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Portfolio Risk Management through Value-at-Risk: An Empirical Study of Stocks and Bonds

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Master in Finance

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May, 2025

BUSINESS
SCHOOL

Department of Finance

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Acknowledgment

First and foremost, I would like to thank my supervisor, Professor António Barbosa, for his guidance, support, and availability, not only throughout this dissertation, but also throughout my academic journey. His insights have undoubtedly been invaluable over the years, and his mentorship was essential to the completion of this work.

I am also deeply grateful to my colleagues, with whom I had the privilege of sharing this academic path. The insightful conversations, mutual support and the countless hours spent together in the study room, library and even in the courtyard were moments that shaped this experience and made it truly memorable.

Furthermore, I would like to sincerely thank ISCTE for being my home over the past few years and to all those who made it possible. To all the professors I had the honor of learning from, the friends I was lucky to meet, the colleagues I had the privilege of working with, and all the people I had the pleasure of seeing each day – thank you.

Finally, I am profoundly thankful to my family and friends for their love, encouragement, and patience throughout this journey. Their support and strength over the years have been essential in overcoming challenges and reaching this milestone. The beginning, the path, and the conclusion of this academic journey would not have been possible without them – this achievement is, in many ways, theirs as well.

Resumo

O Value-at-Risk (VaR) pode ser definido como a perda potencial máxima de uma carteira durante um período específico, com um determinado nível de confiança, e é considerado, atualmente, como a métrica de medição de risco padrão do sector. Este projeto tem como objetivo medir e gerir o VaR de uma carteira diversificada que inclui ações e obrigações, garantindo que o VaR não o excede. Este trabalho considera quatro modelos de VaR, RiskMetrics (RM), Skewed Generalized Student-t (SGSt), VaR Histórico e VaR de Regressão Quantílica (QR) para determinar a abordagem mais adequada para a gestão do risco. O desempenho de cada modelo de VaR é avaliado por meio de Backtesting para entender qual modelo prevê melhor as perdas potenciais. O modelo com o melhor desempenho no Backtesting é então utilizado para calcular o VaR diário da carteira ao longo de um ano e é aplicada uma estratégia de cobertura para manter o VaR dentro do objetivo definido. Este projeto visa sublinhar a eficiência das estratégias de cobertura, a importância do Backtesting para selecionar o melhor modelo de VaR e a importância de melhorar continuamente as ferramentas de gestão do risco para melhor captar os eventos extremos do mercado.

Palavras chave: Value-at-Risk, Economic Capital, Backtesting, Hedging

Classificação JEL: C10, G32

Abstract

Value-at-Risk (VaR) can be defined as the maximum potential loss of a portfolio over a specific period at a given confidence level, and it is considered, nowadays, as the industry standard risk measurement metric. This project aims to measure and manage the VaR of a diversified portfolio including equities and bonds, ensuring the VaR does not exceed a prespecified target value. This work considers four VaR models, RiskMetrics (RM), Skewed Generalized Student-t (SGSt), Historical VaR, and Quantile Regression (QR) VaR to determine the most suitable approach for managing risk. The performance of each VaR model is assessed through Backtesting to understand which model predicts potential losses better. The model with the best Backtesting performance is then used to calculate the portfolio's VaR daily over one year and a hedging strategy is applied to maintain the VaR within the defined target. This project aims to underscore the efficiency of the hedging strategies, the importance of Backtesting to select the best VaR model, and the importance of continually improving risk management tools to better capture extreme market events.

Keywords: Value-at-Risk, Economic Capital, Backtesting, Hedging

JEL Classification: C10, G32

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List of Abbreviations

BCP – Berkowitz, Christoffersen and Pelletier

CAC 40 – Cotation Assistée en Continu

DAX 40 – Deutscher Aktienindex

EC – Economic Capital

EUR – Euro

EWMA – Exponential Weighted Moving Average

FTSE 100 – Financial Times Stock Exchange 100 Index

JPY – Japanese yen

Nikkei 225 – Nikkei Stock Average

PV – Present Value

PV01 – Present Value of a Basis Point

QR – Quantile Regression

RORAC – Return on Risk-Adjusted Capital

RM – RiskMetrics

S&P500 – Standard & Poor´s 500

SGSt – Skewed Generalized Student-t

UC – Unconditional Coverage

US – United States of America

USD – United States Dollar

VaR – Value-at-Risk

CHAPTER 1

INTRODUCTION

History has repeatedly shown that managing risk is essential, and the markets have made it clear that continuous improvement of these tools is equally crucial. The 2008 Global Financial Crisis was one of the biggest examples, characterized by the almost collapse of the banking system, highlighting the failure of risk management practices at the time. Ten years later, the COVID-19 pandemic has reaffirmed the importance of continuous improvement which is why it is so important to address this topic to ensure that these tools continue to evolve.

Over the last few years, measures of market risk have become almost synonymous with the term Value-at-Risk (VaR), being that, the main topic addressed in this work. In this work, the VaR of a portfolio composed of bonds and equities is measured and managed to ensure it remains within a prespecified target. Therefore, the purpose of this work is to answer the following questions: what is the best model for measuring and managing the VaR of a given portfolio? Are hedging strategies efficient for managing the P&L of that portfolio?

To tackle this issue, it is first necessary to understand which model is most suitable for calculating VaR by Backtesting the models. Thus, this project considers 4 models: RiskMetrics (RM) VaR, Skewed Generalized Student-t (SGSt) VaR, Historical VaR and Quantile Regression (QR) VaR.

To meet the proposed objectives, this work starts by computing the VaR of the portfolio every day for the period between 30 January 2023 and 2 February 2024. Analyzing the time series of VaR estimates it established a VaR target number that cannot be exceeded during the sample period. If it happens, hedging positions will be applied to mitigate potential risks and bring the portfolio´s exposure back within the defined target.

In this sense, this work was organized as follows: Chapter 2 covers the literature review, which provides an overview on the Risk Management subject as well as a theoretical contextualization of the VaR models; Chapter 3 presents the data collected and the portfolio composition; Chapter 4 discusses the methodology as well as the description of the Backtesting procedures; Chapter 5, reveals the results of the Backtest and model selection; Chapter 6 dives into hedging strategies and its impact on portfolio; Chapter 7 summarizes the main results taken from the practical application, and the results of this work.

CHAPTER 2

LITERATURE REVIEW

Over the past few years, risk management has undergone a real transformation to keep up with developments in the financial markets. Risk concerns arise with pressure from regulators to control financial risks better, the globalization of financial markets, and technological advances (Jorian, 2007).

The necessity for effective risk management tools has been recognized for years, as demonstrated by significant historical events so understanding the nature of risk is critical in this dynamic and complex environment. According to Jorian (2007), risk is the volatility of unexpected outcomes and can be divided into three categories: operational, credit, and market risks. This work aims to deal with market risk which can be seen as a measure of uncertainty in the future value of a portfolio resulting from unexpected and adverse movements in the prices of financial assets (Alexander, 2005).

The question of how it is possible to manage these risks inevitably arises. The Basel Committee on Banking Supervision aimed to address this question by formulating supervisory standards, guidelines and recommending a best practice statement. Consequently, in 1988, The Basel I Accord emerged as the first international regulatory framework to ensure that banks establish a minimum capital requirement to cover the risks associated with their assets (Shakdwipee and Mehta, 2017).

During the mid-1990s and given the limitations of the simplistic system provided by Basel I, the concept of VaR gained prominence by being introduced by J.P. Morgan and was established as the standard for market risk measurement in Basel II, in 2004 (Shakdwipee and Mehta, 2017). VaR is then, the maximum potential loss of a portfolio over a specific period at a given confidence level. It quantifies the worst expected loss with a certain probability, indicating how much can be lost with x% certainty over the set horizon (J.P. Morgan/Reuters,1996).

Based on the VaR approach, J.P. Morgan developed a model called RiskMetrics which provided a standardized methodology for measuring market risk. The model assumes that the returns on financial assets follow a normal distribution over time, which simplifies the calculation of VaR by using historical market data, of the compound returns of financial instruments such as fixed income instruments, foreign exchange transactions, equity instruments, and commodities positions (J.P. Morgan/Reuters,1996).

Although the model has become an industry standard, its assumption that returns follow a normal distribution has been criticized since it simplifies the complex nature of financial markets into a more manageable framework. This assumption poses a limitation in that it tends to underestimate the risk associated with extreme events indicated by the “fat tails”. Fat tails represent the higher likelihood of extreme events that can lead to substantial financial losses, represented by the left fat tail of the distribution. Financial returns display distributions where significant price fluctuations such as market crashes or booms, happen more often than predicted by a normal distribution (Kondor and Pafkaa, 2001).

For example, at a 95% confidence level, or equivalently, 5% significance level, the model often appears effective because the 5% quantile of many fat-tailed distributions happens to align with that of the normal distribution. However, the model's success is misleading because as the confidence level increases to 99%, that is, a 1% significance level, which are commonly required by regulatory bodies, the model starts to show its limitations. When extreme events happen more often than expected, the RiskMetrics model may fail to capture the true extent of potential losses, leading to insufficient capital reserves during financial crises (Kondor and Pafkaa, 2001). This limitation led to the research and development of more sophisticated models.

To better capture these risks, researchers have tried to overcome these limitations by coming up with new models that can solve the problem of the non-normality of returns. A good example of this is the Skewed Generalized Student-t (SGSt) distribution proposed by Theodossius (1998).

The SGSt distribution is a variation of the student-t distribution developed by McDonald and Newey (1988) that aims to incorporate parameters that control skewness (asymmetry) and excess kurtosis (heavy tails). While the standard Student-t distribution is symmetrical around the mean and has heavy tails, even if it gives greater weight to extreme events than the normal distribution, the SGSt distribution has parameters capable of controlling the degree of asymmetry and the shape of the tails, which can be skewed to the left or right depending on the nature of the data (Theodossiou, 1998).

The key characteristics of this model are the flexibility in tail behavior which makes it capable of modeling a wider range of data, the shape parameters that allow capturing the empirical characteristics of financial returns, and the generalization. The last one implies that the SGSt distribution includes the Gaussian and student's t-distribution within a single framework for modeling under various conditions (Theodossius, 1998).

However, it is a model with some limitations because of its complexity and potential overfitting. The additional parameters make the model more complex to estimate and implement. Also, there is a risk of overfitting the model to historical data, which can reduce its predictive power for future risks.

Another method is the Historical VaR whose main characteristic remains the fact that it leverages the historical returns of the portfolio using not the parametric distributions but the empirical distribution of the returns. This model uses past returns to estimate potential future losses, offering a distribution-free approach that can capture more realistic market conditions, especially during periods of stress that may not be well modeled by traditional statistical assumptions (Vasileiou, 2017).

However, a disadvantage of this model is its reliance on the assumption that returns will behavior always behave the same way, assigning an equal probability weight to each day's returns which may not hold during periods of market stress. This limitation can lead to inaccurate risk predictions, particularly when extreme market conditions arise since volatility is time-varying and periods of high and low volatility cluster together (Bollerslev, 1986).

To tackle this issue, a series of papers from Barone-Adesi et al (1998) and John Hull and White (1998), proposed a procedure for using a new volatility approach in conjunction with historical simulation when computing VaR. The first one proposed a method assigning greater weight to more recent observations since more recent observations are considered more representative of market behavior (Vasileiou, 2017). The second one presented an approach in which past returns are modified to reflect current market volatility levels, that is, the magnitude of past returns is adjusted based on the volatility conditions at the time, so they are aligned with present market volatility (Vasileiou, 2017).

A more robust alternative for the models is the Quantile Regression VaR introduced by Koenker and Bassett (1978) as a method for estimating conditional quantiles of a response variable which can be adapted to estimate specific percentiles of portfolio returns. This model lies in its flexibility in choosing the explanatory variables for the conditional volatility (Steen et al. 2015). This model first applies an exponentially weighted moving average (EWMA) to estimate volatility, like the approach used in RiskMetrics but then uses this volatility measure as an input to a linear quantile regression (QR) model (Steen et al. 2015).

It is important to mention that one of the key points in defining the model is volatility, so the concept of Exponentially Weighted Moving Average volatility will be introduced. Since its launch by RiskMetric from J.P. Morgan and Reuters in 1996, the EWMA forecasting approach has gained importance and become a vital instrument for risk assessment and portfolio management. Based on the VaR concept, RiskMetric was one of the first widely adopted applications to utilize the EWMA model for forecasting variance and covariance estimators. The EWMA model estimates the conditional variance of future price changes as well as the conditional covariance of spot and futures price movements using an exponentially weighted average, in contrast to other approaches that use a simple average.

The EWMA approach offers some advantages over simpler models, as it ensures that estimates are more sensitive to recent events than to older observations, which gives it a very important feature, especially during periods of significant price fluctuations.

Finally, once the portfolio's risk is being managed, the portfolio's risk profile will also change, meaning the risk and potential returns are different after the changes compared to before. Risk-Adjusted Capital (RORAC) is than a key performance measure that evaluates the profitability of a bank relative to the risk it undertakes, determining how effectively financial institutions can manage their capital allocation. By integrating VaR into capital management, banks can optimize their risk-adjusted returns, ensuring they allocate capital where it can generate the highest returns relative to the risks involved (Kang and Poshakwale, 2019).

CHAPTER 3

DATA AND PORTFOLIO COMPOSITION

To begin this project, it was necessary to make a detailed and careful selection of the data to be worked on to create a complete, reliable, and diversified portfolio. Thus, the portfolio presented in this work is composed of part equities and part bonds downloaded from Yahoo Finance (https://finance.yahoo.com/) and Investing.com (https://www.investing.com/) for stocks and Borse Frankfurt (https://www.boerse-frankfurt.de/bonds) for bonds.

