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Measuring and managing the Value-at-Risk of a stocks and bonds portfolio

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Master of Finance

Supervisor:

PhD, António Manuel Rodrigues Guerra Barbosa, Assistant Professor, ISCTE-IUL

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BUSINESS SCHOOL

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Resumo

Esta dissertação analisa a aplicação de diferentes modelos para estimar o Value at Risk (VaR) e avaliar o risco financeiro associado a um portfólio de investimentos diversificado. O VaR é uma ferramenta fundamental de gestão de risco utilizada por instituições financeiras para estimar a perda máxima potencial dentro de um determinado horizonte temporal e nível de confiança. São comparados quatro modelos distintos: o modelo paramétrico (variância-covariância), a simulação histórica, a distribuição Skewed Generalized Student-t (SGSt) e o VaR com regressão quantílica.

Utiliza-se uma base de dados diária de ações e obrigações do governo, cobrindo o período de 2007 a 2024, que inclui eventos significativos como a crise financeira de 2008 e a pandemia da COVID-19. O estudo também considera fatores adicionais, como variações nas taxas de juro e flutuações cambiais.

Como estratégia complementar, são aplicadas técnicas de cobertura (hedging) para reduzir a exposição ao risco sem alterar significativamente a composição do portfólio. A decomposição marginal do VaR permite identificar as principais fontes de risco.

Para testar a precisão dos modelos, são utilizados testes estatísticos como o Proportion of Failures (Kupiec) e o Conditional Coverage (Christoffersen), avaliando a capacidade dos modelos de prever perdas extremas.

Os resultados demonstram que tanto a escolha do modelo como a alocação dos ativos têm impacto relevante na medição do risco, especialmente em períodos de elevada incerteza no mercado.

Palavras-chave: Value at Risk, gestão de risco, regressão quantílica, hedging, testes de backtesting

Códigos de Classificação JEL: C58, G11

Abstract

This thesis examines the use of multiple models to estimate Value at Risk (VaR) and assess the financial risk associated with a diversified investment portfolio. VaR is a key risk management tool commonly used by portfolio managers and financial institutions to estimate the potential maximum loss over a given time frame and confidence level. The analysis focuses on four distinct models: the Parametric (Variance-Covariance) approach, Historical Simulation, the Skewed Generalized Student-t (SGSt) distribution, and Quantile Regression VaR. Each model reflects different assumptions about return behaviour and risk factor sensitivity, offering a well-rounded comparison of risk estimates.

For this research, I used daily data from both stocks and government bonds, covering the period between 2007 and 2024. This stretch of time includes key market events like the 2008 financial crisis and the COVID-19 pandemic, which gives a good chance to see how the models hold up under different types of market stress. To get a fuller picture of the risks involved, I also included other important factors like interest rate changes and currency fluctuations.

To better manage potential losses, the research introduces hedging strategies aimed at reducing exposure without heavily altering the portfolio structure. The contribution of individual risk sources is examined using marginal VaR decomposition to provide a more complete picture of portfolio risk, the study also considers other key drivers such as interest rate movements and foreign exchange volatility.

To evaluate the reliability of the risk models, Backtesting is conducted using statistical tools such as Kupiec's Proportion of Failures (POF) test and Christoffersen's Conditional Coverage test, both of which help determine whether the models consistently identify tail risks.

The overall results show that both the choice of model and how assets are allocated across the portfolio have a significant influence on risk outcomes especially during periods of elevated uncertainty in the financial markets.

Keywords: Value at Risk, financial risk, hedging, quantile regression, backtesting, portfolio management

JEL Classification Codes: C58, G11

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List of Abbreviations

- **BCP** – Berkowitz, Christoffersen and Pelletier
- **DAX** – Deutscher Aktienindex
- **EC** – Economic Capital
- **EUR** – Euro
- **EWMA** – Exponentially Weighted Moving Average
- **FX** – Foreign Exchange
- **JPYEUR** – Japanese Yen to Euro
- **MSFT** – Microsoft Corporation (Ticker Symbol)
- **P/L** – Profit and Loss
- **POF** – Proportion of Failures
- **PV** – Present Value
- **PV01** – Present Value of a Basis Point
- **QR** – Quantile Regression
- **RM** – RiskMetrics
- **RORAC** – Return on Risk-Adjusted Capital
- **S&P500 / GSPC** – Standard & Poor's 500 Index
- **SGSt** – Skewed Generalized Student-t Distribution.
- **UC** – Unconditional Coverage
- **USD** – United States Dollar
- **USDEUR** – US Dollar to Euro
- **VaR** – Value at Risk

Chapter 1

INTRODUCTION

“All of life is the management of risk, not its elimination.”

Walter Wriston, Former Chairman of Citigroup

In the world of finance, risk is inevitable. It stems from the unpredictable nature of financial markets and affects investment decisions, portfolio performance, and even institutional survival. According to Jorion (2007), financial losses typically arise from two factors: the volatility of the underlying financial variable and the exposure to that source of risk. While corporations may not control the volatility itself, they can adjust their exposure to better manage potential losses.

Risk, at its core, refers to the dispersion of outcomes. The greater the uncertainty or variability in returns, the riskier the asset is considered to be (Francis & Kim, 2013). This variability can be statistically measured through the standard deviation or variance of returns. A widely accepted tool that captures this concept is Value at Risk (VaR)**.** VaR estimates the maximum expected loss over a specified time horizon at a given confidence level, making it a central component of modern risk management strategies.

Closely tied to VaR is the concept of Economic Capital (EC)**,** which refers to the capital a firm needs to absorb potential losses from its activities at a specific confidence level. Essentially, EC is the amount required to remain solvent under extreme conditions and is numerically equal to the VaR when used for internal risk monitoring (Alexander, 2009; Jorion, 2007). Financial institutions rely on EC to guide strategic decisions and regulatory compliance, making its accurate estimation a priority.

This study focuses on computing the VaR of a diversified portfolio composed of stocks and bonds. Using historical market data and risk factor sensitivities, the study applies four different VaR models, Parametric (Variance-Covariance), Historical Simulation, Skewed Generalized Student-t (SGSt), and Quantile Regression to estimate risk levels and identify the most robust model under varying market conditions.

The structure of risk management systems often relies on two elements: models for exposure and models for the distribution of risk factors (Jorion, 2007). Exposure models use either local valuation (e.g., delta-normal approximation) or full revaluation (e.g., Monte Carlo simulation) to estimate changes in portfolio value. Each of the VaR models examined in this thesis falls within these broader risk modelling frameworks.

To ensure optimal diversification, the portfolio construction in this study follows the principles of Modern Portfolio Theory, particularly the Markowitz diversification approach. This strategy involves combining assets with low or negative correlation to reduce total portfolio risk without compromising expected return (Francis & Kim, 2013).

"Indeed, judicious use of VaR may have avoided many of the financial disasters experienced over the past years."
— *Jorion (2007)*

The ultimate objective of this thesis is to estimate the VaR of the selected portfolio using four distinct models and to compare their performance based on accuracy, consistency, and predictive power. Through the application of backtesting techniques, the study evaluates how effectively each model forecasts potential losses under real market conditions and identifies the most reliable candidate for practical implementation. Additionally, the research incorporates a hedging strategy by analysing the marginal contribution of individual risk factors to overall portfolio risk. Based on this decomposition, targeted short positions were introduced on the most influential risk exposures in order to maintain the portfolio's projected VaR below the defined Economic Capital threshold of €110,000.

Chapter 2

LITERATURE REVIEW

VaR is an attempt to provide a single number that summarizes the total risk in a portfolio of financial assets. It was pioneered by J.P. Morgan, and it has become widely used by corporate treasurers and fund managers as well as by financial institutions. The VaR measure is used by the Basel Committee in setting capital requirements for banks throughout the world Hull, J. (2007).

Banks usually keep the details about the models they develop internally a secret. However, in 1994 J.P. Morgan made a simplified version of their own system, which they called RiskMetrics, available on the internet. RiskMetrics included variances and covariances for a very large number of different market variables. This attracted a lot of attention and led to debates about the pros and cons of different VaR models. Software firms started offering their own VaR models, some of which used the RiskMetrics database. After that, VaR was rapidly adopted as a standard by financial institutions and some nonfinancial institutions Hull, J. (2007).

VaR are more suitable when they are diverse sources of financial risk like interest rate, exchange rate and commodity prices will benefit from a global risk management system that considers all these risk accounting for correlation, various exposures and volatility across risk factors. Institutions that are exposed to one source of risk only may not require a sophisticated global risk management system. Savings and loans institutions, for instance, are exposed mainly to domestic interest rate risk, in which case a simple duration measure may be sufficient Jorion, P. (2007).

Often the significance level is set by an external body, such as a banking regulator. Under the Basel II Accord, banks using internal VaR models to assess their market risk capital requirement should measure VaR at the 1% significance level, i.e. the 99% confidence level. A credit rating agency may set a more stringent significance level, i.e. a higher confidence level (e.g. the 0.03% significance or 99.97% confidence level). In the absence of regulations or external agencies, the significance/confidence level for the VaR will depend on the attitude to risk of the user Alexander, C. (2009) he further went on to explain the risk horizon as the period over which we measure the potential loss. Different risks (depending on the liquidity and size of the position) are naturally assessed over different time periods. under the Basel banking regulations, the risk horizon for the VaR is 10 days, for illiquid assets like real estate it can be as much as 6 months to 1 year.

2.1 Decomposition of value at risk

Risk attribution involves breaking down the total risk factor VaR into its component VaR, each linked to different risk factors. Risk managers map portfolios to their respective risk factors because analysing the risk components associated with various factors creates an effective framework for hedging risks and allocating capital Alexander, C. (2009). The total risk of a portfolio may be decomposed into systematic risk, i.e. the risk that is captured by mapping the portfolio to risk factors, and specific risk, i.e. the risk that is not captured by the portfolio mapping. We then calculate total VaR by assuming systematic and specific risks are uncorrelated. The total VaR is calculated by combining these risks using the square root of the sum of their squares. It can also be estimated directly, we forget about the risk factor mapping and measure the VaR at the portfolio level, using a univariate series of portfolio returns or P&L. In the simple normal linear historical VaR model we build an empirical distribution using a time series for the portfolio returns or P&L; and in the Monte Carlo VaR model we simulate this distribution using a parametric model for the portfolio's P&L Alexander, C. (2009).

We can also break down the systematic risk of a portfolio into 'stand-alone' components, each linked to fundamental risk factors. The goal is to separate the VaR into risks associated with specific asset classes, such as equity VaR, interest rate VaR, forex VaR, and commodity VaR. This approach enables the individual assessment of foreign exchange and interest rate risks across various securities in international portfolios, which can then be combined and managed by different specialized teams or desks Alexander, C. (2009). An alternative way to disaggregate VaR is to decompose it into marginal VaR components. Marginal VaR. assigns a proportion of the total risk to each component and hence provides the risk manager with a description of the relative risk contributions from different factors to the systematic risk of a diversified portfolio. Unlike stand-alone VaR, marginal VaR is sub-additive Alexander, C. (2009).

2.2 Value at risk models.

Normal Linear VaR approach uses the covariance matrix of risk factor (or asset) returns as a core element, some people call this approach the covariance VaR model, Sometimes, this model is also called Delta-Normal VaR, particularly in the context of options. The term "delta" here refers to the sensitivity of an option's price to changes in the underlying asset. Alexander, C. (2009). The model works well when the relationship between the portfolio's returns and the risk factors (such as stock prices, interest rates, etc.) is linear. This means that small changes in the risk factors cause proportional changes in the portfolio value. Real-world asset returns often exhibit fat tails (more extreme events than predicted by a normal distribution), so this model may underestimate the probability of large losses. If the portfolio includes options or other instruments with nonlinear payoffs, the normal linear VaR model won't capture the risks properly.

The SGSt VaR model is a more advanced and flexible approach for estimating VaR. It addresses some of the limitations of the Normal Linear VaR model by relaxing the assumptions of normality, allowing for skewness and heavy tails in asset return distributions, which are often observed in real-world financial data. The student-t distribution is known for having heavier tails than a normal distribution, which means it gives more weight to extreme values (positive or negative). This allows for the modelling of extreme losses (or gains) that the Normal Linear VaR would underestimate. Skewness is added to the student-t distribution to allow the model to handle asymmetric return distributions, where losses might be more probable than gains (or vice versa). The Generalized part of the model refers to its flexibility in shaping the tails and skewness of the distribution, allowing the model to adjust to a wide range of possible return behaviours.

The historical VaR model assumes that all potential future variations have been observed in the past, meaning the future risk is modelled using past returns. This creates a simulated distribution that is identical to the historical returns over the forecast horizon. However, this assumption may be flawed if future risks differ significantly from past experiences. When using the historical VaR model, simulated movements in risk factors (e.g., stock prices, interest rates) are applied to a risk factor mapping function (like the cash flow model for bonds or the Taylor expansion for options) to translate these into potential portfolio returns Alexander, C. (2009).He also explained that historical returns are used to build an empirical distribution of portfolio performance over the forecast horizon (e.g., 1 day, 10 days). The VaR is then determined by finding the relevant percentile in this empirical distribution. One significant limitation of historical VaR is that it heavily depends on the number of data points available. A limited sample size can lead to unreliable estimates, especially when focusing on extreme losses (i.e., the tails of the distribution)

Quantile Regression VaR (QR VaR) is a non-parametric approach used to estimate VaR, particularly advantageous when the distribution of financial returns follows non normal pattern. The idea is to estimate the conditional quantiles of the return distribution. Quantile Regression is particularly useful for estimating VaR, as it focuses on the tail ends of the return distribution typically the 1% or 5% quantiles to assess potential worst-case losses (Chen & Chen, 2005). In their work, the authors also pointed out that combining Quantile Regression with time varying volatility models, such as the t-GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model, can significantly improve the reliability of VaR estimates. This is because t-GARCH models are designed to capture changes in market volatility over time a key factor when assessing financial risk. However, when estimating very low or high quantiles, like the 1% or 99% levels, a sufficiently large dataset is required to ensure the estimates are statistically meaningful.

As discussed earlier, estimating the coefficients of the α-quantile requires at least 1/α observations. If you lack sufficient data points, the estimates may become unreliable or shift toward a less extreme quantile (e.g., estimating the 2% quantile instead of the 1%).

A shortcoming of VaR is that it does not consider the expected size of a loss in the event that this loss exceeds the VaR of the portfolio. A consequence of this is that when portfolio returns are not drawn from a multivariate elliptical distribution, VaR is not a coherent measure of risk. In particular, it does not have the desirable property of subadditivity, meaning that it is possible for the VaR of a portfolio to exceed the weighted average VaR of the assets that it comprises Harris & Shen, 2006.

