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Error Estimation: A New Approach in Auditing

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Master in Business Administration

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BUSINESS SCHOOL

Department of Marketing, Strategy and Operations

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Finally, I dedicate this thesis to my two grandparents, José and Idalina, who passed away during the development of this thesis. Despite the profound difficulty of that loss, they were a source of inspiration in effort and dedication, and it is in their memories that I conclude this chapter.

Resumo

Esta dissertação explora novas abordagens no âmbito da metodologia em auditoria, com foco na melhoria do método Stringer Bound para uma estimativa mais precisa do erro na população. A revisão da literatura identificou algumas limitações do método Stringer Bound, incluindo a sua natureza excessivamente conservadora, o facto de se focar apenas em sobreavaliações e não considerar as contas sem erros, o que pode resultar em interpretações incorretas por parte dos auditores e na alocação desnecessária de recursos. De modo a mitigar estas limitações, o presente estudo propõe duas modificações ao Stringer Bound: o ajustamento do valor contabilístico no cálculo e a incorporação da proporção de contas sem erros. A implementação destas alterações foi realizada através de uma aplicação empírica com recurso ao software R. As abordagens propostas visam reduzir o conservadorismo na estimativa de erro, aproximando o erro estimado do erro real e permitindo que os auditores tomem decisões mais fundamentadas sobre a aceitação ou rejeição das demonstrações financeiras auditadas. Ao oferecer aos auditores a possibilidade de uma visão mais precisa, este estudo contribui para o avanço da metodologia em auditoria, proporcionando uma estimativa mais rigorosa do erro na população.

Abstract

This thesis explores new approaches to audit methodology, focusing on improving the Stringer Bound method for a more accurate estimation of the error in population. A review of the literature has identified some limitations of Stringer Bound, including its overly conservative nature, its exclusive focus on overstatements, and its neglect of error-free accounts, which can lead to misinterpretation by auditors and unnecessary allocation of resources. In order to mitigate these limitations, this thesis proposes two key modifications to the Stringer Bound: adjusting the book value in the calculation and incorporating the proportion of error-free accounts. An empirical application, conducted using the R software, was employed to implement the proposed changes. These approaches aim to reduce the conservatism in error estimation, thereby approximating the estimated error to the actual error, and enabling auditors to make more informed decisions when deciding whether to accept or reject the financial statement under auditing. By offering auditors more accurate insights, this research contributes to advancing audit methodology by providing a more precise estimation of the error in population.

Key Words: Auditing; Error Estimation; Stringer Bound; New Approach; R software.

JEL Classification System Code: C18; C15.

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Glossary

MUS – Monetary Unit Sampling

CAV – Combined Attributes and Variables

SB – Stringer Bound

1. Introduction

Auditing plays a critical role in ensuring the credibility and integrity of a company's financial statements. When conducting an audit of a company, the primary objective of auditors is to ensure that the values reported in the financial statements are accurate and not materially misstated. This process involves examining every account in the population, but due to the time and cost involved, auditors often opt to use sampling techniques to estimate the total error of the population instead (Higgins & Nandram, 2009). The audit process itself is multifaceted, involving meticulous examination, data scrutiny, and verification measures. It comprises a series of stages, including planning, fieldwork, and reporting, all of which contribute to a comprehensive evaluation of a company's financial health.

Within this auditing process, the selection of representative samples from the population for in-depth analysis is a crucial point. To achieve this, auditors employ the MUS (Monetary Unit Sampling), which is an effective statistical sampling technique to evaluate potential monetary misstatements within an account balance (Wampler & McEacharn, 2005). It is based on sampling techniques for attributes and has the objective of estimating the upper and lower error bounds - the maximum values of overstatement and understatement, respectively - and comparing them to the tolerable error. The accounting entry is validated if both bounds are lower than the tolerable error (Curto, 2019).

Once the sample has been selected, the next critical task is to accurately determine the upper limit of the population. To accomplish this, auditors utilize a statistical method called the Stringer Bound.

The Stringer bound was one of the first CAV (Combined Attributes and Variables) methods to be introduced (Dworin & Grimlund, 1984). This method considers the relative misstatement in the sampled items, called "taintings" (the difference between the book value and the audited value expressed as a percentage of the book value). These taintings are then arranged in descending order based on their magnitudes (Lucassen et al., 1996).

Although the Stringer Bound is the most widely method recognized in auditing and is often used as a benchmark for evaluating other approaches (Clayton, 1994), it has its drawbacks. One of the main concerns is that it leads to overly conservative confidence limits (Dworin & Grimlund, 1984), i.e., the actual confidence level obtained by this method is higher than the nominal confidence level (Burdick and Reneau (1978), Leitch et al (1982), Plante et al (1985), Reneau (1987), as mentioned in (Lucassen et al., 1996).

Several authors have addressed this issue, illustrating the conservatism of the Stringer Bound. For example, as noted by Neter et al. (1977), "All known simulation studies have consistently indicated the method's conservatism." (Lucassen et al., 1996, p. 19). Also, according to Mae Matsumura et al. (1991), the Stringer Bound has an issue of being overly conservative, so the confidence levels calculated often exceed the nominal level, potentially leading to unnecessary additional sampling of accounts. In the groundbreaking paper by Bickel et al. (1992), solid evidence was provided to support the conservatism of the bound, along with an asymptotic expansion of the Stringer Bound's probability, this ultimately resulted in Bickel's assertion that the Stringer Bound is consistently and substantially overestimated in the asymptotic sense.

In terms of errors, this method only considers overstatements and completely ignores the existence of understatements which, although "less important" for the calculation of the Stringer Bound, are still errors. Consequently, it may lead to a less accurate estimate of the true error in the financial statements (Wampler & McEacharn, 2005).

Another shortcoming of this method is that it doesn't take zero errors into account, since the Stringer Bound calculates non-zero upper bounds even if no error is detected in the selected sample. How can an error-free sample result in a substantially non-zero error rate for the population? Again, this may result in unreasonably high confidence values for the error, which can lead to less truthful interpretations. In addition, it fails to recognize the heterogeneous nature of the error distributions within the total population of account errors. These distributions typically include a dominant zero error mass, along with secondary distributions of small errors and 100% errors (Higgins & Nandram, 2009).

According to Anderson and Kraushaar (1986) "The occurrence, magnitude, and direction of non-zero book errors in a sample affect the inferences that can be made" (p.381).

The objective of this study is to propose and evaluate an alternative approach to the Stringer Bound method, taking into account the limitations of the original method and developing a new methodology that can effectively overcome these critical points. We intend to elucidate the potential advantages of these methodological improvements, thereby contributing to both the practical application of auditing and to future research.

The structure of the thesis is as follows: Chapter 1 introduces the study, including a brief contextualization of auditing, a definition of the problem, the study's objectives, and an explanation of its relevance. Chapter 2 provides a literature review on sampling methods and estimating error in the population, focusing specifically on the Stringer Bound method. In this Chapter, we also discuss the limitations of the method, along with the approaches proposed by other authors to overcome these weaknesses. Chapter 3 delineates the methodology employed in the study, detailing the calculation of the Stringer Bound and presenting the two novel approaches proposed, along with their specific calculations, implementation, and testing in the R software. Chapter 4 presents a discussion of the results obtained from the new approaches, with a comparison to the traditional Stringer Bound method and to the proposals of other authors. Finally, Chapter 5 summarizes the research findings and highlights the study's key contributions and implications for the field of auditing.

2. Literature Review

At the beginning of this century, the global economy was affected by scandals and financial crises, and this has led regulators and supervisors to reflect a lot about the auditor – its role and responsibility in ensuring reliable and high-quality information (Quick et al., 2018). This concern is still prevalent today and, as a result, the auditor's role is subject to constant review and improvement.

The relationship between audit firms and their clients is fundamental to trust and confidence in the audit. In this respect, trust in the auditor exists if the auditor is perceived to be independent of the audited entity, has the ability and attitude to perform a high-quality audit, demonstrates adherence to relevant professional principles and rules, and operates in a fair and open market (Van Hoinaru & Mary, 2016). The audit performs an indispensable economic function in serving the public interest by reinforcing trust and confidence in financial reporting (Monroe & Woodliff, 1994).

The auditor's objective is to assess whether the financial statements of the companies under scrutiny comply with the standards, and the opinion is then expressed in a final report Rocha et al. (2020). Given the time needed to carry out an audit in full and, consequently, its cost, the audit must be based on testing a sample representative of all operations. Sampling and testing are essential for the auditor to structure his opinion and issue a judgment on the financial statements. Tests can be either of compliance or substantive. Compliance tests aim to verify that the company's accounting procedures and internal control measures are in place, that they are effective in detecting material misstatements, and that they operate throughout the year. Substantive tests are designed to confirm the accuracy of the accounting process and the supporting documentation for specific balances and transactions (Curto, 2019).

