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Analyzing and managing portfolio risk and performance using Value-at-Risk

Beatriz de Jesus Mendes Martins

Master in Finance

Supervisor:
PhD, António Manuel Rodrigues Guerra Barbosa, Assistant Professor, ISCTE-IUL

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Resumo

A globalização aumentou a necessidade de garantir que as instituições financeiras aplicam uma gestão de risco eficiente. Por isso, as regulamentações reforçaram a monitorização da gestão de risco, através de requisitos mínimos de capital e implementação do método Value-at-Risk (VaR) como uma métrica padrão de risco. O principal objetivo deste trabalho é utilizar o VaR para analisar o desempenho e gerir o risco de um portfólio composto por ações e obrigações de 4 mercados diferentes. Primeiro, precisamos de testar os modelos de VaR, utilizando o processo de Backtesting para identificar o modelo que se adequa mais à composição e caraterísticas do portfólio. Em seguida, usamos o modelo mais preciso para medir e comparar as duas estratégias: o VaR diário do portfolio sem estratégia de gestão de risco e o VaR diário com uma estratégia de hedging, para o período de um ano. Por fim, comparamos o desempenho de ambas as estratégias utilizando uma medida de desempenho, o Return on Risk-Adjusted Capital (RORAC). Os resultados indicam que o portfolio com a estratégia de gestão de risco para limitar o VaR máximo diário apresenta um desempenho superior ao portfolio sem gestão de risco.

Palavras-Chave: Gestão de Risco, Value-at-Risk, Portfólio, Backtesting, Hedging

Classificação JEL: G11, G28

Abstract

Globalization has enhanced the need to guarantee that financial institutions apply efficient risk management. Therefore, regulations strengthened the monitorization of risk management, by enforcing the minimum capital requirements and implementing Value-at-Risk (VaR) approach as a standard risk measurement metric. The main goal in this work is to use the VaR to analyze the performance and manage the risk of a portfolio composed of stocks and bonds from 4 different markets. First, we need to test the VaR models, by using the Backtesting approach to identify the model that provides the best fit, considering the portfolio composition and characteristics. Then, we use the more accurate model to measure and compare two approaches: the daily VaR of the portfolio without a risk management strategy and the daily VaR managed through a hedging strategy, for a period of one-year. Finally, we compare the performance of both strategies using a performance measure, the Return on Risk-Adjusted Capital (RORAC). The results indicate that the portfolio with a risk management strategy to limit the daily maximum VaR outperforms the portfolio without risk management.

Keywords: Risk Management, Value-at-Risk, Portfolio, Backtesting, Hedging

JEL Classification: C10, G11, G28

Contents

List of Abbreviations

AEX	Amsterdam Exchange
BCP	Berkowitz, Christoffersen and Pelletier
CAC40	Cotation Assistée en Continu
DAX	Deutscher Aktienindex
EC	Economic Capital
EUR	Euro
EWMA	Exponential Weighted Moving Average
FTSE100	Financial Times Stock Exchange 100
GBP	Great British Pound
PV	Present Value
PV01	Present Value of a Basis Point
QR	Quantile Regression
RORAC	Return on Risk-Adjusted Capital
S&P500	Standard & Poor's 500
SGSt	Skewed Generalised Student-t
UC	Unconditional Coverage
U.S.	United States
USD	United States Dollar
VaR	Value at Risk

Chapter 1.
Introduction

Financial institutions need to be very cautious about several kinds of risk, this work will focus only on the market risk that arises from the fluctuations in the market price of financial assets. It is essential that the banks implement efficient risk management procedures, so to guarantee this the Basel Accords set a unified regulatory framework with a primarily focus on minimum capital requirement. In 1994, J.P. Morgan (1996) published the RiskMetrics model, enhancing the adoption of the Value-at-Risk (VaR) to measure market risk.

The VaR is a statistical measure that estimates with a certain probability the potential maximum loss of a portfolio, during a given time period. This way financial institutions can measure their risk with the VaR, and then manage it by defining a pre-defined limit for the Economic Capital (EC), that represents the capital at risk originated from the investment activities (Jorion, 2007).

In this work we will study a portfolio composed by bonds from the European and the United States (U.S.) markets and stocks associated with various indices such as, the DAX from German, the CAC 40 from France, the AEX from Netherlands, the S&P500 from the U.S. and FTSE 100 from the United Kingdom.

The total risk of a portfolio can be decomposed into systematic risk, which is captured by mapping the portfolio risk, and the specific risk that can be diversified. We will compute the Total VaR and the Systematic VaR at the risk factor level, so it is possible to decompose it, which will help manage the risks.

Several researchers such as Lee & Su (2011), Pritsker (2006), Hull & White (1998), Xiao et al. (2015) and Steen et al. (2015) have studied the performance of various VaR models, to test which model is the most accurate. However, the choice of a VaR model depends on the portfolio and its characteristics, so it is difficult to encounter a model that outperforms all the others for every portfolio. That is why we need to test the performance of different VaR models with our portfolio.

The models that will be estimated can be categorized into parametric and non-parametric models. In the parametric models we will compute the Normal VaR, and the Skewed

Generalized Student-t (SGSt) VaR from Theodossiou (1998). In the non-parametric models, we will compute the Historical adjusted volatility VaR from Hull & White (1998) and the Quantile Regression from Koenker & Bassett (1978). Furthermore, we will use the EWMA to compute the volatility of returns that represent the current market conditions.

Aftewards, we will Backtest the models to assess the performance of each one and choose the one that has better fit to our portfolio. To do this we will use two statistical tests: the Unconditional Coverage (UC) test from Kupiec (1995) and the Berkowitz, Christoffersen and Pelletier (BCP) test proposed by Berkowitz et al. (2011).

Finally, we will use the best model to go forward one year in time using two approaches. In the first one we compute the daily VaR for that year without any risk management strategy and in the second approach we compute the daily VaR for the year while hedging the main sources of risk, in the days that the portfolio surpasses a pre-defined maximum daily EC. We define this maximum target at 1.6% of the portfolio value on 27 January 2023, which translates to almost 160 000€, and it is in concordance with the historical range of risk. The hedging strategy is defined every day according to the VaR decompositions.

To compare the approaches, we use a risk sensitive performance measure, the Return on Risk-Adjusted Capital (RORAC), which shows that the portfolio achieves a better performance with a risk management strategy.

The rest of work is organized as follows: Chapter 2 presents the relevant literature to this dissertation, Chapter 3 describes the data and the details about the characteristics and composition of the portfolio, Chapter 4 dives into the necessary methodology, Chapter 5 presents the VaR models, Chapter 5 reveals the results of Backtesting, Chapter 6 uncovers the hedging strategy and the impact on the P&L of the portfolio and finally Chapter 7 concludes and summarizes the results of this work.

Chapter 2.

Literature Review

Financial institutions' distress not only impacts the financial markets, but also has a major impact in the real economy (Dell'Ariccia et al., 2008; Hoggarth et al., 2002), which can be proven historically. Efficient risk management can help prevent some of these main consequences (Bessis, 2010). Therefore, it is important that the banks have some degree of regulation (Dow, 1996), to create stability and confidence in the banking system (Hull, 2015). With the globalization, the financial instability started to affect market failures in an international level, so in 1988, the Basel Accords were introduced by the Basel Committee on Banking Supervision as a series of recommendations to reinforce and strengthen the minimum capital requirements by monitoring and reporting the risk (Shakdwipee & Mehta, 2017).

The risk can be categorized into three main categories: the operational risk, the credit risk and the market risk. In this work we will deal with market risk, which measures the uncertainty in the portfolio's profit and loss (Alexander, 2008b). The VaR is a statistical risk measurement metric that is widely used in the financial market (Khindanova & Rachev, 2019), and even with its own limitations, it is recommended by the regulators (Krause, 2003). VaR can be defined as the maximum loss that can be expected that will not be exceeded over a given time horizon for a given level of confidence (Jorion, 2007). This work will follow the Basel guidelines, so we will consider a confidence level of 99% and a daily time horizon.

In the past regulations for financial institutions were more focused on regulatory capital, however since Basel II the regulations are based on risk sensitive measures (Jorion, 2007; Porteous & Tapadar, 2006). The EC can be defined as the amount of capital necessary to protect against a set of risks and absorb potential losses, during a time horizon and up to a certain confidence level (Porteous & Tapadar, 2006). Considering only market risks, the EC is commonly estimated based on the VaR measure (Jorion, 2007). Therefore, the VaR measures the risk of a portfolio, and then we can use a pre-defined maximum value for the EC to manage that risk.

The VaR started gaining faster adoption when J.P. Morgan (1996) released their VaR model RiskMetrics. Afterwards, the VaR was adopted by the regulators and the financial institutions as a standard measure of market risk, that can be calculated based on standardized or internal models (Hull, 2015; Khindanova & Rachev, 2019). The application of different VaR methods provides distinctive VaR estimates (Khindanova & Rachev, 2019), the model that best fits a specific portfolio depends on its characteristics and composition. We will see two main categories of models: the parametric models and the non-parametric models. The parametric methods assume that returns follow a specific distribution, while the non-parametric models do not impose any distribution assumption, it uses the empirical distribution of the returns (Khindanova & Rachev, 2019).

VaR is a forward-looking risk measure which means that the focus must be on the current market conditions, so to compute the VaR models we need to estimate the variances and covariances in a way that the recent observations in the sample have more weight than the older observations (Alexander, 2008b; Hull, 2015). To tackle this issue, diverse researchers proposed volatility models, such as the Autoregressive Conditional Heteroskedasticity (ARCH) proposed by Engle (1982), the Generalized ARCH (GARCH) proposed by Bollerslev (1986) and the Exponentially Weighted Moving Average (EWMA) that is very popular due to its simplicity and its use in the RiskMetrics model (J.P. Morgan, 1996). In this work, we will use the EWMA volatility model.

The RiskMetrics model is categorized as a Normal VaR model, which is the most commonly used parametric model. This parametric model assumes that the returns follow a normal distribution (Krause, 2003), however this contradicts the conclusions of Fama (1965) that normally the financial returns show negative skewness and excessive kurtosis. Therefore, if the returns of our portfolio do not follow a normal distribution, then the Normal VaR will not capture real risk, whereas for higher confidence levels the Normal VaR is likely to underestimate the true VaR (Alexander, 2008b; Stefaniak, 2018).

Taking this into account and still in the parametric methods, Theodossiou (1998) developed a flexible distribution that accommodates both skewness and excess kurtosis, the Skewed Generalizes Student-t (SGSt), which is a skewed extension of the standard Student-t distribution created by Mcdonald & Newey (1988). Lee & Su (2011) compared the VaR estimates of the normal, standard Student-t and SGSt models, using daily data of thirteen stock indices, in North America, Europe and Asia. The authors concluded that the SGSt VaR model was the better fit of the three, followed by standard Student-t VaR and the worst was the Normal VaR model. Despite that, all models leaned to underestimate the real market risk, for all confidence levels, which can indicate that maybe the parametric models were not the best fit for that specific data.

The most used non-parametric method is the Historical VaR model that constructs the distribution of the portfolio from historical data, assuming that the trends of past returns will continue in the future (Khindanova & Rachev, 2019). This model relies completely on historical returns, so the choice of the sample size is crucial, however the larger the sample size, the less the model reflects the current market conditions (Alexander, 2008b; Pritsker, 2006). To tackle this issue, some authors proposed some refinements to the standard Historical model. Barone-Adesi et al. (1998) and Boudoukh et al. (1998) suggested assigning more weight to recent observations, whereas Hull & White (1998) suggested adjusting the volatility of the returns to reflect the current market volatility. Hull & White (1998) studied the three refinements of the historical model, where they compared nine years of daily data on twelve exchange rates and five different stock indices and based on their findings their refinement of the model has better results than the Boudoukh et al. (1998) methodology, for the significance level of 1%.

Another non-parametric method is the Quantile Regression (QR) that was proposed by Koenker & Bassett (1978), which might be a good fit to estimate the VaR, since it summarized by current regulations as a conditional quantile (Xiao et al., 2015). The biggest advantage of the QR VaR methodology consists of its flexibility of choosing the explanatory variables, which expands the modeling options (Xiao et al., 2015). Xiao et al. (2015) studied the five bigger world market indexes, and he concluded that the QR model is more robust than the Normal model, to a series of confidence levels. Steen et al. (2015) studied nineteen different commodity futures, where it concluded that the Quantile Regression model outperforms the Normal and the Historical models in predicting the VaR for most of these commodities.