The equity part of the portfolio is composed of thirty positions, long and short, in stocks from the world's largest indices: S&P500, CAC 40, Nikkei 225, DAX 40, and FTSE 100, representing 30% of the portfolio. The part composed of bonds represents a total of 70% of the portfolio and comprises 3 bonds issued by the US government and 3 European bonds issued by the governments of Luxembourg, Germany and the Netherlands.

The portfolio has a total value of 14 846 573.86 € on 30 January 2023 and demonstrates a well-diversified structure across various asset classes and sectors. With a 30% allocation to stocks and a 70% allocation to bonds, it balances growth potential with risk mitigation. The stock investments are further diversified across different sectors, including Communication Services (14.55%), Consumer Cyclical (15.89%), Healthcare (21.54%), Financial Services (12.66%), Consumer Defensive (16.54), Industrials (7.72%), Technology (7.45%) and Energy (3.65%) and across various markets (see appendix B).

For this work, data was collected on a daily basis. The daily share prices for each stock in the portfolio were downloaded from Yahoo Finance to capture current market information. Furthermore, daily exchange rates for USD/EUR, GBP/EUR, and JPY/EUR, also obtained from Yahoo Finance, were included to account for currency exchange fluctuations as well as for changes in the share price of stocks. In addition, daily interest rates for bonds were analyzed to provide a complete valuation of bond performance. Specifically, EUR interest rates were obtained from the European Central Bank (https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html), while USD daily interest rates were sourced from the Federal Reserve (https://www.federalreserve.gov/releases/h15/).

The sample period spans approximately 17 years, ranging from 4 January 2007 to 2 February 2024, which provides freedom to employ larger sample sizes in VaR models that enhance the overall reliability and robustness of the analysis.

Tables 3.1 to 3.3 below present all the components and details of the portfolio studied in this work.

Stock	Ticker	Currency	Quantity	Share Price	Value (EUR)	Allocation (%)
Alphabet Inc.	GOOGL	USD	1 900	99.37	173 302.28	3.84
McDonald´s Corporation	MCD	USD	1 000	263.21	241 602.27	5.35
Booking Holdings Inc.	BKNG	USD	-65	2 458.22	-146 665.96	-3.25
Johnson & Johnson	JNJ	USD	1 200	160.75	177 058.12	3.92
JPMorgan Chase & Co.	JPM	USD	1 650	135.63	205 420.11	4.55
The Protector & Gamble	PG	USD	1 450	136.20	181 282.46	4.02
L'Oréal S.A.	OR.PA	EUR	600	367.55	220 527.63	4.89
Airbus SE	AIR.PA	EUR	1 500	114.28	171 420.41	3.80
BNP Paribas SA	BNP.PA	EUR	3 250	54.33	176 568.91	3.91
Sanofi	SAN.PA	EUR	2 100	82.89	174 063.01	3.86
LVMH Moet Hennessy	MC.PA	EUR	-150	788.53	-118 279.79	-2.62
Michelin Société	ML.PA	EUR	5 800	27.65	160 341.72	3.55
SAP SE	SAP.DE	EUR	1 700	101.95	173 315.37	3.84
Siemens Aktiengesellschaft	SIE.DE	EUR	1 300	136.09	176 914.26	3.92
Bayer Aktiengesellschaft	BAYN.DE	EUR	3 250	53.57	174 113.72	3.86
Mercedes-Benz Group AG	MBG.DE	EUR	2 900	58.67	170 147.14	3.77
Deutsche Telekom AG	DTE.DE	EUR	8 200	19.01	155 866.16	3.45
Allianz SE	ALV.DE	EUR	950	199.52	189 547.02	4.20
Tesco PLC	TSCO.L	GBP	650	231.04	171 125.98	3.79
Vodafone Group	VOD.L	GBP	1 750	82.77	165 062.96	3.66
AstraZeneca PLC	AZN.L	GBP	25	1 0243.72	291 820.55	6.47
Shell PLC	SHEL.L	GBP	65	2 226.28	164 896.12	3.65
Unilever PLC	ULVR.L	GBP	40	3 810.10	173 665.93	3.85
Persimmon PLC	PSN.L	GBP	125	1 349.74	192 255.80	4.26
Sony Group Corporation	6758.T	JPY	2 000	11 512.79	162 744.83	3.61
Nissan Motor Co., Ltd.	7201.T	JPY	60 000	433.31	183 756.02	4.07
Nikon Corporation	7731.T	JPY	-15 000	1 200.22	-127 247.13	-2.82
Takeda Pharmaceutical	4502.T	JPY	5 650	3 884.68	155 131.45	3.44
SoftBank Group Corp.	9984.T	JPY	3 750	6 128.69	162 440.98	3.60
Subaru Corporation	7270.T	JPY	11 000	2 073.03	161 173.92	3.57
Total					**4 513 372.24**	**100.00**

Table 3.1. Stocks characteristics. This table showcases the stocks that comprise the portfolio studied in this work, as well as the amount invested in each one, converted into EUR. The USD/EUR, GBP/EUR, and JPY/EUR exchange rates at 30 January 2023 are 0.9179, 1.1395 and 0.0071, respectively.

Bond	Currency	Maturity	Coupon Rate	Coupon/ year	Face Value (EUR)	Fair Value (EUR)	Allocation (%)
NL0015001DQ7	EUR	2030-01-15	2.50%	1	1 400 000.00€	1 419 224.23	13.73
DE000BU2Z015	EUR	2033-08-15	2.60%	1	2 500 000.00€	2 593 561.64	25.10
LU1556942974	EUR	2027-02-01	0.63%	1	1 300 000.00€	1 223 984.97	11.85
US91282CJD48	USD	2025-10-31	5.47%	4	2 000 000.00€	1 933 067.34	18.71
US91282CJG78	USD	2030-10-31	4.88%	2	1 800 000.00€	1 817 310.71	17.59
US9128286A35	USD	2026-01-31	2.63%	2	1 500 000.00€	1 346 052.73	13.03
Total						**10 333 201.61**	**100.00**

Table 3.2. Bonds characteristics. This table showcases the characteristics of the bonds that comprise the portfolio studied in this work, as well as the amount invested in each one, converted into EUR. The Fair value of the bond is the sum of the PV of all its future cash flows discounted to 30 January 2023 and convert to EUR where appropriate. The Exchange rate at 30 January 2023 is 0.9179.

Portfolio value		
	Value (EUR)	**Value (%)**
Stocks	4 513 372.24 €	30%
Bonds	10 333 201.61 €	70%
Total	**14 846 573.85 €**	**100%**

Table 3.3. Portfolio value. This table showcases the total value of the portfolio at 30 January 2023 as well as the amount allocated to equity and bonds.

Also, in order to have a more comprehensive view of the portfolio's performance over the years, Table 3.4 below presents the descriptive statistics of both P&L and returns over the entire Backtest period, from 11 February 2013 to 27 January 2023. For a more detailed analysis of the behavior of the portfolio's returns over the years, Appendix A presents the descriptive statistics of the returns for the annual sub-periods of the Backtesting.

	Mean	Median	StdDev	Min	Max	Skew	Kurt
P&L	2 466.42	3 911.42	55 079.65	-429 498.62	335 027.43	-0.2910	7.7561
Returns	0.0166%	0.0263%	0.3710%	-2.8929%	2.2566%	-0.2910	7.7561

Table 3.4. Portfolio P&L and returns descriptive statistics over the global Backtest period. This table showcases the descriptive statistics of both P&L and returns over the entire Backtest period, from 11 February 2013 to 27 January 2023.

CHAPTER 4

METHODOLODY

To address the proposed objective, it is necessary to take the steps that will be covered in this chapter. It begins with the risk factor mapping, followed by the application of the Exponentially Weighted Moving Average model to estimate volatility. Subsequently, the chapter outlines the VaR models studied and concludes with a detailed Backtesting procedure to assess their performance. Therefore, this chapter will cover all these necessary processes required to ensure that everything is prepared for effective VaR management.

4.1. Risk Factor Mapping

Risk factor exposure mapping is the process of identifying, quantifying, and analyzing the various risk factors that influence the value of a portfolio. This consists of choosing a set of risk factors that reflect the main sources of risk to which a portfolio is exposed and then mapping each portfolio position to an equivalent exposure to those factors, in terms of risk.

The portfolio's overall VaR is defined as a function of the vector of risk factor loadings $\Theta = [\theta_1, \theta_2, \ldots \theta_n]^T$, that is, the vector of exposures to each of the *n* risk factors affecting a portfolio. Thus, the Total portfolio VaR is denoted as:

$$\text{VaR} = f(\Theta) \quad (1)$$

In the following sections, the different exposure mapping methodologies for each risk factor type present in this portfolio will be presented: equity risk, interest rate risk and currency risk.

4.1.1. Equity Risk

For each stock in the portfolio, the risk factor loading is computed based on the number of shares held and its current price, allowing for an assessment of the investment amount in that stock. If the stock is priced in a foreign currency, converting to the base currency (EUR) can be done by multiplying the investment's foreign currency value by the current exchange rate. The exposure to price changes for each stock can therefore be calculated as follows:

$$\theta_i = N_i \text{ x } P_i \text{ x } FX_i \quad (2)$$

where N_i is the number of shares held, P_i is the current market price at 30 January 2023 and FX_i is the exchange rate also at 30 January 2023.

4.1.2. Interest Rate Risk

The value of a bond fluctuates primarily due to changes in interest rates, which is the main source of risk for bonds. The discount rate applied to these cash flows changes when interest rates change, causing bond price fluctuations.

This work will apply the PV01 (Present Value of 1 Basis Point) approach to quantify this sensitivity. This approach measures the change in the present value of a bond when the yield curve shifts by one basis point.

To calculate the PV01 for a single cash flow, CF_T, that occurs at time T, it is first determined its present value, which is given by

$$\text{PV}\,(CF_T, R_T) = CF_T e^{-R_T T} \quad (3)$$

where R_T is the continuous compounding interest rate for the maturity of the cash flow.

The PV01 can be approximated using a first-order Taylor expansion as:

$$\begin{aligned} \text{PV01}\,(CF_T, R_T) &\approx \frac{\partial \text{PV}(CF_T, R_T)}{\partial r_T} \times (-0.01\%) \\ &= T \times PV\,(CF_T, , R_T\,) \times 0.01\% \end{aligned} \quad (4)$$

Thus, the change in the present value of the cash flow, which corresponds to its Profit and Loss (P&L), can be defined as:

$$\begin{aligned} \Delta\text{PV}\,(CF_T, R_T) &\approx \frac{\partial \text{PV}(CF_T, R_T)}{\partial R_T} \Delta R_T \\ &\approx -\text{PV01}\,(CF_T, R_T) \times \frac{\Delta R_T}{0{,}01\%} \\ &= -\text{PV01}\,(CF_T, R_T) \times \Delta R_T\,(b.p) \end{aligned} \quad (5)$$

where $\frac{\Delta r_T}{0{,}01\%}$ is the absolute change in interest rate converted to basis points. In the case of a bond with more than one cash flow, its P&L is the sum of the P&Ls of each cash flow. When dealing with a portfolio of bonds, the total PV01 is the sum of the PV01 values of all their cash flows, that is,

$$Portfolio\ PV(CF_T, R_T) = \sum_{i=1}^{n} \Delta\text{PV}\ (CF_{T_i}, R_{T_i}) \tag{6}$$

As mentioned above, in a portfolio context, some complications arise when calculating interest rate risk, as the portfolio may have a large number of bonds and these bonds may have many cash flows. Therefore, obtaining historical interest rate time series for all the cash flow maturities in a portfolio becomes infeasible.

To address this complexity, Alexander (2008) proposed the PV+PV01 invariant mapping approach used in this project, where the cash flows will be mapped to a set of standard maturities called vertices, for which there are data. Considering a single cash flow with a present value computed using the Equation 7, if the maturity *T* of this cash flow lies between two adjacent standard vertices, T_1 and T_2 ($T_1 < T < T_2$), the present value is redistributed between these vertices, in such a way that satisfies two key conditions:

The first condition is the present value invariance, which ensures that the total present value of the mapped cash-flow matches the present value of the original cash-flow:

$$x_{T_1} + x_{T_2} = 1 \tag{7}$$

where x_{T_i} denotes the proportion of the present value of the cash flow that is mapped into the vertex maturity T_i.

The second condition to be satisfied is the PV01 invariance. In a PV01 invariant map, the PV01 of the mapped cash-flows equals PV01 of the original cash-flow, that is,

$$\text{PV01}_{T_1} + \text{PV01}_{T_2} = \text{PV01}_T \tag{8}$$

which ensures that the P&L of the set of mapped CFs is the same as that of the original CF following a parallel shift in the yield curve.

Thus, the PV01 preservation equation can be defined as:

$$x_{T_1} T_1 + x_{T_2} T_2 = T \tag{9}$$

Joining both Equations 7 and 9, the equation to calculate the mapping weights x_{T_1} and x_{T_2} is:

$$\begin{cases} x_{T_1} = \dfrac{T_2 - T}{T_2 - T_1} \\ x_{T_2} = 1 - x_{T_1} \end{cases} \tag{10}$$

Once all cash flows in the portfolio are mapped to the chosen vertices, the PV01 for each vertex is calculated through Equation 4. Aggregating these values across all cash flows in the portfolio gives the PV01 for each vertex, which can then be used to construct the risk factor loadings vector for the portfolio.

Since all the risk factors need to be quantified in EUR, the last step is to convert the −PV01 exposures to EUR using the exchange rate at 30 January 2023.

4.1.3. Currency Risk

To compute the risk factor loading for USD/EUR, GBP/EUR, or JPY/EUR returns, it is essential to first identify the source of this risk, whether from stocks, bonds, or both. Next, the investment amount in each asset denominated in a foreign currency is multiplied by the current exchange rate at 30 January 2023.