The purpose of this literature review is to provide a brief context for auditing, highlighting its role and importance in the financial context. This analysis will focus on one final stage of the audit process, the Stringer Bound method. It will also analyze other approaches that have been developed to complement or replace the Stringer Bound method and assess their advantages and disadvantages.

2.1.1 Theoretical Foundations of Auditing and MUS[1]

An auditor's work involves several steps: defining the target population; selecting the type of sample to be used; determining the confidence level; calculating the sample size based on the confidence level; analyzing the data; and, finally, providing an opinion on the financial statements.

One of the difficulties in the audit process is ensuring that the sample is representative of the population, which is not easy to determine and achieve. In fact, the auditor will never know whether the sample is representative or not unless the entire population is audited. This leads to two types of error: non-sampling error and sampling error. Non-sampling risk occurs when the auditor fails to identify exceptions in the sample, either through the auditor's fault or inadequate procedures. Sampling risk occurs when there are more/fewer errors in the sample than in the population, which can lead to a false rejection/acceptance of the control mechanism.

Audit risk is defined by:

$$RA = IR \times CR \times OP \times RB \tag{2.1}$$

IR (Inherent risk) – The probability of material errors in the financial statements. It is usually considered 100%, reflecting the expectation that such an error is very likely to occur.

CR (Control risk) – The probability that internal control may fail to detect and correct material errors. A value between 10% and 100% is usually set.

OP (Other Procedures Risk) – The probability of failure of (non-statistical) audit procedures to detect errors. This risk may not be less than 50%, i.e. it must vary between 50% and 100%.

RB (Detection Risk) – The probability of failure (statistical) audit procedures in identifying errors.

In the compliance tests mentioned above, the auditor can estimate the percentage of items in the population that have a specific attribute, which is called the deviation rate.

[1] This section is mainly based on Curto (2019)

However, the same process for the sample is called the exception rate and the difference is called the sampling error.

The key issue for auditors is the upper limit of the range of estimates for the exception rate (upper exception rate), which needs to be compared with the tolerable exception rate. This is the highest exception rate that the auditor is willing to accept to confirm that the audit is compliant.

There are different types of sampling techniques, which can be divided into two categories: attribute sampling and variable sampling. In attribute sampling, although the hypergeometric distribution is applicable in most cases, the binomial and Poisson distributions are often considered conservative approximations. Variable sampling is also a source of concern for auditors because it is based on the central limit theorem and most accounting populations are skewed, which will increase the sample size (Gillett, 2000). In an attempt to overcome these shortcomings, Monetary Unit Sampling has been developed (MUS) and is broadly accepted among auditors (Wampler & McEacharn, 2005).

In MUS sampling, the population consists of monetary units (euro, dollar, pound, etc.) and each monetary unit has an equal probability of being included in the sample. However, the units that are the subject of the audit are the logical units to which each monetary unit in the sample belongs. This means that the more monetary units associated with a logical unit, the more likely it is to be included in the sample.

MUS can be considered as an ultimate application of stratification by book value as it relates directly to monetary units, each of which is treated as a sampling unit. Since MUS involves sampling units of the same size, creating subgroups based on value is unnecessary. As a result, MUS has some efficiency advantages similar to those of stratified sampling, but without the need to divide the population into strata, thus simplifying the whole process (Higgins & Nandram, 2009).

However, MUS also has some weaknesses, such as its inability to recognize that a total population of accounting errors is usually made up of many individual distributions. This can lead to exaggerated error estimates and conservative auditor judgments about the fairness of a client's financial statements (Higgins & Nandram, 2009).

To make decisions for different confidence levels, it is important to obtain reliable limits for the total error of the population. There are several methods for calculating these limits in MUS, one of which is the Stringer Bound method (Higgins & Nandram, 2009).

While is based on attribute sampling principles, the goal of MUS diverges from that of conventional attribute sampling, which focuses on estimating the misstatement rate in a population. In particular, MUS requires the evaluation of two key factors:

- The nature of the exception (overstatement or understatement), as this has significant implications for assessing the monetary misstatement within the population.
- The magnitude of the exception, which must be measured and considered in the misstatement estimation (Wampler & McEacharn, 2005).

2.1.2 Stringer Bound – A Critical Analysis

Once the sample size has been determined, the auditor's objective is to estimate the maximum error limit in the accounting records as accurately as possible, taking into account the set level of confidence. If the estimated value exceeds the established tolerance level, the auditor may not be able to express an opinion (Curto, 2019). The MUS enables the calculation of an upper error bound for the accounting records through the application of the Stringer Bound (Lucassen et al., 1996).

The Stringer Bound, introduced by Stringer in 1963, constitutes a non-classical heuristic methodology that employs the Binomial or Poisson distribution to determine the upper bound on the total error of a population (Clayton, 1994). This method gained popularity due to its simplicity and the ready availability of the requisite statistical tables (Swinamer et al., 2004), and so is widely used by auditors (Dworin & Grimlund, 1984).

The Stringer Bound considers the relative misstatement of sampled items, referred to as taintings, which represent the difference between the book value and the audited value expressed as a percentage of the book value. Subsequently, the aforementioned taintings are ordered in descending order of magnitude. This is accomplished through a linear combination of the taintings, with particular coefficients assigned to each tainting. Importantly, these coefficients are designed to decrease as the tainting decreases (Lucassen et al., 1996). This approach helps to quantify the impact of errors in financial records, by assigning greater coefficients to bigger discrepancies, thus providing a strong framework for error estimation.

The Stringer Bound is typically employed when the auditor's principal concern is the overstatement of a specific accounting balance. Applying the Stringer Bound to calculate an upper error bound is appropriate in circumstances where there is a need to estimate the maximum amount of errors within a population to ascertain whether the total of errors within the population is significant. Furthermore, this approach is applicable when the sample result is the primary source of information regarding the population (De Jager et al., 1997).

Nevertheless, this approach has certain limitations. In earlier studies, the confidence level achieved by the Stringer Bound was higher than the nominal confidence level, suggesting that the Stringer Bound may be conservative (Lucassen et al., 1996). De Jager et al. (1997) additionally observed a prevailing view in the literature that the Stringer Bound performs in such a manner that the actual confidence probability is at least equal to the nominal confidence level.

Another limitation of the Stringer Bound is that accommodates only overstatements (Swinamer et al., 2004). This limitation results in tainting distributions that are predominantly situated within the unit interval (Pap & Van Zuijlen, 1996).

It should be noted that the Stringer Bound method does not account for the possibility of zero errors in an audit population. It is, however, typical for a significant proportion of an audit population to be free of errors. This presents a challenge when attempting to utilize Stringer Bound for estimating the upper confidence bound for the population total error, which is based on the assumption of a normal distribution of the taintings (Pap & Van Zuijlen, 1996). Once more, this can result in overly optimistic confidence levels, which may lead to less accurate interpretations. Furthermore, it fails to acknowledge the heterogeneous nature of the error distributions within the total population of accounting errors (Higgins & Nandram, 2009).

2.1.3 Alternative Approaches[2]

The consequence of pursuing high reliability is a reduction in efficiency (Swinamer et al., 2004). It is therefore likely that a significant proportion of auditors do not extrapolate

[2] This section is mainly based in Swinamer et al. (2004)

sample results to populations to circumvent the potential for unrealistic conservatism and client resistance. However, the failure to extrapolate results gives rise to an error in judgment regarding the financial statements as a whole. This is one of the key reasons for the ongoing research and development of new methods (Higgins & Nandram, 2009). A review of the literature has been conducted to identify established approaches that complement conventional methods and provide auditors with greater adaptability and accuracy in their assessments. Next, we describe these approaches highlighting the strengths and weaknesses of each one.

Stringer Bound with Meikle's Adjustment for Understatements (ST-meik)- proposed by Meikle in 1972, expands the applicability of the Stringer Bound by taking into consideration the possibility of understatement errors, an aspect that was previously overlooked in the original Stringer method.

Stringer Bound with the LTA Adjustment for Understatements (ST-lta) - In their 1979 study, Leslie et al. explore a potential improvement to the Stringer Bound, known as the LTA adjustment, which accounts for understatement errors. This involves adjusting the upper bound for total population overstatement by the average understatement error calculated from the sample. Researchers Grimlund and Schroeder (1988) have found the LTA adjustment to be more accurate and consistently superior to the adjustment proposed by Meikle.

Rohrbach Augmented Variance Estimator Bound (AVE) – In his 1993 paper, Rohrbach proposes a bound for probability proportional to size sampling without replacement that improves the jackknife variance of the Horvitz-Thompson estimator. He claims that the traditional unadjusted variance estimator is inadequate because it overestimates the correlation between the book and actual values, leading to an underestimation of the variance of the bound. This is because typical audit populations either have very low error rates or, in the case of high error rates, the majority of units have insignificant monetary error amounts. The AVE bound offers the advantages of simplicity and ease of computation while accommodating both understatement and overstatement scenarios. The bound requires an adjustment parameter, which may vary.