Chapter 3

DATA AND PORTFOLIO COMPOSITION

This study utilizes a daily dataset containing historical prices for a diversified multi-asset portfolio of 36 stocks and 4 sovereign bonds, covering a period of over 17 years, from January 2007 to February 2024. Data was obtained from Yahoo Finance for stocks and Börse Frankfurt for European sovereign bonds. The equity component includes 36 large-cap stocks diversified across major economies (USA, Europe, Japan) and sectors such as Technology, Energy, Financials, Real Estate, Healthcare, and Consumer Goods. These stocks were selected to simulate a real-world diversified institutional portfolio. All bonds assume bullet redemption at par and fixed coupon payments (annual or semi-annual). The total portfolio value is 10,000,000 euro in this study.

Asset weights were based on nominal holdings and converted to EUR-equivalent values, with the euro serving as the base currency for the entire portfolio. All non-euro positions especially USD and JPY-denominated assets were adjusted using corresponding spot FX rates sourced from Yahoo Finance.

Table 1 and 2 below present the content of my portfolio that will be used in this work

Stock Portfolio					
Portfolio	**Stock**	**Ticker**	**Currency**	**Ref Mkt**	**Quantity**
2	Microsoft (MSFT)	MSFT	USD	GSPC	1 999
2	Apple (AAPL)	AAPL	USD	GSPC	3 386
2	ASML Holding (ASML)	ASML	EUR	AEX	547
2	Adobe	ADBE	USD	GSPC	1 057
2	Salesforce (CRM)	CRM	USD	GSPC	1 790
2	Eli Lilly (LLY)	LLY	USD	GSPC	581
2	Goldman Sachs	GS	USD	GSPC	576
2	Walmart (WMT)	WMT	USD	GSPC	3 147
2	Dominion Energy (D)	D	USD	GSPC	3 402
2	ExxonMobil (XOM)	XOM	USD	GSPC	1 339
2	BlackRock (BLK)	BLK	USD	GSPC	135
2	Johnson & Johnson (JNJ)	JNJ	USD	GSPC	610
2	Coca-Cola (KO)	KO	USD	GSPC	3 397
2	Simon Property Group (SPG)	SPG	USD	GSPC	1 666
2	Chevron (CVX)	CVX	USD	GSPC	1 450
2	SAP (Germany)	SAP	EUR	GDAXI	1 637
2	Siemens (SIE, Germany)	SIE	EUR	GDAXI	2 077
2	BNP Paribas (BNP, France)	BNP	EUR	FCHI	3 439
2	Toyota Motor Corp (7203.T, Japan)	7203.T	JPY	N225	-13 972
2	Mitsubishi UFJ Financial (8306.T, Japan)	8306.T	JPY	N225	40 981
2	Intuit	INTU	USD	GSPC	468
2	Vertex Pharmaceuticals (VRTX)	VRTX	USD	GSPC	610
2	Ventas	VTR	USD	GSPC	3 021
2	Federal Realty Investment Trust	FRT	USD	GSPC	1 410
2	ConocoPhillips	COP	USD	GSPC	1 249
2	COP (ConocoPhillips)	COP	USD	GSPC	1 250
2	EOG (EOG Resources)	EOG	USD	GSPC	1 163
2	Healthpeak Properties	DOC	USD	GSPC	5 927
2	Netflix (NFLX)	NFLX	USD	GSPC	407
2	Santander (SAN, Spain)	SAN	EUR	IBEX	42 088
2	Sony Corporation	SONY	JPY	N225	-281 061
2	Panasonic	PCRFF	JPY	N225	-2,808,600
2	Pernod Ricard SA	RI.PA	EUR	FCHI	-1 254
2	Carrefour SA	CA.PA	EUR	FCHI	-10 622

2	Oracle Corporation	ORCL	USD	GSPC	-2 245
2	Pfizer Inc.	PFE	USD	GSPC	-2 391
2	3M Company	MMM	USD	GSPC	-902

Table 1 lists all equity positions held in Portfolio 2*. Including ticker symbols, currencies, quantities, and their reference markets. The portfolio is diversified across geographies (U.S., Europe, Japan) and sectors (Technology, Healthcare, Energy, Financials, Consumer Goods). Currency exposures are handled via spot FX rates aligned to evaluation date.*

Bond Portfolio						
Portfolio	Bond	Face Value	Currency	Maturity	Coupon Rate	Coupons/year
2	LU0905090048	1 370 000	EUR	2028-03-19	2.25%	1
2	DE0001135085	1 925 000	EUR	2028-07-04	4.75%	1
2	US9128286T26	1 375 000	USD	2029-05-15	2.38%	2
2	US9128287B09	822 500	USD	2026-06-30	1.88%	2

Table 2 summarizes the sovereign bond positions in Portfolio 2*. Including their ISINs, face values, currencies, coupon structures, and maturity dates. Coupon payments are annual or semi-annual depending on the bond. All values are reported at nominal face value and were later converted to present value in EUR and USD where applicable.*

Chapter 4

METHODOLGY

This study aims to answer the following empirical question, which VaR model provides the most accurate and robust risk estimate for this specific diversified stock and bond portfolio? To address this question, the models will be evaluated based on Backtesting performance, predictive accuracy, and robustness across different market conditions. The results will offer insights into the reliability of different VaR methodologies and their practical implications for portfolio risk management.

While the four VaR models analysed in this study differ in structure and assumptions, they all rely fundamentally on the portfolio's volatility to generate risk estimates. This chapter outlines the methodological steps used to compute portfolio returns, model volatility, and implement each VaR approach using historical financial data. Chapter 5 presents the Backtesting framework, and the statistical tools used to evaluate each model's effectiveness. Based on these results, one model will be selected and applied in the final stage of the study for a one-year forward-looking risk assessment detailed in Chapter 6.

4.1 Risk Factor Exposure Mapping

To apply VaR models meaningfully, the portfolio must first be translated into exposures to a manageable number of market risk factors. This is achieved through a process known as risk factor exposure mapping, my initial mapping date is 27 January 2023. These risk factors, which include interest rates, equity indices and exchange rates, are essential inputs in the computation of VaR. The way in which exposures are mapped to these risk factors significantly impacts the accuracy and stability of risk estimates.

In the following subsections, we detail the methodology used to map exposures for each asset class included in the portfolio. A summary of the resulting risk factor exposures derived from this mapping can be found in Table 3 at the end of the section.

Equity Mapping

The equity portion of the portfolio was mapped to major indices based on geographical classification (e.g., S&P 500 for U.S. stocks, DAX for German stocks). In theory, the return of each stock can be modelled as a linear function of the index return and its sensitivity to that index, as shown below:

$$\boldsymbol{R}_i = \boldsymbol{B}.\boldsymbol{f}_{\boldsymbol{i}} + \boldsymbol{\mathcal{E}}_{\boldsymbol{i}}$$

(1)

Where $\boldsymbol{R}_i$ vector of asset returns, $\boldsymbol{B}$ matrix of betas (each row is a stock, each column a factor), $\boldsymbol{f}_{\boldsymbol{i}}$ Vector of factor returns (e.g., index returns), $\boldsymbol{\mathcal{E}}_{\boldsymbol{i}}$ vector of residuals.

In this thesis, this relationship was used to compute the systematic exposure of each stock by multiplying the asset's beta with its currency-adjusted value in EUR, thereby capturing its exposure to index risk for VaR estimation.

To quantify the sensitivity of individual stocks to their corresponding equity indices, beta coefficients were estimated using the Exponentially Weighted Moving Average (EWMA) covariance matrix, a technique that will be formally introduced in section 4.3. This estimation is essential for implementing a factor-based VaR framework. For each stock mapped to a specific index (e.g., Microsoft to S&P 500), the beta was computed as the ratio of the covariance between the stock's returns, and the index returns to the variance of the index:

$$\beta_i = \frac{Cov(r_{i,}r_m)}{Var(r_m)} \qquad (2)$$

Here r_i refer to the returns of the individual stock, while r_m represents the returns of the index it is mapped to for example the S&P 500 (GSPC) for U.S. stocks. The values for both covariance and variance were calculated based on historical return data, EWMA matrix using a smoothing parameter λ=0.94, as specified in the configuration sheet. This approach dynamically adjusts the weights of past observations, giving more significance to recent market movements, thereby capturing volatility clustering in financial time series.

Bond Mapping

To estimate the interest rate risk of bonds in the portfolio, this thesis applies the PV + PV01 invariant mapping approach, which has become standard in financial risk management (Alexander, 2008). This method is particularly useful when bond cash flows occur at non-standard maturities for which no direct market interest rate is available.

In such cases, each cash flow is split and mapped to two adjacent standard maturity vertices (e.g., 5 and 7 years), for which interest rate data is available. The goal is to preserve two key properties of the original cash flow:

- PV (Present Value): the current value of the future cash flow, discounted using the yield curve.
- PV01 (Price Value of a Basis Point): the sensitivity of that present value to a one basis point (0.01%) shift in interest rates.

The PV + PV01 invariant mapping ensures that both the economic value and the interest rate sensitivity of the original cash flow are maintained after the mapping. The portion of the cash flow allocated to each vertex is determined using linear interpolation based on time to maturity.

Cash Flow Mapping: PV + PV01 Invariant Interpolation

Let PV_{C_T} represent the present value of a cash flow maturing at time T and let T_1 and T_2 be the two standard maturities immediately below and above T. The exposure to each maturity is calculated using weights w_1 and w_2, such that:

$$w_1 = \frac{T_2 - T}{T_2 - T_1} \tag{3}$$

$$w_2 = (\mathbf{1} - \frac{T_2 - T}{T_2 - T_1}) \tag{4}$$

The cash flow PV_{C_T} is then distributed proportionally to the lower vertices and higher vertices denoted as T_1 and T_2 . Here x_{T_1} and x_{T_2} represent the present value portions of the original cash flow that are mapped to T_1 and T_2 vertices, respectively.

$$x_{T_1} = w_1 . PV_{C_T} \tag{5}$$

$$x_{T_2} = w_2 . PV_{C_T} \tag{6}$$

To ensure both the value and interest rate sensitivity of each bond cash flow are accurately represented, the mapping process is guided by equations (7) and (8). Equation (7) ensures that the total present value (PV) of the cash flow is preserved during the mapping and equation (8) ensures that the PV01, or the interest rate sensitivity, of the original cash flow is also preserved.

$$x_{T_1} + x_{T_2} = PV_{C_T} \tag{7}$$

$$T_1 . x_{T_1} + T_2 . x_{T_2} = T . PV_{C_T} \tag{8}$$

Application to the Portfolio

Each bond’s cash flows were mapped using this interpolation method, with interest rate data sourced for standard EUR vertices such as EUR3M, EUR6M, EUR1Y, EUR2Y, up to EUR20Y. For instance, a bond maturing in 5.433 years was mapped to EUR5Y and EUR7Y vertices using weights of 78.33% and 21.67% respectively. The resulting PV01 values reflect the bond's sensitivity to movements in those specific interest rate points on the curve.

The exposures to each risk factor are summed across all bonds. Additionally, since some bonds are denominated in USD, their PV01 exposures were converted into EUR using the prevailing exchange rate as of 27 January 2023.

This mapping ensures that the bond portion of the portfolio is appropriately integrated into the factor-based VaR framework used in this thesis.

Currency

Since the base currency of this analysis is EUR and the portfolio contains assets denominated in foreign currencies, these positions are exposed not only to their respective market risk factors, but also to fluctuations in foreign exchange rates. Specifically, assets denominated in USD and JPY carry additional exposure to the USD/EUR and JPY/EUR exchange rates, respectively.

To account for this, all foreign-denominated positions whether equity or bond were converted to EUR using the corresponding spot exchange rates as of 27 January 2023. The exposure to each FX risk factor is defined as the total EUR equivalent value of all positions originally denominated in that currency.

For example, assets priced in USD were exposed to the USD/EUR risk factor, and the total exposure was the sum of all USD asset values converted to EUR. The same logic applies to JPY and other currencies. This mapping allows changes in exchange rates to be captured in the Value-at-Risk estimation, ensuring that FX movements are properly reflected in overall portfolio risk.

Summary of Mapped Portfolio Exposures

Table 3 presents the total portfolio exposure across mapped risk factors on the chosen mapping date. The exposures are grouped into three categories: equities, interest rate vertices, and currency pairs. Each value reflects the EUR-equivalent exposure to that specific risk driver. Equity exposures are based on the EUR market value of each stock. Bond exposures represent sensitivities to standard maturity interest rates using the PV+PV01 mapping method. Currency exposures capture the total investment in non-EUR assets converted using the applicable FX rates. This mapping forms the basis for VaR estimation, as it quantifies how the portfolio is exposed to systematic changes in equity markets, interest rates, and exchange rates.

Equity		Interest Rate		Currency	
Risk Factor	Exposure (EUR)	Risk Factor	Exposure (EUR)	Risk Factor	Exposure (EUR)
MSFT	449698.09	EUR3M	-1.71	USDEUR	5941634.62
AAPL	449845.81	EUR6M	-2.65	JPYEUR	-269996.13
ASML	360097.36	EUR1Y	-7.57		
ADBE	359670.36	EUR2Y	-23.28		
CRM	269460.47	EUR3Y	-50.91		
LLY	180036.97	EUR5Y	-1325.7		
GS	179984.59	EUR7Y	-331.78		
WMT	134978.8	EUR10Y	0		
D	179998.46	EUR15Y	0		
XOM	135039.35	EUR20Y	0		
BLK	90279.34	USD3M	-0.35		
JNJ	90004.54	USD6M	-0.72		
KO	179908.01	USD1Y	-1.26		
SPG	180024.86	USD2Y	-8.13		
CVX	225057.32	USD3Y	-198.86		
SAP	180035.21	USD5Y	-6600.54		
SIE	2077	USD7Y	-5521.62		
BNP	225013.77	USD10Y	0		
7203.T	-179997.95	USD20Y	0		
8306.T	-270002.16				
INTU	179985.97				
VRTX	135014.9				
VTR	135043.06				
FRT	144516.65				
COP	269825.97				
EOG	134946.58				
DOC	153015				
NFLX	134778.36				
SAN	180004				
SONY	-180001.94				
PCRFF	-208067.41				
RI.PA	-225048.36				
CA.PA	-180037.71				
ORCL	-180032.73				
PFE	-90045.14				
MMM	-89980.95				

Table 3 Risk Factor Exposure Map in EUR as of 27 January 2023*. The values represent mapped exposures to various equity, bond and currency risk factors as of reference date.*

4.2 Returns

The return on the portfolio is derived from changes in the value of its underlying assets, which are driven by their exposure to specific risk factors namely, equity prices, interest rates, and foreign exchange rates. This section outlines how profit and loss (P&L) is computed for each asset class in the portfolio, assuming the capital allocated to each position remains constant over the time horizon of analysis.