The currency risk can therefore be calculated as follows:

$$\theta_i = V_i \text{ x } FX_i \tag{11}$$

where V_i is the amount invested in assets denominated in that foreign currency.

4.1.4. Portfolio Exposures

The portfolio exposure table below results from the methodology explained in subsections 4.1.1. to 4.1.3.

Stocks		Bonds		Currencies	
Risk Factor	Exposure (EUR)	Risk Factor	Exposure (EUR)	Risk Factor	Exposure (EUR)
GOOGL	173 302.28	EUR3M	-0.39	USDEUR	5 928 430.06
MCD	241 602.27	EUR6M	-2.62	GBPEUR	1 158 827.33
BKNG	-146 665.96	EUR1Y	-7.52	JPYEUR	698 000.07
JNJ	177 058.12	EUR2Y	-20.59		
JPM	205 420.11	EUR3Y	-220.28		
PG	181 282.46	EUR5Y	-401.31		
OR.PA	220 527.63	EUR7Y	-950.61		
AIR.PA	171 420.41	EUR10Y	-1 874.71		
BNP.PA	176 568.91	EUR15Y	-331.19		
SAN.PA	174 063.01	EUR20Y	0.00		
MC.PA	-118 279.79	USD3M	-3.67		
ML.PA	160 341.72	USD6M	-1.63		
SAP.DE	173 315.37	USD1Y	-15.45		
SIE.DE	176 914.26	USD2Y	-119.42		
BAYN.DE	174 113.72	USD3Y	-791.26		
MBG.DE	170 147.14	USD5Y	-70.97		
DTE.DE	155 866.16	USD7Y	-739.86		
ALV.DE	189 547.02	USD10Y	-329.02		
TSCO.L	171 125.98				
VOD.L	165 062.96				
AZN.L	291 820.55				
SHEL.L	164 896.12				
ULVR.L	173 665.93				
PSN.L	192 255.80				
6758.T	162 744.83				
7201.T	183 756.02				
7731.T	-127 247.13				
4502.T	155 131.45				
9984.T	162 440.98				
7270.T	161 173.92				

Table 4.1. Portfolio exposures. This table showcases the portfolio exposures, in EUR, at 30 January 2023.

4.2. Exponentially Weighted Moving Average

In the EWMA framework, the one-day forecast equations assume that the mean of daily returns is zero and incorporate a decay factor, λ, which plays a crucial role in assigning weights to the most recent observations while also accounting for the extent of historical data used in volatility estimation. This decay factor can vary between 0 and 1, ensuring that recent price movements have a more significant impact on the volatility calculations, the lower the λ, the more weight is attributed to recent observations

From the historical daily returns, the EWMA variance, can be computed as:

$$\hat{\sigma}_t^2 = (1\text{-}\lambda)\, r_{t-1}^2 + \lambda \hat{\sigma}_{t-1}^2 \qquad (12)$$

where $\hat{\sigma}_t^2$ is the variance estimated for day t on day t – 1, r_{t-1}^2 is the return observed on day t − 1 and λ is the smoothing factor.

4.3. Value-at-Risk Models

Value-at-Risk can be defined as the maximum potential loss of a portfolio during a given period (h) at a given confidence level (α), if the portfolio's composition remains unchanged during this period. Statistically, the 100α% h-day Value-at-Risk at time t, $VaR_{h,\alpha}$, is minus the α quantile of the h-day discounted P&L distribution, that is:

$$P\,(X_h < -VaR_{h,\alpha}) = \alpha \qquad (13)$$

where X_h represents a continuous random variable representing h-day portfolio returns.

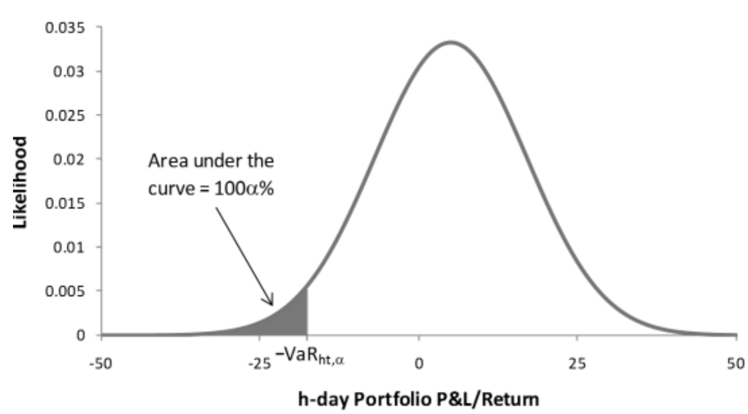

Figure 4.1. VaR illustration. The figure represents the 100α% h-day Value-at-Risk at time t.

Throughout the project, the significance level adopted was α = 1%, or equivalently, a confidence level $(1 - \alpha)$ of 99% and a future time horizon of 1 day (h = 1).

Typically, VaR is reported as a positive number even though it represents potential losses which are, by nature, negative. By convention, it is assumed losses to be negative values, but reporting VaR as a positive figure makes the magnitude of potential losses clearer for interpretation.

4.3.1. RiskMetrics VaR

The RiskMetrics VaR approach calculates the maximum expected loss over a specified time horizon at a given confidence level, assuming normal distribution of portfolio returns, that is, $X_h \sim N(\mu_h, \sigma_h^2)$, where μ_h and σ_h^2 represent the mean and variance estimates, respectively.

In that case $VaR_{h,\alpha}$ can be computed as:

$$VaR_{h,\alpha} = \Phi^{-1}(1-\alpha)\sigma_h - \mu_h \quad (14)$$

with Φ^{-1} denoting the α quantile of the standard normal distribution also known as the inverse cumulative distribution function (CDF) of the standard normal distribution.

When the time horizon is short, the expected return is negligible compared to the volatility, and thus the expected return can be set to zero with minimal impact in the VaR estimation (Alexander, 2008). Since this work is estimating 1-day VaR, the simplified expression becomes:

$$VaR_{h,\alpha} = \Phi^{-1}(1-\alpha)\sigma_h \quad (15)$$

and the σ parameter will be estimated using EWMA volatility model.

4.3.2. Skewed Generalized Student-t VaR

The Skewed Generalized Student-t distribution is an extension of the classical Student t distribution, and it is designed to capture the characteristics of asset return distributions, which typically exhibit negative skewness and excess kurtosis (fat tails), as can be seen in the figure below. These features imply that the probability of extreme negative returns is higher than predicted by normal distributions, making traditional VaR calculations inadequate for capturing potential risks.

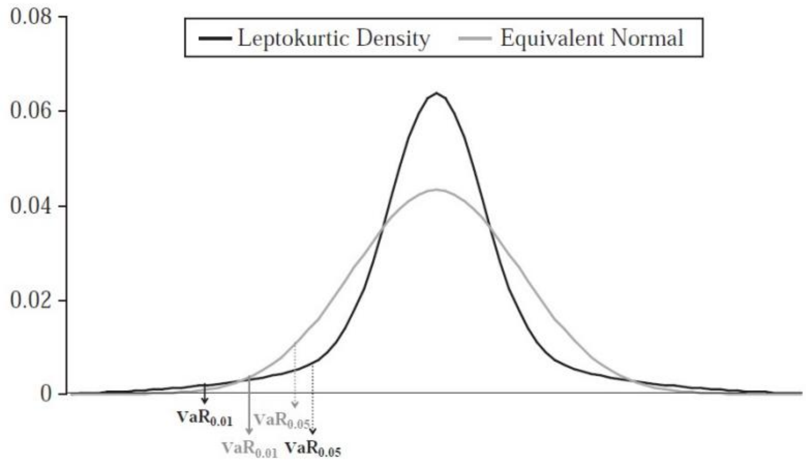

Figure 4.2. Comparative Analysis of the VaR in normal and Leptokurtic Distributions. The graph compares the VaR between normal and leptokurtic distributions, showing that the fat tailed distribution accumulates probability faster than the equivalent normal, resulting in a greater risk of extreme losses.

The SGSt distribution introduces five parameters: the mean (μ), standard deviation (σ), skewness (λ) and two shape parameters $p > 0$ and $q > 0$ that independently control the shape of the central and tail region of the distribution, respectively (Theodossius, 1998). If $\lambda = 0$ the distribution is symmetric; $\lambda>0$, implies positive skewness and $\lambda<0$ indicates negative skewness.

Estimating the parameters of the SGSt distribution is typically achieved using Maximum Likelihood Estimation (MLE). This involves maximizing the likelihood function over observed data points to obtain estimates for μ, σ, λ, p, and q:

$$(\hat{\mu}, \hat{\sigma}, \hat{\lambda}, \hat{p}, \hat{q}) = \arg \max_{\mu,\sigma,\lambda,p,q} \sum_{i=1}^{n} ln[f(x_{t-i}, \mu, \sigma, \lambda, p, q)] \quad (16)$$

Since the VaR is just the symmetric of the distribution´s quantile, the VaR formula can be defined as:

$$VaR_{h,\alpha} = - T^{-1}_{0,1,\lambda,p,q}(\alpha) \text{ x } \sigma - \mu \quad (17)$$

where $T^{-1}_{0,1,\lambda,p,q}$ is the quantile of a standardized SGSt distribution. As with the normal distribution, μ is also assumed to be zero.

4.3.3. Historical VaR

Unlike the parametric models discussed above, the Historical VaR relies on empirical distributions of past returns, letting the empirical distribution shape the risk assessment. Historical VaR calculation involves some key steps starting with the appropriate choice of sample size. Next, daily returns for each of the portfolio´s risk factors across the sample period are calculated, creating the basis for the empirical distribution. To create an accurate daily portfolio P&L distribution, the observed returns are applied to each risk factor while holding the portfolio's risk-factor loadings constant.

Once the returns are calculated, they are sorted from smallest to largest to create an empirical cumulative distribution function (CDF), assigning an equal probability of $\frac{1}{n}$ to each observation. Historical VaR is then defined as the negative of the return at the $100\alpha\%$ quantile of this distribution, where α denotes the significance level chosen.

An important consideration when analyzing a historical model is the size of the sample. The size of the sample is a key consideration in historical models. A larger sample helps mitigate extreme outliers by providing a more robust analysis of past market conditions, but it may include outdated data. A smaller sample, while more responsive to recent market changes, risks overemphasizing extreme events from that period.

Furthermore, one of the limitations of this model is that it assigns the same weight to all observations. Nevertheless, to address this, one refinement to the model is the volatility adjustment using the Exponentially Weighted Moving Average volatility model. While the historical model is based on the premise that all observations have the same weight, the EWMA model has emerged as a way of applying a weighting factor to past returns those decays exponentially, placing more emphasis on recent data than on older observations. For this, the EWMA volatility estimate is calculated using a smoothing parameter, λ, typically set between 0.94 and 0.97. A higher λ value gives more weight to recent observations, while a lower value gives more weight to historical data. The key point of this adjustment is to make the historical model better reflect volatility fluctuations, in order to mitigate the impact of outdated data, while avoiding overemphasis of extreme events in shorter timeframes, and thus responding more swiftly to changes in market conditions.

4.3.4. Quantile Regression VaR

The Quantile Regression VaR model is an advanced risk assessment tool that estimates potential portfolio losses under different scenarios having focused on specific quantiles of the return distribution (Koenker and Bassett, 1978).

The key behind quantile regression VaR is that it allows the estimation of conditional quantiles to predict specific percentiles of the return distribution conditional on certain explanatory variables, such as historical returns, volatility, or economic indicators (Steen et al. 2015). Unlike OLS, which minimizes the sum of squared residuals to determine the average effect of independent variables on a dependent variable, quantile regression minimizes a weighted sum of residuals, where weights differ depending on whether observations fall above or below the chosen quantile.

The α-quantile is the value below which a proportion α of the distribution falls. The quantile regression achieves this by minimizing an asymmetric loss function:

$$\hat{q}_{\alpha} = \arg\min_{q_\alpha} \sum_{i=1}^{n} \alpha(y_i - q_\alpha) I_{y_i - q_\alpha > 0} + (\alpha - 1)(y_i - q_\alpha) I_{y_i - q_\alpha < 0} \tag{18}$$

where $\alpha(y_i - q_\alpha) I_{y_i - q_\alpha > 0}$ are the observations above quantile, $\alpha(y_i - q_\alpha) I_{y_i - q_\alpha < 0}$ are the observations below quantile and $I_{y_i - q_\alpha < 0}$ is an indicator function that takes the value 1 if $y_i < q_\alpha$ and 0 otherwise. Likewise, $I_{y_i - q_\alpha > 0}$ takes a value of 1 if $y_i > q_\alpha$ and 0 otherwise.

For a quantile regression model on a portfolio's returns, if y represents the portfolio returns and x an explanatory variable, in this case the volatility, the quantile regression model becomes:

$$y = a + bx + \epsilon \tag{19}$$

where a and b are parameters estimated by minimizing the quantile-specific loss function. The estimated quantile regression equation for a quantile, $q_{\alpha,y}$ is then:

$$q_{\alpha,y} = \hat{a} + \hat{b}x \tag{20}$$

Therefore, the α-quantile regression VaR can be computed as:

$$\text{VaR}\alpha \equiv -q_{\alpha,y} = -(\hat{a} + \hat{b}x) \tag{21}$$

4.4. Backtest

At this stage, it is imperative to realize a Backtesting procedure, which involves comparing the VaR estimates produced by each model against the empirical data of the portfolio, to assess the model's capacity to predict potential losses. In this case, the Backtesting period covers nearly 10 years, from 11 February 2013 to 27 January 2023. Daily VaR estimates were generated for each model under analysis to evaluate their performance in predicting significant losses.

The number of exceedances is the main performance metric used to understand the model's performance. The UC test evaluates the number of exceedances while the BCP test evaluates the autocorrelation between them. In this work, a 99% VaR model was used, meaning that exceedances should occur on only 1% of the days, given a 1% significance level.