Modified Moment Bound (MM) – Dworin & Grimlund (1984, 1986) introduced a novel concept: a parametric bound that uniquely accounts for both overstatement and understatement. This remarkable proposal incorporates a hypothetical tainting observation, known as z∗, tailored to whether the population of interest is accounts receivable (external population) or inventory (internal population). In calculating the sample moments of the unknown monetary error distribution, z∗ is treated as an additional observed taint. As a result, the resulting moment bound is typically inflated compared to scenarios where this hypothetical observation is not considered. Dworin and Grimlund (1986) noted that this conservative approach serves to compensate for the limited information available due to the small number of non-zero errors.

Multinomial-Dirichlet Bound (MD, MD-lta, MD-ext) – Based on the multinomial sampling model employed by Fienberg et al. (1977), Tsui et al. (1985) present a nonparametric Bayesian method in which all errors are considered to be overstatements, with a maximum error of 1.0. To classify these errors, they are first rounded and then grouped according to their value in cents, ranging from 0 to 100 cents. The researchers also introduce a Dirichlet prior distribution p=(p0, p1, p100...), specifically Dir($K\alpha_0$,…, $K\alpha_{100}$) using alpha values (αi), where each $\alpha i > 0$ and their sum equals 1. According to Tsui et al. (1985), the Multinomial-Dirichlet bound outperforms the Stringer Bound for various populations. However, the researchers also noted that the effectiveness of the bound may vary depending on the pre-existing values. Grimlund and Felix (1987) compared the various Bayesian bounds and found that the Multinomial-Dirichlet bound is the most reliable. By adding negative taint classifications, (Matsumura et al., 1990) expand the multinomial-Dirichlet model to account for understatements.

Parametric Power Bound (PP, PP-lta) – Tamura and Frost (1986) employ a power function density to model the distribution of taints and utilize a parametric bootstrap method (Efron & Tibshirani, 1994) to establish a bound on the total population error. In cases where an audit sample reveals no errors, the maximum potential taints are assumed to be worth $1 and the conservative attributes method is used to obtain the 100 (1-α)% bound, as described by (Johnson et al., 2005). In the event that one or more errors are detected, the parameter λ of the power density can be estimated using the number and value of the taints. It is assumed that all errors are overstatements and that no accounting errors exceed their book value, with error values in the range $z \in (0, 1]$. The reliability of this bound was investigated under limited conditions. The results showed it to be significantly more reliable and restrictive than the Stringer Bound. Nevertheless, it was suggested that the performance of the bound should be investigated in other audit populations.

Overall, this literature review has highlighted the crucial role of auditing in ensuring the accuracy and integrity of financial statements, serving as a safeguard against errors and unethical practices. The widely accepted Stringer Bound method has been utilized by auditors, but it has faced criticism for its overly conservative nature, resulting in confidence levels that surpass nominal levels. This indicates that the audit industry is continuously evolving. In light of these concerns, auditors are actively seeking alternative methods that strike a balance between reliability and efficiency. To achieve the optimal blend of accuracy and efficiency, auditing techniques must be adaptable to diverse populations, and a multi-faceted approach may prove to be the key.

3. Methodology

Having provided an overview of the Stringer Bound, including an assessment of its strengths and weaknesses, we will now elucidate its operational principles and the methodology employed to determine the final result, namely the upper misstatement bound.

$$STR = B \left\{ ps(n;0) + \sum_{i=1}^{k} [ps(n;i) - ps(n;i-1)]\, t_i \right\} \tag{3.2}$$

$X = i$; is the number of exceptions (or misstatements) found in the sample, $1 \leq i \leq \mathrm{k}$

n is the sample size

B is the total book value of the population

ps(n; i) is the exact upper confidence bound for p, which represents the probability in a binomial distribution $X \sim$ Binomial *(n,p)*

$t_1 \geq t_2, \ldots, \geq t_k$ are the taintings, which are the k ordered non-zero *ti*

Σps(n;i)−ps(n;i−1) is the sum of the differences between the confidence bounds for successive exceptions, multiplied by their respective tainting. This process captures the contribution of each error to the overall upper misstatement bound, giving more weight to larger errors (i.e., larger taintings). The final result is multiplied by the book value (B) to give the upper misstatement limit, which represents the maximum possible misstatement for the population.

To calculate the tainting associated with each account or item, it is necessary to consider both the book value (B_i) and the audit value (A_i) to determine the ratio of the discrepancy between the audit and book values to the book value. The tainting is calculated using the following formula (3.3).

$$t = \frac{B_i - A_i}{B_i} \tag{3.3}$$

3.1 New Approaches

Building upon the foundation of the Stringer Bound method, this research aims to address its inherent conservativeness and its disregard for neglect of error-free accounts in the sample. While based in the Stringer Bound principles, the proposed approaches introduce statistical refinements designed to mitigate these limitations. By incorporating a more restrictive treatment of book value and including error-free accounts, these methods strive to deliver more accurate and reliable estimates of audit error.

3.1.1 Approach 1 – Adjust the book value

In the traditional Stringer Bound formula, the book value used to calculate the upper limit of the error is the total book value of the population. We propose a more specific approach, i.e. we propose a modified approach that considers the specific book value of each account containing an error. By replacing the total book value (B), our method aims to provide a more accurate estimate of the total error, particularly in situations where errors are concentrated in a few large accounts. This refinement is expected to enhance the precision of audit risk assessments, and improve the efficiency of audit procedures, as well as provide a more accurate representation of the error distribution within the sample.

Let's assume that the selected sample has a total book value of 18 000 and that in this sample there are two accounts with errors. An account with a book value of 5 500 and an audit value of 5 000 and an account with a book value of 390 and an audit value of 350, represent an overstatement error of 500 and 40 respectively. Under the traditional Stringer Bound, both errors would be calculated using the total book value of 18 000. However, our new approach, as shown in formula (3.4), utilizes the specific book values of the accounts in question, namely 5 500 and 390.

$$STR1 = \sum_{j=1}^{m} B_j \left\{ ps(n;0) + \sum_{i=1}^{k_j} [ps(n;i) - ps(n;i-1)]\, t_i \right\} \tag{3.4}$$

B_j is the specific book value for each account j that has an error

m is the total number of account

k_j is the number of errors found in the account j

t_{ij} is the tainting for the ith exception in the account j

3.1.2 Approach 2 – Incorporate error-free accounts

In the traditional Stringer Bound framework, the treatment of accounts that exhibit zero errors, i.e., accounts where there is no discrepancy between the book value and the audited value, has been largely overlooked. This can lead to overly conservative estimates of the total error, given that a considerable number of accounts are typically error-free.

The proposed modification aims to address this limitation by quantifying the proportion of error-free accounts and integrating this data into the final error estimation. To implement this modified approach, it is first necessary to identify the accounts that exhibit no discrepancies. Let *B* represent the total book value, and *B0* represent the book value of accounts with zero errors. The proportion of these error-free accounts can be expressed mathematically as shown in formula (3.5).

$$P_{zero} = \frac{B_0}{B} \tag{3.5}$$

The calculated proportion P_{zero}, is employed as a tainting factor in the final error estimation. However, unlike conventional to tainting factors, the contribution of this zero-error proportion is treated as a negative value in the calculation. The objective of this adjustment is to reduce the overall calculated error, thereby reflecting the reality that the majority of accounts are indeed error-free.

The incorporation of this zero-error tainting into the Stringer Bound methodology follows the same calculation process as the original formula, with the only distinction being the inclusion of the negative P_{zero} factor. This adjustment recognizes the importance of error-free accounts without overshadowing the contributions of accounts with identified errors.

By positioning the P_{zero} proportion at the end of the error list and applying a negative coefficient, this approach provides a more balanced assessment of the total misstatement. The result is a more refined estimation that more accurately reflects the nature of the audited accounts, thereby enhancing the reliability of the audit conclusions.

$$STR2 = B\left\{ps(n;0) + \sum_{i=1}^{k}[ps(n;i) - ps(n;i-1)]\,t_i\right\} + B[ps(n;k+1) - ps(n;k)](-P_{zero}) \tag{3.6}$$

ps(n; k+1) − ps(n; k) refers to the lowest coefficient

P_{zero} is the proportion of the error-free accounts (formula 3.5), treated as a negative value.

3.2 Pratical Simulation

To illustrate the Traditional Stringer Bound calculation and the proposed new approaches, we present a simple and practical example. The objective of this example is to provide readers with a comprehensive understanding of the methodologies in question. To this end, we utilize data from the article by Higgins & Nandram (2009), which is based on the characteristics of the population described in Table 3.1, originally sourced from Lohr's book *Sampling: Design and Analysis* (1999). Table 3.2 presents the specific data for the four accounts in the sample that contain errors, all of which are overstatements.