Equity Returns

For stocks, the relevant risk factor is the change in market price. The daily profit or loss from holding a stock is calculated by multiplying the invested amount by the percentage change in the stock price between two consecutive trading days. This is given by.

$$P\&L_{stock_t} = M_{stock} * (\frac{P_t}{P_{t-1}} - 1) \tag{9}$$

In this formula,M_{stock} represents the amount of money invested in the stock, while P_t and P_{t-1} are the stock price on day t and the previous day $t-1$, respectively.

Bond Returns

Bonds contribute to P&L through changes in interest rates. Each cash flow of a bond is exposed to a specific point (or vertex) on the yield curve, and its sensitivity to interest rate movements is captured using the PV01 (Present Value of 1 basis point). The daily return from bonds is calculated as the sum of the PV01-weighted interest rate changes across all mapped maturities:

$$P\&L_{bond_t} = \sum_{i=1}^{n} -PV01_{T_i} * \frac{\Delta_{r_{T_i}}}{0.01\%} \tag{10}$$

Where $PV01_{T_i}$ is the PV01 exposure at maturity T_i, $\Delta_{r_{T_i}}$ is the change in the interest rate for that maturity, n is the number of mapped vertices for the bond's cash flows.

Currency Returns

Foreign assets introduce currency risk in addition to market or interest rate risk. The gain or loss from exchange rate movements is computed by applying the change in the currency pair's exchange rate to the euro-equivalent value of the investment. The formula used is:

$$P\&L_{bond_t} = M_{currency} * (\frac{FX_t}{FX_{t-1}} - 1) \tag{11}$$

In this context $M_{currency}$ is the value of the investment made in a foreign currency, while FX_t and FX_{t-1} refer to the exchange rates on the current day and the previous day, respectively.

Aggregated Portfolio Returns

To calculate the overall return of the portfolio, we combine the returns from stocks, bonds, and currency positions. This is done by taking the dot product of the exposure vector and the vector of changes in their respective risk factors.

$$P\&L_{portfolio_t} = \begin{bmatrix} M_{stock} \\ \vdots \\ -PV01_{T_i} \\ \vdots \\ M_{currency} \end{bmatrix}^T * \begin{bmatrix} (\frac{P_t}{P_{t-1}} - 1) \\ \vdots \\ \frac{\Delta r_{T_i}}{0.01\%} \\ \vdots \\ (\frac{FX_t}{FX_{t-1}} - 1) \end{bmatrix} \tag{12}$$

This equation efficiently captures the total P&L of the portfolio at any point in time t, by matching each exposure to the corresponding change in the underlying risk factor.

Return as a Percentage

To express the portfolio P&L in percentage terms, it is normalized by dividing by the total portfolio value on the base date of 27 January 2023

$$R_t(\%) = \frac{P\&L_{portfolio_t}}{Portfolio\ Value}$$

This transformation allows for the return to be interpreted on a relative basis, enabling comparison across time or between different portfolios.

4.3 Volatility and Covariance Estimation Using EWMA

To accurately model volatility and covariances is a fundamental step in estimating portfolio risk. In this study, we adopt the Exponentially Weighted Moving Average (EWMA) approach to estimate the variances and covariances of asset returns. This method is especially relevant in financial modelling because it reflects the reality that more recent market information carries more weight than distant historical data.

The EWMA model is a -looking technique that addresses the limitations of the simple historical variance, which assigns equal weights to all past return observations regardless of their age. By contrast, EWMA assigns declining weights to older returns through a decay factor, allowing the model to capture shifts in market volatility more responsively.

Following the RiskMetrics methodology developed by J.P. Morgan and Reuters (1996), the EWMA variance $\hat{\sigma}_t^2$ for a given asset on day t is computed recursively as:

$$\hat{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2 \tag{13}$$

Where $\hat{\sigma}_t^2$ is the estimated variance on day t, r_{t-1}^2 is the return observed on day $t-1$, λ is the smoothing parameter ranging between 0 and 1.

In this thesis, we did not rely on just one smoothing parameter. Instead, we tested multiple values of λ across the different VaR models to see how changes in the weighting of past data affect volatility estimates. This approach gave us the flexibility to observe how models respond under different market conditions, whether they react quickly to new information with lower λ values or remain more stable with higher ones. It also helped us compare which specification worked best in estimating risk accurately.

It's also important to note that implementing the EWMA model requires a warm-up period**.** Since EWMA calculates variance recursively, the early values can be highly sensitive to initial assumptions. To address this, we used a warm-up phase of 260 trading days approximately one calendar year to ensure that the variance estimates were stable and reflective of actual market dynamics before being used for risk estimation. This step helps eliminate bias from initial conditions and improves the reliability of the model in practice.

Covariance Estimation

To compute the full risk profile of a diversified portfolio, we extend the EWMA approach to estimate the time-varying covariance matrix. Given two assets i and j, their EWMA covariance at time t is given by.

$$\widehat{Cov}_{i,j,t} = (1-\lambda) r_{i,t-1} r_{j,t-1} + \lambda \widehat{Cov}_{i,j.t-1} \tag{14}$$

Here $r_{i,t-1}$ and $r_{j,t-1}$ represent the returns of assets i and j on the previous day, while and $\widehat{Cov}_{i,j.t-1}$ refers to the previous day's covariance estimate. The entire EWMA covariance matrix is updated daily, using historical return series of all the portfolio's risk factors. The initial covariance matrix is calculated over a warm-up period of 260 trading days, which allows for a stable base before applying the recursive EWMA updates.

EWMA Configuration in This Study

As configured in our model:

- Reference Mapping Date: January 27, 2023
- EWMA Smoothing Factor: λ varies between 0 and 1
- Warm-up Period: 260 days

These parameters are chosen to ensure both responsiveness to recent market movements and consistency across all portfolio assets. The resulting EWMA-based volatility and covariance estimates are used as key inputs in the Value-at-Risk (VaR) models implemented later in this thesis.

4.4 Introduction to VaR Methodology

VaR is a widely adopted risk measure used to quantify the potential loss in value of a portfolio over a specified time horizon, given a predefined confidence level. In this thesis, we use a one-day holding period ($h = 1$) and a confidence level of 99%, which corresponds to a significance level of α=1%. In other words, we are 99% confident that under normal market conditions, the portfolio will not experience a loss exceeding the VaR estimate on the following day.

Formally, the VaR over h days is derived from the α-quantile of the distribution of returns over that period. Let X denote the h day portfolio return. The α-quantile, denoted x_α satisfies:

$$P(X < x_\alpha) = \alpha \tag{15}$$

If the cumulative distribution function $F(x)$ of returns is known, then the quantile x_α be computed as the inverse of F at α

$$x_\alpha = F^{-1}(\alpha) \tag{16}$$

Since VaR represents a potential loss, we express it in absolute value terms by taking the negative of the quantile:

$$VaR_{h,\alpha} = -F^{-1}(\alpha) = -x_\alpha \tag{17}$$

This framework provides a consistent method for quantifying market risk.

This study aims to estimate VaR using four different models: the Parametric VaR (Variance-Covariance method), Historical Simulation VaR, Skewed Generalized Student-t (SGSt) VaR, and Quantile Regression VaR. There are numerous VaR models to choose from, each with various potential variations. This raises an important question: which model should be selected for the upcoming one-year period, given the unique characteristics of the portfolio as of today? Identifying the optimal model with confidence requires testing multiple options and evaluating their accuracy.

Normal Linear VaR (RiskMetrics VaR)

The parametric VaR assumes that portfolio returns follow a normal distribution of random variable denoted as $X_h \sim N(\mu_h, \sigma_h)$ where μ_h and σ_h represent the mean and standard deviation of returns over the time horizon h, respectively.

Recalling the generic VaR expression in Equation (17), and applying the assumption of a normal distribution, the VaR can be expressed as:

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha) * \sigma_h - \mu_h \tag{18}$$

Here, $\Phi^{-1}(\alpha)$ denotes the inverse cumulative distribution function (quantile) of the standard normal distribution corresponding to the significance level α.

According to the recommendation by Alexander (2009), it is reasonable to set the expected return $\mu_h = 0$ for short-term horizons, particularly when calculating daily VaR. This assumption simplifies the formula, which then becomes:

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha) * \sigma_h \quad (19)$$

In our study, we adopt this simplification and use a significance level of 1% (i.e., $\alpha = 0.01$). The critical value $\Phi^{-1}(0.01)$ corresponds to approximately −2.326, which implies that we are 99% confident that the portfolio will not lose more than the estimated VaR in a single day.

The volatility σ_h used in this computation is estimated using the EWMA model, as described in Equation (13).

Skewed Generalized Student-t (SGSt) VaR

The SGSt VaR model is an advanced and flexible approach for estimating VaR. It improves on traditional models, such as the Normal Linear VaR, by addressing two key limitations which are fat tail losses and skewness. While the Normal VaR approach relies on the assumption that asset returns are normally distributed, empirical evidence often reveals that financial return series tend to exhibit fat tails and asymmetry. These features deviate from normality and, if ignored, can lead to an underestimation of risk, particularly in the tails of the distribution. In practical terms, this means that models based on the normal distribution may underestimate the likelihood of extreme losses, especially at low significance levels such as 1% or lower.

To address the limitations of the normal distribution in modelling financial returns, the SGSt distribution is adopted due to its enhanced flexibility in capturing the stylized features of asset returns. The SGSt distribution incorporates three additional parameters λ (lambda), p, and q that allow it to better represent deviations from normality. Specifically, λ governs skewness: when λ = 0, the distribution is symmetric; positive or negative values introduce asymmetry. The p parameter influences the distribution's symmetry and central shape, while the q parameter primarily affects the kurtosis by determining the thickness of the distribution’s tails and thus the probability of extreme events.

As part of the SGSt VaR model implementation, these parameters alongside the location (μ) and scale (σ) were estimated using maximum likelihood estimation (MLE). Together, these five parameters allow the SGSt distribution to flexibly match the empirical distribution of financial returns, particularly by accommodating both skewness and heavy tails.

To ensure robustness, the SGSt parameters were estimated using rolling windows of varying lengths 250, 500, 750, and 1000 daily observations allowing the VaR model to reflect both short-term and longer-term market dynamics. This rolling estimation approach helps account for changes in the statistical properties of returns over time and ensures that the SGSt-based VaR remains responsive to evolving market conditions.

Formally, the h-day SGSt VaR is expressed as:

$$VaR_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) * \sigma_h - \mu_h \tag{20}$$

Where $T^{-1}_{0,1,\lambda,p,q}(\alpha)$ denotes the $\alpha -$ quantile of the standardized SGSt distribution. For short horizons (e.g., 1 day), $\mu_h$the mean is often considered negligible, and the formula simplifies to:

$$VaR_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) * \sigma_h \tag{21}$$

By incorporating skewness and fat tails, the SGSt model offers a more conservative and realistic estimation of potential losses, especially when markets are under stress. In this implementation, we do not use the volatility estimate derived from Maximum Likelihood Estimation (MLE), as it treats all past observations with equal weight. Instead, we adopt the EWMA-based volatility estimate, which places greater emphasis on more recent returns. This adjustment is expected to enhance accuracy, particularly under changing market conditions.

Historical VaR (Volatility-Adjusted)

While the RiskMetrics and SGSt Value-at-Risk models rely on the assumption that portfolio returns follow a specific distribution, the Historical VaR approach offers a non-parametric alternative. It does not assume any underlying distribution of returns but rather utilizes actual past return data to estimate the VaR.

To compute the Historical VaR, a rolling window of n days is selected. Within this window, the model collects daily portfolio returns and ranks them from worst to best. The VaR is then defined as the return at the α-quantile (for instance, the 1st percentile if α = 1%). This effectively gives the minimum loss that is expected to be exceeded with a probability of α. However, this method gives equal weight to all observations regardless of how recent they are, thereby not accounting for current market conditions. To address this limitation, we adopt the Volatility-Adjusted Historical

VaR methodology proposed by Hull and White (1998), which adjusts historical returns based on changes in market volatility. The idea is to scale past returns to reflect the current level of volatility, allowing the adjusted historical returns to better align with today's market dynamics.

The adjustment involves computing a time series of volatilities using the EWMA model as shown in equation (13). Let $\hat{\sigma}_t$ be the estimated volatility for day t, and T be the day for which the VaR is being computed. The volatility-adjusted return $\hat{r}_t$ is calculated as:

$$\hat{r}_t = \frac{r_t}{\hat{\sigma}_t} \hat{\sigma}_T \tag{22}$$

This formulation ensures that each return is rescaled to reflect the volatility on the VaR measurement date.

In our implementation, we estimate volatility using the EWMA method with a smoothing parameter λ ($from\ 0\ to\ 1$) and construct the adjusted return series using rolling windows of 250, 500, 750, and 1000 observations. Finally, the VaR is computed by selecting the α-quantile of this adjusted return distribution.

Quantile Regression VaR (QR VaR)

QR VaR provides a non-parametric alternative for estimating the α-quantile of portfolio returns, particularly useful when the distribution of returns does not adhere to standard assumptions like normality. Instead of relying on a predefined distribution, QR VaR estimates the conditional quantile of the return distribution based on selected explanatory variables. In this method, the dependent variable is the portfolio return, and the model seeks to estimate its α-quantile using historical data and chosen predictors.

The general form of the α-quantile regression for estimating VaR is:

$$VaR_\alpha = -q_{\alpha.y} = -(\hat{a} + \hat{b}x_i) \tag{23}$$

Where $\hat{a}$ and $\hat{b}$ are estimated coefficients from quantile regression, y is the portfolio return, x_i is the explanatory variable used (e.g. volatility), $q_{\alpha.y}$ is the α-quantile of the conditional distribution of returns.

The quantile regression model minimizes the asymmetric loss function to obtain the best fit for the α-quantile.

$$(\hat{a}, \hat{b}) = \arg\min_{a,b} \sum_{i=1}^{n} [y_i - (a + bx_i)] * (\alpha - I_{y_i-(a+bx_i)<0}) \tag{24}$$

The term $I_{y_i-(a+bx_i)<0}$ acts as an indicator: it returns 1 if the prediction falls below the actual value (i.e., the residual is negative), and 0 otherwise. This mechanism allows the model to focus more on downside risks, which is especially important for lower quantile estimations like VaR. To improve the robustness of the model, we implemented three specifications of the explanatory variables:

QR Specification 1: includes only the EWMA volatility as a predictor, capturing short term risk through:

$$y_t = b.\sigma_t + \varepsilon_t \tag{25}$$

Where σ_t represents the estimated volatility on day t, calculated using EWMA approach.

QR Specification 2**:** This version uses two EWMA volatility estimates with different decay factors (e.g., λ_1 and λ_2) to capture short-term and medium-term volatility effects represented by σ_{t,λ_1} and σ_{t,λ_2}

$$y_t = b.\sigma_{t,\lambda_1} + c.\sigma_{t,\lambda_2} + \varepsilon_t \tag{26}$$

QR Specification 3**:** And for each of these, specifications both with and without a constant term were considered.

$$y_t = a + b.\sigma_{t,\lambda_1} + c.\sigma_{t,\lambda_2} + \varepsilon_t \tag{27}$$

In each specification, the estimated quantile regression is re-evaluated daily using a rolling window of 260 and 520 observations, ensuring that the most recent market dynamics are reflected in the parameter estimates.

