

MODELLING THE CENTRAL BANK REPO RATE
IN A DYNAMIC GENERAL EQUILIBRIUM
FRAMEWORK

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Modelling the Central Bank Repo Rate in a Dynamic General Equilibrium Framework

Abstract

The present paper adds a central bank to an existing general equilibrium model with banking sector. In our model, the central bank lends reserves to commercial banks and charges its *repo* interest rate. We obtain the usual result of flexible price models that expansionary monetary policy has a negligible effect on real variables such as output, consumption and investment expenditure. However, the composition of total investment is significantly altered as investment by banks increases at the expense of investment by nonbank firms. This result is a consequence of our explicit modelling of the central bank *repo* rate.

Keywords: dynamic general equilibrium; monetary policy; central bank *repo* rate; composition of investment expenditure; price level determinacy.

JEL Classification: E13, E52.

List of Symbols

HOUSEHOLD VARIABLES	
c_t	household's consumption in real terms
l_t	leisure
n_t^s	household's supply of labour
B_{t+1}	household's debt at the beginning of period (t+1)
z_t^f	% of firm f that the household buys at beginning of period (t-1) and sells at beginning of period t
$z_t^{bank,l}$	% of bank l that the household buys at beginning of period (t-1) and sells at beginning of period t
CD_t	amount of checkable deposits the household decides to hold at the beginning of period t
λ_t	shadow price (marginal utility of consumption, in this model)
β	intertemporal discount factor

FIRM VARIABLES	
y_t	firm's output
k_t	firm's stock of capital
n_t^d	firm's labour demand
i_t	firm's investment
Π_t^f	nominal profits of firm f in period t
δ	per period rate of depreciation of the firm's capital stock
α	firm Cobb-Douglas parameter

	BANK VARIABLES
B_t^s	bank's nominal supply of credit at the beginning of period t
b_t^s	bank's real supply of credit at the beginning of period t
k_t^b	bank's capital stock in period t
n_t^b	bank labour demand in period t
i_t^b	bank's investment expenditure in period t
Π_t^{bank}	nominal profits of the typical bank in period t
δ_B	per period rate of depreciation of the bank's capital stock
γ	bank Cobb-Douglas parameter

	PRICE VARIABLES
P_t	price of physical output
W_t	nominal wage rate
R_t	nominal interest rate between the beginning of period t and the beginning of period (t+1)
R_t^{repo}	interest rate the central bank charges when lending to banks
Q_t^f	nominal price of firm f at beginning of period t (amount necessary to buy 100% of firm f)
$Q_t^{bank,l}$	nominal price of bank l at beginning of period t (amount necessary to buy 100% of bank l)

	SHOCK VARIABLES
A_t	firm's technological parameter
D_t	bank's technological parameter
RES_t	reserves
θ_t	currency ratio: (currency in circulation/money supply)
r_t^{req}	required reserve ratio

x	steady-state value of variable x_t
\hat{x}_t	the percentage deviation of variable x_t from its steady-state (in period t)
$E_t [\cdot]$	expectation conditional on information pertaining to the beginning of period t and earlier of the indicated argument
H	number of households
F	number of firms
L	number of banks

Nominal variables appear in capital letters. Lower case letters denote the real equivalent of a nominal variable. Thus, for example, $x_t = \frac{X_t}{P_t}$.

The per capita equivalent of variable X_t is denoted \bar{X}_t . For example, $\bar{k}_t = \frac{F}{H} k_t$ or $\bar{k}_t^b = \frac{L}{H} k_t^b$.

The inflation rate is denoted \tilde{p}_{t+1} . Thus, $1 + \tilde{p}_{t+1} = \frac{P_{t+1}}{P_t}$.

1 Introduction

In modern economies, monetary policy decisions correspond to changes in the central bank *repo* interest rate. Yet, neither traditional macroeconomic models nor dynamic general equilibrium models have so far included this variable. Of course it can be argued that the variable is included in an implicit way because a lower *repo*, for example, tends to be associated with higher liquidity and lower economy interest rates. However, this paper shows that explicit modelling of the central bank *repo* rate produces a result in terms of the composition of investment expenditure which is absent in previous models.

Leao (2003) extends the general equilibrium model of King, Plosser and Rebelo (1988) to explicitly include a banking sector. Here, we add a central bank that lends reserves to commercial banks and charges its *repo* interest rate. Unlike Christiano and Eichenbaum (1995), where cash injections from the central bank to the banking sector are costless lump sum transfers, in our model commercial banks have to pay interest on the amounts borrowed from the central bank. Note that the central bank *repo* rate corresponds in the eurozone to the “main refinancing rate of the European Central Bank” and has an approximate correspondence to what in the U.S. is the “target for the Federal Funds Rate”.

We have log-linearized the competitive equilibrium of our model around the steady-state values of its variables and then calibrated it using Postwar U.S. data. Afterwards, we examined the response of the model to monetary policy changes and other exogenous shocks.

In our general equilibrium model with central bank, increases in liquidity by the central bank are associated with a fall in interest rates. In order to induce commercial banks to hold more liquidity, the central bank must decrease its *repo* rate. This, in turn, allows banks to reduce their lending rate. *In the empirical literature*, the papers of Strongin (1995), Hamilton (1997) and other authors strongly support the existence of a negative relationship between liquidity and interest rates.

In our flexible price model, expansionary monetary policy makes the price level rise and has almost no impact on aggregate real variables such as output, consumption and total investment expenditure. However, explicit modelling of the central bank *repo* rate has the consequence that, unlike in previous models, expansionary monetary policy which is seen as temporary significantly changes the composition of investment expenditure. Specifically, a decrease in the central bank *repo* rate makes investment expenditure by commercial banks become so attractive that significant amounts of investment expenditure are switched from nonbank firms to banking firms. This effect does not appear in papers, such as Christiano and Eichenbaum (1995), that have included modelling of the relationship between the central bank and commercial banks.

It is important to underline that our results were obtained with a model which was built without imposing any special restrictions and calibrated with parameters which were *all* obtained using U.S. data.

The structure of the article is as follows. In section 2, we characterize the economic environment: preferences, technology, resource constraints, and market structure. In section 3, we describe the

typical bank's behaviour and its relation with the central bank. Sections 4 and 5 deal with the typical firm's behaviour and with the typical household's behaviour, respectively. In section 6, we write down the market clearing conditions. Section 7 includes the set of equations that describe the competitive equilibrium and section 8 reports the calibration of the model. In section 9, we look at the impact of expansionary monetary policy and other shocks - and then discuss the main results of the paper. Section 10 makes an overview and conclusion.

2 The Economic Environment

Our model is a closed economy model with no government. There are H homogeneous households, F homogeneous firms, L homogeneous commercial banks, and one central bank in this economy (in this paper, we shall refer to "nonbank firms" simply as "firms"; likewise, we shall refer to "commercial banks" simply as "banks"). There is only one physical good produced in this economy which we denote physical output. Output can either be consumed or used for investment (i.e., used to increase the capital stock). In our model, bank loans are the only source of money to the economy. Banks make loans to households and households then use the money obtained in this way to buy consumption goods from the firms. In performing their role of suppliers of credit, banks incur labour costs, capital costs, and interest costs. They hire people in the labour market and buy capital goods in the goods market. They also pay interest on the amount of reserves they borrow from the central bank.

We next examine the typical household's preferences, the technology available in the economy (production functions and capital accumulation equations), the resource constraints that exist in a given period, and the market structure. Let us suppose that we are at the beginning of period 0 and that households, firms and banks are considering decisions for periods t with $t = 0, 1, 2, 3, \dots$

The household's utility level in period t is given by $u(c_t, \ell_t)$, where c_t denotes consumption and ℓ_t is leisure. The function $u(\cdot, \cdot)$ has the usual properties. The household seeks to maximize lifetime utility given by $U_0 = E_0 \left[\sum_{t=0}^{t=\infty} \beta^t u(c_t, \ell_t) \right]$, where β is a discount factor ($0 < \beta < 1$).

Each firm's production function is described by $y_t = A_t F(k_t, n_t^d)$, where y_t is the output of the firm, A_t is a technological parameter, k_t is the firm's (pre-determined) capital stock, and n_t^d is the firm's labour demand. The firm's capital accumulation equation is $k_{t+1} = (1 - \delta)k_t + i_t$, where i_t is the flow of investment in period t and δ is the per-period rate of depreciation of the capital stock. For each bank, there is a production function which tells how much credit in real terms the bank is able to process for each combination of work hours hired and capital stock available. This technology, which is available to all banks, can be summarized by

$$b_t^s = D_t (k_t^b)^{1-\gamma} (n_t^b)^\gamma \quad (1)$$

where b_t^s is the bank's supply of credit in real terms, D_t is a technological parameter, k_t^b is the (pre-determined) capital stock of the bank, and n_t^b is the number of work hours hired by the bank. The typical bank's capital accumulation equation is $k_{t+1}^b = (1 - \delta_B)k_t^b + i_t^b$, where i_t^b is the flow

of investment by the bank in period t and δ_B is the per-period rate of depreciation of the bank's capital stock.

The resource constraints that exist in this economy are as follows. Each firm enters period t with a stock of capital, k_t , which was determined at the beginning of period $(t-1)$. In other words, the capital stock that will enter the production function in period t cannot be changed by decisions taken at the beginning of period t or during period t . Likewise, each bank enters period t with a capital stock, k_t^b , which is pre-determined. Each household has an endowment of time per period which is normalized to be equal to *one* by an appropriate choice of units. This endowment of time can be used to work or to rest. Therefore, we can write $n_t^s + \ell_t = 1$, where n_t^s is the household's labour supply during period t .

Let us now describe the market structure. There are six markets: the goods market, the labour market, the bank loans market, the market for firm shares, the market for bank shares, and a market where the central bank lends reserves to commercial banks. We assume that each household behaves as a price-taker, each firm behaves as a price-taker, and each bank also behaves as a price-taker. Prices are perfectly flexible and adjust so as to clear all markets in every period.

In each period, the complete description of monetary flows among economic agents in our model is as follows. At the beginning of the period, there are no notes and coins and no checkable deposits in the economy. Then, in the first instants of the period, the central bank lends reserves to banks. Banks need reserves because they must supply notes and coins through the cash-machine during the period and because they must comply with a required reserve ratio. Still at the beginning of the period, banks use the reserves to support loans to households. Each loan implies the creation of new checkable deposits (creation of money) which are a liability to the bank and an asset to the household. These checkable deposits become available at the household's bank account. Households borrow from the banks the amount that they need in order to buy consumption goods from the firms during the period. We assume that, for institutional reasons, part of the consumption expenditure must be paid using notes and coins. As a consequence, each household asks for a certain percentage of her checkable deposits to be converted into notes and coins. During the period, households spend their notes and coins and checkable deposits buying consumption goods from the firms. At the end of the period, households receive back from the firms these notes and coins and checkable deposits (as wage payments and dividend payments). Then, households pay to the banks interest on the amount borrowed at the beginning of the period. However, households immediately receive back from the banks the amount of interest they paid. They receive this amount in the following ways (see definition of bank profits in section 3): (i) banks use part of the interest received from households to pay interest to the central bank, and the central bank then transfers the amount received to the households by means of a lump-sum transfer; (ii) another portion of the interest received by the banks is used by them to pay the physical capital they obtained from the firms and firms then give that amount of money to households in the form of wages and dividends; (iii) another part is received by the households in the form of wages paid by the banks; (iv) finally, the remaining part is paid to the households in the form of bank dividends. We may summarize as

follows what has happened until now in the period (note that we are already at the period's end point). The typical household borrowed from banks the amount she needed to buy consumption goods, then she spent this money buying goods, and then she received it back from the firms as wages and dividends. Afterwards, she used part of this amount to pay interest to banks but then received it back according to (i), (ii), (iii) and (iv). So, at the moment, the household again possesses the principal she borrowed at the beginning of the period (the value of consumption) which she naturally uses now to repay the bank loan. As a consequence, the household is left with no money - the same situation she was in at the beginning of the period. Note that the household makes the payment to the bank *partly* using the notes and coins she had withdrawn through the cash-machine *partly* by debit of her checkable deposits account (by destroying the checkable deposits she has in her account). Let us now see the position banks are in at the moment. They have received back the notes and coins supplied and we also know that, since households are left with no money, banks now have zero checkable deposits on the liabilities side of their balance sheet. This means they no longer need to hold required reserves. Therefore, banks now have available exactly the amount they had borrowed from the central bank at the beginning of the period and they use this amount to repay that loan. The conclusion to be drawn is that, after all these payments, each commercial bank is left with no reserves and each household is left with zero money balances. Hence, at the start of the next period, each bank must again borrow reserves from the central bank and each household must again borrow from commercial banks. Therefore, the pattern of payments will be repeated in the next period and in every period ahead.

