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Assessing and managing Value-at-Risk in a diversified portfolio

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Master in Finance

Supervisor:
PhD, António Manuel Rodrigues Guerra Barbosa, Assistant Professor,
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Sumário

O Value at Risk (VaR) representa uma estimativa da perda potencial máxima que um portefólio pode sofrer num determinado período e nível de confiança. Neste trabalho, é definido um limite para o VaR, que serve como referência em torno da qual são tomadas as decisões de gestão de risco. O portefólio, composto por 160 ações e 15 obrigações, é gerido de forma a assegurar que o seu VaR se mantém abaixo do limite pré-determinado, preservando simultaneamente uma alocação de 60% em ações e 40% em obrigações, com uma margem de tolerância de 5%. Para identificar a metodologia mais adequada para cálculo do VaR, são implementados 4 modelos distintos, cuja precisão é avaliada através de procedimentos de backtesting. O modelo com melhor desempenho é então aplicado para calcular o VaR diário do portefólio, que é gerido ativamente através de uma estratégia de equity-hedging ao longo de um horizonte de um ano. A eficácia desta estratégia é avaliada utilizando o indicador Return on Risk Adjusted Capital (RORAC).

Palavras-chave: Value-at-Risk; Backtest; Hedging; Return on Risk-Adjusted Capital

Classificação JEL: C52; G32

Abstract

Value at Risk (VaR) represents an estimate of the maximum potential loss that a portfolio may experience over a specified time horizon and confidence level. In this work, a threshold for VaR is established, serving as the benchmark around which risk management positions are taken. The portfolio, composed by 160 equities and 15 bonds, is managed to ensure that its VaR remains below the predetermined threshold while maintaining a target allocation of 60% equities and 40% bonds, allowing a 5% deviation. To identify the most suitable VaR methodology, 4 distinct VaR models are implemented, and their accuracy is evaluated through backtesting procedures. The model that presents the better performance is then applied to calculate the portfolio's daily VaR, which is actively managed using an equity-hedging strategy across a one-year horizon. The effectiveness of this strategy is evaluated using the Return on Risk-Adjusted Capital (RORAC) metric.

Key words: Value-at-Risk; Backtest; Hedging; Return on Risk-Adjusted Capital

JEL Classification: C52; G32

Index

Tables Index

Figures Index

Chapter 1 – Introduction

Risk management is a central pillar of portfolio management, not only for investors seeking to balance return and risk, but also for regulators who require institutions to adopt rigorous frameworks to preserve financial stability. Among the wide range of tools available for measuring and controlling market risk, Value at Risk (VaR) has emerged as one of the most widely adopted metrics. VaR provides an estimate of the maximum potential loss of a portfolio over a given time horizon and confidence level, offering a standardized metric for comparing risks across different asset classes and investment strategies. Its ability to condense risk exposures into a single, interpretable measure has made it widely used in regulatory frameworks, institutional policies, and practical asset management.

Despite its popularity, the effectiveness of VaR depends heavily on the methodology used to estimate it. Different statistical approaches can yield significantly different results, particularly during periods of market stress, making the choice of model crucial.

This thesis implements VaR as both a measurement tool and a decision-making threshold in the management of a diversified portfolio composed of 160 equities from various markets and sectors and 15 bonds with a AAA credit rate (Standard & Poor's scale). A target allocation of 60% equities and 40% bonds, with an allowed deviation of 5%, is maintained while ensuring that the portfolio's VaR remains below a predetermined threshold of 800.000€. Four distinct VaR models, namely RiskMetrics VaR, Skewed Generalized Student-t VaR, Historical VaR, and Quantile Regression VaR, are implemented and compared through backtesting to identify the most accurate model. The backtest, based on 10 years of historical data using the portfolio composition as of May 29, 2023, is conducted using the Unconditional Coverage test and the Berkowitz, Christoffersen, and Pelletier test.

The best-performing model is then employed for daily VaR estimation, which serves as the basis for an active equity-hedging strategy over a one-year period from May 30, 2023, to June 3, 2024, intending to maintain the VaR below the 800.000€ threshold.

Finally, the effectiveness of the proposed risk management framework is evaluated by comparing the performance of the Unrebalanced Unhedged Portfolio, Rebalanced Unhedged Portfolio, and the Rebalanced Hedged Portfolio by using the Return on Risk-Adjusted Capital (RORAC), a performance metric that relates profitability to the level of risk undertaken.

This thesis is organized as follows: Chapter 2 provides a review of the literature on Value at Risk and its applications in risk management; Chapter 3 describes the data and portfolio composition, with the details of the selected equities and bonds; Chapter 4 outlines the methodology, including risk factor mapping, volatility, VaR models, backtesting procedures, and hedging techniques; Chapter 5 discusses the process of model selection and presents the results of the backtests; Chapter 6 applies the chosen model to the active management of the portfolio's VaR and evaluates the effectiveness of the hedging strategy using RORAC; The thesis concludes with a summary of the findings.

Chapter 2 - Literature Review

Risk management was always a concern, but over the past few decades, risk management has increasingly gained significance for all companies. Financial institutions are realizing the need to allocate more resources to risk management in order to reduce and avoid avoidable losses (Hull, 2015). The 2007 credit crisis was one of the events that reinforced that idea since it exposed significant flaws in risk management (Jorion, 2011), but this preoccupation was not new. The concerns over the fragility of the global financial system were a key driver behind the establishment of the Basel Committee on Banking Supervision (BCBS) in 1974, after the collapse of Bankhaus Herstatt (Goodhart, 2011). The BCBS is the primary global authority for establishing standards for the prudent regulation of banks, namely risk management guidelines.

Risk is defined as the level of uncertainty of future returns (Jorion, 2007). This uncertainty can take many forms, exposing the participants in the financial markets to multiple types of risks that can be classified according to the source of the underlying uncertainty: market risk, credit risk, operational risk, liquidity risk, and model risk (McNeil, Frey, & Embrechts, 2015). This work is going to focus on market risk. Market risk is the risk to an institution's financial condition resulting from adverse movements in the level or volatility of market prices (Frain & Meegan, 1996).

Before risk can be mitigated, it must be assessed. A market risk metric is a single value that quantifies the uncertainty in a portfolio's profit and loss (P&L) or its return (Alexander, 2008). According to Artzner et al. (1999), a coherent risk metric should satisfy four axioms: monotonicity (preserves stochastic dominance), sub-additivity (accounts for diversification effects), positive homogeneity (scaling a portfolio by a positive factor should scale the risk by the same factor), and translation invariance (adding risk-free assets reduces the risk by the same amount). Among the many risk metrics, there is the Value-at-Risk (VaR). VaR is a quantile risk metric, and it is defined as "a measure of the maximum potential change in the value of a portfolio of financial instruments with a given probability over a pre-set horizon." (J.P. Morgan & Reuters, 1996).

As defined by Klaassen & van Eeghen (2009), "Economic capital represents an estimate of the worst possible decline in the institutions amount of capital at a specified confidence level within a chosen time horizon.". Therefore, Economic Capital may be regarded as an estimation of the capital required by a financial institution to maintain solvency. In financial institutions, Economic Capital is composed by credit risk, market risk, and operational risk (Jorion, 2007). Jorion (2007) demonstrates that VaR may be interpreted as a measure of Economic Capital, as it reflects the maximum loss a financial institution is prepared to tolerate.

VaR offers several advantages as a risk metric. It quantifies potential losses that could occur with a specified probability, evaluates the risk associated with both the underlying risk factors and their sensitivities, it enables comparisons across different markets and types of exposures, making it a versatile tool applicable to all activities and categories of risk, it can be calculated at multiple levels, from individual trades or portfolios to a measure that covers all risks within an organization. Furthermore, when aggregating for larger portfolios or disaggregating to focus on specific risks, it effectively accounts for the interdependencies among different assets or portfolios (Alexander, 2008).

Nevertheless, VaR does not satisfy the subadditivity property, which means that the risk of a combined portfolio can exceed the sum of the individual portfolio risks, therefore discouraging diversification (Artzner et al., 1999). Since it does not respect this property, VaR cannot be classified as a coherent risk metric as classified by Artzner et al. (1999). Additionally, VaR ignores tail risks since it does not provide any information about the magnitude of losses that could occur beyond the threshold (Hull, 2015).

Value at Risk, in contrast to indicators such as return and volatility that can be computed directly from observed historical data, is an estimation rather than a direct financial measure, as it is model-dependent. Its computation requires specific assumptions concerning the distribution of returns, the time horizon, and the confidence level. Various methodological approaches may be adopted to obtain the most accurate estimation, and as a result, VaR outcomes can differ substantially depending on the chosen model (Vasileiou, 2017).

In this work, four VaR models are going to be tested: RiskMetrics (RM) VaR, Skewed Generalized Student-t (SGSt) VaR, Historical VaR, and Quantile Regression (QR) VaR.

The Risk Metrics or parametric linear Value at Risk (VaR) model relies on analytical formulas based on assumptions about risk factor returns, specifically assuming that h-day returns are independent and identically distributed (i.i.d.) and follow a normal distribution (Alexander, 2008).

Financial asset returns may depart from the normal distribution since returns often present leptokurtosis and negatively skewed distributions (Nam & Choi, 2008). A leptokurtic distribution is one whose density function has a higher peak and larger tails than the normal density function of the same variance, while skewness refers to the degree of asymmetry of a distribution around its mean (Hull, 2015). The normal linear VaR, given the leptokurtosis and negative skewness in risk factor return distributions, may underestimate the VaR at high confidence levels (Alexander, 2008).

Theodossiou (1998) developed a skewed General t Distribution designed to capture the skewness and excess kurtosis commonly present in financial data. The Skewed Generalized Student t VaR, based on the distribution proposed by Theodossiou (1998), is a model that accounts for the skewness and excess kurtosis, and its distribution is more flexible than the Student-t and normal distributions.

Historical VaR is the preferred methodology of banks. The historical VaR has some advantages when compared with parametric models since it does not make assumptions about the density function of the risk factor returns, and it applies not only to linear portfolios (Alexander, 2008). To improve the accuracy of the historical Value at Risk and reduce its limitations, the weight of risk factor returns must not be equal. There are two weighting options: exponential weighting of probabilities and volatility adjustment of returns. In this work it will be used the volatility adjustment of returns proposed by Hull and White (1998), which adjusts the volatility of the sample to the current volatility.

The last model will be the quantile regression VaR. Unlike traditional regressions (such as OLS) that focus on estimating the conditional mean, quantile regression estimates specific quantiles of the distribution of financial returns. By using quantile regression, the VaR of a portfolio is computed based on historical data and explanatory variables. This method is flexible in the use of explanatory variables, which brings a subjective component to the VaR (Xiao et al., 2015).

The estimation of the VaR involves making decisions that create model risk. There are two sources of model risk (Cairns, 2000): model choice and parameter uncertainty. To test the model risk, the backtesting methodology is used to assess if the VaR is well specified.

A backtest takes a fixed portfolio and assumes its weights are fixed for the entire backtest. The result of a backtest depends on the portfolio composition, as well as on the evolution of the risk factors and the assumptions made about risk factor return distributions when building the model. The backtest should be performed using a long period, daily data, and it is applied a rolling window without overlapping the data. The backtesting will compare two series, the h-day VaR and the realized return, to observe the exceedances, that is, when the portfolio losses are higher than the estimated VaR (Alexander, 2008).

The Conditional Coverage Test (Christoffersen, 1998) is a method used to evaluate the accuracy of risk models. It tests the unconditional coverage, which checks if the actual exceedances match the model's predicted, and independence, which ensures that exceedances are not clustered.

Berkowitz, Christoffersen, and Pelletier (BCP) (2007) proposed a test similar to the Conditional Coverage test but that also tests autocorrelation in the exceedances at lagged intervals. Engle (1982) emphasizes that accurate interval forecasts should adapt to market volatility changes, ensuring that outliers are distributed across time and not concentrated during specific periods of market turbulence. The clustering of exceedances indicates that the VaR model is not sufficiently responsive to changing market circumstances.

In this work, a hedging strategy will be applied with the objective of keeping VaR below a predetermined threshold and also a portfolio target allocation of 60% stocks and 40% bonds with a 5% allowed deviation. As a result, three portfolios are considered: the unrebalanced unhedged, rebalanced unhedged, and the rebalanced hedged portfolio. To compare their performance, it is necessary to employ an appropriate performance measure. Conventional performance measures, such as Return on Assets (ROA), provide insights into profitability but fail to account for the risks undertaken to achieve those returns (Crouhy, Galai, & Mark, 2023). This limitation is particularly relevant when evaluating the analysed portfolios, given the differences in their risk profiles. In order to overcome this limitation, Risk-Adjusted Performance Measurement (RAPM) frameworks were developed. RAPM techniques represent a class of methodologies designed to evaluate financial performance by relating returns to the capital invested, while accounting for the risks undertaken. Unlike conventional performance measures, which may focus solely on profitability, RAPM models adjust the returns for the risk incurred in generating those returns (Crouhy, Galai, & Mark, 2023). Among the various models, there is Return on Risk-Adjusted Capital (RORAC). RORAC, as described by Matten (2000), is a measurement that relates returns to the level of risk undertaken, with risk quantified through the concept of Economic Capital.

To better understand VaR role in modern risk management, it is important to consider the historical evolution of Value at Risk. The concept of VaR was popularized in the early 1990s, particularly with the "Risk Metrics Technical Document" by JPMorgan. It presented VaR as a standardized risk methodology and set a point of reference for market risks since "Risks are comparable only when they are measured with the same yardstick" (J.P. Morgan & Reuters, 1996). Even though some of the ideas were new, they described practices that were already used (Holton, 2013).

VaR adoption by the Basel Committee in the 1996 Market Risk Amendment to Basel I marked a turning point, as it provided a methodology for calculating capital requirements against market risk, enabling greater comparability between financial institutions. Financial institutions could choose among two methodologies to calculate the capital charge, the standardized measurement method or VaR internal models (McAleer, Jiménez-Martín, & Pérez-Amaral, 2013; Basel Committee on Banking Supervision, 1996)

To assess the accuracy of the internal VaR models, the Basel market risk framework adopted a backtesting procedure that corresponds to comparing actual daily trading results with the corresponding VaR forecasts over a rolling 250-day period to determine whether the model's predictions align with observed outcomes. To interpret the results, the Basel Committee applies a system that classifies the number of VaR exceptions into three zones: green (0–4 exceptions), indicating model performance within statistical expectations; yellow (5–9 exceptions), suggesting potential deficiencies; and red (10 or more exceptions), that leads to an automatic presumption that a problem exists with the model. When the results fall into the yellow or red zones, the Basel framework mandates the application of a higher capital multiplier on the VaR based capital requirement (McAleer, Jiménez-Martín, & Pérez-Amaral, 2013).

Basel II, finalized in 2004, expanded the regulatory framework to incorporate credit risk and operational risk alongside market risk. However, during the 2007 credit crisis, certain limitations were exposed, leading to the realization that changes were necessary in the calculation of capital for market risk. As a response, the BCBS released in 2009 the "Revisions to the Basel II market risk framework", also known as Basel II.5, that required financial institutions to calculate two VaRs, the usual VaR and a stressed VaR (calculated from a stressed period of 250 days), in order to compute the total capital charge (Hull, 2015; Basel Committee on Banking Supervision, 2009).

On Basel III, namely on "Minimum capital requirements for market risk" from 2016 (revised on 2019), the Basel Committee considering the weaknesses of VaR, namely the inability to capture tail risk, changed from VaR at a confidence level of 99% to Expected Shortfall (ES) at a confidence level of 97,5% to compute the regulatory capital requirements (Chang, Jiménez, Maasoumi, McAleer, & Pérez-Amaral, 2019; Basel Committee on Banking Supervision, 2019).

Chapter 3 - Data and Portfolio Composition

The analyzed portfolio is composed of equity and debt securities. The sample period spans from January 2, 2007, to June 3, 2024, with May 29, 2023, treated as the current date, "today". The analysis is based on daily data observations.

With the Euro as the base currency, both equity and debt securities have securities that are denominated in their respective home currencies, including the Euro (EUR), US Dollar (USD), British Pound (GBP), Brazilian Real (BRL), Japanese Yen (JPY), and Hong Kong Dollar (HKD). The exchange rates used in this analysis are USDEUR, GBPEUR, BRLEUR, JPYEUR, and HKDEUR.