Finally, the quantile based VaR is computed as:

$$VaR_{\alpha,t} = -(\hat{b}.x_t + \hat{c}.z_t) \tag{28}$$

Where x_t and z_t are the selected input variables based on the chosen specification, such as current volatility and its short-term averages. This approach allows for more flexibility and accuracy in capturing the dynamic risk profile of the portfolio without assuming a specific return distribution, making it particularly suited for financial time series data with asymmetries and volatility clustering.

For ease of presentation going forward, we assign a number to each VaR model computed. Table 4 below presents the number assigned to each model and its corresponding specification, including the distributional assumption, volatility estimation method, and rolling sample details where applicable. While we explored other numerous configurations of the models, for purpose of this thesis and space, we will be presenting these 13 models.

VaR Model	Description
1	Parametric Normal with EWMA volatility estimations with a lambda of 0.96
2	Parametric Normal with EWMA volatility estimations with a lambda of 0.94
3	Parametric SGSt with EWMA volatility estimations with a lambda of 0.94(Sample of 520 obs. Reestimated daily)
4	Unadjusted Historical VaR using a rolling sample of 260 obs.
5	Unadjusted Historical VaR using a rolling sample of 520 obs.
6	Volatility Adjusted Historical VaR using a rolling sample of 520 obs. with EWMA volatility estimations
7	Volatility Adjusted Historical VaR using a rolling sample of 260 obs. with EWMA volatility estimations
8	Quantile regression with EWMA variance as the explanatory variable 0.96 (sample of 520 obs reestimated daily)
9	Quantile regression with EWMA variance as the explanatory variable 0.96 and a constant (sample of 520 obs reestimated daily)
10	Quantile regression with two EWMA volatility as the explanatory variable 0.96 and 0.9 and a constant (sample of 520 obs. Reestimated daily)
11	Quantile regression with two EWMA volatility as the explanatory variable 0.96 and 0.9 (Sample of 520 obs. Reestimated daily)
12	Quantile regression with two EWMA volatility as the explanatory variable 0.96 and 0.9(Sample of 260 obs. Reestimated daily)
13	Quantile regression with EWMA variance as the explanatory variable 0.94 (sample of 800 obs reestimated daily)

Table 4 Summary of Value-at-Risk models implemented in this study.

The table outlines 13 distinct VaR models, covering both parametric and non-parametric approaches. Parametric models include the Normal and SGSt distributions, estimated using equally weighted and EWMA volatility measures. Historical models, both unadjusted and volatility-adjusted, are computed with sample sizes of 260 and 520 observations. Quantile regression models explore various combinations of explanatory variables such as EWMA volatility with varying decay factor are computed with sample sizes of 260 and 520 observations. Where stated, models are re-estimated daily to reflect evolving market conditions.

Chapter 5

BACKTESTING THE VAR MODELS

In this chapter, the performance of the four VaR models discussed in Chapter 4 is evaluated through Backtesting. The objective is to determine which model best captures the tail risk of the diversified portfolio under analysis and should therefore be adopted for risk monitoring and management.

The Backtesting is conducted over a ten-year historical period, from 11 February 2013 to 27 January 2023, using historical portfolio returns that reflect the portfolio composition on the base date. For each VaR model, daily risk forecasts are generated and compared against actual daily returns. This comparison enables the identification and analysis of VaR breaches (also known as exceedances), providing insight into each model's predictive accuracy and reliability in capturing extreme losses. Two statistical tests are used to assess model adequacy. The Unconditional Coverage (UC) test, introduced by Kupiec (1995), which examines whether the proportion of VaR violations is consistent with the chosen confidence level. The BCP test (Berkowitz, Christoffersen, and Pelletier, 2011), which further assesses the independence of exceedances, indicating whether they occur in clusters or are randomly distributed.

Though both Backtesting models UC and BCP tests assess model performance from different perspectives, our evaluation will primarily rely on the UC test results. The BCP test, however, plays a crucial role when two models show similar performance under the UC test, helping to further distinguish their reliability. For instance, even if a model shows a low number of exceedances, it may still fail the BCP test if these exceedances occur in close succession. On the other hand, a model with more exceedances might pass the BCP test if those events are well spaced over time, even though this depends on the lag used in the test. In situation where the model fails BCP test due to clustering, we must carefully look at the clustering(lags) to determine how it occurs before choosing a model to use in the next one year to estimate VaR.

To conduct a comprehensive evaluation, both tests are applied over the entire historical period (from 11 February 2013 to 27 January 2023), referred to as the global period, as well as on each individual year (260 days) within this range when appropriate. While analysing performance during distinct time periods is useful for understanding how models behave under varying market conditions, the final model selection will be primarily based on performance over the full sample, as it provides more stable and representative results.

Section 5.1 and Section 5.2 explain the methodology behind the UC and BCP tests respectively. Section 5.3 presents the outcomes of the Backtesting and identifies the model selected for forecasting risk in the subsequent one-year period.

5.1 Unconditional Coverage Test

The Unconditional Coverage (UC) Test evaluates whether a VaR model accurately predicts the frequency of losses that exceed the VaR estimate. A model is considered well-specified if the observed proportion of exceedances aligns with the predefined significance level α, which in this study is set at 1% (Alexander, 2009).

We define an exceedance through an indicator function as:

$$I_{\alpha,t} = \begin{cases} 1, & if\ r_t < -VaR_{1,\alpha,t} \\ 0, & otherwise \end{cases}$$

Here r_t refers to the portfolio return at time t while $VaR_{1,\alpha,t}$ represents the 1-day VaR estimate for time t. This function generates a sequence of binary outcomes representing whether a breach has occurred each day.

Under the null hypothesis H_0, the exceedances are assumed to follow an independent and identically distributed (i.i.d.) Bernoulli process with success probability α. The UC test then compares the observed exceedance rate π_{obs} to the expected rate $\pi_{exp} = \alpha$. The hypotheses are:

$$H_0 : \pi_{obs} = \pi_{exp} = \alpha$$
$$H_1 : \pi_{obs} \neq \pi_{\mathrm{exp}}$$

The test statistic is given by:

$$LR_{UC} = \left(\frac{\pi_{exp}}{\pi_{obs}}\right)^{n_1} \left(\frac{1-\pi_{exp}}{1-\pi_{obs}}\right)^{n_0} \tag{29}$$

In this context, n_1 refers the number of VaR exceedances while $n_0 = n - n_1$ represents the number of days when the actual loss did not breach the VaR estimate, The observed exceedance rate is calculated as $\pi_{obs} = \frac{n_1}{n}$ $and\ \pi_{exp} = \alpha$

According to the null hypothesis, the test statistic, $-2\ln(LR_{UC})$ follows a chi-squared (χ^2) distribution with 1 degree of freedom. If this test statistic exceeds the critical value meaning the p-value falls below the chosen significance level, we reject the null hypothesis. This suggests that the model may not be accurately estimating the frequency of extreme losses. At 95% confidence level, the critical threshold is $\chi^2_{0.95}(1)$ =3.8415

$$-2\ln(LR_{UC}) > 3.8415$$

It indicates that the model may be mis specified. The test statistic is evaluated directly and compared against the chi-squared critical value. Alternatively, the corresponding p-value is computed, which provides sufficient information to determine whether the model's exceedance rate significantly deviates from the expected value under the null hypothesis.

5.2. BCP Test (Berkowitz, Christoffersen, and Pelletier)

In assessing the accuracy of VaR models, it is not sufficient to merely check how many times the model underestimates losses (exceedances); it is also crucial to analyse how those exceedances are distributed over time. The BCP Test, proposed by Berkowitz et al. (2011), evaluates whether exceedances are independent from each other**,** which is a desirable property in a well-specified VaR model.

The underlying idea is that if the exceedances are truly random, the occurrence of one exceedance should not increase the probability of another happening soon after. If exceedances tend to cluster (i.e., occur in close succession), it may suggest that the VaR model fails to capture changes in volatility or risk conditions promptly, in other words, the autocorrelation in exceedances should 0 at all lags if the VaR model is accurate.

Null and Alternative Hypotheses, let $\hat{\rho}_k$ denote the autocorrelation at lag k, then:

$$H_0: \hat{\rho}_k = 0, \quad \forall \mathrm{k} \in \{1, \dots, K\}$$
$$H_1: \exists k \in \{1, \dots, K\}\ such\ that\ \hat{\rho}_k \neq 0$$

This means the null hypothesis assumes that exceedances are independent (i.e., no autocorrelation), while the alternative assumes at least one lag exhibits non-zero autocorrelation.

The BCP test statistic is defined as:

$$BCP(K) = n(n+2)\sum_{k=1}^{K}\frac{\hat{\rho}_k^2}{n-k} \qquad (30)$$

Where n is the number of observations, K is the maximum lag considered (we use $K = 10$), $\hat{\rho}_k$ is the sample autocorrelation of exceedance indicators at lag k.

Under the null hypothesis H_0, the BCP statistic follows a chi-squared distribution with K degrees of freedom:

$$BCP(K) \sim \chi_K^2 \qquad (31)$$

Choosing the value of K is essential, a larger K increases test sensitivity to distant autocorrelations but makes the null harder to reject due to increased critical values. A smaller K increases the power of the test at lower lags but may overlook higher-order clustering. In this study, the BCP test is applied using lags from 1 to 10, capturing potential autocorrelation effects over short-term horizons while maintaining reasonable test sensitivity.

At a 5% significance level**,** the critical value for $\chi_{10,0.95}^2$ is 18.31. If the calculated $BCP(K)$ value exceeds 18.31**,** we reject the null hypothesis, concluding that the exceedances show statistically significant autocorrelation an indication that the model is not well-calibrated to the actual behaviour of risk in turbulent conditions.

5.3 Backtest Results and Model Selection

To determine the most suitable VaR model for our portfolio, we conducted backtests over a ten-year global period, spanning from 11 February 2013 to 27 January 2023. During this period, we computed daily VaR estimates for each of the models discussed in Chapter 4. Using the Unconditional Coverage (UC) and BCP tests explained earlier in this chapter, we assessed each model's performance under both the full sample and yearly sub-samples when applicable. This analysis serves as the foundation for selecting the model to be used in future risk monitoring. The portfolio under study had a total value of €9,999,999,92**,** comprised of €4,499,839.58 in stocks and €5,500,160.34 in bonds. This diversified composition was used throughout the analysis to simulate real-world investment exposure.

Given that our global period includes n = 2600 observations, and that each VaR model is estimated at a 1% significance level, a well-specified model is expected to produce approximately $2600 \times 0.01 = 26$ exceedances.

Following standard statistical practice, we reject the null hypothesis of the UC test when the corresponding p-value is below 5%**.** Therefore, a model is considered accepted by the UC test if the p-value exceeds 5%**,** indicating that the observed exceedance rate is not significantly different from the expected rate.

The table below (Table 5) summarizes the UC test results for all models, reporting the number of exceedances, the exceedance rate, and the associated p-value for each.

VaR Model	**Model class**	**exceedance**	**exceedance rate**	**p-value**
1	Parametric Normal	51	1.96%	0.00%
2	Parametric Normal	50	1.92%	0.00%
3	Parametric SGSt	83	3.19%	0.00%
4	Historical	36	1.38%	6.25%
5	Historical Vol Adj.	43	1.65%	0.22%
6	Historical Vol Adj.	39	1.50%	1.70%
7	Historical	53	2.04%	0.00%
8	Quantile	28	1.08%	69.70%
9	Quantile	29	1.12%	56.15%
10	Quantile	44	1.69%	0.12%
11	Quantile	32	1.23%	25.37%
12	Quantile	36	1.38%	6.25%
13	Quantile	28	1.08%	69.70%

Table 5 shows the UC test results for the global Backtesting period*. Models with a p-value greater than 5% pass the test, meaning the number of exceedances is statistically consistent with the model's expected risk level. For reference, see Table 4 for the description of each model specification.*

We observe from Table 5 that the parametric Normal models, represented by model numbers 1 and 2, are clearly rejected by the UC test due to their very low p-values. Specifically, model 1 shows an exceedingly high number of exceedances (51), resulting in an exceedance rate of11.96%, far above the expected 1%. This reflects the fundamental limitation of assuming normally distributed returns, which is likely inappropriate given the non-normal characteristics of the portfolio's return distribution.

Among the SGSt and Historical models (models 3 to 7), model 3 is also rejected due to a p-value of 0%, while models 4 through 7 display p-values ranging from 0% to 6.25%. Notably, model 4 exhibits the highest p-value in this group and remains within a reasonable exceedance rate (1.38%), suggesting it is better specified compared to others. The Quantile Regression models (models 8 to 13) display varying performance. While models 10 is rejected by the UC test, models 8, 9, 11, 12 and 13 show p-values of 69.70%, 56.15%, 25.37%, 6.25% and 69.70% respectively, with exceedance rates close to the expected 1%, indicating good model calibration.

Consequently, for the analysis moving forward, only the models that passed the UC test (p-value > 5%) will be considered specifically, models 4, 8, 9, 11, 12 and 13. Models failing the UC test are excluded as they deviate significantly from the expected exceedance behaviour under the assumed significance level.

Model class	**Model no**	**p-value (%)**									
		Lag1	**Lag2**	**Lag3**	**Lag4**	**Lag5**	**Lag6**	**Lag7**	**Lag8**	**Lag9**	**Lag10**
historical	4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
quantile	8	11.76	25.99	16.14	24.94	34,30	43.61	30.38	37.85	45.29	1.50
quantile	9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
quantile	11	1.80	4.28	7.14	11.28	14.62	19.44	22.79	27.79	32.82	10.31
quantile	12	47.12	59.65	67.12	72.41	76.41	79.64	82.29	84.49	86.36	6.21
quantile	13	19.83	37.46	30.57	41.58	51.56	60.28	51.61	58.97	51.70	45.58

Table 6 presents the p-values from the BCP test across ten lags for selected models. *As noted previously, a model is considered to pass the BCP test if the p-values are consistently above the 5% significance level across the lags, indicating no statistically significant autocorrelation in exceedances.*

From the results of the BCP test, we observe that models 12 and 13 passed the test across multiple lags, as evidenced by their corresponding p-values being above the 5% threshold. When we jointly consider the outcomes of both the BCP and UC tests**,** model 13 stands out as the most reliable. It consistently demonstrates higher p-values in both tests, thereby providing greater statistical confidence in its adequacy to model the tail risk of the portfolio. Consequently, model 13 is selected as the preferred model for risk monitoring and further analysis.