3 The Central Bank and the Behaviour of the Typical Commercial Bank

Compared to the model of Leao (2003), the model of the present paper has one extra market: the market where the central bank lends reserves to commercial banks and charges the *repo* rate. Reserves lent are credited into the account that each commercial bank has at the central bank. Commercial banks then use these reserves to support their loans to the private sector.

The central bank performs an open-market operation at the beginning of each period where it lends reserves for one period. The two reasons why commercial banks need to hold reserve accounts at the central bank are as follows. First, the loans they make to nonbank agents imply the creation of new checkable deposits. Because there is a required reserve ratio, the amount of checkable deposits in the liabilities side of their balance sheet must be partially covered with reserves held at the central bank. Second, banks need notes and coins because they must convert checkable deposits into notes and coins whenever households demand such conversions (through the cash-machine or at the bank's counter). Since the central bank is the only agent in modern economies who can print notes and coins, commercial banks have no alternative but to hold reserve accounts at the central bank which allow conversion into notes and coins.

The nominal supply of credit of the typical bank at the beginning of period t is denoted B_t^s and corresponds to the amount of checkable deposits that the bank creates at the beginning of period t . At the beginning of period t , households ask for a certain percentage (θ_t) of these checkable

deposits to be converted into notes and coins. Currency in circulation is therefore given by $\theta_t B_t^s$ and the amount of checkable deposits during period t will be $(1 - \theta_t) B_t^s$. If we denote the required reserve ratio by r_t^{req} , then the amount of required reserves is $r_t^{req}(1 - \theta_t) B_t^s$. Therefore, the total demand for reserves by the typical bank is given by

$$\theta_t B_t^s + r_t^{req}(1 - \theta_t) B_t^s = [\theta_t + r_t^{req}(1 - \theta_t)] B_t^s$$

This is the amount of reserves that will be demanded by each bank when it goes to the open-market operation that occurs at the beginning of the period.

In period t , the nominal profits of each bank are given by: interest income *minus* interest expense (interest paid to the central bank on the amount of reserves borrowed) *minus* wage payments to the bank's employees *minus* investment in physical capital made by the bank

$$\Pi_t^{bank} = R_t B_t^s - R_t^{repo} [\theta_t + r_t^{req}(1 - \theta_t)] B_t^s - W_t n_t^b - P_t [k_{t+1}^b - (1 - \delta_B) k_t^b] \quad (2)$$

where R_t is the interest rate charged by the bank for loans that start at the beginning of period t and end at the beginning of period $(t + 1)$, R_t^{repo} is the interest rate charged by the central bank, W_t is the nominal wage rate, and P_t is the price of physical output. We assume that the bank pays wages at the end of the period. The profits earned by each bank during period t are distributed to households (the shareholders) at the end of the period in the form of dividends. Using equation (1), the previous equation can be rewritten as

$$\begin{aligned} \Pi_t^{bank} = & [R_t - [\theta_t + r_t^{req}(1 - \theta_t)] R_t^{repo}] P_t D_t (k_t^b)^{1-\gamma} (n_t^b)^\gamma \\ & - W_t n_t^b - P_t [k_{t+1}^b - (1 - \delta_B) k_t^b] \end{aligned} \quad (3)$$

Each bank maximizes the Value of its Assets (VA), i.e., the expected discounted value of its stream of present and future dividends. Therefore, when we are at the beginning of period 0, the typical bank's optimization problem is

$$Max_{n_t^b, k_{t+1}^b} VA = E_0 \left[\sum_{t=0}^{\infty} \frac{1}{1+R_{0,t+1}} \Pi_t^{bank} \right]$$

where Π_t^{bank} is given by (3). Given the economic environment we are working with, we think it is appropriate to assume that $(1 + R_{0,t+1}) = (1 + R_0)(1 + R_1)(1 + R_2) \dots (1 + R_t)$. Note that we are at the beginning of period 0. Therefore, because dividends are only distributed at the end of the period, we discount period 0 dividends by multiplying them by $1/(1 + R_0)$, we discount period 1 dividends by multiplying them by $1/[(1 + R_0)(1 + R_1)]$, and so on.

4 The Typical Firm's Behaviour

In nominal terms, the profits of firm f in period t are given by: income from the sale of output *minus* the wage bill *minus* investment expenditure

$$\Pi_t^f = P_t A_t F(k_t, n_t^d) - W_t n_t^d - P_t [k_{t+1} - (1 - \delta) k_t] \quad (4)$$

The firm pays wages to households at the end of the period. The profits earned by each firm during period t are distributed to households (the shareholders) at the end of the period in the form of dividends. Each firm maximizes the Value of its Assets (VA). Therefore, when we are at the beginning of period 0, the typical firm's optimization problem is

$$\underset{n_t^d, k_{t+1}}{Max} VA = E_0 \left[\sum_{t=0}^{t=\infty} \frac{1}{1+R_{0,t+1}} \Pi_t^f \right]$$

where Π_t^f is given by (4).

5 The Typical Household's Behaviour

The way bank loans work in this model is as follows. We have mentioned that R_t denotes the interest rate between the beginning of period t and the beginning of period $(t+1)$. At the beginning of period t , the household borrows from banks the amount $\frac{B_{t+1}}{1+R_t}$. This means that the household receives $\frac{B_{t+1}}{1+R_t}$ monetary units at the beginning of period t and that she will have to pay $\frac{B_{t+1}}{1+R_t}(1+R_t) = B_{t+1}$ monetary units at the beginning of period $(t+1)$. Hence, B_{t+1} denotes the debt the household has at the beginning of period $(t+1)$.

Shares work as follows. Q_t^f is the nominal price that someone would have to pay to buy 100% of firm f at the beginning of period t . z_t^f is the percentage of firm f [i.e., the share of firm f] that the household bought at the beginning of period $(t-1)$ and sells at the beginning of period t . z_{t+1}^f is the percentage of firm f that the household buys at the beginning of period t . These percentages are measured as a number belonging to the closed interval $[0,1]$. Therefore, $z_t^f Q_t^f$ is the nominal value of the shares of firm f that the household sells at the beginning of period t . On the other hand, $z_{t+1}^f Q_t^f$ is the nominal amount that the household spends buying shares of firm f at the beginning of period t . The shares of banks work in the same way: $Q_t^{bank,l}$ is the nominal price that someone would have to pay to buy 100% of bank l at the beginning of period t . $z_t^{bank,l}$ is the percentage of bank l that the household bought at the beginning of period $(t-1)$ and sells at the beginning of period t . $z_{t+1}^{bank,l}$ is the percentage of bank l that the household buys at the beginning of period t .

In the real world, the interest earned by the central bank by lending to commercial banks is used for many purposes, including financing economic research. In this model, we assume that the interest income of the central bank is transferred to households at the end of the period by means of a lump-sum transfer.

In nominal terms, the typical household's budget constraint for period t is as follows

$$\begin{aligned} W_{t-1} n_{t-1}^s + \sum_{f=1}^{f=F} z_t^f \Pi_{t-1}^f + \sum_{f=1}^{f=F} z_t^f Q_t^f + \sum_{l=1}^{l=L} z_t^{bank,l} \Pi_{t-1}^{bank,l} + \sum_{l=1}^{l=L} z_t^{bank,l} Q_t^{bank,l} + \frac{B_{t+1}}{1+R_t} + \\ + (1/H) LR_{t-1}^{repo} [\theta_{t-1} + r_{t-1}^{req}(1 - \theta_{t-1})] B_{t-1}^s = B_t + P_t c_t + \sum_{f=1}^{f=F} z_{t+1}^f Q_t^f + \sum_{l=1}^{l=L} z_{t+1}^{bank,l} Q_t^{bank,l} \end{aligned} \quad (5)$$

This equation simply states that the total amount of money the household has at the beginning of period t [wage earnings, dividend earnings from firms, money received from selling the shares of firms bought at the beginning of period $(t-1)$, dividend earnings from banks, money received from

selling the shares of banks bought at the beginning of period (t-1), the amount she borrows from the banks at the beginning of period t, and the lump-sum transfer received from the central bank] must be equal to the amount the household spends at the beginning or during period t [payment of the debt contracted from banks at the beginning of period (t-1), consumption expenditure during period t, purchase of shares of firms at the beginning of period t, and purchase of shares of banks at the beginning of period t]. In section 5 of Leao (2003) it is possible to see how this budget constraint can be derived from the combination of a portfolio allocation constraint and a cash-in-advance constraint [using the approach of Lucas (1982)]. Let us now normalize the household's budget constraint (equation 5). We can do this by dividing both sides of the constraint by P_t , rearranging, and then denoting real variables using lower case letters (so that, for example, $b_t^s = \frac{B_t^s}{P_t}$) and denoting the inflation rate by \tilde{p}_{t+1} . We obtain

$$\begin{aligned} & \frac{w_{t-1}}{1 + \tilde{p}_t} n_{t-1}^s + \sum_{f=1}^{f=F} z_t^f \frac{\pi_{t-1}^f}{1 + \tilde{p}_t} + \sum_{f=1}^{f=F} z_t^f q_t^f + \sum_{l=1}^{l=L} z_t^{bank,l} \frac{\pi_{t-1}^{bank,l}}{1 + \tilde{p}_t} + \sum_{l=1}^{l=L} z_t^{bank,l} q_t^{bank,l} + \frac{b_{t+1}}{1 + R_t} + \\ & + (L/H) R_{t-1}^{repo} [\theta_{t-1} + r_{t-1}^{req} (1 - \theta_{t-1})] \frac{b_{t-1}^s}{1 + \tilde{p}_t} = \frac{b_t}{1 + \tilde{p}_t} + c_t + \sum_{f=1}^{f=F} z_{t+1}^f q_t^f + \sum_{l=1}^{l=L} z_{t+1}^{bank,l} q_t^{bank,l} \quad (6) \end{aligned}$$

We use the following initial condition concerning the household's debt position at the beginning of period 0

$$\begin{aligned} B_0 = & W_{-1} n_{-1}^s + \sum_{f=1}^{f=F} z_0^f \Pi_{-1}^f + \sum_{l=1}^{l=L} z_0^{bank,l} \Pi_{-1}^{bank,l} + \\ & + (1/H) L R_{-1}^{repo} [\theta_{-1} + r_{-1}^{req} (1 - \theta_{-1})] B_{-1}^s \quad (7) \end{aligned}$$

This initial condition simply states that the household begins period 0 with a debt which equals the sum of the wage earnings she receives at the beginning of period 0, dividend earnings from firms she receives at the beginning of period 0, dividend earnings from banks she receives at the beginning of period 0, and lump-sum transfer from the central bank she receives at the beginning of period 0. She receives these amounts because of the hours she worked during period (-1), because of the shares of firms and of banks she bought at the beginning of period (-1), and because of the central bank transfer. Note that period 0 is not the period where the household's life starts but rather the period where our analysis of the economy begins (the household has been living for some periods and we catch her in period 0 and try to model her behaviour). In the Appendix, we show that this initial condition is the initial condition that naturally arises when we think back to the initial moment of a closed economy without government and where firms don't borrow. This initial condition (equation 7) can also be normalized by dividing both sides by P_0 giving

$$\begin{aligned} \frac{b_0}{1 + \tilde{p}_0} = & \frac{w_{-1}}{1 + \tilde{p}_0} n_{-1}^s + \sum_{f=1}^{f=F} z_0^f \frac{\pi_{-1}^f}{1 + \tilde{p}_0} + \sum_{l=1}^{l=L} z_0^{bank,l} \frac{\pi_{-1}^{bank,l}}{1 + \tilde{p}_0} \\ & + (L/H) R_{-1}^{repo} [\theta_{-1} + r_{-1}^{req} (1 - \theta_{-1})] \frac{b_{-1}^s}{1 + \tilde{p}_0} \quad (8) \end{aligned}$$

Consequently, at the beginning of period 0, the household is looking into the future and maximizes $U_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t) \right]$ subject to (6), (8) and $n_t^s + \ell_t = 1$. The choice variables are c_t , n_t^s , b_{t+1} , z_{t+1}^f , $z_{t+1}^{bank,l}$ and ℓ_t . There are also initial conditions on holdings of shares [assuming market clearing in the shares market in period (-1), these initial conditions will be $z_0^f = \frac{1}{H}$ and $z_0^{bank,l} = \frac{1}{H}$], a standard transversality condition on the pattern of borrowing, and non-negativity constraints.

