INSTITUTO
UNIVERSITÁRIO
DE LISBOA

Portfolio Risk Dynamics: Managing Value-at-Risk Across Stocks and Bonds

Ricardo Filipe Marques Gomes da Silva

Master in Finance

Supervisor:
PhD, António Manuel Rodrigues Guerra Barbosa, Assistant Professor, ISCTE-IUL

July, 2025

BUSINESS
SCHOOL

Department of Finance

Portfolio Risk Dynamics: Managing Value-at-Risk Across Stocks and Bonds

Ricardo Filipe Marques Gomes da Silva

Master in Finance

Supervisor:
PhD, António Manuel Rodrigues Guerra Barbosa, Assistant Professor, ISCTE-IUL

July, 2025

Resumo

O Value at Risk (VaR) é uma métrica de avaliação de risco utilizado para medir o Economic Capital (EC) que consiste no capital em risco relativo a atividades de investimento. As formulações de estratégias de risco são realizadas através de um limite máximo para o EC definido previamente. Esta dissertação estima e gere o VaR de uma carteira composta por ações e obrigações dos mercados Europeus, Canadianos, Americanos e Asiáticos com o objetivo de não ultrapassar o máximo pré-definido. Através de backtest, foi analisada a performance de 10 modelos diferentes, dada a variedade de modelos existentes, tendo sido utilizado o modelo com melhores estimativas para a carteira em questão. Utilizando o modelo com melhor performance, o Var da carteira é medido diariamente e gerido através de uma estratégia de cobertura aplicada à exposição em ações para um período de um ano considerando limites individuais de contribuição de cada ativo para o risco total da carteira. O Return on Risk-Adjusted Capital (RORAC) é uma métrica de performance utilizada para a análise do resultado da estratégia de cobertura executada.

Palavras Chave: Economic Capital, Value-at-Risk, Backtest, Cobertura, Return on Risk-Adjusted Capital

Abstract

Value-at-Risk (VaR) is a risk measurement metric used to mensurate the Economic Capital (EC), which consists of the capital at risk derived from investment activities. The formulation of risk management strategies is done through a pre-defined maximum target value for the EC. This dissertation measures and manages the VaR of a portfolio composed of equities and bonds from the European, U.S., Canadian and Asian markets with the aim of not exceeding a pre-defined target. Through a Backtest process, given the range of VaR models, 10 different models are analyzed by their performance, to select the model that provides the most thorough estimates for the portfolio. Selecting the best performing model, the VaR of the portfolio is measured daily and managed by an equity exposure hedging strategy for a one-year period, also capping the individual risk contribution of each asset for the total portfolio risk. Return on Risk-Adjusted Capital (RORAC) is a performance metric used to analyse the applied hedging strategy result.

Keywords: Economic Capital, Value-at-Risk, Backtest, Hedging, Return on Risk-Adjusted Capital

CONTENTS

List of Abbreviations

BCP - Berkowitz, Christoffersen and Pelletier

CAD – Canadian Dollar

DAX - Deutscher Aktienindex

EC – Economic Capital

ECB – European Central Bank

EUR – Euro

FTSE 100 – Financial Times Stock Exchange 100 Index

FX – Foreign Exchange

GBP – Great British Pound

JPY – Japanese Yen

NASDAQ – National Association of Securities Dealers Automated Quotations

P&L – Profit and Loss

QR - Quantile Regression

RM – RiskMetrics

RORAC - Return on Risk-Adjusted Capital

S&P500 - Standard & Poor's 500

SGSt - Skewed Generalized Student-t

UC - Unconditional Coverage

U.S. - United States

USD - United States Dollar

VaR – Value at Risk

Chapter 1

INTRODUCTION

During the last few years, the role of financial risk management has become more important for companies and financial institutions. Influential events, such as the 2008 financial crisis or the increased market volatility in the last 5 years due to the 2020 pandemic and macroeconomic tensions, strengthen the role of risk management. In addition, the growth of the financial derivatives markets with high leverage financial instruments and given the role of financial institutions in our system and economies, it is also a legal obligation to control financial risk. This control began in the late-1980s via introduction of the first Basel Accords ***(Shakdwipee & Mehta, 2017)*** where financial supervisors increased the minimum capital requirements with proper risk mensuration and reporting regulations towards the protection of the financial system stability.

The goal of this Master Thesis is to measure and mitigate market risk, that can be defined as a measure of the uncertainty of future value of certain financial assets that arises from changes in market prices and unknown profit and loss (P&L) profiles. To do so, we need to use a risk metric. The Value-at-Risk (VaR) is the industry standard market risk measurement metric, and as a statistical measure, it can be defined as the maximum expected loss we are confident will not be exceeded for a given significance level and over a given future time horizon ***(Alexander, 2009)***. Regarding the administration of financial institutions and the oversight of their internal financial operations, the Economic Capital (EC) consists of the desirable level of capital a firm would like to hold for insurance against its risks ***(Alexander, 2009)***. Hence, mathematically, the EC is equal to the VaR ***(Jorion, 2007)***, and for a specific pre-defined maximum value for the EC, it works as a benchmark in which the development of financial risk management strategies is based on.

This dissertation objective is to assess and manage the VaR of a portfolio composed of equities and bonds from the U.S., Canadian, European and Asiatic markets during a test period of one year from 30 January 2023 to 2 February 2024 such that the value does not exceed the pre-defined maximum for the EC. Also, measure and manage the individual risk contributions of each portfolio asset and make sure it does not surpass a maximum percentage of the total portfolio VaR. As the ultimate goal, we analyze the P&L profile for a one-year period and compare it with the P&L profile if we had not managed the VaR.

The first question that arises when measuring VaR is which model should be used to complete this task. 1996 was the year that the necessity of having a risk measure was met. Thanks to the efforts from financial institutions and regulators, ***J.P. Morgan and Reuters, (1996)*** were able to introduce RiskMetrics (RM) VaR model. While the forerunner, the solution may not lie here, and thus we explore four distinct VaR models in this dissertation: Parametric Normal VaR, Skewed Generalized Student-t (SGSt), Historical VaR and Quantile Regression (QR) and a total of 12 different models are tested. Via a Backtest method, utilizing historical data, we calculate historical VaR estimates for each model using the portfolio composition on 30 January 2023. Sample wise, the series of historical VaR estimates for the Backtest spans over 11 years from 19 December 2011 to 27 January 2023 and we measure the performance of each model through the Unconditional Coverage (UC) test ***(Kupiec, 1995)*** and Berkowitz, Christoffersen and Pelletier (BCP) test ***(Berkowitz et al., 2011)***.

Working with the best performing model and for a time span of a year, an equity exposure hedging strategy assesses and manages the VaR of the portfolio on a daily basis. Supported by historical data ranges, we define the daily EC to a maximum of €525 million, that corresponds to close to 3.9% of the portfolio value on 30 January 2023, a maximum individual contribution of each asset to the total risk of the portfolio of 12.5% and a maximum total currency risk of 10.5%. With this, the daily VaR estimate cannot exceed those boundaries. Therefore, we test the impacts on the one-year return of the portfolio after limiting the VaR and the individual exposure to each asset by using an equity exposure hedging strategy.

For comparison reasons, we name the portfolio with the implemented hedging strategy as Hedged Portfolio and the same portfolio without the strategy as Unhedged Portfolio.

This thesis is structured as follows: Chapter 2 covers the significant literature; Chapter 3 shows the data used, portfolio composition and time span; Chapter 4 defines the path forward and delves into the applied methodology; Chapter 5 discloses the outcomes of the backtesting and outlines the model selection process; Chapter 6 analyzes the hedging and portfolio rebalancing strategies and their impact on portfolio returns; Chapter 7 recaps the results of this thesis.

Chapter 2

LITERATURE REVIEW

Historically, distress in financial institutions has shown its potential to result in negative effects of great magnitude on both global economy and financial markets ***(Hoggarth et al., (2002)*** and ***Dell'Ariccia et al., (2008))***. The massive financial crisis of 2008 works as a major example of such outcomes ***(Baur, 2012)***. Given that many of these institutions operate in the private sector, the concept of regulation to safeguard financial stability naturally emerges ***(Dow, 1996)***. As a solution, financial regulators created the Basel Accords in 1988, being subsequently updated to revised versions in later years, targeting minimum capital requirements by rigorous risk measuring and reporting standards ***(Shakdwipee & Mehta, 2017)***.

Although risk can normally be classified into market, credit, and operational risk ***(Allen et al., 2004)***, this dissertation addresses market risk only, which refers to the uncertainty of the future value of financial assets due to fluctuations in market prices ***(Alexander, 2009)***. Efficient risk management starts with precise risk measurement. Despite its limitations ***(Krause, 2003)***, VaR is still the industry standard metric for measuring market risk. As a statistical model, VaR correspond to the maximum anticipated loss that is not expected to be exceeded, for a certain pre-defined confidence level and for a specific time span ***(Alexander, 2009)***.

Minimum capital requirements, from a regulatory perspective, can be identified as regulatory risk capital ***(Alexander, 2009)*** and are computed using techniques established by financial regulators ***(Bank for International Settlements, 2023)***. Once financial institutions meet these regulatory risk capital requirements in line with the Basel Accords, they have the flexibility to allocate capital internally to various activities and in amounts they deem appropriate ***(Alexander, 2009)***. In this context, Economic Capital (EC) for a financial institution refers to the capital at risk due to its investment activities ***(Porteous & Tapadar, 2005)***. To ensure financial stability within the firm, the capital at risk from investment activities must not exceed the allocated EC limit and, as EC is measured by VaR, its value should be numerically equal to the VaR ***(Jorion, 2007)***. With a predefined EC limit, risk management strategies can be designed to align with this benchmark.

The demand for a standardized metric to measure market risk emerged in the mid-1990s, driven by collaborative efforts between financial regulators and international banks. This led to the development of the RiskMetrics (RM) VaR model by ***J.P. Morgan and Reuters, 1996***. The adoption of VaR as the official measure of market risk under the Basel II Accords, and in subsequent updates, marked a crucial turning point. The creation of RM played a key role in establishing economic capital-based metrics for defining minimum capital requirements ***(Allen et al., 2004)***.

The RM VaR model lies into the parametric class of models. Moreover, this model can be known as Parametric Normal VaR because it accepts that returns follow a normal distribution for a defined testing period ***(J.P. Morgan and Reuters, 1996)***. This assumption does not respect the empirical data on financial returns that normally confirm both excess kurtosis and negative skewness as shown by ***Fama, (1965)*** and ***Peiro, (1994),*** which are characteristics of a non-normal distribution. This aspect is noteworthy because extreme losses are found in the left tail of a return distribution. The lower the significance level chosen for VaR, the further into the tail we assess. Therefore, if the actual distribution of returns deviates from the normal distribution, the VaR estimate achieved by the RM VaR model may be unsuccessful in capturing the risk incurred, possibly making the RM model unfitting. When estimating VaR at a 5% significance level, or a 95% confidence level, we are 95% confident that future losses will not exceed the VaR. Given that this configuration is generally used in the RM model, ***Pafka & Kondor, (2001)*** investigated possible flaws by analyzing the model's performance at significance levels below 5%. Using 4 years of financial data from the 30 stocks composing the Dow Jones Industrial Average index, the authors calculate the RM VaR for both 5% and 1% significance levels. Their findings indicate that at lower significance levels, such as 1%, where the analysis delves deeper into the left tail of the distribution, the non-normality of returns becomes clear. With this, the higher the level of significance chosen, the bigger the probability that VaR estimations are underestimated.

A promising approach to tackle the non-normality of returns and the limitations of the RM model is the Skewed Generalized Student-t (SGSt) distribution ***(Theodossiou, 1998)***. ***McDonald & Newey, (1988)*** introduced this distribution as an extension of the classical Student-t distribution, allowing substantial flexibility in modelling the shape of both tails and central region of the distribution. The SGSt VaR also assumes a parametric distribution for returns but seeks to better fit the distribution shape to the empirical returns. This approach allows it to accommodate for the fat tails usually observed in financial return distributions. ***Lin & Shen, (2006)*** research compared the performance of SGSt VaR with RM VaR utilizing daily data from the S&P 500, NASDAQ, DAX, and FTSE 100 indices with a sample size of 3 years. As anticipated for equity returns, the normality assumption was strongly rejected for all indices by the Jarque-Bera test. The authors then estimated VaR for each index for a range of significance levels from 55% to 0.1%. Results show that while for a 55% significance level the RM model produces satisfactory results, for lower levels the performance begins to worsen. In contrast, the SGSt VaR maintains a robust performance as we decrease the significance level. The authors conclude that the SGSt VaR allows more accurate estimates for lower significance levels, making it, therefore, an ensuring alternative in the scope of the parametric VaR models.

