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CHAOS, ANTI-CHAOS AND RESOURCES: DEALING WITH COMPLEXITY

FILIPE José António (P), FERREIRA Manuel Alberto M. (P), COELHO M., (P), PEDRO Maria Isabel (P)

Abstract: Chaos is ordinarily disorder or confusion; scientifically it represents a disarray connection, but basically it involves much more than that. Change and time are closely linked and they are essential when considered together to understand the foundations of chaos theory. The theories of dynamic systems have been applied to numerous areas of knowledge. In the 80's, several exact sciences (physics, chemistry or biology, for example) and some social sciences (economics or management or even the sociology) still had their own objects of study and their own methods of analysis and each one of them was different from the others. The Science has been branched and specialized, so that each one uses to have its own world. Recently new forms of analysis, looking for an integrated study have emerged (Filipe, 1). It is the case of chaos theory, which is applied here to natural resources, in order to understand the complexity of natural phenomena following this perspective. Anti-chaos theory is also introduced to show how systems in nature often tend to self-organization.

Keywords: Chaos theory, anti-chaos theory, complexity, dynamical systems, complex adaptive systems

1 Introduction

Chaos theory and complexity theory itself reflect the phenomena that in many activities (such as fisheries) are translated into dynamic forms of analysis and reflect a very complex and widespread reality, specific of complex systems. That reality falls within a range of situations integrated in a broader context, which is expressed in the theory itself but also in terms of their own realities (fisheries, for example), dynamic, complex and often chaotic features in its essence.

The chaos theory stresses that the world does not necessarily work as a linear relationship with perfectly defined or with direct relations in terms of expected proportions between causes and effects. The chaos occurs when a system is very sensitive to the initial conditions. These initial conditions are the measured values for a given initial time. The presence of chaotic systems in nature seems to place a limit on our ability to apply physical deterministic laws to predict movements with any degree of certainty. Indeed, one of the most interesting subjects in the study of chaotic systems is the question of whether the presence of chaos may or may not produce ordered structures and patterns on a wider scale. In past times, the dynamic systems showed up completely unpredictable and the only ones who could aspire to be understood were those that were represented by linear relationships, which are not the rule.

Complexity science has the potential to strengthen many areas of several sciences. The use of complexity theory concepts is changing the focus of research in all scientific disciplines and is leading to important practical outcomes.

This century theoretical physics is coming out from the chaos revolution. The study of complexity is the way and the computer the main tool. Thermodynamics, as a vital part of theoretical physics, will be involved in the transformation. In this complexity analysis, Anti-chaos theory conducts to the understanding of how systems perform a self organization and a structured system.

2 Chaos, Complexity and Dynamical Systems

The chaos theory allows realizing the endless alternative ways leading to a new form or new ways that will be disclosed and that eventually emerge from the chaos as a new structure. The reality is a process in which structure and chaos rotate between form and deformation in an eternal cycle of death and renewal. Conditions of instability seem to be the rule and, in fact, a small inaccuracy in the departure conditions tends to grow to a huge scale. Basically, two insignificant changes in the initial conditions for the same system tend to end in two situations completely different. This situation is known as the "butterfly wing effect". A small movement of the wings of a butterfly can have huge consequences.

It is the microscopic turbulence having effects in a macroscopic scale - an effect called by Grabinski (3) as “hydrodynamics”. Mathematically, the "butterfly wing effect" corresponds to the effect of chaos, which can be expressed as follows.

Given the initial conditions

$$x_1, x_2, x_3, \ldots, x_N$$

it is possible to calculate the final condition given by

$$\textit{final result} = f\,(x_1, x_2, x_3, \ldots x_N)$$

If the initial conditions x_i have a margin of error (variation), the final result will be influenced by the existence of this margin. If these margins in x_i are as small as the margin of error in the final result, we have a non-chaotic situation. Otherwise if the margins of error in x_i are small but the final result has a big variation, there is a chaotic situation. Therefore, small variations in initial conditions can lead to a major effect in the final outcome. Sometimes small changes in x_i have exponential effects on the final result due to the passage of time.

