

Article

A New Algorithm to Mitigate Fragmentation and Crosstalk in Multi-Core Elastic Optical Networks

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Abstract: This paper proposes a core and spectrum allocation algorithm for elastic optical networks based on multi-core fibers. In this context, the fragmentation and crosstalk mitigation algorithm (FraCA) is proposed. FraCA implements mechanisms to reduce spectral fragmentation and inter-core crosstalk in the network, proving efficient when compared with six other algorithms reported in the literature. The numerical results show that when compared with the most competitive of the six algorithms, FraCA achieves a gain of request blocking probability of at least 16.87%, a gain of bandwidth blocking probability of at least 43.95%, and a mean increase in spectral utilization of at least 4.36%.

Keywords: elastic optical network; multicore fibers; routing; modulation; core; spectrum assignment; physical layer impairments

1. Introduction

In recent years, the growth of Internet users and the number of devices for each user, combined with new applications like video streaming, the Internet of things (IoT), or telemedicine, has driven the need to use transport networks efficiently [1]. In this context, the scientific community has designed the space-division multiplexed elastic optical networks (SDM-EONs) to compose the infrastructure of transport networks and meet the growing demand for user traffic [2,3]. In SDM-EONs based on multi-core fibers (MCFs), the fiber is divided into several cores, each with its own optical spectrum. Each optical spectrum is divided into spectral slices known as frequency slots. Thus, each client on the network uses a lightpath with one or more frequency slots. Therefore, SDM-EONs can optimize the use of the optical spectrum due to their elasticity and the scalability provided by multi-core fibers.

An essential issue in an MCF-based SDM-EON is allocating new lightpaths based on incoming requests for new connections. This question is related to a problem known as the routing, modulation, core, and spectrum assignment (RMCSA) [3,4]. The RMCSA problem consists of (i) defining a route between a source node (S) and a destination node (D) in the network; (ii) defining a modulation format to be used by the lightpath; (iii) defining which of the fiber cores will be used by the lightpath; and (iv) defining which set of frequency slots will be used by the lightpath.

To solve the RMCSA problem, two constraints must be obeyed [5]: (i) the spectral contiguity constraint, which requires that all frequency slots of a lightpath must be adjacent, and (ii) the spectral continuity constraint, which requires that the set of frequency slots used by a lightpath on a link be the same on all links along the route. Such constraints, added to the dynamic nature of the network (allocation and deallocation of lightpaths) and the heterogeneity of the bitrates requested by each client, make the optical spectrum fragmented. Spectral fragmentation is a problem that creates situations where RMCSA algorithms are not able to find contiguous and continuous lightpath solutions. When the



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RMCSA algorithm cannot find a solution for a given request, the request is blocked and not allocated in the network.

In addition to spectral fragmentation, another problem can cause a request to be blocked: the physical layer impairments (PLIs) [6,7]. The PLIs directly impact the quality of transmission (QoT) of the network's lightpaths. If the QoT of a lightpath is not in accordance with the network's service level agreement (SLA), this lightpath is not allocated. Among the various PLIs, inter-core crosstalk (XT) is multi-core networks' most prominent limiting factor [6]. Inter-core crosstalk occurs when two lightpaths in adjacent cores use the same frequency slot (spectral overlap) [6]. Two approaches can be used to deal with inter-core crosstalk: (i) in the XT-avoid approach, the RMCSA algorithm tries to avoid spectral overlap without dynamically computing the crosstalk level; and (ii) in the XT-aware approach, the RMCSA algorithm is aware of the real levels of crosstalk and uses this information for decision making. Although efficient, XT-aware approaches introduce a high computational cost for RMCSA algorithms. In this context, this paper aims to propose a new algorithm to solve the core and spectrum allocation problem in SDM-EONs.

1.1. Related Works

The spectral allocation problem began to be investigated even before the study of MCF-based networks, when the focus was on single-core fibers. In this context, there are classic policies for spectral allocation, such as first fit, last fit, and random fit [5]. In the first fit policy [8,9], the frequency slots are indexed in a list of available slots. This policy always tries to choose the lowest index slot from the list of available slots and then allocate this slot to serve the connection request. On the other hand, the last fit policy [10] operates opposite to the first fit, choosing the slot with the highest index from the list of available slots. Therefore, occupied slots are concentrated at the end of the spectrum, while the initial portion of the spectrum remains free to serve new requests. In the random fit policy [8], when a connection request arrives on the network, the random fit randomly selects a slot from the list of available slots and allocates it to the lightpath used to fulfill the connection request.

In addition to classical spectrum allocation policies, many works in the literature have investigated more efficient ways to improve resource allocation in SDM-EON. Some of these works propose XT-aware strategies to deal with the problem of inter-core crosstalk [11–22]. On the other hand, the XT-avoid strategies appear as a low computational cost option [13,23–26].

Within the scope of the XT-avoid algorithms, Fujii et al. proposed a core allocation strategy, named core prioritization (CP) [23]. This policy seeks to avoid crosstalk between cores by balancing the use of each fiber's core. Already in [24], the authors propose an intra-area first fit (IAFF) spectrum assignment scheme. The proposal aims to classify requests according to the number of frequency slots requested in several prioritized areas on each core. Then, the scheme groups lightpaths with the same number of frequency slots in the same prioritized area, reducing the spectrum fragmentation.

Lacerda et al. presented the core and spectrum balancing algorithm for SDM networks (ABNE) [25]. The ABNE algorithm presents a balancing strategy in the use of cores to avoid spectral overlap and inter-core crosstalk. For this, the algorithm classifies the fiber cores into priority groups, where each group prioritizes a different spectral region, varying between the first fit and last fit policies.

Liu et al. presented a strategy XT-avoid and another XT-aware [13]. The XT-avoid strategy, named the crosstalk avoidance strategy (CAS), partitions the spectrum into groups where adjacent cores have different prioritization areas. The XT-aware strategy, named the inter-core XT-aware algorithm (ICXTAA), checks the crosstalk for the frequency slots block of the candidate lightpath and the lightpaths already active on the network. For this, the ICXTAA labels candidate blocks of slots with the crosstalk level of the most affected slot (by crosstalk) within the block (or the slot of the adjacent core most affected by crosstalk). Then, the ICXTAA selects the block of slots that is contiguous, continuous, located in the

priority region defined by the CAS strategy, and with a crosstalk level (based on ICXTAA labels) lower than the threshold of crosstalk established for the network.

Araujo et al. proposed the inter-core spectral distancing algorithm (ADEIN) [26]. In order to avoid crosstalk, the ADEIN balances the use of resources through the use of a scoring system and division of priority groups. This scoring system consists of creating a table updated each time a new lightpath is allocated, where this table serves as a reference for choosing future candidate lightpaths.

Table 1 presents a summary of works that investigate resource allocation in SDM-EON. All related works deal with inter-core crosstalk either by the XT-avoid or XT-aware approaches. Related to the spectrum fragmentation issue, some works implement mechanisms to reduce fragmentation in the resource allocation process [12,17–21,23,24]. Some of these works were performed through a static traffic matrix, while others generated arrivals and ends of new requests dynamically. The number of cores per fiber varies between studies. However, in general, the seven-core fiber is the most investigated in the literature. In this sense, the scope of this paper encompasses seven-core fibers, dynamic traffic, and XT-avoid strategies. Therefore, the algorithms compared to the proposed algorithm on performance evaluation (Section 5) are also XT-avoid, and are designed for dynamic traffic.