Rather than imposing a specific returns structure via a parametric approach, as performed by the RM and SGSt models, a much straightforward alternative is presented: utilizing the empirical distribution of returns. This method accurately captures the empirical skewness and kurtosis of the return's distribution, establishing the core of the Historical Simulation VaR, also referred to as Historical

VaR. However, this flexibility comes with some drawbacks. Since Historical VaR is based entirely on the empirical sample of past returns, the selection of the sample size is a subjective yet crucial factor. As ***Pritsker, (2006)*** discusses, a larger sample size broadens the diversity of possible outcomes but, simultaneously, with the growth of the sample size, the model might be less responsive to current market volatility conditions. To tackle this matter, ***Barone-Adesi et al., (1998)*** and ***Boudoukh et al., (1998)*** propose a refinement to the classical methodology by giving greater weight to later observations. This change outcomes in a sample in which recent data points hold more magnitude than those from the bygone years, in that way improving the model's elasticity to changes in market conditions. ***Hull & White, (1998)*** propose a further refinement where past returns are adjusted through a volatility adjustment methodology in order to reflect current market volatility. This method changes the magnitude of past returns based on the market volatility conditions back then, adjusting them to current volatility. ***Hull & White, (1998)*** tested their model using approximately 9 years of daily financial data from 12 distinct exchange rates and 5 different stock indices. ***Hull & White, (1998)*** compared their volatility-adjusted approach with the standard Historical VaR, where the adjustment of the weight of observations does not occur, and with the ***Boudoukh et al., (1998)*** method, that adjusts the weights of later observations. As predictable, the results of this comparison indicate that the volatility adjustment methodology reached better overall results. Hull and White's proposed volatility adjustment, when compared to the ***Boudoukh et al., (1998)*** methodology, delivers much better results specifically for the 1% significance level.

Continuing in the non-parametric framework, VaR can also be defined as a conditional quantile ***(Xiao et al., 2015)***. ***Koenker & Bassett Jr, (1978)*** introduced an alternative technique for estimating VaR known for using quantile regressions. The Quantile Regression (QR) VaR model is comparable to the Historical VaR in that it is based on empirical returns instead of assuming a given parametric distribution. One of the key advantages of QR VaR is the flexibility it offers in choosing explanatory variables, allowing for more fitted and hypothetically more precise risk estimations. ***Steen et al., (2015)*** assessed the performance of the RM and Historical VaR models versus the QR VaR by working with nearly 20 years of daily data for futures contracts of 19 distinct commodities. Their findings side with preceding results of other authors regarding the RM model's performance: it produces reasonable results at the 5% significance level for most commodities. However, its performance weakens when tested at the 1% significance level. While the Historical VaR outperformed the RM model overall, it was nonetheless topped by the QR VaR, which outclassed both models throughout all confidence levels. Although the accuracy of QR VaR estimates depend on the configuration of the model it has the capability of delivering better results when compared with the RM model. Against the Historical VaR, the conclusions are similar with the QR VaR obtaining improved or at least similar estimates. For commodities in particular, QR VaR presents itself as a promising model because of its flexibility and improved performance.

Nevertheless, for two unique portfolios, the same VaR model will produce two completely different estimates as investigated by ***Alexander, (2009)***. This means that we need to test the performance for different models in order to choose the model that fits best our portfolio. That can be achieved through a Backtest model using historical data of portfolio returns and VaR estimates for the same period. The number of exceedances, a vastly used Backtest performance metric, consists of an event in which the VaR estimate is surpassed by the actual loss. In order to test the number of exceedances, we choose the UC test ***(Kupiec, 1995)***, for the time span under analysis. To make a comprehensive assessment, we also use the BCP test ***(Berkowitz et al., 2011)*** to check if the observed exceedances (if there is any) are autocorrelated. There is, it verifies if the exceedances are independent from each other or if they occur in clusters. This metric is important because its by testing the existence of clusters that we can verify the model's ability to adjust for quick variations in market conditions.

When measuring the VaR of a portfolio, efficient management and finding a good balance between returns and risk are two of the main objectives of a risk manager. Whilst EC exceeds VaR, we need to adjust the portfolio by using a risk management strategy. As a consequence, the risk profile of our portfolio changes alongside with its composition. Therefore, and as explored by ***Longley-Cook, (1998)***, we should try to achieve the highest return possible for the lowest level of VaR, creating different risk profiles along the way.

With this in mind, a new metric emerges, the RORAC ***(Matten, 1996)***. This metric yields the ratio between returns and the risk incurred to accomplish them, connecting a non-adjusted return to a risk-adjusted capital base ***(Matten, 1996)***. Hence, we can get a fair assessment regarding the returns of portfolios with different risk profiles by using the RORAC.

Chapter 3

DATA

The structure of this portfolio includes a mixture of equities and bonds, with allocations across U.S., Canadian, Asian and European markets. Euro (EUR) is defined as the local currency, while the U.S. Dollar (USD), Canadian Dollar (CAD), Great British Pound (GBP) and Japanese Yen (JPY), are treated as foreign currencies.

The equity component consists of thirty-three stocks from the U.S., Canadian, Asian and European markets. The daily adjusted closing prices were downloaded from yahoo finance[1], along with the correspondent exchange rates: USD/EUR, CAD/EUR, GBP/EUR and JPY/EUR.

The fixed income component incorporates three fixed coupon government bonds with different maturities, yields and payment dates, and were issued by the U.S., Germany and Netherlands markets. The data for the bonds was taken from the Frankfurt Stock Exchange[2] and the daily interest rates from the Federal Reserve[3] for USD and from the European Central Bank (ECB)[4] for EUR. In this dissertation we work with data between January 3rd of 2007 to February 2nd of 2024.

Table 1 below has the composition of the portfolio used on this dissertation including the value disaggregation of each asset.

[1] https://finance.yahoo.com
[2] https://www.boerse-frankfurt.de/en
[3] https://www.federalreserve.gov/datadownload/Choose.aspx?rel=H15
[4] https://sdw.ecb.europa.eu/browseSelection.do?node=9689726

Asset	Ticker/ISIN	Currency	No. Shares / Face Value	Price	Exchange Rate		Value (EUR)	Allocation (%)
Microsoft Corporation	MSFT	USD	4 450 000	239.70	USDEUR	0.9198	981 162 746 €	7.42
Apple Inc.	AAPL	USD	8 000 000	141.83	USDEUR	0.9198	1 043 698 612 €	7.89
NVIDIA Corporation	NVDA	USD	6 300 000	191.53	USDEUR	0.9198	1 109 891 817 €	8.39
Amazon.com, Inc.	AMZN	USD	10 500 000	100.55	USDEUR	0.9198	971 144 105 €	7.35
Alphabet Inc.	GOOGL	USD	14 000 000	96.94	USDEUR	0.9198	1 248 370 080 €	9.44
UnitedHealth Group Incorporated	UNH	USD	1 200 000	476.71	USDEUR	0.9198	526 191 417 €	3.98
Mastercard Incorporated	MA	USD	1 600 000	368.38	USDEUR	0.9198	542 155 762 €	4.10
Salesforce, Inc.	CRM	USD	1 600 000	164.54	USDEUR	0.9198	242 153 517 €	1.83
Walmart Inc.	WMT	USD	2 400 000	46.35	USDEUR	0.9198	102 328 759 €	0.77
Thermo Ficher Scientific Inc.	TMO	USD	300 000	562.41	USDEUR	0.9198	155 199 268 €	1.17
Adobe Inc.	ADBE	USD	1 100 000	363.42	USDEUR	0.9198	367 717 091 €	2.78
Oracle Corporation	ORCL	USD	1 300 000	85.76	USDEUR	0.9198	102 545 624 €	0.78
Caterpillar Inc.	CAT	USD	950 000	255.36	USDEUR	0.9198	223 143 999 €	1.69
S&P Global Inc.	SPGI	USD	450 000	361.88	USDEUR	0.9198	149 791 231 €	1.13
Booking Holdings Inc.	BKNG	USD	200 000	2430.76	USDEUR	0.9198	447 181 968 €	3.38
Vertex Pharmaceuticals Incorporated	VRTX	USD	400 000	319.98	USDEUR	0.9198	117 732 165 €	0.89
Chipotle Mexican Grill, Inc.	CMG	USD	160 000	1609.86	USDEUR	0.9198	236 930 177 €	1.79
O'Reilly Automotive, Inc.	ORLY	USD	100 000	789.63	USDEUR	0.9198	72 633 326 €	0.55
The Hershey Company	HSY	USD	600 000	215.81	USDEUR	0.9198	119 104 818 €	0.90
LVMH Moet Hennesy - Louis Vuitton	MC.PA	EUR	770 000	786.37	-	1.0000	605 501 936 €	4.58
ASML Holding N.V.	ASML	EUR	1 300 000	641.46	-	1.0000	833 902 472 €	6.31
Christian Dior SE	CDI.PA	EUR	70 000	764.54	-	1.0000	53 517 863 €	0.40
Texas Pacific Land Corporation	TPL	USD	1 000 000	645.78	USDEUR	0.9198	594 018 513 €	4.49
Mitsui O.S.K. Lines, Ltd.	9104.T	JPY	5 000 000	2805.89	JPYEUR	0.0071	99 398 623 €	0.75
Netflix, Inc.	NFLX	USD	850 000	353.11	USDEUR	0.9198	276 083 985 €	2.09
General Electric Company	GE	USD	1 000 000	64.15	USDEUR	0.9198	59 009 548 €	0.45
Moody's Corporation	MCO	USD	350 000	309.86	USDEUR	0.9198	99 759 125 €	0.75
Vulcan Materials Company	VMC	USD	450 000	176.24	USDEUR	0.9198	72 951 203 €	0.55
Shell plc	SHEL.L	GBP	4 000 000	22.31	GBPEUR	1.1405	101 751 027 €	0.77
Wells Fargo & Company	WFC	USD	2 500 000	44.32	USDEUR	0.9198	101 912 254 €	0.77
Canadian National Railway Company	CNR.TO	CAD	2 200 000	153.36	CADEUR	0.6914	233 264 100 €	1.76
McDonald's Corporation	MCD	USD	500 000	263.39	USDEUR	0.9198	121 140 315 €	0.92
JPMorgan Chase & Co.	JPM	USD	2 000 000	134.48	USDEUR	0.9198	247 404 228 €	1.87
Total Equity					-	-	**12 258 691 677 €**	**92.72**
German Bond 2033	DE000BU2Z015	EUR	500 000 000	103.48%	-	1.0000	517 413 420 €	3.91
Dutch Bond 2030	NL0015001DQ7	USD	200 000 000	101.09%	USDEUR	0.9198	202 181 159 €	1.53
U.S. Treasury Bond 2034	US91282CJZ59	EUR	250 000 000	97.28%	-	1.0000	243 192 414 €	1.84
Total Bonds					-	-	**962 786 992 €**	**7.28**
Total Portfolio					-	-	**13 221 478 669 €**	**100.00**

Table 1. Portfolio composition on 30 January 2023. This table illustrates the assets that compose the portfolio used in this dissertation, along with the amount invested in each, converted from USD, GBP, CAD or JPY to EUR where appropriate. The correspondent exchange rates for each currency as of 30 January 2023 are 0.9198, 1.1405, 0.6914 and 0.0071 respectively. Due to rounding, figures may not sum to the total.

Chapter 4

METHODOLOGY

This dissertation objective is to assess and manage the VaR of a portfolio. We will manage the portfolio over a one-year period starting from 30 January 2023, ensuring that it does not exceed pre-defined levels of Economic Capital and individual risk exposures from the constituents of the portfolio.

Considering the several VaR Models available, to decide which one to use we measure the performance and the accuracy of those VaR models with a process called Backtesting. This can be achieved by using historical data and today's portfolio composition. We will assume that our portfolio existed in the past and will compute a historical series of daily VaR estimates from 19 December 2011 to 27 January 2023. We are going to assess four different models: Parametric Normal VaR (RiskMetrics), SGSt VaR, Historical VaR and QR VaR. Inward these models, we explore several assumptions of each one to conclude which one is the better fit to our portfolio. We do that by analyzing the results of known statistical tests that measure the performance of the models. Despite their differences, all four models share one critical input for their calculations: portfolio volatility.

After choosing the VaR Model to apply, we measure the VaR of our portfolio for a period of one year going forward, through a hedging strategy we manage it according to our pre-defined risk boundaries. To evaluate the results of the hedging strategy we compute a performance metric named Return on Risk-Adjusted Capital for both portfolios and analyse the results.

4.1. Risk Factor Mapping

Risk factors are the underlying variables that are responsible for changes in the value of assets within a portfolio. The first step in risk analysis is to identify these factors and quantify the portfolio's exposure to them. This process, referred to as risk factor mapping, involves assigning each portfolio position to its corresponding risk factor exposure. The specific risk factors associated with an asset depend on its type, as different asset classes are affected by different variables.

To estimate VaR in EUR - our local currency - all exposures must be stated in EUR. This requires the conversion of all foreign currency exposures at the relevant exchange rate on the backtesting date, which in this case is 27 January 2023. In the following subsections, we explain the methodology for mapping exposures for each asset class in our portfolio. At the end of this section, Table 2 summarizes the exposures to each risk factor resulting from the risk factor mapping process.