6 The Market Clearing Conditions

With H homogeneous households, F homogeneous firms and L homogeneous banks, the market clearing conditions for period t are as follows. In the goods market, the condition is $Hc_t + Fi_t + Li_t^b = Fy_t$. In the labour market, the condition is $Hn_t^s = Fn_t^d + Ln_t^b$. In the bank loans market, the condition is $H \frac{B_{t+1}}{1+R_t} = LB_t^s$. The market clearing condition in the shares market is that each firm and each bank should be completely held by the households (the only owners of shares in this model). Since households are all alike, each household will hold an equal share of each firm and an equal share of each bank. Therefore, the market clearing conditions in the shares market in period t are $H z_{t+1}^f = 1$ and $H z_{t+1}^{bank,l} = 1$. Finally, the market clearing condition in the reserves market is $L[\theta_t + r_t^{req}(1 - \theta_t)] B_t^s = RES_t$, where RES_t denotes the supply of reserves by the central bank. Note that this market clearing condition in the reserves market embodies the idea of a money multiplier. In fact, it can be rewritten as

$$LB_t^s = \frac{1}{[\theta_t + r_t^{req}(1 - \theta_t)]} RES_t \quad (9)$$

LB_t^s is the amount of credit supplied by the L banks and, because the only source of money in this economy is bank loans, it also corresponds to the money supply of this economy. Therefore, equation (9) means that the money supply of this economy is equal to a multiple of the amount of reserves supplied by the central bank.

7 The Competitive Equilibrium

To obtain the system that describes the competitive equilibrium, we have put together in a system the typical household's first order conditions, the typical firm's first order conditions, the typical bank's first order conditions, and the market clearing conditions. We then assumed Rational Expectations. After all these steps, and if we also assume that the production function of each firm is homogeneous of degree one and denote the per capita counterpart of variable X_t by \bar{X}_t , then we can write the system describing the Competitive Equilibrium with H homogeneous households, F homogeneous firms and L homogeneous banks plus Rational Expectations as

$$u_1(c_t, 1 - n_t^s) = \lambda_t \quad (10)$$

$$u_2(c_t, 1 - n_t^s) = E_t \left[\beta \lambda_{t+1} \frac{w_t}{1 + \tilde{p}_{t+1}} \right] \quad (11)$$

$$\frac{\lambda_t}{1 + R_t} = E_t \left[\beta \lambda_{t+1} \frac{1}{1 + \tilde{p}_{t+1}} \right] \quad (12)$$

$$\lambda_t \bar{q}_t^f = E_t \left[\beta \lambda_{t+1} \left(\frac{\bar{\pi}_t^f}{1 + \tilde{p}_{t+1}} + \bar{q}_{t+1}^f \right) \right] \quad (13)$$

$$\lambda_t \bar{q}_t^{bank,l} = E_t \left[\beta \lambda_{t+1} \left(\frac{\bar{\pi}_t^{bank,l}}{1 + \tilde{p}_{t+1}} + \bar{q}_{t+1}^{bank,l} \right) \right] \quad (14)$$

$$\frac{b_{t+1}}{1 + R_t} = c_t \quad (15)$$

$$A_t F_2(\bar{k}_t, \bar{n}_t^d) = w_t \quad (16)$$

$$E_t \left[\frac{P_{t+1}}{1 + R_{t+1}} [A_{t+1} F_1(\bar{k}_{t+1}, \bar{n}_{t+1}^d) + (1 - \delta)] \right] = P_t \quad (17)$$

$$[R_t - [\theta_t + r_t^{req}(1 - \theta_t)] R_t^{repo}] \gamma D_t \left(\bar{k}_t^b \right)^{1-\gamma} (\bar{n}_t^b)^{\gamma-1} = w_t \quad (18)$$

$$E_t \left[\frac{P_{t+1}}{1 + R_{t+1}} \left[[R_{t+1} - [\theta_{t+1} + r_{t+1}^{req}(1 - \theta_{t+1})] R_{t+1}^{repo}] (1 - \gamma) D_{t+1} \left(\bar{k}_{t+1}^b \right)^{-\gamma} (\bar{n}_{t+1}^b)^{\gamma} + (1 - \delta_B) \right] \right] = P_t \quad (19)$$

$$c_t + [\bar{k}_{t+1} - (1 - \delta)\bar{k}_t] + [\bar{k}_{t+1}^b - (1 - \delta_B)\bar{k}_t^b] = A_t F(\bar{k}_t, \bar{n}_t^d) \quad (20)$$

$$n_t^s = \bar{n}_t^d + \bar{n}_t^b \quad (21)$$

$$\frac{b_{t+1}}{1 + R_t} = D_t \left(\bar{k}_t^b \right)^{1-\gamma} (\bar{n}_t^b)^{\gamma} \quad (22)$$

$$z_{t+1}^f = \frac{1}{H} \quad (23)$$

$$z_{t+1}^{bank,l} = \frac{1}{H} \quad (24)$$

$$H [\theta_t + r_t^{req}(1 - \theta_t)] D_t \left(\bar{k}_t^b \right)^{1-\gamma} (\bar{n}_t^b)^{\gamma} = \frac{RES_t}{P_t} \quad (25)$$

$$\tilde{p}_{t+1} = \frac{P_{t+1}}{P_t} - 1 \quad (26)$$

$$\bar{\pi}_t^f = A_t F(\bar{k}_t, \bar{n}_t^d) - w_t \bar{n}_t^d - [\bar{k}_{t+1} - (1 - \delta)\bar{k}_t] \quad (27)$$

$$\bar{\pi}_t^{bank,l} = [R_t - [\theta_t + r_t^{req}(1 - \theta_t)] R_t^{repo}] D_t (\bar{k}_t^b)^{1-\gamma} (\bar{n}_t^b)^\gamma - w_t \bar{n}_t^b - [\bar{k}_{t+1}^b - (1 - \delta_B) \bar{k}_t^b] \quad (28)$$

for $t = 0, 1, 2, 3, \dots$

Equations 10-15 have their origin in the typical household's first order conditions. Equation (15) is the credit-in-advance constraint which results from combining the household's budget constraint with the household's initial debt condition and then using the market clearing conditions from the shares' market. In the appendix of Leao (2003), it is possible to see how this credit-in-advance constraint appears in period 0 and how, under Rational Expectations, it is propagated into future periods. Equations (16) and (17) have their origin in the typical firm's first-order conditions. Equations (18) and (19) have their origin in the typical bank's first-order conditions. Note that the first order conditions of the typical bank are affected by the central bank *repo* rate. Equations 20-25 have their origin in the market clearing conditions. Equation (26) is the definition of rate of inflation. Equation (27) results from multiplying the definition of firm profits in real terms by (F/H) . Equation (28) results from multiplying the definition of bank profits in real terms by (L/H) . We have 5 exogenous variables (A_t , D_t , RES_t , θ_t and r_t^{req}) and 19 endogenous variables. It is important to point out that the model works even if r_t^{req} is set to zero (if we set $r_t^{req} = 0$, then we still have a perfectly defined system). This happens because the only difference between the case $r_t^{req} = 0$ and the case $0 < r_t^{req} \leq 1$ is that with $r_t^{req} = 0$ the typical bank has one reason to demand reserves from the central bank (the need to supply notes and coins to its customers) whereas with $0 < r_t^{req} \leq 1$ the typical bank has two reasons to demand reserves from the central bank (the need to supply notes and coins to its customers *and* the need to fulfill required reserves). The case $r_t^{req} = 0$ is relevant because there are some countries which actually have $r_t^{req} = 0$. The firm's production function and household's utility function used in the simulations below were $A_t F(k_t, n_t^d) = A_t (k_t)^{1-\alpha} (n_t^d)^\alpha$ and $u(c_t, \ell_t) = \ln c_t + \phi \ln \ell_t$.

8 Calibration

In order to study the dynamic properties of the model, we have log-linearized each of the equations in the system 10-28 around the steady-state values of the variables we have in that system of equations. The log-linearized system was then calibrated with the following parameters. With the specific utility function we are using, we obtain

Elasticity of the MU of consumption with respect to consumption	-1
Elasticity of the MU of consumption with respect to leisure	0
Elasticity of the MU of leisure with respect to consumption	0
Elasticity of the MU of leisure with respect to leisure	-1

where MU denotes "Marginal Utility". From the U.S. data, we obtain

Firms' investment as a % of total expenditure in the s.s. (Fi/Fy)	0.167	Barro (1993)
Firm workers' share of the output of the firm (α)	0.58	King et al. (1988)
Labour supply in the steady-state (n^s)	0.2	King et al. (1988)
Real interest rate in the steady-state (r)	0.00706	FRED and Barro
Banks' share of total hours of work in s.s. (Ln^b/Hn^s)	0.014	BLS
Bank workers' share of the bank's income (γ)	0.271	FDIC
Banks' investment as % of total expenditure in s.s. (Li^b/Fy)	0.00242	BEA
Bank's investment as a % of its income (i^b/RB^s)	0.107	BEA and FDIC

Note that the value used to calibrate the real interest rate is a quarterly value. Note also that, because in our model steady state inflation is zero, the *quarterly* steady state nominal interest rate is equal to the *quarterly* steady state real interest rate (0.00706). The parameters above already existed in the paper of Leao (2003). The data used to calibrate them are described in section 8 of his paper. The parameters which are new to the model of the present paper were calibrated as follows

Ratio (currency in circulation/total money supply) in the steady state (θ)	0.0678	FRED
Steady state value of the required reserve ratio (r^{req})	0.0383	FRED
Steady state value of the central bank <i>repo</i> rate (R^{repo})	0.004525	FRED

As usual, we have taken the post-war average of each variable as representing the steady state value. In our model, the total amount of money corresponds to the checkable deposits that are created by banks at the beginning of each period in the loan supplying process. These checkable deposits are then partially converted into notes and coins. Therefore, the exogenous variable θ_t corresponds to the ratio (currency in circulation/total money supply). To calibrate the steady state value of this ratio, denoted θ , we used data from the Federal Reserve Economic Data (FRED) to compute its average value in the period 1959:01-1986:12. To calibrate the steady state value of the required reserve ratio, r^{req} , we also used data from the FRED and for the same period. As a proxy to calibrate the steady state value of the central bank *repo* rate, R^{repo} , we used Federal Funds Rate data from the FRED for the period 1955-1986. Because in our model we have zero steady state inflation, we have subtracted the average *quarterly* inflation rate in the period 1955-1986 from the average *quarterly* nominal Fed Funds Rate in the same period.

The values in the preceding tables imply

Consumption share of total expenditure in the s.s. (Hc/Fy)	0.833
Household's discount factor (β)	0.993
Per-quarter rate of depreciation of the firm's capital stock (δ)	0.0047
Per-quarter rate of depreciation of the bank's capital stock (δ_B)	0.0012

The simulation results we present below are robust to changes in the parameters. In other words, the qualitative nature of our results does not change when we make reasonable modifications in the parameters.

9 Response of the Model to Exogenous Shocks

The response of the log-linearized model to shocks in the exogenous variables (A_t , D_t , RES_t , θ_t and r_t^{req}) can be obtained using the King, Plosser and Rebelo (1988) method, which is based on Blanchard and Khan (1980). We start by looking at the impact of shocks in the firms' technological parameter (A_t), the standard RBC exercise, just to make sure the model is capable of reproducing the usual RBC results. We then look at the impact of shocks in the banks' technological parameter (D_t) in order to confirm that the results of Leao (2003) are preserved with our extension of his model. Then, we turn to the main focus of the paper by examining the response of the model to an increase in central bank liquidity (increase in RES_t). Finally, we look at the response of the model to shocks in the currency ratio (θ_t) and in the required reserve ratio (r_t^{req}). Note that, in all figures of this paper, 0.01 on the vertical axis corresponds to 1%.

9.1 Technological innovations in firms

As can be seen in figures 1 to 8, our model reproduces the key results obtained by the RBC model of King, Plosser and Rebelo (1988). First, consumption, investment and work hours are procyclical. Second, consumption is less volatile than output and investment is more volatile than output. To perform these exercises, we made the usual assumption in the literature that the firms technological parameter evolves according to $\hat{A}_t = 0.9\hat{A}_{t-1} + \varepsilon_t$, where \hat{A}_t denotes the % deviation of A_t from its steady-state value, and ε_t is a white noise.