Table 1. Summary of works that address the RMCSA problem.

References	Is Crosstalk Considered?	Is Fragmentation Considered?	Crosstalk Approach	Traffic Scenario	Number of Cores
[23]	yes	yes	XT-avoid	dynamic	7, 12, and 19
[24]	yes	yes	XT-avoid	dynamic	7
[11]	yes	no	XT-aware	dynamic	3, 7, and 12
[12]	yes	yes	XT-aware	dynamic	7
[25]	yes	no	XT-avoid	dynamic	7
[13]	yes	no	XT-avoid	dynamic	7 and 19
[14]	yes	no	XT-aware	static	3 and 7
[26]	yes	no	XT-avoid	dynamic	7
[15]	yes	no	XT-aware	static	3, 7, and 19
[16]	yes	no	XT-aware	static	6 and 7
[17]	yes	yes	XT-aware	dynamic	3 and 7
[18]	yes	yes	XT-aware	dynamic	7
[19]	yes	yes	XT-aware	static	7
[20]	yes	yes	XT-aware	dynamic	7 and 12
[21]	yes	yes	XT-aware	dynamic	7
[22]	yes	no	XT-aware	dynamic	7

Many works shown in Table 1 do not jointly deal with fragmentation and crosstalk. On the other hand, works that jointly deal with crosstalk and fragmentation commonly use XT-aware techniques, which causes a higher computational cost. This paper differs from the related works by implementing an XT-avoid algorithm that not only mitigates the crosstalk and fragmentation efficiently, but also ensures a low blocking probability with a low computational cost. Numerical results demonstrate the superior performance of the proposed algorithm over the six other XT-avoid algorithms.

1.2. Contributions and Paper Organization

This paper proposes a new algorithm to solve the core and spectrum allocation problem in SDM-EONs. The main contributions of this work are the following:

1. Development of the fragmentation and crosstalk mitigation algorithm (FraCA). FraCA is an XT-avoid algorithm that generates a low computational cost;
2. Evaluation of the performance of the proposed algorithm. The performance evaluation compares the proposed algorithm with six algorithms proposed by other authors;
3. Analysis of the impact of PLIs on network performance in different network and MCF scenarios assuming the use of a reliable QoT model.

This paper differs from our previous work [22] in that it implements a mechanism to reduce spectrum fragmentation and reduces the computational cost (inherent feature of an XT-avoid approach). This paper is an extended version of [27], including a comprehensive performance evaluation with a new topology, metrics, and complexity analysis.

The remainder of this paper is organized as follows: Section 2 presents the basic concepts of elastic optical networks based on multi-core fibers. System modeling in terms of physical layer impairments is discussed in Section 3. Section 4 presents the proposed algorithm. The performance evaluation of the proposed algorithm is presented in Section 5. Finally, the conclusions and future works are presented in Section 6. All acronyms used in this work are described at the end of the paper.

2. Elastic Optical Networks Based on Multi-Core Fiber

In this paper, the network topology is defined by a graph $G = \{N, E\}$, where N represents the node set of the network, and E represents the set of bidirectional links (each direction is associated with a separate fiber) in the network. In this scenario, requests arrive to form an end-to-end communication. Each request follows the format $R = (S, D, R_{b,i})$, which considers the source node S , destination node D , and the bitrate $R_{b,i}$ requested by R . We consider a transparent network, in which all data are transmitted from the source node to the destination node without optical–electro–optical conversion. Each network link consists of one or more spans, with an erbium-doped fiber amplifier (EDFA) between the two spans. During the signal propagation, several PLIs impact the QoT of lightpaths. As previously mentioned, the inter-core crosstalk is the multi-core networks’ most prominent limiting factor. The crosstalk behavior is illustrated in Figure 1.

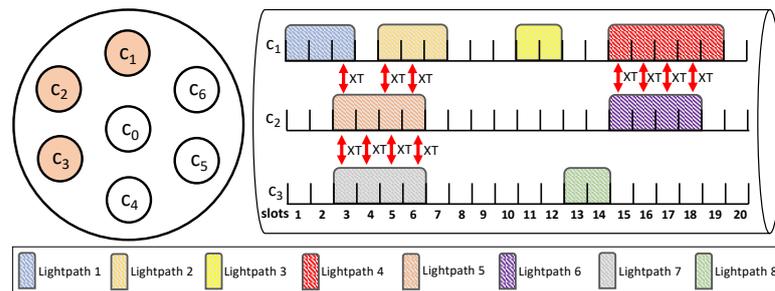


Figure 1. Spectral prioritization for the FraCA algorithm.

According to Figure 1, the inter-core crosstalk occurs between a lightpath that uses the same frequency slots used by another lightpath in a neighboring core [6]. In this example, lightpath 5 (allocated on core 2) is seriously affected by crosstalk, as it suffers (and causes) crosstalk concerning neighboring lightpaths 1 and 2 on core 1 and neighboring lightpath 7 on core 3. In the same example, the spectrum allocated to lightpath 3 does not overlap with the spectrum allocated to any neighboring core, so that it does not suffer from inter-core crosstalk. Figure 1 also highlights that even if two lightpaths use the same frequency slot, inter-core crosstalk is not considered if the cores used by these two lightpaths are not adjacent. An example of this case can be observed between lightpaths 1 and 7, where both use slot 3 on non-adjacent cores. This occurs because, despite existing, the crosstalk values between non-adjacent cores are negligible [6]. Therefore, the resource allocation algorithms must be able to deal with the inter-core crosstalk to guarantee the QoT of lightpaths.

In addition to crosstalk, another fundamental issue in SDM-EONS is the resource allocation. Figure 2 illustrates the arrival of a request R_1 , which needs to connect node n_1 to node n_4 , with a bitrate of 80 Gbps. In the first step, the RMCSA algorithm chooses the route $(e_1-e_2-e_3)$ among the three smallest possible routes. Next, the RMCSA algorithm selects the 4-QAM modulation format to be used by the lightpath to be allocated. Then, the bandwidth necessary to meet the requested bitrate is calculated, and, in this example, two frequency slots will be needed to meet the request demand. Then, the RMCSA algorithm chooses

core c_1 from the two available cores. In the spectral allocation step, the RMCSA algorithm selects slots 3 and 4 because they are free, contiguous, and continuous along the route. In Figure 2, if a request needs three frequency slots, it will be blocked because three slots are not free, contiguous, and continues in this example (even with more than 50% of free slots). Therefore, spectrum fragmentation impacts the spectrum allocation process, increasing the blocking probability in the network.

