HUMANITIES AND SOCIAL SCIENCES: AN APPROACH[1]

FILIPE José António (P), FERREIRA Manuel Alberto M. (P)

Abstract. Chaos theory has been first used in the applied and natural sciences and just then to the humanities and social sciences. Once introduced, chaos theory got rapidly a developing field. The idea of chaos has been then very significant both for empirical and for theoretical research not just in the applied and natural sciences but also in the humanities and social sciences. Being chaos extremely complex and difficult to be identified in the real world, even using the workable information, it is possible, anyway, up to a certain extent, to find specific mathematical relationships for problems to be solved. In this study some issues are presented in order to work this subject in the humanities and social sciences.

Keywords. Chaos theory, dynamical systems, politics, "butterfly effect", "drop of honey effect"

Mathematics Subject Classification: *MSC 2010:* 37N40

1   Introduction

In the 20th century, chaos theory got rapidly a developing field. Much of the progress in this subject was evident after the 1960s. Chaos, in the sense it is studied, is consequently yet a not well known field and is now distant from being completely understood or determined. Chaos is extremely complex and difficult to be identified in the real world, using the available information. However, up to a certain extent, it is possible to present specific mathematical relationships for problems to be solved in several areas. As soon as the idea of nonlinearity[2] was introduced into theoretical models, chaos got more obvious. A very complex structure is observed in the field data and just simple patterns can be found and approximated theoretically; complex patterns to be got through models are another matter. In any event, we cannot just grab a nice little set of data, apply a simple test or two, and declare "chaos" or "no chaos." (Williams, 1997). Chaos occurs as far as known in deterministic, nonlinear, dynamical systems.
The word "chaos" presumes the existence of turbulence and disorder. The predisposition to a profound change in the direction of a phenomenon generates an own force, understood as a deep change that results from small changes in their initial conditions. The chaos is - from this point of view - something extremely sensitive to the initial conditions. The sensitive dependence on initial

---

[1] A part of this material is developed in the chapter Ferreira, M. A. M., Filipe, J. A., Coelho, M., Pedro, M. I., "Chaos Theory in Politics: a Reflection. The 'Drop of Honey Effect", in Banerjee, S., Erçetin, S. S., Tekin, A. (Eds.), "Chaos Theory in Politics", Springer, Germany, forthcoming.
[2] Nonlinear means that output isn't directly proportional to input, or that a change in one variable doesn't produce a proportional change or reaction in the related variable(s).

conditions shows how a small change at one place or moment in a nonlinear system can result in large differences to a later state in the system.

The understanding of inherently nonlinear phenomena present in social phenomena shows that it is possible to use mathematical models in the analysis of the social environment and social issues. Moreover, when this does not happen, some kind of qualitative analysis is yet possible to perform by following the ideas of chaos theory.

The deterministic chaos present in many nonlinear systems can impose fundamental limitations on the human ability of predicting behaviors. Additionally, the exploration of a big number of conditions by a single deterministic result may create the possibility to have a prospective outcome in terms of adaptation and evolution. In the context of artificial life models this has led to the notion of "life at the edge of chaos" expressing the principle that a delicate balance of chaos and order is optimal for successful evolution (Campbell and Mayer-Kress, 1997). Nevertheless, the essence of life may conduct to specific situations that sometimes bring new situations creating a new order even considering extremely difficult situations.

2   Finding Chaos Theory in Some Socio-Political Situations

In social phenomena, chaos may be evidenced in many situations. Historically, simple facts with no visible significant consequences have registered considerable impacts that could not be predictable at the initial moment. Nowadays, such kind of situations continue to occur in many social contexts around the world.

The "Arabian Spring" is an example of the way how the "butterfly effect" can be found when causing a wide spread regional political reform in the political regimes of some countries in that geographical area. The "flapping of the butterfly wings" may be represented by the immolation by fire of a Tunisian salesman that was t[3]he starting point for the regime change in Tunisia first and then the contagion to Egypt and Libya. The consequences would be seen as well in Syria where a civil war is yet in course.

The well known "butterfly effect" could also be named as the "drop of honey effect", expression got from the wonderful tale written by the Armenian poet Hovanés Tumanian (1869-1923).