9.2 Technological innovations in banks

In figures 9 to 16, we confirm that our model approximately reproduces the results of Leao (2003). Note that figures 9 to 16 in the present paper correspond to figures 13, 14, 15, 22, 18, 17, 23 and 24 in Leao (2003), respectively. In terms of the response of consumption, note that although the shapes of figure 10 in the present paper and figure 14 in Leao (2003) are slightly different, the actual value is similar in meaning. To perform these exercises we assumed, like Leao (2003), that the banks technological parameter evolves according to $\hat{D}_t = 0.9\hat{D}_{t-1} + \varphi_t$, where \hat{D}_t denotes the % deviation of D_t from its steady-state value, and φ_t is a white noise.

9.3 Monetary policy

We should start by mentioning that if the central bank performs a 1% increase in reserves which is perceived by all economic agents as a permanent increase, then the price level increases by 1% and there is no impact whatsoever on the real variables of the model. In particular, the composition of investment expenditure does not change in the case of a permanent increase in liquidity.

When the increases in liquidity by the central bank are seen as being temporary, we start having significant effects on the composition of investment expenditure. Figures 17 to 32 show the effect of an increase in central bank liquidity (reserves) when the process through which agents form expectations about future monetary policy is $\widehat{RES}_t = 0.9\widehat{RES}_{t-1} + \xi_t$, where \widehat{RES}_t denotes the % deviation of RES_t from its steady-state value, and ξ_t is a white noise. The central bank

provokes the increases in liquidity by lending more in open market operations. We can see in figures 17 to 22 that the increase in liquidity has a negligible impact on the % deviation from steady state of real output, consumption, total investment expenditure, work hours, real money, and real wage (although the figures show some effect, when we look at the numbers on the vertical axis we conclude the effect is very small which means the impact is almost nil). Figure 23 shows the real interest rate itself, not its deviation from steady state, and we can see that the effect on this variable is also very small. In figure 24 we can see that the 1% increase in reserves makes the price level rise by almost 1%. In figures 25 and 26, we see that interest rates fall sharply following the increase in central bank reserves, the impact being specially strong in the case of the *repo* rate. This means that, in order to persuade banks to hold more liquidity, the central bank must lower its *repo* rate (figure 25). This lower *repo* - lower cost of obtaining liquidity from the central bank - allows commercial banks to decrease their lending rate (figure 26). We may rationalize as follows. The fact that it has become cheaper to obtain loans from commercial banks leads to higher levels of borrowing by the households and, consequently, households have more money to buy goods from firms. As is usual in models without price rigidities, this higher demand for goods makes prices go up (figure 24) and the impact on real output is insignificant (figure 17).

The negative relationship between liquidity and interest rates, shown in figures 25 and 26, corresponds to the results obtained in the empirical literature. Two important empirical studies on this subject are Strongin (1995) and Hamilton (1997). Using U.S. data, Strongin (1995) finds that an increase in liquidity has a strong and persistent negative effect on interest rates. Using daily data, Hamilton (1997) also concludes that more liquidity puts downward pressure on interest rates.

In figures 27 and 28 we have the main finding of the paper: the composition of investment expenditure is significantly changed with investment by banks rising at the expense of investment by nonbank firms (total investment expenditure remains roughly the same as can be seen in figure 19). Our interpretation of this result is as follows. As can be seen in equation (19), the fall in the *repo* rate increases the value of the marginal product of capital in banks and, because this is a temporary opportunity, banks decide to increase investment very strongly. In other words, the temporary fall in the cost of obtaining liquidity from the central bank makes investment expenditure by banks become so attractive that it is in the interest of the shareholders, the households who in this model own both the banks and the firms, to temporarily slow investment in firms to just below replacement levels (see figure 32) in order to allow a strong increase in commercial bank investment expenditure. Note that because the banking industry is very small (see section 8), the 58% increase in investment by banks only requires a 1.2% drop in investment by nonbank firms. The proof that it is the temporary nature of the increase in reserves that is decisive for the change in the composition of investment expenditure is that, as already mentioned, when the increase in reserves is seen as permanent the composition of investment does not change.

Let us now report the results of an empirical study we performed to examine the impact of monetary policy on bank and nonbank investment expenditure. Using annual data for the

period 1947-1997 and using a Hodrick-Prescott filter to estimate the trends, we have obtained a contemporaneous correlation between the “Percentage deviations from trend of the St. Louis adjusted monetary base (in nominal terms)” and the “Percentage deviations from trend of *real* investment in U.S. commercial banks” equal to 0.28. The data were obtained from the FRED’s web site and from a Bureau of Economic Analysis cd-rom entitled “Fixed reproducible tangible wealth of the U.S., 1925-97” (file Tw3ces.xls, row 139), respectively. Using annual data from the FRED, the contemporaneous correlation between the “Percentage deviations from trend of the St. Louis adjusted monetary base (in nominal terms)” and the “Percentage deviations from trend of *real* private fixed investment in the U.S. (in billions of chained 2000 dollars)” that we obtained was 0.16. Although these empirical results do not support the existence of a negative impact of monetary policy on nonbank firms investment, they suggest a stronger link between monetary policy and bank investment than between monetary policy and firm investment. Note that, because in our model some of the reserves supplied by the central bank are converted to circulation, RES_t in our model corresponds in the real world to the monetary base.

9.4 Shocks in the currency ratio

Figures 33 to 40 show the impact of a 1% shock in θ_t , the ratio (currency in circulation/total money supply). To perform these exercises, we assumed that θ_t evolves according to a stochastic process analogous in form to the one used for A_t and D_t . Note that we again have a significant impact on the composition of investment expenditure (figures 37 and 38), in this case with a fall in the weight of bank investment. The explanation for this result is the fact that an increase in θ_t decreases the value of the marginal product of capital in banks (see equation 19).

9.5 Changes in the required reserve ratio

In figures 41 to 48, we have the effect of a 1% shock in the required reserve ratio (r_t^{req}). To perform these exercises, we assumed that r_t^{req} evolves according to a stochastic process analogous in form to the one used for A_t and D_t . Once more, the most relevant result is the change in the composition of investment expenditure. The explanation is again the effect of a change in r_t^{req} on the value of the marginal product of capital in banks (see equation 19).

Before turning to the concluding section of the article, we would like to emphasize that all our results were obtained with a model which was built without imposing any special restrictions and calibrated with parameters which were *all* obtained using U.S. data. It is also worth pointing out that by adding a bank reserves market to the model of Leao (2003), we obtained price level determinacy (which did not exist in his model). In other words, this paper shows that taking consideration of the bank reserves market is enough to create a nominal dimension in his model.

10 Conclusion

This paper represents a first attempt at modelling the central bank *repo* interest rate in a general equilibrium setup. We have arrived at a simple model which yet contains the main ingredients

of a monetary system: (i) a reserves market where the central bank lends reserves to commercial banks; (ii) a bank loans market where commercial banks make loans to nonbank economic agents thereby creating money.

In our model, increases in liquidity by the central bank which are seen as temporary lead to a fall in interest rates. This result, which has been difficult to replicate in general equilibrium models, is in accordance with the findings of the empirical literature.

In the model we have built, when lower interest rates lead to an increase in bank lending, the higher purchasing power of consumers makes prices go up and real output is almost unaffected. Other real aggregates, such as the total amount of consumption expenditure and the total amount of investment expenditure, are also almost unaffected by monetary policy changes, a result which is common in flexible price models. However, we find that monetary policy has a non-negligible effect on the composition of investment expenditure in our model. In particular, expansionary monetary policy which is seen as temporary favours an increase in the weight of commercial banks investment expenditure in total investment expenditure.

Our theoretical framework implies that monetary policy changes which are seen as temporary create a strong incentive for investment expenditure by banks and, in our model calibrated with U.S. data, this happens at the expense of nonbank firms investment. We think these findings point to the need for further research on at least two directions.

First, as usual in the literature, our short empirical study used simple correlations to obtain some indication about the impact of monetary policy on bank and nonbank investment expenditure in the U.S. economy. We think the results obtained with our theoretical model suggest it would be important to carry out a more profound econometric study concerning the impact of monetary policy on the composition of investment expenditure. In particular, this study should control for other influences on investment, besides monetary policy, within a multivariate modelling framework.

Second, in the class of models to which our model belongs, the way investment expenditure is modelled is still rudimentary. In particular, using the market clearing condition in the goods market and taking into account that firms are all alike in the model, it is possible to show that the output of each firm is equal to its own investment *plus* bank investment per firm *plus* consumption per firm. This implies that the investment expenditure of each firm is equal to a certain percentage of its own output. Since the only physical good in the model is the output of the firms (which can either be consumed or used for investment), we conclude that the investment expenditure of each firm corresponds to the firm using part of its own output to build capital. As a consequence, it does not make much sense in these models to have firm investment financed by loans. Therefore, one difficult but important research avenue involves modelling firm heterogeneity in a way that may create in some firms the need to borrow to finance investment - and then see if our simulation results are preserved in this improved setup.

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APPENDIX

In this Appendix, we show that the initial condition we used in the main text (equation 7) is the initial condition which naturally arises when we think back to its initial moment a closed economy without government and where firms don't borrow. We argue that when we start our analysis of the economy at a certain point in time that is in the middle of History (let us denote this point by period 0), the specific initial condition (concerning the household's debt position at the beginning of period 0) that we ought to use is the initial condition we have used in this article.

Let us consider an economy where there are households, firms, banks and one central bank, and where firms don't borrow. Let us imagine that we are at the moment in time where serious economic activity is going to begin and denote this "Beginning of the economy" period by t_B . Since production is only going to take place during period t_B , firms will only receive income from the sales of the final good during period t_B and hence will only pay wages and dividends at the end of period t_B . However, in order to be able to buy consumption goods from the firms during period t_B households must use money. The only possibility they have to obtain this money at the beginning of period t_B is to obtain a loan from a bank. Hence, when the household is at the beginning of period t_B , and considering choices for t_B and future periods, she faces the following set of budget constraints. For period t_B the budget constraint is

$$\frac{B_{t_B+1}}{1 + R_{t_B}} = P_{t_B} c_{t_B} + \sum_{f=1}^{f=F} Q_{t_B}^f (z_{t_B+1}^f - z_{t_B}^f) + \sum_{l=1}^{l=L} Q_{t_B}^{bank,l} (z_{t_B+1}^{bank,l} - z_{t_B}^{bank,l}) \quad (29)$$

This budget constraint for period t_B simply states that, at the beginning of period t_B , the household must borrow from the banks an amount enough to finance her consumption expenditure during period t_B and to finance her net purchase of shares of firms and of shares of banks.

Since the household is looking into the future, when she is at the beginning of period t_B she also takes into account the budget constraints for periods $(t_B + 1), (t_B + 2), (t_B + 3), \dots$ which are given by

$$\begin{aligned} W_{t_B-1+i} n_{t_B-1+i}^s + \sum_{f=1}^{f=F} z_{t_B+i}^f \Pi_{t_B-1+i}^f + \sum_{f=1}^{f=F} z_{t_B+i}^f Q_{t_B+i}^f + \sum_{l=1}^{l=L} z_{t_B+i}^{bank,l} \Pi_{t_B-1+i}^{bank,l} + \sum_{l=1}^{l=L} z_{t_B+i}^{bank,l} Q_{t_B+i}^{bank,l} + \\ + \frac{B_{t_B+1+i}}{1 + R_{t_B+i}} + (L/H) R_{t_B-1+i}^{repo} [\theta_{t_B-1+i} + r_{t_B-1+i}^{req} (1 - \theta_{t_B-1+i})] B_{t_B-1+i}^s = \\ = B_{t_B+i} + P_{t_B+i} c_{t_B+i} + \sum_{f=1}^{f=F} z_{t_B+1+i}^f Q_{t_B+i}^f + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} Q_{t_B+i}^{bank,l} \end{aligned}$$

for $i=1,2,3,\dots$

These budget constraints for periods $(t_B+1), (t_B+2), (t_B+3), \dots$ are identical to the household's budget constraints examined in section 5 of the article and can be explained in the same way.

Note that when the household is at the beginning of period t_B , the budget constraint for t_B is different from the budget constraints for the following periods (this happens because at t_B there is no previous period).