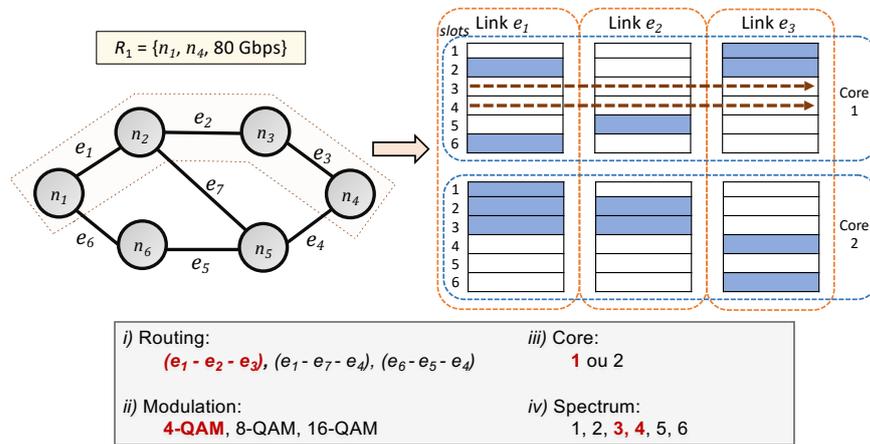


Figure 2. An example of solving the RMCSA problem based on the arrival of a request.

3. Physical Layer Impairments

Due to the PLIs, the QoT of the lightpath degrades as the signal propagates along the fiber and in the network equipment. In this paper, three limiting factors are considered by the physical layer: (i) the amplified spontaneous emission (ASE); (ii) the nonlinear interference (NLI); and (iii) the inter-core crosstalk.

Unlike the crosstalk that occurs between the cores, the ASE noise and NLIs occur within the same core (intra-core). The ASE noise occurs in the optical amplification process, which is generally performed by the EDFA [28,29]. The NLIs manifest due to propagation along the optical fiber, causing a lightpath to cause interference in itself (self-phase modulation (SPM)) and also in other lightpaths, through cross-phase modulation (XPM) and four-wave mixing (FWM). In this paper, the model presented in [30], which is based on an enhanced Gaussian-noise (EGN) model [31], is used to calculate the power spectral density (PSD) of the NLIs (SPM, XPM and FWM).

A method to measure network QoT levels is through the optical signal-to-noise ratio (OSNR). The OSNR establishes a relationship between the PSD of the lightpath and the PSD of the ASE noise and the NLI that affects it. The OSNR is used as one of the QoT metrics in this paper, and is given by [30,32]

$$OSNR_m = \frac{G_m}{G_m^{(ASE)} + G_m^{(NLI)}}, \quad (1)$$

where G_m is the PSD of the lightpath m . The $G_m^{(ASE)}$ is the PSD of the ASE noise from the accumulation of noise generated in all EDFA used in the lightpath m . $G_m^{(ASE)}$ is given by

$$G_m^{(ASE)} = \sum_{a=1}^{|A_m|} G_{a,m}^{(ASE)}, \quad (2)$$

where $|A_m|$ is the number of EDFAs traversed by the lightpath m , and $G_{a,m}^{(ASE)}$ is the PSD of the ASE noise generated by the amplifier a in the lightpath m , given by [33]

$$G_{a,m}^{(ASE)} = 2n_{sp}h\nu(g_a - 1), \tag{3}$$

where n_{sp} is the spontaneous emission factor, which is related to the amplifier noise figure (N_F) by $N_F = 2n_{sp}$. The h parameter is the Planck's constant, ν is the optical carrier frequency, and g_a is the gain of amplifier a . Regarding the NLIs in expression (1), $G_m^{(NLI)}$ is the total PSD of the nonlinear effects that affect the lightpath m . $G_m^{(NLI)}$ is given by the EGN model, which considers the modulation format of the lightpath m and reduces the overestimation of the Gaussian noise (GN) model [32]. Expressions of $G_m^{(NLI)}$ can be found in [30,32].

Another important issue in an SDM-EON, considered as the second QoT criterion in this paper, is the inter-core crosstalk [6]. Using the model reported in [12], the normalized mean crosstalk power in the lightpath m is given by

$$XT_{\mu,m}^{(tot)} = \sum_{e=1}^{|E_m|} XT_{\mu,m}^{(e)}, \tag{4}$$

where $XT_{\mu,m}^{(e)}$ is the mean crosstalk power normalized generated in the link e of the lightpath m , given by [12]

$$XT_{\mu,m}^{(e)} = \frac{P_{XTm}^{(e)}}{P_m}, \tag{5}$$

where P_m is the signal power of the lightpath m . $P_{XTm}^{(e)}$ is the (non-normalized) mean XT power of link e , given by [12]

$$P_{XTm}^{(e)} = \sum_{n=1}^{|N_m^{(e)}|} (I_{SOm,n} \cdot P_n \cdot h_e \cdot L_e), \tag{6}$$

where $|N_m^{(e)}|$ is the number of lightpaths allocated in cores adjacent to the core of the lightpath m , which use frequency slots that overlap totally or partially the frequency slots of lightpath m in the link e . h_e is the power-coupling coefficient (assumed the same for all cores). L_e is the length of the link e . P_n is the signal power of the adjacent lightpath n (n -th lightpath of $N_m^{(e)}$). $I_{SOm,n}$ is the frequency slot overlapping index between the lightpaths m and n , given by [12]

$$I_{SOm,n} = \frac{S_{SOm,n}}{|S_n|}, \tag{7}$$

where $|S_n|$ is the number of frequency slots (carrying signal power) of the lightpath n , given by $|S_n| = \lceil B_n / B_{FS} \rceil$, and $S_{SOm,n}$ is the number of overlapping frequency slots (carrying signal power) between lightpaths m and n . In other words, $S_{SOm,n}$ is the quantity of frequency slots of the lightpath m that have frequency slots with the same index in the lightpath n (disregarding the guard band).

In this paper, QoT is evaluated through the impact of intra- and inter-core effects (OSNR and crosstalk). For each lightpath returned by an RMCSA algorithm, it must be checked whether its OSNR and crosstalk level are in accordance with the OSNR and crosstalk thresholds defined by the network SLA. Otherwise, the request is blocked. QoT levels are also checked for lightpaths already active in the network and the impact suffered by them with the arrival of a new lightpath. If such an impact affects any of the already active optical paths in such a way that its QoT fails to comply with the established threshold,

the candidate lightpath cannot be allocated (request blocking). The OSNR and crosstalk thresholds adopted in this paper are listed in Section 5.

4. Proposed Algorithm

This paper proposes the algorithm for fragmentation and crosstalk mitigation (FraCA) in multi-core SDM-EON networks. FraCA is proposed for core and spectrum allocation, uses an XT-avoid strategy, and is based on two key ideas: reducing inter-core crosstalk and spectral fragmentation.

To reduce the occurrence of crosstalk, the FraCA defines that neighboring cores must prioritize different areas of the spectrum. In this sense, cores 1, 3, and 5 (not adjacent to each other) adopt the first fit spectral allocation policy. Cores 2, 4, and 6 adopt the last fit spectral allocation policy. Since core 0 (central core) generates and suffers crosstalk from all cores, this core prioritizes the central region of the spectrum to avoid spectral overlap with the other cores. At the end of this section, Figure 3 illustrates, among other factors, the division of regions prioritized by each core. The efficiency of this balancing strategy has already been investigated in our previous works [22,25].

