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Dynamic hedging and risk management: a Value-at-Risk analysis in a diversified portfolio

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Master in Finance

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PhD, António Manuel Rodrigues Guerra Barbosa, Assistant Professor, ISCTE-IUL

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Resumo

A volatilidade observada durante a pandemia de COVID-19 e as tensões geopolíticas destacou a necessidade da gestão de risco nos mercados financeiros. Esta dissertação tem como objetivo estimar perdas potenciais e implementar estratégias dinâmicas de cobertura utilizando a métrica de risco Value-at-Risk. O estudo mede e gere o VaR diário de uma carteira diversificada composta por ações e obrigações dos Estados Unidos, Europa e Ásia. Considerando 15 configurações de quatro modelos de VaR, Parametric Normal, Skewed Generalized Student-t, Historical Simulation, and Quantile Regression, os modelos são avaliados através de backtesting. O modelo escolhido é utilizado para estimar o VaR diário ao longo de um horizonte de um ano e para decompor o risco, identificando os principais fatores de risco para o portefólio. Desta forma, é implementada uma estratégia dinâmica de cobertura de ações para mitigar o risco, garantindo que o risco da carteira se mantém dentro de um limite previamente definido. A eficácia da estratégia é avaliada através de métricas de desempenho como o Return on Risk-Adjusted Capital e o Profit and Loss. Os resultados mostram que a carteira coberta apresenta um desempenho superior à não coberta, ao proteger contra perdas adicionais e aumentar o Return on Risk-Adjusted Capital. Para além disto, melhora a diversificação das exposições em ações que foram cobertas e redistribui o risco entre os fatores de risco da carteira.

Keywords: Value-at-Risk, Economic Capital, Backtesting, Marginal VaR Contributions, Equity Hedging, Return on Risk-Adjusted Capital
JEL Classification: G11, G32

Abstract

The volatility observed during the COVID-19 pandemic and geopolitical tensions has highlighted the need for risk management in financial markets. This thesis aims to estimate potential losses and implement dynamic hedging strategies using the Value-at-Risk risk metric. The study measures and manages the daily VaR of a diversified portfolio composed of equities and bonds from the U.S., European, and Asian markets. Considering 15 configurations from four different VaR models, Parametric Normal, Skewed Generalized Student-t, Historical Simulation, and Quantile Regression, the modes are evaluated through backtesting. The chosen model is then used to estimate daily VaR over a one-year horizon and to decompose it by risk factors, identifying the main contributors to risk. By doing so, a dynamic equity hedging strategy is implemented to mitigate risk, ensuring that the portfolio risk remains within a predefined target. The effectiveness of the strategy is assessed using performance metrics such as Return on Risk-Adjusted Capital and Profit and Loss. Results show that the hedged portfolio outperforms the unhedged portfolio by protecting against additional losses and increasing the Return on Risk-Adjusted Capital, while also improving diversification across hedged equity exposures and redistributing risk across portfolio risk factors.

Keywords: Value-at-Risk, Economic Capital, Backtesting, Marginal VaR Contributions, Equity Hedging, Return on Risk-Adjusted Capital

JEL Classification: G11, G32

Contents

List of Abbreviations

AEX: Amsterdam Exchanges index

BCP: Berkowitz, Christoffersen and Pelletier

CAC 40: Cotation Assistée en Continu 40

DAX: Deutscher Aktienindex

DE: Diversification Effect

EC: Economic Capital

EUR: Euro

EWMA: Exponentially Weighted Moving Average

FTSE: Financial Times Stock Exchange

GBP: Great British Pound

HKG: Hong Kong Dollar

HSI: Hang Seng Index

IBEX: Iberian Index

JPY: Japanese Yen

N225: Nikkei 225

P&L: Profit and Loss

PV: Present Value

PV01: Present Value of a Basis Point

QR: Quantile Regression

RORAC: Return on Risk-Adjusted Capital

S&P500: Standard & Poor's 500

SGSt: Skewed Generalized Student-t

UC: Unconditional Coverage

USD: United States Dollar

VaR: Value-at-Risk

CHAPTER 1

INTRODUCTION

In today's highly interconnected and volatile financial markets, effective risk management is fundamental for individual investors and institutions. Market disruptions, such as those caused by the COVID-19 pandemic and ongoing geopolitical tensions, have emphasized the need to accurately measure, manage, and mitigate potential risks.

In financial risk management, quantifying potential losses is essential for making informed investment decisions and protecting portfolios from adverse market movements. In this context, during the 1990s, Value-at-Risk (VaR) was widely adopted for measuring market risk in trading portfolios. By quantifying risk into a single numerical estimate of potential losses over a specified time horizon (Hendricks, 1996), VaR allows financial managers to plan and be more prepared for adverse outcomes (Jorion, 2002).

The motivation for this thesis arises from the critical importance of risk management in protecting portfolios. In this field, VaR is a widely used measure, however, while it provides an understanding of the potential downside risk, it does not indicate how this risk can be reduced. The objective of the study is to measure and actively manage VaR by identifying the sources of risk and implementing a hedging strategy to mitigate them.

The portfolio includes 30 stocks with long and short positions from different sectors and markets, such as S&P500, AEX, DAX, CAC 40, IBEX 35, FTSE 100, Nikkei 225, and Hang Seng index. The bond portion consists of six sovereign bonds rated Aaa on the Moody's scale, issued by different countries, including the U.S., Germany, and the Netherlands.

As emphasized by Jorion (2007), the effectiveness of VaR models depends on the portfolio's composition and the return distributions of its underlying assets. A model that performs well for one portfolio may not be the best for the other. To account for this, we tested 15 configurations across four classes of models: Parametric Normal, Skewed Generalized Student-t (SGSt), Historical and Quantile Regression.

Over a 10-year global backtesting period, from 11 February 2013 to 27 January 2023, we computed a series of daily historical VaR estimates for each model under analysis, as if the current portfolio had existed throughout the past. This approach allows us to backtest all 15 models by comparing predicted losses with actual experienced losses. The accuracy of the estimates is evaluated using the Unconditional Coverage (UC) and the Berkowitz, Christoffersen, and Pelletier (BCP) tests.

After identifying the model that best fits the portfolio, VaR is measured and actively managed going forward from 27 January 2023 to 2 February 2024. By decomposing risk through marginal VaR decompositions, which quantify each component's contribution to total portfolio risk, we implement a dynamic hedging strategy to maintain risk within a predefined target. The target level of capital held to cover the portfolio's risk exposure was based on the average level observed under normal volatility conditions over the past 10 years. This level was defined as € 95 000, corresponding to 1.2% of the portfolio's value as of the mapping date. For one year, whenever VaR exceeds € 95 000, we implement dynamic equity hedging, otherwise, hedging positions are removed.

As highlighted by Jorion (2002), hedging enhances portfolio resilience but can also alter the risk profile, as mitigating some exposures can unintentionally increase others. Therefore, a key question arises: What is the impact of hedging on this portfolio?

To evaluate the effectiveness of this strategy, we compare performance metrics such as Profit and Loss (P&L), marginal VaR, diversification effect, exceedance rates, and RORAC both before and after hedging.

The structure of this thesis is as follows: Chapter 2 provides the theoretical background on VaR and the relevant literature; Chapter 3 describes the composition of the selected portfolio, contextualizes key market events during the analysis period, and presents the descriptive statistics; Chapter 4 outlines the methodology, including the risk factor mapping, the estimation of volatility and covariance, and the VaR models; Chapter 5 presents the model selection process based on backtesting results; Chapter 6 reports the results of actively managing VaR through the equity hedging strategy, and Chapter 7 concludes the study and offers suggestions for future research.

CHAPTER 2

LITERATURE REVIEW

In finance, risk relates to the unpredictable fluctuations that can influence the value of assets, equity, or earnings (Jorion, 2007). This concept, understood in risk management as the difference between actual and expected outcomes, covers both potential gains and losses (Moreira & Muir, 2017). Planning for the consequences of these outcomes is only possible if financial managers understand risk, enabling them to be prepared for inevitable uncertainties (Jorion, 2009), such as the economic crises of the 1990s (for more details on this topic, see Dell'Ariccia et al. (2008)). These events emphasized the impact of market risk, one of the four super-categories identified by Bankers Trust. Liquidity, credit, and operational risks also play a crucial role in maintaining financial stability and must be effectively managed to prevent similar disruptions (Kaplan & Mikes, 2016).

With the growing need for better risk management tools and a standardized metric to measure market risk, in the late 1980s, the Bankers Trust pioneered the adoption of Value-at-Risk (VaR), a statistical risk measure that estimates the maximum potential loss over a given future time horizon and for a given confidence level (Best, 2000). In 1996, J.P. Morgan introduced RiskMetrics (RM), a system that standardized and popularized the use of VaR in financial risk management. Also, the Bank for International Settlements (2023) helped VaR's widespread acceptance as a global standard for measuring and managing risk in financial institutions by allowing banks to use their internal VaR models to calculate the amount of regulatory capital required (Linsmeier & Pearson, 1996). Given its widespread adoption and regulatory relevance, this study adopts VaR as the primary risk measure. Expected Shortfall is acknowledged as a complementary metric that provides additional information into extreme losses beyond the VaR threshold.

In terms of regulation, a financial institution is required by the Market Risk Amendment to have a minimum of capital to cover risk and ensure its solvency. However, regarding management systems, the approach to internal capital allocation differs (Alexander, 2009). Economic capital, estimated using internal methods approved by shareholders (Alexander, 2009), is the amount of capital needed to protect against the potential losses that may happen due to the risk exposures (Tiesset & Troussard, 2005). The allocation of economic capital follows a top-down approach: after determining the total requirement, it is distributed across business units according to their respective risks. The more economic capital allocated to a specific activity, the greater the risks it can take (Alexander, 2009). Thus, VaR is an essential tool for risk managers, as it provides a single quantitative measure of potential losses, summarizing the risk profile of each business unit (Hendricks, 1996). By using VaR models, risk managers can select the most appropriate method to manage the specific risks associated with their portfolios (Jorion, 2007), aligning the economic capital and regulatory capital requirements (Alexander, 2009).

Accurately estimating VaR depends on the distribution of portfolio returns (Trenca et al., 2011). There are various methods for measuring VaR, which can be classified as either parametric or non-parametric models. According to Ammann and Reich (2001), the parametric models are based on statistical parameters of the risk factor distribution, such as the Parametric Normal model and the Skewed Generalized Student-t (SGSt). On the other hand, non-parametric models don't make any assumption about the statistical distribution, like the Historical VaR and the Quantile Regression VaR (QR) models. These are the fours models that will be studied.

Among these categories, the Parametric Normal VaR model is the simplest to use in practice (Dowd, 1998), since it assumes that returns follow a normal distribution over time (Morgan, 1996). This topic is included in several finance studies, however, many of them reach the same conclusion: the Normal VaR model is the one that provides the least accurate VaR estimation. By ignoring fat tails in the distribution, it fails to predict extreme losses, as explored by Hendricks (1996). Hendricks studied 12 approaches across 1,000 randomly selected foreign exchange portfolios and found that the Normal VaR cannot produce reliable estimates for these portfolios, as the normal distribution fails to predict extreme outcomes. Similarly, this problem also arises in stock portfolios, as investigated by Pafka and Kondor (2001). Their study analyzed the performance of the RM model (a parametric approach based on normally distributed returns) for market risk using 4 years of data from the 30 stocks in the Dow Jones Industrial Average. They demonstrated that if returns follow a normal distribution, the reliability of the model is impacted during extreme events, particularly under volatile conditions, since it underestimates extreme risks, especially at higher confidence levels like 99%. Nevertheless, during stable market periods, this issue does not occur, and VaR estimates can be reasonably accurate. The authors also highlighted an important feature of financial markets: financial data frequently exhibit a higher probability of extreme events than normal distribution would predict. This is represented by fat tails, as well as negative skewness.

To better capture the characteristics of financial data, the SGSt distribution can be used. Introduced by Theodossiou (1998), it extends the traditional Student-t distribution initially proposed by McDonald and Newey (1988), by addressing its inability to capture the asymmetry in the asset return distributions. This approach aligns with Giot and Laurent (2003), who not only addressed asymmetry but also incorporated kurtosis in their analysis. They applied the SGSt distribution to estimate VaR using daily returns from stock indices such as the FTSE, NASDAQ, and NIKKEI, as well as individual US stocks like Alcoa, McDonald's, and Merck. Their methodology enhanced the accuracy of VaR predictions for both long and short trading positions by effectively capturing asymmetry and fat tails in the financial data. They found that incorporating skewness and kurtosis into VaR estimation outperforms the Normal VaR model, providing a more accurate representation of reality.

Instead of making assumptions about the statistical distribution of returns, an alternative approach can be used: the Historical Simulation. Also referred to as Historical VaR, this method assumes that all possible future variations have already occurred in the past, which automatically incorporates the fat tails and skewness. Nevertheless, as explored by Hendricks (1996) and van den Goorbergh and Vlaar (1999), the Historical Simulation generally provides more accurate VaR estimations as the sample size increases. However, increasing the sample size means including older and less relevant data, which may not reflect the current market conditions. With this in mind, Boudoukh et al. (1998) and Barone-Adesi et al. (1998) proposed an improvement to the conventional methodology by attributing more weight to recent observations. Rather than prioritizing recent observations over older ones, Hull and White (1998) proposed a refined method that adjusts the volatility of the entire series of returns, while still giving the same weight to each observation. This means that the impact of a past return is modified to reflect current market volatility conditions. The authors analyzed 9 years of daily data from 12 exchange rates and 5 stock indexes, and revealed that the proposed volatility adjustment method outperformed the methodology of Boudoukh et al. (1998), especially at a 99% confidence level. This topic was also explored by van den Goorbergh and Vlaar (1999), who evaluated both the traditional historical model and the one proposed by Hull and White. The empirical evaluation involved a fictitious investment in the Dutch stock index AEX, using 16 years of data. The study demonstrated that volatility adjustment techniques can produce more accurate VaR estimates compared to the traditional historical model, particularly in periods of heavy fluctuations. This happens because it can deal much better with the changing distribution by giving decaying weights to past observations.

However, since estimating VaR is equivalent to identifying the conditional quantile of the return distribution (Christoffersen et al., 2001), it is important to consider methods that naturally capture this, such as Quantile Regression. Introduced by Koenker and Bassett Jr (1978), QR does not assume a specific parametric form for the return distribution. Instead, it directly estimates the conditional quantiles. In 2019, Westgaard, Århus, and Frydenberg compared this method to the RM parametric model and to the HS incorporating volatility adjustments. The purpose of the study was to evaluate whether a model effective in financial markets exhibits the same behavior in power markets, which are known for their volatility (Chan & Gray, 2006; Haugom et al., 2014). The study focused on the European energy markets to determine which model most effectively captured price changes in these markets. The authors analyzed 9 different energy futures over the period from 2007 to 2017, applying the models across 3 different confidence levels: 99%, 95%, 90%. Their findings reveal the QR model significantly outperforms both the Parametric and Historical VaR models, being the only one that provides accurate forecasts across all specified confidence levels. This performance aligns with the theoretical strengths of QR, confirming its effectiveness in capturing complex return distributions, particularly in the energy markets where traditional models may be less effective.

In volatile markets like these, as well as others, accurately estimating return volatility is essential, since VaR is a forward-looking risk measure. To model volatility, several approaches have been proposed, including the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) and the Exponentially Weighted Moving Average (EWMA) model, which became widely used due to its straightforward implementation and integration into the RiskMetrics framework. In this study, we adopt the EWMA model to estimate the volatility and covariances for the four classes of VaR models.

Considering all the VaR models discussed, it is crucial to evaluate how accurately they predict potential future losses. The validity of the effectiveness of these models can be used by backtesting, a common approach where statistical tests are applied to a historical sample of portfolio returns and their corresponding VaR estimates. This procedure compares actual profits and losses against the projected VaR estimates, serving as "reality checks". If discrepancies arise, the models must be reassessed to identify potential issues, such as incorrect assumptions, inaccurate parameters, or errors in modeling (Jorion, 2002).

Several testing methods have been proposed for backtesting. In this study, the test of Kupiec (1995) is used to evaluate how often the losses exceed the VaR estimate. This test checks whether the observed number of exceedances is consistent with the number expected by the VaR model. However, even if the number of exceptions is statistically as expected, exceedances may occur in clusters. To analyze this, we adopt the BCP test (Berkowitz et al., 2011), which checks if the exceedances are independent of each other. If this is the case, the model is well specified, and the model can adapt quickly to market conditions.

When the VaR exceeds its set pre-defined maximum value, risk management actions are needed to meet the EC quotas. In such instances, derivatives become a critical tool for managing financial risks. By using derivatives to hedge a wide range of risks, the portfolio's resilience is enhanced, ensuring it remains aligned with the desired risk profile. However, reducing exposure to some market risks will change the overall risk profile, as some risks may be mitigated while others could potentially increase (Jorion, 2002).

Additionally, when considering portfolio strategies, there is a fundamental principle that higher risk is associated with the potential for higher returns (Sharpe, 1970). Despite this, it is important to consider a risk metric that accounts for both the returns and the risks incurred to achieve them. RORAC addresses this by quantifying the return on risk-adjusted capital. By incorporating both risk exposure and capital requirements, RORAC provides a comprehensive view of how changes in risk exposures affect the returns relative to the capital at risk, ensuring that decisions align with the overall risk management strategy (Alexander, 2008).

CHAPTER 3

DATA

The objective is to determine, as of 27 January 2023, what is the most appropriate Value-at-Risk (VaR) model to evaluate the risk of the portfolio until 2 February 2024, corresponding to a period of 265 trading days. Therefore, to assess the reliability of the models we backtest the different VaR models over a 10-year period.