Although both tests are important, the decision should initially be made based on the results of the UC test, since it assesses the number of exceedances. Models that pass the UC test are then subjected to the BCP test to further differentiate their performance. To be considered successful, models must show *p*-values above either 5% or 10%. This approach will initially be carried out for the overall period in order to get an idea of the model's performance under different market conditions. In addition, if necessary, an assessment will be made of the sub-periods in order to identify possible sensitivities to specific market conditions.

In the Backtesting, four classes of VaR models will be tested, RiskMetrics, Skewed Generalized Student-t, Historical and Quantile Regression VaR. Table 4.2 below presents the models studied in this work as its respective description.

Model Number	Description		
	Model Class	Sample Size	EWMA Smoothing Factor
1	Normal	-	0.94
2	Normal	-	0.975
3	SGSt	800	0.96
4	SGSt	1000	0.96
5	SGSt	300	0.96
6	SGSt	800	0.94
7	Historical VolAdj	300	0.98
8	Historical VolAdj	800	0.98
9	Historical VolAdj	1000	0.98
10	Historical VolAdj	800	0.94
11	Historical VolAdj	800	0.96
12	QR	700	0.96 as explanatory variable
13	QR	300	0.94 as explanatory variable
14	QR	300	0.97 as explanatory variable

Table 4.2. Characterization of the models. The table shows the characteristics of the models chosen from a large sample of models with other specifications. The choice of these models was based firstly on the best model for each model class, and then the models from which the specifications actually started to show the worst results.

It is important to mention that the selection of these models was not done randomly, but by trial and error in order to fine-tune the models and find the best model for the portfolio. To do this, the models were adjusted through combinations and specifications between the EWMA smoothing factor and the sample size, in the models that allowed it.

Starting with the Normal VaR, where it is only possible to define the lambda of the model, various configurations were tested, starting with a lambda of 0.93 up to 0.99 at intervals of 0.005. On the other hand, taking into account the specificities of the other models, their choice was based on combinations between lambda and sample size. Following the same logic as the normal VaR, the EWMA smoothing factor was also set from 0.93 to 0.99 in intervals of 0.005, however, these lambdas were combined with a sample size between 300 and 1000 observations at intervals of 100, which offers a much larger sample of models for these three classes of models.

This way, approximately 104 different configurations were tested for each of the SGSt, Historical and QR models and 13 specifications for Normal. Given the vast sample of models, it would be impractical and unclear to provide visibility of all the specifications, not least because many of them showed very similar results. Therefore, in order to provide a clearer and non-exhaustive view of the models, the choice of models presented in the Table 4.2 above was based firstly on the best model for each model class, and then the models from which the specifications actually started to show the worst results, in order to understand the cause of their poor configuration.

Thus, once the sample of models has been chosen, it is then necessary to carry out a detailed analysis of which of them will be the most appropriate for evaluating the portfolio to be studied, which will be explained in the following sub-chapters.

4.4.1. Unconditional Coverage Test

Introduced by Kupiec in 1995, this test evaluates whether the number of observed exceedances aligns with the expected number, based on the model's confidence level. For VaR Backtesting, exceedances occur when actual portfolio losses exceed the predicted ones. More exceedances than expected should indicate an underestimation of risk, while too few suggest it may be overly conservative (Kupiec, 1995).

The main premise of this test, that is, the null hypothesis, H_0, is that the observed exceedances, π_{obs}, are equal to the expected exceedances, π_{exp}, under the assumption that exceedances follow an independent and identically distributed (i.i.d.) Bernoulli process. On the other hand, the alternative hypothesis, H_a, states that the observed exceedances are different from the expected exceedances, suggesting a misspecification of the model.

To test this, the number of exceedances is modeled as a binomial random variable, where the probability of success (exceedance) under the null hypothesis should match the VaR model's significance level. Formally, the hypothesis is written as:

$$H_0 : \pi_{obs} = \pi_{exp} \equiv \alpha$$

$$H_\alpha : \pi_{obs} \neq \pi_{exp}$$

To test this hypothesis, Kupiec (1995) uses a Likelihood Ratio (LR) approach. The test statistic is expressed as:

$$LR_{uc} = \left(\frac{\pi_{exp}}{\pi_{obs}}\right)^{n_1} \left(\frac{1 - \pi_{exp}}{1 - \pi_{obs}}\right)^{n_0} \tag{22}$$

where n_1 is the number of exceedances, $n_0 = n - n_1$ is the number of non-exceedances, $\pi_{obs} = \frac{n_1}{n}$ and $\pi_{exp} = \alpha$.

The test statistics follow a chi-square distribution with one degree of freedom, reflecting the fact that one parameter is tested, that is, whether the observed exceedance rate matches the expected rate or not:

$$-2 \ln (LR_{uc}) \sim \chi_1^2 \tag{23}$$

Small test statistics indicate that the observed exceedance rate is close to the expected rate, and thus the model pass the test. On the other hand, large test statistics indicate a significant difference in the observed exceedance rates compared to those expected, so that the null hypothesis must be rejected and believed that the model is mispecified.

4.4.2. BCP Test

The BCP test, in contrast, examines the autocorrelations in exceedances over multiple lags. A VaR model is considered well specified when the exceedances occur independently of each other (Berkowitz et al., 2011), being impossible to predict when the next exceedance will occur based on the one that has already happened. Thus, the autocorrelation at any lag should ideally be zero.

Berkowitz, Christoffersen and Pelletier (2011) developed a test based on the first *K* autocorrelations of exceedances, *K* being the maximum autocorrelation lag considered in the test.

In this test, the null hypothesis, H_0, suggests that the autocorrelations at all lags (up to *K* lags) are zero. On the other hand, the alternative hypothesis , H_a, states that at least one autocorrelation is different from zero. If autocorrelations are detected in exceedances, then it indicates that the model does not correctly capture risk, as exceedances are supposed to be random and independent.

Formally, the hypothesis is written as:

$$H_0 : \hat{\rho}_k = 0, \ \forall_k \in \{1, \ldots, K\}$$

$$H_\alpha : \exists k \in \{1, \ldots, K\} \; s.t. \; \hat{\rho}_k \neq 0$$

where $\hat{\rho}_k$ = Corr $(I_\alpha, L^k I_\alpha)$ is the k-th order autocorrelation of the time series of exceedances I_α and L^k is the k-th order lag operator.

The test statistic is given by:

$$BCP_K = T(T+2)\sum_{k=1}^{K}\frac{\hat{\rho}_k^2}{T-k} \tag{24}$$

where T is the sample size of the test, that is, the number of observations.

Under the null hypothesis, where all autocorrelations are zero, the BCP test statistic follows a chi-squared distribution with *K* degrees of freedom:

$$BCP(K) \sim \chi_K^2 \tag{25}$$

The selection of the lag *K* is free but it is important to note the tradeoff underlying the choice for *K.* A larger *K* allows for the detection of higher order autocorrelations but makes the null hypothesis harder to reject due to increased degrees of freedom. On the other hand, a smaller *K*, while more powerful, may not be able to detect patterns that take place over long periods, that is, larger than *K*. Then, finally, if autocorrelations are detected in exceedances, then it indicates that the model does not correctly capture risk, as exceedances are supposed to be random and independent. For this reason, the BCP test will not be performed just once, but multiple times for different values of *K*.

CHAPTER 5

MODEL SELECTION

In the Backtesting procedure described above, four classes of VaR models were tested, and this section makes it possible to understand which of these models is the best to adopt given the portfolio under analysis.

The global Backtest period comprises a total of 2 600 observations. Since the VaR models were calculated with a significance level of 1%, a well-specified model, from the perspective of the UC test, should yield approximately 26 exceedances, calculated as 2 600 multiplied by 1%. In statistical testing, the null hypothesis is typically rejected when the *p*-value associated with the test statistic is below 5%. Therefore, a model is considered acceptable under the UC or BCP tests when the null hypothesis is not rejected.

Table 5.1 below presents the model class, the number of exceedances, the exceedance rate and the *p*-value of the UC test for all the chosen models for its global period. The models highlighted in bold are those that pass to the next test, however, the other models were clearly rejected.

Model Number	Model Class	Global Period		
		Number of exceedances	Exceedance Rate (%)	*p*-value (%)
1	Normal	49	1.88	0.01
2	Normal	44	1.69	0.12
3	**SGSt**	**26**	**1.00**	**100.00**
4	SGSt	56	2.15	0.00
5	**SGSt**	**27**	**1.04**	**84.47**
6	**SGSt**	**30**	**1.15**	**44.15**
7	Historical VolAdj	45	1.73	0.07
8	**Historical VolAdj**	**32**	**1.23**	**25.37**
9	**Historical VolAdj**	**35**	**1.35**	**9.20**
10	Historical VolAdj	49	1.88	0.01
11	Historical VolAdj	41	1.58	0.64
12	**QR**	**28**	**1.08**	**69.70**
13	**QR**	**34**	**1.31**	**13.22**
14	QR	40	1.54	1.06

Table 5.1. UC test results for the global period. The table shows the results of the UC test for the global period for the models shown in Table 4.2. The models highlighted in bold are those that pass the UC test for the global period.

The UC test results show that Normal VaR models generally fail to meet accuracy standards, with high exceedance rates and low *p*-values indicating significant risk underestimation suggesting that Normal VaR models do not reliably capture portfolio risk and are therefore rejected. This is not surprising, as the return/P&L distributions of financial asset portfolios typically exhibit heavy tails, which, at such a low significance level as used here, leads to an underestimation of VaR.

On the other hand, models based on other types of distribution seem to perform better. For example, SGSt models 3,5 and 6 show exceedance rates close to 1% and relatively high *p*-values. Similarly, Historical VaR models 8 and 9 and QR VaR models 12 and 13 also achieved exceedance rates and *p*-values that did not indicate statistically significant deviations from the 1% threshold. The SGSt model with a sample size of 800 represented by model number 3 come as the winner for the UC test as it presents the same number of exceedances as expected.

These models, having passed the UC test, will proceed to further evaluation under the BCP test to assess the independence of exceedances, excluding those who did not pass the UC test.

Table 5.2 below presents the *p*-values of the BCP test for the global period at lags 1, 3, 5 and 10. Only these lags are presented as the results for lag 1 indicates from the outset that the test fails on all models, so no model passes the BCP test.

Model Number	**Model Class**	**BCP *p*-values (%)**			
		Lag 1	**Lag 3**	**Lag 5**	**Lag 10**
3	SGSt	0.00	0.00	0.00	0.00
5	SGSt	0.00	0.00	0.00	0.00
6	SGSt	0.00	0.00	0.00	0.00
8	Historical VolAdj	0.00	0.00	0.00	0.00
9	Historical VolAdj	0.95	0.00	0.00	0.00
12	QR	0.29	0.04	0.05	0.00
13	QR	0.00	0.64	0.00	0.00

Table 5.2. BCP test results for the global period. The table shows the results of the BCP test for the global period for the models that passed the UC test for the global period. The values for lags 1 to 10 have been calculated, however, as the models fail the test at the first lag, only lags 1, 3, 5 and 10 are shown in the table.

As expected, the Backtesting analysis over the 10-year global period shows that while several models passed the UC test, none passed BCP test. Since the test shows such low results, this could indicate a structural break or significant changes in the data, indicating, quite possibly, high market instability.

As shown in the Appendix D, the years 2020-2021, marked by the Covid-19 pandemic, present the worst results in the BCP test. This might be as a result of the pandemic´s massive economic disruptions which generated significant market fluctuations. Similarly, the years 2015-2016 also presented poor results probably due to the severe drop in oil prices in these years, which led to a global economic slowdown and a high level of uncertainty.

Given these results, and to narrow down the model selection, the next step would be to analyze the *p*-value of the UC test using sub-periods The choice should fall on the model that has more consistent results on an annual basis.

Table 5.3 below presents the *p*-values of the UC test for the sub-period, for all the models that passed the UC test for the global period.

Model class		**SGSt**			**Historical VolAdj**		**QR**	
Model number		**3**	**5**	**6**	**8**	**9**	**12**	**13**
2022-2023	Exceedance rate (%)	0.77	0.77	0.38	0.77	1.15	0.38	0.77
	p-value (%)	69.67	69.67	25.44	69.67	80.77	25.44	69.67
2021-2022	Exceedance rate (%)	0.00	0.00	0.00	0.38	0.38	0.00	1.15
	p-value (%)	2.22	2.22	2.22	25.44	25.44	2.22	80.77
2020-2021	Exceedance rate (%)	1.92	1.92	1.92	2.31	2.69	1.92	1.54
	p-value (%)	18.44	18.44	18.44	7.01	2.34	18.44	41.87
2019-2020	Exceedance rate (%)	1.15	1.15	1.54	1.54	1.92	1.15	1.54
	p-value (%)	80.77	80.77	41.87	41.87	18.44	80.77	41.87
2018-2019	Exceedance rate (%)	0.77	0.77	0.77	1.15	1.15	1.15	1.15
	p-value (%)	69.67	69.67	69.67	80.77	80.77	80.77	80.77
2017-2018	Exceedance rate (%)	0.38	0.38	0.77	0.77	0.77	0.38	1.54
	p-value (%)	25.44	25.44	69.67	69.67	69.67	25.44	41.87
2016-2017	Exceedance rate (%)	0.38	0.38	0.77	0.38	0.38	0.38	0.38
	p-value (%)	25.44	25.44	69.67	25.44	25.44	25.44	25.44
2015-2016	Exceedance rate (%)	2.69	2.31	2.69	1.92	1.92	2.31	2.31
	p-value (%)	2.34	7.01	2.34	18.44	18.44	7.01	7.01
2014-2015	Exceedance rate (%)	1.15	1.92	1.92	1.92	1.92	2.31	1.15
	p-value (%)	80.77	18.44	18.44	18.44	18.44	7.01	80.77
2013-2014	Exceedance rate (%)	0.77	0.77	0.77	1.15	1.15	0.77	1.54
	p-value (%)	69.67	69.67	69.67	80.77	80.77	69.67	41.87

Table 5.3. UC test results for the sub-periods. The table shows the results for the UC test for the sub-periods of the models that passed in the UC test for the overall period.