This effect can be demonstrated mathematically[1] using the Lyapunov Exponent[2] (see Grabinski, 4). Given the initial value x_0 and being ε its arbitrarily small variation, we are conducted to an initial value between x_0 and $x_0+\varepsilon$. The general form of Lyapunov indicator is presented by

[1] Several statistics may indicate chaos and can express how chaotic a system is. One of the most important statistics to measure the magnitude of chaos is at present Lyapunov exponents. Other statistics could be presented such as the Kolmogorov-Sinai entropy or the mutual information or redundancy.

$$x_{n+1} = f(x_n)$$

that after N iterations leads to a value for x_N between

$$f^N(x_0) \qquad \text{and} \qquad f^N(x_{0+\varepsilon})$$

being the difference between these two values

$$f^N(x_{0+\varepsilon}) - f^N(x_0) \;\Xi\; \varepsilon . e^{N\lambda(x_0)}$$

where λ is a parameter depending on x_0.

Dividing both sides by the variation ε and assuming the limit $\varepsilon \to 0$, we have a differential quotient. Making its logarithm and assuming the limit $N \to \infty$, we get the final definition of the Lyapunov Exponent

$$\lambda(x_0) = \lim_{N\to\infty} \frac{1}{N} \log \left| \frac{d\, f^N(x_0)}{dx_0} \right|$$

and there is chaos when $\lambda > 0$.

Through this function, the chaos exists when arbitrary small variations in initial conditions grow exponentially with a positive exponent.

Grabinski also points that the nonlinearity is the main characteristic of a chaotic situation. Mathematically, the nonlinear functions to be considered chaotic should be based on variables with some resistance. The author also argues that it is not enough to describe the chaotic situations, such as turbulence, but it is necessary to find ways to better cope with the nonlinearity. A smooth flow of a river (non-chaotic) that can be described in quantities like the flow velocity can reach a chaotic behavior with variations of many situations. The best example is a waterfall where the speed of the flow reaches a certain point. In a smoothly flowing river it is easy to calculate or predict the flow velocity of the river at any point. However, to calculate it in a river with a waterfall, it is necessary to introduce chaos. In an attempt to make this calculation, man has focused on the construction of super computers that have shown to be useless due the infinity of factors that may cause turbulence in the flow of the river. Thus, the analysis of frequency on the change of flow's velocity is much more promising than the analysis of velocities themselves.

Moreover, Grabinski shows the situation in which there is chaos on a microscopic scale but not on a macroscopic scale - the hydrodynamics. An example is a glass of water resting on a table, a non-chaotic event. A slight disturbance on the table causes a small flow on a macroscopic level in the water. However, a microscopic observation reveals a great agitation of millions of molecules, a chaotic event. This is a situation where there is chaos on the microscopic scale but a smooth flow on the macroscopic scale.

Mathematically Grabinski presents hydrodynamics equations which combine the chaos theory with business situations. For that, he presents the value function of a company (*v*) depending on two variables, the revenue (*r*) and the number of employees (*n*). Its general form is

[2] A Lyapunov Exponent is a number that reflects the rate of divergence or convergence, averaged over the entire attractor, of two neighbouring phase space trajectories. Trajectories divergence or convergence have to follow an exponential law, for the exponent to be definable.

$$v(r,n) = v_0 + a_{10}.r + a_{01}.n + a_{11}.r.n + a_{20}.r^2 + a_{02}.n^2 + ...$$

being a_{ij} general parameters. For $n = 0$ (no employees) or $r = 0$ (no revenue) the company doesn't exist because the function value is equal to 0. So some terms of the function must be removed ($v_0 = a_{10} = a_{01} = a_{20} = a_{02} = ... = 0$). The general form comes as

$$v(r,n) = a_{11}.r.n + a_{21}.r^2.n + a_{12}.r.n^2 + a_{22}.r^2.n^2 + ...$$

Now, because of symmetry, *r* and *n* may be negative. A negative employee means that the employee is paying to work and negative revenue means that the company is paying the customer to consume. So the previous formula can lead to negative results if *r* and *n* change signs simultaneously. Only these terms are allowed for which the sums of the powers of *r* and *v* are even numbers. Thus the general expression of the equation is

$$v(r,n) = a_{11}.r.n + a_{22}.r^2.n^2 + ...$$

3 Complexity and Ecological Systems

It is usual to consider that the really complex systems could be the biological ones, particularly the systems in which people are present: human body, human groups, society itself or people culture.

