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Diversification Benefits of a Volatility-linked Asset

Maria Carolina Dias Ferreira

Master in Finance

Supervisor:
PhD Paulo Viegas Ferreira de Carvalho, Invited Assistant Professor,
ISCTE-IUL

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BUSINESS
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Resumo

Esta tese averigua se a adição de um Fundo de Índice Cotado ou *Exchange-Traded Fund* (ETF) do VIX a um portfólio melhora a sua diversificação, especialmente em mercados em alta. A investigação analisa como o VIXY, um ETF baseado em volatilidade, afeta o desempenho do portfólio, utilizando retornos diários de 2012 a 2023. O estudo é baseado na amostra de 2012 a 2019 e fora da amostra, no período de 2021 a 2023, para determinar como o VIXY otimiza os retornos ajustados ao risco, medidos através do Índice de Sharpe Ajustado. Um modelo DCC-GARCH é utilizado para estimar correlações variáveis no tempo entre ativos, capturando a natureza dinâmica das relações nos mercados financeiros. Os resultados revelam que, embora o VIXY obtenha continuamente um peso negativo na otimização do portfólio, a sua inclusão melhora o Índice de Sharpe Ajustado do portfólio, tanto na amostra como fora dela. O VIXY aumenta a volatilidade do portfólio, mas a sua assimetria positiva sugere o potencial para grandes ganhos. O estudo concluiu que o VIXY melhora a diversificação em mercados estáveis ou em alta, mas aumenta a vulnerabilidade a eventos de mercado severos. Este estudo contribui para a literatura sobre negociação de volatilidade e gestão de portfólios, fornecendo a investidores institucionais e gestores de portfólios maior conhecimento sobre como diversificar para além das classes de ativos tradicionais, particularmente durante períodos de baixa volatilidade no mercado.

Palavras-chave: Volatilidade, VIX ETF, Diversificação de Portfólio, alocação ótima de ativos, Índice de Sharpe Ajustado

Classificação JEL: G11 e G17

Abstract

This thesis examines whether adding a VIX Exchange-Traded Fund (ETF) to a portfolio improves diversification, especially in bullish markets. The research examines how VIXY, a volatility-based ETF, affects portfolio performance using 2012–2023 daily return data. The study utilizes in-sample data from 2012 to 2019 and out-of-sample data from 2021 to 2023 to determine how VIXY optimizes risk-adjusted returns, measured through the adjusted Sharpe ratio. A DCC-GARCH model is employed to estimate time-varying correlations between assets, capturing the dynamic nature of relationships in financial markets. The results reveal that although VIXY continuously obtains a negative weight in portfolio optimization, its inclusion improves the portfolio's adjusted Sharpe ratio in- and out-of-sample. VIXY increases portfolio volatility, but its positive skewness suggests the potential for larger gains. The study found that VIXY boosts diversification in stable or bullish markets but increases vulnerability to severe market occurrences. This study contributes to the literature on volatility trading and portfolio management by providing institutional investors and portfolio managers with insights on diversifying beyond traditional asset classes, particularly during periods of low market volatility.

Keywords: Volatility, VIX ETF, Portfolio diversification, Optimal Asset Allocation, Adjusted Sharpe Ratio

JEL Classification: G11, G17

Glossary of Acronyms and Symbols

ASR- Adjusted Sharpe Ratio

ATM- At-the-money

BND- Vanguard Total Bond Market Index Fund

CBOE- Chicago Board Options Exchange

DBC- Invesco DB Commodity Index Tracking Fund

DCC GARCH- Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity

ETF- Exchange Traded Fund

FRED- Federal Reserve Economic Data

GLD- SPDR Gold Shares

GRG Nonlinear- Generalized Reduced Gradient Nonlinear

MPT- Modern Portfolio Theory

OEX- S&P 100 INDEX

OTM- On-the-money

S&P 100- Standard & Poor's 100

S&P 500- Standard & Poor's 500

SPY- SPDR S&P 500 ETF Trust

UK- United Kingdom

US- United States of America

USD- United States Dollar

UUP- Invesco DB US Dollar Index Bullish Fund

VIX- Volatility Index

VIXY- ProShares VIX Short-Term Futures ETF

VNQ- Vanguard Real Estate Index Fund ETF Shares

VXX- iPath Series B S&P 500 VIX Short-Term Futures ETN

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1. Introduction

Within the intricate and always changing realm of financial markets, successful portfolio management requires a profound comprehension of risk and the skill to negotiate various market circumstances.

Historically, diversification has been a fundamental principle of portfolio theory, to mitigate risk by allocating assets across different asset classes. Nevertheless, the efficacy of diversification tactics might greatly differ based on the market environment, especially during times of intense volatility or financial crises.

The Chicago Board of Options Exchange (CBOE) Volatility Index (VIX) Exchange-Traded Fund (ETF) has attracted significant interest in recent years because of its inherent capacity to improve portfolio diversification. The VIX, also known as the "fear gauge," measures market volatility derived from S&P 500 options. ETFs that mirror the VIX index have been extensively researched as instruments for mitigating risk during periods of market decline. Nevertheless, there is a conspicuous deficiency in the existing body of knowledge about the function of VIX ETFs in periods of market optimism, characterized by reduced volatility and strong performance of conventional assets.