4.1.1. Bonds

A bond is a financial instrument that generates a series of cash flows. Its fair value depends on discounting these future cash flows back to the present, making the interest rate the predominant risk factor. When interest rates rise, the bond value drops, and when interest rates fall, the bond value rises. As interest rates fluctuate with market conditions, quantifying the sensitivity of bond positions to changes in interest rates is critical to risk measurement and management.

For a fixed coupon bond, the future cash flows include periodic coupon payments over the bond's lifetime and the final principal repayment at maturity. The coupon payment is calculated as follows:

$$Coupon\ = N \times \frac{c_n}{n}, \tag{1}$$

where N is the monetary amount invested in the bond (face value), c_n is the annual coupon rate (annual interest paid by the bond until the maturity date) and n is the coupon frequency (the number of coupon payments per year until maturity).

The final payment at the bond's maturity consists of the redemption of the face value along with the last coupon payment and is given by:

$$C_m\ = N \times \left(1 + \frac{c_n}{n}\right) \tag{2}$$

Let C_T denote a future cash flow, T the time in years from now until the maturity date of the cash flow and r_T the continuously compounding interest rate for the period between now and T. It follows that the present value (PV) of the cash flow C_T is:

$$PV_{C_T,r_T} = C_T \times e^{-r_T \times T} \tag{3}$$

The present value of a basis point (PV01) quantifies the sensitivity of a given cash flow's present value to a one basis point decrease in the interest rate r_T. It is approximated using a first-order Taylor expansion as:

$$PV01_{C_T,r_T} \approx \frac{\partial PV_{C_T,r_T}}{\partial r_T} \times (-0.01\%)\ = \\ T \times PV_{C_T,r_T} \times 0.01\% \tag{4}$$

where T is the maturity of the cash flow.

Consequently, the first-order approximation of the change in the present value of a cash flow, representing its P&L, can be expressed as a function of $PV01$ as follows:

$$\Delta PV_{C_T,r_T} = -\, PV01_{C_T} \times \frac{\Delta r_T}{0.01\%}, \tag{5}$$

where $\frac{\Delta r_T}{0.01\%}$ is the absolute change in interest rate converted to basis points.

Therefore, for cash flow C_T, its P&L is defined by the sensitivity of the cash flow to a one basis point increase in the interest rate $(-\, PV01_{C_T})$ and the actual variation in the interest rate (Δr_T). If the interest rate increases (positive change), the P&L is negative, while if the interest rate decreases (negative change), the P&L is positive. For a bond with n cash flows, the total P&L is the sum of the individual P&Ls calculated by using Equation 5 for each cash flow.

However, a bond with n cash flows will have n different interest rates as risk factors, with each rate corresponding to the maturity of a particular cash flow. In a portfolio of many bonds, this can rapidly become unmanageable. A further challenge arises when a future cash flow falls on a date for which there is no data for the corresponding interest rate r_T. Without an adequate interest rate, it is impossible to accurately discount the cash flow to the present and thus calculate its $PV01$.

To address these concerns, and following ***Alexander, (2008)***, we adopt a vertex mapping approach. This consists of mapping non-standard maturity cash flows to a set of standard maturity interest rates for which data is available. In this cash flow mapping process, a vertex refers to a standard maturity for which interest rate data exist. The mapping method we use is the PV+PV01 invariant mapping, which ensures that both the present value PV and $PV01$ of the original non-standard maturity cash flow are preserved.

Let PV_{C_T} be the present value of the original cash flow with maturity T and let T_1 and T_2 be the standard maturity vertices immediately below and above T, respectively, for which interest rate data are available. Let x_1 and x_2 denote the proportions of PV_{C_T} assigned to vertices T_1 and T_2 respectively. To preserve the PV of the original cash flow, we apply the following condition:

$$x_{T_1} + x_{T_2} = PV_{C_T}, \tag{6}$$

where x_{T_1} is the PV mapped to vertex T_1 and x_{T_2} the PV mapped to vertex T_2.

For $PV01$ invariant mapping, the sum of the $PV01$ of the mapped cash flows is equal to the $PV01$ of the original cash flow. This ensures that the total $PV01$ of the two mapped cash flows is equivalent to the $PV01$ of the original cash flow after a parallel shift of one basis point[5] in the yield curve. To maintain the $PV01$ of the original cash flow, we apply the following condition:

$$T_1 x_{T_1} + T_2 x_{T_2} = T\, PV_{C_T} \tag{7}$$

[5] when the interest rate curve shifts by one basis point it means all spot rates shift by 0.01%.

Finally, we preserve the PV and $PV01$ conditions at the same time by combining equations 6 and 7. We get the values for x_{T_1} and x_{T_2} that meet both requirements simultaneously as:

$$x_{T_1} = \frac{T_2 - T_1}{T_2 - T_1} \times PV_{C_T} \tag{8}$$

and

$$x_{T_2} = 1 - \frac{T_2 - T_1}{T_2 - T_1} \times PV_{C_T} \tag{9}$$

We repeat the PV+PV01 mapping for each cash flow and each bond in our portfolio, and in terms of exposure, each mapped standard maturity cash flow is sensitive to changes in its corresponding standard maturity interest rate, which is given by its $PV01$.

Recalling Equation 5, in the scope of risk factor mapping, each standard maturity interest rate (vertex) serves as a risk factor. The exposure to each risk factor is the sum of all $PV01$s mapped to that maturity multiplied by -1. For bonds in the US market where cash flows are denominated in USD, we convert the $-PV01$ exposures to EUR using the exchange rate on 27 January 2023.

4.1.2. Equity

The value of an investment in a stock is dependent on the number of shares held and the stock market price. Consequently, the risk factor for any stock in our portfolio is the change in its market price. In terms of risk factor mapping, the exposure to the price change of each stock is the amount of capital invested in it, and for each stock we obtain the amount of capital invested by multiplying the number of shares by the current market price:

$$S_{i_t} = N_{i_t} \times P_{i_t} \times F\,X_t \tag{10}$$

where $F\,X_t$ is the exchange rate on 27 January 2023 used to convert the exposure to local currency.

4.1.3. Currency

As our local currency is EUR and the portfolio under analysis contains positions in assets from foreign markets, these positions are not only exposed to the risk factors of the respective assets, but also to the exchange rate between the foreign currency and local currency. In our case, assets denominated in USD, JPY and GBP are additionally exposed to the USD/EUR, JPY/EUR, GBP/EUR and CAD/EUR exchange rates, respectively. Regarding the mapping, the exposure to each foreign currency is the total amount of capital invested in assets, both bonds and equity, denominated in that specific currency converted to EUR. Again, we use the exchange rates on 27 January 2023.

4.1.4. Portfolio Exposures

Below, Table 2 shows the risk factors and the corresponding mapped exposures as of 27 January 2023. For bonds, the risk factors are the standard maturity interest rates and the exposure to each standard maturity interest rate is the sum of all $-PV01s$ mapped to that maturity. We obtain the $-PV01$ via methodology illustrated in subsection 1.1.. For equities, we compute the exposures presented in the table using the methodology described in subsection 1.2. of this chapter. For currency, we get the exposures to the USDEUR, JPYEUR, GBPEUR and CADEUR exchange rates by using the methodology expressed in subsection 1.3..

Equity		Bonds		Currency	
Risk Factor	**Exposure (EUR)**	**Risk Factor**	**Exposure (EUR)**	**Risk Factor**	**Exposure (EUR)**
9104.T	99 160 123 €	EUR3M	0 €	USDEUR	10 767 175 685 €
AAPL	1 062 837 108 €	EUR6M	-595 €	JPYEUR	99 160 123 €
ADBE	374 302 171 €	EUR1Y	-1 160 €	GBPEUR	101 474 523 €
AMZN	985 383 989 €	EUR2Y	-3 427 €	CADEUR	232 933 303 €
ASML	855 804 425 €	EUR3Y	-7 495 €		
BKNG	451 279 888 €	EUR5Y	-17 440 €		
CAT	225 262 007 €	EUR7Y	-141 305 €		
CDI.PA	53 722 390 €	EUR10Y	-374 943 €		
CMG	237 006 190 €	EUR15Y	-66 239 €		
CNR.TO	232 933 303 €	EUR20Y	0 €		
CRM	241 305 444 €	USD3M	-205 €		
GE	60 633 509 €	USD6M	-19 €		
GOOGL	1 276 964 161 €	USD1Y	-654 €		
HSY	116 494 536 €	USD2Y	-1 691 €		
JPM	248 994 044 €	USD3Y	-3 643 €		
MA	545 254 472 €	USD5Y	-7 676 €		
MC.PA	607 169 579 €	USD7Y	-12 427 €		
MCD	121 585 418 €	USD10Y	-157 494 €		
MCO	101 253 261 €	USD20Y	-33 913 €		
MSFT	1 001 078 848 €				
NFLX	281 478 157 €				
NVDA	1 177 083 556 €				
ORCL	104 250 586 €				
ORLY	70 785 693 €				
SHEL.L	101 474 523 €				
SPGI	152 435 094 €				
TMO	157 435 352 €				
TPL	610 308 932 €				
UNH	525 362 631 €				
VMC	72 846 330 €				
VRTX	118 023 586 €				
WFC	101 323 826 €				
WMT	102 939 016 €				

Table 2. Risk factor exposures map in EUR on 27 January 2023.

4.2. Returns

For the equities in our portfolio, if the invested capital (M_{Stock}) remains unchanged, the P&L is obtained from the change in the market price of its risk factor, i.e., the share price, and is given by:

$$P\&L_{Stock_t} = M_{Stock} \times \left(\frac{P_t}{P_{t-1}} - 1\right) \tag{11}$$

When investing in a bond, each cash flow is exposed to a different interest rate and subsequently to movements in that interest rate. In our portfolio, on the assumption that the invested capital and $-PV01$ values stay unchanged, the total P&L is calculated as the sum of the P&Ls of all the cash flows mapped, as shown in equation 5:

$$P\&L_{Bonds_t} = \sum_{i=1}^{n} -PV01_{T_i} \times \left(\frac{\Delta r_{T_i}}{0.01\%}\right) \tag{12}$$

An allocation to an asset valued in a foreign currency implies an exposure not solely to the specific risk factor of the asset, but also an indirect exposure to fluctuations in the exchange rate between the foreign currency and the local currency. In other words, investing in equities and bonds denominated in a foreign currency gives rise to an exchange rate exposure equal to the value of the investment ($M_{Currency}$). Since the underlying risk factor is the foreign exchange (FX) rate, the P&L resulting from currency exposure is obtained by:

$$P\&L_{Currency_t} = M_{Currency} \times \left(\frac{FX_t}{FX_{t-1}} - 1\right) \tag{13}$$

Given a series of daily equity prices, spot interest rates and foreign exchange rates, we use the above equations together with the mapped exposures to each risk factor as of 27 January 2023 to calculate the historical time series of daily P&Ls for the portfolio structure at that date. In vector form, this is denoted by:

$$P\&L_{Portfolio_t} = \begin{bmatrix} M_{Stock_i} \\ \vdots \\ -PV01_{T_i} \\ \vdots \\ M_{Currency} \end{bmatrix}^{\boldsymbol{T}} \times \begin{bmatrix} \left(\frac{P_{i_t}}{P_{i_{t-1}}} - 1\right) \\ \vdots \\ \frac{\Delta r_{T_i}}{0.01\%} \\ \vdots \\ \left(\frac{FX_t}{FX_{t-1}} - 1\right) \end{bmatrix} \tag{14}$$

where the first vector transposed is constant and represents the exposure to each risk factor, and the second vector reflects the change in the corresponding risk factor.

We get the portfolio P&L as a percentage return via:

$$R_t(\%) = \left(\frac{P\&L_{Portfolio_t}}{Portfolio\,Value}\right), \tag{15}$$

where the denominator is the portfolio value on 27 January 2023.

4.3. Volatility

At the beginning of this chapter, we highlighted the two key components required for successful backtesting: portfolio returns and portfolio volatility modelling to compute the VaR models under assessment. In Section 1, we reviewed the procedure for determining the appropriate risk factors for each asset class and the quantification of the portfolio's sensitivity to these risk factors. Having achieved this, we have taken the portfolio exposures for the portfolio composition on 27 January 2023 (see Table 2) and holding them unchanged, we simulated historical returns using equations 14 and 15, we are left with a series of historical returns for our portfolio.

The next step in computing the VaR models is volatility modeling. Volatility, σ, is the standard deviation of returns. The straightforward method to estimating it implies selecting a historical sample of past returns and computing their standard deviation. This methodology assumes that all observations in the sample carry the same weight, regardless of how recent they are. This may be unfitting since σ is equally influenced by older observations, which may have less relevance to current market conditions, as it is by more recent ones.

As VaR is a forward-looking measure, data from the distant past may be of reduced relevance. The Exponential Weighted Moving Average (EWMA) volatility model deals with this by giving more emphasis to more recent observations, which better capture current market conditions. The weighting factor, λ, ranges from 0 to 1, with lower values giving more weight to recent data. Whilst the choice of λ is subjective, according to the results of the RiskMetrics technical paper produced by ***J.P. Morgan and Reuters, (1996)***, a λ of 0.94 proved to be the best overall fit when handling daily returns, so we will be using this from now on.