The chosen portfolio includes equities and bonds, and the dataset spans from 2 January 2007 to 2 February 2024, covering a total of 17 years and 1 month to give us the flexibility to use large sample sizes for the VaR models in the backtesting.

The objective was to build a well-diversified portfolio, balancing exposure across sectors and individual positions (such as long and short). To guarantee sectorial diversification, we assigned at least one short position per sector. Thus, the portfolio consists of 30 equities, with 19 long and 11 short positions, chosen from 8 different sectors and 5 markets. These stocks belong to major indices in the United States (U.S.), Europe, Japan, UK, and Hong Kong (HK). The investment in each stock was designed to establish a balanced approach, with individual stock weights ranging from -3.5% to 4.5% of the portfolio's overall weight as of the mapping date, 27 January 2023. The Euro (EUR) is the local currency, thereby we identify the foreign currencies as the U.S Dollar (USD), the British Pound (GBP), the Japanese Yen (JPY), and the HK Dollar (HKD). The daily data for stocks, their respective indexes, and currencies were retrieved from Yahoo Finance (https://finance.yahoo.com). This includes daily adjusted closing prices in local currency for each stock, as well as the corresponding exchange rates (USD/EUR, GBP/EUR, JPY/EUR, and HKD/EUR).

The bond portion consists of six sovereign bonds issued by different markets, including the U.S. (denominated in USD), Germany, and the Netherlands (in EUR). These bonds were chosen for their higher coupon rates and low credit risk, as they are issued by countries with the best credit ratings, which means the credit risk events are minimal. According to Moody's scale, the credit rating for these countries is set at Aaa, the highest quality and lowest risk of default. Information on these fixed-income bonds was obtained from Borse Frankfurt (https://www.boerse-frankfurt.de/bonds). Each bond has a fixed coupon rate and a different payment frequency, with maturities of up to 20 years. For the daily USD interest rate data, the source used was the Federal Reserve Economic Data website (https://www.federalreserve.gov/dat adownload/Choose.aspx?rel=H15), while for EUR interest rate data, based on AAA-rated sovereign bonds, was retrieved from the European Central Bank website (https://sdw.ecb.europa.eu/browseSelection.do?node=9689726).

3.1. Portfolio Composition

As of the mapping date, 27 January 2023, the total portfolio value was €7 674 324, with 53.75% allocated to bonds and 46.25% to stocks.

Table 3.1 presents the bond composition of the portfolio, while Table 3.2 details the stock composition.

Bonds	ISIN	Currency	Maturity Date	Coupon (%)	Coupon per Year	Face Value (EUR)	Fair Value (EUR)	Weight (%)
German Bund 2027	DE0001135044	EUR	2027-07-04	6.50	1	800 000	968 505	12.62
German Bund 2030	DE0001135143	EUR	2030-01-04	6.25	1	500 000	627 176	8.17
Dutch State Loan 2028	NL0000102317	EUR	2028-01-15	5.50	1	650 000	747 672	9.74
US Treasury 2025	US912810ET17	USD	2025-02-15	7.63	2	917 900	1 011 251	13.18
US Treasury 2037	US912810PU60	USD	2037-05-15	5.00	2	321 265	374 644	4.88
US Treasury 2028	US91282CJF95	USD	2028-10-31	4.88	2	367 160	395 499	5.15
Total Bonds Value							**4 124 748**	**53.75**

Table 3.1. Bond portfolio composition on the mapping date. The table presents details for each bond, including fair value and its weight in the total portfolio (stocks and bonds).

Stock	Ticker	Currency	Sector	No. Shares	Value (EUR)	Weight (%)
Alphabet Inc.	GOOG	USD	Communication Services	3 567	323 546	4.22
Amazon.com, Inc.	AMZN	USD	Consumer Cyclical	1 500	140 769	1.83
Apple Inc.	AAPL	USD	Technology	-400	-53 142	-0.69
Booking Holdings Inc.	BKNG	USD	Consumer Cyclical	111	225 640	2.94
General Electric	GE	USD	Industrials	-1000	-60 634	-0.79
Mastercard Incorporated	MA	USD	Financial Services	383	102 235	1.33
McDonald's Corporation	MCD	USD	Consumer Cyclical	1 300	316 122	4.12
NIKE, Inc.	NKE	USD	Consumer Cyclical	2 500	287 886	3.75
NVIDIA Corporation	NVDA	USD	Technology	1 331	242 890	3.16
The Goldman Sachs Group	GS	USD	Financial Services	500	155 297	2.02
The Walt Disney Company	DIS	USD	Communication Services	2 800	280 616	3.66
Thermo Fisher Scientific Inc.	TMO	USD	Healthcare	-500	-262 392	-3.42
Walmart Inc	WMT	USD	Consumer Defensive	-5 000	-214 456	-2.79
Airbus SE	AIR.PA	EUR	Industrials	3 000	342 841	4.47
Allianz SE	ALV.DE	EUR	Financial Services	-700	-139 666	-1.82
Bayer Aktiengesellschaft	BAYN.DE	EUR	Healthcare	5 500	294 654	3.84
BNP Paribas SA	BNP.PA	EUR	Financial Services	5 000	271 644	3.54
Danone S.A.	DANOY	EUR	Consumer Defensive	-8 000	-80 381	-1.05
Deutsche Post AG	DHL.DE	EUR	Industrials	6 218	190 704	2.48
Heineken N.V.	HEIA.AS	EUR	Consumer Defensive	4 109	352 025	4.59
Industria de Diseño Textil, S.A.	ITX.MC	EUR	Consumer Cyclical	-400	-10 821	-0.14
Meliá Hotels International, S.A.	MEL.MC	EUR	Consumer Cyclical	25 000	152 000	1.98
Orange S.A.	ORAN	EUR	Communication Services	-6 334	-87 838	-1.14
Siemens Aktiengesellschaft	SIE.DE	EUR	Industrials	-1 100	-149 697	-1.95
Veolia Environment SA	VIE.PA	EUR	Industrials	5 000	127 517	1.66
Volkswagen AG	VOW3.DE	EUR	Consumer Cyclical	3 000	322 888	4.21
Unilever PLC	ULVR.L	GBP	Consumer Defensive	5 000	217 082	2.83
Nintendo Co., Ltd	7974.T	JPY	Communication Services	9 000	340 663	4.44
Sony Group Corporation	6758.T	JPY	Technology	-750	-61 029	-0.80
Hong Kong and China Gas Co., Ltd	0003.HK	HKD	Utilities	-20 000	-17 389	-0.23
Total Equity Value					**3 549 576**	**46.25**

Table 3.2. Stock portfolio composition on the mapping date. The table presents the amount invested in EUR for each stock, its sector, and its weight in the total portfolio (stocks and bonds) on 27 January 2023.

3.2. Contextualization

The data used in this study span periods of economic turbulence. Major events, such as financial crises, pandemics, and geopolitical disruptions, led to changes in market volatility that affected the portfolio risk profile. Thus, it is important to have an overview of the major economic events that occurred during the selected period. The events described were chosen due to their impact on the markets covered in this study, primarily in Europe and the U.S.

Among the most impactful developments was the Brexit referendum in 2016, where the UK's vote to leave the European Union. This event caused significant volatility in currency markets, particularly a depreciation of the British pound (GBP) against the euro (EUR). It also had an immediate negative impact on the liquidity of non-US stocks, particularly those from the UK and EU.

The worldwide health crisis, Covid-19 Pandemic (2020-2021), caused economic downturns and some of the major indices, such as the S&P 500, lost up to 30% of their value between mid-February and mid-March 2020. In terms of inflation, in the Eurozone, it decreased to negative values by late 2020, and in the US, it had also fallen sharply. However, as economies started to reopen, inflation began to rise again. In response, the Federal Reserve (Fed) and the European Central Bank (ECB) shifted from supporting economic recovery to tackling growing inflation by raising interest rates in 2021, with the Fed raising interest rates by more than four percentage points by 2022.

In 2022, the major geopolitical event of the Russian invasion of Ukraine increased volatility in energy prices, currency fluctuations, and impacted the global supply chains. Stock markets globally experienced sharp sell-offs in response to the uncertainty. Central banks, particularly in the US and Europe, were forced to restrict monetary policy in response to war-induced inflation. According to Eurostat, Eurozone inflation peaked at 10.6% in October 2022, while US inflation reached 9.1% in June 2022, the highest level in 40 years. In order to reduce inflation to the 2% target level, the ECB began to raise interest rates in July 2022, marking its increase in over a decade. Meanwhile, the Federal Reserve began its tightening cycle earlier, with a 0.25% hike in March 2022, and further 0.75% hikes in June and July 2022.

3.3. Descriptive Statistics

To better understand the characteristics of the portfolio returns over time, this section covers key statistical measures, including skewness and kurtosis, as well as the p-value of the Jarque-Bera normality test.

Table 3.3 shows these statistics for the daily portfolio returns (or absolute change in interest rates) for each year and the entire period.

Data Range	**Kurtosis**	**Skewness**	**Mean (%)**	**Std Dev (%)**	**Minimum (%)**	**Maximum (%)**	**Jarque Bera p-value (%)**
2023	3.812	0.763	0.170	0.328	-0.302	0.900	41.690
2022-2023	3.487	0.276	-0.029	0.452	-1.043	1.569	6.390
2021-2022	4.256	-0.242	0.030	0.284	-0.910	0.954	$1.0\,\mathrm{e}^{-2}$*
2020-2021	10.536	-0.749	0.013	0.693	-3.780	3.132	$2.2\,\mathrm{e}^{-14}$*
2019-2020	4.344	-0.276	0.045	0.262	-0.932	0.944	$2.1\,\mathrm{e}^{-03}$*
2018-2019	3.500	-0.322	-0.010	0.329	-1.047	0.828	3.291*
2017-2018	4.593	-0.028	0.020	0.243	-0.890	1.154	$3.0\,\mathrm{e}^{-04}$*
2016-2017	5.463	-0.120	0.029	0.455	-2.095	1.694	$2.2\,\mathrm{e}^{-12}$*
2015-2016	4.420	-0.258	0.067	0.551	-2.178	1.775	$8.9\,\mathrm{e}^{-04}$*
2014-2015	4.626	-0.260	0.033	0.441	-1.763	1.528	$3.4\,\mathrm{e}^{-05}$*
2013-2014	3.173	-0.046	0.062	0.491	-1.494	1.313	86.900
Global Backtest Period	9.645	-0.375	0.027	0.440	-3.780	3.132	$2.2\,\mathrm{e}^{-14}$*

Table 3.3. Summary statistics and Jarque-Bera test *p*-values. The Global Backtest Period spans from 11 February 2013 to 27 January 2023. The null hypothesis of normality is rejected at the 95% confidence level when the p-value is below 5%. P-values marked with * indicate rejections at the 5% significance level.

As shown in Table 3.3, the portfolio's skewness is negative in most years (except for 2022 and 2023), meaning the distribution's left tail is longer or fatter than the right tail. Kurtosis changes over time, peaking in 2020–2021, a period characterized also by high volatility (a standard deviation of 0.693%). This indicates that the return distribution has heavy tails and a higher probability of extreme events, which reflects the impact of the COVID-19 pandemic. The maximum and minimum returns for the entire period occur during this time, on 2023-03-24 and 2020-03-12, respectively.

Focusing on the results of the Jarque-Bera test for the global backtest period, the null hypothesis of normality is rejected as the p-value is below the significance level (5%). This result anticipates that the VaR model based on the normal distribution is unlikely to perform well, reinforcing the need to consider alternative models for estimating VaR that account for the non-normality of returns.

CHAPTER 4

METHODOLOGY

The objective of this study is to manage market risk using VaR over one year. This thesis aims to analyze the impact of hedging positions on the overall portfolio and individual classes of risk, as well as the diversification effects within a portfolio of stocks and bonds. However, to manage a portfolio's VaR, it must first be accurately estimated.

For this analysis, we consider today as 27 January 2023. On this date, we map the portfolio to its risk factors so that we can estimate the daily VaR. However, since the primary goal is to measure VaR precisely, it is crucial to consider the following question: Which model can more accurately predict VaR over the next year, assuming the portfolio as of today?

To answer this question, we consider four different classes of VaR models: Parametric Normal, Skewed Generalized Student-t (SGSt), Historical VaR, and Quantile Regression (QR). A total of 15 models with different configurations were submitted to backtesting from the global period (11 February 2013 to 27 January 2023). In this phase, using the global period as the test period, we perform two key tests: the UC test (Kupiec, 1995) and the BCP test (Berkowitz et al., 2011). Based on the backtesting results, we select the most appropriate VaR model for our portfolio and estimate the VaR for the one year period from 30 January 2023 to 2 February 2024.

This chapter covers the steps for measuring VaR considering the different models. These steps include mapping portfolio risk factors, determining portfolio returns, modeling portfolio volatility, and computing each VaR model. Chapter 5 focuses on the results of the backtest, which are used to select the most appropriate VaR model for the one-year forward period starting from 30 January 2023. The final chapter analyzes the impact of hedging on the portfolio, examining its effects on risk, return, and diversification.

4.1. Risk Factor Exposure Mapping

To estimate VaR it is crucial to highlight that the model's performance and the accuracy of backtesting depend on how well the portfolio's risk exposure is mapped to the different risk factors.

Mapping a portfolio into risk factors begins by identifying and quantifying the relevant exposures using a method known as risk factor mapping. This approach links each portfolio position with its corresponding risk factor exposure, ensuring a representation of the portfolio's overall exposure. The risk factors to which each position is mapped depend on the asset type, with different assets requiring specific risk factors. The following Figure 4.1 illustrates how the portfolio is mapped to a set of risk factors:

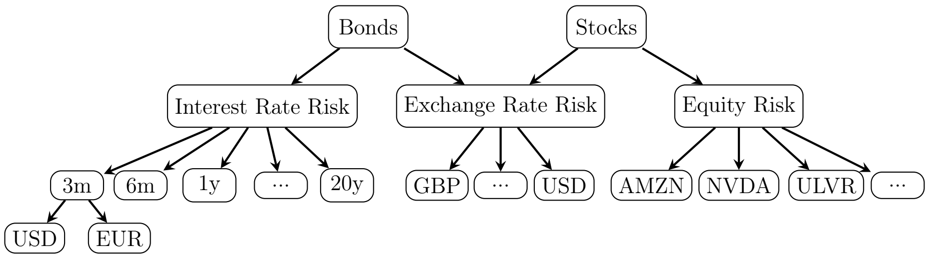

Figure 4.1. Risk factor mapping of the portfolio. The figure illustrates the relationships between assets and their corresponding risk factors.

To summarize, we map our portfolio to its risk factors as of 27 January 2023, according to the classification scheme in Figure 4.1. After quantifying these, we obtain the corresponding vector of risk factor loadings, as described in Section 4.2. This vector is then used to estimate the VaR every day for the 10 years ending on the mapping date. After selecting the best-performing VaR model, the VaR is estimated for the following year.

In the following sections, we dive into the different risk exposure mapping methodologies for each class of assets present in our portfolio. Appendix A displays the portfolio exposures for each risk factor (stocks, bonds, and currencies).

4.1.1. Stock Mapping

To map each asset to its corresponding risk factor, an important question must be addressed: What is the price of this type of asset sensitive to?

For stocks, the value and performance of an equity investment depend on stock price movements. In this type of asset class, the risk factor is the change in the stock's market price, where the risk exposure is determined by the amount of capital invested.

For risk factor mapping, sensitivity to price changes for each stock corresponds to the capital invested, calculated by multiplying the number of shares (N_{i_t}) by the respective market price (P_{i_t}):

$$M_{i_t} = N_{i_t} \times P_{i_t} \times FX_t, \tag{4.1}$$

where FX_t is the exchange rate on the mapping date (27 January 2023), used to convert the foreign stock exposure to the local currency, since the price must be expressed in the currency for which VaR is estimated.

4.1.2. Bond Mapping

A bond is a financial instrument that consists of a series of cash flows that include periodic interest payments (coupon payments) and the repayment of the principal amount (face value) upon maturity. Since these cash flows are predetermined, the value of a bond will fluctuate only due to changes in the appropriate discount rates applied to each cash flow's maturity. As we are calculating market risk, default risk, which constitutes credit risk, is not considered here. As a result, bond volatility is linked to the volatility of the interest rates used to discount their cash flows. When interest rates rise, discount rates rise, reducing the present value (PV) of the bond's future cash flows and, consequently, decreasing its price. Therefore, the primary risk factor is the interest rate used to discount the future cash flows.

The sensitivity of a bond's price to changes in interest rates is captured by the present value of a basis point ($PV01$). Thus, to quantify this metric, it is crucial to know the future cash flows to discount them to present value.

The present value of the cash flow C_T is:

$$PV_{C_T,r_T} = C_T \times e^{-r_T \times T}, \tag{4.2}$$

where T is the time from now until the cash flow date, and r_t is the annualized continuously compounded interest rate for time T.