As can be seen from the table above, some models perform relatively similarly over time, although it is clear that the performance of each model varies considerably from year to year. In some periods, the models show statistically robust results, with high *p*-values and exceedance rates within expectations. However, these same models can fail in other years, with significantly lower performances, which makes crucial, when selecting a model, there is a balance between the results of the overall period and the sub-periods.

Based on this, model 3 was selected. With a total of 2 600 observations and a significance level of 1%, the expected number of exceedances for the portfolio was 26. Model 3 generated exactly 26 exceedances, which is a direct reflection of its good specification. This alignment between the expected and observed exceedances provides strong evidence of the model's reliability, which makes model 3 the only one that perfectly met the expected exceedance rate in the overall period, directly aligning with the 1% significance level, and presenting a *p*-value of 100% (see Table 5.1).

Although the UC test for the global period confirmed that model 3 was correctly specified over the global period, the analysis of UC results for the sub-periods shows some fluctuations. In fact, for instance, in 2021–2022 and 2015–2016, model 3 presented *p*-values of just 2.22% and 2.34%, respectively, highlighting significant deviation from expected exceedance behavior. These fluctuations illustrate that while the model may be well-calibrated overall (as evidenced by the UC test), its performance is not uniformly strong across all sub-periods. However, it also performed exceptionally well in periods like 2018–2019 and 2019–2020, with *p*-values of 69.67% and 80.77% respectively, indicating correct exceedance rates.

Moreover, Table 5.3 also demonstrates that none of the models exhibit consistently superior performance across all sub-periods. While certain models outperform others in a greater number of individual years, this superiority is not maintained throughout the entire period, thus, the choice of model 3, more than any other model, reflects a balance between overall adequacy and an awareness of its limitations under certain market conditions.

In choosing this model, one of the intentions was to evaluate how it performs in a broader scenario, even though it is not the model that has shown the best performance in individual years. In the context of hedging, the ability to predict the exceedance rate is crucial to avoid both underestimate and overestimate risk, and model 3, by achieving a *p*-value of 100% over the overall period, offers exactly this stability.

In addition, the choice of model 3 also reflects the idea of testing a model which, although not necessarily the “best” in absolute terms, performs satisfactorily over the overall period, allowing an assessment of the robustness of a model which, in many practical situations, may be more suitable than those which perform better only in specific sub-periods. Thus, the decision to go with model 3 was an attempt to balance the search for reliability with the acceptance of fluctuations in some years.

Figure 5.1 below, presents the daily VaR estimates for model 3 and the portfolio´s daily P&L over the global period of the Backtest.

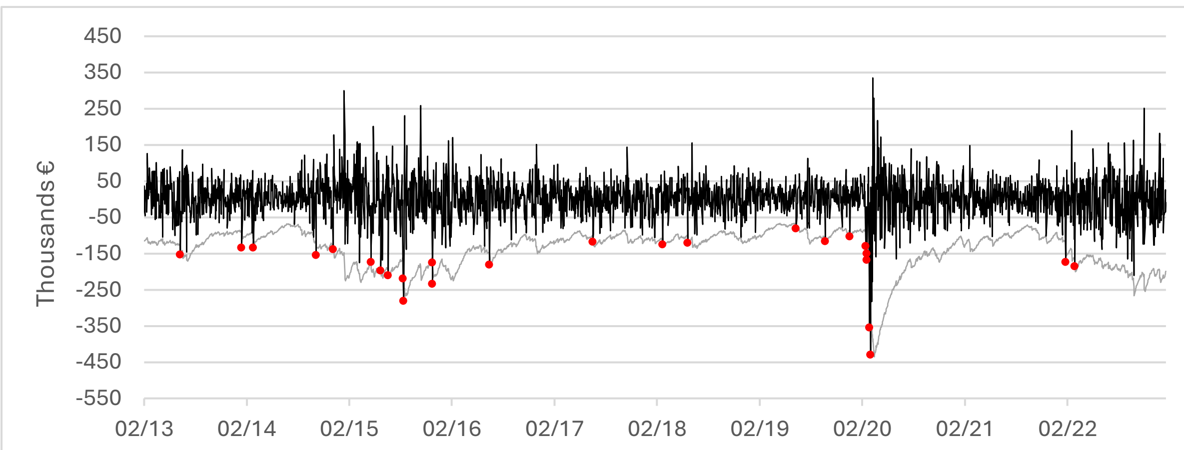

Figure 5.1. SGSt VaR model number 3 global period performance. The grey line shows the daily VaR estimates during the Backtesting period and the black line represents the portfolio's daily P&L during the same period. The red dots represent the observed exceedances.

Date of exceedance	VaR (€)	P&L (€)	Size of exceedance (€)	Size of exceedance (%)
10/03/2022	-176 042.01	-186 216.36	-10 174.35	5.78
04/02/2022	-123 395.66	-174 591.99	-51 196.33	41.49
12/03/2020	-258 157.08	-429 498.61	-171 341.53	66.37
09/03/2020	-176 526.55	-355 000.84	-178 474.30	101.10
28/02/2020	-138 755.69	-168 740.07	-29 984.38	21.61
27/02/2020	-113 935.54	-150 029.76	-36 094.22	31.68
24/02/2020	-86 235.46	-130 431.03	-44 195.58	51.25
30/12/2019	-84 998.65	-103 784.14	-18 785.49	22.10
02/10/2019	-101 408.04	-116 561.94	-15 153.90	14.94
21/06/2019	-76 313.75	-81 627.04	-5 313.29	6.96
31/05/2018	-104 840.84	-121 163.33	-16 322.49	15.57
02/03/2018	-108 176.78	-125 794.03	-17 617.26	16.29
28/06/2017	-102 388.50	-117 766.61	-15 378.10	15.02
24/06/2016	-144 299.76	-181 966.51	-37 666.75	26.10
04/12/2015	-181 808.95	-234 287.82	-52 478.87	28.86
03/12/2015	-160 444.22	-176 017.71	-15 573.50	9.71
24/08/2015	-199 854.94	-280 857.05	-81 002.11	40.53
21/08/2015	-166 402.79	-219 188.76	-52 785.96	31.72
30/06/2015	-190 288.46	-210 810.22	-20 521.76	10.78
03/06/2015	-188 688.08	-197 336.66	-8 648.58	4.58
29/04/2015	-173 489.26	-174 485.59	-996.32	0.57
15/12/2014	-123 905.23	-139 292.73	-15 387.49	12.42
16/10/2014	-112 335.85	-155 074.95	-42 739.10	38.05
07/03/2014	-104 857.96	-134 441.66	-29 583.71	28.21
24/01/2014	-87 810.98	-134 391.08	-46 580.10	53.05
20/06/2013	-126 214.17	-152 914.23	-26 700.06	21.15
Average			**-40 026.75**	**27.54**
Number of Exceedances				**26**
Exceedance Rate				**1.00**

Table 5.4. Model 3 exceedance details. The table shows the characteristics of the exceedances generated by the model 3 during the Backtest period.

Figure 5.1 and Table 5.4 above show that, as expected, model 3 exhibits the 26 expected exceedances. For the majority of the global period, the exceedances happened with a considerable number of days between them, however it is possible to conclude that in 2020 as well as in the second half of 2015, there are some exceedances that occur a few days apart, or even on a day right after the other, as is the case on 03/12/2015 and 04/12/2015. Appendix D shows that the BCP test for the sub-period has captured these occurrences well, since it shows very poor results for the years 2015-2016 and 2020-2021.

CHAPTER 6

VALUE-AT-RISK MANAGEMENT

As previously mentioned, the main objective of this work is to measure and manage the VaR of the portfolio so as not to exceed a prespecified target value. By analyzing the VaR time series during the Backtesting period, it was possible to conclude that the VaR values associated with the portfolio typically fluctuate between 100 000 and 200 000 EUR. With this in mind, a reasonable target for VaR was identified as a value that is exceeded between one third and two thirds of the time. Thus, the VaR target established for this work was set at 125 000 EUR. As such, for the one year going forward, the goal is that the capital at risk from holding the portfolio on the following day, measured by the 1-day VaR, should not exceed 125 000 EUR.

To meet this target, the following process is implemented: at the end of each trading day, the VaR for the next day is estimated based on the current portfolio composition. If the estimated VaR exceeds the 125 000 EUR, a specific hedging strategy is applied to adjust the portfolio composition in such a way that the new VaR estimate remains below the 125 000 EUR target, otherwise, no strategy will be applied and the composition of the portfolio will remain the same. The process is repeated each trading day, starting from 30 January 2023 continuing until 2 February 2024.

6.1. Risk Mitigation and Hedging Strategies

The days when the VaR exceeds economic capital led us to one of the main topics of this work – the concept of hedging strategies. Hedging is a financial strategy used to reduce the risk of adverse movements in the prices of assets or portfolios and according to Black and Scholes (1973), hedging involves taking a position in an asset or derivative in order to offset losses or gains from another position.

Hedging strategies are then implemented to ensure that the VaR remains below this target on all days, with hedging positions removed once they are no longer required. To apply an appropriate hedging strategy, it is first needed to take into account the characteristics of the portfolio, such as its exposure to different risk factors. Thus, it becomes crucial to understand how different risk factors can contribute to the overall portfolio risk, which can be identified through VaR marginal decompositions.

Therefore, the hedging strategy proposed for this work consists of a daily assessment of the Marginal VaR of each of the risk factors to which the portfolio is exposed. Based on this analysis, a hedging strategy will be implemented for the factor with the highest VaR marginal contribution values, in order to mitigate the portfolio's total risk. If the application of this strategy is not enough to reduce the VaR to the target, a new round of hedging will be implemented, targeting the factors which, after the application of the first strategy, have the highest marginal contribution to the total risk.

6.2. VaR Decomposition and Management Strategies

To accurately assess how each risk factor impacts the portfolio, it is essential to decompose the portfolio's overall risk into contributions from specific factors. This involves breaking down the portfolio's returns or exposures to identify sensitivities at the level of each risk factor, which provides a detailed view of how changes in these factors might affect the portfolio's value. In the subsection below, the VaR decomposition analysis will be explored.

A decomposition of the vector of risk factor loadings is any collection $\{\Theta^s\}_{s=1}^{m}$ of exposures to the *n* risk factors, with typical element $\Theta^s = [\theta_1^s, \theta_2^s, \ldots, \theta_n^s]^T$, that satisfies

$$\sum_{s=1}^{m} \Theta^s = \Theta \qquad (26)$$

A key method in VaR decomposition includes Marginal VaR that determines the contribution of each risk factor to the overall VaR by assessing how much the portfolio´s VaR would change in response to a marginal increase in each factor. Mathematically, it is given by:

$$\text{Marginal VaR} = \nabla f(\Theta)^T \Theta^s \qquad (27)$$

where $\nabla f(\Theta)^T$ represents the gradient vector for the portfolio´s VaR, that is, the vector that lists the portfolio VaR's sensitivities to small changes in the exposure to each risk factor away from the current values Θ.

Table 6.1 below, shows the decomposition of the estimated marginal VaR, by asset class, for the first day of analysis, 30 January 2023, which also corresponds to a day when the VaR exceeds the established target – 125 000 EUR. In this way, it will be possible to assess how much each asset class contributes to the portfolio's VaR.

Description	Decomposition by asset class		Total
	Equity	Bonds	
Marginal VaR (%)	35.07	64.93	100
Marginal VaR (EUR)	68 307.24	126 462.28	194 0769.53

Table 6.1. VaR decomposition by asset class. This table showcases how much each asset class is contributing to the VaR estimate, for the first day on which the VaR exceeded the established target, that is, 30 January 2023.

This marginal decomposition of VaR shows that bonds have a significantly higher contribution to the total risk of the portfolio, representing 64.93% of the total VaR, while stocks contribute only 35.07%.

Hedging strategies have to be applied to specific assets, so it is necessary to deconstruct and try to understand further what type of assets may be contributing most to the high VaR. Since bonds make a large contribution to the high VaR value, it is important to dive deeper into bonds. The Tables 6.2 and 6.3 below, show the marginal decomposition by asset within the European and US bond asset classes, respectively.

Description	European Bonds			Total
	Decomposition by Asset			
	NL0015001DQ7	DE000BU2Z015	LU1556942974	
Marginal VaR (%)	6.09	**17.38**	3.13	26.6
Marginal VaR (EUR)	11 853.56	**33 842.91**	6 094.70	51 791.17

Table 6.2. VaR decomposition by asset within European bond asset class. This table showcases how much each European bond is contributing to the VaR estimate, for the first day on which the VaR exceeded the established target, that is, 30 January 2023. The asset with the largest contribution to the portfolio's VaR on the day is highlighted in bold.

Description	US Bonds			Total
	Decomposition by Asset			
	US91282CJD48	US91282CJG78	US9128286A35	
Marginal VaR (%)	12.25	17.18	8.91	38.34
Marginal VaR (EUR)	23 856.51	33 468.72	17 345.88	74 671.11

Table 6.3. VaR decomposition by asset within US bond asset class. This table showcases how much each US bond is contributing to the VaR estimate, for the first day on which the VaR exceeded the established target, that is, 30 January 2023.

Taking into account Table 6.2 above, the marginal decomposition of VaR shows that the German government bond, DE000BU2Z015, is the bond that is applying the greatest pressure on the total risk of the portfolio. This impact can be attributed to the fact that it is a longer maturity bond, which makes it more exposed and sensitive to interest rate fluctuations. As a result, it contributes more significantly to the portfolio's risk, highlighted in the Table 6.2 above.