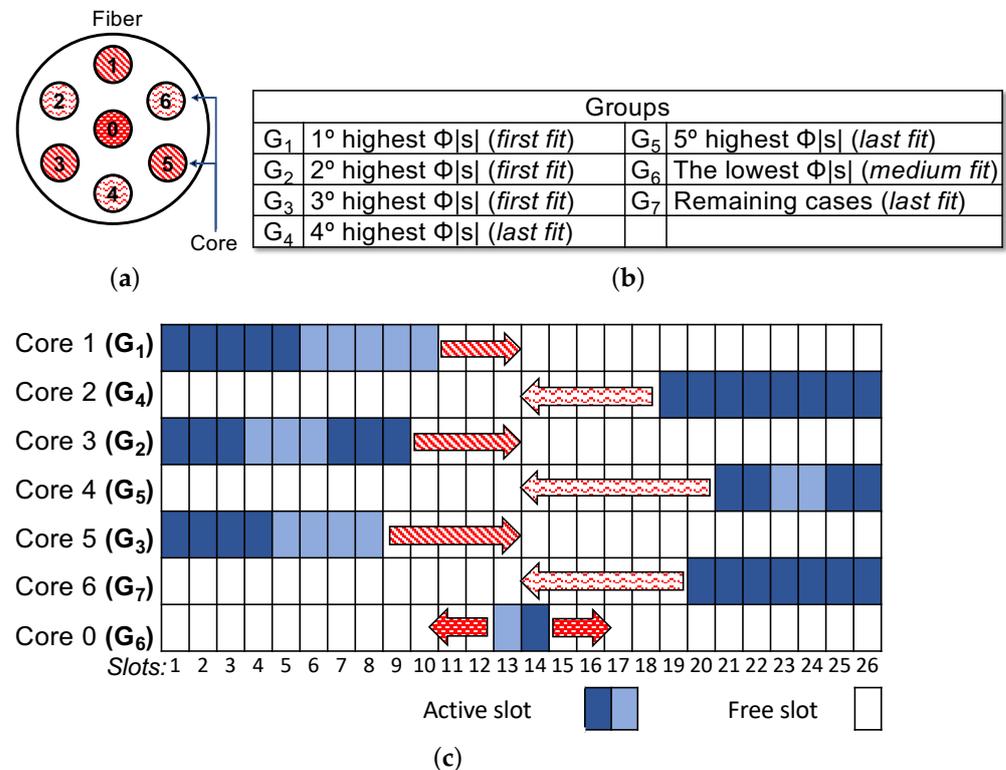


Figure 3. Spectral prioritization for the FraCA algorithm. (a) Fiber architecture. (b) Groups division. (c) Example of spectrum allocation considering the group division.

To reduce spectral fragmentation, FraCA classifies specific fiber cores to only receive lightpaths requiring the same frequency slots. For example, if core c_1 of link e_1 is intended to receive only lightpaths that require three frequency slots, the fragmentation problem will be mitigated for the spectrum of core c_1 of link e_1 . Furthermore, the optical spectrum will be more “organized”, as the first fit and last fit policies will be more effective. This occurs because when a lightpath is closed, the block of free space forming from this deallocation will be the exact size required by the next lightpath to be allocated to that core. A similar idea was presented for the IAFF [24] algorithm. However, unlike IAFF, FraCA classifies the cores more efficiently, in addition to implementing crosstalk mitigation strategies. A comparison between FraCA and IAFF is presented in Section 5.

In classifying the cores to mitigate fragmentation, FraCA considers how many times a certain number of frequency slots is requested and how quickly this type of request increases core occupancy, which is measured through a weight $\Phi_{|s|}$. For this, a vector $G = \{G_1, G_2, G_3, G_4, G_5, G_6, G_7\}$ defines seven groups (one for each core), with each group (except for G_7) receiving only requests with the same number of frequency slots. The groups from G_1 to G_5 store the requests in descending order of $\Phi_{|s|}$. The group G_6 stores the request with the lowest $\Phi_{|s|}$. Finally, the G_7 group stores all requests not classified in the previous groups. The division of groups occurs in decreasing order of weight $\Phi_{|s|}$, given by

$$\Phi_{|s|} = T_{|s|} \cdot |s|, \tag{8}$$

where $T_{|s|}$ is the occurrence rate of requests that require $|s|$ frequency slots and $|s|$ is the number of frequency slots required. The value of $T_{|s|}$ is defined based on a series of previous simulations, in which it is observed how many times a certain amount of frequency slots was requested. For example, if lightpaths with three frequency slots comprised 40% of all lightpaths in the network, the value of $\Phi_{|3|} = 1.2$, since $0.4 \times 3 = 1.2$. If lightpaths that have two frequency slots comprise 15% of all lightpaths in the network, the value of $\Phi_{|2|} = 0.3$, since $0.15 \times 2 = 0.3$. Therefore, in other words, the value $\Phi_{|s|}$ indicates how fast the spectral utilization increases for a given group of requests (according to network histories).

Table 2 presents the distribution of FraCA algorithm groups. $T_{|s|}$ values were obtained from a series of simulations carried out with the parameters adopted in this paper, which are presented in Section 5. In addition to the quantities of frequency slots presented in Table 2, lightpaths with 10 and 7 frequency slots were also requested, presenting occurrence rates of 0.034 and 0.036, respectively. In this case, the lightpaths with $|s| = 10$ and $|s| = 7$ will be assigned to the G_7 group. This paper considers a seven-core fiber. However, FraCA's key idea can be generalized to fiber with another core count by assuming an appropriate number of elements of the G group. Algorithm 1 explains how FraCA works, based on the information in Table 2. Then, Figure 3 provides an example of FraCA's operation.

Table 2. Definition of groups based on weight $\Phi_{|s|}$.

Group	Number of Slots $ s $	Occurrence Rate $T_{ s }$	Weight $\Phi_{ s }$
G_1	5	0.189	0.944
G_2	3	0.214	0.643
G_3	4	0.158	0.631
G_4	8	0.069	0.548
G_5	2	0.256	0.512
G_6	1	0.044	0.044

Figure 3 presents an example of how FraCA works and how the groups are defined (based on Table 2). Requests with 5, 3, and 4 frequency slots are those with the highest $\Phi_{|s|}$, respectively (Table 2). Therefore, requests with five frequency slots will make up G_1 , and be allocated to core c_1 . Requests with three frequency slots will make up G_2 , and be allocated to core c_3 . Requests with four frequency slots will make up G_3 , and be allocated on core c_5 . The allocation of groups G_1 , G_2 , and G_3 on alternate cores (1, 3, and 5) prevents the most requested groups from being in adjacent cores, thus avoiding spectral overlap and crosstalk. The G_6 group comprises requests with the smallest $\Phi_{|s|}$ of all, and is always allocated to the central core (c_0). The allocation of requests with lower $\Phi_{|s|}$ on core c_0 (central core) is because this core is adjacent to all the others; consequently, it has a greater potential to cause and suffer crosstalk. All requests not classified in cores from G_1 to G_6 will make up G_7 and be allocated to core c_6 .

The FraCA time complexity mainly comprises the search for the candidate lightpath (free, contiguous, and continuous) in a core. The core's assignment only considers the number of slots of the request, and its time complexity is constant. Therefore, the FraCA's

time complexity is $O(|S| \cdot |E|)$, where $|S|$ is the number of frequency slots in core c and $|E|$ is the set of links (that comprises the maximum number of hops in a route).