The objective of this thesis is to investigate if including a VIX ETF in a portfolio offers diversification advantages, especially in bullish market conditions. The key research inquiry driving this study then is: Does the inclusion of a VIX ETF in a portfolio enhance diversification, especially in bullish markets, over the long term?

The research employs historical daily returns spanning from 2012 to 2023, including an in-sample data period from 2012 to 2019 and an out-of-sample analysis from 2021 to 2023. The VIXY, chosen for its strong liquidity and strong correlation with the VIX, was assessed in conjunction with a varied collection of prominent index ETFs.

The findings show that including VIXY in the portfolio enhanced the adjusted Sharpe ratio in both periods, even though VIXY was assigned a negative weight, suggesting a short position in volatility. Although the total volatility of the portfolio rose, the skewness of returns indicated more potential benefits.

The results indicate that including VIXY may improve risk-adjusted returns, particularly in bullish markets. However, this comes with the drawback of higher volatility and vulnerability to severe market outcomes, as measured by the increase in kurtosis.

Through its emphasis on positive market circumstances, this study provides a fresh perspective on the strategic usage of VIX ETFs, which have traditionally been seen as instruments for mitigating risk during economic downturns. This paper utilizes econometric models, including the DCC-GARCH model, to accurately represent the dynamic relationships among portfolio assets.

This study is of great significance to portfolio managers, institutional investors, and financial analysts seeking novel methods to improve the long-term performance of portfolios. Investors generally prioritize stocks, bonds, and commodities for the sake of diversification. The incorporation of a VIX ETF represents a new aspect of risk management.

This research emphasizes the benefits of including unconventional assets in portfolio construction, especially for investors aiming to achieve a balance between risk and return in both upward and negative market movements. The insights provided may contribute to more knowledgeable decision-making, particularly for those who oversee diversified portfolios in the long run.

The study is organized as follows. Section 2 examines the existing body of literature about portfolio diversification, volatility trading and the incorporation of volatility as an asset class within asset management. Section 3 delineates the methodology I employ in the dissertation, while the required data and its respective origins are discussed in Section 4. The empirical results are described in Section 5 and discussed in Section 6. An overall conclusion is provided in section 7.

2. Literature review

2.1 Diversification in Portfolio Management

Being a core concept in portfolio management, diversification is well-acknowledged as a key strategy for mitigating risk while preserving possible returns.

The Modern Portfolio Theory (MPT), developed by Markowitz (1952), established a formal concept of diversification and proposed the idea of creating an "efficient" portfolio that optimizes the balance between risk and return. According to MPT, the combination of assets with low or negative correlations enables an investor to construct a portfolio with reduced total risk in comparison to keeping individual assets in isolation. The mitigation of risk is accomplished without necessarily compromising expected returns, hence establishing diversification as a fundamental principle of prudent portfolio management (Elton & Gruber, 1997).

A fundamental observation derived from Markowitz's research is the recognition that portfolio risk is not just determined by the combined weight of individual asset risk but is also affected by the correlations between asset returns. Specifically, the combination of assets with low or negative correlations can decrease the overall volatility of the portfolio by offsetting the impacts of distinct assets moving in opposite directions.

This concept is illustrated by the efficient frontier, which represents the set of optimal portfolios offering the highest expected return for a given level of risk (Fabozzi, Gupta & Markowitz, 2002).

2.2 The role of the Sharpe Ratio in measuring Diversification

The Sharpe Ratio (Sharpe, 1966) is a commonly employed measure for assessing the risk-adjusted return of a portfolio.

$$Sharpe\ Ratio = \frac{E(R_p) - R_f}{\sigma_p} \tag{2.2.1}$$

where $E(R_p)$ is the expected return of the portfolio, R_f is the risk-free-rate and σ_p is the standard deviation of the portfolio, interpreted as a measure of its total risk.

Indicative of effective diversification, a higher Sharpe Ratio suggests that a portfolio is producing greater return per unit of risk. The inclusion of assets with low or negative correlations in a portfolio can effectively decrease the overall volatility of the portfolio without substantially reducing the expected return (Elton et al., 2014). This minimization of risk, while preserving or even augmenting returns, results in a rise in the Sharpe Ratio.

Within the framework of diversification, a greater Sharpe Ratio indicates that the portfolio has an enhanced capacity to provide risk-adjusted returns. This improvement is a direct result of integrating varied assets that exhibit distinct behaviors in different market circumstances. Hence, the Sharpe Ratio serves as both a performance metric and a proxy for the efficiency of diversification techniques implemented in the portfolio (Bodie, Kane & Marcus, 2014).

2.3 The emergence and importance of the VIX

Volatility is a fundamental component of financial market dynamics and is seen as a primary driver in several financial models. Volatility was commonly measured via historical volatility, which computes the standard deviation of past returns. Nevertheless, this retrospective metric fails to include the anticipations of future volatility among market players, thereby giving rise to the emergence of implied volatility as a prospective alternative (Hull, 2012).