An efficient approach to reducing the risk of this bond would be to use financial instruments such as futures, interest rate swaps or options to protect against fluctuations in interest rates, since these are the main risk factors for long-term bonds. However, in this work, a less complex way of managing portfolio risk was applied – the adjustment to the nominal value of the Bond.

The adjustment to the nominal value should be made by selling part of the bond in order to reduce the nominal value of the position, and keep it in cash, thereby reducing the impact of this bond on the total risk of the portfolio. Using the concept of Marginal VaR, it was possible to calculate the amount needed to be sold to achieve the desired risk reduction, using the following:

$$n = \frac{\Delta VaR}{Marginal\ VaR} \times M \qquad (28)$$

where n is the amount needed to be reduced, ΔVaR is the desired change in VaR and M is the current nominal value of the position.

Since the VaR on this day is well above the expected value, reducing the risk associated with the German bond may not be enough on its own, so it will be necessary to adopt a broader hedging approach, applied to other risk factors.

Table 6.4 below, shows the marginal decomposition by risk factor group.

Description	Decomposition by risk factor group							
	USDEUR	**GBPEUR**	**JPYEUR**	**GSPC**	**FCHI**	**GDAXI**	**FTSE**	**N225**
Marginal VaR (%)	**27.60**	1.05	2.56	2.77	6.24	**8.24**	7.32	3.01
Marginal VaR (EUR)	**53 755.87**	2 042.58	4 992.42	5 395.91	12 155.52	**16 052.18**	14 255.13	5 869.37

Table 6.4. VaR decomposition by risk factor group. This table showcases how much currency and stocks from all the indices are contributing to the VaR estimate, for the first day on which the VaR exceeded the established target, that is, 30 January 2023. The assets with the largest contribution to the portfolio's VaR on the day is highlighted in bold.

Given the above, and as expected, the USDEUR exchange rate shows a very large marginal contribution from VaR, so a possible strategy to decrease the VaR is to hedge the USDEUR exchange rate. It can be achieved by taking a position in a USDEUR forward contract, where the exposure to USD is offset by the position in the forward contract, thus adjusting the exposure to changes in the exchange rate between USD and EUR.

Hedging strategies can be applied not just to 1 or 2 risk factors, but to several. To do this, it is possible to establish an arbitrary hedging position on the first risk factor to achieve part of the VaR reduction and then use the incremental VaR to determine the hedging position on the last risk factor to finally achieve the expected reduction.

Given that, stocks also contribute in part to the portfolio's VaR. If necessary, a possible strategy to decrease the VaR is to hedge the equity exposure for European stocks, since this is the stock market with the highest marginal contribution (see appendix C), by adding short positions in stock indices, such as DAX.

The VaR will then be estimated on a daily basis, so the hedging strategies will also be adjusted on a daily basis, repeating this process until 2 February 2024. Whenever, on a given day, the estimated VaR is above the VaR target, the hedging strategy will be implemented for the risk factor with the highest marginal contribution. If this is not enough, other risk factors will be Hedged in order to reach the target. Whenever the positions taken are not guaranteed, meaning that the VaR is below the 125 000 EUR target without them, the positions are removed.

6.3. Value-at-Risk Management Results

Finally, this subchapter aims to compare the initial portfolio, which will henceforth be referred to as the Unhedged portfolio with the portfolio after the implementation of the hedging strategies, which will be defined as the Hedged portfolio. These analyses focus mainly on the VaR, the P&L of the portfolio and its overall performance. Additionally, the impact of the hedging positions and their cumulative effect on the portfolio's performance over time will be examined.

Figure 6.1 below, presents the daily VaR estimates for both Unhedged and Hedged portfolios.

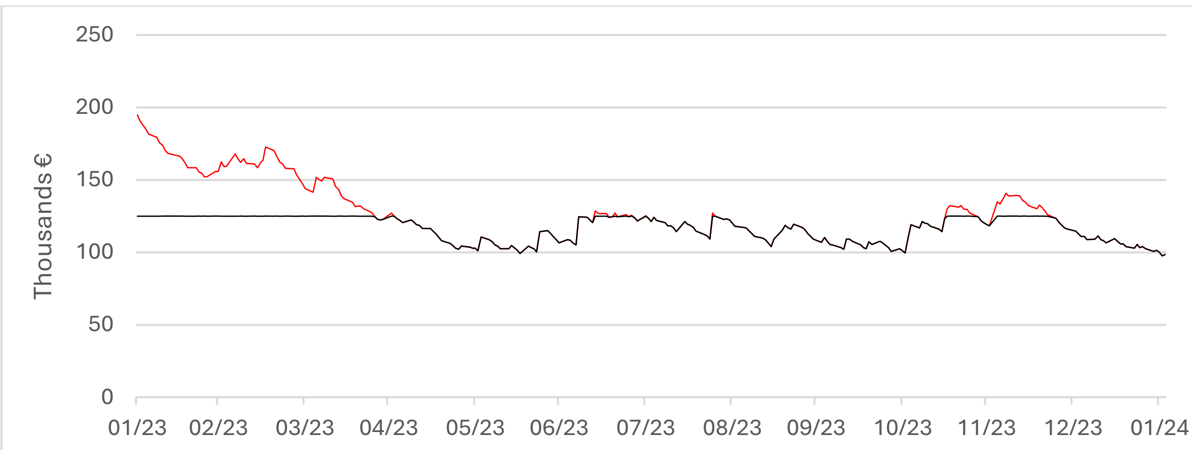

Figure 6.1. Daily VaR estimates from 30 January 2023 to 2 February 2024. The red line represents the Unhedged portfolio and the black line represents the Hedged portfolio.

As expected, by managing the VaR through the efficient implementation of hedging strategies, it is possible to ensure that the VaR remains within the defined limits, never exceeding the target of 125 000 EUR, as is the case with total Unhedged VaR.

Figure 6.2 below presents the hedge position evolution through the one-year period.

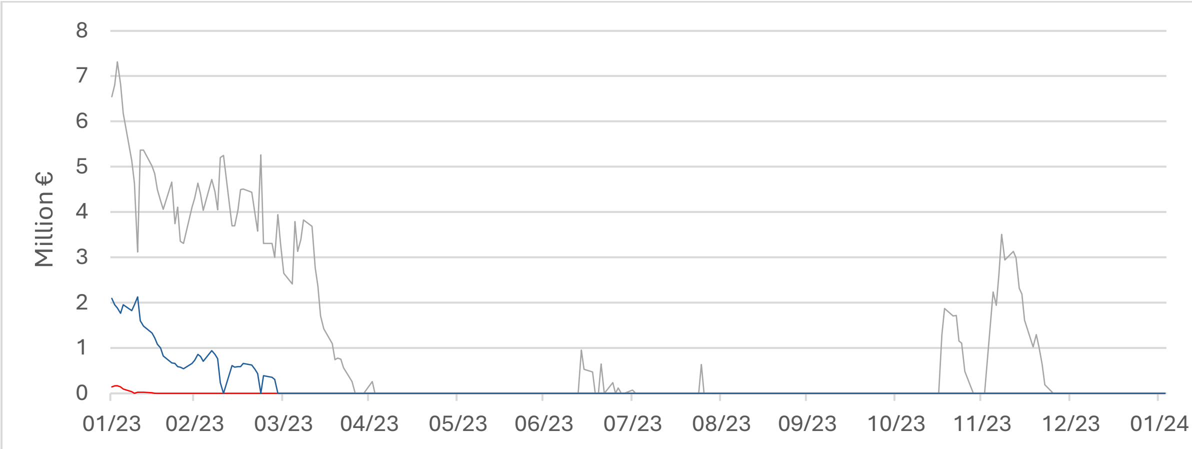

Figure 6.2. Daily hedging position from 30 January 2023 to 2 February 2024. The grey line represents the position in a USDEUR forward contract, the blue line represents the nominal value of bonds sold and the red line represents the short positions in DAX stock indices.

Given the two tables above, it can be concluded that on each of the days when the VaR exceeded the limit, hedging strategies were implemented. Graphically, these two graphs above illustrate that the greater the Unhedged VaR, the greater the hedging position taken. Therefore, when the desired reduction in VaR is smaller, that is, the observed VaR did not exceed the target by a large amount, it was only necessary to apply a hedging strategy to one of the risk factors, in this case the USDEUR, since a position taken in this currency is sufficient to achieve the objective. On the other hand, whenever the VaR is equal to or less than the target, the Figure 6.2 shows the hedging positions set to zero.

Figure 6.3 below presents the daily P&L for Unhedged and Hedged portfolios.

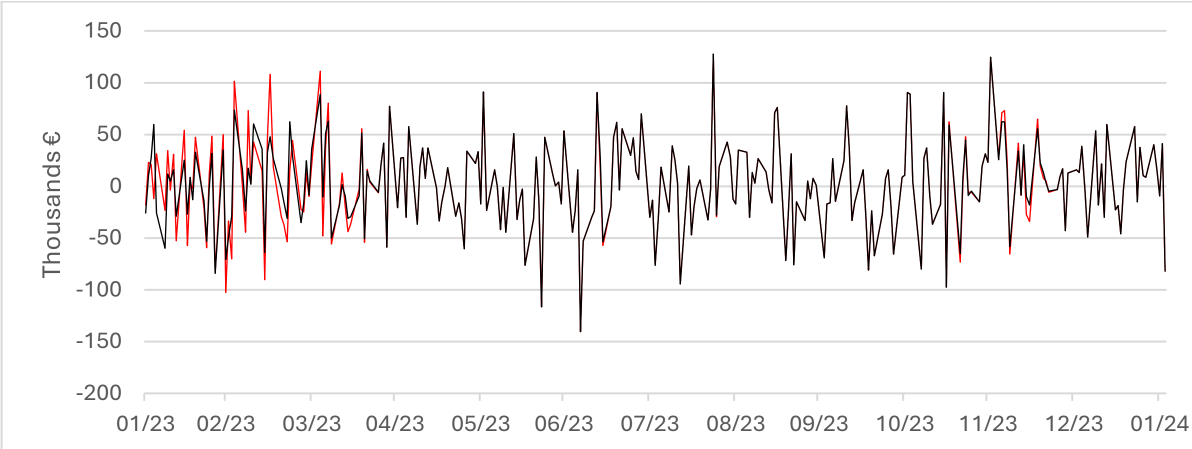

Figure 6.3. Daily P&L from 30 January 2023 to 2 February 2024. The red line represents the Unhedged portfolio and the black line represents the Hedged portfolio.

As it can be concluded from the figure above, the Unhedged portfolio exhibits greater volatility, with larger fluctuations in P&L due to exposure to market movements since the portfolio is fully exposed to interest rate changes and stock price movements

However, while the Unhedged portfolio incurs larger losses on average, it also occasionally experiences higher gains, as it remains fully exposed to market upside potential, without the mitigating effect of the hedge. Also, at the beginning of the year, the Hedged portfolio experienced larger losses, which may have been due to some ineffective hedging during that period. Nevertheless, the global view demonstrates that the Hedged portfolio shows smaller fluctuations in the P&L since hedging mitigates some of the adverse market conditions

Figure 6.4 and 6.5 below presents the Unhedged and Hedged performances for the 1-year period.

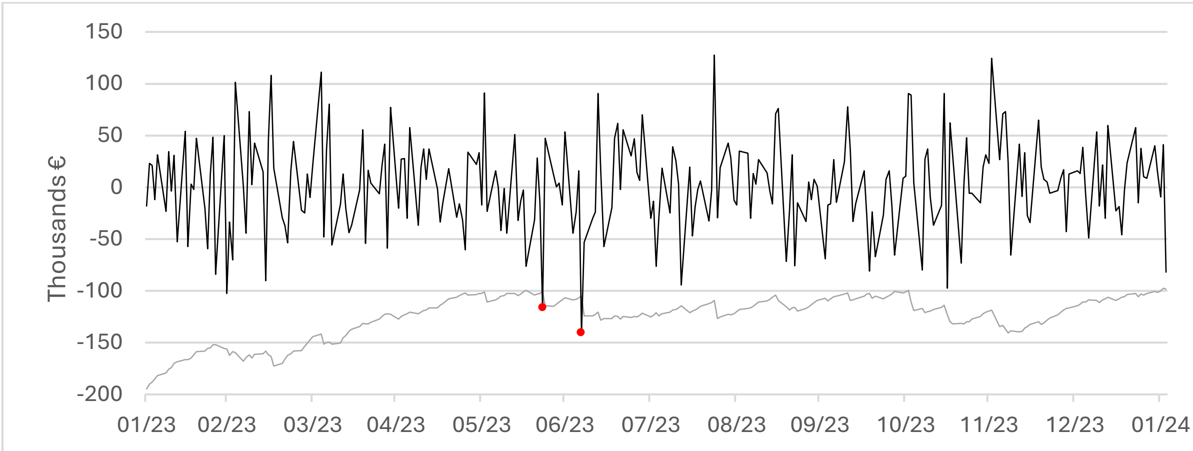

Figure 6.4. Unhedged VaR performance from 30 January 2023 to 2 February 2024. The black line represents the Unhedged daily P&L and the grey line represents the Unhedged daily VaR estimates. The red dots represent the observed exceedances.