Based on the historical daily returns, we recursively estimate the variance of the EWMA as follows:

$$\hat{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2 \tag{16}$$

where $\hat{\sigma}_t^2$ is the variance estimated for day t on day $t-1$, r_{t-1} is the return observed on day $t-1$ and $\lambda \in (0, 1)$ is the smoothing factor.

4.4. Value-at-Risk Models

VaR can be defined as the maximum expected loss over a future time horizon h at a given significance level α. We assume a significance level of $\alpha = 1$, equivalent to a 99% confidence level, and a one-day time horizon ($h = 1$). This implies that, while holding the current portfolio, we are 99% confident that the observed loss over the next day will not exceed the estimated VaR.

Officially, the h-day $100\alpha\%$ VaR ($VaR_{h,\alpha}$) is defined as the negative α-quantile of the h-day return distribution. As ***Alexander, (2009)*** states, for any $0 < \alpha < 1$, the α-quantile of the h-day distribution of a continuous random variable X is a real number (x_α) such that:

$$P(X < x_\alpha) = \alpha, \tag{17}$$

this means that the probability of observing a loss greater than x_α is $1 - \alpha$.

If the distribution function of X is defined, then the α-quantile (x_α) for any chosen value of α is denoted by:

$$x_\alpha = F^{-1}(\alpha), \tag{18}$$

where F^{-1} is the inverse cumulative distribution function of X.

The α-quantile value reached (x_α) is the maximum loss that we expect to be exceeded with probability α. Since VaR is a measure of potential loss, it is conventionally stated in absolute terms as:

$$VaR_{h,\alpha} = -\,F^{-1}(\alpha) \tag{19}$$

This safeguards that the VaR value is always positive, illustrating the magnitude of the potential loss. The subsequent subsections discuss the methodologies needed to build the four models referred to above.

4.4.1. Parametric Normal VaR

Let X denote a continuous random variable which represents portfolio returns. The core assumption of the Parametric Normal VaR model is that it assumes returns follow a normal distribution, that is, $X_h = N{\sim}(\mu_h, \sigma_h)$, where μ_h and σ_h are the estimated mean and standard deviation, correspondingly.

We take note of Equation 19 and, given that we are working with a normal distribution, it follows that:

$$VaR_{h,a} = -\phi^{-1}(\alpha) \times \sigma_h - \mu_h, \tag{20}$$

where $\phi^{-1}(\alpha)$ denotes the α-quantile of the standard normal distribution

With regards to μ_h, ***Alexander, (2009)*** implies applying $\mu_h = 0$ for small time horizons, and as we are working with daily data and, hence daily VaR estimates ($h = 1$), this happens to be a sensible assumption. With this, we adjust the equation above and calculate the h-day $100\alpha\%$ Parametric Normal VaR as:

$$VaR_{h,\alpha} = -\phi^{-1}(\alpha) \times \sigma_h \tag{21}$$

where σ_h is estimated using the EWMA volatility model via Equation 16.

4.4.2. Skewed Generalized Student-t VaR

As previously noted, the distribution of returns on financial assets frequently deviates from the normal distribution by displaying thicker tails. As a result, the normal distribution may underestimate the probability of sharp negative returns. Therefore, by assuming a normal distribution, there is a significant likelihood that VaR will be underestimated at low levels of significance (e.g., 1%).

The shape of the standardized SGSt distribution ***(Theodossiou, 1998)*** seeks to represent the deviations from normality and its density function $T_{0,1,\lambda,p,q}$ is conditional on the parameters λ, p and q: $\lambda \in (-1, 1)$ defines the skewness (if $\lambda = 0$, the distribution is symmetric, if $\lambda > 0$ or $\lambda < 0$, the distribution is positively or negatively skewed, respectively), $p > 0$ controls the shape of the central region of the distribution and $q > 0$ controls the shape of the tail of the distribution.

We use maximum likelihood to estimate these parameters so that the resulting SGSt distribution is as close as possible to the actual return distribution of our portfolio. To ensure that the model reflects current market conditions, we re-estimate the parameters each trading month and compute three different SGSt VaR series, each differing in the size of the rolling sample of portfolio returns used for parameterization: 250, 600 and 800 daily observations.

Officially, we compute the h-day $100\alpha\%$ SGSt VaR as:

$$VaR_{h,\alpha} = -{T^{-1}}_{0,1,\lambda,p,q}(\alpha) \times \sigma_h - \mu_h \tag{22}$$

where ${T^{-1}}_{0,1,\lambda,p,q}(\alpha)$ represents the α-quantile of the standard SGSt distribution. Homogenously to the Paramentric Normal VaR, we adopt $\mu_h = 0$ and estimate σ_h by the EWMA volatility model and we adjust the previous equation to:

$$VaR_{h,a} = -{T^{-1}}_{0,1,\lambda,p,q}(\alpha) \times \sigma_h \tag{23}$$

4.4.3. Volatility-Adjusted Historical VaR

In the RM and SGSt VaR models examined so far, we assume that the portfolio returns comply with a specific defined distribution, which may prove to be unrealistic or, as in the case of the SGSt VaR, computationally complex. Leaving the parametric world behind, Historical VaR offers a more straightforward approach: it uses the empirical distribution of returns directly and relies on the α-quantile of this distribution to estimate the VaR.

We estimate historical VaR by the next steps. First, we select the sample size n, which defines the historical time frame used to estimate VaR. We then calculate the h-day past empirical returns, holding constant the current portfolio exposure to the risk factors over the entire sample horizon. This guarantees that we are simulating how the current portfolio would have performed in past market conditions. Having determined the historical returns, we sort them in ascending order from worst to best. Once sorted, we start accumulating probability from the worst return upwards, where each observation has a probability of $\frac{1}{n}$. Finally, the VaR at significance level α is given by minus the return with α cumulative probability.

As noted in Chapter 2, the selection of the sample size is critical, as a largest sample size increases the diversity of returns, but at the same time, the greater the sample size, the less it reflects current market conditions. This is the major problem with simple historical VaR, as each observation has the same weight, so the current volatility of returns has the same impact as the volatility of the oldest returns in the sample. The volatility-adjusted historical VaR proposed by ***Hull & White, (1998)*** attempts to overcome this shortcoming by adjusting the volatility of the entire series of returns, while still giving each observation the same weight. In this way, the entire sample mirrors current market conditions. To do this, we first get a series of volatility estimates $\hat{\sigma}_t$ and then adjust the series of returns as:

$$\hat{r}_t = \frac{r_t}{\hat{\sigma}_t} \hat{\sigma}_T, \tag{24}$$

where $\hat{r}_t$ is the adjusted return, T is the VaR date and $t < T$. We classify this model as the T volatility-adjusted Historical VaR.

Bearing in mind the relevance of sample size for historical VaR, we compute three different VaR series where the only two distinctions between them are the sample size of the volatility-adjusted returns and if the volatility adjustment is performed or not. These variants use 250, 750 and 1000 daily observations, enabling us to analyze how the selection of sample size impacts the VaR estimates and their sensitivity to market conditions over different time horizons.

Officially, we compute the h-day $100\alpha\%$ volatility-adjusted Historical VaR as minus the α-quantile of the sample of volatility-adjusted returns.

4.4.4. Quantile Regression VaR

The Quantile Regression VaR is an advanced risk assessment model that estimates potential portfolio losses under different scenarios, focusing on specific quantiles of the return distribution ***(Koenker & Bassett Jr, 1978)***.

This VaR model uses quantile regression to estimate conditional percentiles of the return distribution, based on explanatory variables like volatility or economic conditions ***(Steen et al., 2015)***. Unlike ordinary least squares (OLS), which minimizes the sum of squared residuals to capture the average relationship between predictors and response, quantile regression focuses on specific percentiles by minimizing a weighted sum of residuals—using different weights depending on whether observations fall above or below the target quantile.

The α-quantile is the value below which a part of alpha of the distribution falls. The quantile regression achieves this by minimizing the asymmetric loss function:

$$\hat{q}_\alpha = \arg\min_{q_\alpha} \sum_{i=1}^{n} \alpha(y_i - q_\alpha)\, I_{y_i - q_\alpha > 0} + (\alpha - 1)(y_i - q_\alpha) I_{y_i - q_\alpha < 0}, \tag{25}$$

where $\alpha(y_i - q_\alpha)I_{y_i - q_\alpha > 0}$ are the observations above the quantile, $\alpha(y_i - q_\alpha)I_{y_i - q_\alpha < 0}$ are the observations below the quantile and $I_{y_i - q_\alpha < 0}$ is an indicator function that takes the value 1 if $y_i < q_\alpha$ and 0 else. Similarly, $I_{y_i - q_\alpha > 0}$ takes a value of 1 if $y_i > q_\alpha$ and 0 else.

For a quantile regression model on a portfolio's returns, if y represents the portfolio returns and x an explanatory variable, in this case the volatility, the quantile regression model becomes:

$$y = \alpha + bx + \varepsilon, \tag{26}$$

where α and b are parameters estimated by minimizing the quantile-specific loss function. The estimated quantile regression equation for a quantile $q_{\alpha,y}$ is:

$$q_{\alpha,y} = \hat{\alpha} + \hat{b}x \tag{27}$$

Hence, the α-QR VaR can be calculated as:

$$VaR_{h,a} = -q_{\alpha,y} = -(\hat{\alpha} + \hat{b}x) \tag{28}$$

Model ID	Description
1	Parametric Normal VaR
2	SGSt with a rolling sample of 250 observations
3	SGSt with a rolling sample of 600 observations
4	SGSt with a rolling sample of 800 observations
5	Historical with a rolling sample of 250 observations & without Volatility Adjustment
6	Historical with a rolling sample of 750 observations & with Volatility Adjustment
7	Historical with a rolling sample of 1000 observations & without Volatility Adjustment
8	QR with a rolling sample of 1000 observations, with constant & 2 explanatory variables
9	QR with a rolling sample of 1000 observations, with constant & 1 explanatory variables
10	QR with a rolling sample of 1000 observations, without constant & 1 explanatory variables

Table 3. Model numbers and respective description. With the exception of model 8 that has two explanatory variables and for that reason has two λ, that is $\lambda = 0.94$ for variable 1 and $\lambda = 0.85$ for variable 2, the remaining models considered use a EWMA volatility model with $\lambda = 0.94$ for each volatility estimates. The rolling samples on the SGSt and QR models are the sample of returns used to estimate the models' parameters.

Chapter 5

BACKTEST AND MODEL SELECTION

On Chapter 4 we explained the methods utilized to estimate all the models considered for use in our portfolio. In total, we calculated 10 different models with diverse settings and for each we obtained a time series of daily historical VaR estimates over 11 years, between 19 December 2011 and 27 January 2023. We refer to this test period as the global period. The next step is to evaluate the performance of each model and select the best one to use for the next year.

The key performance metric considered for this purpose is the number of exceedances, and since we are working with daily data, we identify an exceedance as an occurrence where the actual return for the day is worse than the VaR estimate for that same day. To assess these exceedances, we use two inference tests: the UC ***(Kupiec, 1995)*** and the BCP ***(Berkowitz et al., 2011)***, where the former measures the number of exceedances and the latter assesses the autocorrelation between exceedances.

While both tests evaluate model performance from distinct standpoints, our main decision criterion is the UC test results. The BCP test is used as a secondary measure to distinguish between models with similar UC test performance. We took this approach because a model with a low number of exceedances might still fail the BCP test if those exceedances are clustered or occur over a short period of time, depending on the test lag. Inversely, a model with a higher number of exceedances may be able to pass the BCP test if those exceedances are more widespread over time.

We perform the UC and BCP tests not only over the entire global period, but also, where relevant, for each individual year within that period. Evaluating model performance over specific time periods allows us to analyze how a model behaves under certain market conditions or to predict its performance if similar conditions occur in the future. Nevertheless, our final decision is based predominantly on the results for the global period, as this provides a more uniform and complete picture.

With this in mind, we have tried to optimize the settings of each model to maximize the results of the UC for the global period.

5.1. Unconditional Coverage Test

The UC test is well specified if the number of exceedances is within the significance level α of the VaR model ***(Alexander, 2009)***. If we recall VaR's definition, there is an α probability that the loss will be worse than the VaR. Since we are estimating the VaR at the 99% confidence level ($\alpha = 1\%$), we expect a number of exceedances of 1% the sample used to compute the VaR ($ex. 500 \text{ observations} \times 1\% = 5 \text{ expected exceedances}$).

Officially, for a sample of n observations, we specify an exceedance for each observation via an indicator function as:

$$I_{\alpha,t} = \begin{cases} 1, & if\ r_t < -VaR_{1,\alpha,t} \\ 0, & \text{otherwise} \end{cases}, \tag{29}$$

where r_t is the return at day t and $VaR_{1,\alpha,t}$ is the VaR estimated at day t. We are thus left with a series of *n* observations, each of which is either 1 or 0, dependent on the indicator function above.