Based on the findings of Alexander (2008b), it is possible that the same VaR model will create different results for two different portfolios, even if they have identical underlying risk factors. Consequently, we need to test the model's performance to evaluate which VaR model is the best fit to the specific portfolio, which will be done by the Backtest method. The Backtest is a statistical framework that uses historical data to verify the accuracy and effectiveness of a model (Zhang & Nadarajah, 2018), by analyzing if the actual losses are in line with the projected losses, when a model is a good fit, the number of observations falling outside VaR should be in line with the confidence level (Jorion, 2007). In this work we will adopt two Backtest methods: the Unconditional (UC) test proposed by Kupiec (1995) and the Berkowitz, Christoffersen and Pelletier (BCP) test proposed by Berkowitz et al. (2011). The UC test analyzes, for a certain period of time, the number of exceedances, which happens when the actual portfolio loss is higher than the VaR estimated in the previous day (Alexander, 2008b;

Kupiec, 1995). The BCP test analyzes if the exceedances are autocorrelated or independent from each other (Berkowitz et al., 2011).

VaR is not only a useful tool for reporting purposes, but also as a risk control tool, where we can define a pre-defined maximum value EC, and then define a risk management strategy (Jorion, 2007). By doing this the adjusted portfolio will have a different risk profile than the original portfolio. The P&L of the adjusted portfolio will be achieved with a lower VaR than the original portfolio, so to make a fair comparison between the two approaches we need to use a risk-adjusted performance metric. To do this, we will use the Risk-adjusted return on capital (RAROC) proposed by Matten (1996) that is a ratio between the P&L and the risk incurred.

Chapter 3.

Portfolio Composition

In this work we will study a portfolio composed of positions in stocks and bonds. The bonds in our portfolio are from the European and the United States (U.S.) markets, whereas the positions in stocks are from European, the U.S. and the United Kingdom markets. Giving this, we define our local currency as Euro (EUR) so we need to convert the assets defined in foreign currency, U.S. dollar (USD) and the British pound (GBP), to our local currency.

This portfolio is composed of 27 positions in stocks from 5 different indices, the S&P500 from U.S., the DAX from German, the CAC 40 from France, the FTSE 100 from United Kingdom and the AEX from Netherlands. The daily adjusted closing prices and the exchange rates were downloaded from the Yahoo Finance website (https://finance.yahoo.com/).

The bond portion consists of 6 fixed coupon government bonds, with different specifications, which can be seen in Table 3.1 and they were obtained from the Borse Frankfurt website (https://www.boerse-frankfurt.de/bonds). The bonds from the European market were issued by Denmark and Netherlands, so we retrieved the daily interest rate of AAA-rate sovereign bonds issued in EUR from the European Central Bank website (https://www.federalreserve.gov/datadownload/Choose.aspx?rel=H15). The daily interest rate data for bonds issued in USD were retrieved from the Federal Reserve Economic Data website (https://sdw.ecb.europa.eu/browseSelection. do?node=9689726).

Bond	Currency	Maturity	Maturity (Years)	Coupon Rate	Coupons per year	Face Value	Face Value (EUR)	Fair Value (EUR)
NL0009446418	EUR	15/01/2042	18.97	3.750%	1	1,300,000.00	1,300,000.00	1,583,529.21
US91282CJJ18	USD	15/11/2033	10.80	4.500%	2	1,150,000.00	1,055,585.00	1,156,041.69
US91282CJU62	USD	02/02/2026	3.02	5.553%	4	1,400,000.00	1,285,060.00	1,363,046.84
US912810QL52	USD	15/11/2040	17.80	4.250%	2	800,000.00	734,320.00	797,081.56
DE000BU2Z023	EUR	15/02/2034	11.05	2.200%	1	1,000,000.00	1,000,000.00	1,008,915.10
DE0001135069	EUR	04/01/2028	4.94	5.625%	1	1,200,000.00	1,200,000.00	1,388,201.44

Table 3.1. Bond Characteristics. The exchange rate on 27 January 2023 and used to convert to EUR the U.S. bonds is 0.9179.

Table 3.2 presents the broad components and specifications of the portfolio on 27 September 2023.

Stock	Ticker	Currency	Market	Quantity	Share Price	Value (EUR)	Allocation (%)
Pfizer, Inc.	PFE	USD	GSPC	15,000.00	41.03	564,900.53	5.68%
Nasdaq, Inc.	NDAQ	USD	GSPC	4,500.00	59.29	244,920.21	2.46%
FedEx Corporation	FDX	USD	GSPC	3,500.00	185.69	596,572.58	6.00%
Electronic Arts, Inc.	EA	USD	GSPC	2,000.00	127.91	234,809.71	2.36%
eBay, Inc.	EBAY	USD	GSPC	-15,000.00	47.62	-655,689.57	-6.59%
CVS Health Corp	CVS	USD	GSPC	1,500.00	84.04	115,706.92	1.16%
Bank of New York Mellon Corporation	BK	USD	GSPC	2,400.00	47.94	105,615.47	1.06%
Apple, Inc.	AAPL	USD	GSPC	3,600.00	144.74	478,276.70	4.81%
American Express Company	AXP	USD	GSPC	-2,800.00	169.31	-435,157.28	-4.38%
Amazon.com, Inc.	AMZN	USD	GSPC	10,500.00	102.24	985,383.99	9.91%
Accenture plc	ACN	USD	GSPC	1,600.00	272.00	399,469.81	4.02%
Heineken N.V.	HEIA.AS	EUR	AEX	-7,000.00	88.01	-616,044.06	-6.19%
Koninklijke Philips N.V.	PHIA.AS	EUR	AEX	-70,000.00	14.40	-1,008,317.52	-10.14%
Aegon Ltd.	AGN.AS	EUR	AEX	-120,000.00	4.81	-577,045.56	-5.80%
ASML Holding N.V.	ASML.AS	EUR	AEX	1,300.00	611.59	795,072.11	8.00%
Shell plc	SHELL.AS	EUR	AEX	-25,000.00	25.39	-634,862.10	-6.38%
Siemens Aktiengesellschaft	SIE.DE	EUR	GDAXI	3,400.00	136.09	462,698.78	4.65%
SAP SE	SAP.DE	EUR	GDAXI	7,200.00	101.95	734,041.57	7.38%
Deutsche Bank Aktiengesellschaft	DBK.DE	EUR	GDAXI	-70,000.00	11.42	-799,371.44	-8.04%
Continental Aktiengesellschaft	CON.DE	EUR	GDAXI	-1,600.00	60.85	-97,367.58	-0.98%
Commerzbank AG	CBK.DE	EUR	GDAXI	100,000.00	9.50	949,986.80	9.55%
Bayer Aktiengesellschaft	BAYN.DE	EUR	GDAXI	-1,800.00	53.57	-96,432.22	-0.97%
Allianz SE	ALV.DE	EUR	GDAXI	2,000.00	199.52	399,046.36	4.01%
adidas AG	ADS.DE	EUR	GDAXI	3,500.00	144.23	504,811.34	5.08%
Crédit Agricole S.A.	ACA.PA	EUR	FCHI	35,000.00	9.99	349,716.05	3.52%
Airbus SE	AIR.PA	EUR	FCHI	-2,100.00	114.28	-239,988.57	-2.41%
Vodafone Group Public Limited Company	VOD.L	GBP	FTSE	-1,200.00	82.77	-113,186.03	-1.14%
Total Equity						**2,647,566.99**	**26.62%**
Bonds	**ISIN**			**Face Value**	**Fair Price**	**Fair Value (EUR)**	
Dutch Bond 2042	NL0009446418	EUR		1,300,000.00	121.81%	1,583,529.21	15.92%
US Treasury 2033	US91282CJJ18	USD		1,150,000.00	109.52%	1,156,041.69	11.63%
US Treasury 2026	US91282CJU62	USD		1,400,000.00	106.07%	1,363,046.84	13.71%
US Treasury 2040	US912810QL52	USD		800,000.00	108.55%	797,081.56	8.02%
German Bond 2034	DE000BU2Z023	EUR		1,000,000.00	100.89%	1,008,915.10	10.15%
German Bond 2028	DE0001135069	EUR		1,200,000.00	115.68%	1,388,201.44	13.96%
Total Bonds						**7,296,815.85**	**73.38%**
Total Equity + Bonds						9,944,382.84	100.00%

Table 3.2. Portfolio composition on 27 September 2023. This table presents the assets and bonds that compose the portfolio as well as the amount invested in each one in EUR and the respective proportion of each asset on the portfolio value.

Chapter 4.

Methodology

The purpose of this work is to estimate the VaR of a portfolio over a period of one-year, starting on 30/01/2023 until 02/02/2024, and then manage the VaR, by implementing a hedging strategy, in order to not exceed the pre-defined value of Economic Capital.

To achieve this goal, we need to first do the risk factor mapping for the Total VaR and the Systematic VaR and select the volatility model that will be applied in the computation of the VaR models. Then we need to select a VaR model and the settings that are more suitable for this portfolio. To do this we need to test different possibilities and evaluate their precision, which is achieved with the Backtesting process.

In this Chapter we will go through the steps of risk factor mapping and the choice and methodology of the volatility model that will be used in the VaR models. In the next chapter, Chapter 5, we will cover the different VaR Models and the settings that we will test. In Chapter 6 we will proceed with the Backtesting of the models.

4.1. Risk Factor

We need to start by identifying the risk factors that contribute to the portfolio risk, to then be possible to manage the portfolio exposure to each risk factor.

The total risk of a portfolio can be decomposed into two types of risk, the systematic risk, which is the risk that is inherent of the market, so it is not reduced by diversifying the portfolio, and the residual risk that can be mitigated through diversification. In this work we will compute the Total VaR and the Systematic VaR at risk factor level, which can help us understand if the portfolio is well diversified, because if it is, then the total risk must be very close to the systematic risk.

The Systematic VaR is computed based on a mapped portfolio, where each portfolio position is mapped into a smaller set of common risk factors. This mapping process preserves, as much as possible, the risk profile of the actual portfolio. Nonetheless, the Systematic VaR is only an approximation of the real VaR that is measured by the Total VaR, which is based on the actual portfolio and its risk factors. It is important to estimate and backtest both of them, because when the portfolio has a large number of risk factors it may be infeasible to compute the Total VaR, whereas the Systematic VaR is, by design, much easier to estimate even for large portfolios. Therefore, with a Systematic VaR model we lose some accuracy in exchange for a less complex and more scalable model.

We will use the process of risk factor mapping, where we will need to quantify the exposure of each risk factor and map each portfolio position to an equivalent exposure, in terms of risk, to those risk factors. To do that we will set a vector that lists all risk factors and the corresponding exposures of the portfolio, which is called vector of risk factor loadings. Let Θ represent the portfolio and $\theta_i, i = 1, \dots, n$ represent one of the n risk factors affecting the portfolio, then the vector of risk factor loadings is given by:

$$\Theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} \tag{1}$$

To estimate the VaR in our local currency, EUR, all exposures will have to be quantified in EUR, so we need to convert each exposure mapped from foreign currency to EUR using the corresponding exchange rate.

Each risk factor has its own exposure mapping methodology and by the characteristics of our portfolio we have three types of risk factors: stocks, interest rates (bonds) and currency. Furthermore, the risk factor exposures for the Total VaR and for the Systematic VaR only differ in the methodology of stocks.

4.1.1. Stocks

For equity we estimate the future volatility of the stock price returns based on past observations of the stock price return. The risk factors of stocks for the Total VaR and the Systematic VaR are different.

For the Total VaR, the risk factor is the change in the market price of the stock, and the exposure to the risk factor is the amount invested in the stock in the currency used to compute the VaR (in our case the EUR). Therefore, the risk factor exposure of stock i at time t ($\theta_{i,t}$) is given by:

$$\theta_{i,t} = M_{i,t} = N_{i,t} \times P_{i,t} \times FX_{i,t} \tag{2}$$

Where $M_{i,t}$ denotes the amount in EUR invested on the stock, $N_{i,t}$ denotes the number of shares, $P_{i,t}$ the stock price per share and $FX_{i,t}$ the spot exchange rate for stock's denomination currency against the EUR.