The need for a comprehensive measure of market volatility, caused by the limitations that historical volatility had in capturing expectations of future volatility, motivated Whaley (1993) to discuss the CBOE Volatility Index (VIX). He presents the VIX as a gauge of the market's 30-day expected volatility, constructed using the prices of the S&P 100 (OEX) index at-the-money (ATM) options. The index became known as the "fear index," as it could be interpreted as a quantification of market sentiment and fear. A higher VIX indicates greater expected volatility, signifying heightened uncertainty and perhaps unfavorable market conditions. In 2003, a revised version of the VIX was introduced, which uses a new formula to extract implied volatilities from a more extensive basket of options on the S&P 500 (CBOE, 2003).

Following the introduction of the VIX, some research has been conducted to examine volatility as an asset class and to offer a variety of tactics for fully using volatility trading, including studies by Caloiero and Guidolin (2017), DeLisle, Doran and Krieger (2010), and Stanton (2011). Caloiero and Guidolin (2017) assess whether long positions in VIX improve portfolio performance, noting the limitations of volatility products like VXX due to transaction costs and rollover effects. DeLisle, Doran and Krieger (2010) explore the potential of VIX to hedge portfolios, highlighting its effectiveness during market downturns. Stanton (2011) discusses the use of VIX as a hedging tool while pointing out the shortcomings of VIX-based exchange-traded notes.

The instruments available for portfolio diversification and risk management have been expanded by the recognition of volatility as an asset class. The introduction of financial instruments such as VIX futures and VIX ETFs has allowed investors to actively trade and invest in volatility as a distinct asset class, a departure from the traditional perception of volatility as a risk factor that should be minimized (Jacob & Rasiel, 2008). These instruments enable the diversification of portfolios that were previously unavailable, by hedging against market downturns or profiting from anticipated market volatility.

In his explanation of the index's composition and function, Whaley (2009) highlights the impossibility of making direct investments or taking positions in it, which is a key limitation. Specifically, while investing in the underlying basket of securities can typically be used to replicate the payoff of any untradable index, it would be very difficult to replicate the VIX index's performance in the same way. This is closely tied to the structure of the new VIX formula, which only considers call and put market prices that are one month out of expiration when they are out-of-the-money (OTM), requiring numerous daily option investments and position rebalancing, thus leading to higher transaction costs.

Despite being difficult to invest in, the VIX's investment benefits have drawn notice right away. Stanton (2011) explains that the S&P 500 index and the VIX have a negatively correlated relationship, displaying an asymmetric profile, since the negative correlation increases more rapidly during bear markets than it decreases during bull markets. This happens because people are more likely to purchase defensive puts to hedge their positions during a market decline, increasing put prices by demand and therefore volatility, than to invest in call options as leverage during a market expansion, due to most investors' risk aversion.

Bekaert and Wu (2000) offer the volatility feedback effect as a contribution to explaining the asymmetric behavior. This effect refers to the concept of changes in asset prices or market volatility influencing investor behavior, leading to further changes in volatility and contributing to the amplification of market movements and the creation of trends or patterns.

Butler and Joaquin (2002) examine the correlations between the US, UK, Japanese, Australian, and European stock market indices between January 1970 and December 2000. They find that return correlations behave abnormally, with bear market correlations being significantly higher than those in calm or bull markets. During periods of market stress, there is a particularly strong correlation between worldwide stock markets, which reduces the advantages of international diversification just when it is most required. This makes VIX's negative correlation all the more appealing.

Following this thought, Alexander, Korovilas and Kapraun (2016) suggest that volatility optimally diversified portfolios only outperform traditional equity-bond portfolios during periods of financial crisis. Moreover, most literature focuses on these periods. Szado (2009) evaluates the diversification effect of a long VIX investment during the 2008 crisis. Chen, Chung and Ho (2011) uses a mean-variance approach to add VIX futures to four US stock portfolios and, again, the sample ends in 2008.

2.4 The Role of VIX ETFs in Diversification

VIX ETFs are specifically created to replicate the performance of the VIX index, therefore enabling investors to get exposure to market volatility. Based on their negative correlation with equities, especially during times of market turmoil, VIX ETFs can function as efficient diversifiers in a portfolio (Daigler & Rossi, 2006).

Incorporating a VIX ETF into a portfolio might reduce the overall risk of the portfolio while simultaneously enhancing its risk-adjusted returns, as seen by an elevated Sharpe Ratio. This is particularly relevant in the context of portfolios that are predominantly allocated to equity assets, which are susceptible to substantial losses during market downturns.

Although prior studies have thoroughly examined the performance of VIX ETFs during periods of market decline, there is relatively little research on their efficacy during periods of market growth. The objective of this study is to fill this research gap by specifically examining the influence of VIXY in a mostly bullish market setting.

3. Methodology

To assess the potential benefits of incorporating volatility as a diversification strategy in a portfolio, I employ a comparative analysis of the risk-adjusted performance of the portfolios.