In order to derive the initial condition, let us start by writing again equation 29 (the budget constraint for period t_B)

$$\frac{B_{t_B+1}}{1 + R_{t_B}} = P_{t_B} c_{t_B} + \sum_{f=1}^{f=F} Q_{t_B}^f (z_{t_B+1}^f - z_{t_B}^f) + \sum_{l=1}^{l=L} Q_{t_B}^{bank,l} (z_{t_B+1}^{bank,l} - z_{t_B}^{bank,l})$$

Imposing here the market clearing conditions in the shares market written for period t_B ($z_{t_B+1}^f = \frac{1}{H}$ and $z_{t_B+1}^{bank,l} = \frac{1}{H}$) and assuming that the world starts with every household owning the same share of each firm and each household owning the same share of each bank so that we also have $z_{t_B}^f = \frac{1}{H}$ and $z_{t_B}^{bank,l} = \frac{1}{H}$ we obtain

$$\frac{B_{t_B+1}}{1 + R_{t_B}} = P_{t_B} c_{t_B} \quad (30)$$

We now write the following tautology

$$B_{t_B+1} = \frac{B_{t_B+1}}{1 + R_{t_B}} (1 + R_{t_B}) \Leftrightarrow B_{t_B+1} = \frac{B_{t_B+1}}{1 + R_{t_B}} + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B}$$

This tautology simply says that the household's debt at the beginning of period $(t_B + 1)$ is equal to the principal borrowed at the beginning of period t_B plus interest on it.

Using equation 30, this last equation becomes

$$B_{t_B+1} = P_{t_B} c_{t_B} + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B}$$

Using the market clearing condition in the goods market, we obtain

$$\begin{aligned} & B_{t_B+1} = \\ & = \frac{F}{H} [P_{t_B} A_{t_B} F(k_{t_B}, n_{t_B}^d) - P_{t_B} [k_{t_B+1} - (1 - \delta)k_{t_B}]] - \frac{L}{H} P_{t_B} [k_{t_B+1}^b - (1 - \delta_B)k_{t_B}^b] + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B} \end{aligned}$$

Using the definition of nominal profits of firm f in period t_B , we obtain

$$\begin{aligned} B_{t_B+1} &= \frac{F}{H} [\Pi_{t_B}^f + W_{t_B} n_{t_B}^d] - \frac{L}{H} P_{t_B} [k_{t_B+1}^b - (1 - \delta_B)k_{t_B}^b] + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B} \Leftrightarrow \\ \Leftrightarrow B_{t_B+1} &= W_{t_B} \frac{F}{H} n_{t_B}^d + \frac{F}{H} \Pi_{t_B}^f - \frac{L}{H} P_{t_B} [k_{t_B+1}^b - (1 - \delta_B)k_{t_B}^b] + \frac{B_{t_B+1}}{1 + R_{t_B}} R_{t_B} \end{aligned}$$

With the market clearing condition in the labour market, this becomes

$$B_{t_B+1} = W_{t_B}(n_{t_B}^s - \frac{L}{H}n_{t_B}^b) + \frac{F}{H}\Pi_{t_B}^f - \frac{L}{H}P_{t_B} [k_{t_B+1}^b - (1 - \delta_B)k_{t_B}^b] + \frac{B_{t_B+1}}{1 + R_{t_B}}R_{t_B}$$

Using the market clearing condition from the bank loans market, we obtain

$$B_{t_B+1} = W_{t_B}(n_{t_B}^s - \frac{L}{H}n_{t_B}^b) + \frac{F}{H}\Pi_{t_B}^f - \frac{L}{H}P_{t_B} [k_{t_B+1}^b - (1 - \delta_B)k_{t_B}^b] + \frac{L}{H}B_{t_B}^s R_{t_B}$$

Rearranging, we obtain

$$B_{t_B+1} = W_{t_B}n_{t_B}^s + \frac{F}{H}\Pi_{t_B}^f + \frac{L}{H} [R_{t_B}B_{t_B}^s - W_{t_B}n_{t_B}^b - P_{t_B} [k_{t_B+1}^b - (1 - \delta_B)k_{t_B}^b]]$$

This is equivalent to writing

$$\begin{aligned} B_{t_B+1} &= W_{t_B}n_{t_B}^s + \frac{F}{H}\Pi_{t_B}^f + \\ &+ \frac{L}{H} [R_{t_B}B_{t_B}^s - R_{t_B}^{repo} [\theta_{t_B} + r_{t_B}^{req} (1 - \theta_{t_B})] B_{t_B}^s - W_{t_B}n_{t_B}^b - P_{t_B} [k_{t_B+1}^b - (1 - \delta_B)k_{t_B}^b]] \\ &+ \frac{L}{H}R_{t_B}^{repo} [\theta_{t_B} + r_{t_B}^{req} (1 - \theta_{t_B})] B_{t_B}^s \end{aligned}$$

Using the definition of nominal profits of bank l in period t_B , we obtain

$$B_{t_B+1} = W_{t_B}n_{t_B}^s + \frac{F}{H}\Pi_{t_B}^f + \frac{L}{H}\Pi_{t_B}^{bank,l} + \frac{L}{H}R_{t_B}^{repo} [\theta_{t_B} + r_{t_B}^{req} (1 - \theta_{t_B})] B_{t_B}^s$$

Finally, using the market clearing conditions in the shares market, this can be written as

$$B_{t_B+1} = W_{t_B}n_{t_B}^s + \sum_{f=1}^{f=F} z_{t_B+1}^f \Pi_{t_B}^f + \sum_{l=1}^{l=L} z_{t_B+1}^{bank,l} \Pi_{t_B}^{bank,l} + \frac{L}{H}R_{t_B}^{repo} [\theta_{t_B} + r_{t_B}^{req} (1 - \theta_{t_B})] B_{t_B}^s$$

Remember that B_{t_B+1} denotes the debt the household has at the beginning of period $(t_B + 1)$ [$\frac{B_{t_B+1}}{1+R_{t_B}}$ is the amount borrowed by the household at the beginning of period t_B and R_{t_B} is the interest rate between the beginning of period t_B and the beginning of period $(t_B + 1)$]. Therefore, this last equation tells us that, in equilibrium (because we have used market clearing conditions to derive it), the amount the household borrows at the beginning of period t_B is such that it implies a debt at the beginning of period $(t_B + 1)$ equal to the sum of the wage earnings, dividend earnings and lump-sum transfer from the central bank that she receives at the beginning of that period [period $(t_B + 1)$]. Hence, when we go on to consider the typical household's optimization problem at the beginning of period $(t_B + 1)$ we should add this equation as an initial condition (describing the debt she carries from the previous period). Therefore, at the beginning of period $(t_B + 1)$ the household faces the following budget constraints

$$\begin{aligned}
W_{t_B+i} n_{t_B+i}^s + \sum_{f=1}^{f=F} z_{t_B+1+i}^f \Pi_{t_B+i}^f + \sum_{f=1}^{f=F} z_{t_B+1+i}^f Q_{t_B+1+i}^f + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} \Pi_{t_B+i}^{bank,l} + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} Q_{t_B+1+i}^{bank,l} \\
+ \frac{B_{t_B+2+i}}{1 + R_{t_B+1+i}} + (L/H) R_{t_B+i}^{repo} [\theta_{t_B+i} + r_{t_B+i}^{req} (1 - \theta_{t_B+i})] B_{t_B+i}^s = \\
= B_{t_B+1+i} + P_{t_B+1+i} c_{t_B+1+i} + \sum_{f=1}^{f=F} z_{t_B+2+i}^f Q_{t_B+1+i}^f + \sum_{l=1}^{l=L} z_{t_B+2+i}^{bank,l} Q_{t_B+1+i}^{bank,l}
\end{aligned}$$

for $i = 0, 1, 2, 3, \dots$

plus the initial condition

$$B_{t_B+1} = W_{t_B} n_{t_B}^s + \sum_{f=1}^{f=F} z_{t_B+1}^f \Pi_{t_B}^f + \sum_{l=1}^{l=L} z_{t_B+1}^{bank,l} \Pi_{t_B}^{bank,l} + \frac{L}{H} R_{t_B}^{repo} [\theta_{t_B} + r_{t_B}^{req} (1 - \theta_{t_B})] B_{t_B}^s$$

Using this initial condition in the period $(t_B + 1)$ budget constraint written with $i = 0$, this period $(t_B + 1)$ budget constraint for $i = 0$ becomes

$$\frac{B_{t_B+2}}{1 + R_{t_B+1}} = P_{t_B+1} c_{t_B+1} + \sum_{f=1}^{f=F} Q_{t_B+1}^f (z_{t_B+2}^f - z_{t_B+1}^f) + \sum_{l=1}^{l=L} Q_{t_B+1}^{bank,l} (z_{t_B+2}^{bank,l} - z_{t_B+1}^{bank,l})$$

Therefore the complete description of the budget constraints faced by the household at the beginning of period $(t_B + 1)$ is

$$\frac{B_{t_B+2}}{1 + R_{t_B+1}} = P_{t_B+1} c_{t_B+1} + \sum_{f=1}^{f=F} Q_{t_B+1}^f (z_{t_B+2}^f - z_{t_B+1}^f) + \sum_{l=1}^{l=L} Q_{t_B+1}^{bank,l} (z_{t_B+2}^{bank,l} - z_{t_B+1}^{bank,l})$$

$$\begin{aligned}
W_{t_B+i} n_{t_B+i}^s + \sum_{f=1}^{f=F} z_{t_B+1+i}^f \Pi_{t_B+i}^f + \sum_{f=1}^{f=F} z_{t_B+1+i}^f Q_{t_B+1+i}^f + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} \Pi_{t_B+i}^{bank,l} + \sum_{l=1}^{l=L} z_{t_B+1+i}^{bank,l} Q_{t_B+1+i}^{bank,l} \\
+ \frac{B_{t_B+2+i}}{1 + R_{t_B+1+i}} + (L/H) R_{t_B+i}^{repo} [\theta_{t_B+i} + r_{t_B+i}^{req} (1 - \theta_{t_B+i})] B_{t_B+i}^s = \\
= B_{t_B+1+i} + P_{t_B+1+i} c_{t_B+1+i} + \sum_{f=1}^{f=F} z_{t_B+2+i}^f Q_{t_B+1+i}^f + \sum_{l=1}^{l=L} z_{t_B+2+i}^{bank,l} Q_{t_B+1+i}^{bank,l}
\end{aligned}$$

for $i = 1, 2, 3, \dots$

These two equations are identical in form to the two equations the household was facing at the beginning of period t_B but written one period ahead. Therefore we can repeat the reasoning and derive an initial condition for the household's optimization problem at the beginning of period

$(t_B + 2)$. We will obviously obtain a condition which has the same form but written one period ahead: we obtain

$$B_{t_B+2} = W_{t_B+1} n_{t_B+1}^s + \sum_{f=1}^{f=F} z_{t_B+2}^f \Pi_{t_B+1}^f + \sum_{l=1}^{l=L} z_{t_B+2}^{bank,l} \Pi_{t_B+1}^{bank,l} \\ + \frac{L}{H} R_{t_B+1}^{repo} [\theta_{t_B+1} + r_{t_B+1}^{req} (1 - \theta_{t_B+1})] B_{t_B+1}^s$$

(and so on and so forth for all periods ahead). The conclusion to be drawn is that, when we start our analysis of the economy in the middle of History (in period 0, for example), we should add to the household's optimization problem the following initial condition

$$B_0 = W_{-1} n_{-1}^s + \sum_{f=1}^{f=F} z_0^f \Pi_{-1}^f + \sum_{l=1}^{l=L} z_0^{bank,l} \Pi_{-1}^{bank,l} + \\ +(L/H) R_{-1}^{repo} [\theta_{-1} + r_{-1}^{req} (1 - \theta_{-1})] B_{-1}^s$$

This is exactly the initial condition we have used in this paper for the household's problem (equation 7). This initial condition was derived by thinking back to its initial moment a closed economy without government and where firms don't borrow. If instead we think back to its initial moment a closed economy without government where firms borrow the wages at the beginning of the period (in order to be able to pay wages in advance at the beginning of the period, before production has taken place), then the initial condition we obtain is $B_0 = \sum_{f=1}^{f=F} z_0^f \Pi_{-1}^f + \sum_{l=1}^{l=L} z_0^{bank,l} \Pi_{-1}^{bank,l} + (L/H) R_{-1}^{repo} [\theta_{-1} + r_{-1}^{req} (1 - \theta_{-1})] B_{-1}^s$. The results we obtained while working with this other initial condition were not very different from the results quoted in this paper.