In the case of the Systematic VaR, the risk factor is replaced by a stock market index and the risk factor exposure is given by:

$$\begin{aligned}\theta_{i,t} &= M_{i,t} \times \beta_{Stock,Index,t} \\ &= M_{i,t} \times \frac{\sigma_{Stock,Index,t}}{\sigma^2_{Index,t}}\end{aligned} \tag{3}$$

Keeping the capital invested in each stock constant, its P&L is given by the change in the market price of its risk factor, which is the respective stock or the index, if we are computing the Total VaR or the Systematic VaR, respectively.

$$P\&L_{i,t} = M_{i,t} \times \left(\frac{P_{i,t}}{P_{i,t-1}} - 1\right) \tag{4}$$

4.1.2. Bonds

In the case of bonds, it does not make sense to use the same methodology of stocks because the volatility of the past returns of the bonds is a bad estimator of the future volatility of those returns. As the bonds have a defined maturity it is expected that the volatility decreases over time as maturity approaches. Furthermore, unlike stocks, which main source of risk is the change in expectations for future cash flows, in the case of bonds these cash flows are fixed and known. Therefore, as only the variations in interest rates can explain the bond's price changes, that is the main source of risk of bonds.

To relate the variations in interest rates with the impact in the bond's price we will use the Present Value of a Basis Point (PV01) to quantify the sensitivity of the fair value of each bond cash flow to changes in interest rates. PV01 measures the change in the sum of the present value of a collection of cash flows, when the yield curve shifts down by one basis point, which means that all spot rates decrease by 0.01%.

To calculate the PV01 of a cash flow (C_T) first we need to compute its present value (PV) converted to EUR, by the exchange rate (FX_i), with continuously compounded interest rate from now until time T (r_T), which is given by:

$$PV_{C_T,r_T} = C_T \times e^{-r_T \times T} \times FX_t \tag{5}$$

The PV01 can then be calculated by a first-order Taylor approximation as follows:

$$\begin{aligned} PV01_{C_T,r_T} &\approx \frac{\partial PV_{C_T,r_T}}{\partial r_T} \times (-0.01\%) \\ &= T \times PV_{C_T,r_T} \times 0.01\% \end{aligned} \tag{6}$$

A portfolio with bonds with many cash flows can become a problem, since it will have as many interest rates for risk factors as different cash flow maturities, and it is possible that not all necessary data of interest rates is available. To surpass this problem Alexander (2008a) proposes that we adopt an approach where we map the cash flows with non-standard maturity to a group of standard maturity interest rates that we have data available, which are called the vertices of the cash flow mapping.

To map the cash flows at nonstandard maturities into standard maturities we will preserve two conditions of the originals cash flows, the present value and the PV01, which is denominated as the PV+PV01 invariant mapping, which we do by using the following condition:

$$\begin{cases} x_{T_2} = 1 - x_{T_1} \\ x_{T_1} = \dfrac{T_2 - T}{T_2 - T_1} \end{cases} \tag{7}$$

Note that x_{T_1} and x_{T_2} represent the proportions of the present value of the original cash flow mapped into the vertex maturities T_1 and T_2, respectively, which are the standard maturities with interest rate data available that are directly below and above T.

We need to repeat the PV+PV01 invariant map for every cash flow of our portfolio, generalizing to n vertices we can compute the PV01 of each vertex as:

$$PV01_{T_i} \approx T_i x_{T_i} \times 0.01\% \tag{8}$$

Given that P&L $\equiv \Delta PV$ and that $\frac{\Delta r_{t_i}}{0.01\%}$ is the absolute change in interest rate in basis point, then we can reformulate Equation 6 to compute the P&L of a bond with n cash flows as:

$$P\&L_{Bond_t} = \sum_{i=1}^{n} -PV01_{t_i} \times \frac{\Delta r_{t_i}}{0.01\%} \tag{9}$$

Therefore, the P&L of a bond t depends on the sensitivity of that bond to an increase of one basis point in the interest rate, $-PV01_t$, and the absolute change in interest rate, Δr_t. Given this, if the change in the interest rate is positive the P&L will be negative, and if the change in the interest rate is negative the P&L will be positive.

4.1.3. Currency

In our portfolio, we have positions in assets from foreign markets, so these positions are exposed not only to the risk factors of the specific asset, but also are exposed indirectly to the exchange rate between the respective foreign and our local currency, EUR.

The methodology of the exposure of the exchange rate risk factor mapping consists of the sum of the capital invested in each stock and bond that is denominated in a specific foreign currency, in our case is either USD or GBP, converted to the local currency EUR. With $FX_{i,t}$ as the exchange rate between foreign currency i and local currency at time t, the P&L generated by the exposure to a foreign currency of size $M_{i,t}$ is given by:

$$P\&L_{i,t} = M_{i,t} \times \left(\frac{FX_{i,t}}{FX_{i,t-1}} - 1 \right) \tag{10}$$

4.1.4. Portfolio

The vector of risk factor loadings is the aggregation of the risk factor loadings identified for each risk factor that is associated with each portfolio's position. Table 4.1 presents the risk factor exposures of our portfolio, for Total VaR and Systematic VaR, on 27 September 2023.

Type of Risk Factor	Risk Factor	Total VaR	Systematic VaR
Stock	PFE	564,900.53	0.00
	NDAQ	244,920.21	0.00
	FDX	596,572.58	0.00
	EA	234,809.71	0.00
	EBAY	-655,689.57	0.00
	CVS	115,706.92	0.00
	BK	105,615.47	0.00
	AAPL	478,276.70	0.00
	AXP	-435,157.28	0.00
	AMZN	985,383.99	0.00
	ACN	399,469.81	0.00
	HEIA.AS	-616,044.06	0.00
	PHIA.AS	-1,008,317.52	0.00
	AGN.AS	-577,045.56	0.00
	ASML.AS	795,072.11	0.00
	SHELL.AS	-634,862.10	0.00
	SIE.DE	462,698.78	0.00
	SAP.DE	734,041.57	0.00
	DBK.DE	-799,371.44	0.00
	CON.DE	-97,367.58	0.00
	CBK.DE	949,986.80	0.00
	BAYN.DE	-96,432.22	0.00
	ALV.DE	399,046.36	0.00
	ADS.DE	504,811.34	0.00
	ACA.PA	349,716.05	0.00
	AIR.PA	-239,988.57	0.00
	VOD.L	-113,186.03	0.00
Index	GSPC	0.00	2,596,274.19
	AEX	0.00	-867,683.71
	GDAXI	0.00	2,333,655.11
	FCHI	0.00	86,720.68
	FTSE	0.00	-149,618.59
Interest Rate	EUR3M	-0.98	-0.98
	EUR6M	0.29	0.29
	EUR1Y	-12.77	-12.77
	EUR2Y	-26.32	-26.32
	EUR3Y	-68.00	-68.00
	EUR5Y	-625.89	-625.89
	EUR7Y	-104.84	-104.84
	EUR10Y	-814.99	-814.99
	EUR15Y	-769.90	-769.90
	EUR20Y	-1,462.58	-1,462.58
	USD3M	-2.05	-2.05
	USD6M	-1.62	-1.62
	USD1Y	-10.69	-10.69
	USD2Y	-27.65	-27.65
	USD3Y	-384.12	-384.12
	USD5Y	-70.78	-70.78
	USD7Y	-106.56	-106.56
	USD10Y	-965.17	-965.17
	USD20Y	-834.85	-834.85
Currency	USDEUR	5,950,979.16	5,950,979.16
	GBPEUR	-113,186.03	-113,186.03

Table 4.1. Risk factor exposures map of the portfolio on 27 September 2023.

4.2. Volatility

Before computing the VaR models we need to define and estimate the volatility of the risk factors. The easiest way of doing this is to select a sample of historical behavior and calculate its standard deviation. Note that historical behavior differs according to the risk factors, for example for the interest rates this behavior refers to the change in basis point of the interest rates, but in the case of the stocks it refers to the returns.

However, this approach implies that the oldest observations will have exactly the same weight as the recent observations, influencing the results with observations that are so far into the past, that have minimal relation with the current market conditions.

VaR is a forward-looking risk measure, so the current market conditions are of more relevance than the distant past, so we need to use a volatility model. In this work we will use the Exponential Weighted Moving Average (EWMA), that assigns more weight to the most recent observations, and as time passes the weight given to a specific observation decreases exponentially. The weight attribution to each observation is given by λ, that is known as the smoothing factor, and it can vary between 0 and 1. The lower the λ, the more overweight are the most recent observations.

The EWMA variance is estimated recursively as follows:

$$\hat{\sigma}_t^2 = (1-\lambda)x_t^2 + \lambda\hat{\sigma}_{t-1}^2 \tag{11}$$

Where $\hat{\sigma}_t^2$ is the variance estimated on day t for day $t+1$ and x_t is the observed behavior of the risk factor on day t.

The EWMA covariance is estimated as follows:

$$\hat{\sigma}_{i,j,t} = (1-\lambda)x_{i,t}x_{j,t} + \lambda\hat{\sigma}_{i,j,t-1} \tag{12}$$

Where $\hat{\sigma}_{i,j,t}$ is the covariance estimated on day t for day $t+1$ and $x_{i,t}$ is the observed behavior of the risk factor i, on day t.

The value chosen for λ is usually between 0.9 and 0.98. However, the decision is subjective (Alexander, 2008b). For instance, J.P. Morgan (1996) in the RiskMetrics model proposes as the optimal choice of λ=0.94, for daily data. In this work, we will test some different values of λ to evaluate the best option to our portfolio.

Chapter 5.

VaR Models

The $VaR_{h,\alpha}$ represents for a future time horizon h, the loss that it is expected to not be exceeded $100(1-\alpha)\%$ of the time. Furthermore, statistically the $100\alpha\%$ h-day VaR is the symmetric of the α-quantile of the h-day discounted P&L distribution (X_h), therefore we have that:

$$P(X_h < -VaR_{h,\alpha}) = \alpha \tag{13}$$

As proposed by the Basel Committee guidelines, we will adopt a confidence level of $(1 - \alpha) = 99\%$ and a future time horizon of $h = 1$. The $VaR_{1,1\%}$, represents the loss that we are 99% confident that will not be exceeded over the next day.

In this Chapter we will use the methodology applied in Chapter 5 to compute four different classes of VaR models. We will compute two parametric models, the Normal VaR and the Skewed Generalized Student-t (SGSt) VaR, and two non-parametric models, the Historical VaR and the Quantile Regression VaR. In the end of this chapter, we show the 17 models that we computed to evaluate the performance of each model in our portfolio, in Chapter 7.

5.1. Normal VaR

Let the h day portfolio returns be given by X_h, where X is a continuous random variable that follows a non-standard normal distribution, $X_h \sim N(\mu, \sigma^2)$, where $\mu_h$is the estimated mean and σ_h^2 is the estimated variation. Therefore, we have that:

$$VaR_{h,\alpha} = -\Phi_{\mu,\sigma}^{-1}(\alpha) \tag{14}$$

where $\Phi_{\mu,\sigma}^{-1}(\alpha)$ denotes the α quantile of the normal distribution with mean μ and standard deviation σ.

Let $Q_X(\alpha)$ denote the α quantile of the random variable X which follows some distribution, and $g(x)$ be an increasing function of x. Then the equivariance property of quantile functions is given by:

$$Q_{g(X)}(\alpha) = g\{Q_X(\alpha)\} \tag{15}$$

Using this property, we can formulate the portfolio return as a linear function of a standard normal random variable, and then rewrite the Equation 14 as:

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha)\sigma - \mu \tag{16}$$

The μ works as a drift adjustment. Since we are working with daily VaR estimates (h = 1), Alexander (2008b) proposes using $\mu = 0$, because when the measurement horizon is small, the expected return is close to zero, so ignoring the drift adjustment has a minor impact on the VaR estimation, besides it is difficult to accurately estimate the expected returns (μ). Therefore, we compute the 100α% h-day Normal VaR by the following:

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha) \times \sigma \tag{17}$$

Where σ is estimated using the EWMA volatility model that results from the Equation 11.