This analysis involves the creation of two distinct portfolios: one comprised solely of Index ETFs and another that includes the same Index ETFs and a VIX ETF. To account for higher moments like the skewness and kurtosis in return distributions, particularly relevant for portfolios that include VIX ETFs exhibiting non-normal return distributions, I choose to assess the risk-adjusted performance with the Adjusted Sharpe Ratio (ASR), first proposed by Pézier (2004). This measure is obtained as follows:

$$ASR = SR\left(1 + \left(\frac{S \times SR}{6}\right) - \left(\frac{(K-3) \times SR^2}{24}\right)\right) \tag{3.1}$$

where S and K are the skewness and kurtosis of the return distribution, respectively, and SR represents the Sharpe Ratio. My decision to use the ASR is supported by Peziér and White (2006), who discuss the significance of considering higher moments of return distribution, notably in the context of non-traditional investments.

I estimate the expected portfolio return for the Sharpe ratio by utilizing historical log returns.

$$Log\ return = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{3.2}$$

where P_t is the price of the asset at time t, and P_{t-1} is the price of the asset at the previous period $t-1$. Log returns possess desirable statistical properties, such as time-additivity and suitability for compounding over multiple periods (Campbell, Lo, & MacKinlay, 1997). By relying on historical data, I aim to capture the long-term trends and patterns in asset performance, which are generally accepted as a reasonable proxy for expected returns in financial analysis (Markowitz, 1952; Bodie, Kane & Marcus, 2014).

To gauge skewness and kurtosis for each portfolio, I use Excel's built-in functions. To calculate skewness, I apply the SKEW function to the historical log returns, indicating the asymmetry of the return distribution. To calculate kurtosis, I apply the KURT function to the same returns. Higher kurtosis implies a greater likelihood of extreme returns, reflecting the presence of fat tails in the distribution.

To estimate the covariance matrix for the portfolio's standard deviation, I use the Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model, developed by Engle (2002). This model captures time-varying correlations between assets, making it well-suited for analyzing the dynamic relationships in financial markets. Furthermore, this method takes into consideration volatility clustering, which is the occurrence of high-volatility events tending to cluster, resulting in prolonged periods of high volatility (Engle, 1982). I choose the widely accepted DCC-GARCH (1,1) specification because it both captures essential volatility dynamics and avoids overfitting. The "1,1" notation indicates that both the GARCH model for individual assets and the dynamic correlation model include one lag of past conditional variances and correlations (Engle, 2002; Bollerslev, 1986).

The DCC-GARCH model works in two stages. First, I fit univariate GARCH (1,1) models to each asset's return series to estimate their conditional variances. This method emphasizes more recent observations, ensuring that the model quickly responds to changes in market conditions (Taylor, 2008). Next, I use the standardized residuals from these models to estimate the time-varying correlations between the assets, capturing the dynamic co-movements typical in financial markets (Chiang, Jeon & Li, 2007). By using the DCC-GARCH (1,1) model, I incorporate the most recent volatility and evolving correlations into the covariance matrix, resulting in a more accurate and responsive portfolio risk assessment. I perform the DCC-GARCH model using RStudio, a strong statistical software environment particularly designed for time-series analysis and finance econometrics. This approach enhances the precision of portfolio optimization, ensuring that it reflects the latest market conditions.

I start the optimization process by assigning equal weights to all assets in the portfolio. This approach provides a neutral baseline, ensuring that no asset is initially favored, allowing for an objective assessment of the optimal configuration of the portfolio without any inherent bias. This method is widely accepted in financial research as an effective starting point when there is no prior information suggesting that any asset should be weighted more heavily than others (DeMiguel, Garlappi, & Uppal, 2009).

Next, I set the asset weights as the decision variables for the optimization problem. My primary objective is to maximize the Adjusted Sharpe Ratio.

I calculate the portfolio's expected return R_p as the weighted average of the individual asset returns:

$$R_p = \sum_{i=1}^{n} \omega_i \times R_i \tag{3.3}$$

where ω_i represents the weight of asset i in the portfolio, and R_i is the expected return of asset i (Bodie, Kane & Marcus, 2014).

To determine the portfolio variance σ_p^2, I calculate the weighted sum of covariances between the assets:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \times \omega_j \times \sigma_{ij} \tag{3.4}$$

where σ_{ij} represents the covariance between asset i and asset j (Markowitz, 1952).

I set constraints on the asset weights, allowing them to range from -200% to 200%. This range permits both leveraged positions and short selling, giving me the flexibility to explore a wider range of portfolio strategies. This approach aligns with the flexibility emphasized in portfolio optimization literature, which suggests that allowing for leverage and short selling can enhance portfolio performance (Luenberger, 1998).

To solve this nonlinear optimization problem, I use the GRG Nonlinear solving method in Excel Solver. I choose this method, originally developed by Lasdon et al. (1978), for its wide use in financial optimization studies due to its effectiveness in handling smooth nonlinear functions (Cornuejols & Tutuncu, 2006). It permits a detailed exploration of the optimization landscape, leading to a robust and practical solution. Moreover, I enable the Multistart option to explore multiple starting points, reducing the risk of converging to a local maximum rather than the global maximum of the optimization.