To compute $PV01$, which measures the change in a bond's price for a one-basis point movement in interest rates, the PV is adjusted to reflect this change. For a single cash flow, the $PV01$ can be approximated using a first-order Taylor expansion as:

$$\begin{aligned} PV01_{C_T,r_T} &\approx \frac{\partial PV_{C_T,r_T}}{\partial r_T} \times (-0.01\%) \\ &= T \times PV_{C_T,r_T} \times 0.01\%. \end{aligned} \tag{4.3}$$

Given this, the first-order approximation of the P&L of a cash flow, which corresponds to the change in its PV, can be written as a function of $PV01$ as follows:

$$\begin{aligned} \Delta PV(c_T, r_T) &\approx \frac{\partial PV(c_T, r_T)}{\partial r_T} \times \Delta r_T \approx -PV01(c_T, r_T) \times \frac{\Delta r_T}{0.01\%} \\ &= -PV01(c_T, r_T) \times \Delta r_T\,(b.p.), \end{aligned} \tag{4.4}$$

where $\Delta r_T\,(b.p.)$ is the absolute change in the interest rate, expressed in basis points (0.01%).

In the case of a bond with several cash flows, the total P&L is the sum of the P&Ls of each cash flow, which can be expressed as:

$$P\&L \equiv \Delta PV(C,R) \approx \sum_{i=1}^{n} -PV01_{T_i} \times \Delta R_{T_i}(b.p.). \tag{4.5}$$

This can also be represented in vector form as:

$$\Delta PV \approx \Theta^T \times \Delta r(b.p.), \tag{4.6}$$

where

$$\Theta = \begin{bmatrix} -PV01_{T_1} & \dots & -PV01_{T_n} \end{bmatrix}^T \tag{4.7}$$

$$\Delta r(b.p.) = \begin{bmatrix} \Delta r_{T_1}(b.p.) & \dots & \Delta r_{T_n}(b.p.) \end{bmatrix}^T. \tag{4.8}$$

Determining the P&L from the sensitivity of bond prices to changes in interest rates may seem relatively straightforward for a small number of cash flows. However, for a bond with n cash flows, there are n different interest rates to consider as risk factors, one for each cash flow maturity. Furthermore, for large bond portfolios, cash flows can occur almost every day, making the portfolio sensitive to changes in interest rates at virtually all maturities, which can be a challenge.

To simplify this process, we use cash flow mapping to align the bond's cash flows with a set of standard maturities, known as the vertices of the cash flow map. For this analysis, the vertices were selected based on the maturities of the bonds in the portfolio, and historical rates available from the European Central Bank and Federal Reserve Economic Data website. The selected vertices of the cash flow map for each currency are detailed in Table 4.1.

Currency	**Risk Factor Vertices**									
	3M	**6M**	**1Y**	**2Y**	**3Y**	**5Y**	**7Y**	**10Y**	**15Y**	**20Y**
EUR	EUR3M	EUR6M	EUR1Y	EUR2Y	EUR3Y	EUR5Y	EUR7Y	EUR10Y	EUR15Y	EUR20Y
USD	USD3M	USD6M	USD1Y	USD2Y	USD3Y	USD5Y	USD7Y	USD10Y	-	USD20Y

Table 4.1. Vertices of the cash flow map by currency and maturity.

Nevertheless, when considering bonds with non-standard maturities or vertices where no interest rate is available, we adopt the vertices mapping approach suggested by Alexander (2008). With this approach, cash flows with non-standard maturities are mapped into standard vertices for which interest rate data is available.

By using the vertices mapping approach, we perform a PV+PV01 invariant cash flow mapping, where the PV of each cash flow is distributed to the lower and upper vertices, while still preserving the PV and $PV01$ of the original cash flow.

The $PV+PV01$ invariant conditions are the following:

$$\begin{cases} x_{T_L} + x_{T_U} = 1 \\ T_L x_{T_L} + T_U x_{T_U} = T \end{cases} . \tag{4.9}$$

The objective of this condition is to solve in order for x_L and x_U, which represent the proportion of PV_{C_T} assigned to vertices T_L and T_U, respectively, being PV_{C_T} determined by applying Equation 4.2, with the interpolated rate r_t between T_L and T_U.

The first condition ensures the preservation of the PV of the original cash flow. In this context, the PV of the original cash flow, denoted as PV_{C_T}, is distributed across two standard maturity vertices, T_L and T_U, where T_L is the nearest lower standard maturity and T_U is the nearest upper standard maturity ($T_L < T < T_U$). The second condition ensures the preservation of the $PV01$, guaranteeing that when there is a parallel shift of all spot rates by 0.01% in the interest rate curve, the combined P&L of the two mapped cash flows is equivalent to the P&L of the original cash flow.

By solving the condition above, the $PV+PV01$ invariant conditions ensure that a non-standard vertex can be mapped to two standard vertices as follows:

$$x_L = \frac{T_U - T}{T_U - T_L} \tag{4.10}$$

and

$$x_U = 1 - x_{T_L}. \tag{4.11}$$

In this way, we assign each proportion of the present value to the upper and lower vertices. The $PV + PV01$ invariant mapping process is repeated for each standard vertex maturity, with each exposure defined as -$PV01$.

4.1.3. Currency Risk

The portfolio includes positions in stocks and bonds from both local and foreign markets. Since VaR is estimated in EUR, there is exposure to exchange rate fluctuations between foreign currencies and the EUR. This means that stocks denominated in USD, JPY, HKG, and GBP are exposed to the USD/EUR, JPY/EUR, HKG/EUR, and GBP/EUR exchange rates, respectively. Additionally, bonds denominated in USD are also exposed to the USD/EUR exchange rate. The approach to measuring currency exposure is the same for stocks and bonds: the exposure in the asset's currency is equal to the position value in that currency. To quantify these exposures, the total amount invested in foreign stocks and bonds is converted to EUR. Assets exposed to the same currency are grouped together to determine the total exposure for each currency. The exchange rate used for these conversions corresponds to the mapping date of 27 January 2023.

4.2. Historical Returns

The portfolio's sensitivity, derived from risk factor mapping, quantifies how changes in underlying risk factors impact the portfolio's value. To determine if these factors result in a profit or a loss for the portfolio, it is necessary to consider the returns of the respective risk factors. These returns are represented by the vector of risk-factor realizations (X), which captures the changes in each risk factor.

Based on the portfolio's composition, the returns for stocks, bonds, and currency are considered. For stocks, returns are determined by the relative change in the market price of each stock from day t-1 to day t:

$$X_{\text{Stock}_i} = \left(\frac{P_{i_t}}{P_{i_{t-1}}} - 1 \right), \tag{4.12}$$

where P represents the adjusted close price of stock i.

For bonds, each cash flow is exposed to its respective interest rates. For interest rate risk factors we consider the change in their values expressed in absolute terms and measured by basis points, that is,

$$\Delta R_{T_i}(b.p.) = (i_t - i_{t-1}) \times 10000. \tag{4.13}$$

For currency exposures, returns are defined by the relative change in their respective exchange rates and are calculated as:

$$X_{FX_i} = \left(\frac{FX_{i_t}}{FX_{i_{t-1}}} - 1 \right), \tag{4.14}$$

being FX_i a currency pair (e.g. USD/EUR). Considering the formulas presented above, the vector of risk factor realizations (X) is the following:

$$X = \begin{bmatrix} \left(\frac{P_{i_t}}{P_{i_{t-1}}} - 1 \right) \\ \vdots \\ \frac{\Delta r_{T_i}}{0.01\%} \\ \vdots \\ \left(\frac{FX_{i_t}}{FX_{i_{t-1}}} - 1 \right) \end{bmatrix}. \tag{4.15}$$

4.2.1. Profit and Loss of the Portfolio

In a generic way, the P&L in EUR recorded by the portfolio can be computed by multiplying the vector of risk-factor loadings (Θ) in EUR by the vector of risk-factor realizations (X). Thus, the historical time series of daily P&Ls for day t can be expressed as follows:

$$\text{P\&L}_{Portfolio_t} = \mathbf{\Theta}^\top \mathbf{X} = \begin{bmatrix} M_{\text{Stock}_i} \\ \vdots \\ -PV01_{T_i} \\ \vdots \\ M_{\text{Currency}_i} \end{bmatrix}^\top \times \begin{bmatrix} \left(\frac{P_{i_t}}{P_{i_{t-1}}} - 1\right) \\ \vdots \\ \frac{\Delta r_{T_i}}{0.01\%} \\ \vdots \\ \left(\frac{FX_{i_t}}{FX_{i_{t-1}}} - 1\right) \end{bmatrix}, \tag{4.16}$$

where Θ represents the portfolio's risk exposures to different risk factors, derived from the risk factor mapping in Section 4.1, and remains constant from the mapping date, 27 January 2023, while X denotes the changes in these risk factors from t-1 to t.

For our study, the data was reviewed to identify and handle outliers by inspecting the daily portfolio P&L values and computing the descriptive statistics for the returns (or absolute change in interest rates) over the selected period. Unexpected measures or P&L fluctuations with a higher deviation from the expected values were analyzed. If a price error was identified, the incorrect price was replaced with the correct daily price. Additionally, linear interpolation was also used to fill the missing data prices for all risk factors when no information was available.

4.3. EWMA Volatility and Covariance Estimation

To measure VaR and with the portfolio risk already mapped, there is still one question remaining: How can the volatility of returns be estimated?

As highlighted in the previous section, the vector of risk factor realizations represents the historical sample of returns. One common approach to measure volatility is to calculate the standard deviation of this sample. However, this method implies that older observations, which may no longer reflect current market conditions, have the same influence as more recent ones (for example, yesterday's returns have the same impact as last year's returns). Consequently, volatility is equally influenced by all observations, regardless of when they occur.

To overcome this, the Exponential Weighted Moving Average (EWMA) volatility model determines today's variance as a function of the previous day's variance. This model assigns greater weight to recent data, while older observations have a lower impact on the estimated volatility.

By introducing a time decay factor (λ), the influence on past data decreases on average exponentially, allowing the model to react faster to recent changes in the time series.

The value of λ is subjective and determines how quickly the weight of past observations decreases. The smaller the λ, the greater weight is assigned to more recent observations, which increases the sensitivity of variance estimates to current market conditions. This allows the model to quickly detect small changes while accounting for volatility clustering, making the model more sensitive to recent changes.

Considering the historical daily portfolio returns, the EWMA variance can be recursively estimated as:

$$\hat{\sigma}_t^2 = (1-\lambda)r_t^2 + \lambda\hat{\sigma}_{t-1}^2, \tag{4.17}$$

where $0 < \lambda < 1$ is the smoothing factor, $\hat{\sigma}_t^2$ is the variance estimated on day t-1 for day t and r_t is the return observed on day t. For covariance estimation between different time series, the same methodology used for variance estimation is applied, as expressed below:

$$\hat{\sigma}_{ij,t} = (1-\lambda)r_{i,t}r_{j,t} + \lambda\hat{\sigma}_{ij,t-1}, \tag{4.18}$$

where $\hat{\sigma}_{ij,t}$ is the estimated covariance between time series i and j at time t, and $r_{i,t}$ and $r_{j,t}$ are the returns (or changes) of time series i and j at time t.

4.4. Value-at-Risk Models

The measure used to quantify potential losses in this study is Value-at-Risk. This represents, in present value terms, the greatest loss we are confident (with a probability of 1-α) will not be exceeded if the portfolio is held static for a given amount of time h. For this study, we adopt a significance level of alpha = 1% (equivalent to a 99% confidence level) and a future horizon of 1 day (represented by h), as recommended by the Basel Committee guidelines. In this scenario, assuming the portfolio remains unchanged over the one-day horizon, we are 99% confident that losses will not exceed the VaR estimation on that day. In other words, under normal conditions, we expect the VaR to be exceeded once in every 100 days (once per $\frac{1}{\alpha}$).

According to Alexander (2009), the α-quantile of the h-day distribution of a continuous random variable X is a real number x_α that corresponds to the cumulative probability of x_α. This relationship can be expressed as follows:

$$P(X < x_\alpha) = \alpha. \tag{4.19}$$

Given the distribution function of a continuous random variable X, the quantile at any given significance level α can be determined as:

$$x_\alpha = F^{-1}(\alpha), \tag{4.20}$$

where F^{-1} represents the inverse distribution function with cumulative distribution of X.

The cumulative probability at a given α level corresponds to the maximum potential loss, with an α-probability of being exceeded. This concept aligns with the definition of VaR. Thus, the α-quantile value can be expressed symmetrically as VaR:

$$\text{VaR}_{h,\alpha} = -F^{-1}(\alpha) = -x_\alpha. \tag{4.21}$$

In mathematical terms, the $100\alpha\%$ VaR of an h-day period is the negative of the α-quantile of the h-day return distribution or P&L distribution. The choice between using the return distribution or the P&L distribution depends on whether VaR is being measured as a percentage of the portfolio value or in the currency in which VaR is being estimated. For this analysis, we used the P&L distribution.

4.4.1. Parametric Normal VaR

In this model, it is assumed that the h-day portfolio returns follow a normal distribution with mean μ_h and standard deviation σ_h, which can be expressed as $X_h \sim N(\mu_h, \sigma_h)$, where X_h represents a continuous random variable corresponding to the portfolio returns. Using the derivation of the formula on Equation 4.21 for the Parametric Normal VaR, we obtain the following:

$$\text{VaR}_{h,\alpha} = -\Phi^{-1}(\alpha) \times \sigma_h - \mu_h, \tag{4.22}$$

where $\Phi^{-1}(\alpha)$ is the inverse cumulative distribution function (or α-quantile function) of the standard normal distribution for a given probability level α.

However, according to Alexander (2008), since we are estimating 1-day VaR ($h = 1$), the drift adjustment (μ_h) can be considered zero for small time horizons [1]. With this additional assumption, the simplified formula for the h-day $100\alpha\%$ Parametric Normal VaR is the following:

$$\text{VaR}_{h,\alpha} = -\Phi^{-1}(\alpha) \times \sigma_h, \tag{4.23}$$

where the mapped portfolio standard deviation is given by:

$$\sigma_h = \sqrt{\Theta^\top \Sigma \Theta}. \tag{4.24}$$

Here, Σ is the covariance matrix determined using the EWMA volatility model (Equations 4.17 and 4.18) and Θ is the vector of risk factor loadings in EUR.

4.4.2. Skewed Generalized Student's VaR

As mentioned in Chapter 2, using a normal distribution to fit the financial returns underestimates VaR at lower significance levels (more extreme confidence levels). Additionally, the results in Section 3.3 indicate that the VaR model based on the normal distribution is unlikely to perform well, highlighting the importance of considering models, such as the Skewed Generalized Student-t (SGSt) distribution, that capture tail risks and account for the skewness and excess kurtosis observed in our portfolio's returns.

[1] In pratice, on very short time horizons any estimated value for the average return would be of almost insignificant dimension, so the assumption does not lead to a significant imprecision on the models.

This distribution, a generalization of the Student-t distribution proposed by Theodossiou (1998), captures the characteristics of financial data by accommodating asymmetry and providing greater flexibility in capturing extreme risks.

For its normalized density function $T_{0,1,\lambda,p,q}$, each parameter controls different aspects:

- λ determines the skewness, where $\lambda \in (-1, 1)$:
  - $\lambda = 0$: symmetric distribution
  - $\lambda > 0$: positively skewed
  - $\lambda < 0$: negatively skewed
- $p > 0$ controls the shape of the central region of the distribution.
- $q > 0$ controls the shape of the tail region of the distribution.

Formally, the h-day $100\alpha\%$ SGSt VaR is defined as:

$$\text{VaR}_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) \times \sigma_h - \mu_h, \tag{4.25}$$

where the α-quantile of the standard SGSt distribution is represented by $T^{-1}_{0,1,\lambda,p,q}(\alpha)$.

Similar to the Parametric Normal VaR discussed in the previous subsection, μ_h is assumed to be 0, and σ_h is estimated using the EWMA volatility model, which leads to the simplified expression:

$$\text{VaR}_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) \times \sigma_h. \tag{4.26}$$

4.4.3. Historical VaR

Instead of assuming that portfolio returns follow a specific distribution, the empirical distribution of actual historical returns can be used to estimate VaR.

It is important to highlight that when computing the distribution from historical data, the sample size has a considerable influence on the precision of the VaR estimate, as a larger dataset provides a broad diversity range of values and the VaR estimation becomes less exposed to outliers. However, as the sample size increases, the representation of current market conditions decreases, since a larger sample includes data from older periods that may not adequately represent the current market behavior. In this approach, referred to as Unadjusted Historical VaR, each observation is given the same weight, meaning that the effect of the volatility from older data is considered the same as the volatility from more recent observations.

The volatility-adjusted Historical VaR aims to address this issue. As proposed by Hull and White (1998), the volatility-adjusted Historical VaR adjusts the entire series of returns to account for current volatility, while still assigning the same weight to each observation. This method guarantees that the whole sample better reflects the current state of the market.

To implement this approach, we first obtain a series of volatility estimates ($\hat{\sigma}_t^2$), using the EWMA model (Equation 4.17), and then adjust the entire series of returns as follows:

$$\hat{r}_t = \frac{\hat{\sigma}_T}{\hat{\sigma}_t} r_t, \tag{4.27}$$

where, $\hat{r}_t$ is the adjusted return, T is the VaR measurement date, $\hat{\sigma}_t$ is the volatility estimate for day t, which is estimated at the end of day $t-1$, and $t < T$.

To estimate both the Unadjusted and Adjusted Historical VaR, the $100\alpha\%$ h-day historical VaR, is the α quantile of the empirical h-day distribution. The procedure for calculating this is as follows: firstly, to replicate the portfolio's returns in previous market situations, we calculate the h-day empirical past returns using the chosen sample size while keeping the portfolio weights constant. Next, we compute the portfolio P&L time series using these risk factor returns and their respective risk factor loadings. We then sort the P&L values from lowest to highest, and by assigning each observation a probability of $\frac{1}{n}$ (where n is the sample size), we create the empirical h-day portfolio P&L distribution by accumulating probabilities from the worst return upwards. Finally, we determine VaR by finding the α-quantile of the cumulative distribution.