Presenting another example and considering the political situation in Greece there is a new stage to be studied for Greek, European and World economy. The political *status quo* has been broken in Greece: a new party has taken an advantage that it has never had. The emergent crisis in Greece that was felt severely after the Greece-*Troika* agreement. Throughout this Program, Greece has to respect an austerity program in order to put national budgeting at acceptable levels and is complied to obey the agreement that conducted Greek people to severe self well being sacrificing. This situation made Greeks to vote in favor of a new situation. All this new situation has imposed a new socio-economic condition to European Union and to the World that has threatened the world economic stability. The possible bankruptcy in Greece has tormented world leaders and a new *status quo* is being redefined for Europe with considerable implications for the world.

3   Chaos in Mathematical Terms

As Williams (1997) says, phenomena happen over time as at discrete, separate or distinct, intervals[4] or as continuously[5]. Discrete intervals can be spaced evenly in time or irregularly in time.

---

[3] It is about how an apparently insignificant and inoffensive drop of honey provokes a war.
[4] Examples are the occurrence of earthquakes, rainstorms or volcanic eruptions.

Continuous phenomena might be measured continuously. However, we can measure them at discrete intervals[6]. Special types of equations apply to each of those two ways in which phenomena happen over time. Equations for discrete time changes are difference equations and are solved by iteration, the most of the times, or analytically. In contrast, equations based on a continuous change (continuous measurements) are differential equations. The term "flow" often is associated to differential equations[7].

It follows a mathematical model that works the concepts of chaos theory and contributes to explain the possible presence of some effects based on the idea of chaos.

So, in Berliner (1992) it is referred that non-invertibility is required to observe chaos for one-dimensional dynamic systems. Additionally it is said "everywhere invertible maps in two or more dimensions can exhibit chaotic behavior". The study of strange attractors shows that in the long term, as time proceeds, the trajectories of systems may become trapped in certain bounded regions of the state space of the system.

The model presented in Berliner (1992) is an example in two dimensions of the Hénon map, displaying the property of having a strange attractor.

The Hénon map appears represented by the equations:

$$x_{t+1} = 1 + y_t - ax_t^2 \qquad (1)$$

And

$$y_{t+1} = bx_t, \qquad (2)$$

for fixed values of a and b with $t = 0, 1, \ldots$

This invertible map possesses strange attractors and simultaneously has strong sensitivity to initial conditions.

The Hénon map representing a transformation from R^2 to R^2 has Jacobian equal to $-b$.

If $0 < b < 1$, the Hénon map contracts the domains to which it is applied. These maps are said to be dissipative. On the contrary, maps that maintain the application domain are said to be conservative.

4   Modelling mathematically dissipative effect on politics

Considering the model in Berliner (1992), it is possible now to suggest a model on this basis for economics politics in the area of fisheries[8].

So, if a general situation is considered, the following equations may represent a system in which fish stocks, at time t, are given by x_t and catches by y_t. The model is as follows:

$$x_{t+1} = F(x_t) - y_t \quad \text{and} \quad y_{t+1} = bx_t. \qquad (3)$$

It is a generalization of Hénon model. The Jacobian is equal to b. As y_{t+1} is a portion of x_t, $0 < b < 1$. So, it is a dissipative model and the values of x_t are restricted to a bounded domain.

---

[5] Examples are air temperature and humidity or the flow of water in perennial rivers.

[6] For example, we may measure air temperature only once per hour, over many days or years.

[7] For some authors (see Bergé and Pomeau, 1984), a flow is a system of differential equations. For others (see Rasband, 1990), a flow is the solution of differential equations. Note that for the Navier–Stokes equations, that describe the motion of fluid substances, surprisingly, given their wide range of practical uses, mathematicians have not yet proven that in three dimensions solutions always exist, or that if they do exist, then they do not contain any singularity.

[8] And also, evidently, in the area of other reproducing and harvesting natural resources.

Considering the particular case below:

$$x_{t+1} = x_t - y_t \quad \text{and} \quad y_{t+1} = bx_t . \tag{4}$$

So,

$$x_{t+2} = x_{t+1} - y_{t+1} \quad \text{and} \quad x_{t+2} - x_{t+1} + bx_t = 0 \tag{5}$$

Now, after solving the characteristic equation associated to the difference equation (see Ferreira and Menezes, 1992) it is obtained:

$k = \dfrac{1 + \sqrt{1 - 4b}}{2}$ or $k = \dfrac{1 - \sqrt{1 - 4b}}{2}$; calling $\Delta = 1 - 4b$ and being $0 < b < 1$, comes that $-3 < \Delta < 1$.