I perform this process for both portfolios to compare the Adjusted Sharpe Ratios, expected portfolio return, portfolio standard deviation, skewness, kurtosis, and any other relevant aspects.

I apply the optimized portfolio weights to an out-of-sample period to assess whether the effects of including the VIX ETF persist beyond the original optimization period. This approach allows me to evaluate the stability and robustness of the VIX ETF's contribution to portfolio performance under different market conditions, ensuring that the observed effects are not only specific to the in-sample data but can be generalized to other periods.

By evaluating and comparing the performance of these two portfolios, I aim to determine whether the inclusion of the VIX ETF results in a more diversified portfolio.

4. Data

To attain this objective, I compile historical adjusted close daily prices, in USD, for the following Index ETFs:

1. Invesco DB Commodity Index Tracking Fund (DBC)
2. SPDR Gold Shares (GLD)
3. SPDR S&P 500 ETF Trust (SPY)
4. Invesco DB US Dollar Index Bullish Fund (UUP)
5. Vanguard Real Estate Index Fund ETF Shares (VNQ)
6. Vanguard Total Bond Market Index Fund (BND)

I select ETFs across various asset classes – equity, bond, commodity, real estate and currency – considering modern portfolio theory, which highlights diversification to reduce risk and optimize risk-adjusted returns. Michaud and Michaud (2008) consider that using index-fund-based ETFs is an effective strategy for achieving broad market exposure and constructing a well-rounded portfolio. Elton et al. (2014) emphasize that diversification across different asset classes reduces portfolio risk by combining assets with low correlations (Appendix A & B). This diversification allows for a clear analysis of the results of adding the VIX ETF, as it ensures that no single asset dominates the portfolio performance.

I also compile historical adjusted close daily prices, in USD, for ProShares VIX Short-Term Futures ETF (VIXY). Because of its High Average Daily Trading Volume, tight Bid-Ask Spread and large Assets Under Management, I determine that it has high liquidity, which lowers transaction costs and enables seamless trade execution without significantly impacting market prices, particularly during periods of heightened volatility (Amihud & Mendelson, 1986; Chordia, Roll & Subrahmanyam, 2001). This liquidity reduces the risk of price slippage and ensures more accurate tracking of the VIX index (Pástor & Stambaugh, 2003), supporting VIXY as the VIX ETF for this analysis.

All the data acquisition process is facilitated through Yahoo Finance.

I calculate the risk-free rate for the Adjusted Sharpe Ratio as the average of the Market Yield on U.S. Treasury Securities at a 3-Month Constant Maturity, quoted on an investment basis, and expressed as a percentage. I adjust this average yield for daily returns by dividing it by the assumed 252 trading days in a year, as the analysis uses daily return data. The 3-Month U.S Treasury bill rate is widely regarded as a proxy for the risk-free rate in financial analysis because of its minimal credit risk as it is backed by the U.S. Government (Bodie, Kane, & Marcus, 2014). This data is gathered from FRED, Federal Reserve Economic Data. For periods where the risk-free rate data is missing, I average the values immediately before and after the missing data points.

The in-sample period for this study spans from 2012 to 2019, as the inception of VIXY was in 2011. I choose this period to capture a substantial amount of market data while ensuring that the ETF has sufficient trading history. The out-of-sample period extends from 2021 to 2023. The year 2020 is excluded from the analysis, as the focus of this research is on more stable markets. It is already well-established that VIX-based instruments perform effectively in short-term bear markets, such as the market downturn seen in 2020.

5. Empirical results

The next section describes the performance metrics of the two portfolios: one that does not include a VIX ETF (VIXY) and another that does include VIXY.

I divide the study into two sections: in-sample findings generated by optimizing the portfolios for the period from 2012 to 2019, and out-of-sample outcomes obtained by applying the improved weights from the in-sample period in the period from 2021 to 2023. In addition to expected portfolio return, standard deviation, skewness, kurtosis, and Adjusted Sharpe ratio are among the performance metrics. These metrics offer valuable information on the risk-return profile and distribution characteristics of the portfolios.

This enables a thorough comparison of their performance, both with and without the incorporation of VIXY. Ultimately, these evaluations aim to address the research question.

5.1 Optimized Portfolio Weights

The following table presents the optimized portfolio weights for both the portfolio without the VIX ETF VIXY and the portfolio that includes VIXY, which I obtained through the GRG Nonlinear solving method in Excel Solver. These weights indicate the proportion of the total portfolio allocated to each asset class. Delta is the difference between both weights in percentage points.

Table 5.1.1: Optimized Portfolio Weights

2012-2019				
Asset Class	ETF	Portfolio without VIX	Portfolio with VIX	Δ
Commodity	DBC	-14%	-15%	-1%
Gold	GLD	-2%	0%	2%
Equity	SPY	31%	15%	-16%
Currency	UUP	8%	14%	6%
Real Estate	VNQ	-6%	-8%	-2%
Bond	BND	83%	100%	17%
VIX	VIXY	-	-6%	-

Within the portfolio devoid of VIXY, the largest proportion is allocated to bonds (BND) at 83%, with equities (SPY) following immediately at 31%. The allocation for Commodities (DBC) is -14%, which signifies a short position. Similarly, gold (GLD) has a -2% allocation, indicating a short position. An 8% allocation is allocated to the U.S. Dollar (UUP), while a -6% allocation is assigned to real estate (VNQ).