Figure 1 shows the chart of exceedance points identified under VaR Model 13. Over the entire sample period of 2,600 days, exceedances occurred only 28 times, as summarized in Table 7. These exceedance points represent instances where actual portfolio losses exceeded the VaR estimates.

Additionally, we calculated the magnitude of each exceedance relative to the model's initial VaR prediction, providing further insight into the severity of unexpected losses.

Among all the models evaluated in this methodology, Model 13 demonstrated superior accuracy in VaR estimation. Based on its performance, we will continue with Model 13 for further VaR forecasting and the development of a corresponding risk management strategy, including hedging approaches to mitigate.

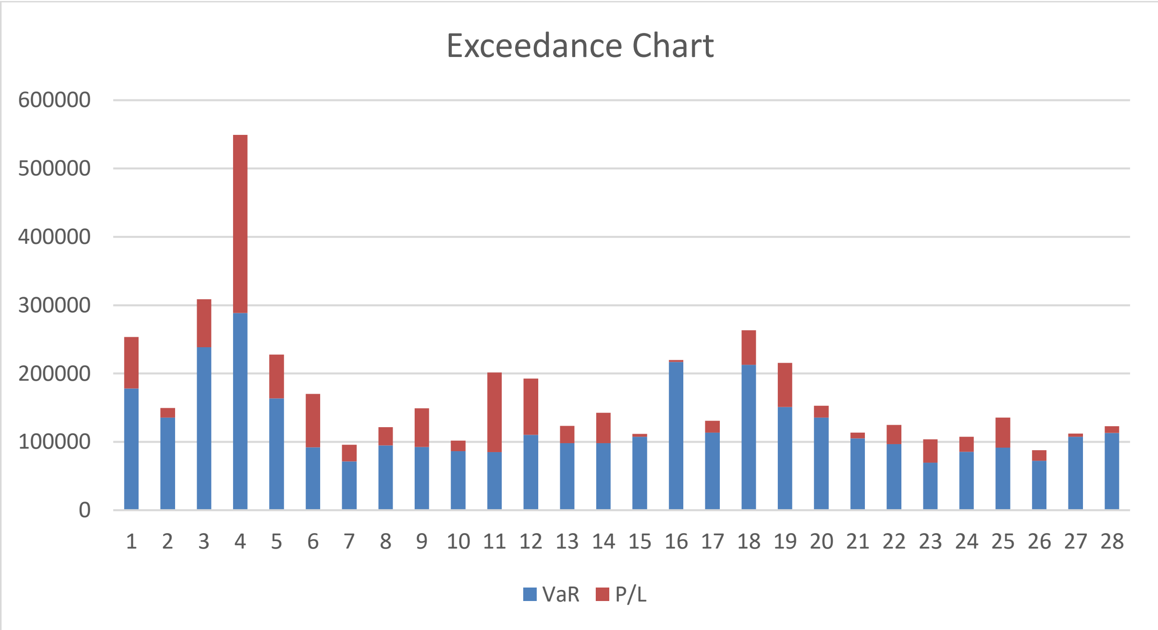

Figure 1, shows exceedance Points Where Actual Losses Surpassed VaR Estimates under Model 13

Table 7 presents the dates on which exceedances were observed instances where actual losses surpassed the VaR estimate under Model 13. Notably, during the COVID-19 market crisis in 2020, several significant exceedances occurred, reflecting the model's difficulty in capturing extreme tail risk during periods of unprecedented volatility. The table also includes the magnitude and percentage of each exceedance, offering deeper insights into the scale of deviations. This information enables institutions to assess the average exceedance rate and implement more robust risk management strategies, particularly during periods of elevated market uncertainty.

Figure 2 below shows the Profit and Loss (P/L) alongside the estimated 1% VaR over the 10-year global sample period (2,600 days). While fluctuations in P/L are observed, the model performed well, with only 28 exceedances recorded throughout the entire period. This suggests that Model 13 maintained strong predictive accuracy and effectively captured downside risk within the expected confidence level.

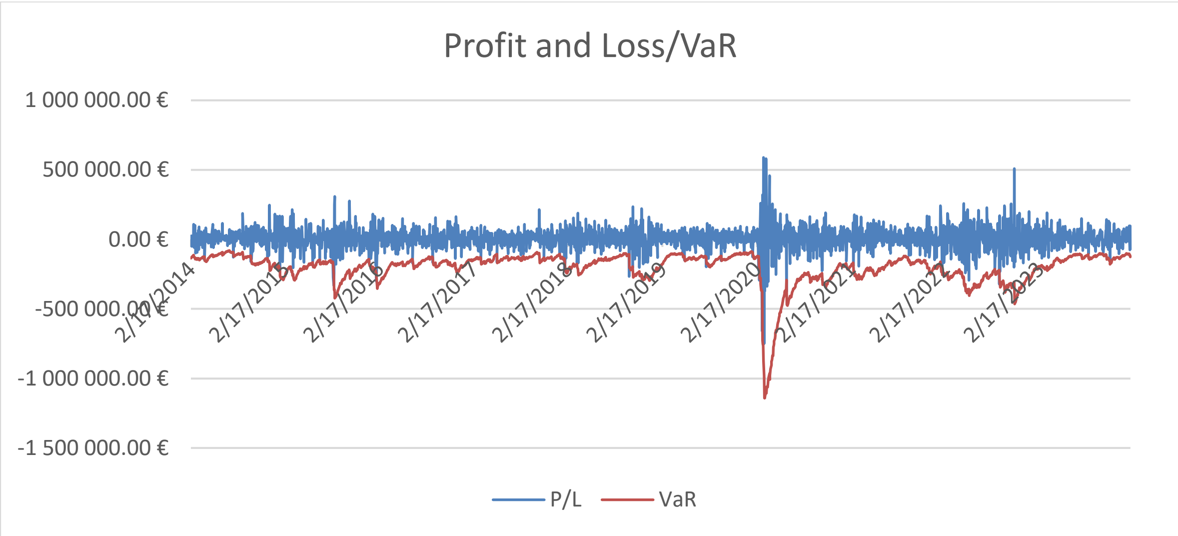

Figure 2 shows graph of profit and loss over 2600 days versus the estimated VaR by model 13.

	Date	P/L	VaR	Exceedance Size	Exceedance
1	9/13/2022	-253,658.80	177,986.33	-75,672.47	-42.52%
2	2/25/2021	-149,561.80	135,698.10	-13,863.65	-10.22%
3	6/11/2020	-308,752.00	238,451.14	-70,300.87	-29.48%
4	3/9/2020	-549,053.40	288,344.92	-260,708.50	-90.42%
5	2/27/2020	-227,659.90	163,399.55	-64,260.35	-39.33%
6	2/24/2020	-169,951.60	91,904.64	-78,046.96	-84.92%
7	1/31/2020	-95,864.66	71,315.36	-24,549.29	-34.42%
8	10/2/2019	-121,395.80	94,961.89	-26,433.90	-27.84%
9	8/5/2019	-148,908.80	92,633.05	-56,275.72	-60.75
10	5/13/2019	-101,679.30	86,473.78	-15,205.47	-17.58%
11	10/10/2018	-201,484.60	84,791.95	-116,692.65	-137.62%
12	2/2/2018	-192,815.20	110,180.72	-82,634.44	-75.00%
13	5/17/2017	-123,516.10	98,113.19	-25,402.91	-25.89%
14	9/9/2016	-142,678.10	98,117.51	-44,560.62	-45.42%
15	6/24/2016	-111,649.90	107,340.92	-4,308.97	-4.01%
16	2/5/2016	-219,817.60	216,905.96	-2,911.63	-1.34%
17	1/7/2016	-130,760.50	113,550.20	-17,210.26	-15.16%
18	8/24/2015	-263,306.20	212,519.88	-50,786.35	-23.90%
19	8/21/2015	-215,701.80	150,993.02	-64,708.82	-42.86%
20	6/3/2015	-152,759.80	135,558.03	-17,201.80	-12.70%
21	12/10/2014	-113,571.50	105,122.60	-8,448.91	-8.04%
22	10/7/2014	-124,914.90	96,478.93	-28,435.94	-29.47%
23	7/31/2014	-103,796.20	69,311.36	-34,484.84	-49.75%
24	4/10/2014	-107,409.90	85,526.01	-21,883.87	-25.59%
25	1/24/2014	-135,423.30	91,720.00	-43,703.33	-47.65%
26	1/13/2014	-87,829.76	72,306.05	-15,523.70	-21.47%
27	6/20/2013	-111,969.60	107,208.65	-4,760.94	-4.44%
28	5/31/2013	-123,067.50	112,835.60	-10,231.89	-9.07%

***Table 7: Summary of VaR Exceedances** — Dates, Magnitudes, and Percentage Deviations under Model 13*

Chapter 6

VALUE-AT-RISK MANAGEMENT

In this chapter, we explore the application of VaR as a core tool for measuring and managing the capital at risk in our €10 million portfolio. As defined earlier, Economic Capital (EC) refers to the capital required to absorb potential losses with a given confidence level and over a specified horizon. In our framework, EC is measured by the 1-day VaR, and we set a risk threshold of 1% of portfolio value, which corresponds approximately to €110,000.

Our focus is on Model 13 Quantile regression with EWMA variance as the explanatory variable 0.94(sample of 800 observation re-estimated daily). This model was selected based on its performance during Backtesting. As summarized in Table 7**,** we observed 28 exceedances instances where actual losses exceeded the model's VaR estimate out of 2600 trading days**.** This is a reasonably acceptable exceedance rate within a 1% VaR confidence level. However, it is worth noting that the COVID-19 crisis in 2020 led to several sharp deviations, exposing the model's limitations in capturing tail risk under extreme market stress.

By quantifying these exceedances, we aim to design a practical risk management process for the next one year. The core objective is to ensure that the daily VaR remains below our defined Economic Capital threshold of €110,000. As seen in Table 7, there were instances particularly during market turmoil when the 1% VaR exceeded this limit. To prevent such breaches going forward, our strategy is to estimate the VaR at the end of each trading day for the next day using the current portfolio composition. If the estimated VaR surpasses €100,000, we will implement a hedging strategy rather than rebalancing the portfolio. This preference stems from the fact that daily rebalancing may incur significant transaction costs, whereas targeted hedging allows us to reduce risk exposure more efficiently without substantially altering the core portfolio structure.

This chapter is structured as follows: Section 1 presents the marginal VaR decomposition methodology used to identify the contribution of each risk factor and design effective hedging strategies. Section 2 discusses the results of this risk control framework over the one-year period, analysing its effectiveness and areas of improvement.

6.1. VaR Decomposition and Management Strategy

To ensure that the VaR of our portfolio remains below the defined Economic Capital threshold of €110,000, we adopt a risk management strategy based on marginal VaR decomposition**.** This approach allows us to identify the specific contribution of each risk factor to the overall VaR estimate, thereby enabling targeted interventions particularly hedging rather than full-scale portfolio rebalancing.

Let ∇ denote the gradient vector, which measures the sensitivity of the portfolio's VaR to small changes in exposure to each of the n risk factors, based on the current values of portfolio risk factor exposures denoted by Θ. Similarly, let S be the decomposition vector that represents the current exposure to each risk factor θ_i, depending on the chosen decomposition logic (e.g., asset class, region, or strategy).

To compute the marginal sensitivity, we apply a perturbation method. For example, to determine the impact of the first risk factor, we slightly increase its exposure by a small value ε (e.g., €1), while keeping all other exposures constant

$$\Theta_1 = \begin{bmatrix} \theta_1 + \varepsilon \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}$$

We then compute the VaR of the perturbed portfolio VaR_{Θ_1} and derive the sensitivity for risk factor 1.

$$\frac{VaR_{\Theta_1} - VaR_{\Theta}}{\varepsilon}$$

Repeating this process for all risk factors provides us with the complete gradient vector ∇. The marginal VaR is then computed as the dot product:

$$Marginal\,VaR = S^T \nabla$$

Description	Decomposition by Risk Factor Type			
	Stocks	**Currency**	**Interest Rate**	**Total**
VaR (EUR)	97,063	37,364	13,673	148,101
VaR Breakdown (%)	65.5	25.22	9.23	100

Table 8. Marginal VaR Decomposition by Risk Factor Type*. This table presents the marginal decomposition of the total VaR on 30 January 2023, identifying how much each risk factor equities, foreign exchange, and bonds contributes to the portfolio's overall VaR.*

This decomposition reveals the extent to which each exposure equities, foreign exchange, or interest rates contributes to the total VaR. As shown in Table 8, on 30 January 2023 (the first day the VaR exceeded our €110,000 threshold), equity risk emerged as the dominant contributor, accounting for 65.5% of the total VaR. This was followed by currency risk at 25.2% and interest rate risk at 9.2%. The results highlight that most of the risk stems from foreign-based equity exposures. Despite the portfolio containing different asset classes, the decomposition suggests that risk is not evenly distributed. Rather, the portfolio is heavily concentrated in equity market risk, highlighting the need to hedge the primary sources of risk, especially the dominant equity exposures, to control portfolio volatility.

To better understand whether equity or bonds are the primary contributors to FX-related risk in our portfolio, we conduct an additional decomposition as shown in Table 9. The results reveal that 84.25% of the VaR is attributable to equity (stocks), while bonds contribute only 15.75%. This strongly indicates that much of the FX risk stems from foreign equity holdings rather than fixed-income instruments.

Given this finding**,** it becomes crucial to identify which specific foreign stocks are driving both the FX linked exposure and the dominant equity risk**,** as highlighted in Table 8**.** This step is essential to ensure that our hedging strategy directly targets the assets contributing most significantly to the portfolio's total VaR.

Description	Decomposition by Asset Class		
	Stocks	Bond	Total
VaR (EUR)	133,805	25,001	158,806
VaR Breakdown (%)	84.25	15.75	100

Table 9. Marginal VaR Decomposition by Asset Class. *This table illustrates the contribution of each asset class stocks and bonds to the total Value-at-Risk on 30 January 2023, highlighting the concentration of portfolio risk.*

Description	Decomposition by risk factor group										
	Row Total	**USDEUR**	**JPYEUR**	**GSPC**	**AEX**	**GDAXI**	**N225**	**IBEX**	**FCHI**	**IR_EUR**	**IR_USD**
VaR	148,101.79 €	36,654.03€	710.85€	89,150.97€	13,431.27€	2,338.24€	2,016.84€	-4,647.03€	-1,192.79€	7,262.08€	6,411.11 €
VaR	100.000%	24.749%	0.480%	60.196%	9.069%	1.579%	-1.362%	-3.138%	-0.805%	4.903%	4.329%

Table 10. Marginal VaR Decomposition by Risk factor group

This table presents the marginal VaR decomposition as of 30 January 2023, identifying how each risk factor including major equity indices, foreign exchange rates, and interest rates contributes to the total VaR. The analysis helps isolate the impact of regional and macroeconomic factors such as U.S. equity markets (GSPC), European indices (AEX, GDAXI, IBEX, FCHI), Japanese markets (N225), and FX and interest rate exposures.