Many scientists see today, with particular interest, the chaos theory as a way to explain environment. Therefore, the chaos theory stresses the fundamental laws of nature and natural processes and requires a course for a constant evolution and recreation of nature.

In biological area and in order to frame some methodological developments, it must be mentioned, first of all, that some characteristics associated with some species support strategic survival features that are exploited by the present theory. Its aim is to find the reasons and the way in which these strategies are developed and the resulting consequences. The species use their biological characteristics resulting from evolutionary ancient processes to establish defense strategies. It is particularly interesting to see the behavior of schooling species and the way they delineate a consistent strategy for the group and specie as a whole, which is self-organizing, an anti-chaos feature, and which can be understood according to a focus based on the systems properties.

The ecology where many things are random and uncertain, in which everything interacts with everything at the same time is, itself, a fertile area for a cross search to the world explanations (Filipe *et al*, 2).

Lansing (5) states that the initial phase of the research of nonlinear systems was based on the deterministic chaos, and it was later redirected to new outbreaks of research focusing on the systems properties, which are self-organizing, the so called anti-chaos. It also says that the study of complex adaptive systems, discussed in the context of non-linear dynamic systems, has become a major focus of interest resulting from the interdisciplinary research in the social sciences and the natural sciences.

The theory of systems in general represents the natural world as a series of reservoirs and streams governed by various feedback processes. However, the mathematical representations were ignoring the role of these adjustment processes.

The theory of complex adaptive systems, part of the theory of systems, has in specific account the diversity and heterogeneity of systems rather than representing them only by reservoirs. It explicitly considers the role of adaptation on the control of the dynamics and of the responses of these heterogeneous reservoirs. This theory allows ecologists to analyze the reasons inherent to the process at the lower levels of the organization that lead to patterns at higher levels of organization and ecosystems. The adaptive systems represent one of the means to understand how the organization is produced to a large scale and how it is controlled by processes that operate at lower levels of organization. According to Lansing (5), came to be a general idea involving physical and mathematical complexity that is hidden behind very simple systems.

Considering a system composed by many interactive parts, if it is sufficiently complex, it may not be practical or even not be possible to know the details of each interaction place. Moreover, the interactions can generate local non-linear effects that, often, it becomes impossible to find a solution even for simple systems. However, diverting us from causal forces that move the individual elements, if we focus on the system behavior as a whole we can highlight certain global behavior standards. However, these behavior standards may hide an associated cost: it can not be expected to understand the causes at the level of individual behavior.

Indeed, the systems do not match the simple decomposition of the whole into parts. Therefore do not correspond to the mere sum of the parts, as living systems are not the juxtaposition of molecules and atoms. Since the molecule to the biosphere, the whole is organized and each level of integration leads to properties that can not be analyzed only from mechanisms that have explanatory value in the lower levels of integration. This corresponds to the appearance of new features to the level of the set that does not exist at the level of the constituent elements. Lansing (5) believes that the adoption in the social sciences of the idea that complex global patterns can emerge with new properties from local interactions had a huge impact here.

The ecological systems are comparable to systems self-organized as they are open systems which arise far from thermodynamic equilibrium. On self-organized and self-regulated systems, the reciprocal interactions within the system between the structures and the processes contribute to the regulation of its dynamics and the maintenance of its organization; partly due to the phenomena of feedback (see Lévêque, 6). These systems seem to develop themselves in accordance with the properties referred to the anti-chaotic systems. Indeed, we have auto-regulated systems that channel different initial conditions for the same stage, instead of what is happening with chaotic systems, which are very sensitive to initial conditions (see Kauffman, 7). These systems would be relatively robust for a particular type of disturbance, to which the components of the system fit, creating a meta-stability that depends not only on the internal interactions within the system but also on external forces that can regulate and strengthen the internal factors of cohesion (see Lévêque, 6).

4 Some Notes about Fisheries

Some people see nature as not casual and unpredictable. The natural processes are complex and dynamic, and the causal relations and sequential patterns may extend so much in time that may seem to be non-periodical. The data appear as selected random works, disorderly, not causal in their connections and chaotic. In nature, for example, for live resources in the sea, the vision provided by nature leads to consider the fish stocks, time, the market and the various processes of fisheries management as likely to be continuously in imbalance rather than behave in a linear fashion and in a constant search for internal balance. It is this perspective that opens the way for the adoption of the chaos theory in fisheries. However, the models of chaos do not deny, for themselves, some of the linearity resulting from the application of usual bionomic models. What is considered is that there are no conditions to implement all significant variables in a predictive model. Moreover, in finding that a slight change in initial conditions caused by a component of the system may cause major changes and deep consequences in the system itself.