So, $0 < \Delta < 1$ if $0 < b < \dfrac{1}{4}$ and $-3 < \Delta < 0$ if $\dfrac{1}{4} < b < 1$, being $\Delta = 0$ when $b = \dfrac{1}{4}$.

Consequently for $0 < b < \dfrac{1}{4}$,

$$x_t = A_1 \left(\frac{1 + \sqrt{1 - 4b}}{2} \right)^t + A_2 \left(\frac{1 - \sqrt{1 - 4b}}{2} \right)^t \tag{6}$$

And for $b = \dfrac{1}{4}$,

$$x_t = \left(A_1 + A_2 t \right) \left(\frac{1}{2} \right)^t \tag{7}$$

Finally, for $\dfrac{1}{4} < b < 1$

$$x_t = \left(\sqrt{b} \right)^t \left[A_1 \cos\left(\left(\arccos \frac{1}{2\sqrt{b}} \right) t \right) + A_2 \, sen\left(\left(\arccos \frac{1}{2\sqrt{b}} \right) t \right) \right] \tag{8}$$

In these solutions, A_1 and A_2 are real constants.

Note that the bases of t powers are always between 0 and 1. So, $\lim_{t \to \infty} x_t = 0$ and whatever the value

of b, the dissipative effect is real, even leading to the extinction of this species. Of course, this is evident according to the hypotheses of this particular situation of the model.

Concluding this approach, the general model does not allow to obtain in general such explicit solutions. But, of course, with simple computational tools it is possible to obtain recursively concrete time series solutions after establishing the initial value x_0 and to check the dissipative effect.

Additionally a new example may be presented for politics, in general, considering the political credibility.

Call x_t the political credibility, of a politician or of a party measured, for instance, in number of votes, or in the number of chamber's members, or even in money, in the year t ; and consider b the credibility rate, $-1 \le b \le 1$.

It is admissible that in the year $t+1$, $x_{t+1} = x_t + bx_t$, that is: in a certain year the political credibility is the one of the former year plus, or minus, a part of it. So:

$$x_{t+1} - (1+b)x_t = 0 \qquad (9)$$

Solving this difference equation (see Ferreira and Menezes, 1992) it is obtained[9]:

$$x_t = x_0(1+b)^t, b \neq 0 \quad \text{and} \quad x_t = x_0, b = 0. \qquad (10)$$

Then, according to this model, if the credibility rate is null the political credibility is kept unchanged, assuming always the initial value. If $0 < b \leq 1$, the political credibility follows an increasing exponential path. If $-1 < b < 0$, the political credibility follows a decreasing exponential path converging to 0. Finally, if $b = -1$, x_t is permanently null. Evidently, values like $-1 \leq b < 0$ define political credibility paths that may lead to people's chaotic behaviors.

5 Conclusions

Along this study it was shown that in politics, the possible existence of chaos may be evidenced for numerous situations. Historically, there are a lot of simple facts, considered insignificant in the moment for the consequences they had, that in a completely unexpected way gave raise to huge impacts that could not be predicted, or even guessed, at the initial moment of its occurrence. In fact, they are situations for which the output is not directly proportional to the input.

Nowadays, such kind of facts continues to occur in many socio-political contexts around the world. It is at least strange that the simple – despite the greatness of the personal sacrifice – immolation by fire of a Tunisian salesman was the starting point for the regime change in Tunisia first and then the contagion to Egypt and Libya. The consequences would then be seen as well in Syria where a bloody civil war is still in course.

In this paper two mathematical models with difference equations were presented. They contribute to identify possible chaotic situations, in politics, through the values of the models' parameters. The more accurate is the evaluation of these parameters, the more is the usefulness of each of the models.

Acknowledgement

References

[1] Berliner, L. M. (1992), Statistics, Probability and Chaos. Statistical Science, 7 (1), 69-122.