Algorithm 1 FraCA’s core and spectrum allocation.

Input: request R

- 1: $|s| \leftarrow$ calculate the number of slots to meet R
- 2: **if** ($|s| == 5$) **then**
- 3: Allocate the lightpath on core c_1 by first fit policy
- 4: **end if**
- 5: **if** ($|s| == 8$) **then**
- 6: Allocate the lightpath on core c_2 by last fit policy
- 7: **end if**
- 8: **if** ($|s| == 3$) **then**
- 9: Allocate the lightpath on core c_3 by first fit policy
- 10: **end if**
- 11: **if** ($|s| == 2$) **then**
- 12: Allocate the lightpath on core c_4 by last fit policy
- 13: **end if**
- 14: **if** ($|s| == 4$) **then**
- 15: Allocate the lightpath on core c_5 by first fit policy
- 16: **end if**
- 17: **if** ($|s| == 1$) **then**
- 18: Allocate the lightpath on core c_0 by medium fit policy
- 19: **end if**
- 20: **if** (otherwise) **then**
- 21: Allocate the lightpath on core c_6 by last fit policy
- 22: **end if**

5. Performance Results and Discussion

The computational simulation technique is used to evaluate the performance of this paper’s algorithm. The simulations are performed by the Slice network simulator (SNetS) [34]. A total of 100,000 requests are generated in each simulation. Request generation is a Poisson process with an average rate of λ , and the average lightpath retention time is exponentially distributed with an average of $1/\mu$. The traffic load is evenly distributed among all source and destination node pairs. The 100, 200, 300, and 400 Gbps requests are generated, following the arrival ratio of 4, 3, 2, and 1, respectively. That is, for each request requesting a bitrate of 400 Gbps, four requests arise requesting a bitrate of 100 Gbps. The load, in Erlangs, can be defined by $\rho = \lambda/\mu$. Five simulations (replications) are carried out for each evaluation scenario with different random variable generation seeds. All results presented have a confidence level of 95%. This paper considers five modulation formats: 4-QAM, 8-QAM, 16-QAM, 32-QAM, and 64-QAM. Table 3 presents the OSNR and crosstalk thresholds for each modulation format [22].

Table 3. OSNR and XT thresholds [22].

Threshold	4-QAM	8-QAM	16-QAM	32-QAM	64-QAM
OSNR (dB)	8.95	13.15	15.49	18.51	21.28
XT (dB)	−19.03	−23.23	−25.57	−28.59	−31.36

In the simulated scenarios, each fiber has seven cores. Each core is divided into 320 frequency slots, where each frequency slot has 12.5 GHz. Between each lightpath, there is a guard band with a bandwidth of one frequency slot. Amplifier gains are adjusted to compensate for device and fiber losses. The topologies used are Cost239, NSFNet, and EURO28, as shown in Figure 4. Other parameters used in the simulations are listed in Table 4 [12,30,35,36].

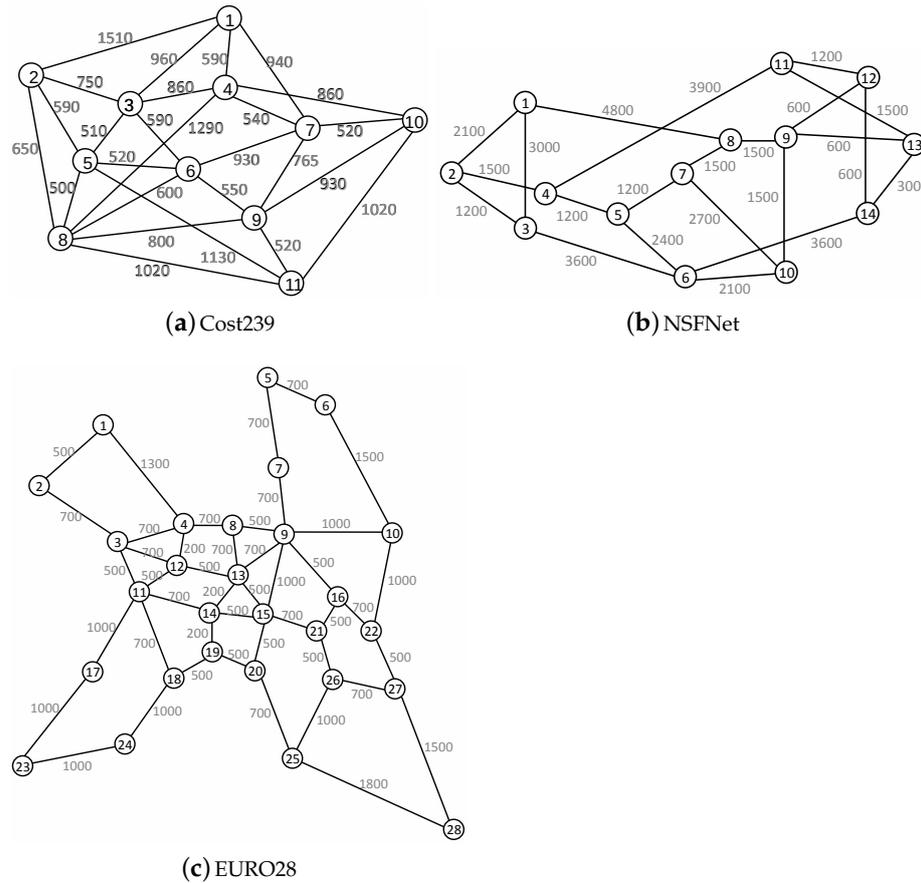


Figure 4. Topologies with the length of links in km.

Table 4. Values of physical layer parameters [12,30,35,36].

Definition	Value
Amplifier noise figure (N_F)	5 dB
FEC overhead (F)	25 %
Fiber field loss coefficient (α)	0.2 dB/km
Fiber nonlinearity coefficient (γ)	$1.3 \text{ (W}\cdot\text{km)}^{-1}$
Group-velocity dispersion (β_2)	$-20 \text{ ps}^2/\text{km}$
Node loss (W_n)	15 dB
Number of FSs per core	320
Optical carrier frequency (ν)	193 THz
Power-coupling coefficient (h_e)	$6.4 \times 10^{-9} \text{ m}^{-1}$
Span length (L_s)	80 km

FraCA does not use XT values information to select new lightpaths, so comparing it to other XT-aware algorithms is unfair. Therefore, the FraCA algorithm is compared with six other XT-avoid algorithms, namely: (i) the CP [23] algorithm for core assignment in conjunction with the random fit algorithm for the spectrum assignment, called CPRF; (ii) the CP [23] algorithm for core assignment in conjunction with the IAFF [24] algorithm for spectrum assignment, called CPIAFF; (iii) the CP [23] algorithm for core assignment in conjunction with the CAS [13] algorithm for spectrum assignment, called CPCAS; (iv) the ABNE algorithm [25]; (v) the ADEIN algorithm [26]; and (vi) the ICXTAA algorithm [13]. The route selection in all algorithms is performed by Dijkstra’s algorithm [37]. From the route definition, the most spectrally efficient modulation format is selected considering the OSNR thresholds.