The equity component of the portfolio has 160 stocks, with both long and short positions. It was constructed by randomly selecting stocks from major indexes across various countries. It includes 20 stocks each from the S&P 500 (GSPC), Dow Jones Industrial Average (DJI), FTSE 100 (FTSE), Ibovespa (BVSP), Nikkei 225 (N225), and Hang Seng Index (HSI). Furthermore, it includes 30 stocks from Eurozone indices, with 5 stocks randomly selected from the Austrian Traded Index (ATX), BEL 20 (BFX), CAC 40 (FCHI), DAX 40 (GDAXI), IBEX 35 (IBEX), and FTSE MIB Index (FTSEMIB.MI).

The exchange rates and the adjusted close prices (prices adjusted for splits and dividends) from stocks and the corresponding stock market indices were retrieved from Yahoo Finance (https://finance.yahoo.com/).

The debt component is composed by 15 government bonds, with an AAA credit rate, based on the Standard & Poor's (S&P) scale. These bonds were randomly selected, being 6 issued by the United States, 5 by the Netherlands, and 4 by Germany. The bond maturities span from 2025 to 2033, being all coupon bonds with different coupon rates and coupon dates.

The bond data, including the maturity, coupon rate, coupon payment frequency, and currency, were obtained from Börse Frankfurt (https://www.boerse-frankfurt.de/en).

To assess the interest rate risk associated with the bonds, risk-free rates from the US and the Eurozone were obtained. Euro Zone risk-free rates are derived from the "Yield curve spot rate – Government bond, nominal, all issuers whose rating is triple A" sourced from the European Central Bank (ECB) (Spot curve | ECB Data Portal) for 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, and 10-year maturities. For the US, the risk-free rates are represented by the "Market Yield on US Treasury Securities at Constant Maturity", obtained from the St. Louis Federal Reserve System (FED) (FRED) (Treasury Constant Maturity | FRED | St. Louis Fed), for the same maturity spectrum.

The initial portfolio, on May 29, 2023, is composed as presented in tables 1 and 2 (stocks and bonds, respectively), with a total portfolio value of 67.506.586€ allocated 60% to stocks and 40% to bonds.

Stock	Ticker	Index	Quantity	Price (29/05/23)	Currency	Value (EUR)	Allocation (%)
NVIDIA Corporation	NVDA	GSPC	782	395,16	USD	288 177	0,71%
NRG Energy, Inc.	NRG	GSPC	(5 029)	32,92	USD	(154 385)	-0,38%
Deckers Outdoor Corporation	DECK	GSPC	(226)	477,15	USD	(100 565)	-0,25%
General Electric Company	GE	GSPC	4 332	81,50	USD	329 255	0,81%
Western Digital Corporation	WDC	GSPC	9 563	39,83	USD	355 211	0,88%
Micron Technology, Inc.	MU	GSPC	(5 576)	72,39	USD	(376 404)	-0,93%
Eli Lilly and Company	LLY	GSPC	877	423,35	USD	346 245	0,85%
Trane Technologies plc	TT	GSPC	(1 172)	164,27	USD	(179 548)	-0,44%
QUALCOMM Incorporated	QCOM	GSPC	3 906	109,89	USD	400 281	0,99%
Leidos Holdings, Inc.	LDOS	GSPC	1 467	78,62	USD	107 558	0,27%
United Rentals, Inc.	URI	GSPC	1 204	344,84	USD	387 186	0,96%
Eaton Corporation plc	ETN	GSPC	3 546	179,59	USD	593 887	1,47%
Domino's Pizza, Inc.	DPZ	GSPC	1 534	291,67	USD	417 250	1,03%
Royal Caribbean Group	RCL	GSPC	5 605	79,63	USD	416 230	1,03%
Amphenol Corporation	APH	GSPC	6 152	75,98	USD	435 931	1,08%
Westinghouse Air Brake Technologies Corporation	WAB	GSPC	(2 970)	94,06	USD	(260 527)	-0,64%
TransDigm Group Incorporated	TDG	GSPC	617	758,58	USD	436 486	1,08%
NetApp, Inc.	NTAP	GSPC	(3 731)	67,83	USD	(236 025)	-0,58%
PulteGroup, Inc.	PHM	GSPC	5 581	65,76	USD	342 248	0,85%
KLA Corporation	KLAC	GSPC	1 024	455,51	USD	434 991	1,07%
Caterpillar Inc.	CAT	DJI	2 319	207,02	USD	447 697	1,11%
Amazon.com, Inc.	AMZN	DJI	(3 854)	120,89	USD	(434 476)	-1,07%
JPMorgan Chase & Co.	JPM	DJI	4 098	133,66	USD	510 795	1,26%
The Goldman Sachs Group, Inc.	GS	DJI	1 597	319,33	USD	475 582	1,17%
Amgen Inc.	AMGN	DJI	2 533	211,13	USD	498 739	1,23%
American Express Company	AXP	DJI	2 909	155,46	USD	421 734	1,04%
Walmart Inc.	WMT	DJI	8 174	48,06	USD	366 381	0,90%
Microsoft Corporation	MSFT	DJI	1 737	329,47	USD	533 704	1,32%
International Business Machines Corporation	IBM	DJI	3 938	123,91	USD	455 062	1,12%
3M Company	MMM	DJI	4 231	75,75	USD	298 882	0,74%
Verizon Communications Inc.	VZ	DJI	(16 105)	32,53	USD	(488 621)	-1,21%
The Travelers Companies, Inc.	TRV	DJI	3 430	169,04	USD	540 696	1,34%
Merck & Co., Inc.	MRK	DJI	(2 249)	107,19	USD	(224 810)	-0,56%
The Procter & Gamble Company	PG	DJI	4 753	140,68	USD	623 585	1,54%
The Home Depot, Inc.	HD	DJI	(2 030)	283,08	USD	(535 907)	-1,32%
Salesforce, Inc.	CRM	DJI	(1 105)	216,87	USD	(223 484)	-0,55%
The Walt Disney Company	DIS	DJI	5 260	87,77	USD	430 535	1,06%
Apple Inc.	AAPL	DJI	4 303	175,43	USD	703 987	1,74%
Honeywell International Inc.	HON	DJI	4 087	189,91	USD	723 821	1,79%
The Coca-Cola Company	KO	DJI	(11 036)	58,17	USD	(598 629)	-1,48%
Rolls-Royce Holdings plc	RR.L	FTSE	1 436	146,72	GBP	242 588	0,60%
Marks and Spencer Group plc	MKS.L	FTSE	2 149	178,41	GBP	441 428	1,09%
Intermediate Capital Group plc	ICG.L	FTSE	232	1 305,51	GBP	348 723	0,86%
3i Group plc	III.L	FTSE	205	1 947,97	GBP	459 778	1,14%
Antofagasta plc	ANTO.L	FTSE	242	1 355,58	GBP	377 703	0,93%
BAE Systems plc	BA.L	FTSE	(105)	923,13	GBP	(111 600)	-0,28%
InterContinental Hotels Group PLC	IHG.L	FTSE	69	5 243,74	GBP	416 583	1,03%
Next plc	NXT.L	FTSE	47	6 210,49	GBP	336 074	0,83%
Associated British Foods plc	ABF.L	FTSE	225	1 807,72	GBP	468 302	1,16%
Barclays PLC	BARC.L	FTSE	2 740	148,90	GBP	469 731	1,16%
Howden Joinery Group Plc	HWDN.L	FTSE	719	648,55	GBP	536 887	1,33%
Diploma PLC	DPLM.L	FTSE	151	2 932,64	GBP	509 856	1,26%
RELX PLC	REL.L	FTSE	204	2 438,52	GBP	572 754	1,41%
Berkeley Group Holdings plc	BKG.L	FTSE	141	3 894,57	GBP	632 252	1,56%
Frasers Group plc	FRAS.L	FTSE	687	681,00	GBP	538 660	1,33%
Melrose Industries PLC	MRO.L	FTSE	(552)	472,50	GBP	(300 295)	-0,74%
Scottish Mortgage Investment Trust PLC	SMT.L	FTSE	691	669,76	GBP	532 854	1,32%
Taylor Wimpey plc	TW.L	FTSE	4 422	107,09	GBP	545 250	1,35%
Experian plc	EXPN.L	FTSE	62	2 784,32	GBP	198 757	0,49%
Lloyds Banking Group plc	LLOY.L	FTSE	7 443	42,86	GBP	367 271	0,91%
BRF S.A.	BRFS3.SA	BVSP	(67 645)	7,38	BRL	(93 154)	-0,23%
Embraer S.A.	EMBR3.SA	BVSP	115 817	18,81	BRL	406 511	1,00%
Petróleo Brasileiro S.A. - Petrobras	PETR4.SA	BVSP	(34 047)	21,65	BRL	(137 528)	-0,34%
Petróleo Brasileiro S.A. - Petrobras	PETR3.SA	BVSP	97 668	24,59	BRL	448 227	1,11%
Companhia de Saneamento Básico do Estado de São Paulo - SABESP	SBSP3.SA	BVSP	58 544	52,02	BRL	568 244	1,40%
Ultrapar Participações S.A.	UGPA3.SA	BVSP	91 455	16,47	BRL	281 100	0,69%
Companhia Paranaense de Energia - COPEL	CPLE6.SA	BVSP	(168 323)	7,37	BRL	(231 636)	-0,57%

Continues on the next page

Stock	Ticker	Index	Quantity	Price (29/05/23)	Currency	Value (EUR)	Allocation (%)
Banco do Brasil S.A.	BBAS3.SA	BVSP	(70 620)	20,83	BRL	(274 436)	-0,68%
CPFL Energia S.A.	CPFE3.SA	BVSP	121 705	28,83	BRL	654 824	1,62%
Itaúsa - Investimentos Itaú S.A.	ITSA4.SA	BVSP	126 858	7,77	BRL	183 895	0,45%
CEMIG - Companhia Energética de Minas Gerais	CMIG4.SA	BVSP	156 109	8,03	BRL	233 800	0,58%
Itaú Unibanco Holding S.A.	ITUB4.SA	BVSP	109 225	25,00	BRL	509 616	1,26%
TIM S.A.	TIMS3.SA	BVSP	117 715	13,42	BRL	294 669	0,73%
Transmissora Aliança de Energia Elétrica S.A.	TRPL4.SA	BVSP	94 734	23,04	BRL	407 256	1,01%
Telefônica Brasil S.A.	VIVT3.SA	BVSP	14 252	39,56	BRL	105 202	0,26%
ENGIE Brasil Energia S.A.	EGIE3.SA	BVSP	82 183	40,71	BRL	624 267	1,54%
Usinas Siderúrgicas de Minas Gerais S.A.	USIM5.SA	BVSP	567 304	7,04	BRL	744 959	1,84%
Cyrela Brazil Realty S.A. Empreendimentos e Participações	CYRE3.SA	BVSP	149 378	17,99	BRL	501 323	1,24%
Companhia Siderúrgica Nacional	CSNA3.SA	BVSP	128 620	11,29	BRL	270 902	0,67%
Suzano S.A.	SUZB3.SA	BVSP	88 503	44,34	BRL	732 309	1,81%
Fujifilm Holdings Corporation	5803.T	N225	38 611	1 054,40	JPY	269 387	0,67%
Kawasaki Kisen Kaisha, Ltd.	9107.T	N225	(45 774)	1 055,44	JPY	(319 678)	-0,79%
Mitsukoshi, Ltd.	3099.T	N225	(14 623)	1 421,40	JPY	(137 535)	-0,34%
Mitsubishi Heavy Industries, Ltd.	7011.T	N225	82 663	602,18	JPY	329 378	0,81%
Daiwa Securities Group Inc.	8601.T	N225	104 068	608,36	JPY	418 930	1,03%
SMC Corporation	7735.T	N225	8 652	7 017,38	JPY	401 747	0,99%
Nomura Holdings, Inc.	8604.T	N225	69 424	484,94	JPY	222 770	0,55%
Tokyo Electric Power Company Holdings, Inc.	9501.T	N225	(116 924)	480,00	JPY	(371 369)	-0,92%
Hitachi, Ltd.	6501.T	N225	10 339	8 005,30	JPY	547 668	1,35%
Idemitsu Kosan Co., Ltd.	5019.T	N225	100 748	537,65	JPY	358 426	0,89%
Sumitomo Mitsui Financial Group, Inc.	8316.T	N225	10 339	5 466,23	JPY	373 962	0,92%
EBARA Corporation	6361.T	N225	10 277	6 115,71	JPY	415 886	1,03%
Tokyo Electron Limited	8035.T	N225	(976)	19 454,84	JPY	(125 643)	-0,31%
The Kansai Electric Power Co., Inc.	9503.T	N225	22 867	1 558,20	JPY	235 773	0,58%
Mitsubishi UFJ Financial Group, Inc.	8306.T	N225	78 024	901,59	JPY	465 478	1,15%
Furukawa Electric Co., Ltd.	5801.T	N225	17 515	2 375,27	JPY	275 286	0,68%
Kawasaki Heavy Industries, Ltd.	7012.T	N225	11 263	3 099,51	JPY	230 998	0,57%
NEC Corporation	6701.T	N225	11 222	6 358,09	JPY	472 126	1,17%
Resona Holdings, Inc.	8354.T	N225	11 747	2 546,16	JPY	197 913	0,49%
Tokio Marine Holdings, Inc.	8766.T	N225	23 965	3 019,19	JPY	478 773	1,18%
CNOOC Limited	0883.HK	HSI	(19 685)	11,15	HKD	(26 119)	-0,06%
Lenovo Group Limited	0992.HK	HSI	140 308	7,10	HKD	118 588	0,29%
China Shenhua Energy Company Limited	1088.HK	HSI	54 063	23,26	HKD	149 664	0,37%
Orient Overseas (International) Limited	0316.HK	HSI	47 473	102,01	HKD	576 425	1,42%
Beijing Capital International Airport Company Limited	2899.HK	HSI	112 385	10,47	HKD	140 041	0,35%
PetroChina Company Limited	0857.HK	HSI	231 847	4,92	HKD	135 734	0,34%
Techtronic Industries Co. Ltd.	0669.HK	HSI	(38 581)	71,68	HKD	(329 166)	-0,81%
China Resources Power Holdings Company Limited	0836.HK	HSI	129 857	17,08	HKD	263 926	0,65%
Shenzhou International Group Holdings Limited	2313.HK	HSI	(69 638)	62,39	HKD	(517 140)	-1,28%
Bank of China Limited	3988.HK	HSI	(815 630)	2,88	HKD	(280 071)	-0,69%
HSBC Holdings plc	0005.HK	HSI	91 726	54,79	HKD	598 228	1,48%
China Mobile Limited	0941.HK	HSI	71 667	57,72	HKD	492 370	1,22%
Tencent Holdings Limited	0700.HK	HSI	17 353	310,52	HKD	641 377	1,58%
CLP Holdings Limited	0002.HK	HSI	92 856	53,96	HKD	596 386	1,47%
China Construction Bank Corporation	0939.HK	HSI	(197 255)	4,69	HKD	(110 067)	-0,27%
China Unicom (Hong Kong) Limited	0762.HK	HSI	640 169	5,47	HKD	416 848	1,03%
CK Infrastructure Holdings Limited	1038.HK	HSI	121 562	40,75	HKD	589 606	1,46%
Geely Automobile Holdings Limited	0175.HK	HSI	361 059	8,98	HKD	385 803	0,95%
Industrial and Commercial Bank of China Limited	1398.HK	HSI	859 584	3,91	HKD	400 007	0,99%
Power Assets Holdings Limited	0006.HK	HSI	(52 790)	40,18	HKD	(252 485)	-0,62%
Infineon Technologies AG	IIA.VI	ATX	26 931	15,26	EUR	410 967	1,01%
Erste Group Bank AG	EBS.VI	ATX	5 173	28,70	EUR	148 445	0,37%
EVN AG	EVN.VI	ATX	21 254	20,38	EUR	433 236	1,07%
Telekom Austria AG	TKA.VI	ATX	47 207	5,43	EUR	256 370	0,63%
DO & CO Aktiengesellschaft	DOC.VI	ATX	5 185	110,11	EUR	570 901	1,41%
UCB SA	UCB.BR	BFX	5 964	81,70	EUR	487 271	1,20%
D'Ieteren Group	DIE.BR	BFX	1 148	161,74	EUR	185 683	0,46%
Solvay SA	SOLB.BR	BFX	20 994	18,55	EUR	389 392	0,96%
Ageas SA/NV	AGS.BR	BFX	16 453	36,10	EUR	593 987	1,47%
Anheuser-Busch InBev SA/NV	ABI.BR	BFX	11 277	52,17	EUR	588 371	1,45%
Renault SA	RNO.PA	FCHI	22 695	31,79	EUR	721 578	1,78%
Safran SA	SAF.PA	FCHI	3 170	136,44	EUR	432 526	1,07%
Compagnie de Saint-Gobain S.A.	SGO.PA	FCHI	9 136	51,71	EUR	472 409	1,17%