From the table, we observe that the dominant contributor to portfolio risk is the GSPC index, accounting for more than 60.196% of the total VaR. To manage this risk and maintain daily VaR below our €110,000 Economic Capital threshold, it is necessary to implement a hedging strategy. This would involve taking a short position on the S&P500 (GSPC) and other contributing indices such as DAX (GDAXI) to reduce the portfolio's sensitivity to movements in the U.S. and Euro equity market.

6.2. VaR Management Results

This section presents the outcome of applying our daily VaR estimation and hedging strategy over the one-year period from 30 January 2023 to 02 February 2024. The strategy is based on Model 13**,** which integrates Quantile Regression with EWMA volatility (λ = 0.94) using an 800-day rolling window.

To assess the effectiveness of our risk management framework, we compare the performance of two portfolio configurations:

- The Hedged portfolio, where the daily VaR estimate exceeding the Economic Capital (EC) threshold of €110,000 triggers a targeted hedging adjustment.
- The Unhedged portfolio, which follows the same investment structure but without any hedging intervention.

Our evaluation focuses on three core metrics: Frequency of exceedance which tells us how often actual losses breached the daily VaR estimate**,** magnitude of breaches; the extent to which losses exceeded the VaR threshold, average daily VaR; the average risk level estimated across the one-year period.

In addition to risk control, we also assess the Profit and Loss (P&L) performance of both portfolios to understand the trade-off between risk reduction and return. The results that follow provide insight into the practical benefits and limitations of the hedging strategy under real market conditions.

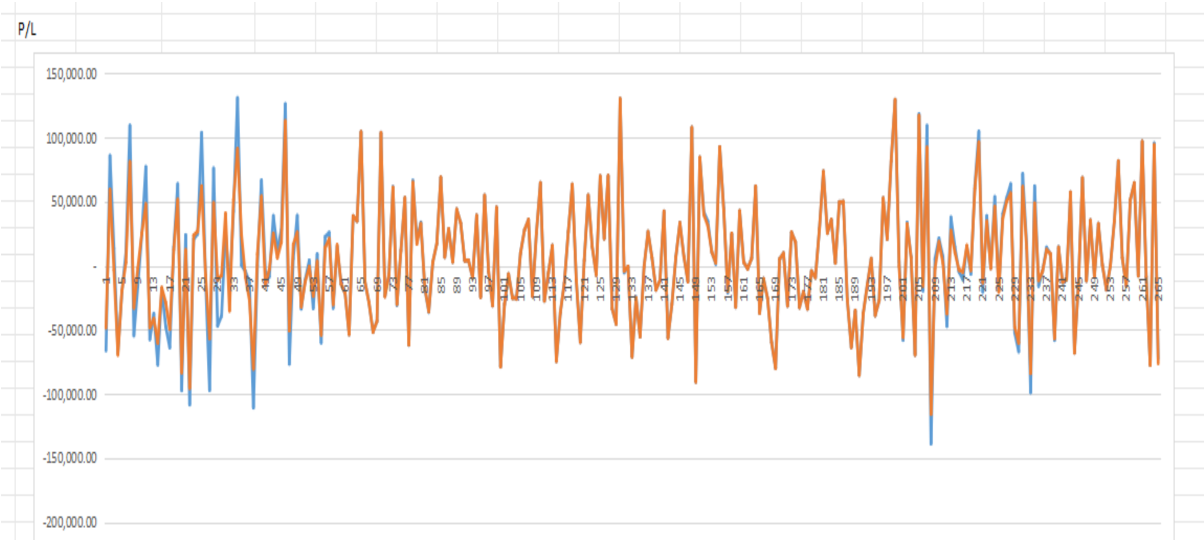

Figure 3 illustrates the daily Profit and Loss (P&L). *This shows the trajectories of the hedged and unhedged portfolios over the one-year risk management period.*

This comparison enables us to evaluate how effectively the hedging strategy reduced loss severity and volatility compared to the unhedged baseline. The visual clearly shows that the hedged portfolio experienced fewer and less severe negative outcomes, aligning with our objective of maintaining daily losses below the €110,000 Economic Capital threshold.

As seen in the chart, most of the actual P&L values remain well below the unhedged profit and loss estimates, indicating that the hedging strategy successfully reduced the occurrence of exceedances while maintaining profit as same time. Compared to the earlier unhedged period, the P&L trajectory now demonstrates lower volatility and fewer extreme negative outcomes, which aligns with our objective of maintaining daily losses below the €110,000 Economic Capital threshold.

This outcome validates the hedging framework implemented in Section 6.1, confirming that the strategy was not only effective in minimizing risk exposure but also preserving portfolio stability under stressed conditions.

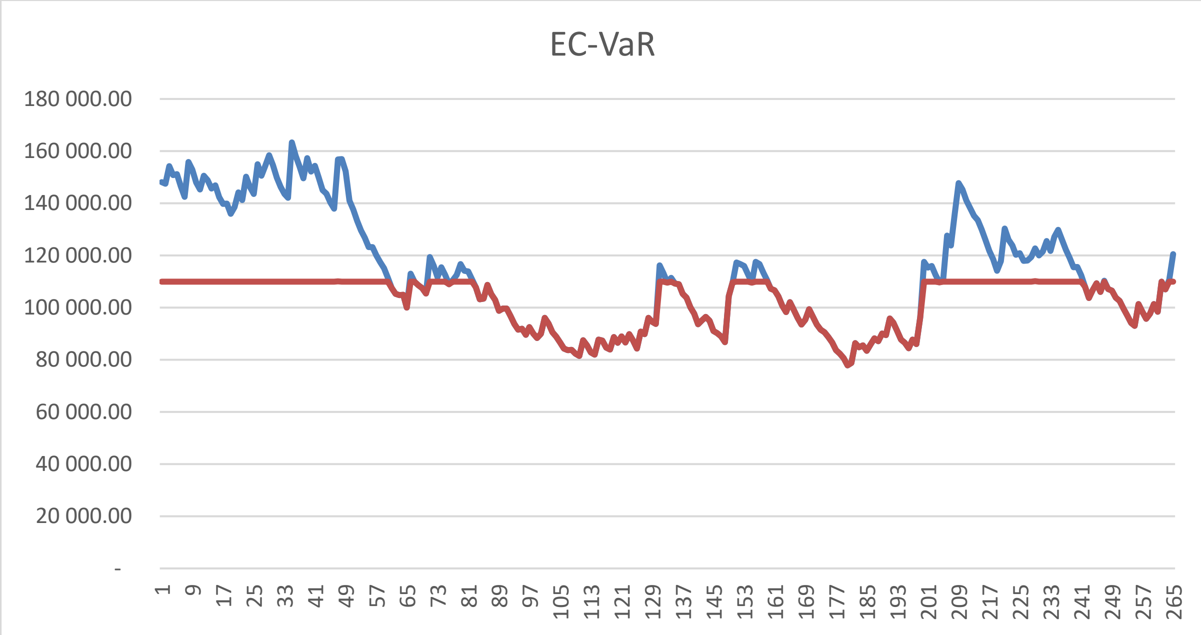

Figure 4 the unhedged portfolio's 1% daily VaR estimates (blue line) against the Economic Capital (EC) threshold of €110,000 (orange line). *This visualization illustrates the natural risk trajectory of the portfolio without any intervention through hedging.*

From the chart, we observe that VaR estimates frequently breach the EC threshold, particularly during periods of heightened market volatility. These breaches indicate moments of excessive risk exposure, where potential losses exceed the portfolio's predefined risk tolerance. The lack of any risk-reducing intervention here highlights the vulnerability of the portfolio to adverse shocks, especially in periods marked by increased uncertainty.

This figure provides an essential benchmark for evaluating the effectiveness of the hedging strategy introduced later. The frequent exceedances in the unhedged portfolio stand in stark contrast to the stability observed post-hedging (as shown in Figure 2), underscoring the importance of active risk management to maintain VaR within acceptable limits.

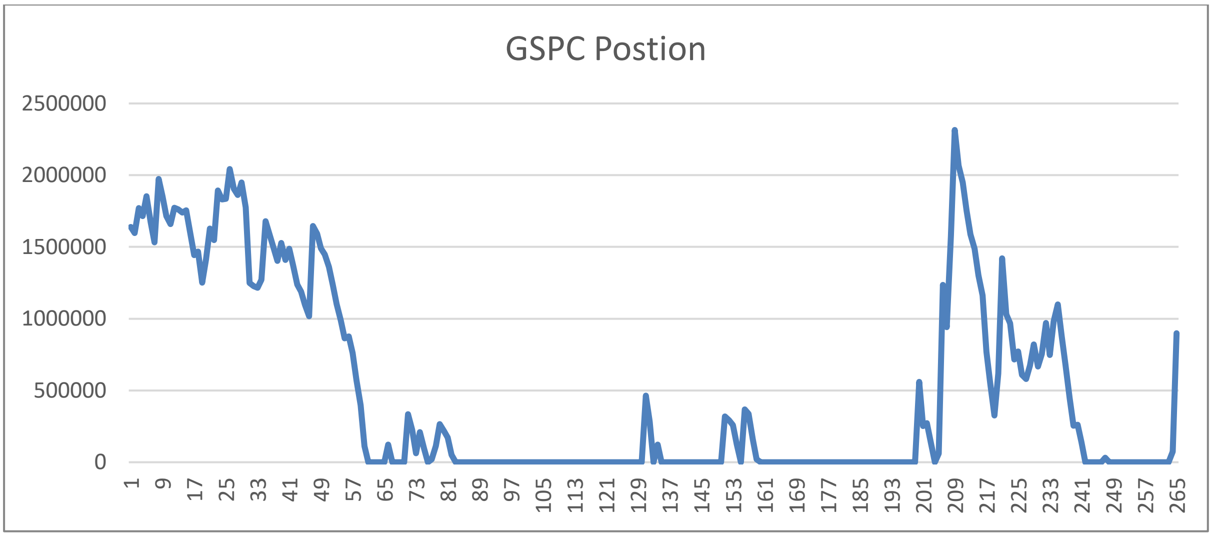

Figure 5 illustrates the evolution of short positions taken on the S&P 500 index (GSPC) *, as part of our hedging strategy over the one-year period. These positions were implemented dynamically based on the VaR decomposition results and were activated only on days when the projected portfolio VaR exceeded the €110,000 Economic Capital threshold.*

From the chart, we observe that the hedging positions were non-continuous, reflecting the strategy's selective nature positions were entered only when risk levels were deemed excessive. Periods of zero exposure indicate days where no hedging was necessary, as the estimated VaR remained within acceptable bounds. Conversely, sharp increases in short positions correspond to days with elevated equity risk, particularly driven by U.S. stock exposure (as indicated in Table 10).

Notably, we see significant hedging activity around days 200–230, which aligns with spikes in the portfolio's unhedged VaR (see Figure 3). These actions demonstrate the strategy's responsiveness and its role in mitigating risk without altering the core portfolio.

Overall, this figure provides evidence that the hedging mechanism was actively deployed during periods of heightened risk and was effective in keeping the portfolio's risk profile aligned with the defined capital constraints.

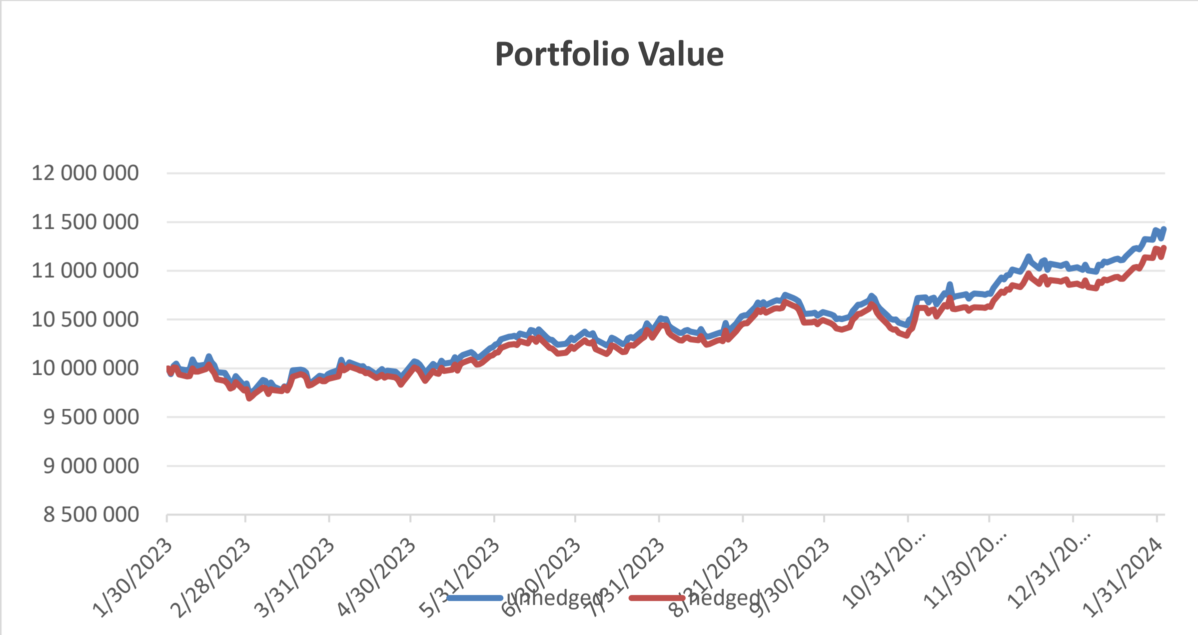

Figure 6 displays the daily evolution of the portfolio value over the one-year*. The management period for both the hedged and unhedged strategies. Despite occasional divergences, both portfolios show a clear upward trajectory, reflecting overall growth in asset value.*

Notably, the hedged portfolio maintained a positive performance throughout the year, demonstrating that the risk-reduction strategy did not eliminate profitability. While the unhedged portfolio slightly outperformed in terms of peak value, the hedged version provided a more stable and consistent growth path, particularly during periods of heightened market volatility.

This outcome reinforces the effectiveness of the hedging strategy in mitigating downside risk without sacrificing long-term returns, aligning with the study's objective of maintaining portfolio value within risk limits while still achieving capital growth.