4.4.4. Quantile Regression VaR

Estimating VaR is equivalent to determining the symmetric of the α-quantile of the return distribution. Accordingly, VaR estimates can be obtained from a QR, which models the portfolio returns as a function of relevant explanatory variables. Consequently, the value of the QR VaR depends on both the parameter estimates and the values of these explanatory variables.

Therefore, α-quantile (q_α) regression VaR can be estimated as follows:

$$VaR_\alpha \equiv q_{\alpha,y} = -(\hat{a} + \sum_{j=1}^{k} \hat{b}_j x_j), \tag{4.28}$$

where y is the dependent variable, x_j is the explanatory variable, and $\hat{a}$ and $\hat{b}_j$ are the estimated parameters of the α-quantile regression of y.

In this study, the dependent variable y represents the portfolio's P&L, while the explanatory variables are the EWMA volatility estimates of the portfolio, calculated using different smoothing factors. The parameters of the α-quantile regression are estimated by minimizing the quantile loss function:

$$(\hat{a}, \hat{b}) = \arg\min_{a,b} \sum_{i=1}^{n} \varepsilon_i \left(\alpha - I_{\varepsilon_i < 0}\right), \quad \varepsilon_i = y_i - (a + bx_i) \tag{4.29}$$

where I_ε is an indicator function of event ε_i:

$$I_{\varepsilon_i < 0} = \begin{cases} 1, & \text{if } \varepsilon_i < 0, \\ 0, & \text{otherwise.} \end{cases} \tag{4.30}$$

To estimate the daily VaR, we considered five configurations, each differing in the number of explanatory variables and the sample size. For our study, none of the configurations include a constant ($\hat{a}$). Typically, when volatility is the only explanatory variable, including a constant restricts the model's adaptability in certain cases. Because volatility cannot be negative, the constant imposes a lower bound on VaR estimates, restricting the model's ability to adapt to low volatility periods. As a result, VaR cannot decrease sufficiently to reflect these market conditions.

The first specification is defined as:

$$y_t = b \times \sigma_{\lambda,t} + \varepsilon_t, \tag{4.31}$$

where, σ_t is the explanatory variable representing the volatility estimate for day t, computed using the EWMA volatility model (Equation 4.17). The second specification, which includes two explanatory variables is defined as:

$$y_t = b_1 \times \sigma_{\lambda_1,t} + b_2 \times \sigma_{\lambda_2,t} + \varepsilon_t, \tag{4.32}$$

where, $\sigma_{\lambda_1,t}$ and $\sigma_{\lambda_2,t}$ represent the two portfolio EWMA volatility estimates for day t, each calculated using different smoothing factors (Equation 4.17). Similar to the SGSt model, the parameters were re-estimated every trading day.

Formally, the h-day $100\alpha\%$ QR VaR can be estimated as:

$$\text{VaR}_{\alpha,t} = -(\hat{b_1} \times x_{1,t} + \hat{b_2} \times x_{2,t}), \tag{4.33}$$

where the explanatory variables are $x_{1,t}$ and $x_{2,t}$.

4.5. Performance Metrics and Tests for VaR Models

After describing the VaR models and estimating their parameters, a critical question arises: How can we guarantee that the VaR models predict risk accurately? To answer this question and to verify that the results of VaR calculations are consistent and accurate, we perform backtesting. The primary goal is to select the best-performing VaR model during the backtesting period, ensuring that the losses predicted by the model reflect the losses experienced.

When measuring a portfolio's greatest potential loss, it is possible to observe a larger loss than the predicted VaR. When this happens, an exceedance occurs. The number of exceedances is the main performance metric used to evaluate a VaR model. To assess the reliability of these exceedances, we adopt two tests: the UC test (Kupiec, 1995), which evaluates whether the number of exceedances observed corresponds to the expected, and the BCP test (Berkowitz et al., 2011), which checks for autocorrelations between them.

4.5.1. Unconditional Coverage Test

Introduced by Kupiec (1995), the Unconditional Coverage test (UC) compares whether the exceedance rate verified throughout the test period (π_{obs}) corresponds to the expected exceedance rate (π_{exp}).

According to the definition of VaR, which represents the maximum loss expected not to be exceeded with a confidence level of $1-\alpha$, there is a probability α that the losses will exceed VaR. Thus, the expected exceedance rate corresponds to the VaR model's significance level α. For example, in a sample of 500 daily observations, if the VaR is estimated with a 99% confidence level ($\alpha = 1\%$), the expected number of exceedances is $500 \times (1 - 0.99) = 5$. This means the VaR will likely be exceeded on approximately 5 days during the sample period. A model performs well if the observed and expected exceedance rates align.

According to the UC test, the realized number of exceedances must statistically correspond with the prespecified confidence level. To identify an exceedance, an indicator function is used for a sample of n observations, defined as:

$$I_{\alpha,t} = \begin{cases} 1, & \text{if } P\&L_t \leq -VaR_{1,\alpha,t} \\ 0, & \text{otherwise} \end{cases}, \tag{4.34}$$

where $VaR_{1,\alpha,t}$ is the estimated VaR for day t. This process results in a binary series of n observations, where 1 indicates an exceedance (a "success"), and 0 indicates a non-exceedance.

The purpose of the UC test is to check the validity of the null hypothesis, which states that the probability of the indicator function is equal to the significance level of the VaR model (α) (Alexander, 2009):

$$\begin{aligned} H_0 &: \pi_{\text{obs}} = \pi_{\text{exp}} \equiv \alpha \\ H_1 &: \pi_{\text{obs}} \neq \pi_{\text{exp}} \end{aligned}, \tag{4.35}$$

where the observed and predicted exceedance rates are denoted by π_{obs} and π_{exp}, respectively.

To determine whether the observed exceedances are statistically consistent with the significance level, the UC test employs a likelihood-ratio test statistic, expressed as:

$$LR_{\text{uc}} = \left(\frac{\pi_{\text{exp}}}{\pi_{\text{obs}}}\right)^{n_1} \left(\frac{1-\pi_{\text{exp}}}{1-\pi_{\text{obs}}}\right)^{n_0}, \tag{4.36}$$

where n_1 is the number of exceedances observed, n_0 is the number of non-exceedances, which means the number of days that an exceedance does not occur ($n_0 = n - n_1$), $\pi_{\text{obs}} = \frac{n_1}{n}$, and $\pi_{\text{exp}} = \alpha$.

Under null hyphotesis ($\pi_{\text{obs}} = \pi_{\text{exp}}$), the UC test statistic follows a chi-squared distribution with one degree of freedom:

$$-2\ln(LR_{\text{UC}}) \sim \chi^2_1. \tag{4.37}$$

If the test statistic falls within the acceptance range, meaning that the null hypothesis is not rejected, the VaR model is considered appropriate. This indicates that the number of observed exceedances aligns with the expected number at the significance level α (neither significantly above nor below the expected value).

However, the UC test raises an important question: What if the model passes the UC test, but the exceedances are temporally dependent? Since Kupiec's UC test considers only the number of observed exceedances without accounting for their temporal clustering, the BCP test is used to address this limitation.

4.5.2. BCP Test

According to the BCP test, a model is well specified if exceedances are independent of each other, meaning it is not possible to predict when the next exceedance will occur (Berkowitz et al., 2011). This implies that the time between VaR exceedances is not related to the time elapsed since the last exceedance. In other words, the BCP test evaluates the independence of exceedances by examining their k-order autocorrelation.

For a well-specified model, the autocorrelation should be 0 at all lags. The BCP test assesses this through the following hypotheses:

$$\begin{aligned} &H_0 : \hat{\rho}_k = 0,\ \forall k \in \{1, \dots, K\} \\ &H_1 : \exists k \in \{1, \dots, K\} \text{ s.t. } \hat{\rho}_k \neq 0 \end{aligned}, \tag{4.38}$$

where $\hat{\rho}$ represents the autocorrelation of k-th order of the time series of exceedances. This time series is derived from the indicator function (where each observation is 1 or 0) as defined in Equation 4.34, and the highest autocorrelation lag considered in the test is denoted by K.

The test statistic can be computed recursively as follows:

$$BCP(K) = n(n+2)\sum_{k=1}^{K} \frac{\hat{\rho}_k^2}{n-k}, \tag{4.39}$$

where n is the sample size of the test.

Under the null hypothesis that $\hat{\rho}_k = 0$, the asymptotic distribution of the test static follows a chi-squared distribution with K degrees of freedom, stated as $BCP(K) \sim \chi^2_K$.

The choice of lag order K involves a trade-off. A larger K captures non-independence at higher-order lags, by including information about the correlations up to lag K. However, as K increases, the critical value from the χ^2_K distribution also rises, reducing the probability of rejecting H_0.

On the other hand, a smaller K increases the power of the test because, by reducing the degrees of freedom and consequently its critical value, it is easier to reject H_0 when it is false, increasing the probability of correctly rejecting H_0. However, a smaller K ignores any dependencies at higher-order lags.

Given this trade-off, we compute the BCP test considering several different lags, ranging from 1 to 10, establishing a balance between capturing higher-order dependencies while maintaining an adequate test power. It is important to be aware of the properties of K when interpreting the results, as the choice of K influences both the power and the scope of the test.

CHAPTER 5

BACKTEST

In the previous chapter we detailed the methodology for mapping the portfolio risk factors, estimating the four VaR models, and explaining the backtesting tests. This chapter focuses on the results and the performance of each model to select the VaR model to be applied for the one year going forward.

During the 10-year global backtesting period, from 11 February 2013 to 27 January 2023, we compute a series of daily historical VaR estimates for each model under analysis, as if the current portfolio had existed throughout the past. This allows us to compare the losses predicted for each VaR model to those that were experienced over the global testing period.

We assessed the accuracy of these estimates using the UC and the BCP tests, as outlined in Subsections 4.5.1 and 4.5.2, respectively. Both tests are important and complement each other because even if the number of exceedances aligns with expectations, the exceedances can still be correlated, which could indicate the clustering of risk events. In this situation, if exceedances are correlated, it implies that extreme losses are not occurring randomly but rather in clusters, meaning that the model cannot respond rapidly enough to changes in market conditions. For example, during periods of high volatility, losses may cluster, revealing that the model fails to capture the speed and magnitude of changes in the market.

We perform the tests on both the global backtesting period and annual sub-periods of one trading year because a model's performance can differ depending on the timeframe considered. For example, a model may underperform during specific sub-periods, such as periods of high volatility, while still performing well overall.

The process for selecting the VaR model is performed over multiple rounds. In the first stage, models are evaluated using the UC test over the entire backtesting period, with those with p-values greater than 5% moving forward. In the second round, the focus is on the BCP test, with models with p-values greater than 5% moving on to the next stage. This test is only computed for the models that were not rejected in the previous round and helps us to distinguish between models with a similar number of expected exceedances (i.e., models with similar UC test performance). In this round, most models can be rejected in the BCP test due to the high volatility observed in the COVID-19 market turmoil. If this happens, our decision will be highly based on the UC test, because the BCP test is sensitive to the timing of exceedances. If exceedances occur in clusters within a short period, the model can be rejected by the BCP test, even if the overall number of exceedances is within acceptable UC limits. Conversely, a model with a higher number of exceedances might pass the BCP test if these exceedances are well-spaced over time. Therefore, we prioritize the UC test in our decision-making process. In the third round, the UC is applied to annual sub-periods. The preferred model is the one with consistent results across the years, being the ones that maintain a steady number of exceedances annually, rather than concentrating exceedances in a few years. The last round evaluates the models in terms of the BCP test, but this time for the annual sub-periods.

This chapter includes Section 5.1, which outlines the parameters, configurations, and specifications of each model selected for further analysis in backtesting, and Section 5.2, which presents and explains the VaR model selection process results.

5.1. Model Parameters and Configurations

The study evaluates 15 different configurations of the models discussed in the previous section: Parametric Normal, SGSt, Historical, and QR. These configurations share a common parameter, the EWMA smoothing factor. However, the optimal configuration for one model may not be the best for another. Jorion (2002) highlighted that if the VaR models are inaccurate, they must be reexamined for inappropriate assumptions or wrong parameters. To identify the best-performing representatives of each model class, we tested multiple variations by adjusting parameters such as sample size and smoothing factors. From this extensive search, we narrowed our analysis to 15 optimized configurations based on backtesting results, including the UC and BCP tests. The objective was to select the best models that fall within the green zone of the traffic light system proposed by the Basel Committee, which corresponds to backtesting results that do not raise any issue about the accuracy or quality of the models. If a model with a given configuration performs well in backtesting, it proceeds to forward analysis. Otherwise, parameters such as the sample size and the EWMA smoothing factor are adjusted to achieve satisfactory backtesting results. The 15 models represent a selected sample from the many variations tested, with the best-performing configurations chosen for each model class.

According to the UC and BCP tests, the configurations of the VaR models for our portfolio, selected for further analysis, are presented in Table 5.1.

Models				
Class	**Number**	**Specifications**	**Sample Size**	**EWMA Smoothing Factor**
Normal	1	Parametric Normal with EWMA volatility estimation	-	0.970
SGSt	2	Parametric SGSt with EWMA volatility estimation	370	0.990
	3	Parametric SGSt with EWMA volatility estimation	500	0.955
	4	Parametric SGSt with EWMA volatility estimation	600	0.945
	5	Parametric SGSt with EWMA volatility estimation	900	0.960
Hist	6	Volatility Adjusted Historical	300	0.975
	7	Volatility Adjusted Historical	400	0.975
	8	Volatility Unadjusted Historical	500	-
	9	Volatility Adjusted Historical	500	0.990
	10	Volatility Adjusted Historical	700	0.970
QR	11	Quantile Regression with EWMA volatility as the explanatory variable	300	0.985
	12	Quantile Regression with EWMA volatility as the explanatory variable	400	0.975
	13	Quantile Regression with EWMA volatility as the explanatory variable	500	0.970
	14	Quantile Regression with 2 EWMA volatility explanatory variables	1000	0.980 and 0.930
	15	Quantile Regression with EWMA volatility as the explanatory variable	700	0.955

Table 5.1. VaR Model Descriptions. Each model adopts a different sample size and EWMA smoothing factors. These values are determined based on the backtesting results, according to the specification for each model.

5.2. Backtest Results

As described in the previous section, selecting a VaR model for further analysis involved adjusting the configurations of each model class to identify the best fit, allowing them to proceed to backtest. The backtesting period is from 11 February 2013 to 27 January 2023, corresponding to 2600 trading days (520 weeks). For a well-specified model at the 1% significance level, the UC test expects approximately 26 exceedances (1% of 2600). Generally, the null hypothesis is rejected at a 95% confidence level of the test statistic when the p-value is below 5%. This way, the UC and BCP tests are not rejected when the p-value is above 5%. Table 5.2 below presents the exceedance rate and the p-value of the UC test for all the models.

Class	**Number**	**Exceedance Rate (%)**	**UC Test p-value (%)**
Normal	1	1.50	1.70*
SGSt	2	1.00	100.00
	3	1.08	69.70
	4	1.12	56.15
	5	1.00	100.00
Hist	6	1.12	56.15
	7	1.08	69.70
	8	1.38	6.25
	9	1.19	33.88
	10	1.23	25.37
QR	11	1.23	25.37
	12	1.19	33.88
	13	1.27	18.54
	14	1.38	6.25
	15	1.19	33.88

Table 5.2. UC test results for the different models over the global period. P-values marked with * are rejected at the 5% significance level.

As observed in Table 5.2, we reject model 1, as it presents a higher number of exceedances, leading to a rejection of the Unconditional Coverage (UC) test. The Parametric Normal model is rejected due to its assumption that returns follow a normal distribution, which fails to capture the observed data characteristics in Section 3.3. Over the global period, the model that best captures the negative skewness and kurtosis of the portfolio's returns is the SGSt. This distribution consistently shows a higher p-value over the different configurations. This result is consistent with the analysis presented in Section 3.3 and the findings of Pafka and Kondor (2001), which highlighted that the financial data with the characteristics of our portfolio (higher kurtosis and negative skewness) is better captured by the SGSt instead of the Normal model. Following this, the Historical distribution seems to be a good fit with certain configurations, particularly for the return's distribution in models 6 and 7. All models, except model 1, are within the acceptance range. Among the accepted models, the ones that stood out were models 2 through 7 because they present an exceedance rate close to the expected (1%). Specifically, models 2 and 5 present an exceedance rate exactly equal to what is expected. To proceed with the analysis, the models rejected by the UC test were excluded. For the remaining models, Table 5.3 presents the results of the BCP test for the global period.

Class	Number	BCP p-value (%)									
		Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag 9	Lag 10
SGSt	2*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3*	0.18	0.33	0.01	0.02	0.06	0.11	0.12	0.22	0.37	0.00
	4*	0.29	0.57	0.02	0.06	0.13	0.25	0.29	0.49	0.78	0.05
	5*	0.06	0.09	0.00	0.00	0.01	0.02	0.02	0.03	0.05	0.00
Hist	6*	0.29	0.57	0.02	0.06	0.13	0.25	0.29	0.49	0.78	0.05
	7*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8*	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
QR	11*	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	12*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	13*	0.01	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.00
	14*	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
	15*	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.00

Table 5.3. BCP test results for the global period, for lags from 1 to 10. A model passes the BCP test if the p-value exceeds 5%, indicating that the exceedances are independent across the specified lags. The lags represent the time intervals considered for assessing the independence of exceedances generated by the models. VaR models marked with * in the Number column are rejected at least at one of the lags.

