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SOLVING LOGISTICS PROBLEMS USING M|G|∞ QUEUE SYSTEMS BUSY PERIOD

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Abstract. In the M|G|∞ queuing systems customers arrive according to a Poisson process at rate λ. Each of them receives immediately after its arrival a service whose length is a positive random variable with distribution function G(.) and mean value α. An important parameter of the system is the traffic intensity $\rho = \lambda\alpha$. The service of a customer is independent of the services of the other customers and of the arrival process. The busy period of a queuing system begins when a customer arrives there, finding it empty, and ends when a customer leaves the system letting it empty. During the busy period there is always at least one customer in the system. Therefore in a queuing system there is a sequence of idle and busy periods. For these systems with infinite servers the busy period length distribution is difficult to derive, except for a few exceptions. But formulae that allow the calculation of some of the busy period length parameters for the M|G|∞ queuing system are presented. These results can be applied in logistics (see, for instance, Ferreira [4,5] and Ferreira, Andrade and Filipe [9]). For instance, they can be applied to the failures which occur in the operation of an aircraft, shipping or trucking fleet. The customers are the failures. And their service time is the time that goes from the instant at which they occur till the one at which they are completely repaired. Here a busy period is a period in which there is at least one failure waiting for reparation or being repaired. The formulae referred allow the determination of measures of the system performance.

Key Words: M|G|∞, Busy Period, Failures

Mathematics Subject Classification: Primary 60E05, 60G07; Secondary 60K25.

1 Introduction

In the M|G|∞ queuing system (see, for example, Ferreira [2,6] and Ferreira and Andrade [7])
- The customers arrive according to a Poisson process at rate λ,
- Each of them receives a service whose length is a positive random variable with distribution function G(.) and mean value α. So

$$\alpha = \int_{0}^{\infty}[1 - G(t)]dt \qquad (1),$$

- There are infinite servers. So when a customer arrives it always finds a server available,
- The service of a customer is independent of the services of the other customers and of the arrival process.

An important parameter is the traffic intensity called ρ, being

$$\rho = \lambda\alpha \qquad (2)$$

It is obvious that in an M | G | ∞ queuing system there are neither losses nor waiting. In fact there is no queuing in the formal sense of the word.

For these systems it is not so important to study the population process as for other systems with losses or waiting. Generally it is much more interesting the study of some other processes as, for instance, the busy period.

The busy period of a queuing system begins when a customer arrives there, finding it empty, and ends when a customer leaves the system letting it empty. During the busy period there is always at least one customer in the system.

Therefore in a queuing system there is a sequence of idle and busy periods.

It will be shown in the next section that these concepts can be applied in logistics, particularly to the failures that occur in the operation of a fleet of aircraft, of shipping or of trucking.

The results related to the busy period length of the M | G | ∞ queuing system, that is a random variable, allow the evaluation of performance measures of the fleet. In consequence it is possible to identify ways of improving the performance of the fleet.

The theory will be illustrated with a very simple and short numerical example.

2 Results and applications

Let us call B the M | G | ∞ queuing system busy period length (see Ferreira and Andrade [8]).

The mean value of B, whatever is G(.), is given by Takács [11]

$$\mathrm{E}[B] = \frac{e^{\rho} - 1}{\lambda} \qquad (3)$$

But VAR [B], the variance of B, depends largely on the form of B. But Sathe [10] showed that

$$\lambda^{-2}\max\left[e^{2\rho} + e^{\rho}\ \rho^{2}\gamma_s^{2} - 2\rho e^{\rho} - 1;0\right] \le VAR[B] \le \lambda^{-2}\left(2e^{\rho}\left(\gamma_s^{2} + 1\right)\left(e^{\rho} - 1 - \rho\right) - \left(e^{\rho} - 1\right)^{2}\right) \qquad (4),$$

where γ_s is the variation coefficient of G(.). And, after (4), the bounds to SD [B] and the standard deviation of B can be very easily computed.