Figure 1
1% shock in A
Output as % dev. from steady state

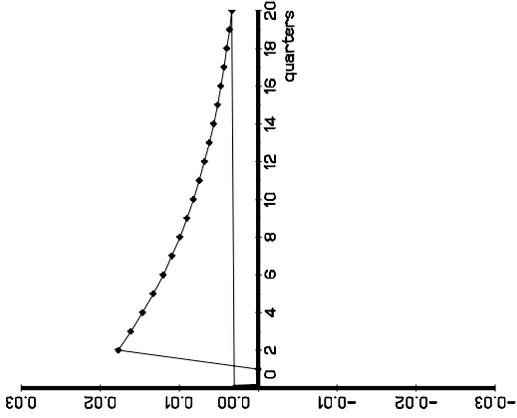


Figure 2
1% shock in A
Consumption as % dev. from steady state

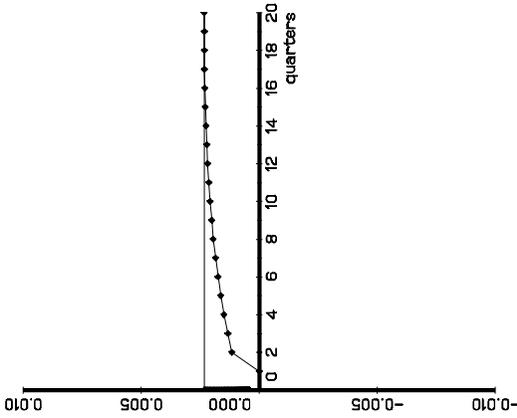


Figure 3
1% shock in A
Firms investment as % dev. from s.s.

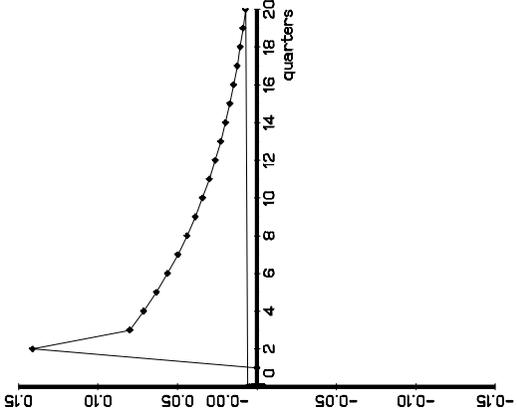


Figure 4
1% shock in A
Firms labour demand as % dev. from s.s.

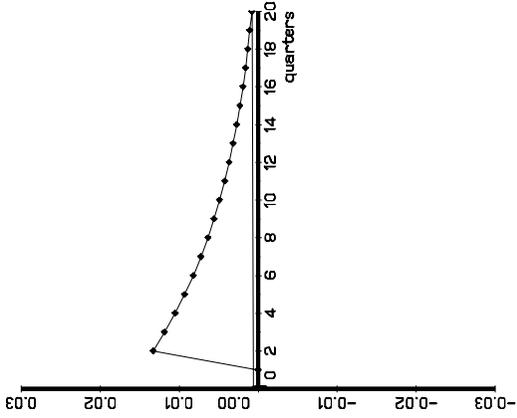


Figure 5
1% shock in A
Labour supply as % dev. from steady state

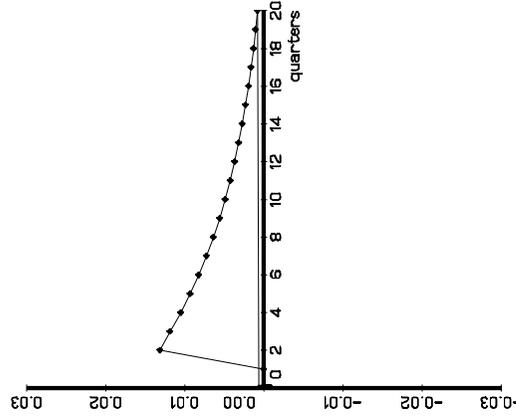


Figure 6
1% shock in A
Real wage as % dev. from steady state

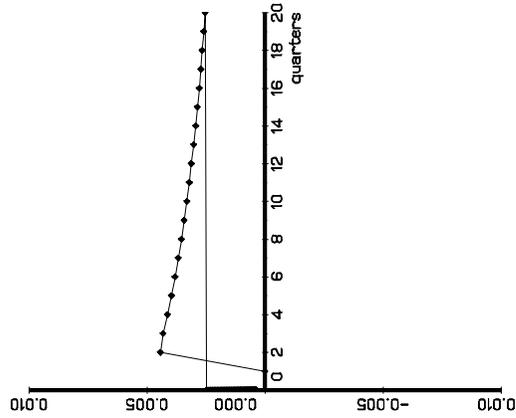


Figure 7
1% shock in A
Expected real interest rate

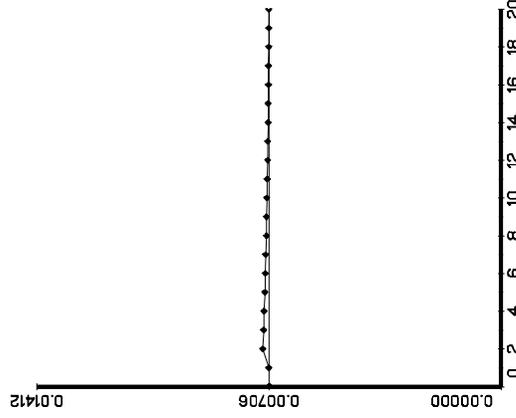


Figure 8
1% shock in A
Firms capital stock % dev. from s.s.

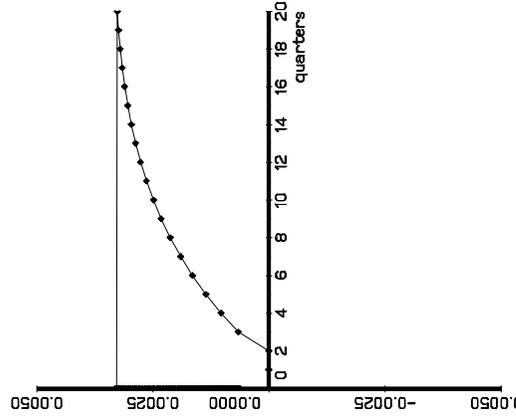


Figure 9
1% shock in D
Output as % dev. from steady state

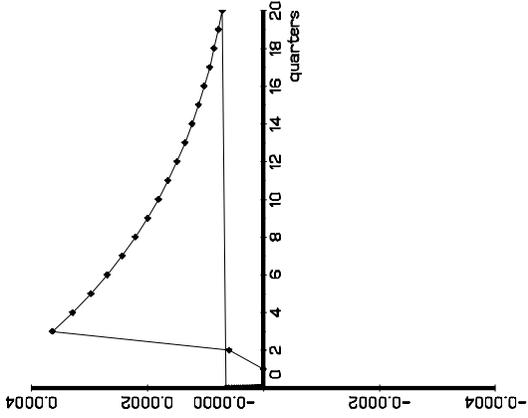


Figure 10
1% shock in D
Consumption as % dev. from steady state

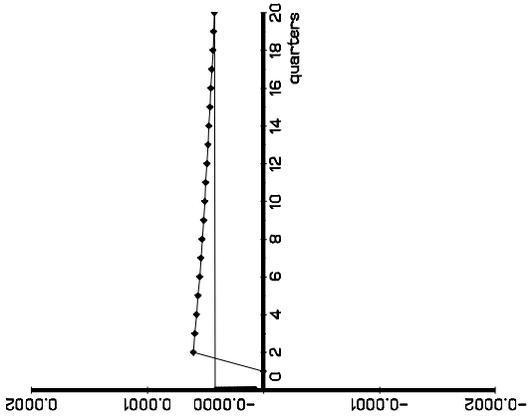


Figure 11
1% shock in D
Firms investment as % dev. from s.s.

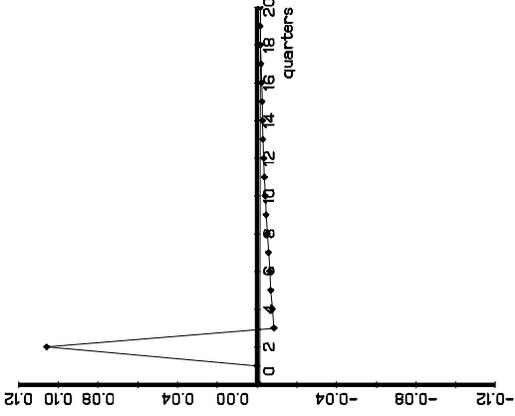


Figure 12
1% shock in D
Banks investment as % dev. from s.s.

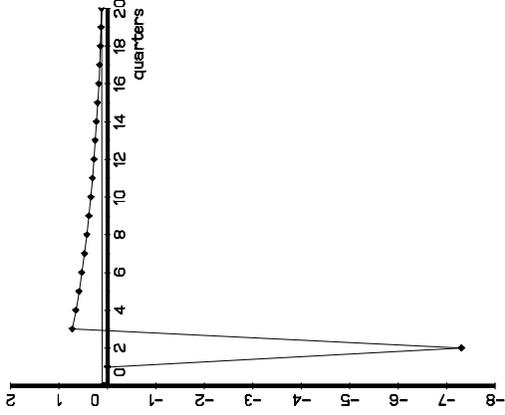


Figure 13
1% shock in D
Firms labour demand as % dev. from s.s.

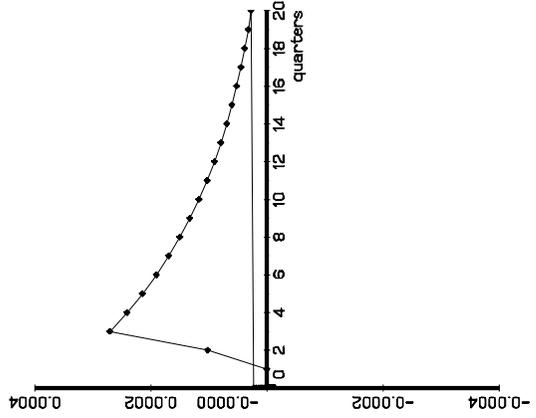


Figure 14
1% shock in D
Banks labour demand as % dev. from s.s.

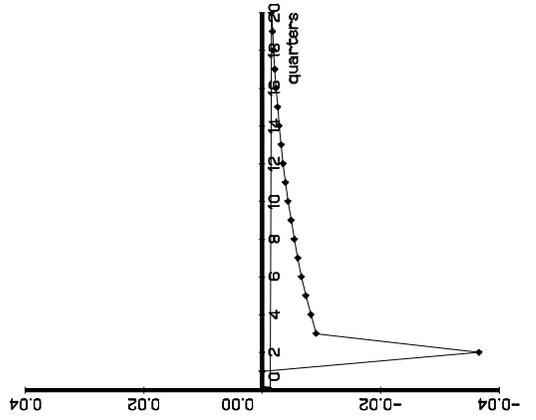


Figure 15
1% shock in D
Firms capital stock as % dev. from s.s.

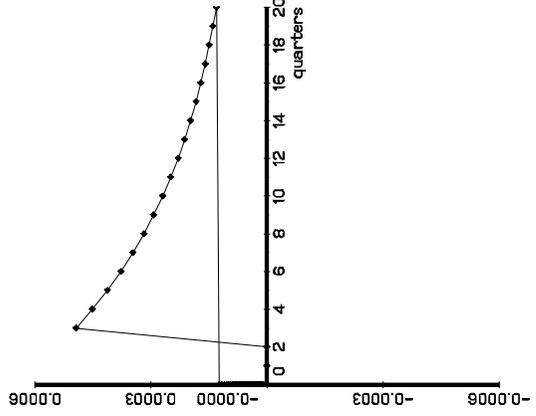


Figure 16
1% shock in D
Banks capital stock as % dev. from s.s.