For the portfolio including VIXY, the bond allocation (BND) is set at 100%, while the allocation to SPY is reduced to 15%. Commodities (DBC) continue to be held in a short position with a 15% allocation, while real estate (VNQ) likewise maintains a short position with a corresponding 8% allocation. The proportion of the allocation to the U.S. Dollar (UUP) rises to 14%, but gold (GLD) eliminates all allocations, being set at 0%. Indicating a short position in the VIX ETF, a fresh allocation to VIXY at -6% is introduced.

5.2 In-Sample Performance Metrics

Next, Table 5.2.1 summarizes the performance metrics for the in-sample analysis for both portfolios—without VIXY and with VIXY. The metrics include the expected return, portfolio standard deviation, skewness, kurtosis, and adjusted Sharpe ratio. The last column indicates the variation, in percentage points, between the portfolios with and without VIXY.

Table 5.2.1: Performance Metrics Comparison

	2012-2019		
	Portfolio without VIX	Portfolio with VIX	Δ
Expected Portfolio Return	0.027935%	0.038929%	0.010994%
Portfolio Standard Deviation	0.232815%	0.297496%	0.064682%
Skewness	-0.472344	1.353429	1.825774
Kurtosis	6.072941	30.452026	24.379085
Adjusted Sharpe Ratio	10.782678%	12.348039%	1.565361%

The portfolio without VIXY has an expected return of 0.027935%, whereas the portfolio that incorporates VIXY has an expected return of 0.038929%.

The portfolio without VIXY has a standard deviation of 0.232815%, which assesses the volatility of returns. However, the inclusion of VIXY results in a standard deviation of 0.297496%.

The portfolio without VIXY has a skewness of -0.472344, which indicates the tail of the distribution of returns is more pronounced on the left, whereas the portfolio with VIXY shows a skewness of 1.353429, indicating that the most extreme returns are on the right side of the distribution.

The kurtosis of the portfolio without VIXY is 6.072941, while the kurtosis of the portfolio with VIXY is 30.452026, which is significantly higher.

Similarly, the adjusted Sharpe ratio, which takes into consideration skewness and kurtosis, rises from 10.782678% in the absence of VIXY to 12.348039% in the presence of VIXY.

5.3 Out-of-sample Performance Metrics

Table 5.3.1 presents the out-of-sample performance metrics for the portfolios with and without VIXY, during the 2021-2023 period. Just like Table 5.2.1, it compares expected return, standard deviation, skewness, kurtosis, and adjusted Sharpe ratio between the two portfolios. The difference between them is once again in percentage points.

Table 5.3.1: Out-of-sample Performance Metrics Comparison

2021-2023			
	Portfolio without VIX	Portfolio with VIX	Δ
Expected Portfolio Return	-0.007856%	0.005578%	0.013434%
Portfolio Standard Deviation	0.476448%	0.532514%	0.056066%
Skewness	-0.358022	1.202642	1.560663
Kurtosis	5.010038	19.183578	14.173540
Adjusted Sharpe Ratio	-3.641120%	-0.727203%	2.913917%

Utilizing the optimized weights from the in-sample period (2012-2019), the out-of-sample analysis uncovers variations in portfolio performance indicators for the years 2021-2023.

The portfolio without VIXY exhibits an expected return of -0.007856%, whilst the portfolio including VIXY has an enhanced expected return of 0.005578%.

The standard deviation, which measures the volatility of returns, is 0.476448% for the portfolio without VIXY. However, when VIXY is included, the standard deviation rises to 0.532514%.

The skewness for the portfolio without VIXY is -0.358022, reflecting a slight negative skew, while the skewness for the portfolio with VIXY is 1.202642, displaying a positive skew.

The kurtosis for the portfolio without VIXY is 5.010038, compared to a greater kurtosis of 19.183578 for the portfolio with VIXY.

The adjusted Sharpe ratio, which considers skewness and kurtosis, is -3.641120% for the portfolio without VIXY and -0.727203% for the portfolio with VIXY.

6. Discussion

The objective of this research is to examine whether including a VIX ETF in a portfolio may provide diversification benefits, particularly during periods characterized by favorable market circumstances. The research included both the time frame of 2012-2019 and the out-of-sample time frame of 2021-2023, to comprehend the influence of VIXY on portfolio performance, risk characteristics, and overall risk-adjusted returns.

6.1 Consistent negative VIX allocation

Throughout both the in-sample and out-of-sample periods, the optimization model consistently gave a negative weight to VIXY. The model indicated a preference for a short position in VIXY, predicting declining or stable volatility, often linked to bull markets. The objective of the portfolio was to capitalize on a fall in market volatility by shorting VIXY, since the VIX index often declines during times of market stability or upward trends (Whaley, 2000).