5.2. SGSt VaR

The standardized Skewed Genralized Student-t (SGSt) distribution proposed by Theodossiou (1998), is a generalization of the classical Student-t distribution that allows for asymmetry and extra flexibility in the shape of its tail and central regions. The SGSt density function $T_{\mu,\sigma,\lambda,p,q}$, depends on the μ, σ, λ, p and q parameters, where μ is the mean of the distribution, $\sigma > 0$ is the standard deviation, $-1 < \lambda < 1$ determines the skewness, $p > 0$ controls the shape of the central region of the distribution and $q > 0$ controls the shape of the tail region of the distribution. To estimate these parameters, we will use the maximum likelihood method.

To compute the SGSt VaR, we use the same process used for the Normal VaR, where the only difference is that $\mathrm{T}_{\mu,\sigma,\lambda,p,q}^{-1}(\alpha)$ represents the α quantile of the SGSt distribution. Therefore, we compute the h day $100\alpha\%$ SGSt VaR as:

$$VaR_{h,\alpha} = -\,\mathrm{T}_{\mu,\sigma,\lambda,p,q}^{-1}(\alpha) \tag{18}$$

Similar to the Normal VaR, we can use the equivariance property and assume $\mu = 0$ to rewrite the SGSt VaR as:

$$VaR_{h,\alpha} = -\,\mathrm{T}_{0,1,\lambda,p,q}^{-1}(\alpha) \times \sigma \tag{19}$$

5.3. Historical VaR

To estimate the Historical VaR we need to follow some steps, first we need to choose a sample size (n), then compute non-overlapping h-day returns or interest rate change in basis point, in the case of the interest rates, for the portfolio risk factors over the sample period, by keeping these portfolio risk factor loading constant to compute the empirical h-day portfolio P&L distribution from the risk factor returns. Afterwards we need to obtain the empirical cumulative function by summing the probabilities from the smallest P&L of the portfolio to the largest, where each observation has a probability of $\frac{1}{n}$. Finally, the VaR is obtained by minus the $100\alpha\%$ smallest portfolio P&L.

The main problem of the basic Historical VaR model is that it gives the same weight to all observations, and the larger the sample size, the less this model reflects the current market conditions. On the Normal and SGSt models we solved this problem by using EWMA volatility estimates, however in the case of the basic Historical model this is not possible, because the EWMA can be used to estimate the covariance matrix, but not the whole distribution. Hull and

White (1998) proposed the volatility-adjusted Historical VaR with the purpose of solving this problem, where it gives the same weight to every observation but adjusts the volatility of the series of returns to match the current volatility. Therefore, we are adjusting the sample, so it reflects the current market conditions. To do this adjustment in a single series of portfolio returns (or P&Ls), we need to obtain a series of volatility estimates, $\hat{\sigma}_t$, knowing that T is the VaR measurement date ($t < T$), we can adjust the series of returns ($\hat{r}_t$) by:

$$\hat{r}_t = \frac{r_t}{\hat{\sigma}_t}\hat{\sigma}_T = \frac{\hat{\sigma}_T}{\hat{\sigma}_t}r_t \quad (20)$$

All things considered we compute the h day $100\alpha\%$ volatility adjusted Historical VaR as minus the α-quantile of the sample of volatility adjusted returns given by Equation 20.

5.4. Quantile Regression VaR

Since the VaR is defined as the symmetric of the α quantile, we can estimate the VaR through a quantile regression of the portfolio return onto some explanatory variables that we choose.

Let y represent the P&L of a portfolio, $\hat{a}$ and $\hat{b}$ represent the estimated parameters of the α-quantile regression of x onto y, then we can estimate the α-quantile regression ($q_{\alpha,y}$) as:

$$VaR_\alpha = -q_{\alpha,y} = -\left(\hat{a} + \hat{b}x_i\right) \quad (21)$$

The estimates parameters, $\hat{a}$ and $\hat{b}$, can be determined by solving the following minimization problem:

$$\left(\hat{a}, \hat{b}\right) = arg \min_{a,b} \sum_{i=1}^{n} [y_i - (a + bx_i)]\left(\alpha - I_{y_i-(a+bx_i)<0}\right) \quad (22)$$

where $I_{y_i-(a+bx_i)<0}$ is an indicator function of event:

$$I_{y_i-(a+bx_i)<0} = \begin{cases} 1, if\ y_i - (a + bx_i) < 0 \\ 0, otherwise \end{cases} \quad (23)$$

To compute the h -day $100\alpha\%$ QR VaR, we tested different parameters, where we choose between some specifications, like whether we used a constant or not and whether it had one or two independent variables, which were always the estimated volatility of the portfolio through the EWMA method with different smoothing factors.

5.5. Summary of the VaR Models

We tested multiple configurations for each class of model, but it was not viable to present them all, so in Table 5.1 we present the best settings of each model class. To note that the models are numerated to facilitate the analysis, and each model is computed for the Total VaR and Systematic VaR.

Model number:	Description
1	Normal, with EWMA smoothing factor 0.93
2	Normal, with EWMA smoothing factor 0.94
3	Normal, with EWMA smoothing factor 0.96
4	Normal, with EWMA smoothing factor 0.98
5	SGSt, with EWMA smoothing factor 0.93 and sample size 300
6	SGSt, with EWMA smoothing factor 0.93 and sample size 800
7	SGSt, with EWMA smoothing factor 0.98 and sample size 300
8	SGSt, with EWMA smoothing factor 0.98 and sample size 800
9	Historical with volatility adjustment, EWMA significance level 0.94 and sample size 300
10	Historical with volatility adjustment, EWMA significance level 0.94 and sample size 800
11	Historical with volatility adjustment, EWMA significance level 0.96 and sample size 300
12	Historical with volatility adjustment, EWMA significance level 0.96 and sample size 800
13	Historical with volatility adjustment, EWMA significance level 0.98 and sample size 300
14	Historical with volatility adjustment, EWMA significance level 0.98 and sample size 800
15	Quantile Regression, with EWMA volatility with 0.94 smoothing factor as independent variable, sample size 800
16	Quantile Regression, with EWMA volatility with 0.94 smoothing factor and EWMA volatility with 0.90 smoothing factor as independent variables, sample size 800
17	Quantile Regression, with a constant and EWMA volatility with 0.94 smoothing factor and EWMA volatility with 0.90 smoothing factor as independent variables, sample size 800

Table 5.1 Description of the VaR models.

Chapter 6.

Backtesting

In the previous Chapter we refer to the necessary methodology to compute the four types of VaR models. Altogether we computed 17 different models with different settings to test the performance of the Total VaR and Systematic VaR and choose the best one. For each model we computed a series of daily historical VaR estimates for over 10 years from 11 February 2013 to 27 January 2023, which we define as the global period.

An exceedance is an event where the actual P&L for the day is worse than the VaR estimate for that day, and it is considered one of the main performance metrics. We will adopt two tests: the Unconditional Coverage test (UC) proposed by Kupiec (1995), which evaluates the number of exceedances and the BCP test proposed by Berkowitz et al. (2011) that evaluates the autocorrelation between those exceedances.

For the same number of exceedances in close proximity, a model with fewer exceedances in total has a higher probability of failing the BCP. Therefore, we need to primarily focus on the results for the UC test to the global period and then differentiate the models with the best performance using the BCP test to the global period, finally we will apply the UC to annual subperiods to assess the model that has more consistent results on an annual basis, and that will be the model chosen to go forward one year.

6.1. Unconditional test

Remembering the definition of VaR as the worst loss we are $1-\alpha$ confident will not be exceeded, so there is a chance that the loss will be worse than the VaR. For example, for a sample of 500 daily VaR estimates at 99% confidence level ($\alpha = 1\%$), we expect that the VaR to be exceeded $500 \times \alpha = 5$ times, which means that we expect 5 exceedances to occur.

For each observation of a sample, we can identify an exceedance through an indicator function. Let $VaR_{t,\alpha}$ be the VaR estimated for the day t, then we have a time series with elements given by:

$$I_{t,\alpha} = \begin{cases} 1, if\ P\&L_t < -VaR_{t,\alpha} \\ 0, otherwise \end{cases} \tag{24}$$

Let π_{obs} and π_{exp} be the observed and the expected exceedance rates, respectively, then the null and alternative hypothesis, for the UC test are given by:

$$\begin{gathered} H_0: \pi_{obs} = \pi_{exp} = \alpha \\ H_\alpha: \pi_{obs} \neq \pi_{exp} \end{gathered} \tag{25}$$

Let $n_1$and n_0 be the number of exceedances and non-exceedances, respectively, then the test statistic is given by:

$$LR_{uc} = \left(\frac{\pi_{exp}}{\pi_{obs}}\right)^{n_1} \left(\frac{1-\pi_{exp}}{1-\pi_{obs}}\right)^{n_0} \tag{26}$$

This test statistic under the null hypothesis ($\pi_{obs} = \pi_{exp}$) follows a chi-squared distribution with one degree of freedom: $-2\ln(LR_{uc}) \sim \chi_1^2$.

Therefore, the model is well specified if the null hypothesis defined at Equation 25 is not rejected at 95% confidence level, meaning that the model passes the UC test if the exceedance rate is within the expected value of $\alpha = 1\%$.

6.2. BCP test

Berkowitz, Christoffersen and Pelletier (BCP) test, assesses that a model is well specified if the exceedances are independent from each other, which means that the exceedances do not follow a pattern. Therefore, the autocorrelation in exceedances is 0 at all lags.

Let $\hat{\rho}_k = Corr(I_\alpha, L^k I_\alpha)$ be the k-th order autocorrelation of the time series of exceedances given by Equation 24 and K is the maximum autocorrelation lag considered in the test. Then, the null and alternative hypothesis, for the BCP test are given by:

$$\begin{gathered} H_0: \hat{\rho}_k = 0, \forall k \in \{1, \dots, K\} \\ H_\alpha: \exists k \in \{1, \dots, K\}\ s.t.\ \hat{\rho}_k \neq 0 \end{gathered} \tag{27}$$

Let the n be the sample size, then the test statistic is:

$$BCP_K = T(T+2) \sum_{k=1}^{K} \frac{\hat{\rho}_k^2}{T-k} \tag{28}$$

This test statistic under the null hypothesis ($\hat{\rho}_k = 0$) follows a chi-squared distribution with K degrees of freedom: $BCP_K \sim \chi_K^2$.

There is a tradeoff to consider when choosing the number of lags K. On one hand, a larger K helps detect autocorrelation present at higher-order lags. But on the other hand, a larger K makes it harder to reject the null hypothesis when autocorrelation is only present at lower-order lags. This happens because with a larger K the critical value for rejection increases but, if there is no evidence of autocorrelation at higher-order lags, the value of the test statistic will not increase enough to compensate for this increase in the critical value, resulting in a failure to reject the model. Knowing this, we compute the BCP test for a range of lags, from the 1 to 10 lags.

6.3. Results of Backtesting

In this section, we will use the UC and BCP tests to evaluate the performance of each model to conclude the best model for our current portfolio. Recalling that we computed the daily VaR estimates for every model for the global period of 10 years, it totalizes n = 2600 observations. Since, the VaR models were computed at a significance level of α = 1%, in the perspective of the UC test, a model will be considered well specified if it has near to $2600 \times 1\% \approx 26$ exceedances.

Usually, the null hypothesis is rejected when the p-value of the test statistic is less than 5%, so a model will be accepted by the UC and BCP tests when their p-value is higher than 5%.

Table 6.1 presents the UC test for Total VaR estimated by each model, for the global period.