From Table 5.3, we observe that all models are rejected at a 95% confidence level of the BCP test across all lags. Failing this test means that the models tested are not generating independent exceedances over time, tending to generate consecutive exceedances or exceedances separated by intervals within the range of 1 to 10 days.

Although all models failed the global BCP test, models 4 and 6 were the closest to passing. In both cases, their rejection was mainly due to exceedances with 3- and 10-day intervals, since the p-value decreases at these lags. For the remaining lags, the p-value increased due to the higher degrees of freedom in the test distribution. Since no exceedances occurred at these intervals for both models, the test statistics remained unchanged, and the p-value rises as the degrees of freedom increase.

Since all the models are in the rejection range of the BCP test, we consider the ones with the best global UC test results. According to Table 5.2, models 2 through 7 meet this criterion, so we narrow our selection to these models.

Tables 5.4 and 5.5 show UC test results across sub-periods for models 2 to 7. The results for the remaining models are presented in Tables B.2 and B.1, located in Appendix B.

Date Range	Model 2			Model 3			Model 4		
	Exc. Rate (%)	Number of Exc.	p-value (%)	Exc. Rate (%)	Number of Exc.	p-value (%)	Exc. Rate (%)	Number of Exc.	p-value (%)
2022-2023	0.00	0	2.22*	0.38	1	25.44	0.38	1	25.44
2021-2022	0.38	1	25.44	1.15	3	80.77	1.15	3	80.77
2020-2021	2.69	7	2.34*	1.92	5	18.44	2.31	6	7.01
2019-2020	0.77	2	69.67	1.15	3	80.77	1.15	3	80.77
2018-2019	1.54	4	41.87	1.15	3	80.77	0.77	2	69.67
2017-2018	0.77	2	69.67	0.77	2	69.67	0.77	2	69.67
2016-2017	0.38	1	25.44	0.38	1	25.44	0.77	2	69.67
2015-2016	1.15	3	80.77	1.15	3	80.77	1.15	3	80.77
2014-2015	1.15	3	80.77	1.54	4	41.87	1.54	4	41.87
2013-2014	1.15	3	80.77	1.15	3	80.77	1.15	3	80.77

Table 5.4. UC test results for models 2, 3, and 4 across one-year sub-periods. P-values marked with * are rejected at the 5% significance level.

Date Range	Model 5			Model 6			Model 7		
	Exc. Rate (%)	Number of Exc.	p-value (%)	Exc. Rate (%)	Number of Exc.	p-value (%)	Exc. Rate (%)	Number of Exc.	p-value (%)
2022-2023	0.00	0	2.22*	0.77	2	69.67	0.38	1	25.44
2021-2022	1.15	3	80.77	0.77	2	69.67	0.38	1	25.44
2020-2021	1.92	5	18.44	1.54	4	41.87	1.54	4	41.87
2019-2020	1.15	3	80.77	1.15	3	80.77	1.15	3	80.77
2018-2019	1.15	3	80.77	0.77	2	69.67	1.54	4	41.87
2017-2018	0.77	2	69.67	1.92	5	18.44	1.92	5	18.44
2016-2017	0.38	1	25.44	0.77	2	69.67	0.38	1	25.44
2015-2016	1.15	3	80.77	1.15	3	80.77	1.15	3	80.77
2014-2015	1.15	3	80.77	1.15	3	80.77	1.15	3	80.77
2013-2014	1.15	3	80.77	1.15	3	80.77	1.15	3	80.77

Table 5.5. UC test results for models 5, 6, and 7 across one-year sub-periods. P-values marked with * are rejected at the 5% significance level.

The choice of the model should fall on the model that has more consistent results on an annual basis. For instance, it is preferable to have a small number of exceedances distributed each year rather than having all the exceedances concentrated in specific years and none in the others.

From Tables 5.4 and 5.5, it is possible to observe that the choice of the model is challenging, since all models perform very similarly in the earlier sub-periods, with only slight differences in the number of exceedances. The most significant difference is seen in the sub-period 2020-2021, where most models have increased exceedances. This period was extraordinarily demanding for the models due to all the turbulence caused by COVID-19, a period of high volatility.

Despite the overall similarities among the models, the exceedances for model 2 show inconsistencies over some periods. The highest exceedance rate (2.69%) occurred in 2020-2021, while no exceedances were observed in 2022-2023. Consequently, model 2 was rejected at a 5% significance level for this period. Similarly, model 5, which had no exceedances in the 2022-2023 subperiod, was also rejected by the UC test. This highlights that even when a model meets the expected number of exceedances (as seen with models 2 and 5), it can still be not optimal if those exceedances are not evenly distributed (clustering of exceedances). Consequently, both models were excluded from further analysis.

Nevertheless, selecting the best model remains challenging because the exceedances appear to be compensated over periods. For example, when comparing models 3 and 6, model 6 has one fewer exceedance in the 2020-2021 sub-period but one more in the 2022-2023 sub-period. This balancing effect complicates the decision-making process, as no single model consistently outperforms the others across all periods.

Overall, the remaining models (models 3, 4, 6, and 7) perform very similarly over the annual sub-periods. To break the tie and select the most suitable model, we consider again the exceedance rate over the global period, based on the global UC test results. Although models 4 and 6 were the closest to passing the global BCP test, they also show the highest exceedance rate over the global period (1.12%) among the four models. This narrows the selection to models 3 and 7, exhibiting an exceedance rate of 1.08%. To make the final decision, we examine the annual sub-periods for the BCP test for each model.

Tables 5.6 and 5.7 present the p-values of the BCP test for the annual sub-periods across all ten lags for models 3 and 7, respectively.

Date Range	BCP *p*-value %									
	Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**	**Lag 6**	**Lag 7**	**Lag 8**	**Lag 9**	**Lag 10**
2022-2023	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2021-2022	**84.93**	**96.43**	**99.07**	**99.74**	**99.93**	**99.98**	**99.99**	100.00	100.00	100.00
2020-2021	**74.95**	**90.26**	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
2019-2020	84.93	96.43	99.07	99.74	99.94	99.99	100.00	100.00	100.00	100.00
2018-2019	**84.93**	**96.43**	**99.07**	**99.74**	**99.93**	**99.98**	**99.99**	**100.00**	**100.00**	**0.12***
2017-2018	**92.90**	**99.21**	**99.90**	**99.99**	**100.00**	**100.00**	**100.00**	**100.00**	**100.00**	**100.00**
2016-2017	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2015-2016	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
2014-2015	**79.92**	**93.70**	**97.82**	**99.20**	**99.70**	**99.89**	**99.96**	**99.98**	**99.99**	100.00
2013-2014	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00

Table 5.6. BCP test results for annual sub-periods across 10 Lags for model 3. Bold values indicate *p*-values that differ between model 3 and model 7. *P*-values marked with * are rejected at the 5% significance level.

Date Range	BCP *p*-value %									
	Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**	**Lag 6**	**Lag 7**	**Lag 8**	**Lag 9**	**Lag 10**
2022-2023	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2021-2022	**94.99**	**99.60**	**99.97**	**100.00**	**100.00**	**100.00**	**100.00**	100.00	100.00	100.00
2020-2021	**79.92**	**93.70**	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
2019-2020	84.93	96.43	99.07	99.74	99.94	99.99	100.00	100.00	100.00	100.00
2018-2019	**82.60**	**95.25**	**0.01***	**0.04***	**0.09***	**0.20***	**0.40***	**0.74***	**1.29***	**2.13***
2017-2018	**77.57**	**92.16**	**96.97**	**98.77**	**99.49**	**99.78**	**99.90**	**99.96**	**99.98**	**99.99**
2016-2017	94.99	99.60	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2015-2016	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*	0.00*
2014-2015	**84.93**	**96.43**	**99.07**	**99.74**	**99.93**	**99.98**	**99.99**	**100.00**	**100.00**	100.00
2013-2014	84.93	96.43	99.07	99.74	99.93	99.98	99.99	100.00	100.00	100.00

Table 5.7. BCP test results for annual sub-periods across 10 Lags for model 7. Bold values indicate *p*-values that differ between model 3 and model 7. *P*-values marked with * are rejected at the 5% significance level.

From the tables above, we can observe that the global BCP test results for both models were strongly influenced by the rejection in the subperiods 2015-2016 and 2020-2021. Overall, the two models exhibit a similar performance, however, the main difference is observed in the sub-period 2018-2019, where model 7 is rejected for lags from 3 to 10, while model 3 passes the BCP test for all lags, except for lag 10. This is illustrated in Appendix B, which presents two graphs showing the temporal distribution of exceedances for both models. In each graph, triangles represent the P&L on days when exceedances occurred for both models, while circles indicate the P&L on days with an exceedance in only one model. It can be observed that model 7 shows consecutive exceedances in early 2018, but almost none between 2020 and 2022, while for model 3 these exceedances are more distributed over time.

Therefore, to summarize and justify the final model selection we need to recap the decision process. Models 3 and 7 have the exact same exceedance rate and are both rejected by the global BCP test, due to cluster of exceedances in the 2015-2016 and 2020-2021 sub-periods. This means that both models are slow to adapt and tend to generate clustered exceedances. Even though both models fail the test, model 7 performs worse because its exceedances occur over several consecutive days, which is more problematic than model 3, whose exceedances occur at three-day intervals (as observed in the 2020-2021 subperiod in Table 5.6). Considering all of this we proceed with model 3, the Parametric SGSt with EWMA volatility estimation, a sample size of 500 and λ =0.955.

Figure 5.1 presents the details for the exceedances in the selected model and complements the BCP test results. In the figure, the grey portion represents the size of each exceedance, while the corresponding percentage indicates the exceedance relative to the VaR.

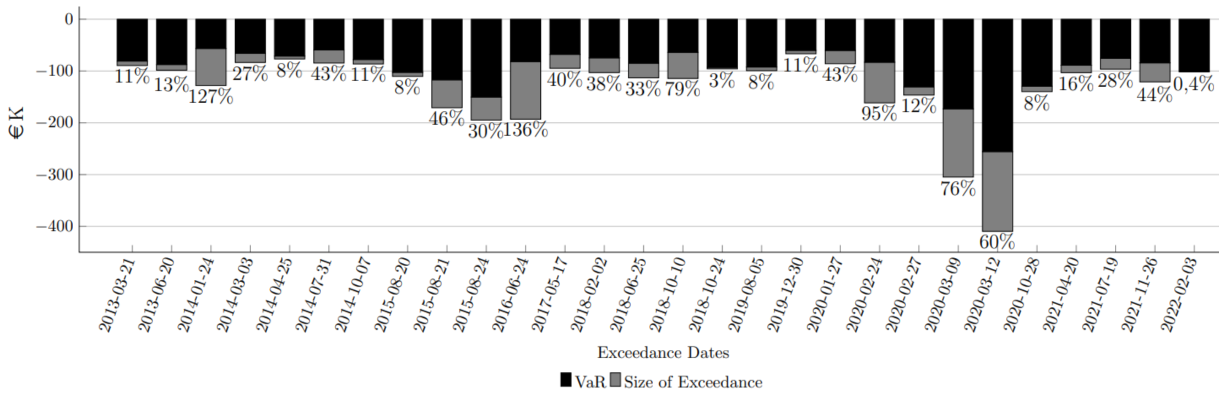

Figure 5.1. Exceedances details for model number 3 over the global backtesting period.

As shown in Figure 5.1, during the 2015 period, exceedances occurred on three consecutive trading days (20, 21, and 24 August 2015). This clustering explains the rejection of the global BCP test for lags up to 10 in this period, as the test detects autocorrelation at any lag within the specified range. Similarly, during the 2018-2019 period, the exceedance that occurred 10 trading days apart (from 10 October to 24 October 2018) was also well captured by the BCP test, since lag 10 was the only rejected lag. Additionally, in 2020, the exceedance on February 24 was followed by exceedances with gaps of 3, 7, and 10 trading days, occurring on February 27, March 9, and March 12, respectively. These exceedances account for the rejection of the global BCP test at lags from 3 to 10.

Appendix B provides further information on the backtesting results for the other models, with detailed UC test results for each sub-period.

CHAPTER 6

VaR MANAGEMENT

The selected model, Parametric SGSt with EWMA volatility estimation (sample size of 500, λ=0.955) is used from now on. VaR is measured and managed over one period going forward from the mapping date of 27 January 2023 until 2 February 2024.

An important consideration in managing portfolio exposure is to define the acceptable level of risk. This aligns with the concept of Economic Capital (EC), which represents the desirable level of capital to hold that is sufficient to cover the risk exposure with a given degree of confidence over a specified time horizon. This is quantified using VaR, meaning that, numerically, Economic Capital is equivalent to VaR.

To define the level of EC we established the following goal: we want the daily exposure to be below the average VaR level measured over the 10 years, under normal volatility conditions. We designated this threshold as the stop-loss limit. Observing Figure B.1 from Appendix B, during the Backtesting period, VaR ranged mostly between € 50 000 and € 200 000, excluding the high volatility period in 2020 (Covid-19), when VaR was higher than this. Since we want our stop-loss limit to be based on normal market conditions, VaR during these periods was on average € 96 000. To manage VaR over a one year we want the stop-loss limit to be slightly below this, at € 95 000, which corresponds to approximately 1.2% of the portfolio value as of the mapping date. The approach for limiting losses is as follows: each day throughout one year, we estimate the next day's VaR based on the portfolio composition at the end of each day. If the estimated loss exceeds € 95 000, we implement a hedging strategy to keep the VaR below this limit. Otherwise, no hedging is applied. This process is repeated daily from 27 January 2023 to 2 February 2024. This means that hedging is dynamic, since the positions are only taken whenever VaR is higher than € 95 000. In other words, whenever the VaR without hedging is below the stop-loss limit we remove the hedging position. Additionally, during this period when bond coupons are received, they are reinvested, as detailed in Table C.1.

This chapter focuses on the results of VaR management and details the impact of hedging at the portfolio level, including its effects on portfolio risk, diversification, and P&L. First, we analyze the portfolio risk on the first day the loss limit is exceeded. Then, we implement a hedging strategy based on risk decomposition and compare its impact on both the hedged and unhedged portfolios.

6.1. VaR Decomposition

To measure how much a position contributes to the overall VaR of a portfolio, we decompose VaR into its marginal VaR components. By assigning a proportion of the total risk to each component, marginal VaR is used to disaggregate VaR while considering diversification benefits among the risk components. Marginal VaR can also be defined as the diversified VaR and is expressed by the following formula:

$$\text{Marginal VaR}^s = \nabla f(\Theta)^T \Theta^s, \tag{6.1}$$

where $\nabla f(\Theta)^T$ is the gradient vector, which represents the sensitivity of each risk factor against the portfolio to small changes in the exposure to each risk factor.

For our study, we decompose marginal VaR contributions by asset class (stocks and bonds) to analyze how different asset categories influence the portfolio's risk profile. Table 6.1 shows how much each risk factor contributed to the total VaR on 30 January 2023, the first day the VaR threshold was exceeded.

Category	**Equity**	**Bonds**	**FX**		**Total**
			Equity	**Bonds**	
Marginal VaR (€)	82 328.95	13 311.69	9 577.28	9 576.05	114 793.97
Marginal VaR (%)	71.72	11.60	8.34	8.34	100.00
Excess VaR (%)					20.84

Table 6.1. VaR decompositions for stocks and bonds. Equity VaR reflects stock price risk, while FX Equity captures additional currency risk from foreign stocks. Bond VaR represent interest rate risk, and FX Bonds account for currency risk in foreign bond holdings. All values are in EUR.

On 30 January 2023, stocks accounted for 72% of the portfolio's total risk, representing the main source of overall risk. Stocks and bonds contributed equally to the portfolio's currency exposure, which represented 16.68% of the VaR on that day. Since stocks are the asset class with the highest marginal VaR, Table 6.2 details the exposures from stocks, including both stock risk and the currency risk arising from investments in different regions.

Category	**Equity**			**Equity FX**				**Total**
	U.S.	**Europe**	**Asia**	**USD**	**GBP**	**JPY**	**HKD**	
Marginal VaR (€)	51 201.16	30 687.03	440.77	7 979.41	149.68	1 538.43	-90.23	91 906.24
Marginal VaR (%)	55.71	33.39	0.48	8.68	0.16	1.67	-0.10	100.00

Table 6.2. Equity risk decomposition by region and currency exposure.

Table 6.2 shows a significant concentration of risk in two regions: U.S. and Europe. Together, these regions contribute to 97.94% of the portfolio's stock VaR, with U.S. being the largest contributor. The remaining 2.06% of VaR comes from Asian equity. Our objective is to reduce this main risk. However, when hedging positions are implemented, it is important to recognize that reducing VaR to the target level with only one position can be impossible. If we hedge VaR using only one position, the required size of that position would likely be so large (the VaR reduction would be larger than the marginal VaR of the risk factor) that it would significantly alter the portfolio's overall sensitivity to the risk factor. These large sensitivity variations would make it difficult to hedge the portfolio exposure with only a single position. Therefore, we hedge the two main risk sources. Given the geographic concentration observed on this day, we implemented hedging strategies to mitigate exposure to these dominant regions. Specifically, we implement hedging strategies by taking short positions in the indices with the highest marginal VaR in each region using futures contracts over the indexes with a 1-day maturity. As a result, for North American equities (the main source of stock risk), we shorted the S&P 500 index, while for European equities (the second largest source of risk), we used the CAC 40 to adjust the final position and reach the desired target (€ 95 000). CAC 40 was chosen because it represents the largest risk exposure (45%) from Europe.