Continues on the next page

Stock	Ticker	Index	Quantity	Price (29/05/23)	Currency	Value (EUR)	Allocation (%)
Publicis Groupe S.A.	PUB.PA	FCHI	6 393	67,51	EUR	431 573	1,07%
Schneider Electric S.E.	SU.PA	FCHI	2 653	161,68	EUR	428 946	1,06%
Rheinmetall AG	RHM.DE	GDAXI	1 419	244,89	EUR	347 505	0,86%
Commerzbank AG	CBK.DE	GDAXI	(35 534)	9,29	EUR	(330 245)	-0,82%
Deutsche Bank AG	DBK.DE	GDAXI	49 103	9,41	EUR	462 304	1,14%
adidas AG	ADS.DE	GDAXI	(1 144)	153,85	EUR	(176 005)	-0,43%
SAP SE	SAP.DE	GDAXI	4 229	120,36	EUR	508 991	1,26%
Banco de Sabadell, S.A.	SAB.MC	IBEX	30 255	0,92	EUR	27 892	0,07%
Indra Sistemas, S.A.	IDR.MC	IBEX	29 164	11,29	EUR	329 367	0,81%
Banco Bilbao Vizcaya Argentaria, S.A.	BBVA.MC	IBEX	67 841	6,04	EUR	410 066	1,01%
Banco Santander, S.A.	SAN.MC	IBEX	36 916	3,01	EUR	111 155	0,27%
Industria de Diseno Textil, S.A.	ITX.MC	IBEX	11 626	30,33	EUR	352 667	0,87%
Leonardo S.p.a.	LDO.MI	FTSEMIB.MI	(27 359)	10,64	EUR	(291 100)	-0,72%
Banca Monte dei Paschi di Siena S.p.A.	BMPS.MI	FTSEMIB.MI	112 666	1,97	EUR	221 776	0,55%
UniCredit S.p.A.	UCG.MI	FTSEMIB.MI	19 717	17,71	EUR	349 190	0,86%
BPER Banca S.p.A.	BPE.MI	FTSEMIB.MI	88 557	2,32	EUR	205 672	0,51%
UnipolSai Assicurazioni S.p.A.	UNI.MI	FTSEMIB.MI	80 595	4,54	EUR	366 266	0,90%
					Total Equity	**40 500 051**	**100%**

Table 1. Equity Portfolio on May 29, 2023. This table presents a summary of stock holdings, including the company name, the market identifier code (ticker), the index in which it is listed, the number of shares held (quantity), where the negative values correspond to short positions, the share price in its base currency, the value in euros, and the portfolio allocation.

Bond ISIN	Face Value	Currency	Maturity	Coupon Rate	Coupons/year	Face Value (EUR)	Fair Value (EUR)	Allocation (%)
US912828J272	2 752 400	USD	15/02/25	2,00%	2	2 566 806	2 469 688	9,14%
US9128286A35	1 365 900	USD	31/01/26	2,63%	2	1 273 797	1 232 881	4,57%
US912828X885	2 016 300	USD	15/05/27	2,38%	2	1 880 341	1 771 203	6,56%
US91282CBP59	2 153 400	USD	29/02/28	1,13%	2	2 008 196	1 776 053	6,58%
US9128286B18	2 632 200	USD	15/02/29	2,63%	2	2 454 711	2 322 181	8,60%
US912828Z948	3 017 600	USD	15/02/30	1,50%	2	2 814 123	2 446 750	9,06%
DE000BU22007	2 018 500	EUR	13/03/25	2,50%	1	2 018 500	2 014 041	7,46%
DE0001102390	1 798 500	EUR	15/02/26	0,50%	1	1 798 500	1 697 190	6,28%
NL0012171458	2 431 300	EUR	15/07/27	0,75%	1	2 431 300	2 274 376	8,42%
NL0000102317	1 520 300	EUR	15/01/28	5,50%	1	1 520 300	1 743 439	6,46%
DE0001102622	1 577 700	EUR	15/11/29	2,10%	1	1 577 700	1 556 140	5,76%
DE0001135143	1 679 500	EUR	04/01/30	6,25%	1	1 679 500	2 096 141	7,76%
DE0001135176	1 073 600	EUR	04/01/31	5,50%	1	1 073 600	1 314 223	4,87%
NL0015000RP1	1 446 900	EUR	15/07/32	0,50%	1	1 446 900	1 213 568	4,49%
NL0015001AM2	1 062 800	EUR	15/07/33	2,50%	1	1 062 800	1 078 663	3,99%
					Total Bonds	**27 607 074**	**27 006 535**	**100%**

Table 2. Bond Portfolio on May 29, 2023. This table summarizes the bond holdings, including the International Securities Identification Number (ISIN), the face value, its base currency, the maturity date, the coupon rate, the number of coupon payments per year, the face value in euros, and the fair value in euros.

Chapter 4 – Methodology

This work aims to evaluate and manage the risks of a diversified portfolio composed by stocks and bonds over a one-year period, using historical data from 2 January 2007 to 3 June 2024.

A risk factor mapping approach is used to align individual asset exposures with the relevant underlying risk factors. The returns of the risk factors are computed based on the historical data, and volatility is estimated using the Exponentially Weighted Moving Average (EWMA) method.

Four VaR models are implemented: RiskMetrics VaR, Skewed Generalized Student-t VaR (SGSt VaR), Adjusted Historical VaR, and Quantile Regression VaR (QR VaR). Each is tested under different specifications to assess robustness.

The model's performance is assessed through backtesting, using the Conditional Coverage test and Berkowitz, Christoffersen, and Pelletier (BCP) test. These tests verify the accuracy and reliability of VaR forecasts, and their results help identify the model that best captures the portfolio's risk profile.

4.1. Risk factor mapping

In order to address market risk, each asset exposure is mapped onto a set of predefined risk factors. Risk factors represent the market variables that drive changes in the value of the assets within the portfolio.

Table 3 presents the risk factor mapping as of May 29, 2023. In the following sections it is addressed the methodology used to map the different risk factors.

4.1.1. Equity

For the equity assets, the relevant risk factor is the market price of each stock, as the value of an equity position depends on its market price, which reflects all publicly available information and investor sentiment at a given point in time. Any fluctuation in this price affects the position's value and, in turn, the overall portfolio value.

The exposure to this risk factor is determined by the size of the position held in each stock. Therefore, the capital at risk is calculated by multiplying the number of shares held (N_{i_t}), by the stock's current market price(P_{i_t}). Given that the portfolio's base currency is the euro, and some stocks are in foreign currencies, the market prices are converted using the applicable exchange rate (FX_t) to ensure consistency across all positions:

$$E_{i_t} = N_{i_t} * P_{i_t} * FX_t. \tag{1}$$

Equity					
Risk Factor	**Exposure (EUR)**	**Risk Factor**	**Exposure (EUR)**	**Risk Factor**	**Exposure (EUR)**
NVDA	288 177	HWDN.L	536 887	0883.HK	(26 119)
NRG	(154 385)	DPLM.L	509 856	0992.HK	118 588
DECK	(100 565)	REL.L	572 754	1088.HK	149 664
GE	329 255	BKG.L	632 252	0316.HK	576 425
WDC	355 211	FRAS.L	538 660	2899.HK	140 041
MU	(376 404)	MRO.L	(300 295)	0857.HK	135 734
LLY	346 245	SMT.L	532 854	0669.HK	(329 166)
TT	(179 548)	TW.L	545 250	0836.HK	263 926
QCOM	400 281	EXPN.L	198 757	2313.HK	(517 140)
LDOS	107 558	LLOY.L	367 271	3988.HK	(280 071)
URI	387 186	BRFS3.SA	(93 154)	0005.HK	598 228
ETN	593 887	EMBR3.SA	406 511	0941.HK	492 370
DPZ	417 250	PETR4.SA	(137 528)	0700.HK	641 377
RCL	416 230	PETR3.SA	448 227	0002.HK	596 386
APH	435 931	SBSP3.SA	568 244	0939.HK	(110 067)
WAB	(260 527)	UGPA3.SA	281 100	0762.HK	416 848
TDG	436 486	CPLE6.SA	(231 636)	1038.HK	589 606
NTAP	(236 025)	BBAS3.SA	(274 436)	0175.HK	385 803
PHM	342 248	CPFE3.SA	654 824	1398.HK	400 007
KLAC	434 991	ITSA4.SA	183 895	0006.HK	(252 485)
CAT	447 697	CMIG4.SA	233 800	IIA.VI	410 967
AMZN	(434 476)	ITUB4.SA	509 616	EBS.VI	148 445
JPM	510 795	TIMS3.SA	294 669	EVN.VI	433 236
GS	475 582	TRPL4.SA	407 256	TKA.VI	256 370
AMGN	498 739	VIVT3.SA	105 202	DOC.VI	570 901
AXP	421 734	EGIE3.SA	624 267	UCB.BR	487 271
WMT	366 381	USIM5.SA	744 959	DIE.BR	185 683
MSFT	533 704	CYRE3.SA	501 323	SOLB.BR	389 392
IBM	455 062	CSNA3.SA	270 902	AGS.BR	593 987
MMM	298 882	SUZB3.SA	732 309	ABI.BR	588 371
VZ	(488 621)	5803.T	269 387	RNO.PA	721 578
TRV	540 696	9107.T	(319 678)	SAF.PA	432 526
MRK	(224 810)	3099.T	(137 535)	SGO.PA	472 409
PG	623 585	7011.T	329 378	PUB.PA	431 573
HD	(535 907)	8601.T	418 930	SU.PA	428 946
CRM	(223 484)	7735.T	401 747	RHM.DE	347 505
DIS	430 535	8604.T	222 770	CBK.DE	(330 245)
AAPL	703 987	9501.T	(371 369)	DBK.DE	462 304
HON	723 821	6501.T	547 668	ADS.DE	(176 005)
KO	(598 629)	5019.T	358 426	SAP.DE	508 991
RR.L	242 588	8316.T	373 962	SAB.MC	27 892
MKS.L	441 428	6361.T	415 886	IDR.MC	329 367
ICG.L	348 723	8035.T	(125 643)	BBVA.MC	410 066
III.L	459 778	9503.T	235 773	SAN.MC	111 155
ANTO.L	377 703	8306.T	465 478	ITX.MC	352 667
BA.L	(111 600)	5801.T	275 286	LDO.MI	(291 100)
IHG.L	416 583	7012.T	230 998	BMPS.MI	221 776
NXT.L	336 074	6701.T	472 126	UCG.MI	349 190
ABF.L	468 302	8354.T	197 913	BPE.MI	205 672
BARC.L	469 731	8766.T	478 773	UNI.MI	366 266
				Total	**40 500 051**

Bonds	
Risk Factor	**Exposure (EUR)**
USD3M	(3)
USD6M	(3)
USD1Y	(85)
USD2Y	(457)
USD3Y	(614)
USD5Y	(2 007)
USD7Y	(1 845)
USD10Y	0
EUR3M	(2)
EUR6M	(11)
EUR1Y	(65)
EUR2Y	(469)
EUR3Y	(860)
EUR5Y	(1 742)
EUR7Y	(2 398)
EUR10Y	(1 913)
Total	**(12 475)**

Currency	
Risk Factor	**Exposure (EUR)**
USDEUR	20 527 511
GBPEUR	7 583 555
BRLEUR	6 230 351
JPYEUR	4 740 277
HKDEUR	3 989 955
Total	**43 071 649**

Table 3. Risk factor exposure mapping on May 29, 2023. This table presents the asset exposure mapping, in euros, to its corresponding risk factor: equity, bonds and currency.

4.1.2. Bonds

Bonds are considered fixed-income securities since they provide predetermined cash flows, consisting of coupon payments

$$C_{i_t} = N * \frac{c_n}{n}, \tag{2}$$

where N is the face value, c_n is the annual coupon rate, n is the coupon frequency, and the repayment of principal at maturity (T)

$$C_{i_T} = N * \left(1 + \frac{c_n}{n}\right). \tag{3}$$

The fair value of a bond (PV_{i_t}) is determined by continuously discounting its future cash flows (C_{i_t}), using a discount rate that reflects current market interest rates (r_t),

$$PV_{i_t} = \sum_{t}^{T} C_{i_t} * e^{-r_t * t}. \tag{4}$$

When interest rates rise (fall), the discount factor $(e^{-r_t * t})$ increases (decreases), which reduces (increases) the present value of future payments and, consequently, the bond's price. As a result, interest rates represent the key source of risk for bonds, and each bond is exposed to as many interest rate risk factors as cash flow maturities (t).

However, in a large portfolio, cash flow maturities may occur on a daily basis, implying the need to monitor the volatilities and correlations of interest rates for a wide range of maturities, which could become unmanageable. Additionally, the interest rate data may not be available for all relevant maturities. To address these aspects, Alexander (2008) proposes that the cash flows are mapped to standard maturities of a term structure of interest rates.

Interest rates at various maturities are then computed by applying linear interpolation between the closest larger and smaller standard maturities, the upper and lower vertices, respectively,

$$\begin{cases} w_{t_L} = \dfrac{t_U - t}{t_U - t_L}, \\ w_{t_U} = 1 - w_{t_L} \end{cases} \tag{5}$$

where t_L and t_U are the maturities of the lower and upper vertices, respectively, w_{t_L} and w_{t_U} are the weights allocated to the lower and upper vertices, respectively, and t is the cash-flow maturity. The interpolated interest rates are then calculated with the formula:

$$r_t = r_L * w_{t_L} + r_U * w_{t_U}, \tag{6}$$

where r_L and r_U are the interest rates of the lower and upper vertices.

The present value of a basis point (PV01) measures the sensitivity of a cash flow to a decrease of 1 basis point in market interest rates. Given the small change of the interest rate (1 basis point), the PV01 of a cash flow (C_{i_t}) can be approximated using a first-order Taylor expansion expressed as

$$\begin{aligned} PV01_{C_{i_t}} &\approx \frac{\partial PV_{i_t}}{\partial r_T} * (-0{,}01\%) \\ &= t * PV_{i_t} * 0{,}01\%. \end{aligned} \tag{7}$$

The exposure to the interest rate risk that each vertex represents is then

$$-PV01_{j_t} = PV_{i_t} * w_{j_t} * t_j * (-0{,}01\%), \tag{8}$$

where w_{j_t} is the weight allocated to the vertex and t_j is the maturity of the vertex.

4.1.3. Currency risk factor

The portfolio under analysis has positions in assets that are not denominated in the base currency (EUR) therefore, these positions are not only exposed to the risk factor of the asset but also to the exchange rate risk factor. The assets denominated in USD, GBP, BRL, JPY, and HKD are exposed to the USDEUR, GBPEUR, BRLEUR, JPYEUR, and HKDEUR risk factors, respectively, by the total amount of the investment in EUR.

4.2. Volatility

Volatility represents the degree of uncertainty in asset returns, and it is a critical component in assessing risk, as it directly influences the VaR calculation. Since volatility in financial markets is not constant over time, the choice of the estimation method can significantly influence the quality and responsiveness of the risk assessment.

The most common approach to estimate volatility is the historical method, where the standard deviation is calculated based on a fixed window of past returns, assigning an equal weight to each observation. This method assumes a stable market environment and can lag in capturing sudden changes in volatility, particularly during periods of market stress.

To address this limitation, this analysis adopts the Exponentially Weighted Moving Average (EWMA) model. Unlike the equal-weighted approach, EWMA assigns more weight to recent return observations, allowing the volatility estimate to adjust more rapidly to current market dynamics. The variance at time t is computed as

$$\hat{\sigma}_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \hat{\sigma}_{t-1}^2, \tag{9}$$

where λ is the smoothing factor, r_{t-1}^2 is the squared return from the previous day, and $\hat{\sigma}_{t-1}^2$ is the variance from the previous period. The parameter λ takes a value between 0 and 1 and determines how quickly the influence of past observations declines over time. A higher λ gives more weight to past variances, causing the model to react more smoothly and slowly to new information. In contrast, a lower λ gives more importance to recent returns, making the volatility estimate more responsive to sudden market changes. The adopted smoothing factor is $\lambda = 0{,}94$, as recommended by the RiskMetrics Technical Document (J.P. Morgan, 1996). This value was empirically chosen for daily volatility estimation since it provides a trade-off between responsiveness to new market conditions and stability of the volatility estimate.

By recursively substituting σ, the formula becomes:

$$\hat{\sigma}_t^2 = (1-\lambda) \sum_{i=1}^{\infty} \lambda^{i-1} r_{t-i}^2 . \tag{10}$$

Equation 10 demonstrates that, although all past observations contribute to the estimate, over time, their weight decreases exponentially.