	Model	Exceedances	Exceedance Rate (%)	p-value (%)
Normal	1	48	1.85%	0.00% ***
	2	47	1.81%	0.00% ***
	3	43	1.65%	0.20% ***
	4	38	1.46%	2.70% **
SGSt	5	27	1.04%	84.47%
	6	28	1.08%	69.70%
	7	23	0.88%	54.64%
	8	23	0.88%	54.64%
Historical	9	44	1.69%	0.12% ***
	10	50	1.92%	0.00% ***
	11	38	1.46%	2.69% **
	12	39	1.50%	1.70% **
	13	30	1.15%	44.15%
	14	35	1.35%	9.20% *
Quantile Regression	18	30	1.15%	44.15%
	19	32	1.23%	25.37%
	20	40	1.54%	1.06% **

Table 6.1. UC test results for the global period, for the Total VaR. The *** indicates rejection with 99% confidence level, ** show rejection with 95% confidence level and * rejection with 90% confidence level. The models with *** and ** are rejected because their p-value is lower than 5%

As we can see from the table above, we reject all the Normal VaR models, which is not a surprise, as the main weakness of this model is that it assumes a normal distribution for the returns, which is normally not the case of the financial assets (Fama, 1965). As expected, the Normal VaR models produce too many exceedances, which is a clear sign that the distribution of our portfolio has fatter tails, which implies that the Normal distribution underestimates the VaR when it is computed at a high confidence level. Furthermore, we can observe in Appendix A.1 that presents the descriptive statistics of the portfolio during the Backtesting period, that our portfolio has negative skewness and a leptokurtic distribution.

Model 2 is the RiskMetrics model and as we can observe it has one of the worst results, which means that the proposal of J.P.Morgan (1996), that the optimal EWMA smoothing factor is $\lambda = 0.94$, is not the best choice in all cases.

Table 6.2 presents the results of the UC test for the Systematic VaR against the actual portfolio P&Ls. Note that the Systematic VaR is computed by the mapped portfolio, but it has to be backtested against the actual portfolio.

	Model	**Exceedances**	**Exceedance Rate (%)**	**p-value (%)**
Normal	1	96	3.69%	0.00% ***
	2	90	3.46%	0.00% ***
	3	86	3.31%	0.00% ***
	4	82	3.15%	0.00% ***
SGSt	5	79	3.04%	0.00% ***
	6	59	2.27%	0.00% ***
	7	77	2.96%	0.00% ***
	8	60	2.31%	0.00% ***
Historical	9	84	3.23%	0.00% ***
	10	83	3.19%	0.00% ***
	11	74	2.85%	0.00% ***
	12	70	2.69%	0.00% ***
	13	62	2.38%	0.00% ***
	14	58	2.23%	0.00% ***
Quantile Regression	15	55	2.12%	0.00% ***
	16	62	2.38%	0.00% ***
	17	61	2.35%	0.00% ***

Table 6.2. UC test results for the global period, for the Systematic VaR against the actual portfolio. The *** indicates rejection with 99% confidence level, ** show rejection with 95% confidence level and * rejection with 90% confidence level. The models with *** and ** are rejected because their p-value is lower than 5%

All the Systematic VaR models are rejected by the UC test because the actual portfolio is considerably different from the mapped portfolio. In Appendix B.1, we can observe the UC test results for the Systematic VaR against the mapped portfolio, which has relatively better results than the Systematic VaR against the actual portfolio. Therefore, the Systematic VaR is not a good solution for our portfolio, because the actual portfolio is considerably different from the mapped portfolio, which implies that our portfolio is not diversified enough. On the other hand, since the portfolio is not very large, there is no problem estimating the Total VaR, and so from now on we will only consider the Total VaR models.

Advancing in the backtesting, we exclude the models that were rejected by the UC test. Table 6.3 presents the p-value of the BCP test with the lowest p-value among the 10 BCPs conducted for each model (all of them Total VaR models), for the global period.

	Model number	**Worst p-value**	**Lag**
SGSt	5	59.22%	1
	6	7.66%*	5
	7	37.47%	2
	8	18.40%	2
Historical	13	3.15%**	5
	14	15.72%	10
Quantile Regression	15	55.13%	1
	16	50.63%	2

Table 6.3. BCP test results for the global period. It represents only the worst p-value among the 10 BCP tests conducted and the respective lag it corresponds to. The *** indicates rejection with 99% confidence level, ** show rejection with 95% confidence level and * rejection with 90% confidence level. The models with *** and ** are rejected because their p-value is lower than 5%

We observe that the only model that passes the UC test and does not pass the BCP test is the model 13. In Appendix B.2 we have the BCP test for all the Total VaR models, even the ones that were rejected by the UC test and we can see that the models 4 and 12 passed the BCP test, even though they did not pass the UC test.

To help with a deeper analysis Table 6.4 presents the p-value of the UC test for each model, for the annual subperiods.

	SGSt				Historical	Quantile Regression	
	5	**6**	**7**	**8**	**14**	**15**	**16**
2023-2022	0.38%	0.38%	0.00%	0.00%	0.77%	0.38%	0.38%
	(25.44%)	(25.44%)	(2.22%) **	(2.22%) **	(69.67%)	(25.44%)	(25.44%)
2022-2021	0.77%	0.77%	0.77%	0.77%	1.15%	0.77%	1.15%
	(69.67%)	(69.67%)	(69.67%)	(69.67%)	(80.77%)	(69.67%)	(80.77%)
2021-2020	1.54%	1.54%	1.15%	1.54%	2.31%	1.92%	1.54%
	(41.87%)	(41.87%)	(80.77%)	(41.87%)	(7.01%) *	(18.44%)	(41.87%)
2020-2019	0.38%	0.00%	0.38%	0.00%	0.38%	1.54%	1.54%
	(25.44%)	(2.22%) **	(25.44%)	(2.22%) **	(25.44%)	(41.87%)	(41.87%)
2019-2018	0.77%	0.77%	0.77%	0.77%	1.92%	0.77%	0.77%
	(69.67%)	(69.67%)	(69.67%)	(69.67%)	(18.44%)	(69.67%)	(69.67%)
2018-2017	1.15%	0.77%	1.15%	0.77%	1.54%	1.15%	1.15%
	(80.77%)	(69.67%)	(80.77%)	(69.67%)	(41.87%)	(80.77%)	(80.77%)
2017-2016	1.54%	0.77%	1.54%	1.15%	1.54%	1.15%	1.54%
	(41.87%)	(69.67%)	(41.87%)	(69.67%)	(41.87%)	(80.77%)	(41.87%)
2016-2015	1.54%	1.15%	2.69%	1.92%	1.54%	1.54%	2.31%
	(41.87%)	(80.77%)	(2.34%) **	(18.44%)	(41.87%)	(41.87%)	(7.01%) *
2015-2014	1.54%	1.54%	1.54%	1.15%	1.54%	1.54%	1.92%
	(41.87%)	(41.87%)	(41.87%)	(80.77%)	(41.87%)	(41.87%)	(18.44%)
2014-2013	0.77%	0.77%	0.77%	0.77%	0.77%	0.77%	0.77%
	(69.67%)	(69.67%)	(69.67%)	(69.67%)	(69.67%)	(69.67%)	(69.67%)

Table 6.4. UC test for annual subperiods. The p-value is in parenthesis and below the exceedance rate for each model and for the respective subperiod. The *** indicates rejection with 99% confidence level, ** rejection with 95% confidence level and * rejection with 90% confidence level. The cells with *** and ** are rejected because their p-value is lower than 1% and 5%, respectively, so do not pass the UC test. The cells with * denote the p-value range between 5% and 10%, so it passes the UC test, but is not consistent enough.

As we can see from the table above, model 5 and model 15 are clearly two good candidates to be chosen. Remembering Table 6.1, in the global period the model 5 has 27 exceedances which is a closer number to the supposed (26 exceedances), while the model 15 has 30 exceedances. Therefore, we will choose the model 5, which corresponds to the Total VaR of SGSt with EWMA smoothing factor of $\lambda = 0.93$ and sample size of 300.

Figure 6.1 shows the daily P&L, daily VaR and the exceedances that occurred during the global period of 10 years of the SGSt model number 5.

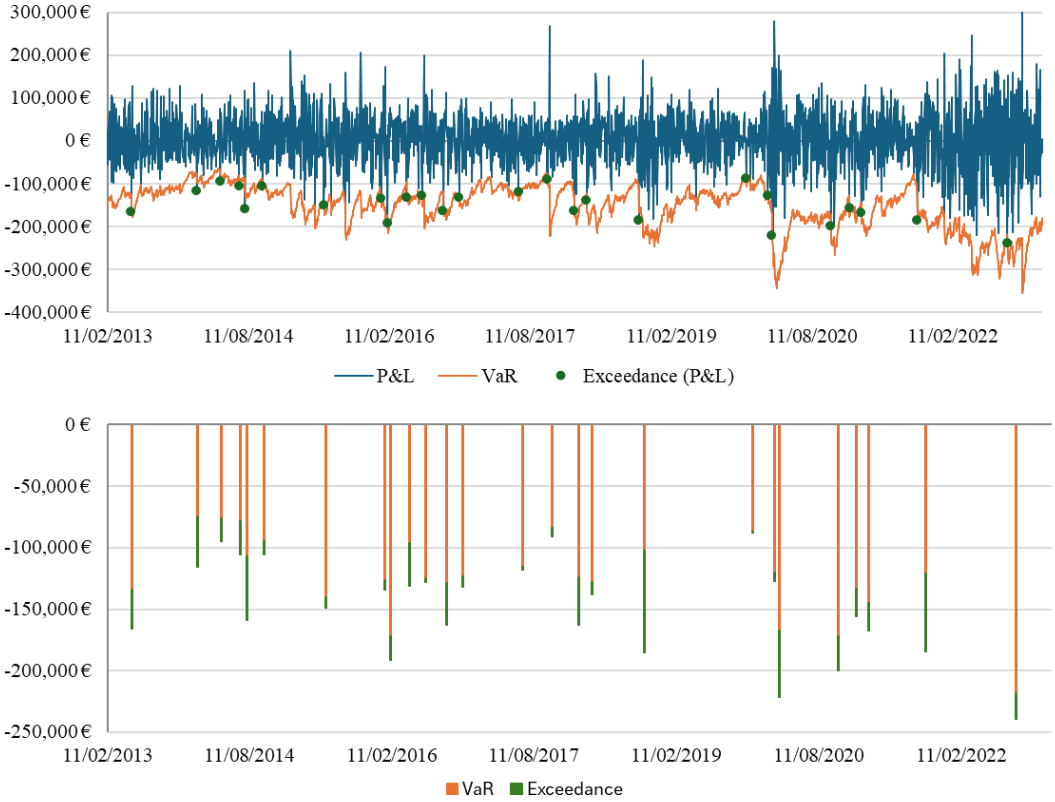

Figure 6.1. Global period performance of SGSt VaR model number 5. The green dots in the 1st panel represent the exceedances. In the 2nd panel we can visualize by how much the VaR is exceeded whenever an exceedance occurs. In the Appendix B.3 is the table with the specifications of the exceedances

We can observe that during this period it did not occur any exceedance clustering. The exceedances that occurred more closely to each other were in 2020 with 17 trading days of interval between two exceedances and in 2014 and 2016 both with 23 trading days of interval between two exceedances. In Appendix B.4 we have the global period performance of the Historical VaR model number 13 and there we can clearly see the exceedance clustering that occurred in 2020. Both models have a similar number of exceedances, but the model 13 failed the BCP test because of the exceedance clustering that we can visualize in Appendix B.4.

Chapter 7.

VaR Management

As mentioned in the previous chapter, we will use from now on model 5, the SGSt with EWMA smoothing factor of $\lambda = 0.93$ and sample size of 300, to measure and manage the VaR. From now on, we will define two approaches.

In the first approach we go forward one year without using any risk management strategy, and in the second we go forward one year, by defining a maximum target of VaR for our portfolio. Therefore, in the second approach we will employ a risk management strategy every day the estimated VaR surpasses the pre-determined target. This strategy will consist of hedging, using a future contract, the risk factors that contribute more to the risk, which are the risk factors that have the larger Marginal VaR,

In both approaches we need to reinvest the coupon payments of the bonds as they occur, which will be done by reenforcing the long positions of some stocks. The specific details of the coupon reinvestments are presented in Appendix C.1.

As we can see in Figure 6.1, during the Backtesting period the VaR of our portfolio ranged primarily between 100 000€ and 200 000€, except during the Covid-19 period (2020-2022), where we can observe more volatility. Therefore, knowing that on 27 January 2023 our portfolio value is almost €10 million, we will set our maximum daily VaR to 1.6% of the portfolio value, translating in 159 110€, which is in conformity with the historical data.