As it has been explained previously, the butterfly effect represents the more sensitive dependence on initial conditions in chaos theory. Small variations on initial conditions in a dynamical system may produce large variations in the system long term behavior.

So, the application of chaos theory to fisheries is considered essential, by many researchers. The chaos theory depends on a multitude of factors, all major (and in the prospect of this theory all very important at the outset) on the basis of the wide range of unpredictable effects that they can cause.

Given the emergence of new forms of predation, species got weaker and weaker because they are not prepared with mechanisms for effective protection for such situations. In fisheries there is a predator, man, with new fishing technologies who can completely destabilize the ecosystem. By using certain fisheries technologies, such as networks of siege, allowing the capture of all population individuals which are in a particular area of fishing, the fishers cause the breakdown of certain species, particularly the pelagic, the ones normally designated by schooling species.

To that extent, with small changes in ecosystems, this may cause the complete deterioration of stocks and the final collapse of ecosystems, which in extreme cases can lead to extinction. These species are concentrated in high density areas in small space. These are species that tend to live in large schools.

Usually, large schools allow the protection against large predators. The mathematical theory, which examines the relationship between schools and predators, due to Brock and Riffenburgh (see Clark, 8), indicates that the effectiveness of predators is a reverse function of the size of the school. Since the amount of fish that a predator can consume has a maximum average value, overcoming this limit, the growth of school means a reduction in the rate of consumption by the predator. Other aspects defensive for the school such as intimidation or confusing predators are also an evidence of greater effectiveness of schools.

However this type of behavior has allowed the development of very effective fishing techniques. With modern equipment for detecting schools (sonar, satellites, etc.) and with modern artificial fibers' networks (strong, easy to handle and quick placement), fishing can keep up advantageous for small stocks (Bjorndal, 9; Mangel and Clark, 10). As soon as schools become scarce, stocks become less protected. Moreover, the existence of these modern techniques prevents an effect of stock in the costs of businesses, as opposed to the so-called search fisheries, for which a fishery involves an action of demand and slow detection. Therefore, the existence of larger populations is essential for fishermen because it reduces the cost of their detection (Neher, 11). However, the easy detection by new technologies means that the costs are not anymore sensitive to the stock size (Bjorndal and Conrad, 12).

This can be extremely dangerous due to poor biotic potential of the species subject to this kind of pressure. The reproductive capacity requires a minimum value below which the extinction is inevitable (Filipe *et al*, 13). Since the efficiency of the school is proportional to its size, the losses due to the effects of predation are relatively high for low levels of stocks. This implies non-feedback in the relation stock-recruitment, which causes a break in the curves of income-effort, so that an infinitesimal increase on fishing effort leads to an unstable condition that can lead to its extinction (Filipe *et al*, 14).

Considering the fished value function of a company (v) depending on two variables, the fishing effort (r) and the fish stock (n), a simple model for fisheries, analogous to the presented in section 2, can be built

$$v(r,n) = v_0 + a_{10}.r + a_{01}.n + a_{11}.r.n + a_{20}.r^2 + a_{02}.n^2 + ...$$

being a_{ij} general parameters. Now it makes no sense to consider negative values for the variables. For $n = 0$ (no fish stock) or $r = 0$ (no fishing effort) the company fished value doesn't exist because the function value is equal to 0.

Consequently, $v_0 = a_{10} = a_{01} = a_{20} = a_{02} = ... = 0$ and now

$$v(r,n) = a_{11}.r.n + a_{21}.r^2.n + a_{12}.r.n^2 + a_{22}.r^2.n^2 + ...$$

5 Conclusion

Chaos theory got its own space among sciences and has become itself an outstanding science. However there is much left to be discovered. Anyway, many scientists consider that chaos theory is one of the most important developed sciences on the twentieth century.

Aspects of chaos are shown up everywhere around the world and chaos theory has changed the direction of science, studying chaotic systems and the way they work.

It is not possible to say yet if chaos theory may give solutions to problems that are posed by complex systems. Nevertheless, understanding the way chaos discusses the characteristics of complexity and analyzes open and closed systems and structures is an important matter of present discussion.