6.2. Hedging strategy

To determine the size of the hedging positions, we analyze the incremental effect on VaR of adding a new instrument to the portfolio. This concept, Incremental VaR, measures how much risk a position adds to the portfolio. For example, if it is positive, increasing the size of the position will increase the portfolio risk. With this in mind, for a given target change in the VaR, we determine the required hedging position by the following procedure:

$$\Delta \text{VaR} \equiv f(\Theta') - f(\Theta) \approx \nabla f(\Theta) \times (\Theta' - \Theta). \tag{6.2}$$

As highlighted previously, we implement the hedging positions based on the two main marginal contributions to portfolio VaR. However, it is important to consider the foreign currency exposure if the index is denominated in a foreign currency. For example, hedging U.S. market exposure with the S&P500 index creates both an S&P500 futures contract exposure and a USD/EUR currency exposure. Therefore, to adjust a hedging position for a desired change in VaR, the required exposure is calculated based on the sensitivities to risk factors such as the S&P 500 and USD/EUR exchange rate, expressed as below:

$$\Delta \text{VaR} \approx \frac{\partial \text{VaR}}{\partial \text{S\&P500}} \times \theta_{\text{S\&P500}} + \frac{\partial \text{VaR}}{\partial \text{USDEUR}} \times \theta_{\text{S\&P500}}. \tag{6.3}$$

Finding $\theta_{\text{S\&P500}}$ determines the amount that is needed to hedge the U.S. stock exposure to guarantee the stop-loss limit for the next estimated day does not exceed € 95 000.

Although, after applying the hedging position on the S&P500, the target VaR of € 95 000 was not fully achieved, since the sensitivities to the risk factor change as the risk factor exposure changes (by introducing a hedging position). To address this, after applying the initial hedging position, as stated in Equation 6.3, and achieving a partial reduction in VaR, we once again use Incremental VaR to determine the required hedging position in the CAC 40 to reach the target VaR reduction.

The following formula determines the size of the hedging position in the CAC40:

$$\Delta \text{VaR} \approx \frac{\partial \text{VaR}}{\partial \text{CAC40}} \times \theta_{\text{CAC40}}. \tag{6.4}$$

Thus, for the one year, we first determined $\theta_{\text{S\&P500}}$ using Equation 6.3 to achieve a partial reduction, and then applied Equation 6.4 to calculate θ_{CAC40} in order to reach the target VaR.

Figure 6.1 shows the short positions taken in the S&P500 and CAC40.

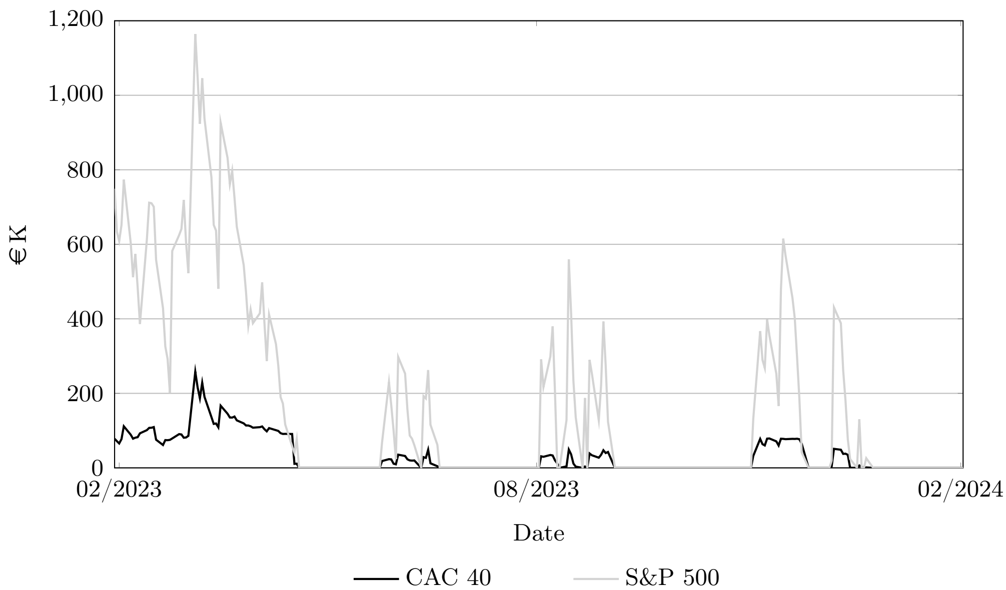

Figure 6.1. Daily short positions on the CAC 40 and S&P 500 indexes.

As expected, since the primary risk comes from the U.S. market, the hedging positions in the S&P500 are larger than the CAC 40, the index with the highest marginal VaR in Europe. Additionally, on 6th March, both positions reached a peak, with the S&P500 exceeding CAC 40 by 351%. It is interesting to observe that the size of S&P500 position shows large variations compared to CAC 40, which remains relatively stable. This is in line with the strategy taken: we first hedge the position with the highest marginal VaR to achieve a larger reduction in overall risk and then adjust the second position to reach the desirable target level. The following subsections present the results and the impact of these strategies on the overall portfolio, including risk, diversification effects, and returns.

6.3. VaR Management Results

In the following section, the Hedged Portfolio refers to the portfolio with hedging positions shown in Figure 6.1, while the Unhedged Portfolio represents the portfolio without them. Figure 6.2 illustrates the VaR management results for both the Hedged and Unhedged portfolios over one year, while Figure C.1 in Appendix C presents these results with a focus on stocks, the main driver of portfolio risk.

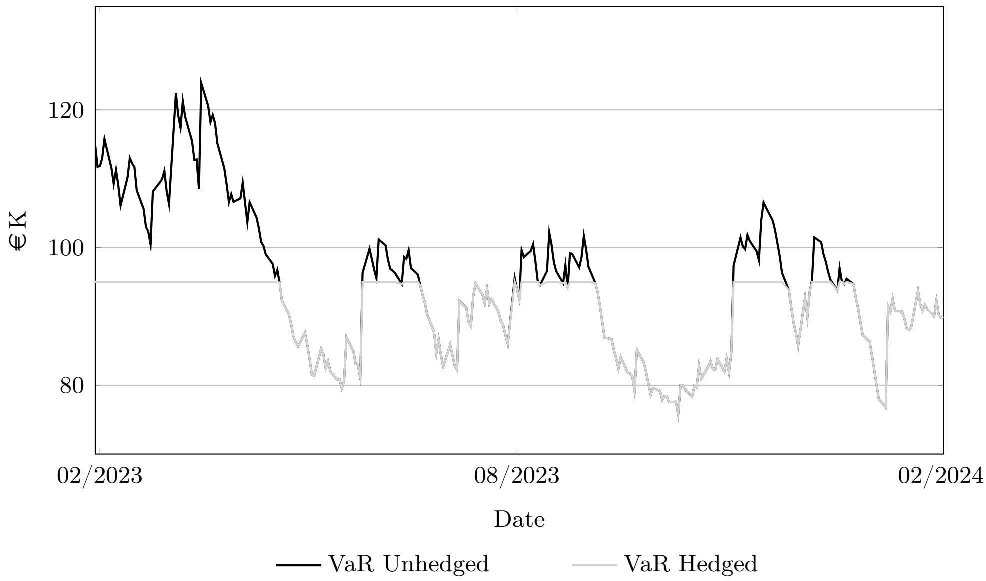

Figure 6.2. Daily VaR estimates comparison between Unhedged and Hedged Portfolios.

Figure 6.2 demonstrates that the portfolio's overall risk decreased. On average, both total portfolio risk and stock risk declined by approximately 4.37% over the one year considered (the total portfolio risk declined from € 94 522 to € 90 390 while the stock risk declined from € 78 972 to € 75 518). However, despite this reduction, it is important to focus specifically on days when hedging was applied. While including all days (both with and without hedging) provides a broad overview of the portfolio's performance, it dilutes the actual impact of the hedging, since risk on non-hedging days offsets the observed risk reduction on hedging days. To isolate the true effect of hedging, Table 6.3 presents the average results exclusively for the days when hedging was applied, which corresponds to the 120 out of 266 days on which VaR exceeded € 95 000.

Risk Factor	Unhedged Marginal VaR		Hedged Marginal VaR		Variation in EUR (%)	Variation in portfolio weight (%)
	EUR	Pct (%)	EUR	Pct (%)		
U.S.	56 480.43	54.22	49 595.70	52.21	-12.19	-2.02
S&P500	56 480.43	54.22	49 595.70	52.21	-12.19	-2.02
Europe	27 782.24	26.67	26 895.84	28.31	-3.19	1.64
DAX	10 137.95	9.73	10 366.68	10.91	2.25	1.18
AEX	2 123.51	2.04	2 119.74	2.23	-0.18	0.18
CAC 40	12 077.98	11.60	10 989.40	11.57	-9.01	-0.03
IBEX	2 911.89	2.80	2 950.47	3.11	1.32	0.31
FTSE 100	530.91	0.51	470.15	0.49	-11.45	-0.01
Asia	1 950.22	1.87	2 064.32	2.17	5.85	0.30
N225	1 919.61	1.84	2 037.23	2.14	6.13	0.30
HSI	30.61	0.03	27.09	0.03	-11.50	0.00
Total Equity	86 212.90	82.77	78 555.87	82.69	-8.88	-0.08
Total Currency	14 879.79	14.29	12 556.45	13.22	-15.61	-1.07
Total Bonds	3 067.40	2.94	3 887.68	4.09	26.74	1.15
Total Portfolio	**104 160.08**	**100.00**	**95 000.00**	**100.00**	**-8.79**	—

Table 6.3. Average Marginal VaR before and after hedging by region and index. The table presents the average marginal VaR for the unhedged and hedged portfolios, considering only the 120 out of 266 days on which hedging was applied.

Before analyzing Table 6.3, let's remember our main goal: the objective of hedging was to decrease both total portfolio and stock risk. On days when hedging was applied, total portfolio risk was reduced by 8.79%, and total stock risk decreased by 8.88%, which is consistent with our main objective. However, by analyzing the variation in portfolio weight for each risk element, we can observe that the risk has been redistributed. As expected, while hedging reduced exposure to the S&P500 and CAC 40 indices, it also changed the correlations between assets, resulting in changes in how risks were spread across the portfolio. Indeed, the largest variations in risk occurred in factors that were not directly hedged, such as currency and interest rates. On the one hand, currency risk saw the largest decrease. By taking short positions, the currency contribution dropped by 1.07% in portfolio weight, since the position in the S&P500 reduced exposure to the U.S. currency market. On the other hand, interest rate risk had the largest increase, both in terms of absolute value and portfolio weight. This can be explained by the fact that hedging altered the portfolio's exposure by also changing correlations and sensitivities between assets. Specifically, the increase was mostly caused by changes in the correlation of short-term rates, which included EUR3M, EUR6M, EUR1Y, USD3M, USD6M, and USD1Y. Among them, EUR6M and USD1Y showed the largest increase in correlation, rising by 46%. Thus, the hedging strategy shifted the relative importance of the various risk factors, making interest rates a more dominant source of portfolio risk.

Regarding the hedged risk factors, the S&P500 had a larger decrease in both portfolio weight and absolute terms compared to the CAC40. This can be explained by the fact that, in the unhedged portfolio, the risk associated with the U.S. equities was, on average, 5.2 times greater than the risk from French stocks (represented by the CAC 40 index). Consequently, the hedging strategy had a more significant impact on the U.S. market. Additionally, both Europe and Asia experienced an increase in their risk weights within the overall portfolio. In Europe, despite a decrease in marginal VaR, the increase was driven by a higher risk weight in practically all unhedged indexes, except for the FTSE. Among them, the DAX contributed the most to the rise in Europe's overall risk weight. In Asia, the N225 was the main index responsible for the increase in the region's risk weight. Overall, the reduction in U.S. equity risk, combined with the rise in European and Asian stocks, resulted in a more balanced distribution of stock weights within the portfolio, since the total stock weight remained nearly unchanged (a decrease of just 0.08%).

From the analysis of Table 6.3, we conclude that hedging altered the distribution of each risk factor exposure while decreasing some risk. In the previous table, our focus was solely on changes in diversified VaR. To complement this analysis, the following table introduces the concept of undiversified VaR, presenting the diversification effects on both the worst VaR day and on average hedged days across the main risk factors. We considered these two scenarios to compare portfolio dynamics under extreme versus average market conditions. To calculate the diversification effect, we consider both standalone and marginal VaR. We define the diversification effect as:

$$\text{Diversification Effect (D.E.)} = \frac{\text{Standalone VaR} - \text{Marginal VaR}}{\text{Standalone VaR}}, \tag{6.5}$$

where the standalone VaR measures the risk of each asset in isolation, independent of its correlations with other portfolio components. The Marginal VaR is defined in Equation 6.1. The resulting value quantifies the reduction in total risk due to the interactions between assets within the portfolio.

Table 6.4 presents the diversification effects between stocks, interest rates and currencies on both the worst day of VaR (the day with the highest total VaR within the one year) and the average hedged days. The worst VaR day occurred on 17 March 2023, when VaR exceeded the limit by 30.4%. Additional details on the diversification effects for each risk index are provided in Table C.2 in Appendix C.

Risk Factor	Worst VaR day			On average hedged days		
	Unhedged D.E.	Hedged D.E.	Variation	Unhedged D.E.	Hedged D.E.	Variation
Equity (%)	17.14	50.61	195.35	14.67	29.77	102.96
Bonds (%)	41.15	43.50	5.71	87.82	84.56	-3.71
Currency (%)	111.62	100.81	-9.69	66.46	68.01	2.34

Table 6.4. Diversification effects for equity, interest rate, and currency. A 20% diversification effect means 20% of the total VaR (undiversified or stand-alone VaR) is reduced due to risk factor interactions. A larger diversification effect is better since it indicates a greater risk reduction.

According to the results presented in Table 6.4, the asset class with the greatest diversification benefits on average (measured by the largest difference between undiversified and diversified VaR) are bonds, followed by currencies, and then stocks. Building on the findings from Table 6.3, an inverse relationship between risk and diversification was observed across all portfolio components.

By observing Table 6.4, the significant increase in stock diversification stands out immediately, both on average days and worst day. As expected, by taking hedging positions on indexes, the total stand-alone of stocks increases while the marginal VaR decreases, which amplifies the diversification effect. As detailed in Table C.2 in Appendix C, the hedged indices saw the largest increase in diversification after hedging, both on the worst day and on average. This occurred because the hedged indices experienced the greatest reduction in marginal VaR, resulting in a wider gap between diversified and undiversified VaR. This effect is also observed in the currency risk factor, however, with a much smaller variation. For the unhedged risk factor (interest rate), hedging did not alter its risk factor exposure. Therefore, its stand-alone VaR remained unchanged, and the slight decrease in diversification was solely due to an increase in its marginal VaR after hedging. On the worst day, the interest rate risk did not exhibit the same behavior because its marginal VaR decreased. Additionally, while equity risk consistently shows a significant increase in diversification after hedging in both situations, interest rate and currency exposures showed an inverse pattern when comparing the worst VaR day and the average hedged days.

From this analysis, we conclude that hedging impacts risk and diversification. However, while risk is reduced, the portfolio's profit and loss can also be affected, since the lower volatility caused by the short positions can limit potential gains. Therefore, the next section analyses the trade-off between risk reduction and its impact on profitability.

6.4. Performance Analysis

Figure 6.3 illustrates the daily P&L differences between the hedged and unhedged portfolios over one year.

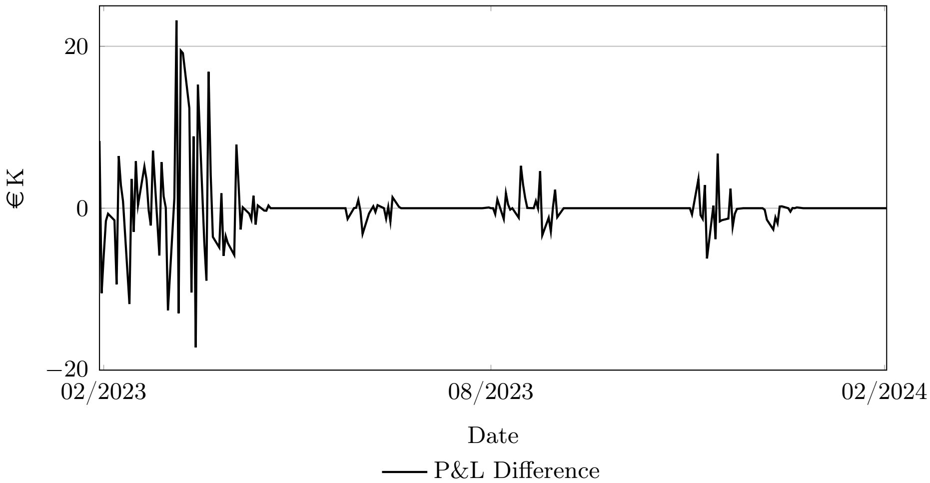

Figure 6.3. Daily P&L difference between Hedged and Unhedged Portfolios.

As displayed in Figure 6.3 above, the period with the largest variation in the P&L difference coincides with the largest decrease in VaR observed in previous Figure 6.2. This can be explained because when hedging positions are implemented, the portfolio's risk profile changes, leading to fluctuations in the P&L. This is evidenced by the fact that, on some days, hedging increased P&L, while on others, it limited profits. Specifically, on nearly half of the days, the P&L increased (58 out of the 120 hedged days). On the remaining days, the P&L of the hedged portfolio was lower than that of the unhedged portfolio, meaning that hedging limited profits on 62 days. Overall, the standard deviation of the P&L decreased by 4.4%. To better understand this trade-off, we analyze the results by distinguishing between days when the unhedged P&L was positive and those when it was negative. Table 6.5 presents the difference between the hedged and unhedged P&L, distinguishing between positive and negative unhedged P&L days to show the trade-off in the impact of hedging.