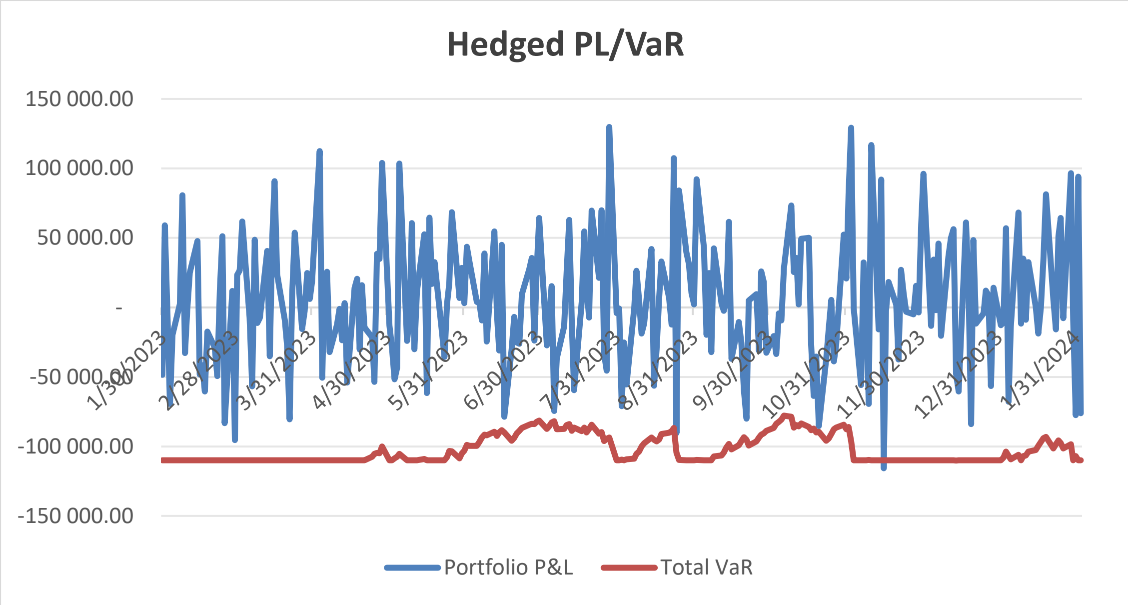

Figure 7 Daily Profit and Loss (P/L) vs. Economic Capital (EC) Threshold

Figure 7 presents the daily profit and loss of the hedged portfolio overlaid with the Economic Capital threshold of €110,000. The EC line represents the maximum acceptable daily loss the portfolio can sustain, based on the 1% Value-at-Risk criterion for a €10 million portfolio.

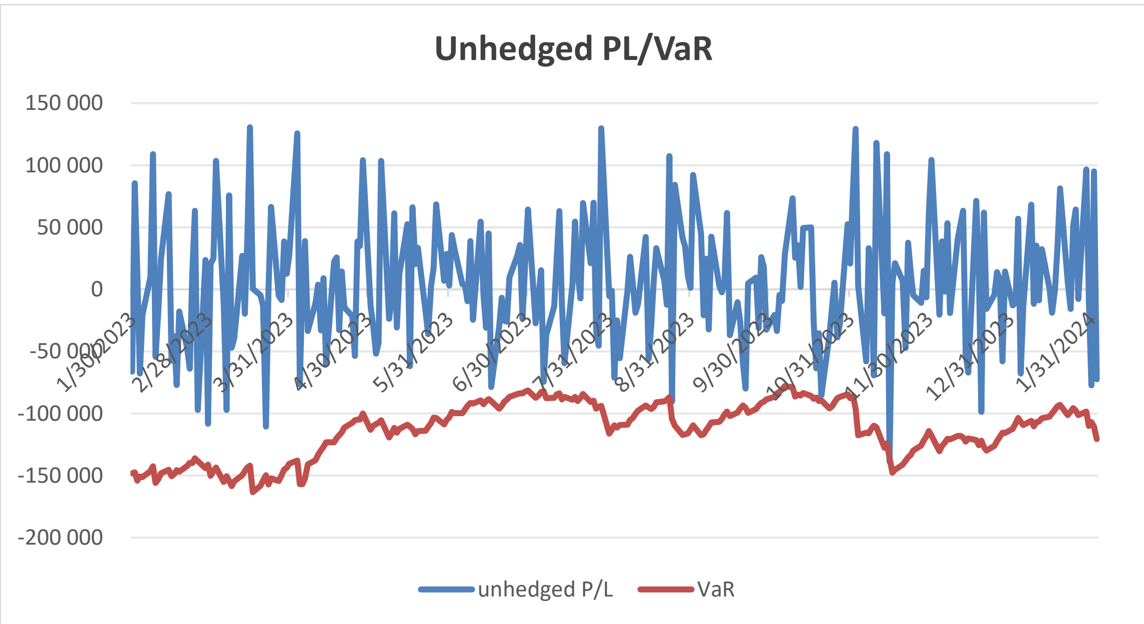

Figure 8 Daily Profit and Loss (P/L) vs. VaR estimates of unhedged portfolio.

This chart displays the one-year performance of the unhedged portfolio**,** plotting the daily P&L (blue line) against the VaR estimates (orange line) for the same period as the hedged version:

	P/L	**EC**	**RORAC**
Unhedged	1,183,349.72	29,857,966.93	3.96%
Hedged	1,159,449.59	27,112,847.73	4.28%

Table 11: Comparison of Portfolio Performance Hedged vs. Unhedged (Profit, Economic Capital, and RORAC)

To further assess the effectiveness of the hedging strategy, we compare both portfolios using the RORAC metric. RORAC is calculated as the ratio of total profit (P&L) to the cumulative EC consumed over the one-year period. As shown in Table 11, the unhedged portfolio achieved a total profit of approximately €1.18 million, with a corresponding EC of €29.86 million, resulting in a RORAC of 3.96%. In contrast, the hedged portfolio generated a slightly lower profit of €1.16 million but consumed less risk capital €27.11 million leading to a higher RORAC of 4.28%.

While the unhedged portfolio earned a slightly higher profit overall, the hedged portfolio still performed impressively especially when you consider it was taking on less risk. This shows that the hedging strategy didn't "cut into" profits but instead helped preserve returns in a more controlled and stable way.

What's even more important is that the hedged portfolio required less EC to support it. That means it was less risky and better aligned with capital constraints, something that matters a lot for banks and financial institutions operating under rules like Basel III. By reducing exposure to highly volatile areas, such as U.S. equities, the hedged portfolio didn't need to set aside as much capital to guard against extreme losses.

Even though it earned slightly less in absolute euro terms, the hedged portfolio made better use of its capital, delivering a higher return per unit of risk. That's exactly what financial institutions aim for: maximizing performance without overstretching risk budgets. In this case, hedging not only protected the portfolio but helped it operate more efficiently under realistic capital requirements.

Chapter 7

CONCLUSION

This thesis set out to evaluate and manage the VaR of a diversified portfolio composed of equities and sovereign bonds across major economies, with a focus on identifying the most suitable VaR model through a structured Backtesting framework. The study employed a comprehensive dataset spanning from 2007 to early 2024, incorporating turbulent periods such as the 2008 financial crisis and the COVID-19 pandemic, thereby enabling a robust evaluation of model performance under various market conditions.

Four categories of VaR models were considered: Parametric (Normal and SGSt), Historical Simulation (standard and volatility-adjusted), and Quantile Regression (QR). A total of thirteen different model specifications were tested using daily data, with performance assessed using Kupiec's Unconditional Coverage (UC) test and the Berkowitz-Christoffersen-Pelletier (BCP) test. These tests helped evaluate the frequency and independence of VaR exceedances, critical metrics in determining the reliability of each model.

Among the thirteen models, Model 13 (Quantile Regression VaR with EWMA volatility as the explanatory variable, using a rolling window of 800 observations re-estimated daily) was selected as the best-performing model based on its ability to maintain exceedances within the 99% confidence interval and demonstrate minimal autocorrelation. This model provided consistent results across the 10-year global test window and also performed well during high-stress periods.

This section presents the outcome of applying our daily VaR estimation and hedging strategy over the one-year period from 30 January 2023 to 30 January 2024. The strategy is based on Model 13, which integrates Quantile Regression with EWMA volatility ($\lambda = 0.94$) using an 800-day rolling window.

To assess the effectiveness of our risk management framework, we compare the performance of two portfolio configurations:

The Hedged portfolio, where the daily VaR estimate exceeding the Economic Capital (EC) threshold of €100,000 triggers a targeted hedging adjustment.

The Unhedged portfolio, which follows the same investment structure but without any hedging intervention.

Our evaluation focuses on three core metrics: the frequency of exceedance, which tells us how often actual losses breached the daily VaR estimate; the magnitude of breaches, which reflects the extent to which losses exceeded the VaR threshold; and the average daily VaR, representing the average risk level estimated across the one-year period.

In addition to risk control, we also assess the Profit and Loss (P&L) performance of both portfolios to understand the trade-off between risk reduction and return. The results that follow provide insight into the practical benefits and limitations of the hedging strategy under real market conditions.

Finally, a comparative performance analysis between the hedged and unhedged portfolios using RORAC (Return on Risk-Adjusted Capital) showed that the hedged strategy yielded less negative returns while taking on significantly less risk. Although both portfolios experienced losses during the year, the hedging approach protected the downside more effectively, highlighting the practical value of VaR-guided risk management.

Recommendations for Future Research:

- Explore advanced machine learning-based VaR models that can capture nonlinear interactions between risk factors.
- Extend the analysis to intraday VaR estimation to manage high-frequency trading portfolios.
- Incorporate stress testing and scenario analysis to supplement the VaR framework.
- Evaluate Expected Shortfall (ES) alongside VaR for a more coherent risk measure.
- Investigate the use of mathematical portfolio weighting methods, such as minimum variance optimization, to further reduce overall portfolio risk.

In conclusion, the study demonstrates the critical importance of selecting an appropriate VaR model tailored to portfolio characteristics and market environments. It also validates the application of quantile regression in capturing tail risks and guiding effective hedging strategies. The methodology and findings offer practical insights for institutional risk managers and set the stage for further research into dynamic and data-driven risk management tools.

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Appendices

Appendix A

Backtest model performance details

A.1. Unadjusted historical model 4

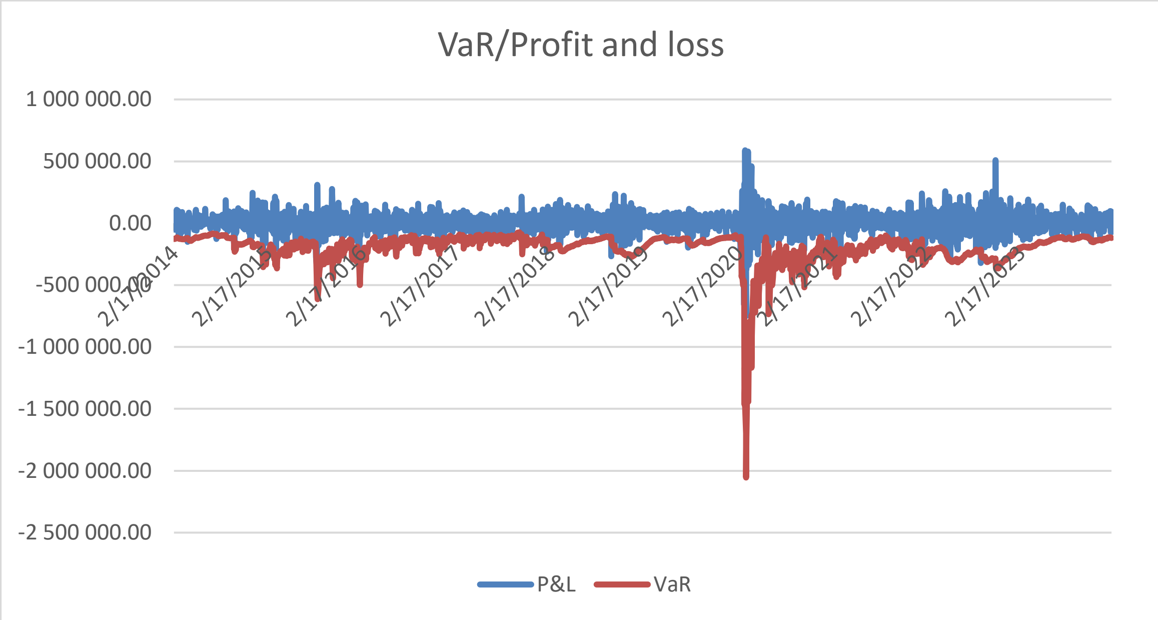

Figure 9 shows graph of profit and loss over 2600 days versus the estimated VaR by model 4

Date	P&L	VaR	Exceedance Size (EUR)	Exceedance Size (%)
11/15/2023	-146 836.70	136 278.12	-10 558.57	-7.75
9/13/2022	-321 765.64	221 547.58	-100 218.06	-45.24
5/18/2022	-293 812.89	197 418.63	-96 394.26	-48.83
5/9/2022	-221 547.58	176 857.35	-44 690.22	-25.27
5/5/2022	-241 819.81	166 866.15	-74 953.66	-44.92
4/29/2022	-176 857.35	164 564.84	-12 292.52	-7.47
4/22/2022	-164 564.84	159 169.79	-5 395.05	-3.39
4/21/2022	-159 169.79	147 162.49	-12 007.30	-8.16
4/11/2022	-147 162.49	146 512.59	-649.90	-0.44
2/3/2022	-197 418.63	146 512.59	-50 906.04	-34.75
3/16/2020	-750 162.86	294 229.03	-455 933.84	-154.96
3/12/2020	-559 195.66	236 253.79	-322 941.87	-136.69
3/11/2020	-236 253.79	231 169.83	-5 083.96	-2.20
3/9/2020	-660 903.32	223 231.76	-437 671.56	-196.06
3/3/2020	-231 169.83	197 810.89	-33 358.94	-16.86
2/27/2020	-294 229.03	164 405.56	-129 823.47	-78.97
2/25/2020	-164 405.56	154 181.92	-10 223.64	-6.63
2/24/2020	-223 231.76	149 327.49	-73 904.27	-49.49
10/24/2018	-203 732.66	170 358.34	-33 374.32	-19.59
10/10/2018	-266 611.29	168 604.80	-98 006.50	-58.13
3/27/2018	-170 358.34	164 838.06	-5 520.28	-3.35
3/22/2018	-168 604.80	164 586.48	-4 018.32	-2.44
2/5/2018	-164 838.06	132 996.40	-31 841.66	-23.94
2/2/2018	-230 140.85	129 902.73	-100 238.11	-77.16
5/17/2017	-164 586.48	137 829.79	-26 756.69	-19.41
2/5/2016	-300 479.95	244 198.59	-56 281.36	-23.05
8/25/2015	-244 198.59	221 376.64	-22 821.95	-10.31
8/24/2015	-332 087.43	204 918.95	-127 168.49	-62.06
8/21/2015	-272 811.44	165 163.24	-107 648.20	-65.18
4/30/2015	-165 163.24	161 206.03	-3 957.21	-2.45
3/19/2015	-204 918.95	160 283.88	-44 635.07	-27.85
2/6/2015	-160 283.88	149 310.77	-10 973.12	-7.35
1/27/2015	-221 376.64	142 366.94	-79 009.70	-55.50
10/7/2014	-161 206.03	127 705.89	-33 500.14	-26.23
7/31/2014	-127 705.89	114 070.36	-13 635.53	-11.95
4/10/2014	-149 310.77	131 496.05	-17 814.72	-13.55

Table 12 5ummary of VaR Exceedances — Dates, Magnitudes, and Percentage Deviations under model 4.