This paper considers three metrics to compare the algorithms performance: request blocking probability (RBP), bandwidth blocking probability (BBP), and spectrum utilization

(SU). The RBP is the ratio of the number of blocked lightpaths to the total number of requested lightpaths. The BBP is defined as the volume of rejected traffic divided by the volume of total traffic offered to the network [11], and is given by

$$BBP = \frac{\sum_n \text{(blocked lightpaths)} R_{b,i,n} \cdot T_n}{\sum_n \text{(all requested lightpaths)} R_{b,i,n} \cdot T_n} \quad (9)$$

where $R_{b,i,n}$ is the information bitrate of the n -th requested lightpath and T_n is the time that the n -th requested lightpath would be active. The third metric is the SU, which is defined as the ratio of the number of FS allocated to the total number of the network's FS. Each metric is computed considering a replication, then the average of the five replications is calculated and shown in the following results.

Figure 5 presents the results in terms of request blocking probability. The FraCA algorithm obtained a lower RBP compared to all other algorithms in the three topologies evaluated. This result indicates that FraCA can handle more requests under the same traffic conditions than other algorithms. Such a performance occurs because FraCA efficiently implements mechanisms that reduce the occurrence of inter-core crosstalk and spectral fragmentation.

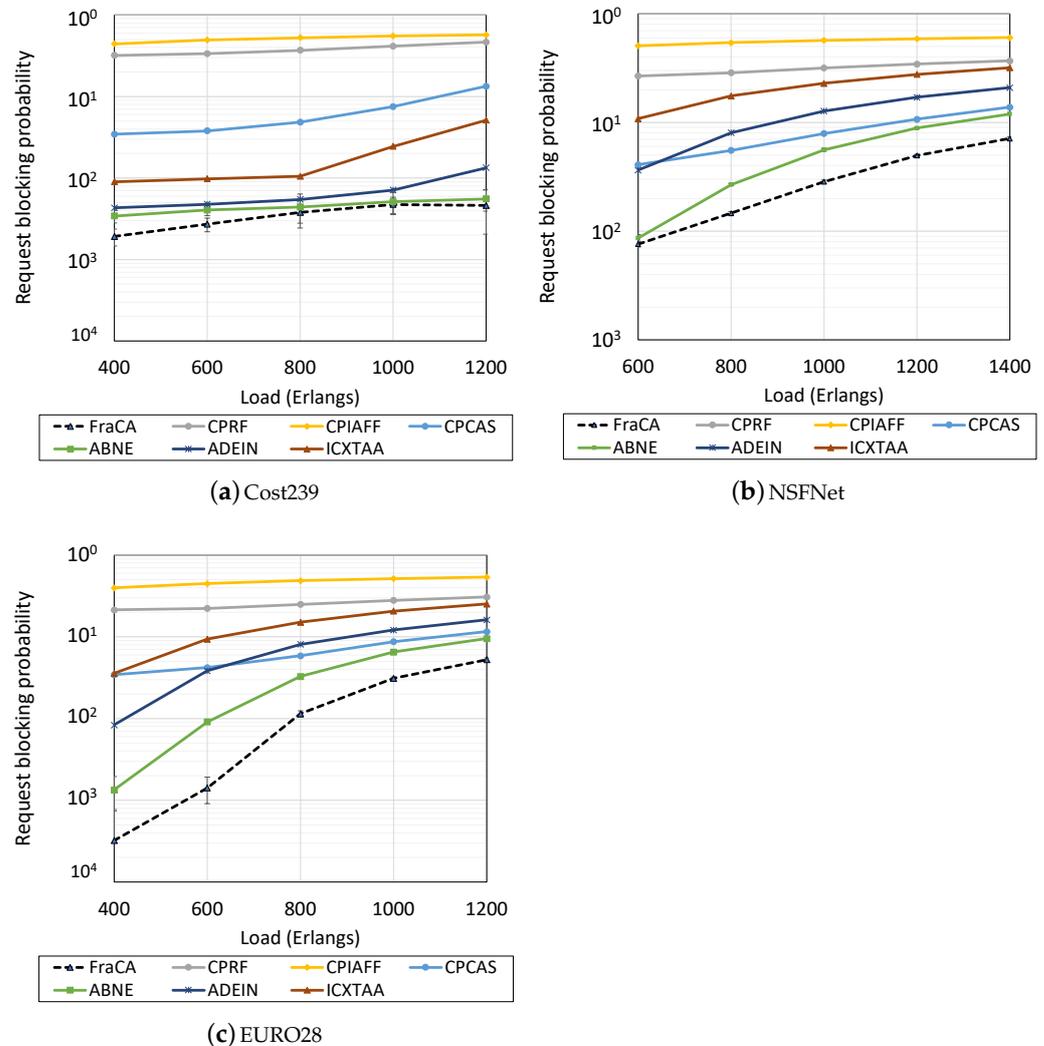


Figure 5. Request blocking probability.

Figure 6 presents the results regarding bandwidth blocking probability. Like RBP, the BBP metric indicates a better FraCA performance than other algorithms in the three

topologies evaluated. This result shows that FraCA ensures that less data are blocked on the network. In other words, the results indicate that FraCA ensures that more traffic can be carried on the network. To numerically evaluate the performance of the proposed algorithm, Table 5 presents the FraCA gain, in terms of RBP and BBP, concerning the other algorithms.