Table 3.1 - Population characteristics, an example taken from Higgins & Nandram (2009)

Total Book Value	612 824
Mean Book Value	7 044
Total Accounts	87
Sample Dimension	20

Table 3.2 - Characteristics of the errors in the sample, Higgins & Nandram (2009)

Account Number	Book Value	Audit Value	Error	Tainting
24	7 090	7 050	40	0,56%
36	2 399	2 149	250	10,42%
46	69 540	69 000	540	0,78%
75	2 291	2 191	100	4,36%
	81 320	80 390	930	

The columns *Δps*, *B,* and *t* represent parcels multiplied in the last column. *Δps* corresponds to the coefficient, *B* to the book value, and *t* to the tainting, which is represented in descending order.

Table 3.3 shows the application of the traditional Stringer Bound. In Table 3.4, which illustrates Approach 1, the distinction lies in the book value. In the initial row, the book value to be utilized is calculated by subtracting the book value of the accounts with errors from the total book value. In the subsequent rows, the taintings are multiplied by the book value corresponding to the account in question. For instance, the 10.42% tainting will be multiplied by 2 399.

In Table 3.5, corresponding to Approach 2, the distinction lies in the final component of the sum, which incorporates the error-free accounts. To achieve this, the proportion of accounts without error is calculated and a negative value is derived. This negative value is then incorporated into the final calculation to effectively "remove" the weight of the accounts without error.

Table 3.3 - Example of the application of Stringer Bound

Error	ps	Δps	B	t	Traditional SB
0	2,95%	2,95%	612 824	100,00%	18 086
1	4,66%	1,70%	612 824	10,42%	1 089
2	6,16%	1,51%	612 824	4,36%	403
3	7,57%	1,41%	612 824	0,78%	67
4	8,92%	1,35%	612 824	0,56%	47
					19 691

Table 3.4 - Example of the application of Approach 1

Error	ps	Δps	B	t	Approach 1
0	2,95%	2,95%	531 504	100,00%	15 686
1	4,66%	1,70%	2 399	10,42%	4
2	6,16%	1,51%	2 291	4,36%	2
3	7,57%	1,41%	69 540	0,78%	8
4	8,92%	1,35%	7 090	0,56%	1
					15 700

Table 3.5 - Example of the application of Approach 2

Error	ps	Δps	B	t	Approach 2
0	2,95%	2,95%	612 824	100,00%	18 086
1	4,66%	1,70%	612 824	10,42%	1 089
2	6,16%	1,51%	612 824	4,36%	403
3	7,57%	1,41%	612 824	0,78%	67
4	8,92%	1,35%	612 824	0,56%	47
5	10,23%	1,31%	612 824	-86,73%	-6 940
					12 752

3.3 Design of Study

The implementation of the proposed methodologies and the traditional Stringer Bound was conducted in R statistical software. To ensure consistency with previous studies, the populations utilized are based on those defined by Neter John & K. Loebbecke (1975) in the American Institute of Certified Public Accountants (AICPA) study. These populations have been employed in numerous prior studies, including those conducted by Mae Matsumura et al. (1991) and Lucassen et al. (1996).

The article defined four populations, of which two - designated as Population 1 and Population 3- were selected for this study. These populations were chosen for their substantially different characteristics, allowing for a more comprehensive and representative study. The selection of two similar populations would not have contributed to the study's value, as the objective is to assess the proposed methodology in diverse contexts.

The principal distinctions between the selected populations pertain to the magnitude of the transactions and the nature of the error (overstatement or understatement). As stated by Neter John & K. Loebbecke (1975), Population 1 consists of “accounts receivable of a freight company” (p. 6) where “the actual error rate is high, and the errors tend to be balanced between overstatements and understatements” (p. 7). In contrast, Population 3 comprises “accounts receivable of a medium-size manufacturer” (p. 7) where “the actual error rate is moderate, with all errors being overstatements.” (p. 7).

In our study, Population 1 is further distinguished by the inclusion of smaller transactions and a combination of overstatements and understatements. On the other hand, Population 3 is characterized by the prevalence of larger transactions, comprising overstatement errors.

To initiate the simulation of the populations, starting with Population 1, we used Table 3.6 from (Neter John & K. Loebbecke, 1975), which describes the frequency of book values. This table defines intervals of book values and the respective frequency of accounts within each interval. In R, we developed a code (Annex A - R script) that randomly generated book values for each account. For example, 2 039 accounts were simulated with random values ranging from 0 to 13.50. The process was repeated for all intervals to create a simulated population that closely resembled that described by Neter John & K. Loebbecke (1975).

Once the population was established, comprising the book values, the subsequent step was to calculate the errors or audit values. It should be noted that Population 1 was configured following the specifications set forth by Neter John & K. Loebbecke, with five distinct error rates: 30%, 10%, 5%, 1%, and 0.5%. The calculation of the errors begins with the 30% error rate, after which the remaining lower error rates are derived by randomly removing the previously defined errors until the expected error rate is reached.

Table 3.6 - Frequency Distribution of Population 1 Book values (Neter John & K. Loebbecke, 1975)

Book Amount	Number of Accounts
0 - 13.50	2,039
13.51 - 22.50	2,455
22.51 - 36.00	1,867
36.01 - 63.00	852
63.01 - 105.00	494
105.01 - 195.00	335
195.01 - 345.00	136
345.01 - 675.00	79
675.01 - 945.00	24
945.01 - 1,545.00	16
1,545.01 - 6,945.00	12
	8,309

In accordance to the methodology proposed by Neter John & K. Loebbecke (1975), an error rate of 30% in the defined population requires the selection of 2 493 random accounts. Subsequently, a random error must be assigned to each account according to the range specified in Table 3.7, based on the value of the account that has been selected. To illustrate, if the first account selected has a book value of 50, it falls within the range [25.00; 99.99], thus a random value will be generated within the range [-12.38; 20.88], which will represent the error.

Table 3.7 - Range and Mean of Error Amounts of Population 1 (Neter John & K. Loebbecke, 1975)

		Error Range		
Error Pool	Book Amount	Minimum	Maximum	Mean
1	0 - 9.99	- 3.60	0.84	- 0.71
2	10.00 - 24.99	- 33.60	10.09	- 1.42
3	25.00 - 99.99	- 12.38	20.88	1.29
4	100.00 - 399.99	- 31.26	14.76	- 0.96
5	400.00 - or more	- 42.18	55.40	9.93

At this stage of the process, Population 1 has been completed and Population 3 is now being constructed. The initial phase for establishing the base population of book values for Population 3 is consistent with that of Population 1, with the exception of the specific values employed, as outlined in Table 3.8.

Table 3.8 - Frequency Distribution of Population 3 Book values (Neter John & K. Loebbecke, 1975)

Book Amount	Number of Accounts
0 - 40.00	1,334
40.01 - 136.00	1,438
136.01 - 400.00	1,475
400.01 - 800.00	878
800.01 - 1,400.00	539
1,400.01 - 3,000.00	548
3,000.01 - 5,000.00	278
5,000.01 - 10,000.00	239
10,000.01 - 49,000.00	258
49,000.01 -100,000.00	39
	7,026

Regarding the generation of errors, the methodology employed for Population 3 is markedly distinct from that used for Population 1, as Population 3 comprises solely overstatements. A random sample of 2 107 accounts was selected from a total of 7 026 accounts. Subsequently, values are assigned in accordance with the probabilities illustrated in Table 3.9. The corresponding overstatement percentage is then applied to each selected account, according to the amount of the account in question. If the calculated error value exceeds 500, a correction is made to this amount in accordance with the proposal put forth by Neter John & K. Loebbecke (1975). To facilitate interpretation, an illustrative example is provided of a first account selected with a value of 300. This account has a probability of 0.017 of exhibiting an error representing 5% of the total book value, a probability of 0.08 of exhibiting an error representing 10%, and so on, following the established error distributions.

Once the populations had been constructed, the next step in the R code was to define the sample size and the number of simulated samples. For the sample sizes, values 100 and 200 were considered, with 100 regarded as the minimum acceptable size for substantive audit tests and 200 viewed as a moderate sample size. Regarding the number of repetitions, 600 was selected, as it yields reasonable estimates of the confidence coefficient (Neter John & K. Loebbecke, 1975).

Table 3.9 - Overstatement Percentages for Population 3 (Neter John & K. Loebbecke, 1975)

Percentage	Probability
Book Amount Under 200	
1	0.06
2	0.13
5	0.06
50	0.13
75	0.13
100	0.50
Book Amount Between 200 and 1.000	
5	0.17
10	0.08
15	0.17
25	0.08
50	0.25
95	0.17
100	0.08
Book Amount Exceeding 1.000	
0.01	0.17
0.05	0.17
0.1	0.17
0.2	0.17
0.3	0.17
0.5	0.17

After defining the populations and delineating the samples, we proceeded with coding the traditional Stringer Bound and the new proposed approaches, in accordance with the methodology outlined at the beginning of this chapter. Furthermore, to guarantee the reliability of the outcomes and to evaluate the efficacy of the novel approaches, we utilize the R package JFA (Justifiable Financial Auditing). This package, in addition to the conventional Stringer Bound, encompasses alternative methodologies previously proposed (Meikle, Lta, Rohrbach, and Moment). Comparing our methods against these established approaches was invaluable in determining whether they contribute positively to the auditing community.