4.3. Value-at-Risk Models

Value at Risk (VaR) summarizes the worst expected loss, in present value terms, over a time horizon, h, that will not be exceeded with a given confidence level, 1-α. It answers the question: "What is the worst expected loss over a given time period under normal market conditions, at a certain confidence level?"

Formally, VaR corresponds to the negative of the lower α-quantile of the projected return distribution, discounted over the time horizon

$$VaR_{h,\alpha} = -F^{-1}(\alpha)\ , \tag{11}$$

where $F^{-1}(\alpha)$ is the inverse cumulative distribution function (CDF) of the returns of the portfolio over the time horizon h. Since losses are negative returns, the inverse CDF is multiplied by -1 to express VaR as a positive potential loss.

On the following sections are presented the 4 VaR models: RiskMetrics VaR, Skewed Generalized Student-t VaR (SGSt VaR), Adjusted Historical VaR, and Quantile Regression VaR (QR VaR).

4.3.1. RiskMetrics VaR

The RiskMetrics VaR is a VaR model that assumes that the portfolio's discounted returns over an h-day period (X_h), are normally distributed and independent across time

$$X_h \overset{\text{i.i.d.}}{\sim} N\left(\mu_h, \sigma_h^2\right)\ , \tag{12}$$

where (μ_h) represents the expected return and $\left(\sigma_h^2\right)$ the variance of returns over the h period.

The RiskMetrics VaR is then computed by the formula:

$$VaR_{h,\alpha} = -\Phi^{-1}_{\mu_h,\sigma_h}(\alpha), \tag{13}$$

where $-\Phi^{-1}_{\mu_h,\sigma_h}(\alpha)$ denotes the α quantile of the normal distribution with mean μ_h and standard deviation σ_h. By transforming the normal variable to a standard normal variable, the formula becomes:

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha)\sigma_h - \mu_h\ , \tag{14}$$

where $\Phi^{-1}(\alpha)$ is the inverse standard normal cumulative distribution function.

The expected return of the portfolio (μ_h) is assumed to be zero since for short-term horizons such as daily VaR $(h = 1)$, the expected excess return is usually small. By assuming an expected return of zero, it is assumed that the average portfolio return equals the risk-free rate, consistent with the risk-neutral measure commonly used in financial modelling and regulatory capital calculations. The standard deviation (σ_h) is estimated using the EWMA volatility model (equation 10). Under these assumptions, the final formula for RiskMetrics VaR becomes:

$$VaR_{1,\alpha} = -\Phi^{-1}(\alpha)\sigma_1. \tag{15}$$

4.3.2. Skewed Generalized Student t VaR

The probability density function (PDF) of the Skewed Generalized t-distribution is:

$$f(x,\mu,\sigma,\lambda,p,q) = \frac{p}{2v\sigma q^{\frac{1}{p}} B\left(\frac{1}{p},q\right)} \frac{1}{\left[\frac{1}{q}\left(\frac{|x-\mu+m|}{v\sigma(\lambda sign(x-\mu+m)+1)}\right)^{p}+1\right]^{\frac{1}{p}+q}}\,, \tag{16}$$

and

$$m = \frac{2v\sigma\lambda q^{\frac{1}{p}} B\left(\frac{2}{p},1-\frac{1}{p}\right)}{B\left(\frac{1}{p},q\right)} \tag{17}$$

$$v = q^{-\frac{1}{p}}\left[(3\lambda^2+1)\frac{B\left(\frac{3}{p},q-\frac{2}{p}\right)}{B\left(\frac{1}{p},q\right)} - 4\lambda^2\left(\frac{B\left(\frac{2}{p},q-\frac{1}{p}\right)}{B\left(\frac{1}{p},q\right)}\right)^2\right]^{-\frac{1}{2}} \tag{18}$$

$$sign(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} \tag{19}$$

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt\,, \tag{20}$$

where μ is the mean of the distribution, $\sigma > 0$ is the standard deviation, $-1 < \lambda < 1$ determines the skewness, $p > 0$ controls kurtosis and tail thickness, $q > 0$ controls the decay rate of the distribution tails, m is the skewness shift adjustment (ensures the distribution remains centred at μ), and v is normalization parameter (ensures unit variance).

The n-th moment of the distribution is only defined if $pq > n$. Therefore, in order to use the mean/standard-deviation parameterization $pq > 2$.

We estimate the parameters by maximizing the likelihood function

$$\left(\hat{\mu},\hat{\sigma},\hat{\lambda},\hat{p},\hat{q}\right) = \max_{\mu,\sigma,\lambda,p,q} \sum_{i=1}^{n} \ln[f(x_{t-i},\mu,\sigma,\lambda,p,q)]\,. \tag{21}$$

The VaR formula, when the portfolio returns follow a SGSt distribution, is

$$VaR_{h,\alpha} = -T^{-1}_{\mu,\sigma,\lambda,p,q}(\alpha)\,, \tag{22}$$

where $T^{-1}_{\mu,\sigma,\lambda,p,q}(\alpha)$ is the inverse of the SGSt distribution. By using the equivariance property of the quantile function, the VaR formula becomes

$$VaR_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) * \sigma - \mu. \tag{23}$$

Following the same reasoning of the RiskMetrics VaR, $\mu = 0$, σ is estimated using the EWMA volatility model (equation 10) and $h = 1$ (daily VaR). Under these assumptions, the final formula for SGSt VaR becomes:

$$VaR_{1,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) * \sigma. \tag{24}$$

4.3.3. Historical VaR

While Risk Metrics VaR and Skewed Generalized Student t VaR models rely on specific assumptions about the distribution of returns, Historical VaR is a non-parametric model that uses directly the empirical distribution of past returns estimating VaR based on the α-quantile of this distribution.

The sample size on this model is crucial since the entire distribution is based on it. The minimum size must be at least $\frac{1}{\alpha}$, which in this analysis corresponds to at least 100 observations. Using the minimum number of observations can make the VaR highly sensitive to outliers. On the other hand, using a very large sample size may include outdated market conditions that are no longer relevant to current risks, especially since the model assumes constant risk sensitivities over the period. The choice of sample size is also influenced by data frequency, since low-frequency data would lead to going further into the past, which might reduce relevance to the current risks, and the data might not be available. To ensure consistency across the various models, it is adopted a sample size of 800 observations using daily data.

Historical VaR assumes that past volatility is representative of current risk. But if market volatility increases or decreases, the unadjusted historical returns might under- or overestimate actual risk. To improve the sensitivity of the historical VaR to current market conditions, as suggested by Hull and White (1998), the volatility of the series of returns can be adjusted to the current volatility so that the whole sample reflects the current market conditions. To compute the adjusted returns $(\hat{r}_t)$, each historical return (r_t) is scaled by the ratio of current volatility $(\hat{\sigma}_T)$ to historical volatility $(\hat{\sigma}_t)$,

$$\hat{r}_t = \frac{\hat{\sigma}_T}{\hat{\sigma}_t} r_t\,, \tag{25}$$

where volatilities are computed by the EWMA model (equation 10).

To compute the VaR, the time series of unadjusted and adjusted historical returns over the past 800 days are first sorted in ascending order. The historical VaR at the 99% confidence level corresponds to the 8th (1% of 800) most negative return observation in these sorted distributions.

4.3.4 Quantile Regression Var

Unlike traditional regressions (such as OLS) that focus on estimating the conditional mean, quantile regression estimates specific quantiles of the distribution of financial returns. By using quantile regression, the VaR of a portfolio is computed based on historical data and explanatory variables

$$VaR_{\alpha} \equiv -q_{y,\alpha} = -\left(\hat{a} + \hat{b}x_i\right), \tag{26}$$

where y is the portfolio returns (dependent variable), x_i is the EWMA volatility estimates (explanatory variable), and $\hat{a}$ and $\hat{b}$ are the estimated intercept and slope coefficients from the quantile regression. These coefficients are obtained by solving the following minimization, as introduced by Koenker and Bassett (1978):

$$\left(\hat{a}, \hat{b}\right) = \arg \min_{a,b} \sum_{i=1}^{n} [y_i - (a + bx_i)]\left(\alpha - I_{y_i-(a+bx_i)<0}\right), \tag{27}$$

where $I_{y_i-(a+bx_i)<0}$ is an indicator function of event

$$I_{y_i-(a+bx_i)<0} = \begin{cases} 1 & \text{if } y_i - (a + bx_i) < 0 \\ 0 & \text{if } y_i - (a + bx_i) \geq 0 \end{cases}. \tag{28}$$

The parameters are estimated using a sample size of 800 daily observations, and they are re-estimated every trading day.

4.4. Backtest

While VaR provides a theoretical estimate of potential losses under normal market conditions, it is essential to evaluate the accuracy of these estimates when compared to observed outcomes. Backtesting compares the VaR values forecasts with realized portfolio losses over a historical period and records the number of exceedances. A VaR exceedance occurs when the loss on a portfolio exceeds the symmetric of the VaR estimate for that day. To identify if an exceedance has occurred it is used the indicator function

$$I_{\alpha,t} = \begin{cases} 1 & \text{if } r_t < VaR_{1,\alpha,t} \\ 0 & \text{if } r_t \geq VaR_{1,\alpha,t} \end{cases}, \tag{29}$$

where $r_t$is the return observed for day t and $VaR_{1,\alpha,t}$ is the VaR estimated for day t.

In this analysis, backtesting is conducted using daily VaR estimates computed for the portfolio held on May 29th, 2023. The underlying risk factor returns are calculated using historical data spanning the past 10 years, from June 13, 2013, to May 31, 2023.

The following sections detail the tests used to backtest the VaR Models: the Unconditional Coverage Test and the Berkowitz, Christoffersen, and Pelletier (BCP) Test.

4.4.1. Unconditional Coverage Test

The Unconditional Coverage (UC) Test evaluates whether the number of VaR exceedances aligns with the expected number given the chosen confidence level. This test is based on a likelihood ratio framework.

To test the unconditional coverage, as proposed by Kupiec (1995), the hypotheses are formulated

$$\begin{aligned} H_0&: \pi_{obs} = \alpha \\ H_1&: \pi_{obs} \neq \alpha \end{aligned}. \tag{30}$$

If H_0 is not rejected, the model's exceedances are consistent with the expected level, if H_0 is rejected the model underestimates or overestimates risk, as the number of exceedances are statistically inconsistent with the assumed VaR confidence level.

The likelihood under the null hypothesis H_0 is

$$L(\alpha, I_1, I_2, \dots, I_n) = (1-\alpha)^{n-n_e} * \alpha^{n_e}, \tag{31}$$

and under the alternative hypothesis H_1

$$L(\pi_{obs}, I_1, I_2, \dots, I_n) = (1-\pi_{obs})^{n-n_e} * \pi_{obs}^{n_e}, \tag{32}$$

where n is the number of observations, n_e is the number of exceedances, $n - n_e$ is the number of non-exceedances and π_{obs} is the exceedance rate in the backtesting period $\left(\frac{n_e}{n}\right)$ and α is the VaR significance level.

The likelihood ratio test is then:

$$LR_{uc} = -2\ln\left[\frac{L(\alpha)}{L(\pi_{obs})}\right] = -2[\ln L(\alpha) - \ln L(\pi_{obs})] \sim \chi^2(1), \tag{33}$$

thus:

$$\begin{aligned} LR_{uc} &= -2[(n-n_e)\ln(1-\alpha) + n_e\ln(\alpha) - (n-n_e)\ln(1-\pi_{obs}) - n_e\ln(\pi_{obs})] = \\ &= -2[n_e(\ln\alpha - \ln\pi_{obs}) + (n-n_e)(\ln(1-\alpha) - \ln(1-\pi_{obs}))] \end{aligned}. \tag{34}$$

4.4.2. Berkowitz, Christoffersen, and Pelletier (BCP) Test

The Unconditional Coverage Test only checks frequency, not the independence or clustering of exceedances. As a result, models that exhibit clustering of exceedances may still pass the UC test despite being misspecified. The Berkowitz, Christoffersen, and Pelletier test addresses this limitation by testing if the exceedances are independent. As proposed by Berkowitz, Christoffersen, and Pelletier (2007), the hypotheses are formulated

$$\begin{aligned} H_0&: \hat{\rho}_k = 0, \forall k \in \{1, \dots, K\} \\ H_1&: \exists k \in \{1, \dots, K\} \text{ such that } \hat{\rho}_k \neq 0 \end{aligned}, \tag{35}$$

where $\hat{\rho}_k$ is the k-th order autocorrelation of the time series of exceedances, and K is the maximum autocorrelation lag considered in the test, If the null hypothesis H_0 is not rejected, the VaR model is well specified, and exceedances are independent at all lags. If the null hypothesis H_0 is rejected, then at least one autocorrelation is significantly different from 0, indicating a model misspecification.

The exceedances sequence has a first-order autocorrelation of zero under the null hypothesis. The BCP test can be computed through the Ljung-Box statistics

$$LB(K) = T(T+2)\sum_{k=1}^{K}\frac{\hat{\rho}_k^2}{T-k}, \tag{36}$$

where T is the total number of observations (sample size). The test statistic has an asymptotic chi-square distribution with k degrees of freedom,

$$LB(K) \sim \chi_k^2. \tag{37}$$

Although the number of lags k is not fixed, a higher K captures longer-range autocorrelation but increases the degrees of freedom, raising the critical value and reducing the test's power. On the other hand, a lower k improves the test's ability to reject the null under strong short-term dependence but limits its ability to capture autocorrelation beyond the chosen horizon.

To balance these trade-offs, the BCP test is computed for $K = 1$ to $K = 10$, testing the autocorrelation until the 10th lag. For each test period, it is selected the lag K that yields the lowest p-value among the tests.

4.5 Hedging

To implement a hedging strategy, it is necessary to evaluate how individual risk factors contribute to the total VaR. A Marginal VaR decomposition allows to isolate and quantify the portion of portfolio VaR attributable to each specific exposure. Therefore, the Marginal VaR allows to determine the positions that can effectively reduce the Value at Risk by identifying the most impactful risk factor exposures.

4.5.1 Gradient Vector

The gradient vector, denoted by ∇VaR, is a vector that captures the first-order sensitivities of the portfolio's VaR with respect to small changes in exposure to each of the n underlying risk factors,

$$\nabla VaR = \begin{bmatrix} \frac{\partial VaR}{\partial \theta_1} \\ \frac{\partial VaR}{\partial \theta_2} \\ \vdots \\ \frac{\partial VaR}{\partial \theta_n} \end{bmatrix}, \tag{38}$$

where the vector of risk factors is

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}. \tag{39}$$

4.5.2 Marginal VaR

The Marginal VaR quantifies the contribution of the decomposition risk factor (Θ^S) to the total portfolio VaR. The Marginal VaR it is computed using the gradient vector (equation 38)

$$Marginal\ VaR^S = \nabla VaR^T \Theta^S\ , \tag{40}$$

where

$$VaR = \sum_{S=1}^{m} Marginal\ VaR^S.$$

A decomposition of the vector of risk factor loadings (Θ) can be any that satisfies:

$$\sum_{S=1}^{m} \Theta^S = \Theta \tag{41}$$

4.5.3 Hedging position

Following the Marginal VaR decomposition, the risk factors that exhibit the highest marginal contributions to the total portfolio VaR are identified as primary candidates for hedging. These are the exposures whose small changes result in the most significant impact on portfolio risk. Given a target change in the VaR (ΔVaR) the required hedging position (Θ^S) is computed with the formula:

$$\Delta VaR \approx \frac{\partial VaR}{\partial \Theta^S} * \Theta^S + \frac{\partial VaR}{\partial \Theta_{FX}^S} * \Theta^S\ , \tag{42}$$

where $\frac{\partial VaR}{\partial \Theta^S}$ is the gradient of the decomposition risk factor, Θ_{FX}^S is the currency risk factor associated with the hedging position, given that if the hedge is held in a foreign currency, the exposure to that currency will vary proportionally with the position, and $\frac{\partial VaR}{\partial \Theta_{FX}^S}$ is the gradient of the currency risk factor. Solving equation 42 for the hedging position Θ^S:

$$\Theta^S \approx \frac{\Delta VaR}{\frac{\partial VaR}{\partial \Theta^S} + \frac{\partial VaR}{\partial \Theta_{FX}^S}}\ . \tag{43}$$

4.6. Return on Risk-Adjusted Capital (RORAC)

The objective for optimal capital allocation is to achieve the best risk-adjusted performance. Return on Risk-Adjusted Capital (RORAC) is a Risk-adjusted performance measurement that relates returns (R) to the level of risk undertaken. The level of risk undertaken is measured by an internal measure of capital at risk, namely Economic Capital (EC),

$$RORAC = \frac{R}{EC}. \tag{44}$$

According to Jorion (2007), VaR can be viewed as a measure of Economic Capital, therefore

$$RORAC = \frac{R}{VaR}. \tag{45}$$

Chapter 5 - Model Choice

The backtesting window is of 2600 trading days, covering the period from June 11, 2013, to May 29, 2023. This ten-year period includes a diverse range of market conditions, including periods of stability and stress.