In this chapter we will first present the methodology of VaR decompositions and then the hedging strategy that we will use to reduce the VaR when it surpasses the pre-determined target. At the end of this chapter, we present the results of the strategy and compare them to the approach where we do not define any daily maximum EC.

7.1. VaR decompositions

We compute the VaR at the risk factor level, to make it possible to disaggregate the portfolio by decomposing the VaR, which supports an effective risk management strategy.

In this work we will use the Marginal VaR that will assist with the selection of the risk factor that will be hedged to then determine the quantity of the required hedging position.

The Marginal VaR for slice Θ^s of the vector of risk factor loadings Θ representing the whole portfolio measures the proportion of the portfolio's VaR that can be attributed to the risk factor exposures in slice Θ^s. The Marginal VaR for slice Θ^s is given by:

$$Marginal\ VaR^s = \nabla f(\Theta)^T \Theta^s = \sum_{i=1}^{n} \frac{\partial VaR}{\partial \theta_i} \times \theta_i^s \tag{29}$$

The gradient vector, $\nabla f(\Theta)$, represents the set of sensitivities of the portfolio VaR to small changes in the exposure to each risk factor, which is given by the following condition:

$$\nabla f(\Theta) \equiv \frac{\partial f(\Theta)}{\partial \Theta} = \begin{bmatrix} \frac{\partial VaR}{\partial \theta_1} \\ \vdots \\ \frac{\partial VaR}{\partial \theta_n} \end{bmatrix} \tag{30}$$

7.2. VaR management strategy

The VaR estimated for 30 January 2023 of 174 374€ is the first day that surpasses the maximum daily value of 159 110€, so we need to hedge it, reducing the VaR by 15 264€ (9.6%). To do this we computed the Marginal VaR to see from where most of the risk came, which is represented on Table 7.1.

Risk Factor Type	Equity					Currency		Interest Rate	
Marginal VaR (EUR)	79,721.10					27,142.46		67,511.15	
Marginal VaR (%)	45.72%					15.57%		38.72%	
Risk Factor Group	**S&P500**	**AEX**	**DAX**	**CAC**	**FTSE**	**USDEUR**	**GBPEUR**	**IR_EUR**	**IR_USD**
Marginal VaR (EUR)	30,694.45	10,391.52	39,207.15	-860.18	288.15	27,263.45	-120.99	43,288.23	24,222.92
Marginal VaR (%)	17.60%	5.96%	22.48%	-0.49%	0.17%	15.63%	-0.07%	24.82%	13.89%

Table 7.1. Marginal VaR decomposition by risk factor. This table shows how much each risk factor is contributing to the VaR on 30 September 2023. The decomposition by the risk factor group is a decomposition of the risk factor type.

As we can see from the table above, the risk factor type with the highest Marginal VaR is equity, and consequently the index that is contributing more to that is the DAX. Therefore, to lower the VaR on this day we will add a short position on a future contract on the DAX of €1.7 million.

Since the VaR is estimated daily, this strategy is also adjusted daily, including the choice of the risk factor that will be hedged, which is based on updated marginal VaR decompositions. Note that whenever the VaR is below the target, or the main risk factor that contributes to the VaR differs we remove their hedging position. We keep on repeating this process for 265 trading days until the estimation of VaR for 2 February 2024.

By hedging a particular risk factor, we are changing the portfolio exposure to it which changes the sensitivity of the VaR to all risk factors. Sometimes those sensitivities have big variations, making it impossible to decrease the VaR to the target value with only one risk factor, making it necessary to hedge two risk factors to be possible to reach the target. To do this, we first reduce a part of the VaR with the risk factor that is the main source of risk and then we reduce the VaR to the target value with the risk factor that has the second highest Marginal VaR.

Figure 7.1 shows the hedging strategy that throughout the year is used to manage the VaR.

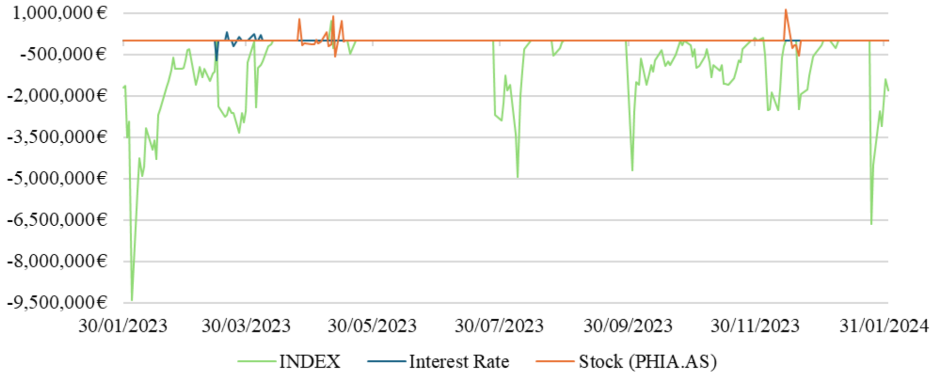

Figure 7.1. Hedging strategy by risk factor and its quantification by day. Note that the index represents the sum of all indices hedged by day.

As we can see from the figure above, some days we had to hedge directly a stock. This happened because on two different occasions the stock PHIA.AS from the AEX index, increased the price significantly from one day to another, changing the correlations with the AEX and decreasing the sensitivity of the Total VaR to that index. Therefore, on those occasions it was not sufficient to hedge with an index position, we had to hedge directly in a position on that stock. As we have short positions in that stock, we need to add long positions financed in the short term. Note that we have to do this not only on the day of the price change but also on the next days until the situation is enough in the past that the correlations and sensitivities normalized. When everything is minimally normalized, we return to the original position.

From Figure 7.1 we can observe that we mostly hedged indices, so in Figure 7.2 we show the decomposition hedging strategy, by index.

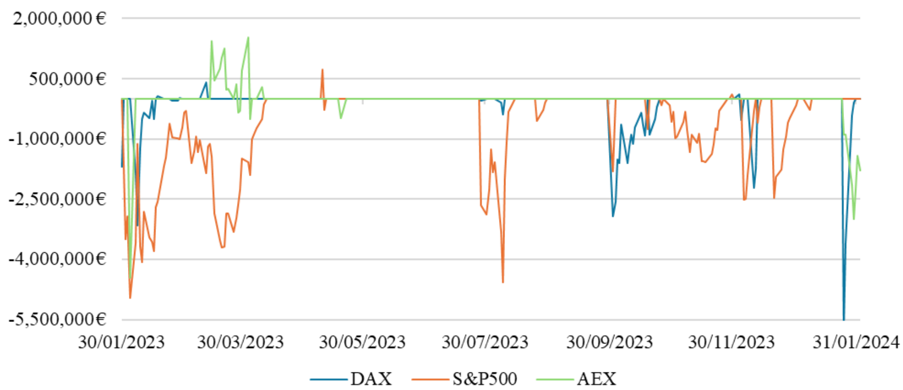

Figure 7.2. Decomposition of the hedging strategy by each index hedged and its value

From the figure above we can observe that the S&P500 was the index that we hedged more frequently, which makes sense knowing from the composition of our portfolio that the positions in stocks from the S&P500 represented almost 25% of the portfolio value on 27 January 2023. As the S&P500 is from the U.S. market, it has associated a currency exposure, so when we hedge the S&P500 it involves a change in the exposure to the USDEUR risk factor.

Overall, the portfolio has a long exposure to the risk of each index via the respective stock positions, then in most cases the hedging is done by a short position on a future contract of the respective index. However, as we can observe the hedging positions of each index in Figure 7.2, in some days the hedging is done with a long position on a future contract, which is explained by the sensitivities and the correlations between the stocks and the respective index, for example when their correlation is negative it is only possible to hedge the index by a long position on a future contract.

7.3. Comparison and Results of VaR management

In Figure 7.3 we can observe the daily VaR estimates of the portfolio for the two approaches: the VaR without hedging and the VaR with hedging, for the one-year period.

In the days that we do the hedging, which are 151 days, the VaR is kept at a maximum of close 160 000€, whereas in the case where we left the portfolio unhedged, the VaR reaches a maximum of more than 220 000€ (40.8% above the target), on 3 February 2024.

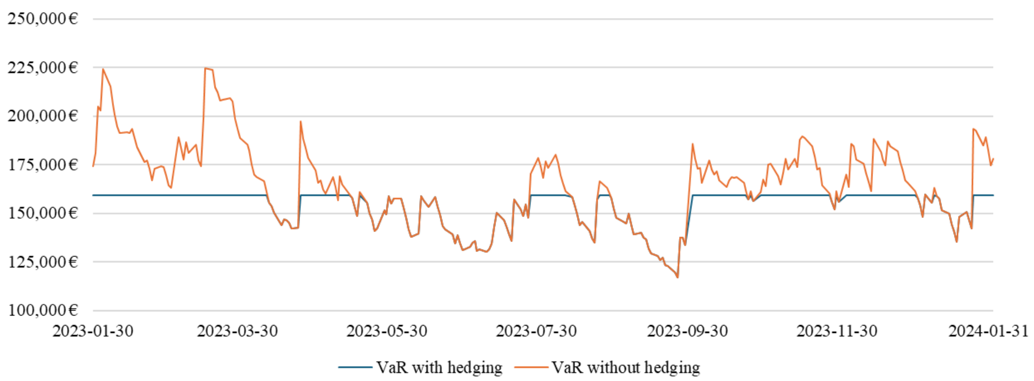

Figure 7.3. Daily VaR of the portfolio with hedging and without hedging.

Figure 7.4 shows the daily P&L for both approaches for the one-year period, and the difference between the P&L of the portfolio with hedging and the portfolio without hedging.

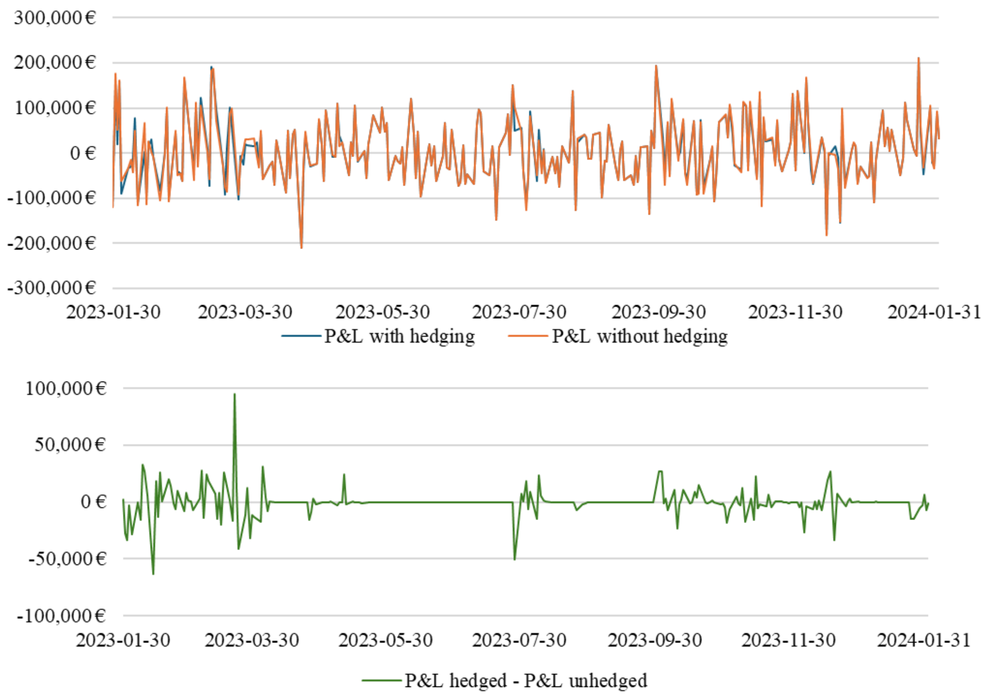

Figure 7.4. Daily P&L of the portfolio with hedging and the portfolio without hedging. In the 2nd section we represent the difference between the P&L of the portfolio hedged and the P&L of the portfolio unhedged

As we can observe from the figure above, some days the strategy increases the P&L, and in another days, it decreases even more the P&L. This happens because it is normal that by hedging, we are mitigating our losses, but we are also limiting our profits.