The utilization of the JFA not only enabled the validation of our approaches but also facilitated a comparison of our results with those of established methodologies, thereby providing a robust analysis of the effectiveness of the proposed approaches. This comparison was crucial for determining whether the novel methodologies offer significant improvements to auditing practices.

In the JFA package, we employed the "evaluation" function, which calculates the population error based on an audited statistical sample. The function enables the user to specify certain parameters, including materiality and level of precision. Upon completion of the function, an object of the JFAEvaluation class is returned, which can be analyzed using the summary() and plot() functions, thus facilitating the interpretation of the results (Derks, 2024).

After executing the code for all scenarios and populations, the result consisted of an Excel table for each scenario, totaling 20 tables. These were subsequently compiled into just 4 summaries. For each method, we calculated the overall mean error, the proportion of errors, the number of values below the actual error, and the average of the errors that were under the actual error. These calculations were essential for evaluating the performance of each approach, enabling a more robust comparative analysis. These summaries can be found in the Results Chapter.

4. Results

This chapter presents the results of applying various methods to estimate the maximum error in the population, with a primary focus on the two novel approaches proposed. The study is based on two populations, designated as Population 1 and Population 3, extracted from the article by Neter John & K. Loebbecke (1975). Additionally, the sample sizes of 100 and 200 were derived from the same source. The main objective is to assess the predictive ability of the novel approaches (SB Approach 1 and SB Approach 2) in comparison to the traditional Stringer Bound method and the existing alternatives (Meikle, Lta, Rohrbach, and Moment).

Below we present in Table 4.1 and Table 4.2, the characteristics of the book values of the populations previously defined in R, as we consider this essential for a better analysis of the study.

Table 4.1 - Major Characteristics of Population 1 Book Values

Total book value	420 896,00
Mean	50,66
Standard deviation	201,76
Skewness	20,62
Kurtosis	535,45
Maximum	6 458,33
Minimum	0,00

Table 4.2 -Major Characteristics of Population 3 Book Values

Total book value	16 719 884,00
Mean	2 379,72
Standard deviation	8 557,19
Skewness	6,62
Kurtosis	52,75
Maximum	99 864,85
Minimum	0,04

The results are presented according to population and sample size, error rate, and the method employed. The primary objective is to assess whether the proposed novel approaches offer improvements over existing methods. To achieve this, a method is required that can get as close as possible to the real error without ever falling below it, i.e. the ideal method must meet two criteria: reduce the conservatism observed in methods such as the Traditional Stringer Bound (SB Traditional), while simultaneously ensuring that the estimated error does not fall below the real observed error.

4.1 Population 1

To facilitate the analysis of the results, we will begin by examining Population 1, analyzing each method by error rate, and sample size.

For an error rate of 30%, the real population error is 9 966. According to Table 4.3, the traditional SB significantly overestimates the error, with estimates of 46 147 and 38 365 for sample sizes of 100 and 200, respectively. Approach 1, on the other hand, has lower estimates than the traditional method, of 12 399 and 6 234, for sample sizes of 100 and 200, respectively. Despite this, the estimate for the sample size of 200 (Table 4.4) underestimates the error, which is undesirable. As for Approach 2, with estimations of 41 454 and 36 081, they are closer to the real error, in a less conservative way than Traditional, which is positive. The other methods analyzed, although they have lower estimations than the traditional SB, also underestimate the error in several cases and to a greater extent. Meikle method significantly underestimates the error, with an average estimated error that is 941% lower than the real error. In contrast, while Approach 1 also underestimates the error, is less severe, at only about 37%.

For an error rate of 10%, the real population error is 3 183. The traditional SB method overestimates the error, with average estimates of 24 422 and 17 996, for sample sizes of 100 and 200, respectively. For example, for a sample size of 100, the traditional SB estimate is approximately 8 times higher than the real error (Proportion column). Approaches 1 and 2 provide estimates lower than the traditional SB without underestimating the real error. Approach 1 is closer to the real value, with estimates of 12 414 and 6 250 for the 100 and 200 sample dimensions, respectively. The Meikle and Rohrbach methods produce negative mean estimates, making them unsuitable. The Moment method, although it approximates the actual error with a mean of 5 768, presents 29 negative estimates out of 600 repetitions, with the average of these negative values being approximately 371% lower than the real error.

For an error rate of 5%, the actual error is 1 655 and the traditional SB method estimates an error of 18 748 and 12 312 for sample sizes of 100 and 200 respectively. This is 11 and 7 times higher than the real error, respectively. Approaches 1 and 2 are the most accurate, with Approach 1 outperforming the others by providing the lowest error estimate for both sample sizes. The remaining methods, except for Rohrbach, yield estimates that are lower than the traditional SB method but higher than the real error, however, they produce negative estimates which compromise their reliability.

For lower error rates of 1% and 0.5%, the actual error is 314 and 140, respectively. In these cases, as in the entire analysis, the traditional SB method overestimates the real error, reaching a proportion 96 times higher for an error rate of 0.5% and a sample size of 100. In this case, the real error is 140 and the traditional SB method estimates it 13 356. Approach 2 outperformed Approach 1 and all other methods. The remaining methods continue to underestimate the true error, as previously observed.

Comparing all simulations of Population 1, the remaining methods perform better in proportion, i.e. they can effectively reduce conservatism. However, although these methods reduce conservatism, it is very important to check that they do not fall below the true error and, if they do, to quantify this. This is what the last two columns of the table are for: they show how often the methods have a result below the real one, and then their average. These columns are essential and perhaps even the "key piece" by which we distinguish the methods.

Methods such as Meikle, Lta, Rohrbach, and Moment have a significant number of estimated errors below the true error, with the Meikle and Rohrbach methods being the most problematic. These results indicate that although Meikle reduces conservatism in this case, it also carries a significant risk of underestimating the error, which is unacceptable from an audit point of view. The Lta and Moment methods perform reasonably well, although they occasionally underestimate the true error, the number of repetitions is lower compared to the Meikle and Rohrbach methods.

In terms of sample size, it can be seen that the methods behave similarly in both dimensions, with the methods showing errors closer to the true error in samples of 200. This is to be expected as the larger the sample, the more accurate the representation of the population.

In summary, Approaches 1 and 2 performed best in Population 1. For higher error rates (5% to 30%), Approach 1 demonstrated superiority, while for lower error rates (0.5% and 1%), Approach 2 stands out. Approach 1 only had one instance of underestimating the true error, at an error rate of 30% and a sample size of 200. All the other methods, without exception, failed to meet the requirement of not underestimating the true error.

This makes the proposed approaches clearly superior, as this type of error cannot be accepted from an audit perspective.

Table 4.3 - Results for Population 1 and sample size 100

Error Rate	Real error	Method	Total Mean	Proportion	Nº of values under the real error	Mean of errors under the real error
30%	9 966	SB traditional	46 147	5	0	-
		SB Approach 1	12 399	1	0	-
		SB Approach 2	41 454	4	0	-
		Meikle	-24 871	-2	192	-350%
		Lta	22 056	2	48	121%
		Rohrbach	573	0	334	-94%
		Moment	-370	0	96	-104%
10%	3 183	SB traditional	24 422	8	0	-
		SB Approach 1	12 414	4	0	-
		SB Approach 2	19 036	6	0	-
		Meikle	-9 370	-3	122	-3219%
		Lta	14 911	5	23	-3079%
		Rohrbach	-1 622	-1	334	-371%
		Moment	5 768	2	29	-8169%
5%	1 655	SB traditional	18 748	11	0	-
		SB Approach 1	12 419	8	0	-
		SB Approach 2	12 863	8	0	-
		Meikle	8 979	5	88	-549%
		Lta	15 761	10	3	-136%
		Rohrbach	985	1	345	-196%
		Moment	15 057	9	8	-1923%
1%	314	SB traditional	14 002	45	0	-
		SB Approach 1	12 422	40	0	-
		SB Approach 2	7 255	23	0	-
		Meikle	8 219	26	19	-38234%
		Lta	12 700	40	8	-16011%
		Rohrbach	-361	-1	393	-669%
		Moment	10 005	32	8	-61618%
0,5%	140	SB traditional	13 356	96	0	-
		SB Approach 1	12 422	89	0	-
		SB Approach 2	6 424	46	0	-
		Meikle	8 359	60	12	-151554%
		Lta	12 274	88	9	-36156%
		Rohrbach	-527	-4	463	-1066%
		Moment	9 237	66	9	-142125%