Specifically, we test whether the null hypothesis that the indicator function, that is assumed to conform to an i.i.d. Bernoulli process, has a probability equal to α of the VaR model ***(Alexander, 2009)***.

The formulation of the null and alternative hypotheses for the UC test is as follows:

$$\begin{gathered} H_0 : \pi_{obs} \equiv \pi_{exp} \equiv \alpha \\ H_1 : \pi_{obs} \neq \pi_{exp}, \end{gathered} \tag{30}$$

where π_{obs} and π_{exp} are the observed and expected exceedance rates, correspondingly.

The test statistic is defined as:

$$LR_{UC} = \left(\frac{\pi_{exp}}{\pi_{obs}}\right)^{n_1} \left(\frac{1-\pi_{exp}}{1-\pi_{obs}}\right)^{n_0}, \tag{31}$$

where n_1 and $n_0 = n - n_1$ are the number of exceedances and non-exceedances.

This test under the null hypothesis follows a chi-squared distribution with one degree of freedom: $-2\ln(L\,R_{UC}) \sim X_1^2$.

5.2. BCP Test

The BCP test is well specified when the exceedances are independent from each other ***(Berkowitz et al., 2011)***, this means that we are unable to predict when the following exceedance will occur by looking at an exceedance that has previously incurred. That is, for all lags, autocorrelation must be 0. The BCP is useful when, for instance, we estimate VaR on a daily basis and have an exceedance during a period where we observe a spike on market volatility. This event might lead to further exceedances over the next trading days, a so-called exceedance clustering, and suggest that the model is not fast enough in adapting to recent volatility increases. The BCP test signals this type of market events.

The formulation of the null and alternative hypotheses for the BCP test is as follows:

$$\begin{gathered} H_0: \hat{\rho}_k = 0, for\ all\ k \in \{1,\ldots,K\} \\ H_1: \exists k \in \{1,\ldots,K\}\ such\ that\ \hat{\rho}_k \neq 0, \end{gathered} \tag{32}$$

where $\hat{\rho}_k$ is the lag k autocorrelation of the series of n observations where each observation is either 1 or 0, specified by the indicator function of the UC test and K is the maximum autocorrelation lag accounted for in the test.

The test statistic is defined as:

$$BCP\ (K) = n(n+2)\sum_{k=1}^{K}\frac{\hat{\rho}_k^2}{n-k}, \tag{33}$$

where n is the sample size of the test.

This test under the null hypothesis follows a chi-squared distribution with K degrees of freedom: $BCP\ (K) \sim X_K^2$. We have the freedom to pick the lag K, but it is essential to understand the implications of opting for a larger or smaller K. A larger K gives information about higher order autocorrelations. Nevertheless, because the test statistic under the null hypothesis follows a chi-squared distribution with K degrees of freedom, an increase in K leads to an upward shift in the critical value, which makes it more difficult to reject the null hypothesis. By contrast, choosing a smaller K increases the sensitivity of the test, but ignores autocorrelations for lags larger than K. Recognizing these tradeoffs, we compute the BCP test for $K = 1$ up to $K = 10$, which allows us to detect autocorrelations up to the 10th lag while preserving an acceptable level of sensitivity.

5.3. Backtest Results

In this section we evaluate the performance of each model for the global period of 11 years from 19 December 2011 and 27 January 2023 and sub-periods when relevant, using the UC and BCP tests described earlier and considering the different characteristics of each model mentioned on Chapter 4. With this, we will conclude which model to use for our current portfolio composition.

For the chosen global period we have $n = 2900$ observations, which means that for a VaR model with a significance level of $\alpha = 1\%$, we expect $2900 \times 1\% = 29$ exceedances. In statistics, normally we reject the null hypothesis when the p-value is below 5%, with that, both the UC and BCP tests are accepted when the null hypothesis is not rejected, that is, when the p-value is above 5%.

Table 4 presents the results of the UC test for each model considered for the global period.

Model	Model ID	Global Period		
		No. of Exceedances	Exc. Rate (%)	*p-value (%)*
Parametric Normal	1	59	2.03%	0.00%
SGSt	2	71	2.45%	0.00%
	3	**33**	**1.14%**	**46.51%**
	4	**33**	**1.14%**	**46.51%**
Historical	**5**	**35**	**1.21%**	**27.81%**
	6	54	1.86%	0.00%
	7	**34**	**1.17%**	**36.37%**
Quantile Regression	**8**	**37**	**1.28%**	**15.22%**
	9	**28**	**0.97%**	**85.11%**
	10	**34**	**1.17%**	**36.37%**

Table 4. UC Test results for each model. The bold models pass the test (p-value > 5%).

Through Table 4, we can, not surprisingly, reject the Parametric Normal model. Its wrong assumption of a normal distribution of returns is the reason for the high number of exceedances. The UC Test also rejects Model 2, which assumes a T-distribution with a rolling window of only 250 observations, and Model 6, which uses the historical empirical distribution with the volatility adjustment with a rolling window of 750 observations.

Looking at the other models, we can see that Model 9 (QR VaR with constant and 1 explanatory variable) clearly outperforms the others, by presenting a similar number of exceedances to the predicted ones and lower than the other models.

We are now going to confirm the choice of model by looking at the results of the BCP test:

Model	Model ID	*p-value (%)*									
		Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
SGSt	3	0.00%	0.00%	0.02%	0.03%	0.06%	0.09%	0.16%	0.28%	0.35%	0.04%
	4	0.00%	0.00%	0.02%	0.04%	0.08%	0.12%	0.21%	0.36%	0.45%	0.06%
Historical	5	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	7	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Quantile Regression	**8**	0.34%	0.68%	1.61%	3.08%	**5.15%**	**5.42%**	**7.94%**	**7.94%**	**7.88%**	0.74%
	9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	10	0.00%	0.01%	0.03%	0.06%	0.12%	0.23%	0.39%	0.64%	0.81%	0.13%

Table 5. BCP Test Results. The bold value has a p-value superior to 5% and indicates which model passes the BCP Test.

By looking at Table 5, we can clearly see that all models performed poorly, except for Model 8 that managed to pass the test from Lag 5 to Lag 9. With this, we feel that, to make the best decision, we need to analyze the results of each sub-period of the UC Test.

Model ID	2023-2022		2022-2021		2021-2020		2020-2019		2019-2018		2018-2017	
	Exc.	*p-value*	Exc.	*p-value*	Exc.	*p-value*	Exc.	*p-value*	Exc.	*p-value*	Exc.	*p-value*
3	0.4%	**25.4%**	1.2%	**80.8%**	1.9%	**18.4%**	1.2%	**80.8%**	1.5%	**41.9%**	0.8%	**69.7%**
4	0.4%	**25.4%**	1.2%	**80.8%**	1.9%	**18.4%**	1.2%	**80.8%**	1.9%	**18.4%**	0.8%	**69.7%**
5	2.7%	2.3%	0.4%	**25.4%**	1.9%	**18.4%**	0.0%	2.2%	2.3%	**7.0%**	0.8%	**69.7%**
7	1.2%	**80.8%**	0.0%	2.2%	3.5%	0.2%	0.4%	**25.4%**	3.1%	0.7%	0.8%	**69.7%**
8	0.8%	**69.7%**	0.4%	**25.4%**	2.3%	**7.0%**	0.4%	**25.4%**	2.3%	**7.0%**	0.4%	**25.4%**
9	0.4%	**25.4%**	0.0%	2.2%	1.5%	**41.9%**	0.8%	**69.7%**	2.7%	2.3%	0.8%	**69.7%**
10	**0.4%**	**25.4%**	**0.8%**	**69.7%**	**1.5%**	**41.9%**	**1.2%**	**80.8%**	**1.5%**	**41.9%**	**0.8%**	**69.7%**

Model ID	2017-2016		2016-2015		2015-2014		2014-2013		2013-2012	
	Exc.	*p-value*	Exc.	*p-value*	Exc.	*p-value*	Exc.	*p-value*	Exc.	*p-value*
3	0.8%	**69.7%**	1.9%	**18.4%**	1.9%	**18.4%**	1.2%	**80.8%**	0.0%	2.2%
4	0.8%	**69.7%**	1.9%	**18.4%**	1.2%	**80.8%**	1.5%	**41.9%**	0.0%	2.2%
5	0.4%	**25.4%**	1.5%	**41.9%**	1.9%	**18.4%**	1.2%	**80.8%**	0.4%	**25.4%**
7	0.8%	**69.7%**	3.1%	0.7%	0.4%	**25.4%**	0.0%	2.2%	0.0%	2.2%
8	0.8%	**69.7%**	2.7%	2.3%	1.2%	**80.8%**	0.8%	**69.7%**	0.0%	2.2%
9	0.8%	**69.7%**	1.9%	**18.4%**	1.5%	**41.9%**	0.4%	**25.4%**	0.0%	2.2%
10	**0.8%**	**69.7%**	**1.9%**	**18.4%**	**1.9%**	**18.4%**	**1.5%**	**41.9%**	**0.8%**	**69.7%**

Table 6. UC test results for each annual sub-period. The bold entries in the dates column indicate the tests that pass that specific sub-period, while the bold entries in the models column indicate the tests that pass all of the sub-periods considered.

As we can clearly see in Table 6, the different models exhibit similar behavior, but there is an outperformance of consistency over the different time periods by Model 10, being the only one that successfully passes the UC test for all periods. Therefore, Model 10 (QR VaR with a rolling sample of 1000 observations, without constant and 1 explanatory variable) is the VaR Model chosen for our portfolio.

On Figure 1 we can observe the daily VaR estimates for model 10 compared to the portfolio's daily P&L for the global period of Backtest.

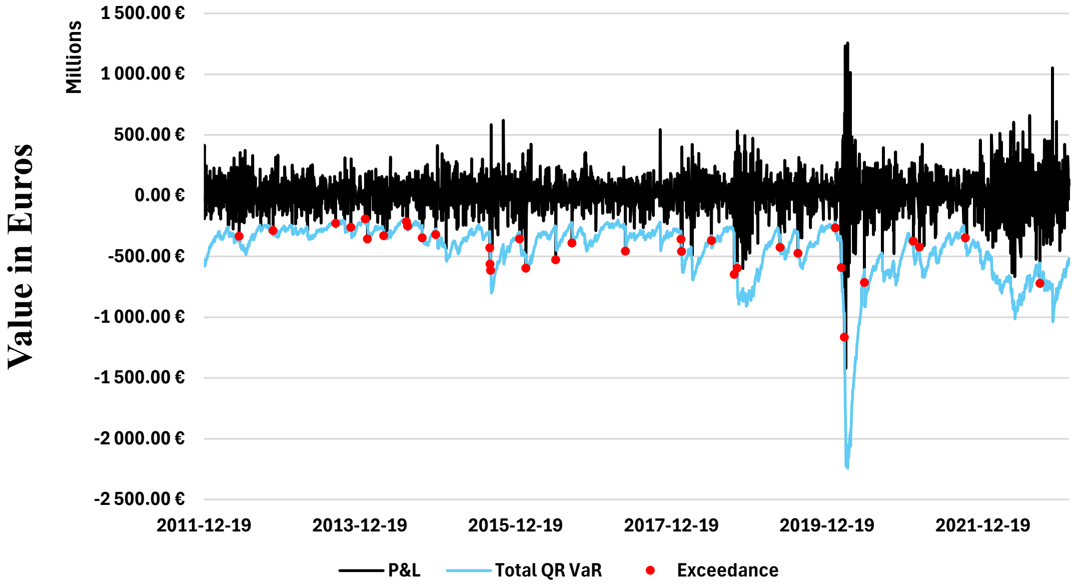

Figure 1. QR VaR Backtesting Performance. The red dots correspond to the exceedances of the model across the global period of backtesting.

We can conclude that overall, the exceedances are decently spread across the observations apart from the last quarter of 2014, the first quarter of 2018 and the third quarter of 2021 where the model underperforms with several exceedances happening within a low number of days. On Table 7 we can get a more detailed overview of the exceedances of the backtesting period.