A.2. Model 11 VaR estimation using Quantile Regression.

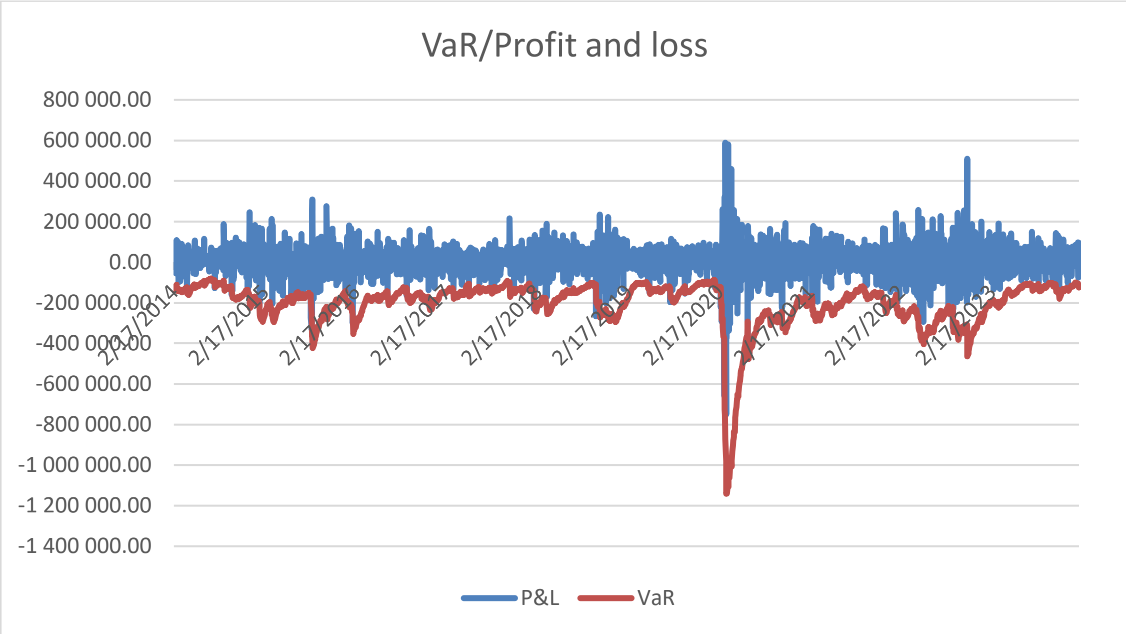

Figure 10 shows graph of profit and loss over 2600 days versus the estimated VaR by model 11

Date	P&L	VaR	exceedance size (EUR)	exceedance size (%)
11/15/2023	-146 836.70	135 858.26	-10 978.44	-8.08
9/13/2022	-321 765.64	213 377.27	-108 388.37	-50.80
8/26/2022	-213 542.06	213 412.83	-129.24	-0.06
10/26/2020	-226 338.88	193 685.34	-32 653.54	-16.86
6/11/2020	-378 192.81	189 592.93	-188 599.88	-99.48
3/9/2020	-660 903.32	471 194.12	-189 709.21	-40.26
2/27/2020	-294 229.03	189 176.41	-105 052.62	-55.53
2/25/2020	-164 405.56	107 911.09	-56 494.47	-52.35
2/24/2020	-223 231.76	119 283.83	-103 947.93	-87.14
1/31/2020	-126 715.07	104 612.15	-22 102.92	-21.13
10/2/2019	-154 181.92	132 149.82	-22 032.10	-16.67
8/5/2019	-197 810.89	125 165.74	-72 645.15	-58.04
5/13/2019	-149 327.49	121 173.84	-28 153.65	-23.23
10/24/2018	-203 732.66	179 964.86	-23 767.81	-13.21
10/10/2018	-266 611.29	109 752.32	-156 858.98	-142.92
2/5/2018	-164 838.06	134 477.75	-30 360.31	-22.58
2/2/2018	-230 140.85	135 576.27	-94 564.57	-69.75
1/15/2018	-99 843.28	92 342.56	-7 500.72	-8.12
11/15/2017	-129 902.73	108 859.67	-21 043.07	-19.33
8/17/2017	-112 034.30	104 840.60	-7 193.69	-6.86
5/17/2017	-164 586.48	100 906.27	-63 680.21	-63.11
12/30/2016	-125 999.12	125 976.49	-22.64	-0.02
9/9/2016	-166 121.46	115 406.24	-50 715.21	-43.94
6/24/2016	-169 971.94	133 873.37	-36 098.57	-26.96
1/7/2016	-170 098.71	143 377.39	-26 721.32	-18.64
8/21/2015	-272 811.44	214 330.43	-58 481.00	-27.29
6/3/2015	-161 617.68	147 346.65	-14 271.03	-9.69
12/10/2014	-141 013.87	140 012.13	-1 001.74	-0.72
10/7/2014	-161 206.03	117 042.91	-44 163.11	-37.73
7/31/2014	-127 705.89	86 452.83	-41 253.06	-47.72
4/10/2014	-149 310.77	119 373.20	-29 937.57	-25.08

Table 13 6ummary of VaR Exceedances — Dates, Magnitudes, and Percentage Deviations under model 11

A.3. Model 8 VaR estimation using Quantile Regression.

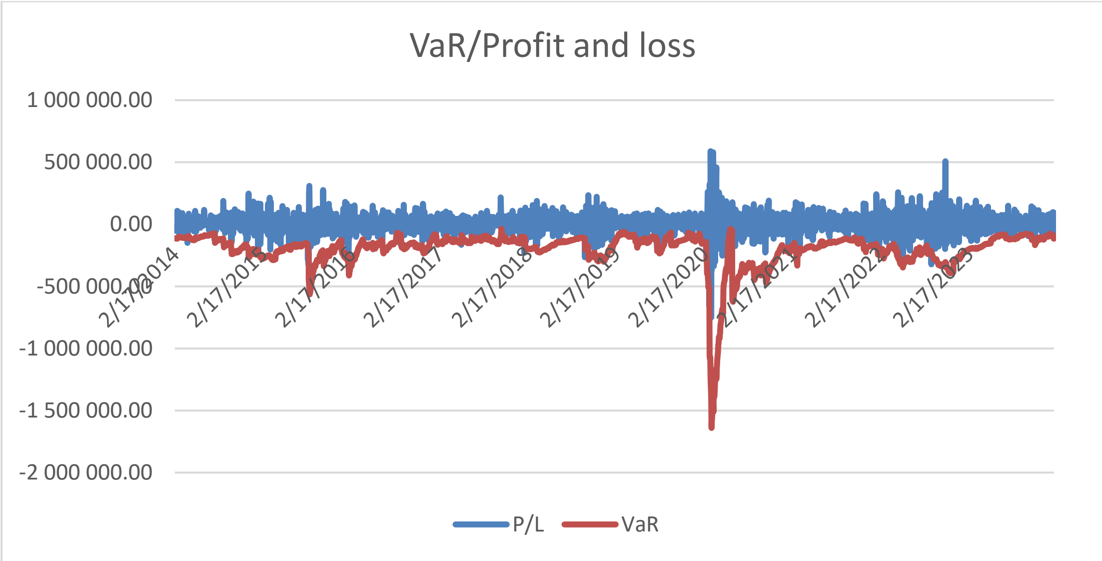

Figure 11 shows graph of profit and loss over 2600 days versus the estimated VaR by model 8

Date	P/L	VaR	exceedance size (EUR)	exceedance size (%)
11/15/2023	-146 836.70	138 573.30	-8 263.40	-5.96
9/13/2022	-321 765.64	214 131.10	-107 634.53	-50.27
5/18/2022	-293 812.89	288 851.16	-4 961.73	-1.72
2/25/2021	-190 899.10	189 067.97	-1 831.13	-0.97
3/9/2020	-660 903.32	370 326.98	-290 576.34	-78.46
2/27/2020	-294 229.03	203 476.46	-90 752.57	-44.60
2/24/2020	-223 231.76	114 190.84	-109 040.92	-95.49
1/31/2020	-126 715.07	94 583.36	-32 131.70	-33.97
10/2/2019	-154 181.92	125 430.64	-28 751.28	-22.92
8/5/2019	-197 810.89	123 713.65	-74 097.24	-59.89
5/13/2019	-149 327.49	118 782.72	-30 544.77	-25.71
10/24/2018	-203 732.66	175 273.21	-28 459.45	-16.24
10/10/2018	-266 611.29	109 969.78	-156 641.51	-142.44
2/2/2018	-230 140.85	136 109.52	-94 031.33	-69.09
5/17/2017	-164 586.48	116 081.97	-48 504.50	-41.78
9/9/2016	-166 121.46	121 940.84	-44 180.62	-36.23
6/24/2016	-169 971.94	143 265.49	-26 706.45	-18.64
2/5/2016	-300 479.95	245 894.30	-54 585.66	-22.20
1/7/2016	-170 098.71	166 808.80	-3 289.90	-1.97
8/24/2015	-332 087.43	231 749.39	-100 338.05	-43.30
8/21/2015	-272 811.44	174 172.51	-98 638.93	-56.63
1/27/2015	-221 376.64	204 386.85	-16 989.79	-8.31
10/7/2014	-161 206.03	127 154.49	-34 051.54	-26.78
7/31/2014	-127 705.89	93 157.59	-34 548.29	-37.09
4/10/2014	-149 310.77	126 205.45	-23 105.32	-18.31

Table 14 7ummary of VaR Exceedances — Dates, Magnitudes, and Percentage Deviations under model 8

A.4. Model 12 VaR estimation using Quantile Regression.

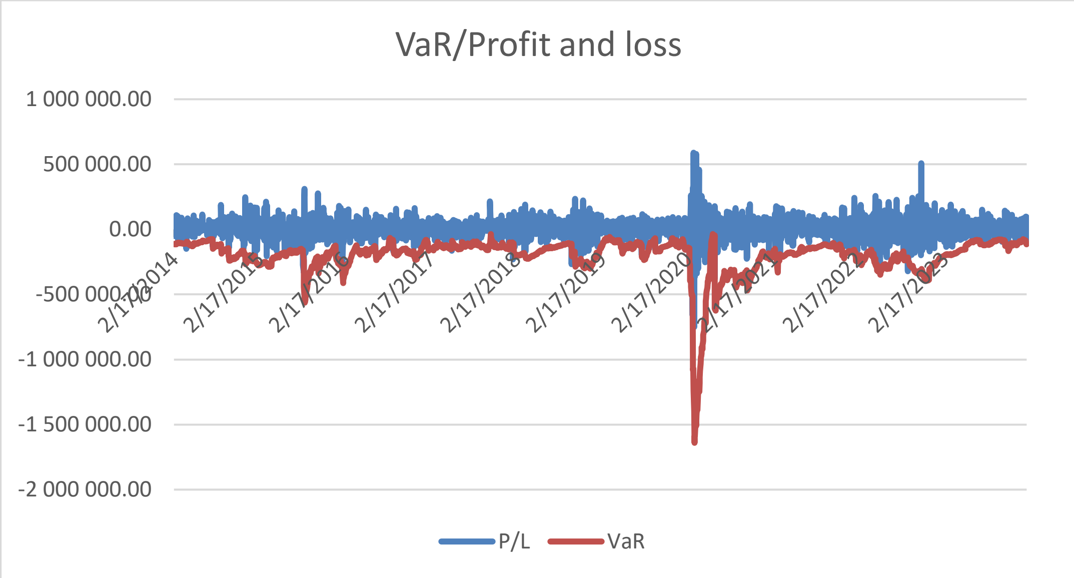

Figure 12 shows graph of profit and loss over 2600 days versus the estimated VaR by model 12

Date	P/L	VaR	exceedance size (EUR)	exceedance size (%)
11/15/2023	-146 836.70	139 494.10	-7 342.60	-5.26
10/20/2023	-94483.567	81553.55155	-12930.01505	-15.8546
9/21/2023	-88321.157	85885.85608	-2435.300617	-2.83551
9/13/2022	-321 765.64	215 102.18	-106 663.45	-49.59
8/26/2022	-213542.06	169511.2633	-44030.79913	-25.9751
5/18/2022	-293 812.89	287 713.85	-6 099.04	-2.12
2/3/2022	-197418.63	179922.0898	-17496.54344	-9.72451
3/18/2021	-194046.07	190034.0339	-4012.038545	-2.11122
2/25/2021	-190 899.10	185 096.93	-5 802.17	-3.13
6/11/2020	-378192.81	46355.18051	-331837.6249	-715.859
6/4/2020	-50886.232	36213.27073	-14672.96174	-40.5182
3/9/2020	-660 903.32	512 672.24	-148 231.09	-28.91
2/27/2020	-294 229.03	274 085.50	-20 143.52	-7.35
2/24/2020	-223 231.76	143 479.20	-79 752.56	-55.58
1/31/2020	-126 715.07	104 698.49	-22 016.57	-21.03
12/30/2019	-89635.358	82827.67185	-6807.686333	-8.2191
10/2/2019	-154 181.92	69 177.86	-85 004.06	-122.88
8/5/2019	-197 810.89	119 931.91	-77 878.98	-64.94
5/13/2019	-149 327.49	120 125.09	-29 202.40	-24.31
5/7/2019	-112422.03	109098.0282	-3323.999597	-3.0468
10/24/2018	-203 732.66	172 616.60	-31 116.07	-18.03
10/10/2018	-266 611.29	109 729.08	-156 882.21	-142.97
2/5/2018	-164838.06	107888.2702	-56949.78968	-52.7859
2/2/2018	-230 140.85	119 252.08	-110 888.76	-92.99
10/31/2017	-58711.537	50822.81998	-7888.717347	-15.522
5/17/2017	-164 586.48	147 795.52	-16 790.96	-11.36
3/16/2017	-132996.4	101134.6612	-31861.74124	-31.5043
9/9/2016	-166 121.46	90 055.72	-76 065.74	-84.47
6/24/2016	-169 971.94	131 962.64	-38 009.30	-28.80
1/7/2016	-170 098.71	125 581.43	-44 517.27	-35.45
8/24/2015	-332 087.43	240 322.77	-91 764.66	-38.18
8/21/2015	-272 811.44	180 875.99	-91 935.44	-50.83
1/27/2015	-221 376.64	205 700.36	-15 676.29	-7.62
10/7/2014	-161 206.03	155 069.30	-6 136.73	-3.96
7/31/2014	-127 705.89	75 759.24	-51 946.65	-68.57
4/10/2014	-149 310.77	108 242.42	-41 068.34	-37.94

Table 14 8ummary of VaR Exceedances — Dates, Magnitudes, and Percentage Deviations under model 12