Table 4.4 - Results for Population 1 and sample size 200

Error Rate	Real error	Method	Total Mean	Proportion	Nº of values under the real error	Mean of errors under the real error
30%	9 966	SB traditional	38 365	4	0	-
		SB Approach 1	6 234	1	600	-37%
		SB Approach 2	36 081	4	0	-
		Meikle	-24 508	-2	253	-941%
		Lta	14 747	1	82	-496%
		Rohrbach	1 046	0	370	-179%
		Moment	-5 615	-1	160	-958%
10%	3 183	SB traditional	17 996	6	0	-
		SB Approach 1	6 250	2	0	-
		SB Approach 2	15 455	5	0	-
		Meikle	-13 289	-4	181	-2174%
		Lta	8 670	3	48	-1437%
		Rohrbach	-1 251	0	326	-333%
		Moment	-784	0	58	-4090%
5%	1 655	SB traditional	12 312	7	0	-
		SB Approach 1	6 254	4	0	-
		SB Approach 2	9 561	6	0	-
		Meikle	4 154	3	152	-310%
		Lta	9 358	6	13	-70%
		Rohrbach	966	1	365	-146%
		Moment	9 024	5	16	-969%
1%	314	SB traditional	7 795	25	0	-
		SB Approach 1	6 257	20	0	-
		SB Approach 2	4 537	14	0	-
		Meikle	3 707	12	34	-12286%
		Lta	6 836	22	9	-8014%
		Rohrbach	-38	0	317	-525%
		Moment	5 130	16	9	-30516%
0,5%	140	SB traditional	7 241	52	0	-
		SB Approach 1	6 257	45	0	-
		SB Approach 2	3 838	27	0	-
		Meikle	1 941	14	27	-71519%
		Lta	6 103	44	19	-17738%
		Rohrbach	-558	-4	359	-1375%
		Moment	3 074	22	19	-68965%

4.2 Population 3

Prior to undertaking a comprehensive analysis, it is worth noting that in Population 3, the Meikle and Lta methods yield highly similar results. Given that both methods are designed to address understatements, but Population 3 has only overstatement, their performance converges towards the traditional Stringer Bound method, as adjustments for understatement are not applicable.

For an error rate of 30%, with a real value of 164 354, the traditional SB method significantly overestimated the actual error, with estimates of 3 288 649 for a sample size of 100 (Table 4.5) and 2 901 847 for a sample size of 200 (Table 4.6). It is important to note that none of the methods underestimated the actual error. Therefore, to identify the most accurate method, we analyzed the 'proportion' parameter, with the lowest value belonging to Approach 1 with a proportion of 3 and 1, respectively, for sample sizes of 100 and 200, and associated estimated errors of 491 409 and 246 517.

For an error rate of 10%, with an actual population error of 53 202, the traditional SB method overestimated the error, providing estimates of 1 542 295 for a sample size of 100 and 1 224 798 for a sample size of 200. The Rohrbach method underestimated the real error in 6 out of 600 estimates, with the average of these values being 64% below the actual error. Among the methods analyzed, Approach 1 demonstrated the best performance for this error rate, with an estimate of 492 668. Approach 1 overestimated the error to a lesser extent when compared to the traditional SB method, indicating greater accuracy.

For an error rate of 5%, we observed a similar pattern to that described above. With an actual error of 24 900, the traditional SB method significantly overestimated the actual error, with estimates 42 and 30 times greater than the actual error for sample sizes of 100 and 200, respectively. Approach 1 demonstrated the best performance, with estimates of 493 197 and 248 289 for sample sizes of 100 and 200, respectively. The Rohrbach method underestimated the actual error in 31 of the estimates, with an average 77% below the real error.

For error rates of 1% and 0.5%, with actual errors of 5,136 and 2,871, respectively, the traditional SB method significantly overestimated the real error. The ratio of estimated error to real error varied between 70 and 193 times, indicating that the estimates were significantly higher than the actual values. The Rohrbach method, as previously observed, underestimated the actual error, with this underestimation becoming more pronounced as the error rate decreased. Approach 2 demonstrated the best performance for rates of 1% and 0.5%, providing the estimates closest to the actual error when compared to the other methods.

When analyzing the results for Population 3, they follow a similar pattern to that observed for Population 1. Traditional SB remains the most conservative method, with the highest proportions in terms of real error, both for sample sizes and for all error rates.

The new approaches proposed, stand out for producing estimates that are significantly closer to the real error, reducing the conservatism with respect to Traditional SB and without underestimating the error. In this population, only the Rohrbach method produces results that are lower than the true error, and this is not the case for all error rates. It happens more at lower error rates, i.e. the number of underestimations increases as the error rate in the population decreases. This may be due to the fact that Population 3 has higher book values than Population 1.

Overall, it can be seen that only the Rohrbach method produces errors that are lower than the actual error, because as Population 3 has higher book values, the margin of error for the methods decreases, resulting in more conservative estimates with values that are closer to each other.

Comparing the results of the two populations, we can affirm that approaches 1 and 2 significantly outperform the traditional Stringer Bound. Considering the criteria of lower conservatism (lower estimated error) and the absence of underestimation of the actual error, the new approaches have superior performance. However, it is important to note that approach 1 did not prove to be entirely robust, underestimating the actual error in a specific scenario: Population 1, error rate of 30%, and population size of 200. Approach 2, on the other hand, demonstrated consistency in all analyzed scenarios, meeting both criteria.

Analyzing the performance of the approaches at different simulated error rates, we observe a distinct behavior. For higher error rates, Approach 1 proved to be more accurate, providing lower error estimates. On the other hand, for lower error rates, Approach 2 proved to be more accurate. This behavior can be easily explained by the modifications made for each approach. Approach 1, by modifying the book value considered in the calculation, tends to impact all error rates equally, while Approach 2, by considering error-free accounts, is more sensitive to variations in the error rate, benefiting when there are more error-free accounts (i.e., lower error rate).

Comparing the new approaches with the remaining (Meikle, Lta, Rohrbach, and Moment), we observe that the latter consistently underestimate the actual error in Population 1, regardless of the error rate or sample size. In Population 3, only the Rohrbach method presents this same limitation. Although the average of the estimated errors by these methods is lower than that of the traditional Stringer Bound, this average is distorted by the presence of underestimations of the error, that is, the average is influenced by values that are systematically smaller than the actual error, which compromises the reliability of the estimate.

Table 4.5 - Results for Population 3 and sample size 100

Error Rate	Real error	Method	Total Mean	Proportion	Nº of values under the real error	Mean of errors under the real error (%)
30%	164 354	SB traditional	3 288 649	20	0	-
		SB Approach 1	491 409	3	0	-
		SB Approach 2	3 114 648	19	0	-
		Meikle	3 288 649	20	0	-
		Lta	3 288 649	20	0	-
		Rohrbach	2 162 982	13	0	-
		Moment	3 218 956	20	0	-
10%	53 202	SB traditional	1 542 295	29	0	-
		SB Approach 1	492 668	9	0	-
		SB Approach 2	1 344 406	25	0	-
		Meikle	1 542 295	29	0	-
		Lta	1 542 295	29	0	-
		Rohrbach	722 290	14	6	-64%
		Moment	1 448 863	27	0	-
5%	24 900	SB traditional	1 035 583	42	0	-
		SB Approach 1	493 197	20	0	-
		SB Approach 2	819 716	33	0	-
		Meikle	1 035 583	42	0	-
		Lta	1 035 583	42	0	-
		Rohrbach	350 374	14	31	-77%
		Moment	905 083	36	0	-
1%	5 136	SB traditional	613 801	119	0	-
		SB Approach 1	493 327	96	0	-
		SB Approach 2	356 361	69	0	-
		Meikle	613 801	119	0	-
		Lta	613 801	119	0	-
		Rohrbach	72 344	14	294	-93%
		Moment	521 953	102	0	-
0,5%	2 871	SB traditional	554 187	193	0	-
		SB Approach 1	493 410	172	0	-
		SB Approach 2	284 386	99	0	-
		Meikle	554 187	193	0	-
		Lta	554 187	193	0	-
		Rohrbach	35 988	13	405	-98%
		Moment	498 239	174	0	-

Table 4.6 - Results for Population 3 and sample size 200

Error Rate	Real error	Method	Total Mean	Proportion	Nº of values under the real error	Mean of errors under the real error
30%	164 354	SB traditional	2 901 847	18	0	-
		SB Approach 1	246 517	1	0	-
		SB Approach 2	2 815 759	17	0	-
		Meikle	2 901 847	18	0	-
		Lta	2 901 847	18	0	-
		Rohrbach	2 151 220	13	0	-
		Moment	2 887 640	18	0	-
10%	53 202	SB traditional	1 224 798	23	0	-
		SB Approach 1	247 765	5	0	-
		SB Approach 2	1 129 847	21	0	-
		Meikle	1 224 798	23	0	-
		Lta	1 224 798	23	0	-
		Rohrbach	706 605	13	0	-
		Moment	1 200 107	23	0	-
5%	24 900	SB traditional	756 651	30	0	-
		SB Approach 1	248 289	10	0	-
		SB Approach 2	654 784	26	0	-
		Meikle	756 651	30	0	-
		Lta	756 651	30	0	-
		Rohrbach	342 363	14	3	-81%
		Moment	708 918	28	0	-
1%	5 136	SB traditional	359 864	70	0	-
		SB Approach 1	248 424	48	0	-
		SB Approach 2	237 194	46	0	-
		Meikle	359 864	70	0	-
		Lta	359 864	70	0	-
		Rohrbach	67 142	13	194	-78%
		Moment	296 933	58	0	-
0,5%	2 871	SB traditional	310 701	108	0	-
		SB Approach 1	248 540	87	0	-
		SB Approach 2	179 628	63	0	-
		Meikle	310 701	108	0	-
		Lta	310 701	108	0	-
		Rohrbach	36 777	13	305	-94%
		Moment	266 335	93	0	-

5. Conclusion

This thesis presents a detailed analysis and a comprehensive comparison of error estimation methods in audit populations, focusing on the development and evaluation of two proposed new approaches aimed at improving the performance and accuracy of these estimates. The main motivation was to reduce the excessive conservatism of the Stringer Bound method while maintaining the essential criterion of not underestimating the real error, in accordance with auditing standards. As identified in the literature review, the Stringer Bound method has some weaknesses, including excessive conservatism, failure to consider understatements, and error-free accounts. These limitations served as the basis for developing the new approaches, both of which aim to reduce conservatism, with Approach 2 also considering error-free accounts.