Date of Exceedance	VaR €	P&L €	Exceedance €	Exceedance (% of VaR)
2022-09-13	-552 002 198 €	-721 367 099 €	-169 364 901 €	30.68%
2021-09-28	-337 288 709 €	-347 881 579 €	-10 592 870 €	3.14%
2021-02-25	-419 414 953 €	-422 738 934 €	-3 323 980 €	0.79%
2021-01-27	-372 848 420 €	-375 612 590 €	-2 764 170 €	0.74%
2020-06-11	-605 966 863 €	-715 587 303 €	-109 620 440 €	18.09%
2020-03-09	-961 959 599 €	-1 165 218 939 €	-203 259 340 €	21.13%
2020-02-24	-379 919 053 €	-593 806 062 €	-213 887 009 €	56.30%
2020-01-27	-216 591 191 €	-265 946 349 €	-49 355 158 €	22.79%
2019-08-05	-316 013 375 €	-474 990 989 €	-158 977 613 €	50.31%
2019-05-13	-322 178 540 €	-425 542 992 €	-103 364 451 €	32.08%
2018-10-24	-554 932 948 €	-597 300 648 €	-42 367 700 €	7.63%
2018-10-10	-306 570 486 €	-649 367 920 €	-342 797 434 €	111.82%
2018-06-25	-328 619 570 €	-369 768 605 €	-41 149 035 €	12.52%
2018-02-05	-397 160 642 €	-458 861 883 €	-61 701 241 €	15.54%
2018-02-02	-292 560 190 €	-358 658 372 €	-66 098 182 €	22.59%
2017-05-17	-251 559 035 €	-457 662 471 €	-206 103 436 €	81.93%
2016-09-09	-218 900 712 €	-390 976 716 €	-172 076 004 €	78.61%
2016-06-24	-251 833 805 €	-529 410 429 €	-277 576 624 €	110.22%
2016-02-05	-485 639 886 €	-597 204 936 €	-111 565 050 €	22.97%
2016-01-07	-296 871 848 €	-356 718 917 €	-59 847 068 €	20.16%
2015-08-24	-515 568 759 €	-615 105 367 €	-99 536 608 €	19.31%
2015-08-21	-387 697 356 €	-565 096 559 €	-177 399 204 €	45.76%
2015-08-20	-291 659 970 €	-429 578 679 €	-137 918 708 €	47.29%
2014-12-10	-272 504 554 €	-322 113 192 €	-49 608 638 €	18.20%
2014-10-07	-243 913 172 €	-347 955 137 €	-104 041 965 €	42.66%
2014-07-31	-238 911 479 €	-249 601 003 €	-10 689 523 €	4.47%
2014-07-25	-201 536 000 €	-214 827 858 €	-13 291 858 €	6.60%
2014-04-10	-286 794 081 €	-330 487 539 €	-43 693 458 €	15.24%
2014-01-24	-218 039 597 €	-357 995 204 €	-139 955 607 €	64.19%
2014-01-13	-172 227 636 €	-193 152 577 €	-20 924 941 €	12.15%
2013-11-07	-248 245 759 €	-260 677 700 €	-12 431 941 €	5.01%
2013-08-27	-224 786 815 €	-226 882 967 €	-2 096 151 €	0.93%
2012-11-07	-283 760 260 €	-287 500 983 €	-3 740 723 €	1.32%
2012-06-01	-330 704 978 €	-335 301 853 €	-4 596 875 €	1.39%

Table 7. QR VaR (Model 10) backtesting period exceedance details.

We can observe that in 2014, 2015 and 2018 several exceedances occur within only a few days of distance, which is captured by the BCP test.

Chapter 6

VALUE-AT-RISK MANAGEMENT

Risk managers define EC as the portion of capital exposed to potential loss based on the portfolio's behavior over a specific time frame and confidence level. Since VaR quantifies this exposure, EC and VaR hold the same numerical value. By setting a target EC for the portfolio, firms can proactively shape their risk management strategies around that benchmark.

Going forward we measure the VaR with model 10 chosen in Chapter 5 that is: QR VaR with a rolling sample of 1000 observations, without constant and 1 explanatory variable (EWMA with $\lambda = 0.94$). Considering the latest 10 observations before 30 January 2023 the VaR estimates were in the range between €500 million and €600 million, so, if we consider a daily EC below €525 million, it corresponds to 3.91% of the initial portfolio value which will be the percentage of VaR of the total portfolio that we will aim for. Additionally, since we actively manage our portfolio, we also impose limits of contribution to the VaR estimate of 12.5% per asset and 10.5% of currency risk.

In order to comply with the EC target we adopt the following procedure: each day, just before markets close, we assess the portfolio's current composition to estimate the next day's VaR. If this estimate exceeds 3.91% of portfolio value, we immediately adjust the portfolio using a predefined strategy to bring the new VaR below the threshold. We repeat this process daily from 30 January 2023 through 2 February 2024, continuously monitoring and modifying the portfolio to maintain risk within the set limit.

When it comes to the individual risk limits, using the Marginal VaR, which is the decomposition of the VaR into individual contributions, we can verify if the violation is from currency risk, if so, we adopt a hedging strategy. If the issue comes from an individual asset, we rebalance our portfolio by selling a percentage of that asset and investing the same amount into another asset. Both strategies decrease the total risk of the portfolio, reducing the VaR estimation.

Section 1 presents the marginal VaR decomposition methodology.

6.1. VaR Decomposition and Management Strategy

We define the gradient vector ∇ to capture how the portfolio's VaR responds to small shifts in each of the n risk factor exposures, starting from their current values, denoted by Θ. The decomposition vector S reflects the portfolio's present exposure to each individual risk factor θ_i tailored to the decomposition method we've chosen. To calculate the first sensitivity in ∇, we introduce a small perturbation ε (e.g. €1). We then slightly adjust only the first risk factor exposure θ_i by ε as follows:

$$\Theta_1 = \begin{bmatrix} \theta_1 + \varepsilon \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}, \tag{34}$$

with this we generate the return time series for the perturbed portfolio Θ_1, which reflects the ε-adjusted exposure to the first risk factor. We then compute the new VaR, denoted VaR_{Θ_1}. The first sensitivity component in the gradient vector ∇ is calculated as: $\frac{VaR_{\Theta_1} - VaR_{\Theta}}{\varepsilon}$. We repeat this procedure for each exposure Θ_i perturbing one factor at a time until we derive the full set of sensitivities across all *n* risk factors.

Finally, we obtain the marginal VaR as:

$$MarginalVaR = S^T \nabla \tag{35}$$

The decomposition done is up to the analyst and in our case, we will first decompose the VaR estimate by asset class.

Description	Asset Class		Total
	Equity	Bonds	
VaR (%)	2.57	0.00	2.57
VaR (EUR)	507 961 442	242 969	508 204 411
VaR Decomposition (%)	99.95	0.05	100.00

Table 8. VaR decomposition by asset class. This table illustrates each asset class contribution to the VaR estimate for 2 February 2024.

We observe that equity exposure is the main contributor to a higher VaR which is in line with the portfolio composition showcased before. With this, and since we have individual limits, we will disaggregate the contribution of each asset for the total VaR and the currency risk factor.

Asset	Contribution (EUR)	Contribution %
MSFT	41 969 154	8.26%
AAPL	27 917 364	5.49%
NVDA	155 784 854	30.65%
AMZN	46 811 015	9.21%
GOOGL	70 115 097	13.80%
UNH	-4 846 388	-0.95%
MA	9 447 789	1.86%
CRM	11 704 732	2.30%
WMT	1 331 650	0.26%
TMO	4 065 095	0.80%
ADBE	18 648 729	3.67%
ORCL	3 711 451	0.73%
CAT	5 919 361	1.16%
SPGI	3 164 310	0.62%
BKNG	13 854 574	2.73%
VRTX	1 693 853	0.33%
CMG	6 590 711	1.30%
ORLY	1 669 366	0.33%
HSY	1 683 113	0.33%
MC.PA	9 099 405	1.79%
ASML	42 456 295	8.35%
CDI.PA	875 730	0.17%
TPL	5 284 686	1.04%
9104.T	477 814	0.09%
NFLX	13 806 109	2.72%
GE	1 776 609	0.35%
MCO	2 391 888	0.47%
VMC	2 006 369	0.39%
SHEL.L	935 930	0.18%
WFC	987 162	0.19%
CNR.TO	1 793 293	0.35%
MCD	1 981 958	0.39%
JPM	2 852 364	0.56%
NL0015001DQ7	-132 804	-0.03%
US91282CJZ59	958 965	0.19%
DE000BU2Z015	-583 192	-0.11%
Currency	47 450 837	9.34%
Total (excl currency)	**508 204 411**	**100%**

Table 9. VaR decomposition by asset and currency risk factor. This table illustrates each individual asset contribution and the currency risk factor contribution to the VaR estimate for 2 February 2024.

Given the above, we can conclude that, although the VaR limit is not exceeded, we have breaches of individual contributions on NVDA and GOOGL. We will tackle this issue by rebalancing the portfolio, that is, by selling a certain number of shares of those companies and reinvesting that amount in different assets. If the currency limit is also breached, we can solve both problems with two trades if we reinvest in an asset quoted in EUR. This way we reduce both the individual exposure to those companies and currency risk.

On the other hand, if we only have a non-compliant exposure to currency risk, we will add short positions on FX futures contracts to hedge and bring both the portfolio risk and currency risk down.

Finally, whenever the VaR estimate for the next trading day is above €525 million (or 3.91% of the total portfolio value) we can use both strategies defined above depending on the size of the exceedance. Both options are managed and adjusted daily so that, if a short position is not needed, we will remove it. We repeat this process daily until 2 February 2024.

6.2. VaR Management

For evaluation reasons, we define the portfolio with the hedging strategy implemented as Hedged Portfolio and the same portfolio without the strategy as Unhedged Portfolio.

Figure 2 below illustrates the daily VaR estimates for both the Unhedged and Hedged portfolios for estimation period.

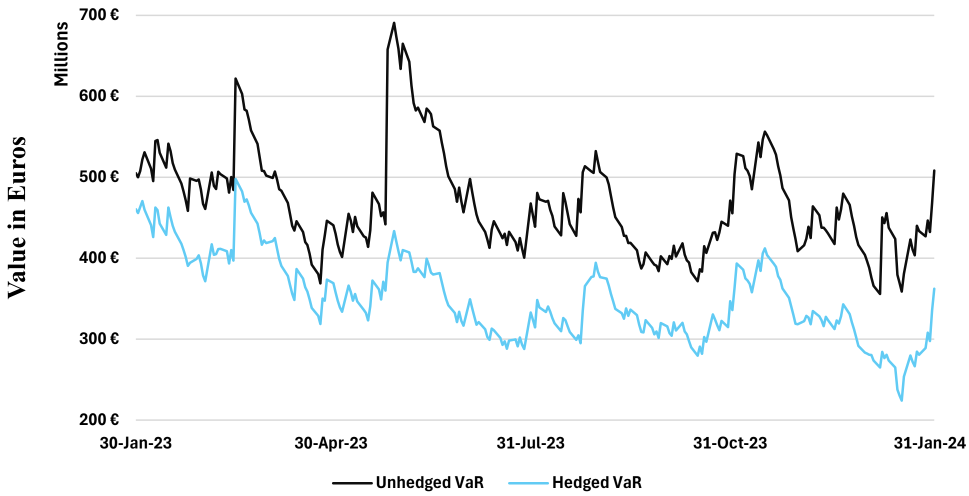

Figure 2. Daily VaR estimates in euros for both portfolios.

In Figure 3 below we can observe the hedging positions we took throughout the year to reduce FX risk.

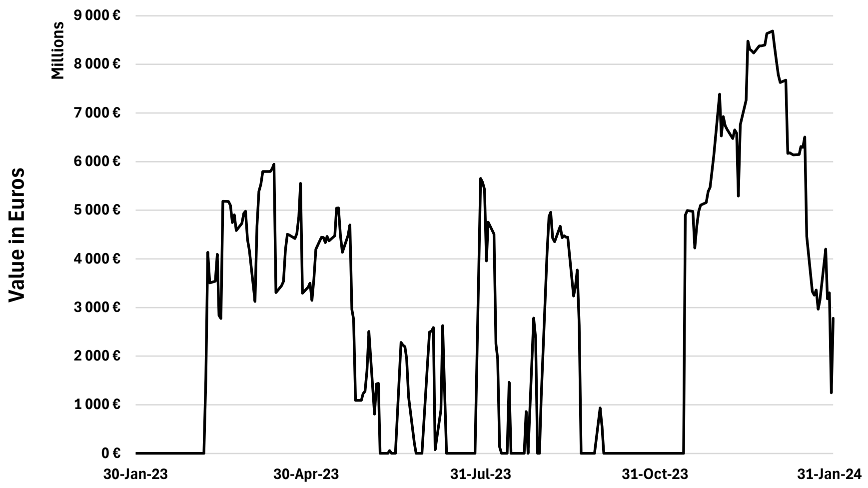

Figure 3. Currency risk hedging positions

Figure 4 below displays the maximum daily Marginal VaR from all risk factors across the estimation period.