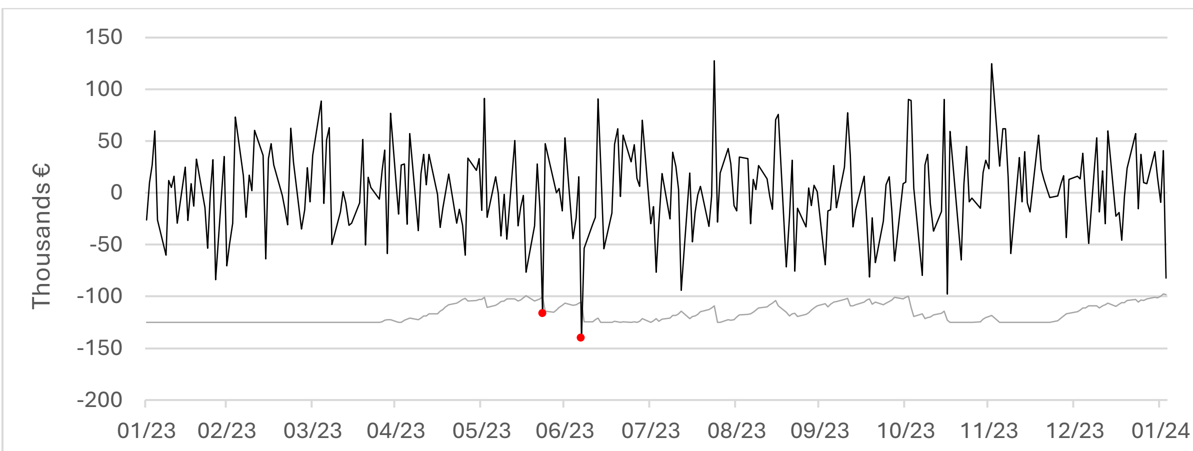

Figure 6.5. Hedged VaR performance from 30 January 2023 to 2 February 2024. The black line represents the Hedged daily P&L and the grey line represents the Hedged daily VaR estimates. The red dots represent the observed exceedances.

As expected, the Unhedged portfolio exhibits a daily P&L with greater volatility over time. The black line typically experiences significant movements, including deep drawdowns, showing greater exposure. The observation of these two graphs suggests that the implementation of hedging strategies enhances financial stability by aligning actual losses closer to VaR estimates. Although it is possible to conclude that both graphs experience the same number of exceedances, these occur when the VaR value does not exceed the target, so no hedging strategy was applied on that day.

Figure 6.6 below presents the daily cumulative portfolio P&L for both the Unhedged and Hedged portfolios.

Figure 6.6. Daily cumulative P&L from 30 January 2023 to 2 February 2024. The red line represents the Unhedged portfolio and the black line represents the Hedged portfolio.

By analyzing the graph above, it is possible to conclude that both portfolios ended the year with a positive cumulative P&L, despite a slight decline in the P&L in the final days. Although the Hedged portfolio experienced more pronounced initial losses at the beginning of the year, the Unhedged portfolio consistently remained below it throughout the rest of the year, with the Hedged portfolio returning a higher gain than the Unhedged portfolio at the end of the year. Therefore, this highlights the role and importance of hedging in reducing volatility and protecting against significant losses.

Finally, the Return on Risk-Adjusted Capital (RORAC) analysis is a very important key point in evaluating these results, since it is a financial indicator used to measure the return on an investment in relation to the risk assumed, thus evaluating the efficiency of the portfolio.

In this work, in order to comprehensively assess the risk-adjusted performance of the portfolios analyzed, RORAC was calculated using two different methods, the average of the daily RORAC and the direct annual RORAC. At first, it was calculated the RORAC value on a daily basis and then obtained the average RORAC for the entire portfolio during the year under analysis, to capture the consistency of risk-adjusted performance over time.

Since the RORAC relates the observed return to the risk incurred in order to obtain it, it is possible to calculate the metric using the following formula:

$$RORAC = \frac{P\&L}{VaR} \tag{29}$$

where P&L is the Profit and Loss of the portfolio on that day, and VaR is the Value-at-Risk calculated for the portfolio on that same day, starting from 30 January 2023.

Figure 6.7 below presents the daily RORAC for the one-year period.

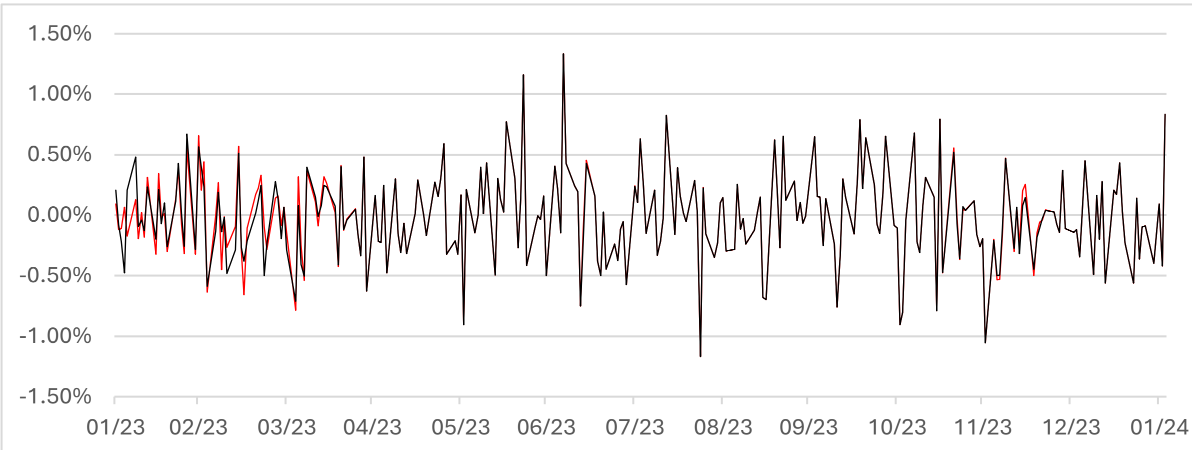

Figure 6.7. Daily RORAC from 30 January 2023 to 2 February 2024. The red line represents the daily RORAC for the Unhedged portfolio and the black line represents the daily RORAC for the Hedged portfolio.

The Figure 6.7 above shows that the portfolio's daily RORAC is highly volatile in both the Unhedged and Hedged portfolios, although the hedging strategies seem to slightly reduce the amplitude of the variations, which makes the risk-adjusted returns less extreme.

To complement the analysis of the individual RORAC of each portfolio, Figure 6.8 below shows the daily difference between the RORAC of the Hedged portfolio and the RORAC of the Unhedged portfolio, that is, the RORAC of the Hedged portfolio minus the RORAC of the Unhedged portfolio, in order to make a direct assessment of the impact of the hedging strategy on risk-adjusted performance over time.

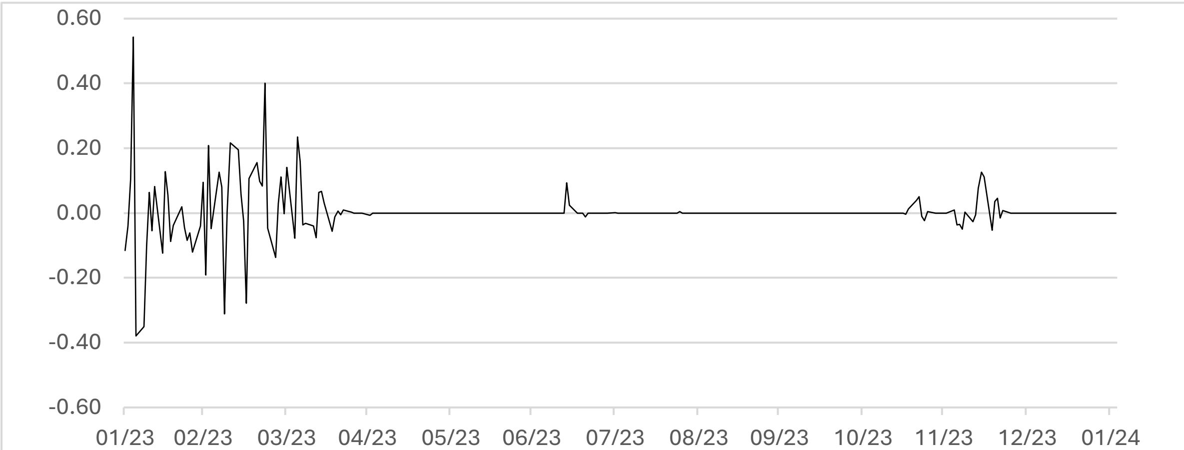

Figure 6.8. Daily difference in RORAC between the Hedged portfolio and the Unhedged portfolio from 30 January 2023 to 2 February 2024.

By analyzing the figure above, it is possible to conclude that the greatest fluctuations in difference between the RORACs occurred at the beginning of the year, where there are both positive and negative peaks, which was expected given the high volatility in the market, as shown in the graphs before. Over time, the difference stabilizes around zero, with small variations, which indicates that under more stable market conditions, hedging had a residual effect on RORAC. Nevertheless, it is possible to conclude that there is more of positive peaks over negative peaks, which may suggest that at times of greater instability, the Hedged portfolio tends to show better risk-adjusted performance than the Unhedged portfolio.

In addition to the average of the daily RORAC, the annual RORAC was also calculated directly. To do this, it is just need to calculate the ratio between the cumulative P&L result over the year and the average daily VaR over the same period, according to the following equation:

$$RORAC = \frac{\sum P\&L}{\frac{1}{n}\sum VaR} \tag{30}$$

where $\sum P\&L$ is the cumulative Profit and Loss of the portfolio over the analyzed period and $\frac{1}{n}\sum VaR$ is is the average VaR over the period, with *n* being the number of days observed.

Finally, Table 6.5 below presents the RORAC for the year for the Unhedged and Hedged portfolios, obtained by averaging the daily RORAC values over the year under analysis, and also by calculating it from the overall period.

Metric	Portfolio	
	Unhedged	**Hedged**
RORAC (%)	1.25	1.64
	1.42	1.85

Table 6.5. RORAC for the one-year period. The values 1.25 and 1.64 were obtained by averaging the daily RORAC values over the 1-year period under analysis, using Equation 29 and the values 1.42 and 1.85 were obtained from the overall period through Equation 30.

By analyzing the table above, it is possible to conclude that the Hedged portfolio shows a better performance than the Unhedged portfolio in both methods. The first method used to calculate the RORAC, shows a result of 1.25 for the Unhedged portfolio and 1.64 for the Hedged portfolio, reflecting risk-adjusted performance on an average daily basis. The annual RORAC, calculated as the ratio between the total annual P&L and the average annual VaR, showed slightly higher values: 1.42 and 1.85 respectively. Since each approach captures different risk perspectives, it is expected that there will be a slight difference between the values, resulting from the fact that the average of the daily ratios can be sensitive to occasional fluctuations in VaR, especially in periods of low volatility, while the direct annual RORAC more robustly reflects aggregate performance, thus presenting higher values. Nevertheless, it is possible to conclude from both analyses that hedging contributed to an improvement in portfolio efficiency in terms of risk-adjusted return.

CHAPTER 7

CONCLUSIONS

The aim of this work was to answer two central questions: what is the best model for measuring and managing the Value-at-Risk of a given portfolio? Are hedging strategies efficient for managing the P&L of a portfolio? To answer these, the main premise of this project was to measure and manage, on a daily basis, the Value-at-Risk of a portfolio composed of stocks and bonds, from 30 January 2023 to 2 February 2024, so that the daily Value-at-Risk did not exceed the established target.

In answering the first question, various VaR models were tested, including RiskMetrics, SGSt, Historical, and QR VaR. A Backtesting process over a 10-year period identified the best-performing models by comparing their VaR estimates to empirical data.

Given that there are various combinations and specifications of these models, it was necessary to carry out a Backtest analysis in order not only to choose a good VaR calculation model, but also the best one for calculating the portfolio's VaR, so 14 models were presented to be analyzed through the UC test and the BCP test. The SGSt model with a sample size of 800 and an EWMA of 0.96 emerged as the best model in the UC test, as it aligned perfectly with the expected exceedance rate.

After choosing the model, it was finally possible to start measuring and managing the VaR of this portfolio, thus being prepared to answer the second question of this work: Are hedging strategies efficient for managing the P&L of a portfolio?

The VaR was then calculated on a daily basis for one year, starting on 30 January 2023 and ending on 2 February 2024, and whenever the VaR value exceeded the established target on any given day, risk management strategies were implemented.

The first time the VaR exceeded the established target of 125 000 EUR was exactly on the first day of analysis, 30 January 2023. The VaR marginal decomposition showed that the risk derives predominantly from bonds, mainly long-term German government bonds. As such, in order to decrease VaR to a maximum of 125 000 EUR, an adjustment was made to its nominal value by partially selling the bond. Additionally, it was necessary to fine-tune the risk management strategy and apply several hedging strategies on the same day, including hedging the USD/EUR exchange rate through a forward contract and hedging the exposure to European stocks, particularly through short positions in stock indices like the DAX.

The dynamic nature of VaR estimation implies that hedging strategies must be continuously adjusted as new data becomes available so this methodology was used throughout the rest of the year, and the hedging strategies were constantly adjusted to the conditions of the portfolio each day, being withdrawn when no longer necessary.

In conclusion, the analysis of both the Unhedged and Hedged portfolios over the one-year period showed that the hedging strategy effectively kept the VaR within the 125 000 EUR thresholds. Without this management, however, the VaR would have exceeded this limit for a significant portion of the year. The comparison between these two portfolios showed that while the Unhedged portfolio experienced higher volatility and larger fluctuations in P&L, the Hedged portfolio exhibits more stable performance with smaller losses during periods of market instability.

Finally, when comparing the returns achieved for the Hedged and Unhedged portfolios, using the RORAC metric, the analysis showed that both portfolios had a positive RORAC, with the Hedged portfolio always showing higher annual values than the Unhedged portfolio. Despite both portfolios having experienced periods of loss, the higher RORAC of the Hedged portfolio indicates that these losses were better managed relative to the level of risk incurred, suggesting that the hedging strategy was effective in enhancing the portfolio's risk-adjusted returns.