Finally, some words to say that anti-chaos theory is directed to research focusing on the systems properties, which are self-organizing. In nature, systems seem to look for a durable organization and stability. This may be seen, for example, in ecological systems (see schooling fish species, for example).

This work shows in fact how natural resources are complex and how complexity theory deals with ecological phenomena.

References

[1] FILIPE, J. A. (2006), *O Drama dos Recursos Comuns. Um caso de aplicação da Teoria dos Jogos aos comuns da pesca. PhD thesis*. Lisboa: ISCTE.

[2] FILIPE, J. A., COELHO, M. AND FERREIRA, M. A. M. (2005), *Sistemas Dinâmicos, Caos e os Comuns da Pesca*. Revista de Economia Global e Gestão. N.º 2/2005. Lisboa: ISCTE.

[3] GRABINSKI, M. (2004), *Is There Chaos in Management or Just Chaotic Management?*. Complex Systems, intelligence and Modern Technology Applications. Paris.

[4] GRABINSKI, M. (2008), *Chaos – Limitation or Even End of Supply Chain Management*. High Spees Flow of Material, Information and Capital. Istanbul.

[5] LANSING, J. S. (2003), *Complex adaptive systems*, Annual Review Anthropology. http://www.ic.arizona.edu/lansing/GompAdSys.pdf.
[6] LÉVÊQUE, G. (2002), *Ecologia: do ecossistema à biosfera*. Lisboa: Instituto Piaget.
[7] KAUFFMAN, S. (1993), *The origins of order: self-organization and selection in evolution*. New York: Oxford Univ. Press.
[8] CLARK, C. W. (1974), *Possible effects of schooling on the dynamics of exploited fish populations*, Journal du Conseil Internatinal pour L'Exploration de la Mer, 36(1).
[9] BJORNDAL, T. (1987), *Production economics and optimal stock size in a North Atlantic fishery*, Scandinavian Journal of Economics, 89(2).
[10] MANGEL, M. and CLARK, G. (1983), *Uncertainty, search and information in fisheries*, Journal du Conseil International pour L'Exploration de la Mer, 41.
[11] NEHER, P. (1990), *Natural Resource Economics: Conservation and Exploitation*. Cambridge: Cambridge University Press.
[12] BJORNDAL, T. and CONRAD, J. (1987), *The dynamics of an open access fishery*, Canadian Journal of Economics, 20(1).
[13] FILIPE, J. A., FERREIRA, M. A. M. and COELHO, M. (2008), *The Relevance of Chaos Theory to Explain Problems of Overexploitation in Fisheries*. Working Paper, WP/24/2008/DE/SOCIUS. Lisboa: ISEG.
[14] FILIPE, J. A., FERREIRA, M. A. M., COELHO, M. and PEDRO, M. I. C. (2009), *Complexity, Theory of Chaos and Fishing*". In Porath, D. and Bayer, A., „International Suplement" of the „update" of the scentific bi-annual magazine of FH Mainz, University of Applied Sciences. FH Mainz, University of Applied Sciences. Mainz.

Current address

José António Filipe, Professor Auxiliar
ISCTE - IUL – Lisbon University Institute
UNIDE – Unidade de Investigação e Desenvolvimento Empresarial
Av. Forças Armadas 1649-026 Lisboa, Portugal
Tel.+351 217 903 000, e-mail: jose.filipe@iscte.pt

Manuel Alberto M. Ferreira, Professor Catedrático
ISCTE – IUL – Lisbon University Institute
UNIDE – Unidade de Investigação e Desenvolvimento Empresarial
Av. Forças Armadas 1649-026 Lisboa, Portugal,
Tel. +351 217 903 000, e-mail: manuel.ferreira@iscte.pt

Manuel Coelho, Professor Auxiliar
SOCIUS/ISEG – Portugal, Rua do Quelhas, 6, 1200-781, LISBOA, Portugal
Phone: +(351) 213925800, Fax: +(351) 213922808, e-mail: coelho@iseg.utl.pt

Maria Isabel Pedro, Professor Auxiliar
CEGIST/IST - Portugal
Av. Rovisco Pais, LISBOA, Portugal
Phone: +(351) 214233267, Fax: +(351) 218417979, e-mail: ipedro@ist.utl.pt