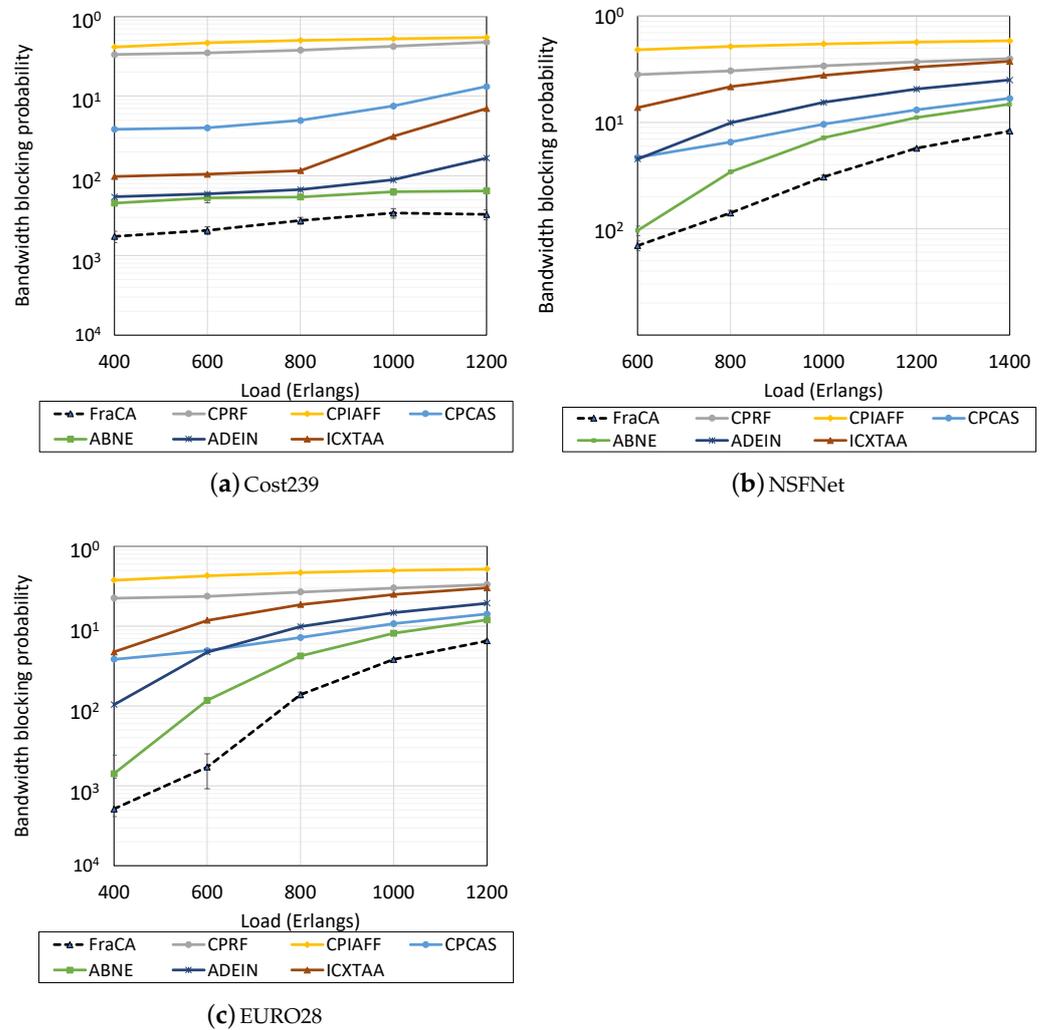


Figure 6. Bandwidth blocking probability.

The FraCA gain is calculated by $GAIN = (R_n - R_m) / R_n$, where R_m is the result (RBP or BBP) of FraCA and R_n is the result (RBP or BBP) of another evaluated algorithm (CPRF, CPIAFF, CPCAS, ABNE, ADEIN, or ICXTAA). According to Table 5, it is observed that FraCA obtained a minimum RBP gain of 16.87% in the Cost239 topology, 40.45% in the NSFNet topology, and 44.77% in the EURO28 topology, in all cases compared with the ABNE algorithm. In the best RBP case, FraCA achieved a gain of 99.19% over the CPIAFF algorithm in the Cost239 topology, 88.21% over the CPIAFF algorithm in the NSFNet topology, and 90.20% over the CPIAFF algorithm in the EURO28 topology. In relation to BBP, FraCA obtained a minimum gain of 49.47% in the Cost239 topology, 43.95% in the NSFNet topology, and 45.30% in the EURO28 topology. In the best BBP case, FraCA achieved a gain of 99.40% over the CPIAFF algorithm in the Cost239 topology, 85.85% over the CPIAFF algorithm in the NSFNet topology, and 87.38% over the CPIAFF algorithm in the EURO28 topology. To understand the reason for the FraCA gain, it is necessary to investigate the causes of blockages for each algorithm in each topology.

Table 5. FraCA gain in relation to other algorithms at the highest load point of the three topologies evaluated.

Algorithm	RBP Cost239	RBP NSFNet	RBP EURO28	BBP Cost239	BBP NSFNet	BBP EURO28
CPRF	99.01%	80.69%	82.84%	99.31%	79.16%	80.33%
CPIAFF	99.19%	88.21%	90.20%	99.40%	85.85%	87.38%
CPCAS	96.55%	48.50%	54.57%	97.52%	50.71%	53.95%
ABNE	16.87%	40.45%	44.77%	49.47%	43.95%	45.30%
ADEIN	65.47%	65.97%	67.38%	80.41%	66.90%	66.27%
ICXTAA	91.01%	77.65%	79.16%	95.32%	77.91%	78.28%

Figure 7 presents the percentage of each cause of RBP observed at the highest load of each topology. In this figure, there are two blocking types: (i) by “XT”, when the request is blocked because the lightpath does not have adequate crosstalk levels; and (ii) by “other”, when other issues, such as absence of resources or inadequate OSNR levels, block the request. Figure 7 shows that the FraCA algorithm reduces the XT blocking concerning other algorithms, mainly in Cost239 and EURO28 topologies. In this way, the FraCA algorithm reduces the XT impact, decreasing the blocking probability.

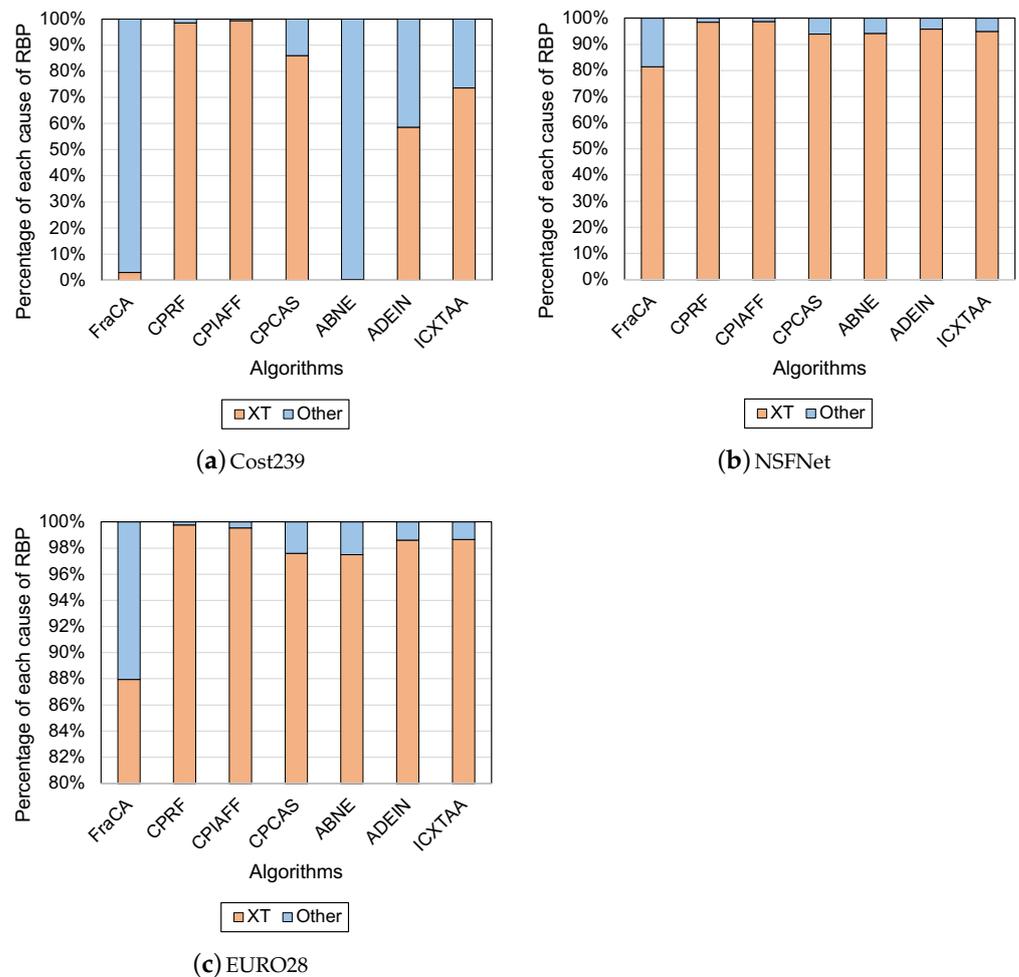


Figure 7. Percentage of each cause of request blocking probability.