The model's continuous suggestion for a short VIXY position over several market periods highlights its dependence on historical data, which likely suggests that minimizing the impact of market volatility might improve portfolio returns during times of strong market performance.

However, this strategy decision adds a speculative aspect, as it presupposes that market circumstances will persist in displaying low or declining volatility (Brière, Burgues, & Signori, 2010). The speculative character of the portfolio is made clear by its susceptibility to unforeseen increases in volatility, which may result in significant losses on the short VIXY position (Carr & Wu, 2006).

6.2 Higher portfolio volatility

Although the model predicted a decrease in volatility, the data indicated that the total volatility of the portfolio rose during both the in-sample and out-of-sample periods. The observed rise indicates that the short VIXY position, initially designed to mitigate risk by taking advantage of low volatility, instead heightened the portfolio's vulnerability to market fluctuations (Whaley, 2009).

The contradictory result may be ascribed to the intrinsic risks associated with shorting volatility. Strategic profitability is contingent upon low volatility; nonetheless, any unforeseen surge in volatility may result in substantial losses, increasing the overall portfolio volatility.

This result underscores a crucial element of risk management: while the early intention was to match the short VIXY position with a market with low volatility, the actual unpredictability of the market made the portfolio more vulnerable to sudden increases in volatility, hence raising its overall risk profile (Fabozzi, Gupta, & Markowitz, 2002). Portfolio volatility exhibited comparable levels of fluctuation in both the in-sample and out-of-sample periods, suggesting that this risk remained consistent across various market conditions.

6.3 Skewness and Kurtosis

Incorporating VIXY into the portfolio not only impacted volatility but also substantially modified the return distribution, especially in terms of skewness and kurtosis.

The transition from negative to positive skewness, when VIXY is included, indicates that the portfolio is now more prone to experiencing sporadic significant wins rather than frequent small losses. This may be advantageous in bullish markets when there is a desire for possible upside. Positive skewness indicates that the portfolio was strategically positioned to profit from positive market moves, even in the presence of higher volatility, by capturing the maximum gain in a way that considers the level of risk involved (Harvey & Siddique, 2000).

Nevertheless, the rise in kurtosis suggests that the portfolio also exhibited a higher susceptibility to extreme outcomes, including both positive and negative. An elevated kurtosis, often linked to "fat tails," indicates that the portfolio has a higher probability of encountering infrequent but significant events that might either significantly boost returns or result in considerable losses (Jondeau & Rockinger, 2003).

The presence of both positive skewness and high kurtosis underscores a significant trade-off: while the portfolio is superiorly positioned to exploit substantial profits, it is also more susceptible to severe market situations. The similar variation in the measures of skewness and kurtosis across the in-sample and out-of-sample periods strengthens the continuous influence of VIXY on the risk profile of the portfolio, irrespective of the market phase.

6.4 Improved Adjusted Sharpe Ratio

Notably, the portfolio's Adjusted Sharpe ratio improved in both the in-sample and out-of-sample periods, despite the heightened volatility. By including skewness and kurtosis with conventional risk and return, the adjusted Sharpe ratio offers a more thorough assessment of risk-adjusted performance.

The enhancement in this measure indicates that the increased risk resulting from the short VIXY position was more than offset by the superior returns attained throughout these periods. This means that, when considering the level of risk, the incorporation of VIXY, despite its negative weight and the resulting rise in volatility, was advantageous. An elevated adjusted Sharpe ratio suggests that the portfolio's returns, after accounting for the heightened risk and possibility of extreme outcomes (as seen by greater kurtosis), were more advantageous compared to a portfolio without VIXY (Bodie, Kane, & Marcus, 2014).

For investors, this implies that while the approach included assuming more risk, it also presented the possibility of greater gains, especially in the setting of optimistic markets where volatility was generally anticipated to decrease. The continuous enhancement in the adjusted Sharpe ratio seen in both the in-sample and out-of-sample periods indicates that this advantage remained throughout time, therefore providing further confirmation for the deliberate incorporation of VIXY.

6.5 Practical implications for investors

These research results have many practical implications for investors.

Across all periods, the persistent negative weight of VIXY indicates that although shorting volatility may boost returns, it also amplifies portfolio volatility. Investors must meticulously evaluate the compromises between possible profits and the increased vulnerability to dramatic market fluctuations. The enhanced Adjusted Sharpe ratio suggests that, in this particular scenario, the increased level of risk was justified by the corresponding positive returns. Nevertheless, this may not always hold, especially in markets where volatility deviates from anticipated patterns (Markowitz, 1952).

Prudent consideration should be given to the strategic inclusion of VIXY in a portfolio. Although the short position in VIXY enhanced the portfolio's risk-adjusted returns, it also had an impact on volatility. For investors contemplating comparable tactics, it is crucial to possess a comprehensive knowledge of market circumstances and be ready to adapt their holdings in the event of an unforeseen rise in volatility (Michaud & Michaud, 2008). The continuous enhancement in the Adjusted Sharpe ratio indicates that using such tactics might be advantageous, but only under the condition of meticulous risk management.