The results of the analysis indicate that both Approach 1 and Approach 2 performed best, meeting the two criteria of reducing the estimated error value while never underestimating the real error. Approach 2, in particular, consistently met both criteria across all tested scenarios, making it a promising and robust alternative to the traditional Stringer Bound and other methods. Approach 1, while meeting the criteria in most situations, transgressed only in a specific scenario, suggesting that future adjustments may further enhance its robustness.

The comparison of all methods revealed that approaches such as Meikle, Lta, Rohrbach, and Moment, although they may reduce conservatism, frequently fail and estimate values below the real error, making them less reliable from an auditing perspective. This result underscores the importance of research and the development of new approaches, as carried out in this thesis.

It is essential to acknowledge that this study has certain limitations. One of the primary limitations is the use of simulated data, which, while covering a variety of amounts, error rates, and sample dimensions, does not fully reflect reality. In future investigations, it would be pertinent to expand the scope of the data to be analyzed, assessing the effectiveness of the methods in diverse contexts. However, the selection of the Higgins & Nandram (2009) populations, along with the choice of populations 1 and 3, which have distinctly different characteristics in terms of value magnitude and types of error, aimed precisely to mitigate this limitation, encompassing a broader range of scenarios.

Another significant limitation to highlight relates to the weaknesses of the Stringer Bound method identified in the literature review, specifically, that it does not consider understatements. Although our new approaches were developed with the intention of improving existing limitations, this particular aspect was not addressed. Future studies could consider incorporating an analysis of understatements.

In conclusion, this thesis, by developing new methods that are less conservative and more reliable, provides a solid foundation for the potential practical application of the new approaches in auditing, without undermining the need for further research that could deepen and expand the obtained results.