Figure 8 presents information about the spectrum utilization of the algorithms. This paper chooses the most spectrally efficient modulation format for each of the evaluated algorithms. Therefore, in this way, a high SU means better management of network

resources [12,38]. In this context, FraCA obtained the highest spectrum utilization of the evaluated scenarios. FraCA obtained 0.75% more spectrum utilization than the second-best algorithm (ABNE) in the Cost239 topology (considering the average of the five load points). In the NSFNet topology, FraCA obtained 7.04% more spectrum utilization than the second-best algorithm (ABNE). In the EURO28 topology, FraCA obtained 5.30% more spectrum utilization than the second-best algorithm (ABNE). Thus, the average increment of spectrum utilization over the second-best algorithm (considering all three topologies) is 4.36%. In relation to CPIAFF (lower spectrum utilization), FraCA obtained an average SU increase of 115.49% in the Cost239 topology, 109.83% in the NSFNet topology, and 86.03% in the EURO28 topology. Considering the use of the most spectrally efficient modulation format possible by all algorithms and the same load on the network, these results show that FraCA is capable of dealing more efficiently with the optical spectrum, reducing idle resources, and serving more customers (or customers with a higher bitrate demand).

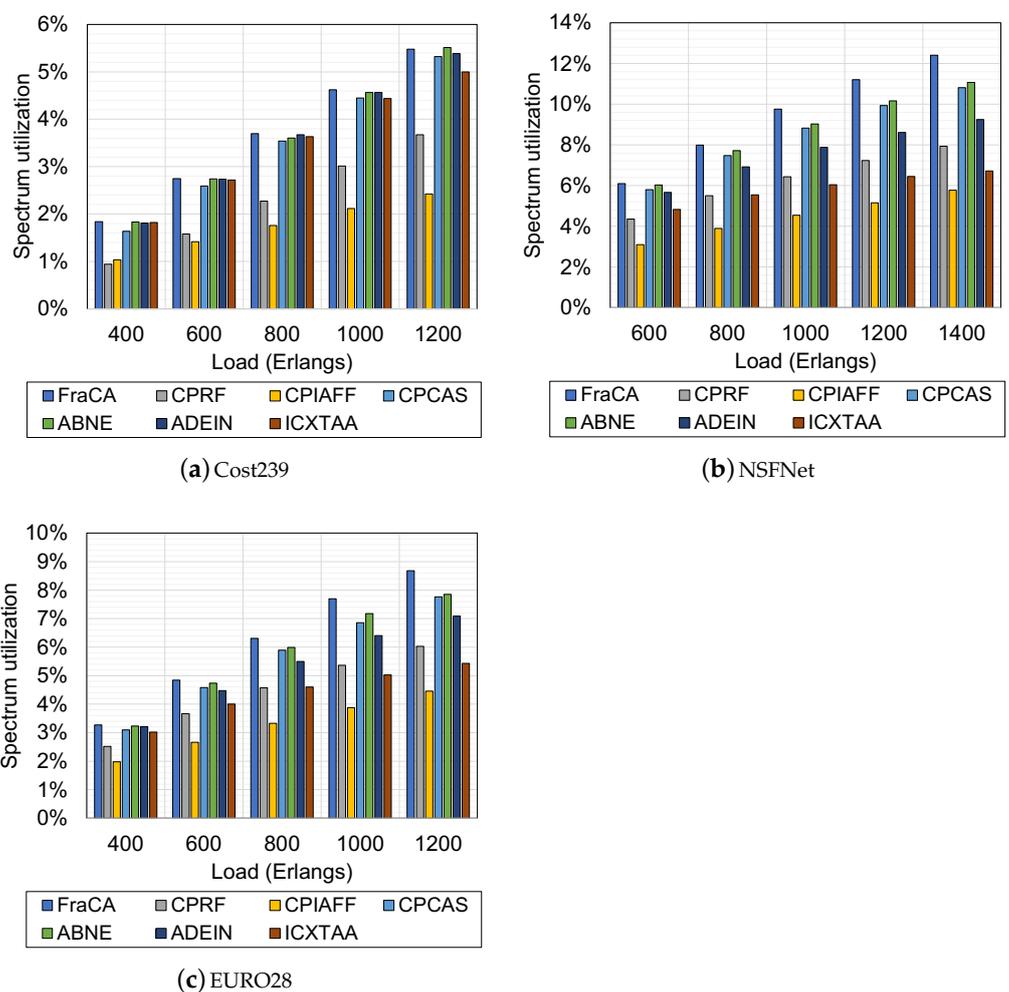


Figure 8. Spectrum utilization.

6. Conclusions

In this paper, the FraCA algorithm was proposed for core and spectrum allocation in multi-core elastic optical networks. FraCA is compared with six other algorithms in the literature, and it obtained the best performance in the Cost239, NSFNet, and EURO28 topology. The numerical results show that FraCA achieves a gain in terms of RBP of at least 16.87% compared to the second-best algorithm, a gain in terms of BBP of at least 43.95% compared to the second-best algorithm, and a mean increase in spectral utilization of 4.36% compared to the second best algorithm. These results show the benefits of FraCA’s

spectral allocation mechanisms. By effectively reducing inter-core crosstalk and spectral fragmentation, the proposed algorithm ensures more simultaneous active lightpaths and, consequently, more clients in the network.

In future work, we intend to investigate the resource allocation problem in SDM-EON, jointly considering the routing and modulation problem. Furthermore, we plan to improve the algorithm in topologies that have nodes with a high number of adjacent nodes, such as the Cost239 topology. Finally, we plan to expand the performance evaluation considering specific fragmentation metrics, such as external fragmentation or Shannon entropy. These metrics are important to measure the algorithm's performance regarding spectrum fragmentation, quantifying how much the fragmentation reduces by using the proposal algorithm.

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Abbreviations

The following abbreviations are used in this manuscript:

ABNE	Core and spectrum balancing algorithm
ADEIN	Inter-core spectral distancing algorithm
ASE	Amplified spontaneous emission
BBP	Bandwidth blocking probability
CAS	Crosstalk avoidance strategy
CP	Core prioritization
EDFA	Erbium doped fiber amplifier
EGN	Enhanced Gaussian-noise
FraCA	Fragmentation and crosstalk mitigation algorithm
FWM	Four-wave mixing
GN	Gaussian noise
IAFF	Intra-area first fit
ICXTAA	Inter-core XT-aware algorithm
MCF	Multi-core fiber
NLI	Nonlinear interference
OSNR	Optical signal to noise ratio
PLI	Physical layer impairments
PSD	Power spectral density
QoT	Quality of Transmission
RMCSA	Routing, modulation, core, and spectrum assignment
RBP	Request blocking probability
SDM-EON	Space-division multiplexed elastic optical networks
SLA	Service level agreement

SMP	Self-phase modulation
SNetS	Slice network simulator
SU	Spectrum utilization
XPM	Cross-phase modulation
XT	Crosstalk

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