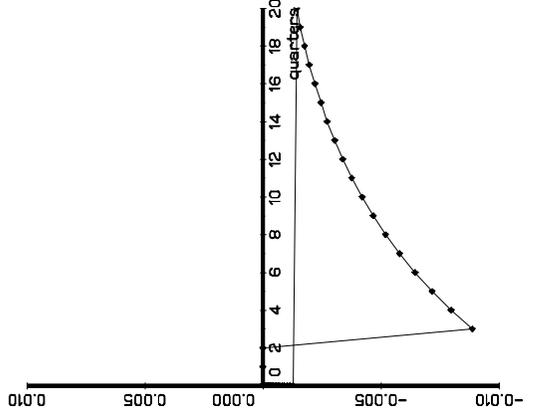


Figure 17
1% shock in RES
Output as % dev. from steady state

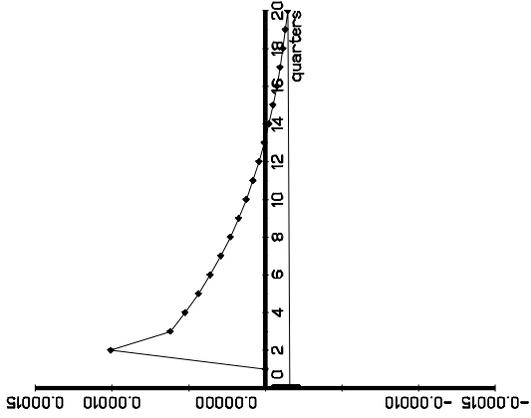


Figure 18
1% shock in RES
Consumption as % dev from steady state

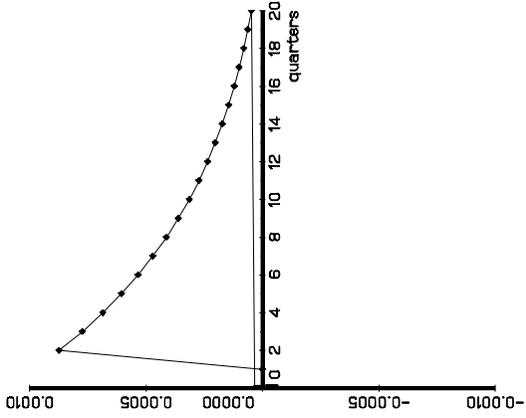


Figure 19
1% shock in RES
Investment as % dev from s.s.

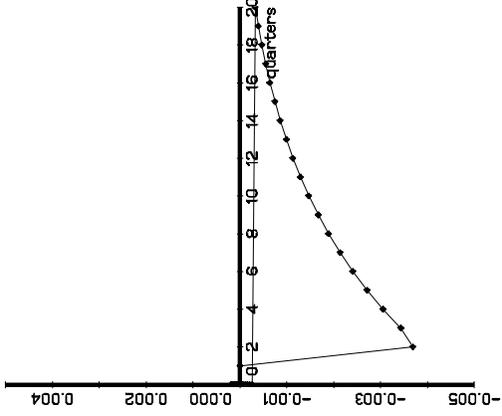


Figure 20
1% shock in RES
Labour supply as % dev. from s.s.

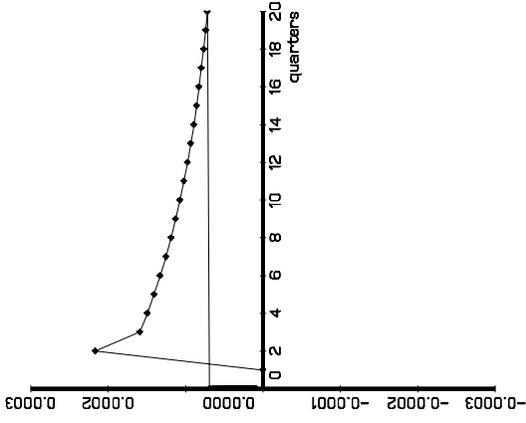


Figure 21
1% shock in RES
Real money balances % dev. from s.s.

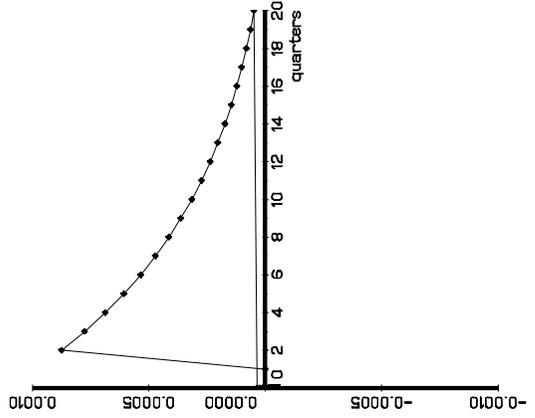


Figure 22
1% shock in RES
Real wage as % dev. from steady state

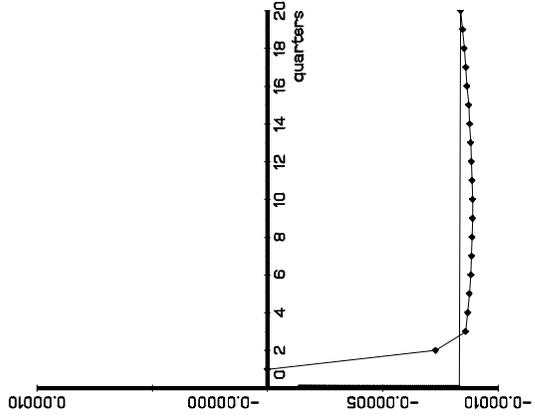


Figure 23
1% shock in RES
Expected real interest rate

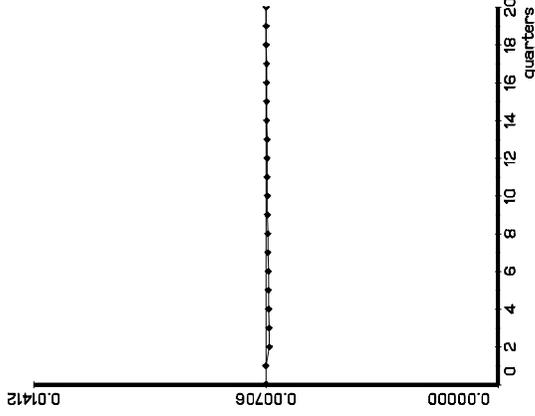


Figure 24
1% shock in RES
Price level as % dev. from s.s.

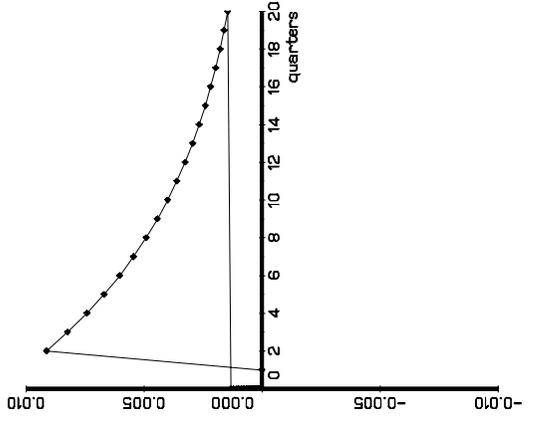


Figure 25
1% shock in RES
Repo rate as % dev. from steady state

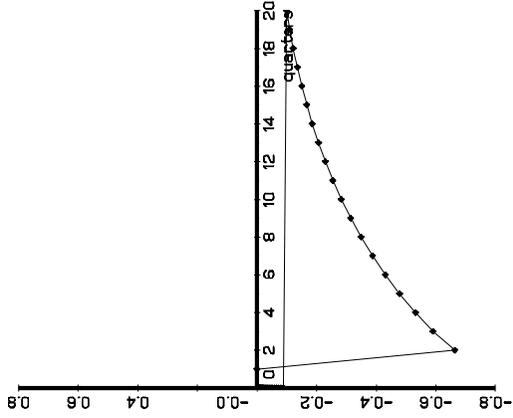


Figure 26
1% shock in RES
Banks lending rate % dev. from s.s.

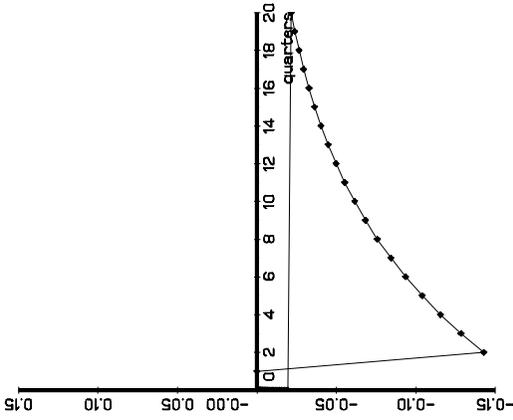


Figure 27
1% shock in RES
Banks investment as % dev. from s.s.

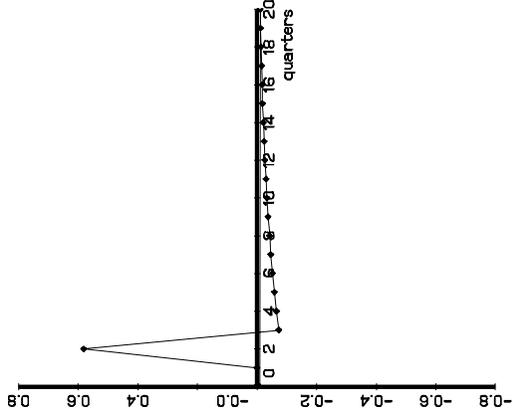


Figure 28
1% shock in RES
Nonbank firms investment % dev from ss

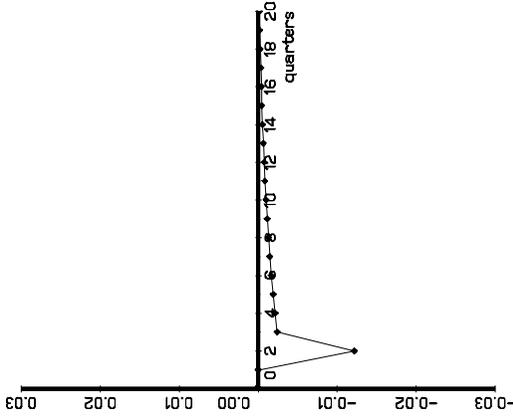


Figure 29
1% shock in RES
Firms labour demand % dev. from s.s.

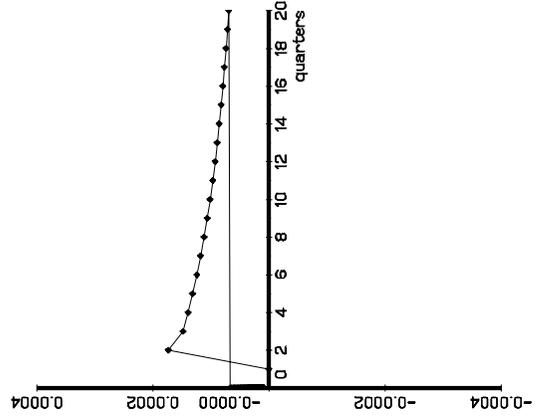


Figure 30
1% shock in RES
Banks labour demand % dev. from s.s.

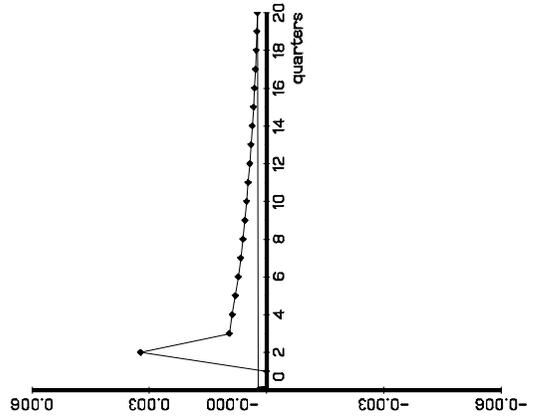


Figure 31
1% shock in RES
Banks capital stock % dev. from s.s.

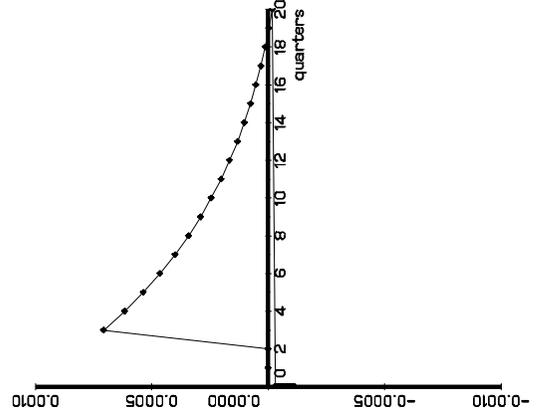


Figure 32
1% shock in RES
Firms capital stock % dev from s.s.