The backtesting is conducted using a consistent set of parameters across all models, as presented in Table 4. The EWMA smoothing factor is set to 0,94 as proposed by J.P. Morgan (1996), ensuring both responsiveness to recent market changes and robustness of volatility estimates.

In order not to overestimate the Historical VaR and the Quantile Regression VaR, the product of the confidence level and the sample size must be an integer:

$$\alpha * n \in \mathbb{Z} \tag{46}$$

Therefore, a rolling sample size of 800 observations is used for all models except for RiskMetrics, whose estimation methodology does not rely on a predefined sample window. The 800 daily observations correspond approximately to 3 years of data, assuming that one year has 260 trading days.

A daily estimation frequency is adopted for models where time-series updating is relevant, namely the SGSt and Quantile Regression specifications. This frequency aligns with the availability of financial market data and ensures that model parameters are regularly updated to reflect the latest market information.

	EWMA smoothing factor	Sample Size	Estimation Frequency
RiskMetrics VaR	0,94	n.a.	n.a.
SGSt VaR	0,94	800	Daily
Unadjusted Historical VaR	0,94	800	n.a.
Adjusted Historical VaR	0,94	800	n.a.
Quantile Regression VaR	0,94	800	Daily

Table 4. Backtesting model parameters. This table presents the parameters applied to each model backtesting, namely the EWMA smoothing factor, the sample size, and the estimation frequency.

Table 5 resumes the backtesting results, including the number of exceedances, the exceedance rate, the p-value for the Unconditional Coverage Test, and the p-value, lag, and clustering for the BCP test.

When comparing the VaR forecasts with the realized portfolio losses over the backtesting period from June 11, 2013, to May 29, 2023, it was expected to observe approximately 26 exceedances $(n * \alpha)$ based on the 2600 observations (n) at a confidence level (α) of 1%.

All models exceed this threshold, indicating a tendency to underestimate the risk. Nevertheless, Risk Metrics, SGSt, and Adjusted Historical Var models recorded exceedances notably above the expected level with 49, 45, and 43 exceedances registered, respectively.

The Unadjusted Historical and Quantile Regression VaR models registered only 35 and 32 exceedances, respectively, closely aligned with the theoretical expectation.

The Unconditional Coverage test corroborates this interpretation. At a 5% significance level, the test rejects the null hypothesis (equation 30) for Risk Metrics, Adjusted Historical, and SGSt VaR models. On the other hand, the Unadjusted Historical and Quantile Regression VaR models yield a p-value of 9,2% and 25,37% respectively, therefore the null hypothesis is not rejected, and the number of exceedances is statistically consistent with the expected level.

All models fail the Berkowitz, Christoffersen, and Pelletier (BCP) test, rejecting the null hypothesis (equation 35), implying that at least one autocorrelation of exceedances is different from 0. As a result, none of the models can ensure the independence of the exceedances.

			UC Test	BCP Test		
Model	Number of exceedances	Exceedance rate	p-value	p-value	Lag	Clustering
RiskMetrics VaR	49	1,88%	0,01%	0,00%	1	6
SGSt VaR	45	1,73%	0,07%	0,00%	1	5
Unajusted Historical VaR	35	1,35%	9,20%	0,00%	10	62
Adjusted Historical VaR	43	1,65%	0,22%	0,00%	1	6
Quantile Regression VaR	32	1,23%	25,37%	0,00%	1	6

Table 5. Backtesting Model Evaluation Results. This table summarizes the performance of the models under backtesting, including the number of exceedances, the exceedance rate, and the results of the Unconditional Coverage (UC) test and the Berkowitz, Christoffersen, and Pelletier (BCP) test.

To assess the temporal robustness of model performance for the VaR models that performed better at the Unconditional Coverage Test, namely the Unadjusted Historical model and the Quantile Regression model (table 5), the 2600 day backtesting window was decomposed into ten non-overlapping sub-periods of 260 trading days each (approximately one year) in order to analyse the stability and consistency of model performance over time, particularly across different market regimes (table 6).

In terms of the Unconditional Coverage test, the expected exceedance rate would be approximately 3 exceedances $(260 * 1\%)$. The Unadjusted Historical Var performs satisfactorily in 5 out of 10 sub-periods, with p-values exceeding 5%. In the remaining 5 subperiods, the null hypothesis is rejected (equation 30), and in 3 out of those 5 periods, the risk is overestimated, producing fewer exceedances than expected. A notable underestimation of risk occurs during the period from June 4, 2019, to June 3, 2020, where 17 exceedances were recorded, reflecting a model's failure to predict extreme market movements during periods of elevated volatility, namely the COVID-19 market shock. The Quantile Regression VaR model shows a more stable performance over time in terms of the Unconditional Coverage test, where, across the 10 sub-periods, it produces acceptable exceedance rates in 8 out of 10 sub-periods, with p-values above 5%, suggesting stronger alignment with the theoretical exceedance frequency. Similar to the Unadjusted Historical model, it performs poorly during the 2019-2020 window, with 7 exceedances recorded and a p-value of 2,34%, thereby underestimating portfolio risk during one of the most turbulent market regimes in the sample.

			Unadjusted Historical VaR					
Backtest Period		Number of	Number of	Exceedance	Unconditional	BCP test		
Begin	End	observations	exceedances	rate	Coverage Test	p-value	Lag	Clustering
11-06-2013	**29-05-2023**	**2600**	**35**	**1,35%**	**9,2%**	**0,00%**	**10**	**62**
31-05-2022	29-05-2023	260	0	0,00%	2,22%	100%	1	0
01-06-2021	30-05-2022	260	0	0,00%	2,22%	100%	1	0
02-06-2020	31-05-2021	260	1	0,38%	25,44%	94,99%	1	0
04-06-2019	01-06-2020	260	17	6,54%	0,00%	0,00%	10	55
05-06-2018	03-06-2019	260	1	0,38%	25,44%	94,99%	1	0
06-06-2017	04-06-2018	260	0	0,00%	2,22%	100%	1	0
07-06-2016	05-06-2017	260	1	0,38%	25,44%	94,99%	1	0
09-06-2015	06-06-2016	260	10	3,85%	0,04%	0,65%	1	2
10-06-2014	08-06-2015	260	4	1,54%	41,87%	79,92%	1	0
11-06-2013	09-06-2014	260	1	0,38%	25,44%	94,99%	1	0

			Quantile Regression VaR					
Backtest Period		Number of	Number of	Exceedance	Unconditional	BCP test		
Begin	End	observations	exceedances	rate	Coverage Test	p-value	Lag	Clustering
11-06-2013	**29-05-2023**	**2600**	**32**	**1,23%**	**25,37%**	**0,00%**	**1**	**6**
31-05-2022	29-05-2023	260	0	0,00%	2,22%	100%	1	0
01-06-2021	30-05-2022	260	2	0,77%	69,67%	89,96%	1	0
02-06-2020	31-05-2021	260	1	0,38%	25,44%	94,99%	1	0
04-06-2019	01-06-2020	260	7	2,69%	2,34%	5,32%	3	2
05-06-2018	03-06-2019	260	3	1,15%	80,77%	0,00%	1	1
06-06-2017	04-06-2018	260	4	1,54%	41,87%	0,01%	1	1
07-06-2016	05-06-2017	260	5	1,92%	18,44%	0,28%	1	1
09-06-2015	06-06-2016	260	5	1,92%	18,44%	0,00%	2	3
10-06-2014	08-06-2015	260	2	0,77%	69,67%	0,00%	10	1
11-06-2013	09-06-2014	260	3	1,15%	80,77%	84,93%	1	0

Table 6. Yearly Backtesting Results for the Unadjusted Historical and Quantile Regression VaR. This table summarizes the performance of the Unadjusted Historical and Quantile Regression VaR, including the number of exceedances, the exceedance rate, and the results of the Unconditional Coverage (UC) test and the Berkowitz, Christoffersen, and Pelletier (BCP) test, per year.

Regarding the BCP test, which evaluates the independence of exceedances, both models exhibit a weak performance. The Unadjusted Historical VaR model rejects the null hypothesis of the BCP test (equation 35) in 2 out of the 10 sub-periods, whereas the Quantile Regression VaR model rejects the null hypothesis in 6 out of 10 sub-periods. Despite the lower failure frequency, the Unadjusted Historical model displays substantially higher clustering intensity, accumulating a total of 62 clustered exceedances across all lags ($K = 1$ to $K = 10$) for the 10-year period (table 7), compared to only 10 clusters observed for the Quantile Regression model (table 7). This suggests that while the Quantile Regression model is rejected more frequently, the severity of exceedances dependence is higher in the Unadjusted Historical model.

Backtest Period		Unadjusted Historical VaR											Quantile Regression VaR										
Begin	End	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10	Sum Lag 1 to 10	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10	Sum Lag 1 to 10
11/06/13	**29/05/23**	**7**	**6**	**7**	**6**	**6**	**4**	**8**	**6**	**7**	**5**	**62**	**6**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**2**	**10**
31/05/22	29/05/23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
01/06/21	30/05/22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
02/06/20	31/05/21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
04/06/19	01/06/20	5	5	7	6	6	3	7	5	6	5	55	1	0	1	0	0	0	0	0	0	1	3
05/06/18	03/06/19	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
06/06/17	04/06/18	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
07/06/16	05/06/17	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
09/06/15	06/06/16	2	1	0	0	0	1	1	1	1	0	7	2	1	0	0	0	0	0	0	0	0	3
10/06/14	08/06/15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
11/06/13	09/06/14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 7. Yearly Lag Dependence Results for Unadjusted Historical and Quantile Regression VaR. This table presents the backtesting results of lag dependence for the unadjusted Historical VaR and the Quantile Regression VaR models, recording for each period the frequency of exceedances across lags 1 to 10 and the cumulative sum of exceedances across all lags.

The model selected to perform the risk assessment is the Quantile Regression VaR since it has a better performance at the Unconditional Coverage test and displays a lower clustering intensity on the BCP test.

Figure 1 presents the daily VaR estimates computed by the Quantile Regression VaR, and the returns observed for the backtesting period.

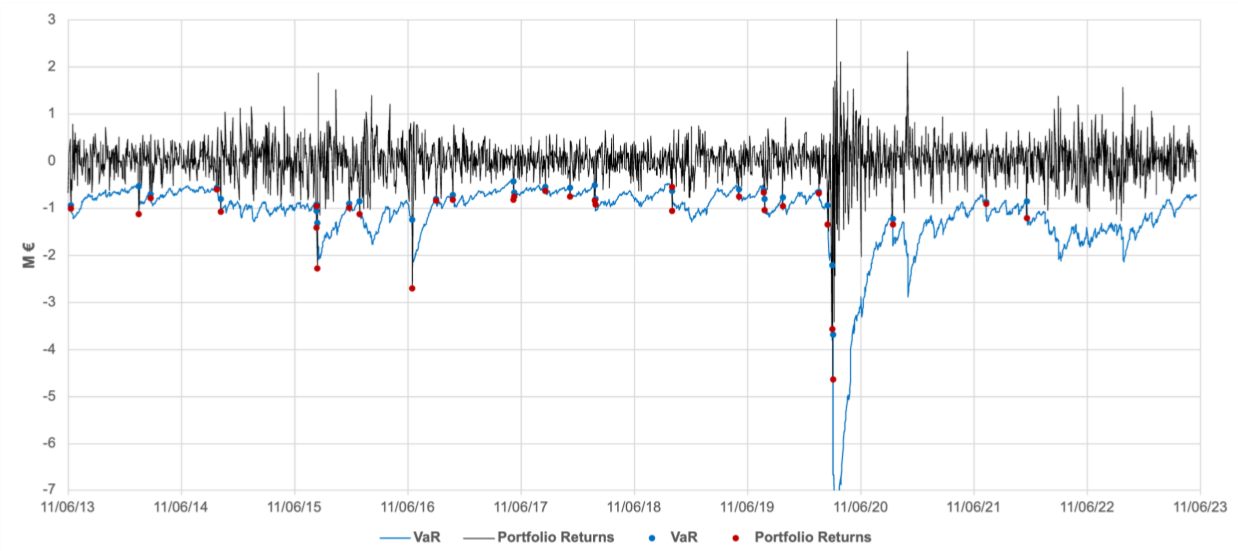

Figure 1. Quantile Regression VaR backtesting period performance. This figure presents the portfolio returns and the corresponding 1-day VaR estimates over the sample period. The black line represents portfolio returns, the blue line the VaR estimates, and the blue and red dots highlight exceedances.

Chapter 6 - VaR Management

For the portfolio held on March 29, 2023 (table 1 and table 2), the one-year Value at Risk (VaR) was computed for the period spanning from May 30, 2023, to June 3, 2024. The VaR was calculated on a daily basis using the Quantile Regression model (chapter 5) and was estimated before market close each day to forecast the potential loss for the following trading day.

All coupon payments received during the year were reinvested into the originating securities, thereby maintaining consistency in the reinvestment policy.

Throughout this period, the portfolio was required to maintain a target allocation of 60% in equities and 40% in bonds, with a permitted deviation of up to 5%. As observed in Figure 2, on February 6, 2024, just before the market closed, the portfolio allocation exceeded this threshold, reaching an allocation of 65,1% in equities and 34,9% in bonds.

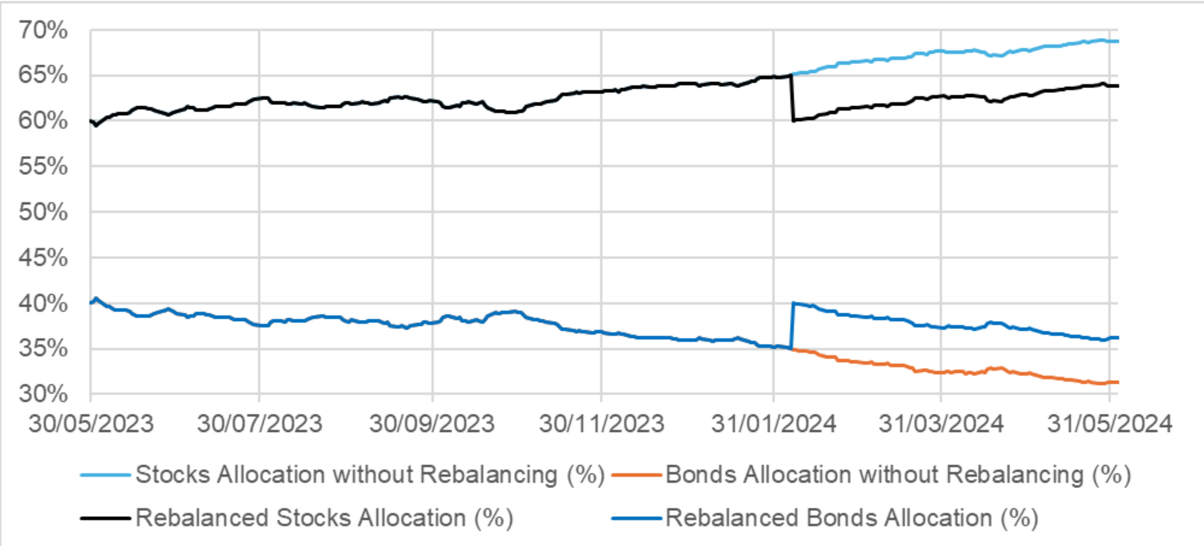

Figure 2. Evolution of portfolio allocation with and without Rebalancing (Rebalancing on 6 February 2024). The figure shows stock and bond allocations over the 1-year period, with and without rebalancing. The rebalancing on February 6, 2024, reset the portfolio to its target allocation of 60% stocks and 40% bonds, while the unrebalanced allocations drifted due to market movements.

As a result, the portfolio was proportionally rebalanced shortly before the market close (see Annex A), on the same day, to restore the target allocation of 60% equities and 40% bonds (Table 8).

	Portfolio value	Portfolio value (%)	New Portfolio value	New Portfolio value (%)
Stocks	51 599 881	65,10%	47 556 236	60,00%
Bonds	27 660 349	34,90%	31 703 985	40,00%
Total	**79 260 230**	**100%**	**79 260 221**	**100%**

Table 8. Rebalanced Portfolio Value on February 6, 2024. This table provides the total value of the investment portfolio, with the allocation by stocks and bonds before and after the rebalancing.