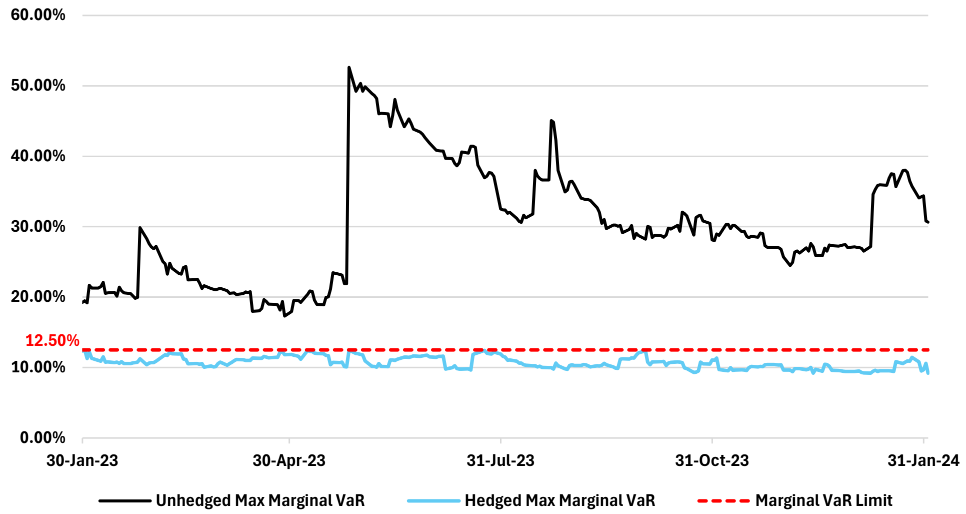

Figure 4. Maximum daily Marginal VaR as a percentage of the Total VaR for both portfolios across the estimation period.

Table 10 illustrates the rebalancing trades of the Hedged portfolio during the period.

Date	Ticker	Trade Signal	No. of Shares	New Positions	Trade Signal	No. of Shares
30-Jan	**NVDA**	Sell	-2 946 932	**ORCL**	Buy	1 300 000
				VRTX		400 000
				GE		1 000 000
				MCD		500 000
1-Feb	**GOOGL**	Sell	-2 054 526	**SHEL.L**	Buy	6 388 334
				WFC		3 217 562
				MCO		451 160
				VMC		627 765
3-Feb	**GOOGL**	Sell	-2 677 117	**MC.PA**	Buy	1 057 308
	MSFT		-1 155 930	**MA**		1 858 339
	AMZN		-2 559 625	**CDI.PA**		364 813
	NVIDIA		-592 566	**WMT**		6 837 054
9-Feb	**GOOGL**	Sell	-2 137 454	**AMZN**	Buy	9 784 360
24-Feb	**NVDA**	Sell	-568 472	**MSFT**	Buy	3 767 964
27-Apr	**AMZN**	Sell	-1 877 204	**ADBE**	Buy	2 000 724
28-Apr	**MSFT**	Sell	-193 594	**VRTX**	Buy	934 128
8-May	**AAPL**	Sell	-1 178 257	**GE**	Buy	5 011 855
26-May	**NVDA**	Sell	-1 082 605	**GOOGL**	Buy	10 007 512
29-May	**ASML**	Sell	-76 859	**ORCL**	Buy	3 255 820
27-Jul	**MSFT**	Sell	-172 178	**AMZN**	Buy	8 275 907
31-Jul	**MSFT**	Sell	-189 597	**CRM**	Buy	1 834 400
7-Aug	**AMZN**	Sell	-1 969 989	**ORCL**	Buy	5 280 653
3-Oct	**GOOGL**	Sell	-1 843 693	**CMG**	Buy	278 689
26-Oct	**GOOGL**	Sell	-1 261 303	**GE**	Buy	6 606 163
25-Jan	**ASML**	Sell	-436 612	**GE**	Buy	9 626 106

Table 10. Rebalancing trades.

We observe from Figures 2 and 3 and from Table 10 that whenever the VaR exceeds the target of €525 million, a short position on the pair EUR/USD is opened or a portfolio rebalancing is done. Due to this combined strategy, the VaR of the Hedged portfolio is kept at a maximum of €525 million and the individual contributions to the VaR inside the pre-established boundaries of 12.5% per stock and 10.5% of currency risk.

From Figure 4 we can conclude that the combined strategy was clearly successful in reducing the individual risk contributions of each risk factor and maintaining the values inside the boundaries.

Figure 5 below presents the daily P&L for both portfolios for the one year period. Figures 6 and 7 illustrate the Unhedged and Hedged VaR (as a loss) performance for the period.

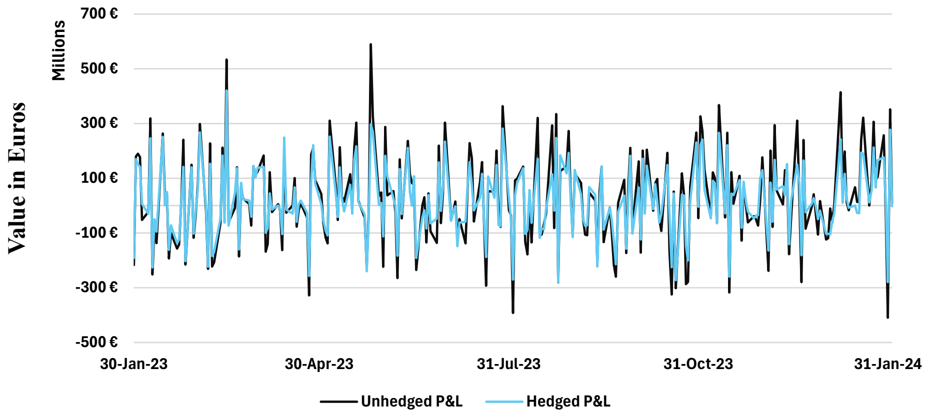

Figure 5. Daily P&L of both portfolios for the one year period.

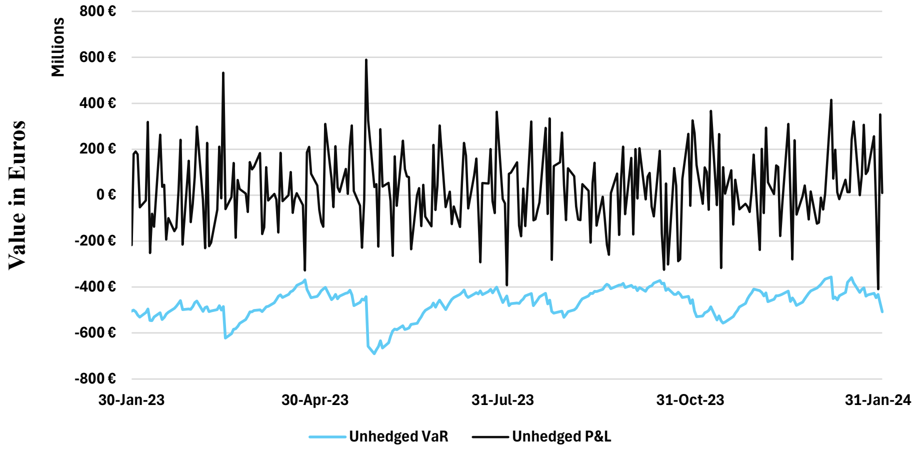

Figure 6. VaR Performance for the Unhedged portfolio.

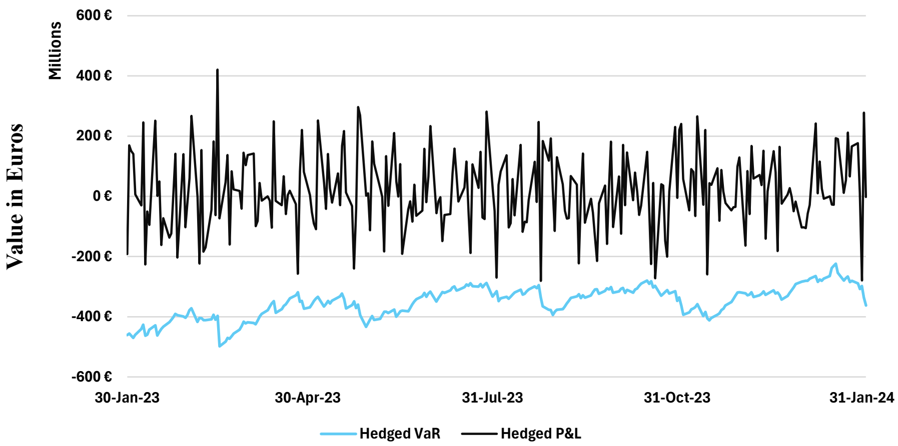

Figure 7. VaR Performance for the Hedged portfolio.

We observe from Figure 5 that the Unhedged portfolio has more volatility on its daily P&Ls than the Hedged Portfolio (see Appendix B for the 22-day rolling volatility process plot). This discrepancy is more obvious for periods with larger short positions or portfolio rebalancing, for instance between April 2023 and July 2023.

From Figures 6 and 7 we can conclude that, for the estimation period, both Unhedged and Hedged VaR models did not have any exceedance, indicating that model was effective in capturing the risk of both portfolios.

Figure 8 below shows the daily cumulative P&L for both portfolios for the one year period.

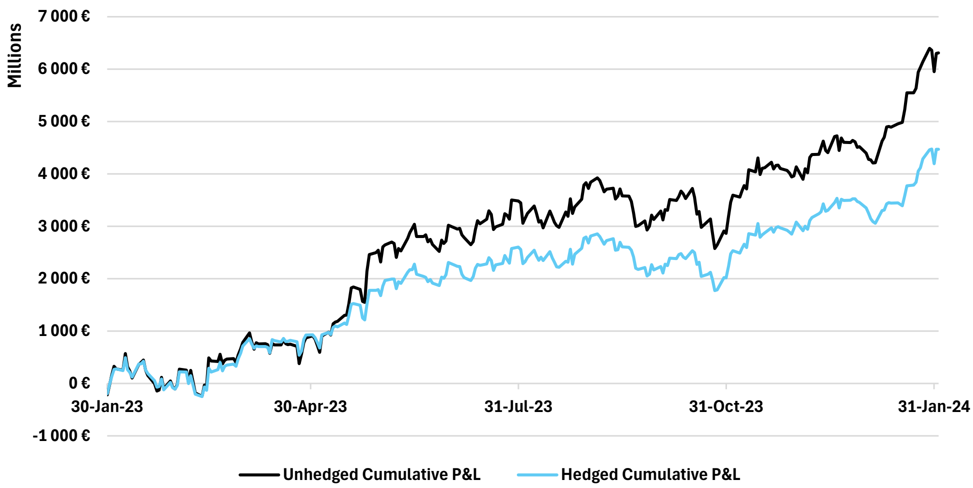

Figure 8. Cumulative P&L of both portfolios.

We can see from Figure 8 that both portfolios return a profit for the year. However, the profit for the Unhedged was higher than the Hedged. This can be explained by the reduction of exposure to certain assets on the Hedged portfolio through the portfolio rebalancing to comply with the individual exposure limits and increase diversification. This indicates that, for the bull market in question, the Unhedged portfolio has a higher profit than the Hedged portfolio, although it has a higher risk profile.

Table 11 below illustrates the returns for both portfolios.

Indicator	Portfolio	
	Unhedged	Hedged
P&L (€M)	6 324 €	3 894 €
Return %	47.94%	29.13%

Table 11. Returns in euros and in % for both portfolios.

We can conclude that the Hedged portfolio underperformed the Unhedged portfolio as previously shown by Table 11. Nevertheless, both annualized volatility and max drawdown (see Appendix B.1.)

illustrate that both portfolios have different risk profiles with the Unhedged portfolio being riskier than the Hedged portfolio, as it should. This states the necessity for a risk-adjusted performance metric to make this comparison fair.

The RORAC ***(Matten, 1996)*** compares the performance with the risk incurred to attain it. We compute the RORAC as:

$$RORAC = \frac{P\&L}{EC}, \tag{36}$$

where P&L is the P&L for the estimation period, in our case, one year, and EC is the sum of the daily EC (measured by the VaR) throughout the year.

Table 12 below shows the RORAC for the estimation year for both Unhedged and Hedged portfolios.

	Portfolio	
	Unhedged	**Hedged**
P&L (€M)	6 324 €	3 894 €
EC(€M)	123 941 €	93 015 €
RORAC %	**5.10%**	**4.19%**

Table 12. RORAC for the one year period.

We can conclude that the Unhedged portfolio, although it has a riskier profile, proved to be more efficient than the Hedged portfolio due to the bullish market conditions during the estimation period, presenting a difference in performance of 0.92% against the latter.

Chapter 7

CONCLUSION

This thesis aimed to actively monitor and control the daily VaR of the portfolio over a one year period - from 30 January 2023 to 2 February 2024. The core objective was to keep VaR within a predetermined limit while also managing the individual risk contributions from each component within the portfolio.

The portfolio studied was composed of part equities and part bonds from U.S., European and Asian markets. To measure the market risk there are several classes of VaR models, so, to define the better suited model to our portfolio, we computed four different models: Parametric Normal VaR, SGSt, Historical and QR. Inside these classes, we calculated 10 different models so that we had a broad range to analyze and choose from. We evaluated each model's performance using a backtesting procedure that incorporated both the UC and BCP tests.