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7. Annexes

7.1 Annex A - R script

```
rm(list = ls())
library(writexl)
library(dplyr)
library(e1071)
library("jfa")

# Population 1
Book_Value <- c(runif(2039,0,13.5), runif(2455,13.51,22.5),
runif(1867,22.51,36),
                runif(852,36.01,63), runif(494,63.01,105),
runif(335,105.01,195),
                runif(136,195.01,345), runif(79,345.01,675),
runif(24,675.01,945),
                runif(16,945.01,1545), runif(12,1545.01,6945))
sum(Book_Value)
sd(Book_Value)
skewness(Book_Value)
kurtosis(Book_Value)
summary(Book_Value)

#Error Rate
Error_Rate <- 0.3
Length_Book_Value <- length(Book_Value)
error_indices <- sample(1:Length_Book_Value, Error_Rate *
Length_Book_Value,
                        replace = FALSE)
Audit_Value <- Book_Value
Audit_Value

Add_Error <- function(x){
  if(x <= 9.99){
    Audit_Value[indice_i] <- x + runif(1, -3.6, 0.84)
  } else if(x <= 24.99){
    x + runif(1, -33.60, 10.09)
  } else if(x <= 99.99){
    x + runif(1, -12.38, 20.88)
  } else if(x <= 399.99){
    x + runif(1, -31.26, 14.76)
  } else{
    x + runif(1, -42.18, 55.40)
  }

}

for(indice_i in error_indices){
  book_value_i <- Book_Value[indice_i]
  Audit_Error_i <- -1
  while(Audit_Error_i <= 0){
    Audit_Error_i <- Add_Error(book_value_i)
  }
  Audit_Value[indice_i] <- Audit_Error_i
}

#------------------------------------------------------------------
# Population 3
```

```
Book_Value <- c(runif(1334,0,40), runif(1438,40.01,136),
runif(1475,136.01,400),
                runif(878,400.01,800), runif(539,800.01,1400),
runif(548,1400.01,3000),
                runif(278,3000.01,5000), runif(239,5000.01,10000),
                runif(258,10000.01,49000),
runif(39,49000.01,100000))
sum(Book_Value)
mean(Book_Value)
sd(Book_Value)
skewness(Book_Value)
kurtosis(Book_Value)
summary(Book_Value)

#Error Rate
Error_Rate <- 0.3
Length_Book_Value <- length(Book_Value)
error_indices <- sample(1:Length_Book_Value, Error_Rate *
Length_Book_Value, replace = FALSE)
Audit_Value <- Book_Value
Audit_Value

Add_Error <- function(x){
  if(x <= 200){
    overstatement_perc <- sample(c(.01, .02, .05, .50, .75, 1), size
= 1, prob = c(.06, .13, .06, .13, .13, .50))
  } else if(x <= 1000){
    overstatement_perc <- sample(c(.05, .10, .15, .20, .50, .95, 1),
size = 1, prob = c(.17, .08, .17, .08, .25, .17, .08))
  } else{
    overstatement_perc <- sample(c(.01, .05, .1, .2, .3, .5)/100,
size = 1)
  }
  x - x * overstatement_perc
}

for(indice_i in error_indices){
  book_value_i <- Book_Value[indice_i]
  audit_value_i <- Add_Error(book_value_i)
  if(book_value_i - audit_value_i > 500) audit_value_i <-
book_value_i - 500
  Audit_Value[indice_i] <- audit_value_i
}


#-------------------------------------------------------------------
#Errors
Audit_Value_30 <- Audit_Value

#Error rate 10%
n_errors_10 <- round(0.1 * Length_Book_Value)
error_indices_10 <- sample(error_indices, n_errors_10)
Audit_Value_10 <- Book_Value
Audit_Value_10[error_indices_10] <- Audit_Value_30[error_indices_10]
table(Audit_Value_10 != Book_Value)/Length_Book_Value #Verificar

#Error rate 5%
n_errors_5 <- round(0.05 * Length_Book_Value)
error_indices_5 <- sample(error_indices, n_errors_5)
Audit_Value_5 <- Book_Value
Audit_Value_5[error_indices_5] <- Audit_Value_30[error_indices_5]
```

```
table(Audit_Value_5 != Book_Value)/Length_Book_Value

#Error rate 1%
n_errors_1 <- round(0.01 * Length_Book_Value)
error_indices_1 <- sample(error_indices, n_errors_1)
Audit_Value_1 <- Book_Value
Audit_Value_1[error_indices_1] <- Audit_Value_30[error_indices_1]
table(Audit_Value_1 != Book_Value)/Length_Book_Value

#Error rate 0.5%
n_errors_05 <- round(0.005 * Length_Book_Value)
error_indices_05 <- sample(error_indices, n_errors_05)
Audit_Value_05 <- Book_Value
Audit_Value_05[error_indices_05] <- Audit_Value_30[error_indices_05]
table(Audit_Value_05 != Book_Value)/Length_Book_Value


#Sampling
sample_size <- 100 #100,200
n_samples <- 600
population_size <- Length_Book_Value

create_sample <- function(population_size, sample_size){
  sample_indices <- sample.int(population_size, sample_size, replace
= FALSE)
  sample_indices
}

Stringer_Bound <- function(Book_Value, Audit_Value, sample_indices){
  Book_Value_sample <- Book_Value[sample_indices]
  Audit_Value_sample <- Audit_Value[sample_indices]

  # Taiting
  Errors <- Book_Value_sample - Audit_Value_sample
  Tainting <- Errors / Book_Value_sample

  # Select positive taintings
  Positive_Tainting <- Tainting[Tainting > 0]
  Positive_Tainting <- sort(Positive_Tainting, decreasing = TRUE)
  Positive_Tainting <- c(1, Positive_Tainting)

  coefs_ps <- sapply(0:(sample_size - 1 ), function(x) {
    qbeta_val <- qbeta(0.95, 1 + x, sample_size - x)
    if (is.nan(qbeta_val)) qbeta_val <- 0.0001
    return(qbeta_val)
  })
  delta_coefs_ps <- c(qbeta(0.95, 1, sample_size), diff(coefs_ps))

  Total_error <- sum(delta_coefs_ps[1:length(Positive_Tainting)] *
sum(Book_Value) * Positive_Tainting)

  Total_error
}

#-------------------------------------------------------------------
#Approach 1

Stringer_Bound1 <- function(Book_Value, Audit_Value,
sample_indices){
  Book_Value_sample <- Book_Value[sample_indices]
  Audit_Value_sample <- Audit_Value[sample_indices]
```

```
  # Taiting
  Errors <- Book_Value_sample - Audit_Value_sample
  Tainting <- Errors / Book_Value_sample

  # Select positive taintings
  indices_Tainting_g0 <- which(Tainting > 0)
  Positive_Tainting <- Tainting[indices_Tainting_g0]
  Order_Positive_Tainting <- order(Positive_Tainting, decreasing =
TRUE)
  indices_Tainting_g0 <-
indices_Tainting_g0[Order_Positive_Tainting]
  Positive_Tainting <- Positive_Tainting[Order_Positive_Tainting]
  Positive_Tainting <- c(1, Positive_Tainting)

  coefs_ps <- sapply(0:(sample_size - 1 ), function(x) {
    qbeta_val <- qbeta(0.95, 1 + x, sample_size - x)
    if (is.nan(qbeta_val)) qbeta_val <- 0.0001
    return(qbeta_val)
  })
  delta_coefs_ps <- c(qbeta(0.95, 1, sample_size), diff(coefs_ps))

  Book_Value_specific <- Book_Value_sample[indices_Tainting_g0]
#Book Value especifico de cada erro
  Book_Value_rest <- sum(Book_Value)-sum(Book_Value_specific) #Book
value da 1º linha
  Book_Value_1 <- c(Book_Value_rest,Book_Value_specific)

  Total_error <- sum(delta_coefs_ps[1:length(Positive_Tainting)] *
Book_Value_1 * Positive_Tainting)

  Total_error
  }

#------------------------------------------------------------------
#Approach 2

Stringer_Bound2 <- function(Book_Value, Audit_Value,
sample_indices){
  Book_Value_sample <- Book_Value[sample_indices]
  Audit_Value_sample <- Audit_Value[sample_indices]

  # Taiting
  Errors <- Book_Value_sample - Audit_Value_sample
  Tainting <- Errors / Book_Value_sample

  # Select positive taintings
  indices_Tainting_g0 <- which(Tainting > 0)
  Positive_Tainting <- Tainting[indices_Tainting_g0]
  Order_Positive_Tainting <- order(Positive_Tainting, decreasing =
TRUE)
  indices_Tainting_g0 <-
indices_Tainting_g0[Order_Positive_Tainting]
  Positive_Tainting <- Positive_Tainting[Order_Positive_Tainting]
  Positive_Tainting <- c(1, Positive_Tainting)

  coefs_ps <- sapply(0:(sample_size), function(x) {
    qbeta_val <- qbeta(0.95, 1 + x, sample_size - x)
    if (is.nan(qbeta_val)) qbeta_val <- 0.0001
    return(qbeta_val)
  })
```

```
  delta_coefs_ps <- c(qbeta(0.95, 1, sample_size), diff(coefs_ps))

  Book_Value_specific <- Book_Value_sample[indices_Tainting_g0]
#Book Value - specific error
  Book_Value_rest <- sum(Book_Value)-sum(Book_Value_specific) #Book
value da 1° linha

  Tainting0 <- Book_Value_rest/sum(Book_Value)

  Total_error <- sum(delta_coefs_ps[1:(length(Positive_Tainting) +
1)] * sum(Book_Value) * c(Positive_Tainting, -Tainting0))

  Total_error
}

#-----------------------------------------------------------------
Data_Population <- data.frame(Book_Value, Audit_Value_30,
Audit_Value_10,
                              Audit_Value_5, Audit_Value_1,
Audit_Value_05)

sample_indices_i <- create_sample(population_size, sample_size)
Data_Population_sample <- Data_Population[sample_indices_i,]

Stringer_Bound_JFA <- evaluation(
  method = "stringer", data = Data_Population_sample,
  values = "Book_Value", values.audit = "Audit_Value_30"
)

Stringer_Bound_meik <- evaluation(
  method = "stringer.meikle", data = Data_Population_sample,
  values = "Book_Value", values.audit = "Audit_Value_30"
)

Stringer_Bound_lta <- evaluation(
  method = "stringer.lta", data = Data_Population_sample,
  values = "Book_Value", values.audit = "Audit_Value_30"
)

Stringer_Bound_rohrbach <- evaluation(
  method = "rohrbach", data = Data_Population_sample,
  values = "Book_Value", values.audit = "Audit_Value_30", N.units =
sample_size,
)

Stringer_Bound_moment <- evaluation(
  method = "moment", data = Data_Population_sample,
  values = "Book_Value", values.audit = "Audit_Value_30"
)

summary(Stringer_Bound_JFA)
Stringer_Bound_JFA$ub*sum(Book_Value)
Stringer_Bound_meik$ub*sum(Book_Value)
Stringer_Bound_lta$ub*sum(Book_Value)
Stringer_Bound_rohrbach$ub*sum(Book_Value)
Stringer_Bound_moment$ub*sum(Book_Value)

Stringer_Bound(Book_Value, Audit_Value_30, sample_indices_i)
Stringer_Bound1(Book_Value, Audit_Value_30, sample_indices_i)
Stringer_Bound2(Book_Value, Audit_Value_30, sample_indices_i)
```

```
#-----------------------------------------------------------------
#Output

Audit_Value_now <- Audit_Value_30
Errors <- Book_Value-Audit_Value_now
Total_positive_errors <- sum(Errors[Errors>0])
Total_positive_errors

Errors_list <- c()
Errors_list1 <- c()
Errors_list2 <- c()
Errors_list3 <- c()
Errors_list4 <- c()
Errors_list5 <- c()
Errors_list6 <- c()
Errors_list7 <- c()


for (i in 1:n_samples){
  sample_indices_i <- create_sample(population_size, sample_size)
  Error_i <- Stringer_Bound(Book_Value, Audit_Value_now,
sample_indices_i)
  Error_i1 <- Stringer_Bound1(Book_Value, Audit_Value_now,
sample_indices_i)
  Error_i2 <- Stringer_Bound2(Book_Value, Audit_Value_now,
sample_indices_i)

  Data_Population_i <- data.frame(Book_Value,
Audit_Value_now)[sample_indices_i, ]
  Stringer_Bound_JFA <- evaluation(method = "stringer", data =
Data_Population_i,
    values = "Book_Value", values.audit = "Audit_Value_now"
  )
  Error_i3 <- Stringer_Bound_JFA$ub*sum(Book_Value)

  Stringer_Bound_meik <- evaluation(method = "stringer.meikle", data
= Data_Population_i,
    values = "Book_Value", values.audit = "Audit_Value_now"
  )
  Error_i4 <- Stringer_Bound_meik$ub*sum(Book_Value)

  Stringer_Bound_lta <- evaluation(method = "stringer.lta", data =
Data_Population_i,
    values = "Book_Value", values.audit = "Audit_Value_now"
  )
  Error_i5 <- Stringer_Bound_lta$ub*sum(Book_Value)

  Stringer_Bound_rohrbach <- evaluation(method = "rohrbach", data =
Data_Population_i,
    values = "Book_Value", values.audit = "Audit_Value_now", N.units
= sample_size,
  )
  Error_i6 <- Stringer_Bound_rohrbach$ub*sum(Book_Value)

  Stringer_Bound_moment <- evaluation(method = "moment", data =
Data_Population_i,
    values = "Book_Value", values.audit = "Audit_Value_now"
  )
  Error_i7 <- Stringer_Bound_moment$ub*sum(Book_Value)

  Errors_list <- c(Errors_list, Error_i)
```

```
  Errors_list1 <- c(Errors_list1, Error_i1)
  Errors_list2 <- c(Errors_list2, Error_i2)
  Errors_list3 <- c(Errors_list3, Error_i3)
  Errors_list4 <- c(Errors_list4, Error_i4)
  Errors_list5 <- c(Errors_list5, Error_i5)
  Errors_list6 <- c(Errors_list6, Error_i6)
  Errors_list7 <- c(Errors_list7, Error_i7)

}

today <- Sys.time() %>% stringr::str_replace_all(c("-" = "", ":" =
"", " " = "_")) %>% substr(1,15)

write_xlsx(data.frame("JFA"= Errors_list3,
                      "Meikle" = Errors_list4,
                      "Lta" = Errors_list5,
                      "Rohrbach" = Errors_list6,
                      "Moment" = Errors_list7,
                      "Stringer Bound Traditional" = Errors_list,
                      "Stringer Bound Approach 1" = Errors_list1,
                      "Stringer Bound Approach 2" = Errors_list2,
                      "Total Positive Error" =
Total_positive_errors),
                      paste0("Output_", today, ".xlsx"),

           )
```