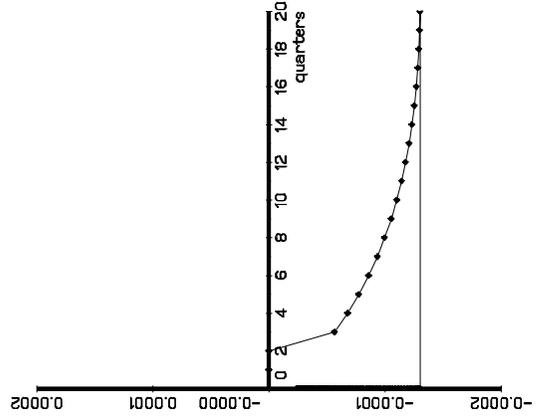


Figure 33
1% shock in θ
Output as % dev. from s.s.

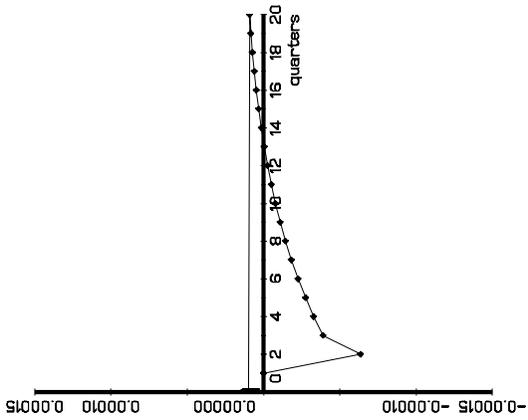


Figure 34
1% shock in θ
Consumption as % dev. from s.s.

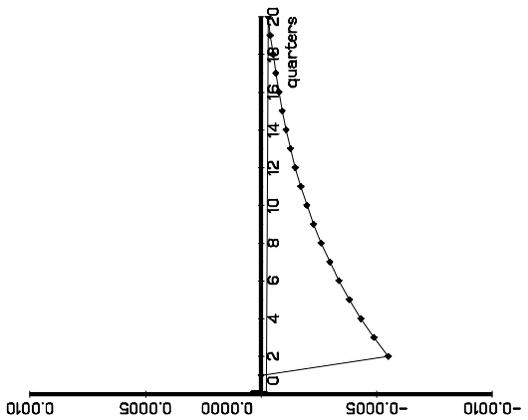


Figure 35
1% shock in θ
Investment as % dev. from s.s.

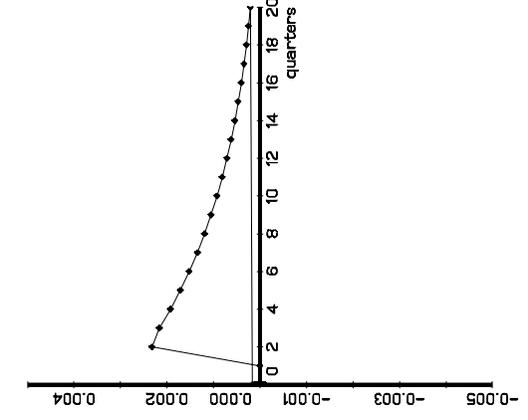


Figure 36
1% shock in θ
Labour supply % dev. from s.s.

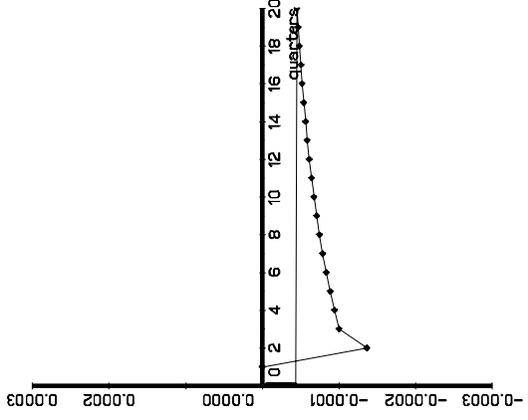


Figure 37
1% shock in θ
Banks investment % dev from s.s.

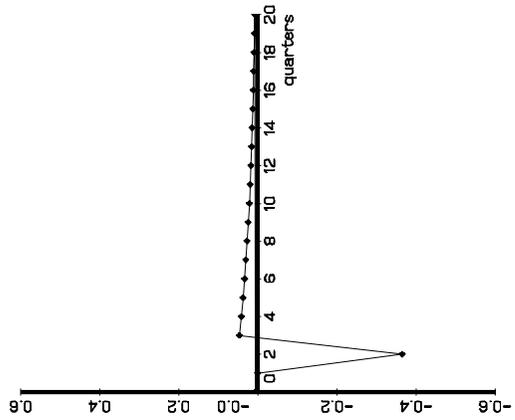


Figure 38
1% shock in θ
Firms investment % dev from s.s.

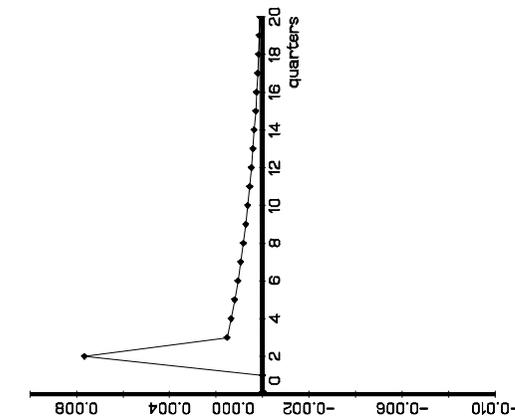


Figure 39
1% shock in θ
Banks lending rate % dev s.s.

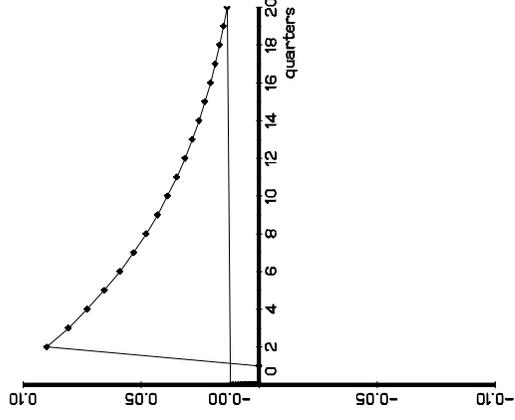


Figure 40
1% shock in θ
Price level as % dev. from s.s.

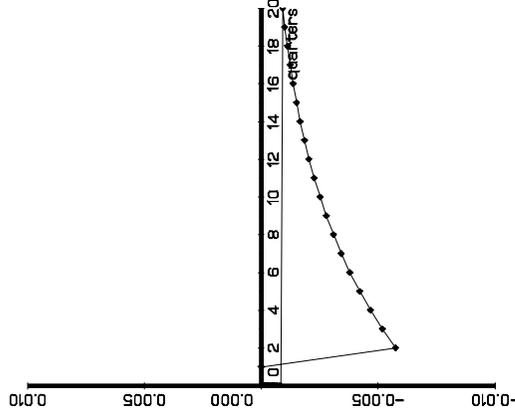


Figure 41
1% shock in r^m
Output as % dev. from s.s.

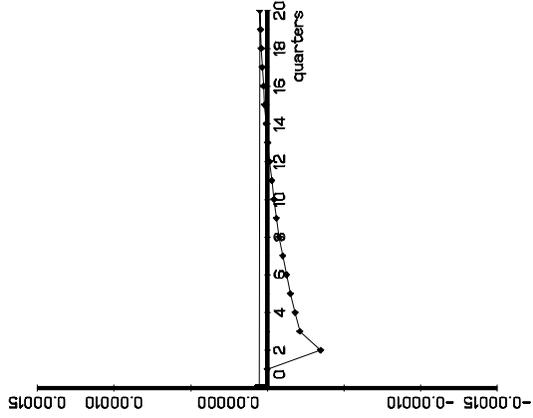


Figure 42
1% shock in r^m
Consumption as % dev. from s.s.

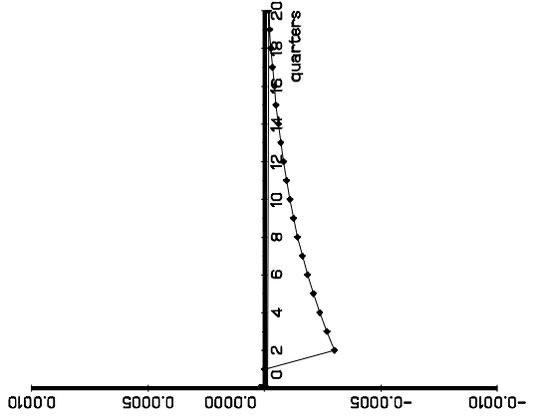


Figure 43
1% shock in r^m
Investment as % dev. from s.s.

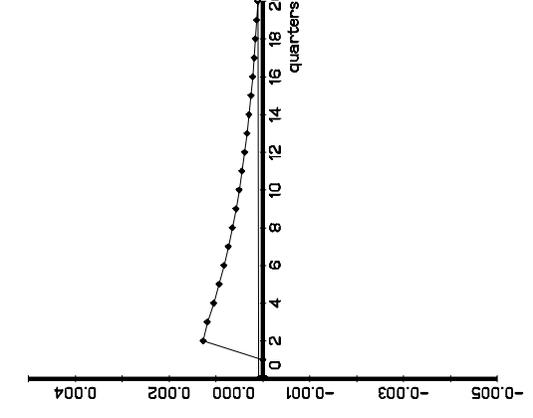


Figure 44
1% shock in r^m
Labour supply as % dev. from s.s.

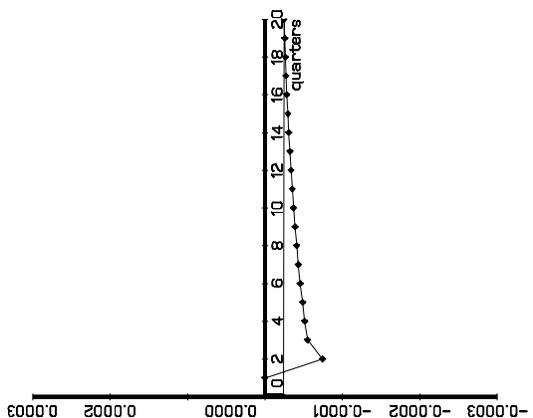


Figure 45
1% shock in r^m
Banks investment % dev. from s.s.

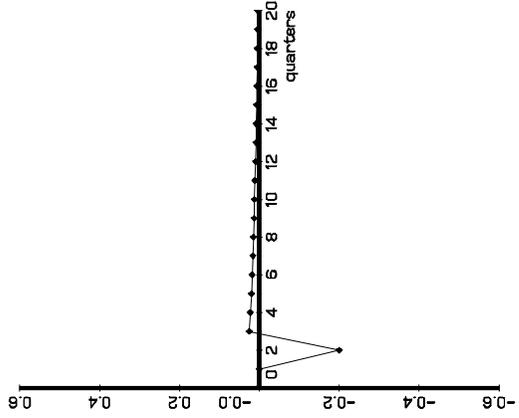


Figure 45
1% shock in r^m
Firms investment % dev. from s.s.

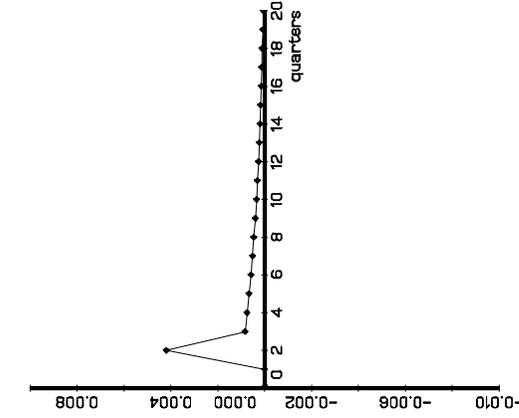


Figure 47
1% shock in r^m
Banks lending rate % dev from s.s.

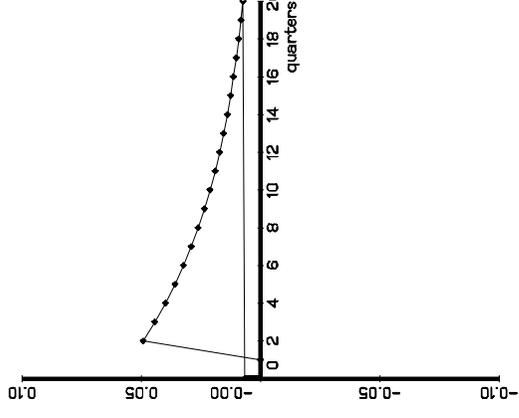


Figure 48
1% shock in r^m
Price level as % dev. from s.s.