REFERENCES

Aas, K. (2006). *The generalized hyperbolic skew student's t-distribution. Journal of Financial Econometrics, 4*(2), 275–309. https://doi.org/10.1093/jjfinec/nbj006

Alexander, C. (2005). The present and future of financial risk management. Journal of Financial Econometrics, 3(1), 3–25. https://doi.org/10.1093/jjfinec/nbi003

Alexander, C. (2008). Market risk analysis: Quantitative methods in finance. John Wiley & Sons.

Barone-Adesi, G., Burgoin, F. and Giannopoulos, K. (1998) Don't Look Back. Risk, 11, 100-104

Berkowitz, J., Christoffersen, P., and Pelletier, D. (2011). Evaluating value-at-risk models with desk-level data. Management Science, 57(12):2213–2227

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, *81*(3), 637–654. http://www.jstor.org/stable/1831029

Bollerslev, T. (1986). *Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307–327.* doi:10.1016/0304-4076(86)90063-1

Chen, M.-Y., & Chen, J.-E. (n.d.). *Application of Quantile Regression to Estimation of Value at Risk*

Christoffersen, P., & Pelletier, D. (2003). *Backtesting Value-at-Risk: A Duration-Based Approach.*

ElBannan, M. A. (2017). *The financial crisis, Basel accords and bank regulations: An overview. International Journal of Accounting and Financial Reporting, 7*(2), 225. https://doi.org/10.5296/ijafr.v7i2.12122

Gaglianone, W., Lima, L., Linton, O., & Smith, D. (2011) *Evaluating Value-at-Risk Models via Quantile Regression, Journal of Business & Economic Statistics*, *29*(1), 150-160, DOI:10.1198/jbes.2010.07318

Halkos, G., & Tsirivi, A. (2019). *Energy Commodities: A Review of Optimal Hedging Strategies. Energies* 2019, *12*(20), 3979; https://doi.org/10.3390/en12203979

Hull, J. and White, A. (1998). Incorporating volatility updating into the historical simu lation method for value-at-risk. Journal of risk, 1(1):5–19.

J.P. Morgan, & Reuters. (1996). Riskmetrics.

Jorion, P. (2007). *Value-at-Risk: The new benchmark for managing financial risk* (3rd ed.). McGraw-Hill.

Kang, W.-Y., & Poshakwale, S. (2019). A new approach to optimal capital allocation for RORAC maximization in banks. *Journal of banking & Finance*, 106, 153-165. https://doi.org/10.1016/j.jbankfin.2019.06.006

Koenker, R. and Bassett Jr, G. (1978). Regression quantiles. Econometrica: journal of the Econometric Society, pages 33–50

Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. The Journal of Derivatives, 3(2)

Lin, C., & Shen, S. (2006). Can the student-*t* distribution provide accurate value at risk?. *The Journal of Risk Finance*, *7*(3), 292–300. https://doi.org/10.1108/15265940610664960

Luiz, R., Lima, B., De Andrade, P., & Néri, S. (2005). *Comparing Value-at-Risk Methodologies*.

McDonald, J. B. and Newey, W. K. (1988). Partially adaptive estimation of regression models via the generalized t distribution. Econometric theory, 4(3):428–457

Pafka, S., & Kondor, I. (2001). Evaluating the riskmetrics methodology in measuring volatility and Value-at-Risk in financial markets. Physica A: Statistical Mechanics and its Applications, 299(1-2), 305–310.

Phelan, M.J. (1997*). Probability and Statistics Applied to the Practice of Financial Risk Management: The Case of J.P. Morgan's RiskMetrics™.* *Journal of Financial Services Research* 12, 175–200 (1997). https://doi.org/10.1023/A:1007926803627

Pritsker, M. (2006). The hidden dangers of historical simulation. Journal of Banking & Finance, 30(2), 561–582. https://doi.org/10.1016/j.jbankfin.2005.04.013

Shakdwipee, P., & Mehta, M. (2017). From Basel I to Basel II to Basel III. International Journal of New Technology and Research (IJNTR), 3(1), 66–70.

Steen, M., Westgaard, S., & Gjølberg, O. (2015). Commodity value-at-risk modeling: comparing RiskMetrics, historic simulation and quantile regression. *The Journal of Risk Model Validation*, *9*(2), 49–78. https://doi.org/10.21314/jrmv.2015.146

Theodossiou, P. (1998). Financial data and the skewed generalized t distribution. Management Science, 44(12-part-1), 1650–1661. https://doi.org/10.1287/mnsc.44.12.1650

Vasileiou, E. (2017). Value-at-Risk (VaR) historical approach: Could it be more historical and representative of the real financial risk environment? Theoretical Economics Letters, 7(4), 951–974. https://doi.org/10.4236/tel.2017.74065

Zhang, Y., & Nadarajah, S. (2017). A review of backtesting for value at risk. *Communications in Statistics - Theory and Methods*, *47*(15), 3616–3639. https://doi.org/10.1080/03610926.2017.1361984

APPENDICES

Appendix A. Portfolio returns descriptive statistics

	Mean	Median	Min	Max	StdDev	Skew	Kurt
2013	0.0236%	0.0068%	-1.0300%	0.9197%	0.3240%	0.0115	3.317
2014	0.0469%	0.0573%	-1.0445%	1.2016%	0.2920%	-0.1979	5.246
2015	0.0369%	0.0569%	-1.8917%	2.0222%	0.5359%	-0.0719	4.597
2016	0.0212%	0.0297%	-1.2256%	1.1475%	0.3689%	-0.1399	3.483
2017	-0.0048%	0.0199%	-0.7932%	0.9680%	0.2731%	-0.1016	3.207
2018	-0.0021%	0.0082%	-0.8473%	1.0516%	0.2817%	-0.0576	3.473
2019	0.0432%	0.0563%	-0.7851%	0.7604%	0.2335%	-0.3712	3.803
2020	0.0055%	0.0336%	-2.8929%	2.2566%	0.5217%	-0.8562	9.832
2021	0.0279%	0.0193%	-0.6968%	0.9986%	0.2525%	0.0943	3.678
2022	-0.0386%	-0.0163%	-1.4155%	1.6898%	0.4517%	0.0738	3.771
2023	0.1050%	-0.0045%	-0.6273%	1.2252%	0.5077%	0.7486	2.877

Table A.1. Portfolio returns descriptive statistics during the Backtest period. The table showcases the portfolio´s returns descriptive statistics for the annual sub-periods of the Backtest ranging from 11 February 2013 to 27 January 2023.

Appendix B. Stock investments across sectors

Stock	Allocation (%)	Sector
Alphabet Inc.	3.84	Communication Services
McDonald´s Corporation	5.35	Consumer Cyclical
Booking Holdings Inc.	-3.25	Consumer Cyclical
Johnson & Johnson	3.92	Healthcare
JPMorgan Chase & Co.	4.55	Financial Services
The Protector & Gamble	4.02	Consumer Defensive
L'Oréal S.A.	4.89	Consumer Defensive
Airbus SE	3.80	Industrials
BNP Paribas SA	3.91	Financial Services
Sanofi	3.86	Healthcare
LVMH Moet Hennessy	-2.62	Consumer Cyclical
Michelin Société	3.55	Consumer Cyclical
SAP SE	3.84	Technology
Siemens Aktiengesellschaft	3.92	Industrials
Bayer Aktiengesellschaft	3.86	Healthcare
Mercedes-Benz Group AG	3.77	Consumer Cyclical
Deutsche Telekom AG	3.45	Communication Services
Allianz SE	4.20	Financial Services
Tesco PLC	3.79	Consumer Defensive
Vodafone Group	3.66	Communication Services
AstraZeneca PLC	6.47	Healthcare
Shell PLC	3.65	Energy
Unilever PLC	3.85	Consumer Defensive
Persimmon PLC	4.26	Consumer Cyclical
Sony Group Corporation	3.61	Technology
Nissan Motor Co., Ltd.	4.07	Consumer Cyclical
Nikon Corporation	-2.82	Consumer Cyclical
Takeda Pharmaceutical	3.44	Healthcare
SoftBank Group Corp.	3.60	Communication Services
Subaru Corporation	3.57	Consumer Cyclical
Total	100.00	

Table B.1. Portfolio composition by sector on 30 January 2023. The table showcases the portfolio´s investment in stocks across different sectors, demonstrating the portfolio's diversification.

Appendix C. Decomposition by asset class and market

Description	Decomposition by asset class and market					Total
	Equity			Bonds		
	US	European	Asian	US	European	
Marginal VaR (%)	6.64	**22.85**	5.58	38.34	26.60	**38.34**
Marginal VaR (EUR)	12 940.04	**44 505.41**	10 861.79	74 671.11	51 791.17	**74 671.11**

Table C.1. VaR Decomposition by asset class and market at 30 January 2023. This table showcases how much stocks and bonds from the U.S. and European markets contribute to the VaR. The asset class and market with the largest contribution to the portfolio's VaR on the day is highlighted in bold.

Appendix D. BCP test results for the sub-periods, for all the models that passed UC test

	***p*-value (%)**									
	Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**	**Lag 6**	**Lag 7**	**Lag 8**	**Lag 9**	**Lag 10**
2022-2023	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00
2021-2022	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2020-2021	0.28	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2019-2020	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2018-2019	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00
2017-2018	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2016-2017	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2015-2016	0.00	0.01	0.03	0.07	0.16	0.32	0.59	1.00	1.61	2.46
2014-2015	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2013-2014	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00

Table D.1. Model number 3 BCP test results for the sub-periods.

	p-value (%)									
	Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**	**Lag 6**	**Lag 7**	**Lag 8**	**Lag 9**	**Lag 10**
2022-2023	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00
2021-2022	84.93	96.43	99.07	99.74	99.94	99.99	100.00	100.00	100.00	100.00
2020-2021	14.28	0.67	0.22	0.08	0.07	0.03	0.02	0.02	0.00	0.00
2019-2020	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2018-2019	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2017-2018	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2016-2017	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2015-2016	0.00	0.01	0.03	0.07	0.16	0.32	0.59	1.00	1.61	2.46
2014-2015	74.95	90.26	95.83	5.35	9.30	14.54	20.94	28.23	36.10	44.18
2013-2014	79.92	93.70	97.82	99.20	99.70	99.89	99.96	99.98	99.99	100.00

Table D.2. Model number 5 BCP test results for the sub-periods.

	p-value (%)									
	Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**	**Lag 6**	**Lag 7**	**Lag 8**	**Lag 9**	**Lag 10**
2022-2023	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00
2021-2022	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2020-2021	0.28	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2019-2020	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2018-2019	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00
2017-2018	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2016-2017	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2015-2016	0.00	0.00	0.00	0.00	0.01	0.01	0.03	0.06	0.10	0.19
2014-2015	74.95	90.26	95.83	5.35	9.30	14.54	20.94	28.23	36.10	44.18
2013-2014	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00

Table D.3. Model number 6 BCP test results for the sub-periods.

	p-value (%)									
	Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**	**Lag 6**	**Lag 7**	**Lag 8**	**Lag 9**	**Lag 10**
2022-2023	0.01	0.06	0.18	0.45	0.98	1.89	3.34	5.45	8.35	12.09
2021-2022	11.53	25.27	38.79	50.87	61.65	0.32	0.58	0.98	1.56	2.35
2020-2021	0.28	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2019-2020	79.92	93.70	97.82	99.20	99.70	99.89	99.96	99.98	99.99	100.00
2018-2019	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2017-2018	3.15	9.07	17.34	27.15	37.59	47.87	57.41	65.87	73.11	79.11
2016-2017	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2015-2016	0.28	1.09	2.75	5.53	9.59	14.95	21.45	28.84	36.78	44.91
2014-2015	79.92	93.70	0.17	0.42	0.92	1.76	0.01	0.02	0.03	0.00
2013-2014	79.92	93.70	97.82	99.20	99.70	99.89	99.96	99.98	99.99	100.00

Table D.4. Model number 8 BCP test results for the sub-periods.

	p-value (%)									
	Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**	**Lag 6**	**Lag 7**	**Lag 8**	**Lag 9**	**Lag 10**
2022-2023	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00
2021-2022	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2020-2021	1.72	0.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2019-2020	79.92	93.70	97.82	99.20	99.70	99.89	99.96	99.98	99.99	100.00
2018-2019	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2017-2018	92.90	99.21	99.90	99.99	100.00	100.00	100.00	100.00	100.00	100.00
2016-2017	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2015-2016	0.28	1.09	2.75	5.53	9.59	14.95	21.45	28.84	36.78	44.91
2014-2015	74.95	90.26	2.69	5.41	9.40	14.68	0.95	1.64	2.66	0.18
2013-2014	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00

Table D.5. Model number 9 BCP test results for the sub-periods

	p-value (%)									
	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
2022-2023	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00
2021-2022	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00
2020-2021	0.28	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2019-2020	74.95	90.26	95.83	98.12	99.13	99.59	99.80	99.90	99.95	99.98
2018-2019	79.92	93.70	97.82	99.20	99.70	99.89	99.96	99.98	99.99	100.00
2017-2018	87.69	97.62	99.49	99.88	99.97	99.99	100.00	100.00	100.00	100.00
2016-2017	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2015-2016	0.00	0.01	0.03	0.07	0.16	0.32	0.59	1.00	1.61	2.46
2014-2015	65.24	81.53	24.54	9.51	14.98	21.46	9.69	13.69	18.37	8.97
2013-2014	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00

Table D.6. Model number 12 BCP test results for the sub-periods

	p-value (%)									
	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
2022-2023	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2021-2022	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2020-2021	0.28	1.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2019-2020	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2018-2019	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00
2017-2018	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2016-2017	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2015-2016	1.72	5.43	11.27	18.96	27.94	37.55	47.15	56.25	64.49	71.70
2014-2015	70.05	86.17	11.13	18.76	27.69	37.26	9.28	13.41	18.32	4.87
2013-2014	89.96	98.41	99.72	99.95	99.99	100.00	100.00	100.00	100.00	100.00

Table D.7. Model number 13 BCP test results for the sub-periods