Statistic	$\mathbf{P\&L_{unhedged} > 0}$	$\mathbf{P\&L_{unhedged} < 0}$
Average (€)	-2 747.29	3 576.72
Maximum (€)	3 617.46	23 197.86
Minimum (€)	-17 204.92	-4 843.30
3Q	-311.65	5 235.20

Table 6.5. Profit and Loss difference between hedged and unhedged portfolios. The table presents some statistics for the P&L difference ($P\&L_{hedged}$-$P\&L_{unhedged}$) based on whether the unhedged P&L was positive or negative.

As highlighted in Table 6.5, hedging reduced on average P&L by € 2 747 on days with positive unhedged P&L. This reflects the trade-off in risk reduction as lower risk typically limits the potential for higher profits. Conversely, on days with negative unhedged P&L, hedging improved performance by increasing the average P&L by € 3 577, since the risk reduction helped mitigate losses.

Looking at the maximum differences, the largest difference between the hedged and unhedged portfolio during a positive P&L scenario was € 3 617. However, when unhedged P&L was negative, the maximum benefit reached € 23 198. This reinforces that, during downturns ($P\&L_{unheged}<0$), hedging can significantly limit losses, as the largest difference occurs when the unhedged result is negative. On the other hand, the minimum difference observed when unhedged P&L was positive was −€ 17 205, reinforcing that in some cases, the hedge reduced profits. The third quartile values further support these findings. In 75% of positive P&L cases, hedging reduced profits by at least € 312, while in 75% of negative P&L cases, it improved results by at least € 5 235.

Figure 6.4 presents the performance of VaR and the exceedances observed over the one year for both the unhedged and hedged portfolios.

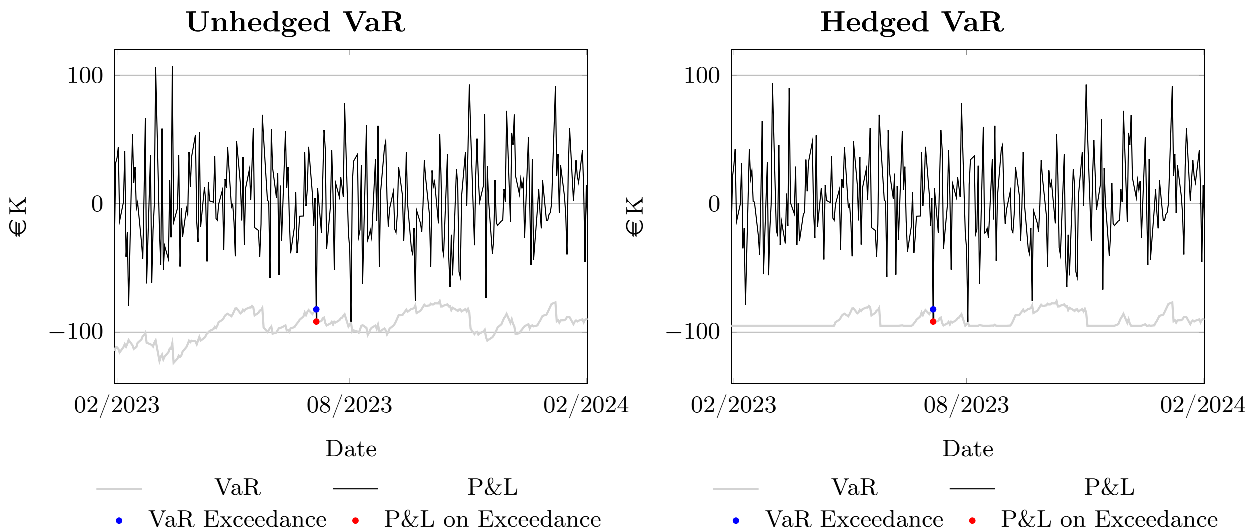

Figure 6.4. Comparison of VaR and P&L evolution with and without hedging over one year.

Figure 6.4 shows that the hedging strategy did not impact the number of exceedances, as both hedged and unhedged portfolios experienced only one exceedance throughout the year. Although hedging was designed to mitigate losses, the exceedance occurred on a day when the strategy was not implemented (6 July 2023). On this day, the VaR was 13% below the target of € 95 000, meaning the portfolio was within the acceptable risk limit, and there was no need to apply the hedging strategy. Therefore, while a fixed VaR threshold is effective in managing and reducing overall portfolio risk, it does not prevent exceedances on days when the strategy is not applied.

Figure 6.5 presents the daily cumulative P&L for the hedged and unhedged portfolios, over one year.

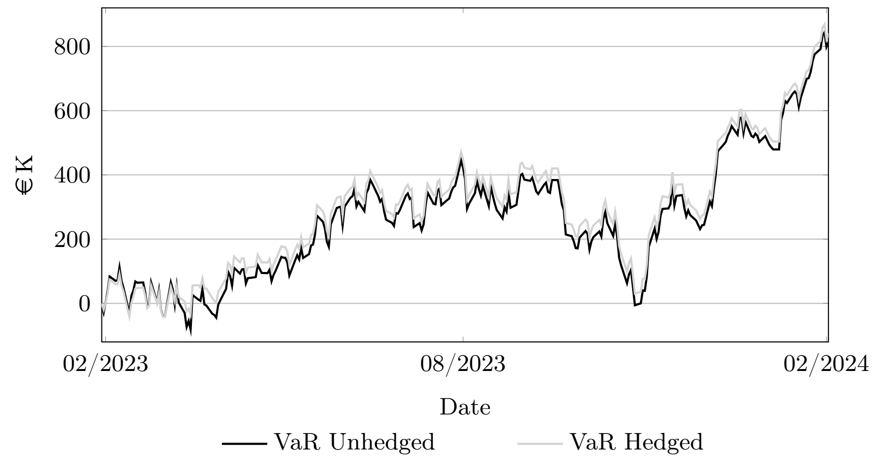

Figure 6.5. Daily cumulative P&L for Hedged and Unhedged Portfolios.

As shown in Figure 6.5, both portfolios exhibited a similar performance. From the beginning of the observation period until 21 April 2023, when the short positions were larger, the daily cumulative P&L remained relatively stable. Furthermore, it was only until early March 2023 that the cumulative daily unhedged P&L was higher than the hedged P&L. Overall, both portfolios generated a profit by the end of the year. The unhedged portfolio achieved a profit of € 795 964, while the hedged portfolio recorded a profit of € 820 433, resulting in an additional gain of 3.07%. However, as discussed in the previous subsection, the hedging strategy altered the portfolio's risk profile. Consequently, the additional profit of 3.07% does not reflect the level of risk taken to achieve this return. To consider the trade-off between the return achieved and the risk taken, we use the concept of RORAC (Matten, 1996). This risk-adjusted performance metric is calculated as follows:

$$\text{RORAC} = \frac{\text{P\&L}}{\text{EC}}, \tag{6.6}$$

where the numerator represents the daily cumulative P&L for the one-year, and the denominator corresponds to the Economic Capital for the same period, calculated as the sum of the daily EC.

Table 6.6 presents the RORAC for both strategies.

Metric	Unhedged	Hedged	Variation (%)
P&L (€ K)	795.96	820.43	3.07
Average P&L (€ K)	3.00	3.10	3.07
Return (%)	10.37	10.69	3.07
EC (€ K)	25 143.01	24 043.80	-4.37
RORAC (%)	3.17	3.41	7.79

Table 6.6. Portfolio Performance Comparison (Hedged vs. Unhedged).

Table 6.6 highlights that the hedging not only improved profits and returns but also enhanced the risk-adjusted performance of the portfolio. The additional return is higher before adjusting for risk, since hedging limits both the potential for large gains and the possibility of significant losses. The increase in RORAC is consistent with the rise in P&L and the reduction in economic capital. Specifically, the 3.07% increase in the P&L of the hedged portfolio corresponds to a 7.79 percentage point increase in return when adjusted for the risk taken, reflecting the combined effect of the improvement in P&L and the reduction in economic capital.

Therefore, while hedging mitigated the portfolio's overall risk, redistributed risk exposure across risk factors, and increased the diversification effects in stocks, it also improved the P&L, resulting in a higher risk-adjusted return.

CHAPTER 7

CONCLUSION

This study demonstrated how VaR management can be integrated into an active risk management strategy, influencing the exposure, diversification, and performance of a portfolio.

Using a portfolio of equities and bonds from global markets, and after testing 15 model configurations across four model classes, the SGSt model with EWMA volatility (λ=0.955, n=500) was identified as the most effective for our portfolio to capture the distributional characteristics of returns, particularly negative skewness and excess kurtosis. However, while the SGSt model outperformed the others overall, it is important to highlight that the BCP rejected the time independence of exceedances for all models.

With the model selected and to manage VaR, we determined the desirable capital to cover exposure, the Economic Capital. Therefore, we define the stop-loss limit as €95 000, slightly below the 10-year average of VaR under normal conditions. Whenever the estimated VaR exceeded this threshold, we implemented a hedging strategy through short positions in futures on the S&P 500 and CAC 40 indices, selected according to their marginal VaR contributions.

The results showed that the hedging strategy met the main objective, reducing both total portfolio risk and stock risk by approximately 8.8%. However, this came with a redistribution of risk across factors. While exposure to S&P 500 and CAC 40 indexes was reduced, the portfolio dynamics shifted, slightly increasing interest rate risk, particularly due to stronger correlations in short-term maturities. The currency risk, in contrast, declined because of the short positions in the U.S. index. Furthermore, equity risk was the decomposition with the smallest variation in the risk weight of the portfolio, but the one with the largest variation in the diversification effect.

Regarding portfolio performance, hedging reduced P&L on average by € 2 747 on days with positive unhedged P&L but mitigated losses by increasing the average P&L by € 3 577 on negative days. Therefore, taking less risk limited the potential for higher profits but improved the RORAC of the portfolio due to the combined effect of the reduction in economic capital and the increase in the portfolio's P&L.

In conclusion, given the limitations observed in the global performance of VaR models, particularly in failing to capture exceedance clustering, future research should explore the use of Expected Shortfall as an alternative risk measure. Expected Shortfall is more sensitive to extreme events than VaR since it considers the average loss in the tail beyond the selected quantile. This is particularly important in portfolios such as the one analyzed here, where return distributions are non-normal and subject to fat tails. Furthermore, since the hedging strategy was applied during a relatively stable period, assessing its robustness under stressed market conditions, such as financial crises or geopolitical shocks where correlations between assets can shift, would provide a better understanding of its effectiveness.

Bibliography

Alexander, C. (2008). *Market risk analysis, pricing, hedging and trading financial instruments*. John Wiley & Sons.

Alexander, C. (2009). *Market risk analysis, value at risk models*. John Wiley & Sons.

Ammann, M., & Reich, C. (2001). Var for nonlinear financial instruments—linear approximation or full monte carlo? *Financial Markets and Portfolio Management*, *15*(3), 363–378.

Bank for International Settlements. (2023). *Basel iii: International regulatory framework for banks*. https://www.bis.org/basel_framework/index.htm

Barone-Adesi, G., Bourgoin, F., & Giannopoulos, K. (1998). Don't look back. *Risk*, *11*, 100–103.

Beder, T. S. (1995). Var: Seductive but dangerous. *Financial Analysts Journal*, *51*(5), 12–24. https://doi.org/10.2469/faj.v51.n5.1932

Berkowitz, J., Christoffersen, P., & Pelletier, D. (2011). Evaluating value-at-risk models with desk-level data. *Management Science*, *57*(12), 2213–2227. https://doi.org/10.1287/mnsc.1080.0964

Best, P. (2000). *Implementing value at risk*. John Wiley & Sons.

Boudoukh, J., Richardson, M., & Whitelaw, R. (1998). The best of both worlds. *Risk*, *11*(5), 64–67.

Chan, K. F., & Gray, P. (2006). Using extreme value theory to measure value-at-risk for daily electricity spot prices. *International Journal of forecasting*, *22*(2), 283–300. https://doi.org/10.1016/j.ijforecast.2005.10.002

Christoffersen, P., Hahn, J., & Inoue, A. (2001). Testing and comparing value-at-risk measures. *Journal of empirical finance*, *8*(3), 325–342. https://doi.org/10.1016/S0927-5398(01)00025-1

Dell'Ariccia, G., Detragiache, E., & Rajan, R. (2008). The real effect of banking crises. *Journal of Financial Intermediation*, *17*(1), 89–112. https://doi.org/10.1016/j.jfi.2007.06.001

Dowd, K. (1998). *Beyond value at risk: The new science of risk management*. Wiley, New York.

Eurostat. (2023). *Inflation in the euro area*. Statistical Office of the European Union. https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Inflation_in_the_euro_area

Giot, P., & Laurent, S. (2003). Value-at-risk for long and short trading positions. *Journal of Applied Econometrics*, *18*(6), 641–663. https://doi.org/10.1002/jae.710

Haugom, E., Langeland, H., Molnár, P., & Westgaard, S. (2014). Forecasting volatility of the us oil market. *Journal of Banking & Finance*, *47*, 1–14. https://doi.org/10.1016/j.jbankfin.2014.05.026

Hendricks, D. (1996). Evaluation of value-at-risk models using historical data. *Economic policy review*, *2*(1). https://doi.org/10.2139/ssrn.1028807

Hull, J., & White, A. (1998). Incorporating volatility updating into the historical simulation method for value-at-risk. *Journal of risk*, *1*(1), 5–19. https://doi.org/10.21314/JOR.1998.001

Jorion, P. (2002). How informative are value-at-risk disclosures? *The Accounting Review*, *77*(4), 911–931. https://doi.org/https://doi.org/10.2308/accr.2002.77.4.911

Jorion, P. (2007). *Value at risk: The new benchmark for managing financial risk*. McGraw-Hill, New York.

Jorion, P. (2009). Risk management lessons from the credit crisis. *European Financial Management*, *15*(5), 923–933. https://doi.org/10.1111/j.1468-036X.2009.00507.x

Kaplan, R. S., & Mikes, A. (2016). Risk management—the revealing hand. *Journal of Applied Corporate Finance*, *28*(1), 8–18. https://doi.org/10.1111/jacf.12155

Koenker, R., & Bassett Jr, G. (1978). Regression quantiles. *Econometrica: journal of the Econometric Society*, 33–50. https://doi.org/10.2307/1913643

Kupiec, P. H. (1995). *Techniques for verifying the accuracy of risk measurement models* (Vol. 95). Division of Research; Statistics, Division of Monetary Affairs, Federal.

Linsmeier, T. J., & Pearson, N. D. (1996). Risk measurement: An introduction to value at risk. https://doi.org/10.22004/ag.econ.14796

Matten, C. (1996). *Managing bank capital: Capital allocation and performance measurement*. Wiley.

McDonald, J. B., & Newey, W. K. (1988). Partially adaptive estimation of regression models via the generalized t distribution. *Econometric theory*, *4*(3), 428–457. https://doi.org/10.1017/S0266466600013384

Moreira, A., & Muir, T. (2017). Volatility-managed portfolios. *The Journal of Finance*, *72*(4), 1611–1644. https://doi.org/10.1111/jofi.12513

Morgan, J. P. (1996). *Riskmetrics- technical document*. https://www.msci.com/documents/10199/5915b101-4206-4ba0-aee2-3449d5c7e95a

Pafka, S., & Kondor, I. (2001). Evaluating the riskmetrics methodology in measuring volatility and value-at-risk in financial markets. *Physica A: Statistical Mechanics and its Applications*, *299*(1-2), 305–310. https://doi.org/10.1016/S0378-4371(01)00310-7

Sharpe, W. F. (1970). Portfolio theory and capital markets. *McGraw-Hill*. https://doi.org/10.1080/00137917108902694

Theodossiou, P. (1998). Financial data and the skewed generalized t distribution. *Management science*, *44*(12-part-1), 1650–1661. https://doi.org/10.1287/mnsc.44.12.1650

Tiesset, M., & Troussard, P. (2005). Regulatory capital and economic capital. *Banque de France Financial Stability Review*, *7*, 59–74.

Trenca, I., Mutu, S., & Dezsi, E. (2011). Advantages and limitations of var models used in managing market risk in banks. *Finance–Challenges of the Future*, *13*, 32–43.

van den Goorbergh, R. W., & Vlaar, P. J. (1999). *Value-at-risk analysis of stock returns historical simulation, variance techniques or tail index estimation?* De Nederlandsche Bank NV.

Westgaard, S., Århus, G. H., Frydenberg, M., & Frydenberg, S. (2019). Value-at-risk in the european energy market: A comparison of parametric, historical simulation and quantile regression value-at-risk. https://doi.org/10.21314/JRMV.2019.213

APPENDIX A

Portfolio Exposures

Equity Risk Factor	Exposure (EUR)	Bond Risk Factor	Exposure (EUR)	Currency Risk Factor	Exposure (EUR)
AMZN	140 769	EUR3M	-0.34	USDEUR	3 330 864
GS	155 297	EUR6M	-2.20	GBPEUR	217 082
NVDA	248 682	EUR1Y	-9.06	JPYEUR	279 634
NKE	287 886	EUR2Y	-22.66	HKDEUR	-17 389
TMO	-262 392	EUR3Y	-114.03		
DIS	280 616	EUR5Y	-624.55		
WMT	-214 456	EUR7Y	-316.83		
AAPL	-53 142	EUR10Y	0.00		
MCD	316 122	EUR15Y	0.00		
GOOG	329 740	EUR20Y	0.00		
MA	130 520	USD3M	-1.94		
GE	-60 634	USD6M	-0.60		
BKNG	250 460	USD1Y	-7.40		
VOW3.DE	322 888	USD2Y	-176.19		
ALV.DE	-139 666	USD3Y	-27.39		
DHL.DE	237 159	USD5Y	-119.53		
BAYN.DE	294 654	USD7Y	-103.91		
SIE.DE	-149 697	USD10Y	-164.73		
HEIA.AS	361 618	USD20Y	-186.84		
BNP.PA	280 065				
DANOY	-80 381				
AIR.PA	342 841				
VIE.PA	127 517				
ORAN	-61 818				
MEL.MC	152 000				
ITX.MC	-10 821				
ULVR.L	217 082				
7974.T	340 663				
6758.T	-61 029				
0003.HK	-17 389				

Table A.1. Portfolio risk factor exposure map by asset class on 27 January 2023.