Active portfolio management is crucial due to the increased volatility seen in both the in-sample and out-of-sample periods. It is important for investors to diligently observe market circumstances and be prepared to adjust their strategy in response to fluctuating levels of volatility. During times of increasing volatility, it may be necessary to decrease the short position in VIXY or use supplementary hedging techniques to minimize possible losses (Fabozzi, Gupta, & Markowitz, 2002).

7. Conclusions

The objective of this thesis was to investigate the potential diversification advantages of integrating a VIX ETF, namely VIXY, into a portfolio, especially during periods of strong market performance. The study was carried out over two periods: a period of data collection from 2012 to 2019 and a period of data sampling from 2021 to 2023. The key research inquiry that drove this work was: Does the inclusion of a VIX ETF in a portfolio enhance diversification, especially in bullish markets, over the long term?

The results of this study provide a nuanced response to this inquiry. The incorporation of VIXY into the portfolio consistently led to a negative allocation, suggesting a strategic short position on volatility. This strategy is based on the anticipation of decreasing or consistent volatility in positive markets, which corresponds to the overall market circumstances seen over the analyzed periods.

The findings indicate that the incorporation of VIXY not only raised the cumulative volatility of the portfolio, but also resulted in an enhancement of the adjusted Sharpe ratio, which considers the non-normal distribution of returns, characterized by skewness and kurtosis. It may be inferred that the increased risk brought about by VIXY was more than offset by greater returns.

The analysis revealed that the addition of VIXY results in a change from negative to positive skewness and an escalation in kurtosis, suggesting that the portfolio saw a higher probability of substantial returns but also increased vulnerability to extreme outcomes. Both the in-sample and out-of-sample periods provide similar results, indicating that the impacts of incorporating VIXY are robust and persistent. Nevertheless, it is crucial to acknowledge that the study's emphasis on bullish markets implies that these findings may not be immediately relevant to more unpredictable or bearish situations, when the behavior of VIXY might vary considerably.

This study asserts that the incorporation of a VIX ETF into a portfolio may effectively enhance diversification in bullish markets, mainly by augmenting risk-adjusted returns. Nevertheless, there is a trade-off associated with this: while the portfolio may attain greater profits, it also becomes more vulnerable to the risk of severe market shocks. Hence, it is crucial to carefully evaluate the investor's risk tolerance, market forecast, and the possibility of unforeseen volatility before deciding to add a VIX ETF.

Although this study provides useful insights into the role of VIX ETFs in bullish situations, it is important to acknowledge a few limitations. The study's dependence on VIXY data, which is provided only from 2011 forward, restricts the capacity to carry out a more extensive analysis over a larger period or include additional out-of-sample periods. The premise that past trends will endure is a prevalent framework, however it may not consistently apply during times of significant economic instability. Furthermore, while the model simplifies elements like transaction costs, liquidity, and leverage, these aspects might impact the actual performance of a portfolio based on market circumstances and investor behavior.

This study's results enhance the more comprehensive knowledge of how volatility-linked assets such as VIX ETFs might be strategically used in portfolio management, especially during non-crise periods. Potential additional research might expand upon this analysis by investigating the function of VIX ETFs in other market scenarios, such as extended bear markets, or by integrating other volatility-related products to evaluate their relative efficacy in portfolio diversification.

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Appendix

Appendix A: 2012-2019 correlations, calculated in Microsoft Excel

	DBC	GLD	SPY	UUP	VNQ	BND	VIXY
DBC	1	0,258672	0,368855	-0,227343	0,158659	-0,097461	-0,299936
GLD	0,258672	1	-0,056786	-0,434055	0,083358	0,358725	0,062587
SPY	0,368855	-0,056786	1	-0,027936	0,588821	-0,235414	-0,840928
UUP	-0,227343	-0,434055	-0,027936	1	-0,077966	-0,208001	0,032824
VNQ	0,158659	0,083358	0,588821	-0,077966	1	0,174745	-0,510908
BND	-0,097461	0,358725	-0,235414	-0,208001	0,174745	1	0,204638
VIXY	-0,299936	0,062587	-0,840928	0,032824	-0,510908	0,204638	1

Appendix B: 2021-2023 correlations, calculated in Microsoft Excel

	DBC	GLD	SPY	UUP	VNQ	BND	VIXY
DBC	1	0,344294	0,230359	-0,250616	0,181296	-0,013218	-0,202524
GLD	0,344294	1	0,136854	-0,522600	0,191834	0,431083	-0,044545
SPY	0,230359	0,136854	1	-0,425100	0,755112	0,223802	-0,753118
UUP	-0,250616	-0,522600	-0,425100	1	-0,378063	-0,381675	0,288405
VNQ	0,181296	0,191834	0,755112	-0,378063	1	0,340300	-0,566438
BND	-0,013218	0,431083	0,223802	-0,381675	0,340300	1	-0,092570
VIXY	-0,202524	-0,044545	-0,753118	0,288405	-0,566438	-0,092570	1