The maximum that the hedging strategy increased the P&L was on 22 February 2023 by an increase of almost 100 000€. On the other hand, on 13 February 2023 the hedging strategy caused a lower P&L than the portfolio without hedging, by almost 60 000€. Furthermore, considering only the days that we do hedging, the average difference between the P&L hedged and the P&L unhedged is -122.5€ and the median is -470.7€, which indicates that on average the P&L hedged is slightly lower than the P&L unhedged. Overall, the hedging strategy contributes to lower P&Ls, suggesting that the hedging strategy is not working as intended.

To investigate this issue, we analyze the P&L differences between the two strategies separately on days with positive and negative P&Ls.

Figure 7.5 shows that difference in those two different scenarios.

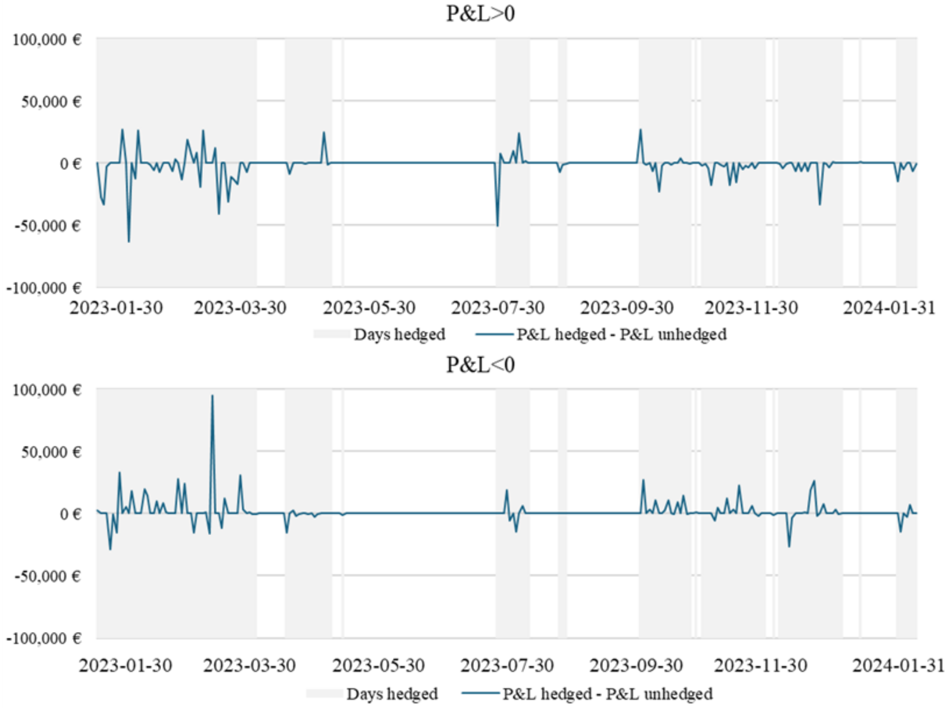

Figure 7.5. Difference of the P&L hedged and unhedged, when the P&L is positive and when the P&L is negative. The grey area represents the days that the hedging happens.

As we can observe in the graphs above, when we hedge, we are reducing the risk, therefore when the P&L is negative, most of the days the hedging improves the result, by having a positive difference and reducing the loss. However, when the P&L is positive, it is normal that by reducing the risk, we will have a worst P&L result, reducing the profit.

Table 7.2 shows the statistics for the P&L differences represented in Figure 7.5

Statistics	P&L hedged – P&L unhedged		
	Total	When P&L>0	When P&L<0
Positive Difference (%)	43.7%	26.0%	62.2%
Average (€)	-122.51	-4,725.10	4,666.67
Median (€)	-470.76	-2,153.58	1,062.65
Maximum (€)	95,417.65	27,017.96	95,417.65
Minimum (€)	-63,309.16	-63,309.16	-28.486,49

Table 7.2. Statistics of the difference between the P&L hedged and unhedged, when the P&L is positive and when it is negative.

We can conclude from Table 7.2 that when the P&L is positive, the hedging only increases the profit by 26% of the days we hedge, whereas when the P&L is negative, the hedging reduces the loss 62.2% of the hedged days, improving the P&L. In the days that we hedge, we are improving our losses on average by 4 666€, but we are also limiting our profits on average by 4 725€. When the P&L is positive, the median of the difference is -2 153.58€ and when the P&L is negative the median is 1 062.65€

Therefore, as expected in the days that we hedge, we are reducing our risk and in contrast we are on most days limiting our profits and improving our losses.

Figure 7.6 presents the comparison between the Unhedged and Hedged VaR (shown as a loss) performance for the one-year period.

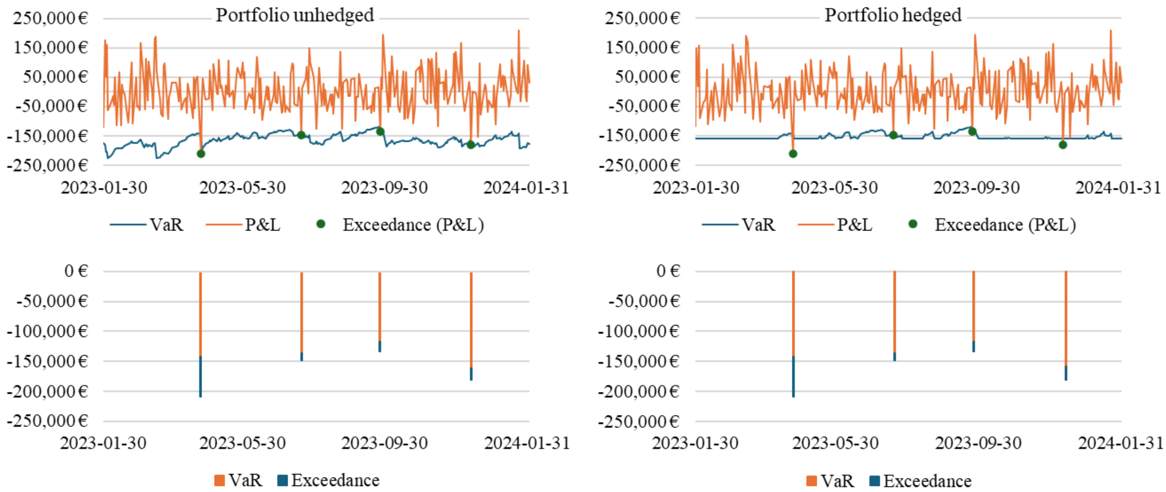

Figure 7.6. Comparison of performance of the Portfolio Unhedged and the Portfolio Hedged. The VaR is shown as a loss. In the panels below we show the exceedances occurring and by how much they exceed the VaR of that day.

We observe from Figure 7.6 that the unhedged portfolio and the hedged portfolio presented the same number of exceedances for the year. Considering this it is important to note that three out of the four exceedances occurred during days that was not necessary a risk management strategy, because the VaR was below the pre-defined maximum. The fourth exceedance occurred on 14 December 2023, a day that the hedging strategy even increased the P&L, but just slightly and not enough to compensate for the decrease of the VaR. As we can see in Appendix C.2. that presents the exceedances that occur during the one-year period, for both strategies (with and without hedging), the increase in the P&L was lower than the decrease in the VaR, so the exceedance became even bigger, but only slightly so, with the hedging strategy.

Figure 7.7 presents the daily cumulative P&L of both approaches

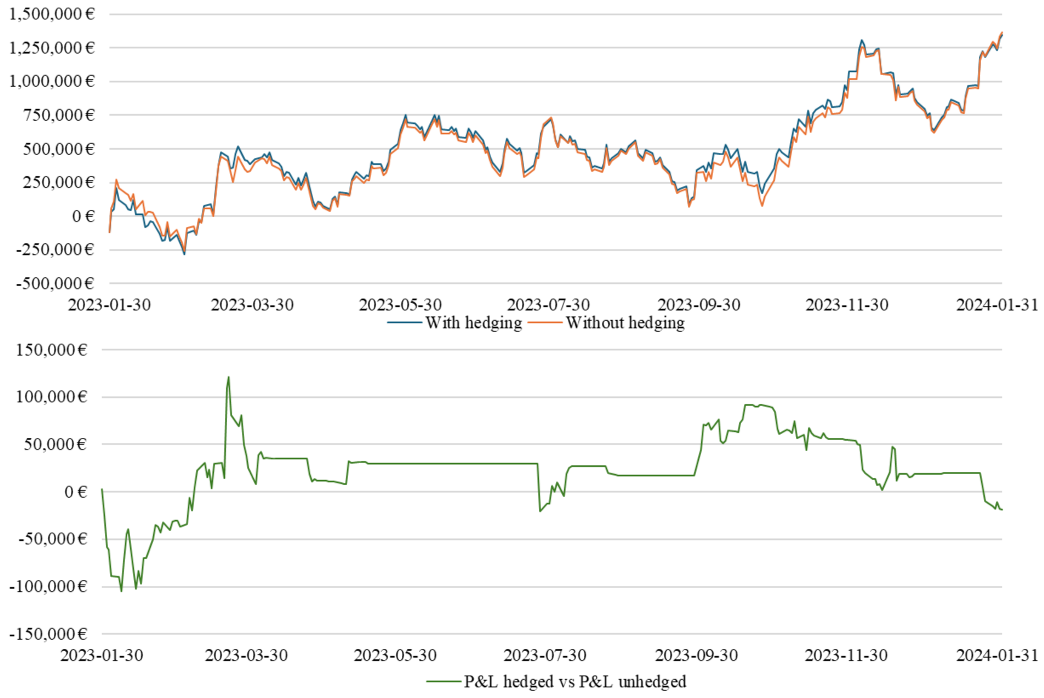

Figure 7.7. Daily cumulative P&L of the portfolio with hedging and the portfolio without hedging. In the 2nd panel we represent the difference between the daily cumulative P&L of the portfolio hedged and the P&L of the portfolio unhedged

As we can see from the figure above, both strategies generated a profit at the end of the year, and even though at the end of the period in analysis there is a difference in the cumulative P&L, that difference is not very significant.

Both approaches studied have distinct risk profiles due to the hedging, so it is not fair to compare the P&L of each approach without considering the risk profile. Therefore, we need to use a risk-adjusted performance metric to make this comparison possible. The RORAC measure associates the P&L with the risk that is incurred to achieve that P&L, and it is given by:

$$RORAC = \frac{P\&L}{EC} \tag{31}$$

We will compute the average RORAC by two different methodologies. The first one we computed by:

$$Average\ RORAC = \frac{Average\ P\&L}{Average\ EC} \tag{32}$$

The second method consists of computing for the year, the daily RORAC by Equation 31, and then computing the average of the RORAC, which is given by:

$$Average\ RORAC = \frac{\sum_{i=1}^{n} \frac{P\&L_n}{EC_n}}{n} \tag{33}$$

In Table 7.3 we present the RORAC for the portfolio with hedge and the portfolio without hedging, where we used two methodologies to compute the average RORAC.

	Without hedge	**With hedge**
P&L (€)	1,367,969	1,349,470
Average P&L (€)	5,162	5,092
EC (€)	43,428,314	40,547,292
Average EC	163,880	153,009
Average RORAC (1)	3.15%	3.33%
Average RORAC (2)	3.01%	3.16%

Table 7.3. Average RORAC. (1) is computed by Equation 32 and (2) is computed by Equation 33.

The results of the RORAC in the two methodologies were different, but their conclusions are the same. We can observe that even though the portfolio with the hedging strategy had a lower P&L, it still provided a better RORAC than the portfolio without hedging because of the reduction in the risk incurred. So, we can conclude that by implementing a risk management strategy it benefits the performance of the portfolio.

Chapter 8.

Conclusion

The main purpose of this work was to measure and manage the daily VaR of the portfolio, such that it did not exceed a pre-defined maximum value, during the period of one-year.

Firstly, we had to measure our portfolio that consisted of positions on stocks from the S&P500, the DAX, CAC40, FTSE100 and the AEX and a part of bonds from U.S. and European markets. To do this, we had to choose one model to measure the VaR. Given the large range of choices, we computed four classes of VaR models with various settings: Normal VaR, SGSt VaR, Historical VaR and Quantile Regression VaR, which in total we computed 17 different models to test which one is the best fit to our portfolio. To tackle this topic, we analyzed the performance of each model applying the UC and BCP tests through the Backtesting assessment.