Being R(t) the mean number of busy periods that begin in [0,t] (being t = 0 the beginning of a busy period), see Ferreira [2],

$$e^{-\rho}(1+\lambda t) \leq R(t) \leq 1+\lambda t \tag{5}$$

Let us call N_B the mean number of the customers served during a busy period in the M|G|∞ queuing systems. After Ferreira [3],

- If G(.) is exponential

$$N_B^M = e^{\rho} \qquad (6),$$

- For any other distribution function

$$N_B \cong \frac{e^{\rho(\gamma_s^2+1)}\left(\rho\left(\gamma_s^2+1\right)+1\right)+\rho\left(\gamma_s^2+1\right)-1}{2\rho\left(\gamma_s^2+1\right)} \tag{7}$$

These results can be applied to logistics. They are applied, for instance, to the failures that occur in the operation of a fleet of aircraft, of shipping or of trucking. The customers are the failures. And its service time is the time that goes from the instant at which they occur till the one at which they are completely repaired. For examples of applications of this kind see, for instance, Carrillo [1]. So

- A busy period is a period, in which there is at least one failure waiting for a reparation or being repaired,
- An idle period is a period in which there are no failures present.

Here some simple expressions that allow computing the mean and bounds to the variance of the busy period were given. And also simple bounds to the mean number of busy periods that begin in a certain length of time. And finally, expressions to the mean number of failures that occur in a busy period were presented.

These formulae are very simple and of evident application. They only require the knowledge of α, λ, ρ and γ_s that are very easy to compute. The only problem is to test the hypothesis of that the failures occur according to a Poisson process.

Only to conclude note that, calling I (t) the idle period of the M|G|∞ queuing system distribution function,

$$I(t) = 1 - e^{-\lambda t} \qquad (8),$$

as it happens with any queue with arrival Poisson process. In this application it gives the probability of that the length of time with no failures is lesser or equal to t.

3 Examples

Suppose a fleet where the failures occur at a rate of 20 per year. So λ = 20/year. Suppose too that the mean time to repair a failure is 4 days (α = 4 day = (4/365) year). In consequence $\rho \cong 0.22$.

Possibly ρ maybe decreased to 0.11. It is enough to make λ = 10/year, for instance buying more vehicles and decreasing, in consequence, the use intensity of each one. Or making α = 2 day. For instance increasing the teams that repair the failures.

On other side, if nothing is changed, things can get worse and maybe ρ can increase to 0.44.

If it is supposed that the repair services times are exponential (a very frequent supposition for this kind of services), $\gamma_s=1$, and after (3), (4), (5) and (6), with $t=1$ year, being SD $[B]=\sqrt{Var[B]}$,

Table 1

ρ	**E[B]**	SD [B] (Lower Bound)	SD [B] (Upper Bound)	R (1) (Lower Bound)	R (1) (Upper Bound)	N_B^M
0.11	2.12 day	2.16 day	2.2 day	18	21	1.12
0.22	4.5 day	4.65 day	4.82 day	16	21	1.25
0.44	10 day	10.72 day	11.46 day	13	21	1.60

And it is possible to conclude that when ρ increases, less busy periods in one year occur, with more failures in each one, of course in mean values.

The mean of the length of the busy period and its dispersion increase with ρ too.

If it is supposed now that the repair service times are constant (D = deterministic), $\gamma_s = 0$, and after (3), (4) (in this case the lower bound is equal to the upper bound and so the real value of VAR[B] is got), (5) and (7), with $t = 1$ year

Table 2

ρ	**E[B]**	SD [B]	R (1) (Lower Bound)	R (1) (Upper Bound)	N_B^D
0.11	2.12 day	0.41 day	18	21	1.59
0.22	4.5 day	1.22 day	16	21	1.68
0.44	10 day	3.85 day	13	21	1.90

E [B] and the R (1) bounds are the same that in the former case, evidently. The behavior of the parameters with the increase of ρ is similar to the one of the exponential situation. But now the busy period length dispersion is much lesser and the mean value of failures in each busy period is greater.

4 Conclusion

Of course, in the operation of a fleet, one is interested in big idle periods and in little busy periods. And if these busy periods occur it is good that they are as rare as possible, with a short number of failures.

Knowing α, λ, ρ and γ_s the manager of the fleet can evaluate the conditions of the operation, namely:

- The mean length of a period with failures,
- The length dispersion of a period with failures,
- The mean number of periods with failures that will occur in a certain length if time,
- The mean number of failures that occur in a period with failures.

As the expressions depend only on a few parameters and are very simple they show very simple ways to improve the operation, although they may be hard to implement.

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