In this rebalancing, 8,73€ were allocated to a cash account. Without rebalancing, the unrebalanced portfolio allocation on June 3, 2024, would have diverged substantially from the target weights, reaching 68,7% in stocks and 31,3% in bonds (Figure 2).

Since there was a portfolio change, as done in chapter 5, the new rebalanced portfolio was backtested, with the same parameters presented in table 4, with a backtesting window of 2600 trading days, from February 19, 2014, to February 6, 2024. As observed in Annex 2, the results were similar to the results analysed in Chapter 5, with the best-performing model continuing to be the Quantile Regression VaR.

Figure 3 presents he results of the daily VaR estimations compared with the actual P&L of the rebalanced portfolio. As illustrated by the blue and red dots (Figure 3), two exceedances were recorded during the period, on July 6, 2023, and April 16, 2024. On these dates, the estimated VaR was 700.112,98€ and 708.390,30€, respectively, while the actual portfolio losses were 728.993,46€ and 886.678,87€, respectively.

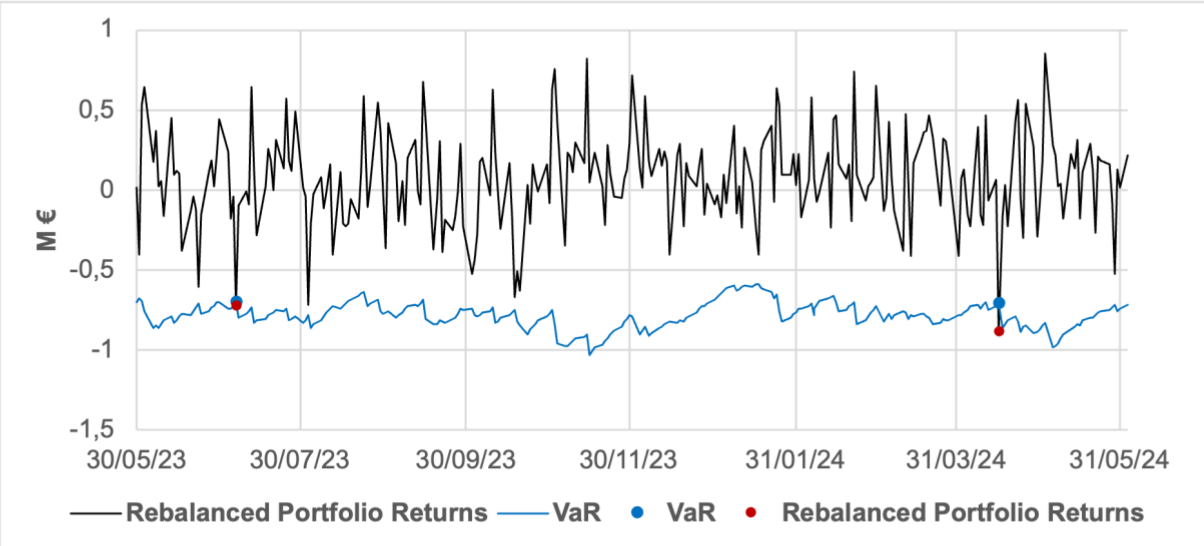

Figure 3. Rebalanced Portfolio Returns and Total Value-at-Risk (VaR). This figure shows the rebalanced portfolio returns (black line) against the VaR estimates (blue line). The dots highlight the exceedances.

An Economic Capital threshold of 800.000€ was established as the upper bound for daily VaR. To ensure that the rebalanced portfolio remained within the limit, before market closing and after estimating the VaR for the next trading day, hedging positions were taken whenever the estimated VaR exceeded the threshold.

The hedging strategy was based on a decomposition of the portfolio's marginal VaR (Chapter 4.5.2) by currency, equity index, and interest rate exposures, as illustrated in Table 9.

Index	Currency	Interest Rate
GSPC	USDEUR	IR_USD
DJI	GBPEUR	IR_EUR
FTSE	BRLEUR	
BVSP	JPYEUR	
N225	HKDEUR	
HSI		
ATX		
BFX		
FCHI		
GDAXI		
IBEX		
FTSEMIB.MI		

Table 9. Risk Factor Decomposition. This table presents the decomposition of risk factors by index, currency, and interest rate.

On the days where the estimated VaR exceeded the 800.000€ threshold, hedging positions were taken in the decomposition (currency, index, or interest rate) with the highest marginal contribution to the overall VaR. As shown in the graph below (Figure 4), index components, namely GSPC, FTSE, BVSP, and N225, were the chosen hedging positions since they consistently exhibited the highest marginal VaR. All hedging positions (Figure 4) were short positions and were computed as described in Chapter 4.5.3. When a short position was carried over from the previous day and the estimated VaR for the next day exceeded the threshold, the marginal VaR of that position was evaluated to determine whether to maintain it as the sole hedge or to keep it and add a new short position that presents a higher marginal VaR. On the other hand, if maintaining existing short positions brought the following day's VaR below the threshold, the position with the lowest marginal VaR was bought back until the total VaR was reduced to or below the threshold.

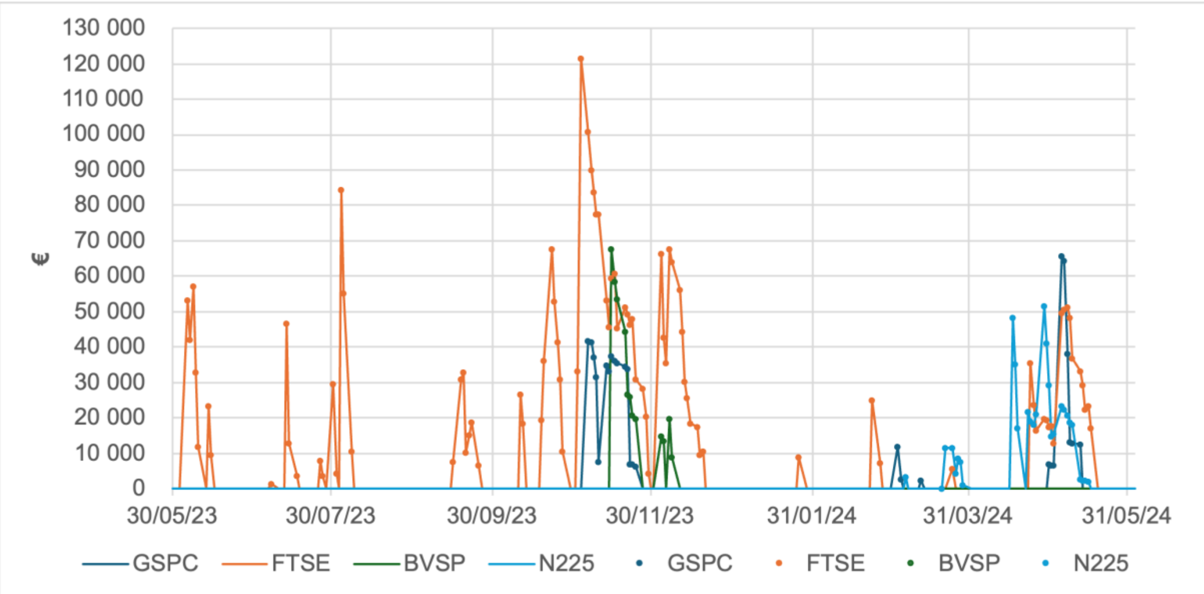

Figure 4. Daily hedging with short positions. This figure shows the daily evolution of short positions in major indices (GSPC, FTSE, BVSP, and N225) employed as hedging positions.

The hedging strategy proved effective in keeping the total VaR within the 800.000€ threshold throughout the period, as illustrated in Figure 5. Figure 5 also shows that the unrebalanced unhedged portfolio exhibits a higher VaR compared to the rebalanced portfolio without hedging, and consequently, to the rebalanced portfolio with hedging.

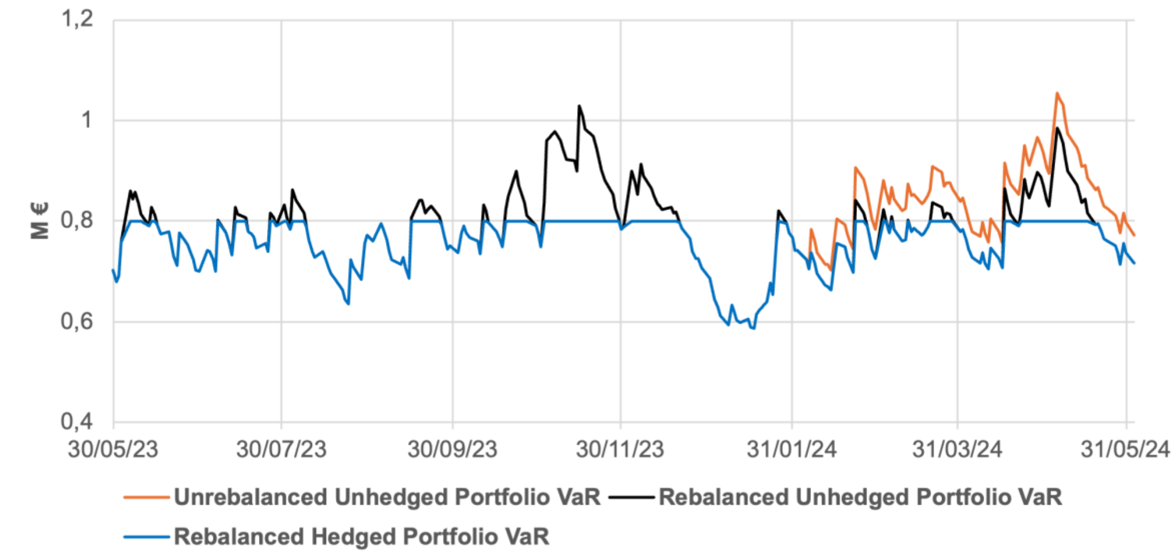

Figure 5. Portfolio VaR under three different strategies. This figure compares the total one-day portfolio VaR under three strategies: unrebalanced unhedged, rebalanced unhedged, and rebalanced hedged portfolios.

While on the hedged portfolio the VaR is lower, with a maximum VaR of 800.000€, the impact on returns is mixed. Figure 6 highlights the trade-off between risk and return, comparing the returns of the rebalanced hedged and rebalanced unhedged portfolios. Although hedging effectively controlled risk exposure, as shown in Figure 5, it occasionally came at the cost of lower returns. However, the presence of positive values suggests that the hedging strategy also captured upside in certain market conditions. Out of the 103 days on which hedging positions were implemented, the strategy resulted in lower returns on 51 days (negative values of Figure 6) and higher returns on 52 days (positive values of Figure 6).

On the three scenarios, unrebalanced unhedged, rebalanced unhedged, and rebalanced hedged, the cumulative returns over the one-year period were positive (Figure 7). However, the hedged portfolio outperformed the unhedged portfolio up until December 7, 2023 (black crossover on Figure 7). From that date onward, the unhedged portfolio delivered higher cumulative returns, except during the period from April 19 to April 23, 2024 (green crossovers on Figure 7), when the hedged portfolio temporarily outperformed the unhedged portfolio. On June 3, 2024, the cumulative return for the unhedged portfolio was 18.673.733€, while on the hedged portfolio the cumulative return was 18.381.306€. On the other hand, the unrebalanced unhedged portfolio outperformed the rebalanced unhedged portfolio from the rebalancing date (February 2, 2024) onward, reaching a cumulative return of €19,437,740 on June 3, 2024.

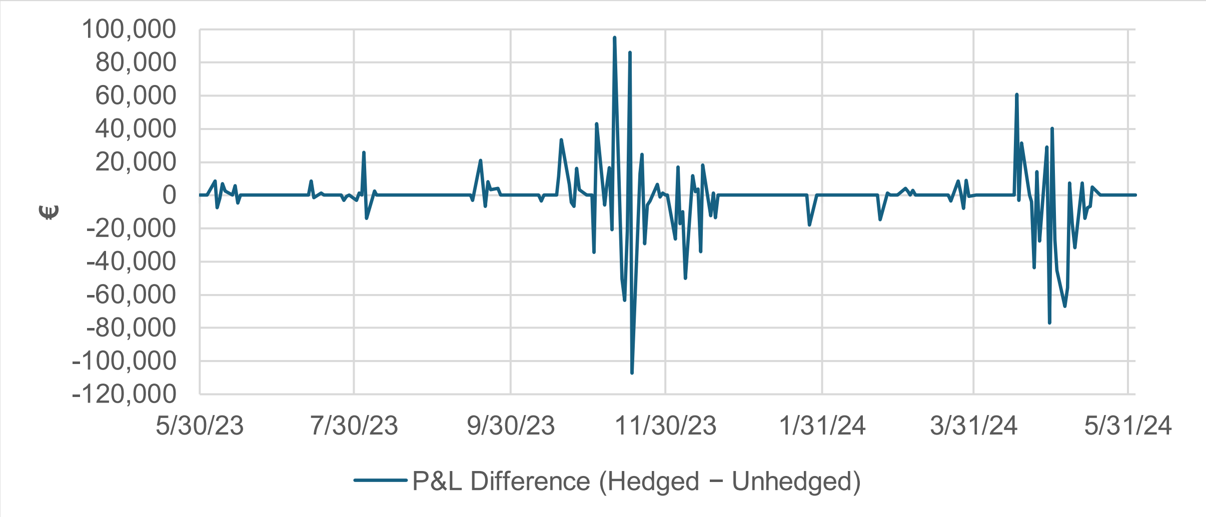

Figure 6. Daily Profit and Loss (P&L) difference between the hedged and unhedged portfolio. This figure illustrates the daily Profit and Loss (P&L) difference between the rebalanced hedged and rebalanced unhedged portfolio, where positive values indicate days where the hedged portfolio outperformed the unhedged, whereas negative values represent periods where the unhedged portfolio outperformed the hedged portfolio.

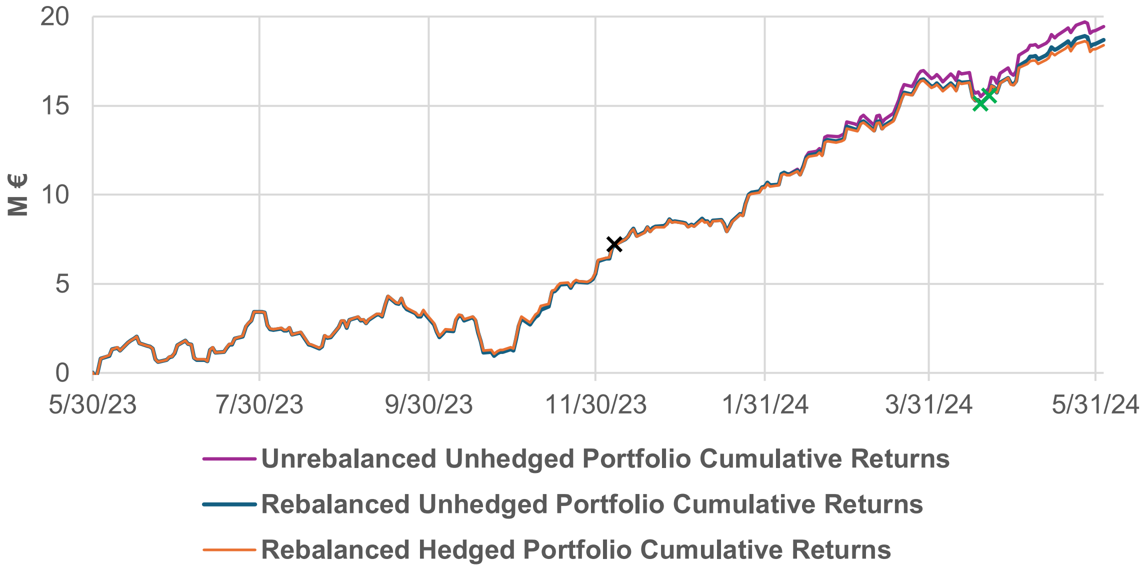

Figure 7. Portfolio cumulative returns under different strategies. This figure compares the cumulative returns over the sample period of the portfolio under three strategies: without rebalancing and without hedging, rebalanced without hedging, and rebalanced with hedging.

Since the three portfolios were exposed to different levels of risk, a direct comparison of their performance based solely on returns could be misleading. To account for this, the performance is measured using the Return on Risk-Adjusted Capital (Equation 45).

Table 10 shows that the unrebalanced unhedged portfolio generated the highest one-year return of 19,44 million euros, but it also carried the highest risk, as reflected in the cumulative VaR of 212,17 million euros. In contrast, rebalancing reduced the overall return to 18,67 million, and although risk also declined to 207,10 million, the RORAC dropped to its lowest level at 9,02%, suggesting that rebalancing on its own does not provide an improvement in efficiency.