The Backtest analysis using the portfolio composition on 30 January 2023 presented expected results. We computed the VaR at the 1% significance level and the Parametric Normal VaR, which assumes returns follow a normal distribution, failed the Backtest, indicating that the returns of our portfolio were not normally distributed and that the model is unfit. Still in the parametric world, the SGSt VaR model aims to solve the limitations of the Parametric Normal model, passed the Backtest and became one of the possible models to suit our portfolio. Out of the parametric landscape, the Historical VaR models passed the Backtest but did not impress, so they were not considered. Finally, the QR VaR models passed the Backtest and presented solid results for our portfolio and, similarly to the SGSt Models, was considered for our portfolio. After extra analysis, we decided on the QR VaR due to its more flexible adaptation to market conditions, something useful for our rebalancing strategy. With this, we used the QR VaR with a rolling sample of 1000 observations, without constant and 1 explanatory variable model.

To manage the VaR, based on more recent observations, we chose a daily limit of €525 million and individual contributions limits of 12.5% per asset and 10.5% of currency risk. From 30 January 2023, every time the VaR or the assets surpassed those limits, a hedging strategy would be applied to the portfolio.

The first time one of those boundaries was crossed was on the first date, 30 January 2023, where we performed a portfolio rebalancing by selling an individual asset and reinvesting the same amount in other assets to solve that exposure limit, increase diversification and reduce the risk profile of our portfolio. The other hedging strategy used, when needed, was shorting the EUR/USD pair every time the VaR surpassed the €525 million limit. The latter was revised daily and, if not necessary, removed entirely. This methodology was repeated daily for the whole one year period and, for comparison purposes, we defined the portfolio without the hedging strategy as Unhedged portfolio and the one with the strategy as Hedged portfolio.

Results displayed that the hedging strategy was successful in keeping the VaR at a maximum of €525 million and the individual contribution per asset and the currency risk below 12.5% and 10.5% respectively throughout the year. If the hedging strategy was not applied, all the pre-established limits would have been broken several times across the estimation period. Finally, to be able to fairly compare the returns from both portfolios, we used the RORAC risk-adjusted performance metric that allowed us to conclude that while the hedging strategy was able to reduce the risk profile of the portfolio, the Unhedged portfolio was more efficient given the bullish market conditions, generating a difference in performance of 0.92% against the Hedged portfolio.

References

Alexander, C. (2009). *Market risk analysis, value at risk models.* John Wiley & Sons.

Allen, L., Boudoukh, J., & Saunders, A. (2004). *Understanding market. Credit and Operational Risk, The Value at Risk Approach.* John Wiley & Sons.

Bank for International Settlements. (2023). *Basel iii: international regulatory framework for banks.* https://www.bis.org/basel_framework/index.htm.

Barone-Adesi, G., Bourgoin, F., & Giannopoulos, K. (1998). *Don't look back. 11*, 100–103.

Baur, D. G. (2012). Financial contagion and the real economy. *Journal of Banking & Finance*, *36*(10), 2680–2692.

Berkowitz, J., Christoffersen, P., & Pelletier, D. (2011). Evaluating value-at-risk models with desk-level data. *Management Science*, *57*(12), 2213–2227.

Boudoukh, J., Richardson, M., & Whitelaw, R. (1998). *The best of both worlds. 11*(5), 64–67.

Dell'Ariccia, G., Detragiache, E., & Rajan, R. (2008). The real effect of banking crises. *Journal of Financial Intermediation*, *17*(1), 89–112.

Dow, S. C. (1996). Why the banking system should be regulated. *The Economic Journal*, *106*(436), 698–707.

Fama, E. F. (1965). The behavior of stock-market prices. *The Journal of Business*, *38*(1), 34–105.

Hoggarth, G., Reis, R., & Saporta, V. (2002). Costs of banking system instability: some empirical evidence. *Journal of Banking & Finance*, *26*(5), 825–855.

Hull, J., & White, A. (1998). Incorporating volatility updating into the historical simulation method for value-at-risk. *Journal of Risk*, *1*(1), 5–19.

Jorion, P. (2007). Value at risk: the new benchmark for managing financial risk. In *Theoretical Economics Letters* (Vol. 3). McGraw-Hill Companies, Inc.

J.P. Morgan and Reuters. (1996). *Riskmetrics - technical document.* https://www.msci .com/documents/10199/5915b101-4206-4ba0-aee2-3449d5c7e95a.

Koenker, R., & Bassett Jr, G. (1978). Regression quantiles. *Econometrica: Journal of the Econometric Society*, 33–50.

Krause, A. (2003). Exploring the limitations of value at risk: how good is it in practice? *The Journal of Risk Finance*, *4*(2), 19–28.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. *The Journal of Derivatives*, *3*(2).

Lin, C.-H., & Shen, S.-S. (2006). Can the student-t distribution provide accurate value at risk? *The Journal of Risk Finance*, *7*(3), 292–300.

Longley-Cook, A. G. (1998). Risk-adjusted economic value analysis. *North American Actuarial Journal*, *2*(1), 87–98.

Matten, C. (1996). *Managing bank capital: capital allocation and performance measurement*. Wiley.

McDonald, J. B., & Newey, W. K. (1988). Partially adaptive estimation of regression models via the generalized t distribution. *Econometric Theory*, *4*(3), 428–457.

Pafka, S., & Kondor, I. (2001). Evaluating the riskmetrics methodology in measuring volatility and value-at-risk in financial markets. *Physica A: Statistical Mechanics and Its Applications*, *299*(1–2), 305–310.

Peiro, A. (1994). The distribution of stock returns: international evidence. *Applied Financial Economics*, *4*(6), 431–439.

Porteous, B., & Tapadar, P. (2005). *Economic capital and financial risk management for financial services firms and conglomerates*. Springer.

Pritsker, M. (2006). The hidden dangers of historical simulation. *Journal of Banking & Finance*, *30*(2), 561–582.

Shakdwipee, P., & Mehta, M. (2017). From basel i to basel ii to basel iii. *International Journal of New Technology and Research (IJNTR)*, *3*(1), 66–70.
Steen, M., Westgaard, S., & Gjølberg, O. (2015). Commodity value-at-risk modeling: comparing riskmetrics, historic simulation and quantile regression. *Journal of Risk Model Validation*, *9*(2), 49–78.
Theodossiou, P. (1998). Financial data and the skewed generalized t distribution. *Management Science*, *44*(12-part-1), 1650–1661.
Xiao, Z., Guo, H., & Lam, M. S. (2015). Quantile regression and value at risk. In *Handbook of Financial Econometrics and Statistics* (pp. 1143–1167). Springe.

Appendices

Appendix A

Cash Reinvestments

In the table below we present the reinvestments of each bond cash flows that matured across the one year period.

Date	Stock	Number of Shares
16-Jan-23	ASML	7 070
17-Feb-23	UNH	8 916
16-Aug-23	MC.PA	16 295
18-Aug-23	SPGI	11 042

Table 13. Cash reinvestments.

Appendix B

Additional Risk Indicators

Figure 9 shows a 22 day rolling volatility process for both Unhedged and Hedged portfolios across the one year period. This delivers further information on the risk profile of each portfolio and if the hedging strategy successfully reduced the risk of the Hedged portfolio.

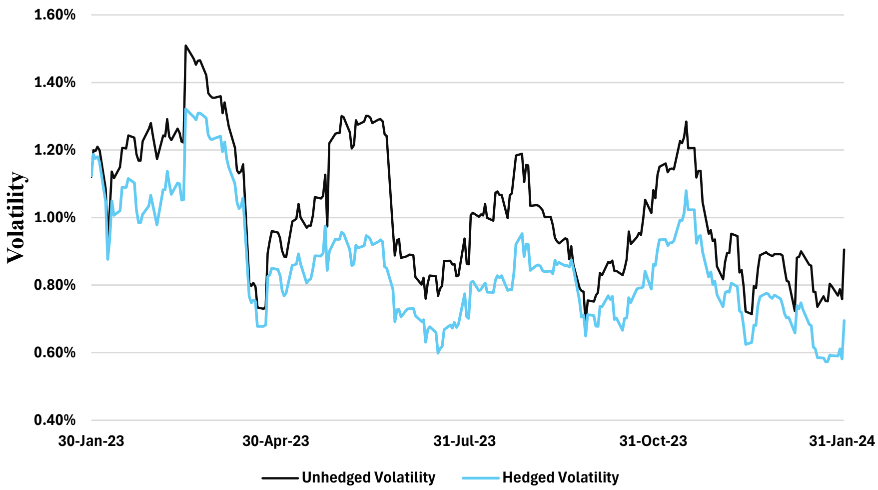

Figure 9. Rolling Volatility process for both portfolios.

B.1. Annualized Volatility and Max Drawdown

With the same purpose, Table 14 presents the annualized volatility and the max drawdown for both portfolios to verify the effectiveness of the hedging strategy and the difference of risk incurred between both portfolios.

Indicator	Portfolio	
	Unhedged	Hedged
Volatility (%)	16.73%	13.91%
Max Drawdown (%)	-7.80%	-6.81%

Table 14. Annualized Volatility and Max Drawdown for both portfolios across the estimation period.

Appendix C

VaR vs Pre-defined Economic Capital Limit

In Figures 10 and 11 we can observe the VaR behaviour for both Unhedged and Hedged portfolios compared to the pre-defined EC. With this we can conclude that the hedging strategy successfully maintains the VaR below the pre-defined EC limit for the Hedged portfolio.

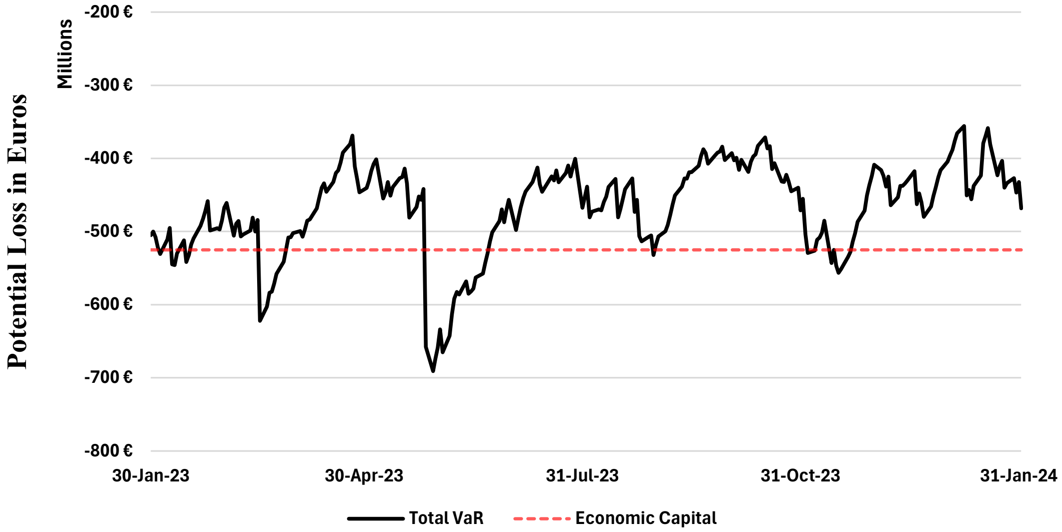

Figure 10. Unhedged Portfolio: Daily Total VaR vs EC Limit.

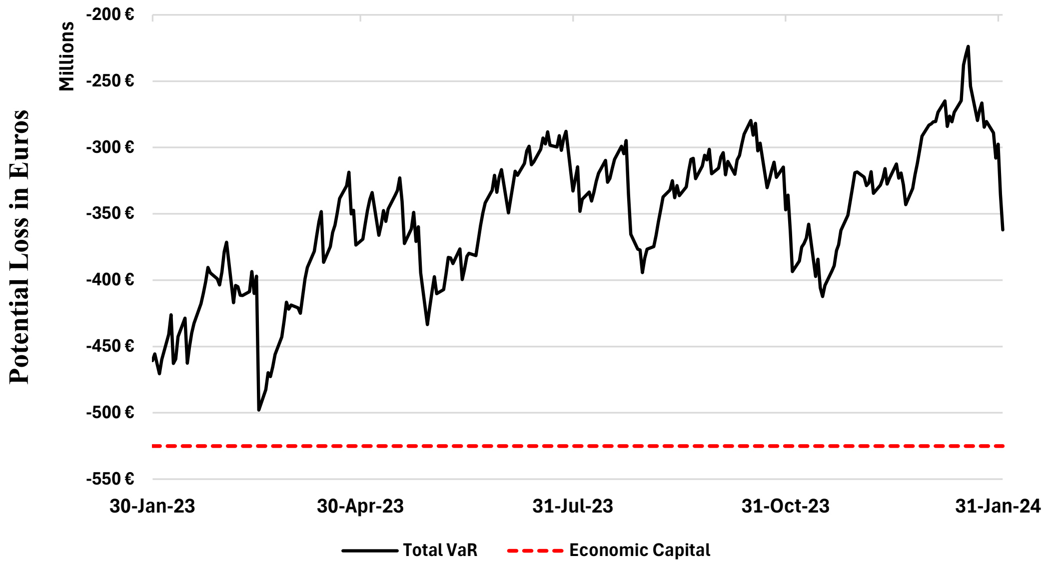

Figure 11. Hedged Portfolio: Daily Total VaR vs EC Limit.