[2] Bjorndal, T. (1987), Production economics and optimal stock size in a North Atlantic fishery. Scandinavian Journal of Economics, 89 (2), 145-164.

[3] Bjorndal, T. and Conrad, J. (1987), The dynamics of an open access fishery. Canadian Journal of Economics, 20(1), 74-85.

---

[9]Evidently, this is the compound interest capitalization formula, at interest rate b used for financial purposes.

[4] Campbell, D. K., Mayer-Kress, G. (1997), Chaos and politics: Applications of nonlinear dynamics to socio-political issues. In Grebogi, C. and Yorke, J. A., The Impact of Chaos on Science and Society. United Nations University Press.

[5] Capra, F. (1996), The web of life: a new scientific understanding of living systems. New York : Anchor Books.

[6] Clark, C. W. (1974), Possible effects of schooling on the dynamics of exploited fish populations. Journal du Conseil Internatinal pour L'Exploration de la Mer, 36 (1), 7-14.

[7] Farazmand, A. (2003), Chaos and transformation theories: A theoretical analysis with implications for organization theory and public management. Public Organization, 3 (4), 339-372 December.

[8] Ferreira, M. A. M., Menezes, R. (1992), Equações com Diferenças – Aplicações em problemas de Finanças, Economia, Sociologia e Antropologia. Sílabo. Lisboa

[9] Ferreira, M. A. M., Filipe, J. A., Coelho, M., Pedro, M. I. (2010), Fishing Policies and the Contribution of Chaos Theory for Fisheries Management. International Conference on Technology and Business Management. Proceedings.

[10] Ferreira, M. A. M., Filipe, J. A., Coelho, M., Pedro, M. I. C. (2011), Chaos Effect in Fisheries Management. Journal of Economics and Engineering, 2 (1), 36-43.

[11] Ferreira, M. A. M., Filipe, J. A., Coelho, M., Pedro, M. I. C. (2011), Modelling the Dissipative Effect of Fisheries. China-USA Business Review, 10 (11), 1110-1114.

[12] Ferreira, M. A. M., Filipe, J. A., Coelho, M., Pedro, M. I. C. (2013), Managing Fisheries in Light of Complexity and Chaos Theories. In Banerjee, S. (2013), Chaos and Complexity Theory for Management: Nonlinear Dynamics.

[13] Filipe, J. A. (2006), O Drama dos Recursos Comuns. Um caso de aplicação da Teoria dos Jogos aos comuns da pesca. PhD thesis. Lisboa: ISCTE.

[14] Filipe, J. A., Coelho, M., Ferreira, M. A. M. (2005), Sistemas Dinâmicos, Caos e os Comuns da Pesca. Revista de Economia Global e Gestão. N.º 2/2005. Lisboa: ISCTE.

[15] Filipe, J. A., Ferreira, M. A. M., Coelho, M. (2007), O Drama dos Recursos Comuns nas Sociedades Actuais: à procura de soluções para os Ecossistemas em perigo. Edições Sílabo. Lisboa.

[16] Filipe, J. A., Ferreira, M. A. M., Coelho, M. (2008), The Relevance of Chaos Theory to Explain Problems of Overexploitation in Fisheries. Working Paper, WP/24/2008/DE/SOCIUS. ISEG. Lisboa.

[17] Filipe, J. A., Ferreira, M. A. M., Coelho, M., Pedro, M. I. C. (2009), Complexity, Theory of Chaos and Fishing. In Porath, D. and Bayer, A., "International Suplement" special "update". FH Mainz, University of Applied Sciences. Mainz, Germany.

[18] Filipe, J. A., Ferreira, Coelho, M., Pedro, M. I. C., (2010), Chaos, Anti-chaos and Resources: Dealing with Complexity. Aplimat-Journal of Applied Mathematics, 3 (2), 83-90.

[19] Filipe, J. A., Ferreira, M. A. M., Coelho, M., Pedro, M. I. (2010), Managing Complexity: a Problem of Chaos in Fisheries Policy. China-USA Business Review. David Publishing Company, 9 (3).

[20] Filipe, J. A., Ferreira, M. A. M., Coelho, M., Pedro, M. I., Andrade, M. (2010), Analysing Fisheries Management through Complexity and Chaos Theories Framework, Journal of Mathematics and Technology, 1(2).