APPENDIX B

Backtest Performance Details

	2018–2017		2017–2016		2016–2015		2015–2014		2014–2013	
	Exc. Rate (%)	*p*-value (%)	Exc. Rate (%)	*p*-value (%)	Exc. Rate (%)	*p*-value (%)	Exc. Rate (%)	*p*-value (%)	Exc. Rate (%)	*p*-value (%)
1	1.15	80.77	0.38	25.44	1.15	80.77	1.54	41.87	1.15	80.77
2	0.77	69.67	0.38	25.44	1.15	80.77	1.15	80.77	1.15	80.77
3	0.77	69.67	0.38	25.44	1.15	80.77	1.54	41.87	1.15	80.77
4	0.77	69.67	0.77	69.67	1.15	80.77	1.54	41.87	1.15	80.77
5	0.77	69.67	0.38	25.44	1.15	80.77	1.15	80.77	1.15	80.77
6	1.92	18.44	0.77	69.67	1.15	80.77	1.15	80.77	1.15	80.77
7	1.92	18.44	0.38	25.44	1.15	80.77	1.15	80.77	1.15	80.77
8	0.38	25.44	0.38	25.44	2.69	2.34	1.54	41.87	0.77	69.67
9	1.15	80.77	0.38	25.44	1.15	80.77	1.92	18.44	0.77	69.67
10	1.54	41.87	0.38	25.44	1.15	80.77	1.92	18.44	1.15	80.77
11	1.92	18.44	0.38	25.44	1.15	80.77	1.15	80.77	1.92	18.44
12	1.92	18.44	0.38	25.44	1.15	80.77	1.92	18.44	1.15	80.77
13	1.54	41.87	0.77	69.67	1.15	80.77	2.31	7.01	1.15	80.77
14	1.15	80.77	0.77	69.67	2.31	7.01	1.54	41.87	1.15	80.77
15	1.15	80.77	0.77	69.67	1.54	41.87	1.54	41.87	1.15	80.77

Table B.1. UC test results for all models over the annual sub-period (2013-2018).

	2023–2022		2022–2021		2021–2020		2020–2019		2019–2018	
	Exc. Rate (%)	*p*-value (%)	Exc. Rate (%)	*p*-value (%)	Exc. Rate (%)	*p*-value (%)	Exc. Rate (%)	*p*-value (%)	Exc. Rate (%)	*p*-value (%)
1	1.15	80.77	1.54	41.87	3.46	0.18	1.54	41.87	1.92	18.44
2	0.00	2.22	0.38	25.44	2.69	2.34	0.77	69.67	1.54	41.87
3	0.38	25.44	1.15	80.77	1.92	18.44	1.15	80.77	1.15	80.77
4	0.38	25.44	1.15	80.77	1.15	7.01	0.38	80.77	2.69	69.67
5	0.00	2.22	1.15	80.77	1.92	18.44	1.15	80.77	1.15	80.77
6	0.77	69.67	0.77	69.67	1.54	41.87	1.15	80.77	0.77	69.67
7	0.38	25.44	0.38	25.44	1.54	41.87	1.15	80.77	1.54	41.87
8	3.08	0.69	0.38	25.44	2.69	2.34	0.00	2.22	1.92	18.44
9	1.54	41.87	0.00	2.22	2.31	7.01	0.77	69.67	1.92	18.44
10	0.00	2.22	0.77	69.67	1.92	18.44	1.54	41.87	1.92	18.44
11	0.77	69.67	0.77	69.67	1.54	41.87	0.77	69.67	1.92	18.44
12	0.38	25.44	0.38	25.44	1.54	41.87	1.54	41.87	1.54	41.87
13	0.38	25.44	0.77	69.67	1.54	41.87	1.54	41.87	1.54	41.87
14	0.00	2.22	1.15	80.77	2.69	2.34	1.92	18.44	1.15	80.77
15	0.38	25.44	0.77	69.67	1.92	18.44	1.54	41.87	1.15	80.77

Table B.2. UC test results for all models over the annual sub-period (2018-2023).

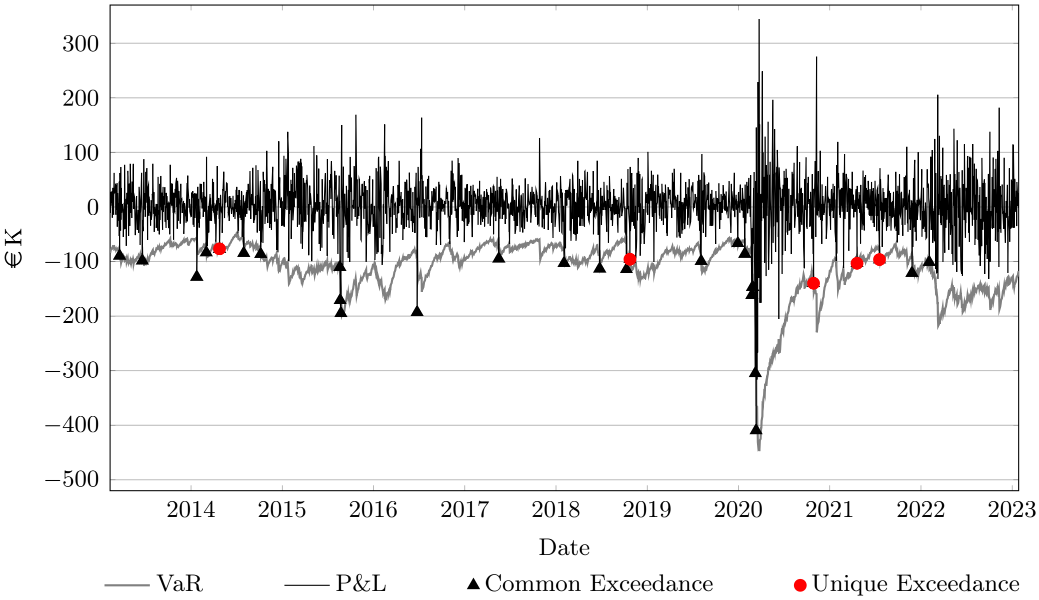

Figure B.1. VaR Model number 3 global perfomance. Triangles represent the P&L on days when exceedances occurred for both Model 3 and Model 7, while circles indicate the P&L on days when an exceedance was observed only in Model 3.

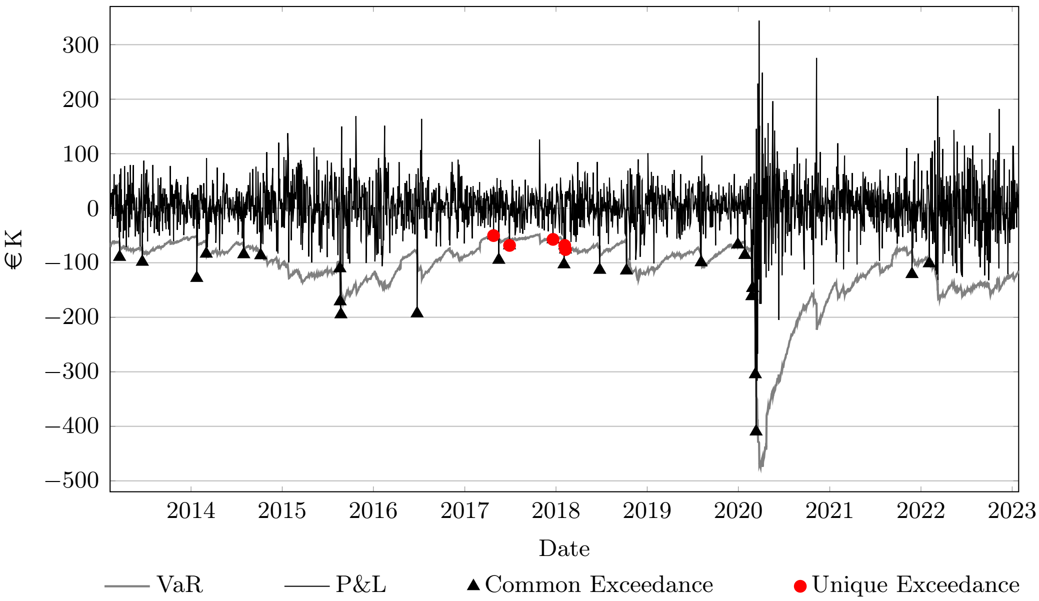

Figure B.2. VaR Model number 7 global perfomance. Triangles represent the P&L on days when exceedances occurred for both Model 3 and Model 7, while circles indicate the P&L on days when an exceedance was observed only in Model 7.

Exceedance Date	P&L (€K)	Total VaR (€K)	Exceedance (€K)	Exceedance (% of VaR)
2022-02-03	-101.21	100.8	0.4	0.4
2021-11-26	-120.98	84.21	36.76	43.7
2021-07-19	-96.42	75.58	20.84	27.6
2021-04-20	-103.22	88.84	14.38	16.2
2020-10-28	-139.86	129.47	10.39	8.0
2020-03-12	-409.66	255.64	154.02	60.2
2020-03-09	-304.74	173.00	131.74	76.2
2020-02-27	-146.51	130.56	15.95	12.2
2020-02-24	-161.52	83.04	78.48	94.5
2020-01-27	-85.94	60.28	25.66	42.6
2019-12-30	-66.71	60.15	6.56	10.9
2019-08-05	-99.32	92.34	6.98	7.6
2018-10-24	-95.88	93.04	2.84	3.1
2018-10-10	-114.40	63.97	50.43	78.8
2018-06-25	-113.19	84.92	28.26	33.3
2018-02-02	-103.08	74.64	28.44	38.1
2017-05-17	-94.81	67.85	26.96	39.7
2016-06-24	-193.02	81.76	111.26	136.1
2015-08-24	-194.78	150.26	44.53	29.6
2015-08-21	-170.98	117.03	53.95	46.1
2015-08-20	-110.36	102.54	7.82	7.6
2014-10-07	-86.46	77.78	8.68	11.2
2014-07-31	-84.63	59.11	25.52	43.2
2014-04-25	-76.77	71.20	5.57	7.8
2014-03-03	-83.54	66.02	17.51	26.5
2014-01-24	-128.06	56.50	71.56	126.7
2013-06-20	-98.42	87.49	10.94	12.5
2013-03-21	-89.46	80.87	8.59	10.6
Number of exceedances				**28**
Exceedance rate (%)				**1.08**

Table B.3. Exceedance details for model 3: P&L, Total VaR, and Size of Exceedance.

Exceedance Date	P&L (€K)	Total VaR (€K)	Exceedance (€K)	Exceedance (% of VaR)
2022-02-03	-101.21	92.90	8.31	8.94
2021-11-26	-120.98	76.97	44.00	57.17
2020-03-12	-409.67	223.19	186.48	83.55
2020-03-09	-304.74	160.15	144.59	90.28
2020-02-27	-146.51	112.21	34.30	30.57
2020-02-24	-161.52	78.59	82.93	105.52
2020-01-27	-85.94	60.80	25.14	41.36
2019-12-30	-66.71	65.22	1.49	2.28
2019-08-05	-99.32	86.32	13.00	15.05
2018-10-10	-114.40	62.01	52.39	84.50
2018-06-25	-113.19	71.57	41.61	58.14
2018-02-08	-75.80	67.77	8.03	11.85
2018-02-05	-68.30	65.94	2.36	3.58
2018-02-02	-103.08	57.03	46.05	80.73
2017-12-20	-57.15	54.74	2.42	4.42
2017-06-29	-68.20	55.61	12.58	22.62
2017-05-17	-94.81	51.10	43.72	85.56
2017-04-26	-50.16	49.80	0.36	0.73
2016-06-24	-193.02	77.20	115.82	150.04
2015-08-24	-194.78	137.54	57.24	41.62
2015-08-21	-170.98	110.36	60.62	54.93
2015-08-20	-110.36	98.23	12.13	12.35
2014-10-07	-86.46	77.25	9.21	11.92
2014-07-31	-84.63	65.29	19.34	29.62
2014-03-03	-83.54	58.64	24.90	42.46
2014-01-24	-128.06	50.71	77.35	152.54
2013-06-20	-98.42	72.73	25.69	35.32
2013-03-21	-89.46	63.62	25.84	40.61
Number of exceedances				**28**
Exceedance rate (%)				**1.08**

Table B.4. Exceedance details for model 7: P&L, Total VaR, and Size of Exceedance.

APPENDIX C

VaR Management Details

Investment Date	Coupon Value (EUR)	Invested Stocks	Share Price (EUR)	Stocks Purchased
2023-04-30	8 855	Nvidia	282	31
2023-05-15	8 063	Alphabet	120	67
2023-07-04	52 000	DHL.DE	43	1 218
2023-08-15	34 964	Booking	3 104	11
2023-10-31	9 185	Heineken	84	109
2023-11-15	8 185	BNP Paribas	52	155
2024-01-04	31 250	Orange	12	2 666
2024-01-15	35 750	Mastercard	428	83

Table C.1. Reinvestment of coupon payments from bonds in the portfolio.

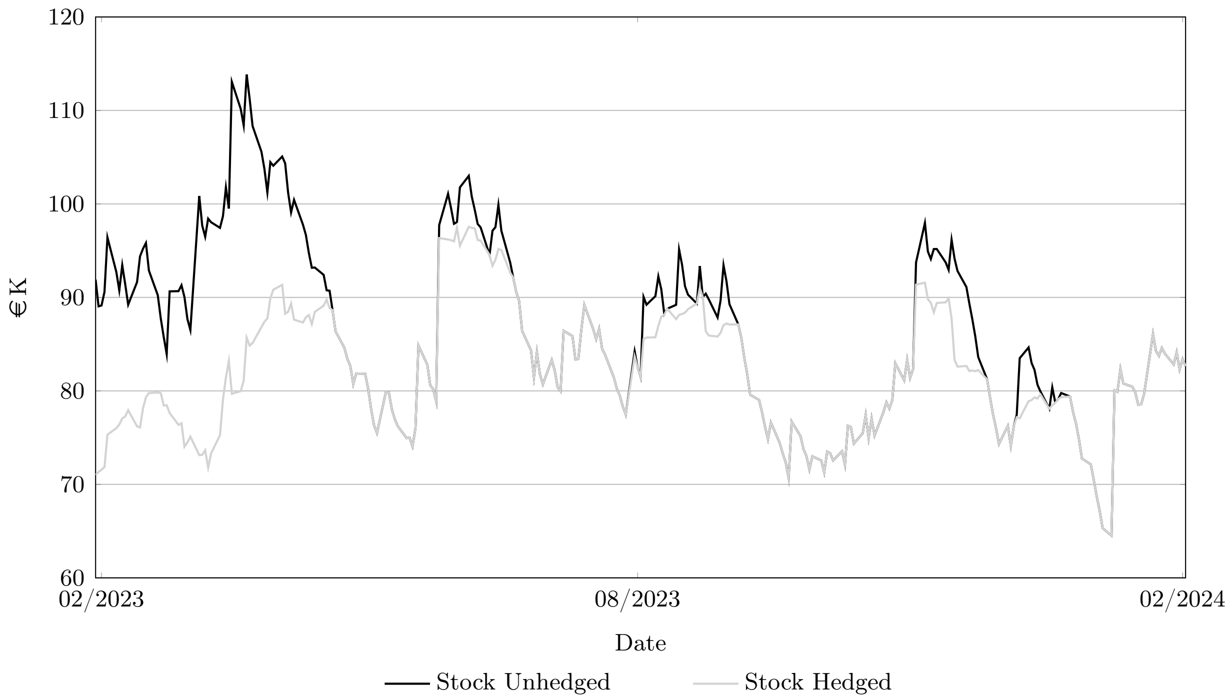

Figure C.1. Stock marginal contribution comparison between Unhedged and Hedged Portfolios.

Risk Factor	Worst VaR day			On average hedged days		
	Unhedged D.E.	Hedged D.E.	Variation	Unhedged D.E.	Hedged D.E.	Variation
S&500 (%)	17.48	19.62	12.26	17.45	17.92	2.67
DAX (%)	43.36	41.23	-4.91	52.01	50.93	-2.08
AEX (%)	66.96	69.56	3.88	82.01	82.04	0.04
CAC 40 (%)	57.56	65.07	13.05	51.35	52.69	2.61
IBEX (%)	60.77	63.96	5.24	61.16	60.64	-0.84
FTSE 100 (%)	81.76	86.72	6.06	89.79	90.96	1.30
N225 (%)	97.65	95.97	-1.72	83.52	82.51	-1.21
HSI (%)	77.59	75.34	-2.90	93.99	94.68	0.74

Table C.2. Diversification effects by index: comparison before and after hedging on the worst VaR day and on average hedged days.