In the Backtesting analysis we had some expected and unexpected results. As expected, Normal VaR failed the Backtesting, which is in accordance with the assumption that the returns of the financial assets do not follow a normal distribution, indicating that this class of model is unfit for our portfolio. The Risk Metric model with EWMA $\lambda = 0.94$, proposed by J.P.Morgan (1996), had was rejected by the UC test, which entails the necessity of testing various models and settings to a specific portfolio, because what is a best fit to a portfolio might not be to another portfolio, even if they are similar.

The Systematic VaR requires a less complex computation but implies less accuracy. In the case of our portfolio, the Systematic VaR is not adequate because the mapped portfolio computed for the Systematic VaR is very different from the actual portfolio, which means that our portfolio is not very diversified.

We measure the VaR applying the model chosen, the SGSt with EWMA smoothing factor of 0.93 and sample size of 300. To manage the VaR we choose a pre-determined daily maximum for VaR of close to 160 000€, and during the period of one-year, we implement a hedging strategy based on the VaR decompositions every day that the VaR exceeds the pre-determined limit.

The hedging strategy consists of a position on futures contract in the source or sources that represent higher risk according to the Marginal VaR. Generally, the main risk came from the equity part of the portfolio, which is expected because a stock is riskier than a bond. Therefore, we mostly hedged the market risk of stocks, which we did by entering into a future contract of the corresponding index. The primary source of risk that we needed to hedge was mostly the stocks from the U.S. market, which on 27 January 2023 represented almost 25% of the portfolio. Even though, some days we had to hedge an European bond, the unexpected was that on certain days we had to hedge directly a stock, because its price changed drastically from one day to another.

Results reveal that in the approach where we did not manage the VaR it reached a maximum of almost 225 000€, which is 40% higher than the target. In addition, the hedging clearly helped minimize the losses, which were improved on average by 4 666€, although it was at the expense of reduced profits, which were limited on average by 4 725€. Lastly, by analyzing and comparing the performance of the portfolio with and without management using the average of RORAC we can conclude that even though the cumulative P&L had decreased with the risk management strategy, the limit of VaR, and therefore the decrease of EC offset it, causing a better risk-adjusted performance to the portfolio with a risk management strategy.

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Appendices

Appendix A: Descriptive Statistics

Appendix A.1. Descriptive Statistics of P&L and return of the portfolio, during the global period

	Mean	Median	Maximum	Minimum	Standard Deviation	Skewness	Kurtosis
P&L	3,533.25	4,802.83	808,339.79	-989,938.41	69,929.14	-0.35	18.05
Return	0.036%	0.048%	8.129%	-9.955%	0.703%	-0.35	18.05

Appendix A.1. Descriptive Statistics of the P&L and return of the portfolio, during the global test period, from 11 February 2013 a 27 January 2023

Appendix B: Backtesting Details

Appendix B.1. UC test results of the Systematic VaR against the mapped portfolio

	Model number	Exceedances	Exceedance Rate (%)	p-value (%)
Normal	1	51	1.96%	0.00%***
	2	51	1.96%	0.00%***
	3	49	1.88%	0.01%***
	4	45	1.73%	0.07%***
SGSt	5	48	1.85%	0.01%***
	6	35	1.35%	9.20%*
	7	47	1.81%	0.02%***
	8	30	1.15%	44.15%
Historical	9	42	1.62%	0.38%***
	10	43	1.65%	0.22%***
	11	35	1.35%	9.20%
	12	39	1.50%	1.70%**
	13	32	1.23%	25.37%
	14	35	1.35%	9.20%*
Quantile Regression	15	32	1.23%	25.37%
	16	33	1.27%	18.54%
	17	39	1.50%	1.70%**

Appendix B.1. UC test results for the global period, for the Systematic VaR against the mapped portfolio. The *** indicates rejection with 99% confidence level, ** show rejection with 95% confidence level and * rejection with 90% confidence level. The models with *** and ** are rejected because their p-value is lower than 5%

Appendix B.2. BCP test for all the Total VaR models

	Model	Worst p-value	Lag
Normal	1	4.60% **	2
	2	0.15% ***	2
	3	1.30% **	3
	4	28.89%	10
SGSt	5	59.22%	1
	6	7.66% *	5
	7	37.47%	2
	8	18.40%	2
Historical	9	2.78% **	2
	10	0.68% ***	2
	11	4.07% **	10
	12	5.88% *	10
	13	3.15% **	5
	14	15.72%	10
Quantile Regression	15	55.13%	1
	16	50.63%	2
	17	0.20% ***	1

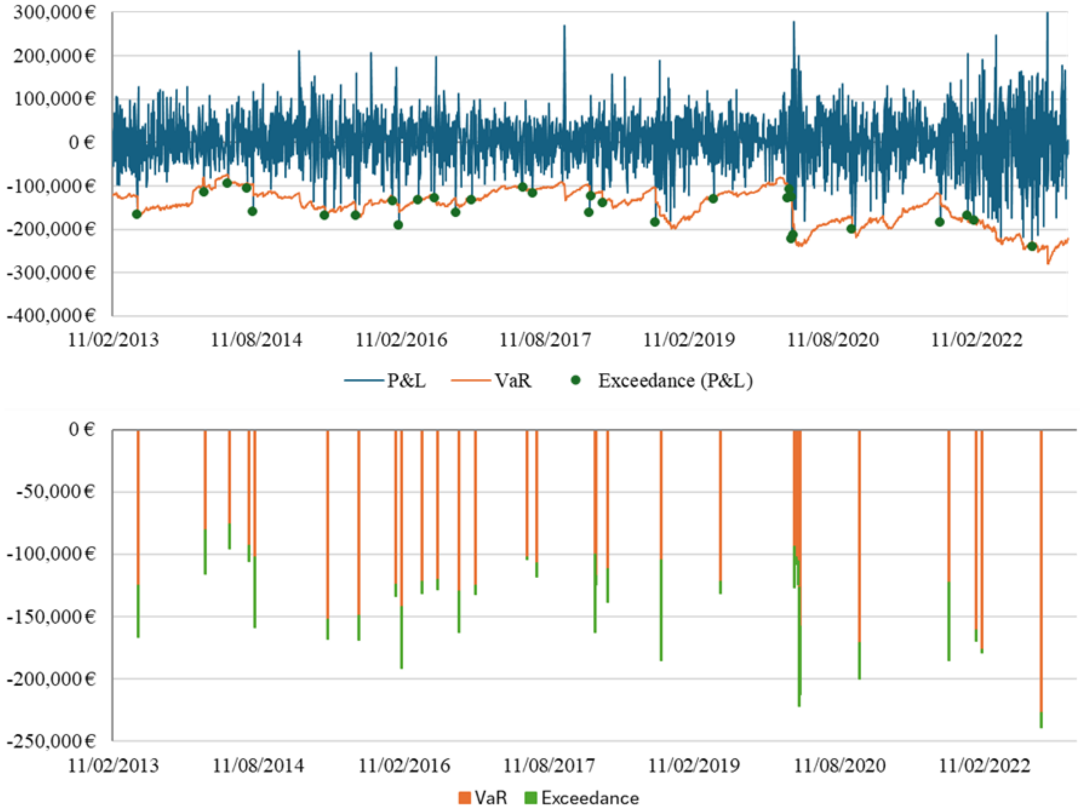
Appendix B.2. BCP test for all 17 Total VaR models, for the global period. The *** indicates rejection with 99% confidence level, ** show rejection with 95% confidence level and * rejection with 90% confidence level. The models with *** and ** are rejected because their p-value is lower than 5%

Appendix B.3.: Global period exceedances of model 5

Date	P&L	Total VaR	Exceedance	% of VaR
13/09/2022	-238,828 €	218,072 €	20,756 €	9.5%
28/09/2021	-184,718 €	120,927 €	63,792 €	52.8%
22/02/2021	-167,666 €	145,287 €	22,379 €	15.4%
06/01/2021	-155,867 €	133,569 €	22,298 €	16.7%
26/10/2020	-199,572 €	171,952 €	27,620 €	16.1%
12/03/2020	-221,653 €	167,727 €	53,926 €	32.2%
24/02/2020	-126,855 €	120,298 €	6,557 €	5.5%
02/12/2019	-87,695 €	87,275 €	420 €	0.5%
10/10/2018	-184,880 €	102,622 €	82,258 €	80.2%
22/03/2018	-138,075 €	127,951 €	10,124 €	7.9%
01/02/2018	-162,452 €	124,141 €	38,311 €	30.9%
20/10/2017	-90,588 €	84,232 €	6,356 €	7.5%
29/06/2017	-117,809 €	115,263 €	2,546 €	2.2%
10/11/2016	-131,746 €	123,543 €	8,204 €	6.6%
09/09/2016	-162,525 €	129,106 €	33,419 €	25.9%
20/06/2016	-127,771 €	124,830 €	2,941 €	2.4%
20/04/2016	-131,411 €	96,143 €	35,268 €	36.7%
05/02/2016	-191,044 €	172,287 €	18,757 €	10.9%
13/01/2016	-133,918 €	126,619 €	7,299 €	5.8%
03/06/2015	-149,022 €	140,689 €	8,332 €	5.9%
07/10/2014	-105,751 €	94,904 €	10,847 €	11.4%
31/07/2014	-158,879 €	106,995 €	51,884 €	48.5%
08/07/2014	-105,350 €	78,939 €	26,411 €	33.5%
25/04/2014	-95,092 €	75,988 €	19,104 €	25.1%
24/01/2014	-115,417 €	74,916 €	40,501 €	54.1%
15/05/2013	-166,071 €	133,914 €	32,157 €	24.0%

Appendix B.3. Global period exceedances of the model 5

Appendix B.4.: Global period of model 13



Appendix B.4. Global period performance of Historical Var model number 13. The green dots in the 1st panel represent the exceedances. In the 2nd panel we can visualize by how much the VaR is exceeded whenever an exceedance occurs.

Appendix C: VaR Management Details

Appendix C.1. Table of coupon payments of the bonds that compose the portfolio

Date	**Bond**	**Value of Coupon (EUR)**	**Stock invested**	**Quantity invested**
2023-02-03	US91282CJU62	17,648.02	Electronic Arts, Inc.	170
2023-02-16	DE000BU2Z023	22,000.00	ASML Holding N.V.	35
2023-05-05	US91282CJU62	17,558.61	CVS Health Corp	283
2023-05-17	US91282CJJ18 US912810QL52	23,795.94 15,634.05	Bank of New York Mellon Corporation	1,090
2023-08-04	US91282CJU62	17,758.80	Accenture plc	63
2023-11-06	US91282CJU62	18,302.41	Apple, Inc.	110
2023-11-16	US91282CJJ18 US912810QL52	23,785.85 15,627.42	Pfizer, Inc.	1,480
2024-01-04	DE0001135069	67,500.00	Pfizer, Inc.	2,612
2024-01-16	NL0009446418	48,750.00	Siemens Aktiengesellschaft	311

Appendix C.1. Table of coupon payments. In each day that we receive a coupon payment from a bond we have to reinvest it in a determined stock.

Appendix C.2. Exceedances that occur during 30 January 2023 and 2 February 2024

	Date	24/04/2023	20/07/2023	26/09/2023	14/12/2023
Without Hedging	P&L (€)	-209,049.02	-148,248.01	-133,963.54	-181,152.03
	VaR (€)	142,738.66	135,732.45	116,815.43	161,367.79
	Exceedance (€)	66,310.36	12,515.55	17,148.11	19,784.23
	% of VaR	46.46%	9.22%	14.68%	12.26%
With Hedging	P&L (€)	-209,049.02	-148,248.01	-133,963.54	-180,078.30
	VaR (€)	142,738.66	135,732.45	116,815.43	159,110.13
	Exceedance (€)	66,310.36	12,515.55	17,148.11	20,968.18
	% of VaR	46.46%	9.22%	14.68%	13.18%

Appendix C.2. Exceedances that occur during the one-year period, from 30 January 2023 until 2 February 2024. Value of which exceedance in the strategy without hedging and the strategy with hedging.