[21] Galtung, J. (1975), Entropy and the general theory of peace. Peace: Research Education Action, Essays in Peace Research, 1, Copenhagen.

[22] Geyer, R. (2003), Europeanisation, Complexity, and the British Welfare State. Paper presented to the UACES/ESRC Study Group on The Europeanisation of British Politics and Policy-Making, Department of Politics, University of Sheffield, September 19, 2003.

[23] Grabinski, M. (2004), Is There Chaos in Management or Just Chaotic Management?. Complex Systems, intelligence and Modern Technology Applications. Paris.

[24] Grabinski, M. (2008), Chaos – Limitation or Even End of Supply Chain Management. High Speed Flow of Material, Information and Capital. Istanbul.

[25] Hastings, A., Hom, C. L., Ellner, S., Turchin, P., Godfray, H. C. J. (1993), Chaos in Ecology: Is Mother Nature a Strange Attractor?, Annual Review of Ecology and Systematics, 24 (1), 1-33.

[26] I Font, J. P. P., Régis, D. (2006), Chaos Theory and its Application in Political Science. (Draft), IPSA – AISP Congress, Fukuoka.

[27] Kauffman, S. (1993), The origins of order: self-organization and selection in evolution. New York: Oxford Univ. Press.

[28] Lansing, J. S. (2003), Complex adaptive systems. Annual Review Anthropology. http://www.ic.arizona.edu/lansing/GompAdSys.pdf.

[29] Lévêque, G. (2002), Ecologia: do ecossistema à biosfera. Lisboa: Instituto Piaget.

[30] Levin, S. (2003), Complex adaptive systems: exploring the known, the unknown and the unknowable. Bulletin of the American Mathematical Society, 40.

[31] Mangel, M., Clark, G. (1983), Uncertainty, search and information in fisheries. Journal du Conseil International pour L'Exploration de la Mer, 41.

[32] Maynard Smith, J. (1968), Mathematical Ideas in Biology. Cambridge: Cambridge University Press.

[33] Neher, P. (1990), Natural Resource Economics: Conservation and Exploitation. Cambridge: Cambridge University Press.

[34] Olsen, L. F., Degn. H. (1985), Chaos in biological systems. Quarterly Review of Biophysics, 18 (2).

[35] Peled, A. (2000), The New Sciences, Self-organization and Democracy. Democratization, 7 (2), 19-35.

[36] Prigogine, I. (1993), Les Lois du chaos. Paris: Flammarion.

[37] Prigogine, I., Nicolis, G. (1989), Exploring complexity : an introduction. New York: W.H. Freeman and Company.

[38] Prigogine, I., Stenglers, I. (1984), Order out of chaos. Boulder (CO, USA): New Science Library.

[39] Radu, M. (2000), Festina Lente: United States and Cuba after Castro. What the experience in Eastern Europe suggests. Probable realities and recommendations, Studies in Comparative International Development, 34 (4), Winter.

[40] Rasband, N. S. (1990), Chaotic dynamics of nonlinear systems. New York: John Wiley.

[41] Scones, I. (1999), New ecology and the social sciences: what prospects for a fruitful engagement?, Annual Review of Anthropology, 28, 479-507.

[42] Thrift, N. (2008), Non-Representational Theory, Routledge, New York, USA.

[43] Williams, G. P. (1997), Chaos Theory Tamed. Washington, D. C.: Joseph Henry Press.

Current address

José António Filipe, Professor Auxiliar
INSTITUTO UNIVERSITÁRIO DE LISBOA (ISCTE-IUL)
BRU - IUL
AV. DAS FORÇAS ARMADAS
1649-026 LISBOA, PORTUGAL
Telefone: + 351 21 790 34 08
Fax: + 351 21 790 39 41
E-mail: jose.filipe@iscte.pt

Manuel Alberto M. Ferreira, Professor Catedrático
INSTITUTO UNIVERSITÁRIO DE LISBOA (ISCTE-IUL)
BRU - IUL
AV. DAS FORÇAS ARMADAS
1649-026 LISBOA, PORTUGAL
Telefone: + 351 21 790 37 03
Fax: + 351 21 790 39 41
E-mail: manuel.ferreira@iscte.pt