The addition of hedging, however, improved efficiency. Although the rebalanced hedged portfolio produced the lowest return of 18,38 million euros, it was offset by the most significant reduction in risk, with cumulative VaR falling to 200,99 million euros. As a result, the RORAC of 9.15% nearly matched the unrebalanced portfolio RORAC of 9,16%.

Metric	Unrebalanced Unhedged Portfolio	Rebalanced Unhedged Portfolio	Rebalanced Hedged Portfolio
1 Year Return	19 437 740	18 673 733	18 381 306
1 Year Cumlative VaR	212 174 641	207 100 214	200 990 600
RORAC	**9,16%**	**9,02%**	**9,15%**

Table 10. Unrebalanced unhedged, unhedged and hedged portfolios performance. This table presents the returns (P&L), VaR and RORAC for the unrebalanced unhedged, rebalanced unhedged, and rebalanced hedged portfolios.

Conclusion

The primary objective of this work was to measure and control the portfolio's daily Value at Risk (VaR) throughout a one-year horizon, spanning from 30 May 2023 to 3 June 2024, ensuring that the estimated VaR consistently remained below a predefined maximum threshold of 800.000€, thereby aligning portfolio risk exposure with defined risk tolerance levels. Additionally, the portfolio should also keep a target allocation of 60% stocks and 40% bonds, with an allowed 5% deviation.

Four VaR models, namely RiskMetrics VaR, Skewed Generalized Student-t VaR, Historical VaR, and Quantile Regression VaR were tested since the accuracy of the portfolio risk metric depends critically on the chosen methodology. Their performance was evaluated through backtesting using the Unconditional Coverage (UC) Test and the Berkowitz, Christoffersen, and Pelletier (BCP) Test, making it possible to compare alternative approaches before selecting the most appropriate one.

Both parametric models, RiskMetrics VaR and SGSt VaR, were rejected by the UC Test at the 5% significance level, indicating that the portfolio returns are not adequately captured by either the normal distribution or the skewed generalized Student-t distribution. Within the non-parametric models, the Historical VaR was tested both with and without a volatility adjustment. Although the adjusted Historical VaR was expected to improve accuracy, the results indicated otherwise, as the unadjusted VaR performed better while the adjusted model was rejected by the UC Test at the 5% significance level. Among all the models considered, the Quantile Regression VaR, a non-parametric model, delivered the strongest performance in the UC Test.

The outcomes of the BCP Test were less favorable, as all models failed to pass at the 5% significance level. This indicates that, despite the differences observed in the UC Test, none of the models were able to adequately capture the dependence structure and clustering of the portfolio returns. This limitation reflects the challenges of VaR modeling, which become even more evident in periods of high uncertainty such as the COVID-19 crisis.

After selecting Quantile Regression VaR as the best-performing model, the portfolio was managed to maintain its Value at Risk below the 800.000€ threshold through an equity-hedging strategy. The hedging positions were determined using the gradient vector and marginal VaR, which identified the most effective assets for reducing overall portfolio risk. Accordingly, short positions were taken in major equity indices such as the S&P 500 (GSPC), FTSE 100, Bovespa (BVSP), and Nikkei 225 (N225), and were adjusted on a daily basis in response to changes in the portfolio's risk profile. These positions operated as expected, ensuring that the VaR did not exceed the predefined limit.

Over the one-year horizon, the Quantile Regression VaR model registered two exceedances, which is aligned with the theoretical exceedance frequency implied by the chosen confidence level. This outcome suggests that the model was appropriately specified for the period under analysis.

On February 6, 2024, the portfolio was proportionally rebalanced after exceeding the 5% allowed deviation threshold, restoring the target allocation of 60% equities and 40% bonds.

The effectiveness of the risk management was then evaluated through the Return on Risk-Adjusted Capital (RORAC). All three portfolios under analysis, unrebalanced unhedged, rebalanced unhedged, and rebalanced hedged achieved positive returns over the one-year period. The unrebalanced unhedged portfolio unexpectedly delivered the highest RORAC, showing that under certain market conditions, avoiding intervention altogether can be more efficient. However, this comes with considerably higher exposure to losses, underscoring that effective risk management does not necessarily aim to maximize performance, but rather to optimize the trade-off between returns and stability. Nevertheless, the rebalanced hedged portfolio achieved a RORAC very close to the unrebalanced unhedged portfolio level while offering significantly lower risk.

Bibliographical references

Alexander, C. (2008). *Market Risk Analysis: Practical Financial Econometrics*. John Wiley & Sons.

Alexander, C. (2008). *Market Risk Analysis: Quantitative Methods in Finance*. John Wiley & Sons.

Alexander, C. (2008). *Market Risk Analysis: Value at Risk Models.* John Wiley & Sons.

Artzner, P., Delbaen, F., Eber, J., & Heath, D. (1999). *Coherent measures of risk. Mathematical Finance*, 9(3), 203–228. https://doi.org/10.1111/1467-9965.00068

Bank for International Settlements. (1996, January 4). *Amendment to the Capital Accord to Incorporate Market Risks* (Basel Committee Publication No. 24). Bank for International Settlements. https://www.bis.org/publ/bcbs24.htm

Basel Committee on Banking Supervision. (2019). *Minimum capital requirements for market risk.* Bank for International Settlements. https://www.bis.org/bcbs/publ/d457.htm

Basel Committee on Banking Supervision. (2009). *Revisions to the Basel II market risk framework.* Bank for International Settlements. https://www.bis.org/publ/bcbs158.htm

Bank for International Settlements. (1996, January 4). *Supervisory framework for the use of "backtesting" in conjunction with the internal models approach to market risk capital requirement* (Basel Committee Publication No. 22). Bank for International Settlements. https://www.bis.org/publ/bcbs22.htm

Berkowitz, J., Christoffersen, P., & Pelletier, D. (2007). *Evaluating Value-at-Risk models with desk-level data. Management Science*, 57, 2213–2227. https://doi.org/10.1287/mnsc.1080.0964

Cairns, A. J. (2000). *A discussion of parameter and model uncertainty in insurance*. Insurance Mathematics and Economics, 27, 313–330. https://doi.org/10.1016/s0167-6687(00)00055-x

Chang, C. L., Jiménez, J. A., Maasoumi, E., McAleer, M., & Pérez-Amaral, T. (2019). *Choosing expected shortfall over VaR in Basel III using stochastic dominance*. International Review of Economics & Finance, 60, 95–113. https://doi.org/10.1016/j.iref.2018.12.016

Christoffersen, P. F. (1998). *Evaluating interval forecasts*. International Economic Review, 39, 841–862.

Crouhy, M., Galai, D., & Mark, R. (2023). *The Essentials of Risk Management* (3rd ed.). McGraw Hill Professional.

Engle, R. F. (1982). *Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation*. Econometrica, 50, 987–1007. https://doi.org/10.2307/1912773

Frain, J., & Meegan, C. (1996). *Market risk: An introduction to the concept & analytics of value-at-risk.* Research Technical Papers 7/RT/96, Central Bank of Ireland.

Goodhart, C. A. E. (2011). *The Basel Committee on Banking Supervision: A history of the early years, 1974–1997*. Cambridge University Press

Holton, G. A. (2013). *Value-at-Risk: Theory and Practice* (2nd ed.). https://www.value-at-risk.net

Hull, J. C. (2015). *Risk Management and Financial Institutions* (4th ed.). John Wiley & Sons.

Jorion, P. (2011). Financial risk manager handbook + test bank: FRM Part I / Part II (6th ed.). Wiley.

Jorion, P. (2007). *Value at risk: The new benchmark for managing financial risk* (3rd ed.). McGraw-Hill.

J.P. Morgan & Reuters. (1996). *RiskMetrics – Technical Document.* https://www.msci.com/documents/10199/5915b101-4206-4ba0-aee2-3449d5c7e95a

Klaassen, P., & van Eeghen, I. (2009). *Economic capital: How it works and what every manager needs to know*. Elsevier Science.

Matten, C. (2000). *Managing bank capital: Capital allocation and performance measurement* (2nd ed.). John Wiley & Sons.

McAleer, M. J., Jiménez-Martín, J.-Á., & Pérez-Amaral, T. (2013). *Has the Basel Accord improved risk management during the global financial crisis?.* The North American Journal of Economics and Finance, 26, 250–265. https://doi.org/10.1016/j.najef.2013.02.004
McNeil, A. J., Frey, R., & Embrechts, P. (2015). *Quantitative risk management: Concepts, techniques and tools (Revised ed.)*. Princeton University Press.
Nam, K., & Choi, P. (2008). *Asymmetric and leptokurtic distribution for heteroscedastic asset returns: The SU-normal distribution*. Journal of Empirical Finance, 15, 41–63. https://doi.org/10.1016/j.jempfin.2006.06.009
Theodossiou, P. (1998). *Financial Data and the Skewed Generalized T Distribution*. Management Science, 44, 1650–1661. https://doi.org/10.1287/mnsc.44.12.16
Vasileiou, E. (2017). *Value at Risk (VaR) Historical Approach: Could It Be More Historical and Representative of the Real Financial Risk Environment?.* Theoretical Economics Letters, 7, 951–974. https://doi.org/10.4236/tel.2017.74065

Annex A – Portfolio Rebalancing on February 6, 2024

Ticker	Quantity	Closing Price (06/02/24)	Value (EUR)	Allocation (%)	Quantities to Buy/Sell	New Quantity	New Value (EUR)	Allocation (%)
NVDA	782	682,20	496 637	0,96%	(61)	721	457 897	0,96%
NRG	(5 029)	53,22	(249 182)	-0,48%	394	(4 635)	(229 660)	-0,48%
DECK	(226)	825,35	(173 647)	-0,34%	18	(208)	(159 817)	-0,34%
GE	4 332	109,61	442 021	0,86%	(339)	3 993	407 430	0,86%
WDC	9 563	58,45	520 356	1,01%	(749)	8 814	479 600	1,01%
MU	(5 576)	84,52	(438 728)	-0,85%	437	(5 139)	(404 345)	-0,85%
LLY	877	702,60	573 626	1,11%	(69)	808	528 495	1,11%
TT	(1 172)	269,43	(293 964)	-0,57%	92	(1 080)	(270 889)	-0,57%
QCOM	3 906	142,79	519 208	1,01%	(306)	3 600	478 533	1,01%
LDOS	1 467	111,83	152 721	0,30%	(115)	1 352	140 749	0,30%
URI	1 204	651,37	730 094	1,41%	(94)	1 110	673 094	1,42%
ETN	3 546	269,47	889 544	1,72%	(278)	3 268	819 805	1,72%
DPZ	1 534	420,17	600 033	1,16%	(120)	1 414	553 095	1,16%
RCL	5 605	120,57	629 124	1,22%	(439)	5 166	579 850	1,22%
APH	6 152	102,56	587 394	1,14%	(482)	5 670	541 372	1,14%
WAB	(2 970)	134,38	(371 555)	-0,72%	233	(2 737)	(342 406)	-0,72%
TDG	617	1140,60	655 149	1,27%	(48)	569	604 181	1,27%
NTAP	(3 731)	87,19	(302 836)	-0,59%	292	(3 439)	(279 135)	-0,59%
PHM	5 581	102,39	531 978	1,03%	(437)	5 144	490 323	1,03%
KLAC	1 024	599,07	571 080	1,11%	(80)	944	526 465	1,11%
CAT	2 319	321,55	694 174	1,35%	(182)	2 137	639 693	1,35%
AMZN	(3 854)	169,15	(606 884)	-1,18%	302	(3 552)	(559 328)	-1,18%
JPM	4 098	174,08	664 131	1,29%	(321)	3 777	612 109	1,29%
GS	1 597	379,98	564 923	1,09%	(125)	1 472	520 705	1,09%
AMGN	2 533	311,41	734 319	1,42%	(199)	2 334	676 629	1,42%
AXP	2 909	205,01	555 201	1,08%	(228)	2 681	511 686	1,08%
WMT	8 174	56,22	427 781	0,83%	(641)	7 533	394 234	0,83%
MSFT	1 737	404,01	653 306	1,27%	(136)	1 601	602 155	1,27%
IBM	3 938	179,97	659 765	1,28%	(309)	3 629	607 995	1,28%
MMM	4 231	76,33	300 659	0,58%	(332)	3 899	277 067	0,58%
VZ	(16 105)	40,45	(606 384)	-1,18%	1 262	(14 843)	(558 867)	-1,18%
TRV	3 430	212,04	677 084	1,31%	(269)	3 161	623 984	1,31%
MRK	(2 249)	126,08	(263 972)	-0,51%	176	(2 073)	(243 315)	-0,51%
PG	4 753	157,94	698 847	1,35%	(372)	4 381	644 151	1,35%
HD	(2 030)	351,69	(664 621)	-1,29%	159	(1 871)	(612 565)	-1,29%
CRM	(1 105)	285,46	(293 646)	-0,57%	87	(1 018)	(270 527)	-0,57%
DIS	5 260	99,29	486 198	0,94%	(412)	4 848	448 115	0,94%
AAPL	4 303	188,80	756 312	1,47%	(337)	3 966	697 080	1,47%
HON	4 087	191,57	728 866	1,41%	(320)	3 767	671 798	1,41%
KO	(11 036)	59,46	(610 928)	-1,18%	865	(10 171)	(563 044)	-1,18%
RR.L	1 436	317,80	532 482	1,03%	(113)	1 323	490 580	1,03%
MKS.L	2 149	243,38	610 268	1,18%	(168)	1 981	562 560	1,18%
ICG.L	232	1783,50	482 789	0,94%	(18)	214	445 331	0,94%
III.L	205	2316,00	553 973	1,07%	(16)	189	510 736	1,07%
ANTO.L	242	1749,32	493 949	0,96%	(19)	223	455 168	0,96%
BA.L	(105)	1188,60	(145 621)	-0,28%	8	(97)	(134 526)	-0,28%
IHG.L	69	7491,67	603 148	1,17%	(5)	64	559 442	1,18%
NXT.L	47	8366,00	458 788	0,89%	(4)	43	419 742	0,88%
ABF.L	225	2255,36	592 099	1,15%	(18)	207	544 731	1,15%
BARC.L	2 740	141,71	453 049	0,88%	(215)	2 525	417 499	0,88%
HWDN.L	719	765,97	642 593	1,25%	(56)	663	592 544	1,25%

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Ticker	Quantity	Closing Price (06/02/24)	Value (EUR)	Allocation (%)	Quantities to Buy/Sell	New Quantity	New Value (EUR)	Allocation (%)
DPLM.L	151	3280,60	577 998	1,12%	(12)	139	532 064	1,12%
REL.L	204	3245,36	772 485	1,50%	(16)	188	711 898	1,50%
BKG.L	141	4745,78	780 769	1,51%	(11)	130	719 858	1,51%
FRAS.L	687	799,00	640 472	1,24%	(54)	633	590 129	1,24%
MRO.L	(552)	588,31	(378 917)	-0,73%	43	(509)	(349 400)	-0,73%
SMT.L	691	782,00	630 494	1,22%	(54)	637	581 223	1,22%
TW.L	4 422	142,03	732 805	1,42%	(347)	4 075	675 301	1,42%
EXPN.L	62	3297,00	238 510	0,46%	(5)	57	219 276	0,46%
LLOY.L	7 443	40,43	351 108	0,68%	(583)	6 860	323 606	0,68%
BRFS3.SA	(67 645)	14,85	(187 947)	-0,36%	5 301	(62 344)	(173 219)	-0,36%
EMBR3.SA	115 817	21,83	473 042	0,92%	(9 076)	106 741	435 972	0,92%
PETR4.SA	(34 047)	38,77	(246 992)	-0,48%	2 668	(31 379)	(227 638)	-0,48%
PETR3.SA	97 668	40,20	734 591	1,42%	(7 654)	90 014	677 023	1,42%
SBSP3.SA	58 544	78,91	864 336	1,68%	(4 588)	53 956	796 600	1,68%
UGPA3.SA	91 455	27,96	478 412	0,93%	(7 167)	84 288	440 921	0,93%
CPLE6.SA	(168 323)	10,22	(321 876)	-0,62%	13 191	(155 132)	(296 652)	-0,62%
BBAS3.SA	(70 620)	29,47	(389 452)	-0,75%	5 534	(65 086)	(358 933)	-0,75%
CPFE3.SA	121 705	33,99	774 024	1,50%	(9 538)	112 167	713 364	1,50%
ITSA4.SA	126 858	10,14	240 660	0,47%	(9 942)	116 916	221 799	0,47%
CMIG4.SA	156 109	8,37	244 445	0,47%	(12 234)	143 875	225 288	0,47%
ITUB4.SA	109 225	33,38	682 113	1,32%	(8 560)	100 665	628 656	1,32%
TIMS3.SA	117 715	17,54	386 246	0,75%	(9 225)	108 490	355 977	0,75%
TRPL4.SA	94 734	26,85	475 909	0,92%	(7 424)	87 310	438 614	0,92%
VIVT3.SA	14 252	52,49	139 954	0,27%	(1 117)	13 135	128 985	0,27%
EGIE3.SA	82 183	39,29	604 167	1,17%	(6 440)	75 743	556 823	1,17%
USIM5.SA	567 304	8,91	946 007	1,83%	(44 458)	522 846	871 871	1,83%
CYRE3.SA	149 378	22,06	616 659	1,20%	(11 706)	137 672	568 334	1,20%
CSNA3.SA	128 620	17,06	410 613	0,80%	(10 080)	118 540	378 433	0,80%
SUZB3.SA	88 503	53,71	889 379	1,72%	(6 936)	81 567	819 678	1,72%
5803.T	38 611	1213,54	293 271	0,57%	(3 026)	35 585	270 287	0,57%
9107.T	(45 774)	2228,69	(638 520)	-1,24%	3 587	(42 187)	(588 483)	-1,24%
3099.T	(14 623)	1967,57	(180 082)	-0,35%	1 146	(13 477)	(165 969)	-0,35%
7011.T	82 663	1065,92	551 491	1,07%	(6 478)	76 185	508 273	1,07%
8601.T	104 068	993,75	647 289	1,25%	(8 156)	95 912	596 560	1,25%
7735.T	8 652	15984,02	865 581	1,68%	(678)	7 974	797 751	1,68%
8604.T	69 424	805,88	350 175	0,68%	(5 441)	63 983	322 731	0,68%
9501.T	(116 924)	765,90	(560 507)	-1,09%	9 163	(107 761)	(516 581)	-1,09%
6501.T	10 339	11745,62	760 080	1,47%	(810)	9 529	700 532	1,47%
5019.T	100 748	793,99	500 674	0,97%	(7 895)	92 853	461 440	0,97%
8316.T	10 339	7516,32	486 395	0,94%	(810)	9 529	448 288	0,94%
6361.T	10 277	9459,00	608 438	1,18%	(805)	9 472	560 779	1,18%
8035.T	(976)	28452,47	(173 810)	-0,34%	76	(900)	(160 276)	-0,34%
9503.T	22 867	1891,45	270 712	0,52%	(1 792)	21 075	249 498	0,52%
8306.T	78 024	1381,24	674 531	1,31%	(6 115)	71 909	621 665	1,31%
5801.T	17 515	2695,24	295 470	0,57%	(1 373)	16 142	272 308	0,57%
7012.T	11 263	3405,25	240 054	0,47%	(883)	10 380	221 234	0,47%
6701.T	11 222	9317,99	654 482	1,27%	(879)	10 343	603 217	1,27%
8354.T	11 747	3599,44	264 647	0,51%	(921)	10 826	243 898	0,51%
8766.T	23 965	3814,25	572 126	1,11%	(1 878)	22 087	527 292	1,11%
0883.HK	(19 685)	14,70	(34 440)	-0,07%	1 543	(18 142)	(31 741)	-0,07%
0992.HK	140 308	8,48	141 609	0,27%	(10 996)	129 312	130 511	0,27%
1088.HK	54 063	30,00	193 034	0,37%	(4 237)	49 826	177 906	0,37%
0316.HK	47 473	117,95	666 436	1,29%	(3 720)	43 753	614 214	1,29%

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Ticker	Quantity	Closing Price (06/02/24)	Value (EUR)	Allocation (%)	Quantities to Buy/Sell	New Quantity	New Value (EUR)	Allocation (%)
2899.HK	112 385	11,55	154 482	0,30%	(8 807)	103 578	142 376	0,30%
0857.HK	231 847	5,72	157 837	0,31%	(18 169)	213 678	145 468	0,31%
0669.HK	(38 581)	83,47	(383 282)	-0,74%	3 023	(35 558)	(353 250)	-0,74%
0836.HK	129 857	15,88	245 430	0,48%	(10 177)	119 680	226 196	0,48%
2313.HK	(69 638)	68,02	(563 757)	-1,09%	5 457	(64 181)	(519 579)	-1,09%
3988.HK	(815 630)	3,02	(293 165)	-0,57%	63 919	(751 711)	(270 191)	-0,57%
0005.HK	91 726	59,05	644 638	1,25%	(7 188)	84 538	594 121	1,25%
0941.HK	71 667	64,63	551 304	1,07%	(5 616)	66 051	508 103	1,07%
0700.HK	17 353	288,31	595 451	1,15%	(1 360)	15 993	548 784	1,15%
0002.HK	92 856	65,44	723 207	1,40%	(7 277)	85 579	666 530	1,40%
0939.HK	(197 255)	4,76	(111 750)	-0,22%	15 458	(181 797)	(102 993)	-0,22%
0762.HK	640 169	5,27	401 448	0,78%	(50 168)	590 001	369 988	0,78%
1038.HK	121 562	44,95	650 311	1,26%	(9 526)	112 036	599 350	1,26%
0175.HK	361 059	8,19	351 945	0,68%	(28 295)	332 764	324 364	0,68%
1398.HK	859 584	3,92	401 039	0,78%	(67 363)	792 221	369 611	0,78%
0006.HK	(52 790)	44,67	(280 652)	-0,54%	4 137	(48 653)	(258 658)	-0,54%
IIA.VI	26 931	21,75	585 749	1,14%	(2 111)	24 820	539 835	1,14%
EBS.VI	5 173	37,56	194 286	0,38%	(405)	4 768	179 075	0,38%
EVN.VI	21 254	23,80	505 845	0,98%	(1 666)	19 588	466 194	0,98%
TKA.VI	47 207	8,06	380 488	0,74%	(3 699)	43 508	350 674	0,74%
DOC.VI	5 185	128,20	664 717	1,29%	(406)	4 779	612 668	1,29%
UCB.BR	5 964	85,76	511 457	0,99%	(467)	5 497	471 409	0,99%
DIE.BR	1 148	177,70	204 000	0,40%	(90)	1 058	188 007	0,40%
SOLB.BR	20 994	22,90	480 861	0,93%	(1 645)	19 349	443 183	0,93%
AGS.BR	16 453	36,72	604 184	1,17%	(1 289)	15 164	556 849	1,17%
ABI.BR	11 277	57,29	646 041	1,25%	(884)	10 393	595 398	1,25%
RNO.PA	22 695	34,15	775 093	1,50%	(1 779)	20 916	714 336	1,50%
SAF.PA	3 170	175,22	555 446	1,08%	(248)	2 922	511 991	1,08%
SGO.PA	9 136	65,95	602 519	1,17%	(716)	8 420	555 299	1,17%
PUB.PA	6 393	92,64	592 248	1,15%	(501)	5 892	545 835	1,15%
SU.PA	2 653	190,35	505 011	0,98%	(208)	2 445	465 417	0,98%
RHM.DE	1 419	329,86	468 065	0,91%	(111)	1 308	431 451	0,91%
CBK.DE	(35 534)	10,47	(372 067)	-0,72%	2 785	(32 749)	(342 906)	-0,72%
DBK.DE	49 103	11,94	586 254	1,14%	(3 848)	45 255	540 311	1,14%
ADS.DE	(1 144)	175,66	(200 960)	-0,39%	90	(1 054)	(185 150)	-0,39%
SAP.DE	4 229	164,32	694 914	1,35%	(331)	3 898	640 524	1,35%
SAB.MC	30 255	1,12	33 971	0,07%	(2 371)	27 884	31 309	0,07%
IDR.MC	29 164	16,06	468 374	0,91%	(2 286)	26 878	431 661	0,91%
BBVA.MC	67 841	8,78	595 737	1,15%	(5 317)	62 524	549 046	1,15%
SAN.MC	36 916	3,67	135 633	0,26%	(2 893)	34 023	125 004	0,26%
ITX.MC	11 626	38,41	446 514	0,87%	(911)	10 715	411 525	0,87%
LDO.MI	(27 359)	16,85	(460 999)	-0,89%	2 144	(25 215)	(424 873)	-0,89%
BMPS.MI	112 666	3,21	362 077	0,70%	(8 829)	103 837	333 703	0,70%
UCG.MI	19 717	27,30	538 266	1,04%	(1 545)	18 172	496 088	1,04%
BPE.MI	88 557	3,30	292 552	0,57%	(6 940)	81 617	269 626	0,57%
UNI.MI	80 595	5,56	448 268	0,87%	(6 316)	74 279	413 139	0,87%
		Total	51 599 881	100%		Total	47 556 236	100%

Table 11. Equity Portfolio Rebalancing on February 6, 2024. This table presents a summary of the rebalancing by stock (ticker), including the number of shares held before and after the rebalancing (negative values correspond to short positions), the share price in its base currency, the exchange rate, the value in euros, and the portfolio allocation before and after rebalancing and the number of shares bought/sold.

Bond ISIN	Face Value	Currency	Maturity	Coupon Rate	Face Value (EUR)	Fair Value (EUR)	New Face Value	New Face Value (EUR)	New Fair Value (EUR)
US912828J272	2 781 100	USD	15/02/25	2,00%	2 589 037	2 542 968	3 187 700	2 967 557	2 914 753
US9128286A35	1 403 400	USD	31/01/26	2,63%	1 306 481	1 264 433	1 608 600	1 497 510	1 449 314
US912828X885	2 041 900	USD	15/05/27	2,38%	1 900 886	1 812 186	2 340 400	2 178 772	2 077 105
US91282CBP59	2 167 300	USD	29/02/28	1,13%	2 017 626	1 808 883	2 484 100	2 312 548	2 073 292
US9128286B18	2 669 900	USD	15/02/29	2,63%	2 485 517	2 361 944	3 060 200	2 848 863	2 707 225
US912828Z948	3 044 500	USD	15/02/30	1,50%	2 834 247	2 476 164	3 489 600	3 248 608	2 838 174
DE000BU22007	2 018 500	EUR	13/03/25	2,50%	2 018 500	2 049 765	2 313 600	2 313 600	2 349 436
DE0001102390	1 798 500	EUR	15/02/26	0,50%	1 798 500	1 731 638	2 061 400	2 061 400	1 984 765
NL0012171458	2 451 000	EUR	15/07/27	0,75%	2 451 000	2 332 756	2 809 300	2 809 300	2 673 771
NL0000102317	1 594 600	EUR	15/01/28	5,50%	1 594 600	1 787 621	1 827 700	1 827 700	2 048 936
DE0001102622	1 611 700	EUR	15/11/29	2,10%	1 611 700	1 605 856	1 847 300	1 847 300	1 840 601
DE0001135143	1 764 900	EUR	04/01/30	6,25%	1 764 900	2 158 847	2 022 900	2 022 900	2 474 436
DE0001135176	1 122 100	EUR	04/01/31	5,50%	1 122 100	1 355 634	1 286 100	1 286 100	1 553 767
NL0015000RP1	1 455 500	EUR	15/07/32	0,50%	1 455 500	1 256 464	1 668 300	1 668 300	1 440 164
NL0015001AM2	1 089 500	EUR	15/07/33	2,50%	1 089 500	1 115 189	1 248 800	1 248 800	1 278 245
				Total	**28 040 095**	**27 660 349**	**Total**	**32 139 258**	**31 703 985**

Table 12. Bond Portfolio Rebalancing on February 6, 2024. This table summarizes the bond portfolio rebalancing, including the bond ISIN, face value in its base currency, maturity, coupon rate, and exchange rate. Additionally, it includes the face value and fair value in euros before and after the rebalancing.

Annex B – Model choice after the rebalancing on February 6, 2024

Model	Number of exceedances	Exceedance rate	UC Test p-value	BCP Test p-value	BCP Test Lag	BCP Test Clustering
RiskMetrics VaR	50	1,92%	0,00%	0,00%	1	6
SGSt VaR	46	1,77%	0,04%	0,00%	1	6
Unajusted Historical VaR	36	1,38%	6,25%	0,00%	10	64
Adjusted Historical VaR	43	1,65%	0,22%	0,00%	1	6
Quantile Regression VaR	31	1,19%	33,88%	0,00%	1	6

Table 13. Backtesting Model Evaluation Results after the rebalancing on February 6, 2024. This table summarizes the performance of the models under backtesting, including the number of exceedances, the exceedance rate, and the results of the Unconditional Coverage (UC) test and the Berkowitz, Christoffersen, and Pelletier (BCP) test.

			Unadjusted Historical VaR					
Backtest Period Begin	Backtest Period End	Number of observations	Number of exceedances	Exceedance rate	Unconditional Coverage Test	BCP test p-value	BCP test Lag	BCP test Clustering
19-02-2014	**06-02-2024**	**2600**	**36**	**1,38%**	**6,3%**	**0,00%**	**10**	**64**
08-02-2023	06-02-2024	260	0	0,00%	2,22%	100%	1	0
09-02-2022	07-02-2023	260	0	0,00%	2,22%	100%	1	0
10-02-2021	08-02-2022	260	0	0,00%	2,22%	100%	1	0
12-02-2020	09-02-2021	260	16	6,15%	0,00%	0,00%	10	55
13-02-2019	11-02-2020	260	2	0,77%	69,67%	89,96%	1	0
14-02-2018	12-02-2019	260	1	0,38%	25,44%	94,99%	1	0
15-02-2017	13-02-2018	260	0	0,00%	2,22%	100%	1	0
17-02-2016	14-02-2017	260	1	0,38%	25,44%	94,99%	1	0
18-02-2015	16-02-2016	260	13	5,00%	0,00%	7,68%	1	2
19-02-2014	17-02-2015	260	3	1,15%	80,77%	84,93%	1	0

			Quantile Regression VaR					
Backtest Period Begin	Backtest Period End	Number of observations	Number of exceedances	Exceedance rate	Unconditional Coverage Test	BCP test p-value	BCP test Lag	BCP test Clustering
19-02-2014	**06-02-2024**	**2600**	**31**	**1,19%**	**33,88%**	**0,00%**	**1**	**6**
08-02-2023	06-02-2024	260	1	0,38%	25,44%	94,99%	1	0
09-02-2022	07-02-2023	260	0	0,00%	2,22%	100%	1	0
10-02-2021	08-02-2022	260	1	0,38%	25,44%	94,99%	1	0
12-02-2020	09-02-2021	260	4	1,54%	41,87%	0,01%	10	2
13-02-2019	11-02-2020	260	5	1,92%	18,44%	0,28%	1	1
14-02-2018	12-02-2019	260	2	0,77%	69,67%	0,00%	1	1
15-02-2017	13-02-2018	260	6	2,31%	7,01%	0,00%	1	2
17-02-2016	14-02-2017	260	3	1,15%	80,77%	84,93%	1	0
18-02-2015	16-02-2016	260	6	2,31%	7,01%	0,00%	2	3
19-02-2014	17-02-2015	260	3	1,15%	80,77%	0,00%	10	1

Table 14. Yearly Backtesting Results for the Unadjusted Historical and Quantile Regression VaR after the rebalancing on February 6, 2024. This table summarizes the performance of the the Unadjusted Historical and Quantile Regression VaR, including the number of exceedances, the exceedance rate, and the results of the Unconditional Coverage (UC) test and the Berkowitz, Christoffersen, and Pelletier (BCP) test, per year.

Backtest Period		Unadjusted Historical VaR											Quantile Regression VaR										
Begin	End	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10	Sum Lag 1 to 10	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10	Sum Lag 1 to 10
19/02/14	**06/02/24**	**7**	**6**	**7**	**6**	**6**	**4**	**9**	**6**	**8**	**5**	**64**	**6**	**1**	**1**	**0**	**0**	**0**	**0**	**0**	**0**	**2**	**10**
08/02/23	06/02/24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
09/02/22	07/02/23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10/02/21	08/02/22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12/02/20	09/02/21	5	5	7	6	6	3	7	5	6	5	55	0	0	1	0	0	0	0	0	0	1	2
13/02/19	11/02/20	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
14/02/18	12/02/19	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
15/02/17	13/02/18	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	2
17/02/16	14/02/17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18/02/15	16/02/16	2	1	0	0	0	1	2	1	2	0	9	2	1	0	0	0	0	0	0	0	0	3
19/02/14	17/02/15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

Table 15. Yearly Lag Dependence Results for Unadjusted Historical and Quantile Regression VaR after the rebalancing on February 6, 2024. This table presents the backtesting results of lag dependence for the unadjusted Historical VaR and the Quantile Regression VaR models, recording for each period the frequency of exceedances across lags 1 to 10 and the cumulative sum of exceedances across all lags.