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The interaction between equity-based compensation and debt in managerial risk choices

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Abstract
This paper examines the risk incentives of traditional and non-traditional call options in the context of a levered firm where managers under-invest due to risk aversion. Our results contrast with those presented in the literature inasmuch as lookback calls do not always induce higher risk taking than regular calls, and managers always prefer a combination of regular calls and shares of stock in their compensation package as opposed to only company shares. We also show that Asian options outperform both plain-vanilla and other nonstandard options in inducing higher risk taking and, thereby, are a superior remedy for alleviating the agency costs of deviating from the optimal volatility level. Finally, we shed new insights that better clarify the incorrect arguments found in the literature regarding the delta of regular and lookback calls.

Keywords Executive compensation · Debt · Asian calls · Lookback calls · Risk-shifting

JEL Classification G32 · G34

1 Introduction

It is well known that corporate governance deals with different types of conflict of interests, among them the conflict between the providers of finance (the shareholders) and the managers (the agents), as well as the one between the shareholders and

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bondholders. These conflicts arise because the contracting parties are asymmetrically informed.

On the one hand, shareholders are uninformed about the level of effort exerted by their managers to increase firm value. As a result, they link manager's pay to the firm's overall performance so that the manager acts more on their (i.e., the shareholders) interest reducing, therefore, the agency cost stemming from the separation between ownership and control. The signals of performance may include stock price, accounting targets, performance-vesting equity, subjective assessments, among others that provide incremental information about the manager's efforts over and above that already conveyed in the output (Hölmstrom, 1979; Chaigneau, Edmans, & Gottlieb, 2022; and Budde, 2023). On the other hand, high financial leverage may increase shareholder-bondholder conflicts. This is because a compensation designed to solely align managerial incentives with those of shareholders may induce risk-shifting that favors equity holders over debtholders. To put it another way, equity investors hold convex claims over firm assets which causes their expected payoff to rise exponentially with firm risk while debtholder payoffs are concave due to limited upside potential of their claims (Jensen & Meckling, 1976). Hence, high risk taking implies a higher probability of losses for debtholders without the same potential for gains that equity holders benefit from (Srivastav et al., 2014; and Hernández-Lagos, Povel, & Sertsios, 2017).

Given that high managerial risk taking is hurtful to bondholders, a body of literature—see, for instance, Jensen and Meckling (1976); Sundaram and Yermack (2007); Edmans and Liu (2011) and Kabir, Li, and Veld-Merkoulova (2013)—argues that inside debt is an efficient form of compensation because it is associated with lower agency costs of debt given that, just like the value of debt held by outside investors, it is sensitive to both the incidence of bankruptcy and the liquidation value of the firm in the event of bankruptcy.[1] Implicit in these studies is that a mix of equity-based compensation and inside debt is optimal in mitigating the foregoing conflicts of interests.

However, most of the erstwhile studies ignore the important fact that managers with undiversified human capital are, typically, risk averse. As such, equity-based compensation does not always induce higher risk taking as it is commonly assumed, i.e., even when compensated with equity-like securities, the manager may prefer to forgo risky but positive net present value (NPV) projects for more certainty (Carpenter, 2000; Ross, 2004; and Tian, 2004). Moreover, inside debt can aggravate the manager's risk aversion which, ultimately, affects both shareholders and bondholders' wealth. In this connection, assuming that for each firm volatility level the manager chooses the investment policy that yields the highest firm value, a combination of leverage and equity-based compensation might be optimal in inducing higher (and more desirable) risk taking.

We examine this issue by extending the utility-maximization framework of Ju, Leland, and Senbet (2014), who study the case of an unlevered firm, to the more

[1] "Inside debt", in the language of Jensen and Meckling (1976), is primarily associated with defined benefit pensions and deferred compensation.

realistic case of a levered firm whose capital structure includes equity, options and debt. Given that manager's risk aversion is costly to shareholders, we argue that leverage can be important in countervailing this effect through risk-shifting. We thus show that risk-shifting can be less costly than under-investment (induced by risk aversion) in our model. Nonetheless, equity-based compensation is utterly important for risk-shifting to be effective because if managers' interests are not aligned with those of shareholders, they might choose any investment policy which is not necessarily the optimal one.

The introduction of debt in the firm's capital structure has some potential effects on the manager's welfare. First, if the financing decision has no effect on the total value of the firm (in the spirit of Modigliani and Miller (1958)), then any increase (resp., decrease) in the value of debt caused by a positive (resp., negative) signal to bondholders leads to a decrease (resp., an increase) in the value of equity, as well as in the value of executive stock options. Second, given that the firm has risky debt, from option pricing theory (e.g., Merton, 1974), the value of common stock rises when firm's variance goes up. As a result, the value of manager's stock holdings and stock option holdings increases with volatility. Following Crouhy and Galai (1994), we assume that the executive stock options have shorter maturity than debt and that the firm reinvests the proceeds from options exercise. This is important because most studies implicitly assume that the firm "gets rid" of the proceeds from the option exercise by either paying dividends, repurchasing stock or retiring its debt. Unlike those studies, we take into account the potential future increase in the size of the firm's assets as a result of options exercise, consistent with previous research (see Babenko, Lemmon, & Tserlukevich, 2011).

In addition, the assumption that the maturity of debt is greater than that of the option implies that events that are expected to occur after the call option expires, but before debt expiration date, can implicitly affect the value of the three claims (i.e., options, stock and debt). In our setting, the exercise of the options might not be rational when the stock price immediately prior to the expiration date is greater than the option's strike price, the reason being that the exercise of the options reduces the probability of default, which causes an increase in the value of debt and, consequently, a reduction in the share price. Thus, options should only be exercised when the stock price immediately after the expiration date is greater than the strike price. Third, given the convexity of the equity-like payoffs, the introduction of debt, which, *ceteris paribus*, causes a reduction in the value of manager's stock and options holdings, leads to a decrease in the manager's pay-performance sensitivity. This lower pay-performance sensitivity is important as a "precommitment device" to minimize the agency costs of debt related to the risk-shifting problem (John & John, 1993).

The vast majority of firms grant traditional call options (i.e., regular or plain-vanilla call options) as opposed to non-traditional stock options (Johnson & Tian, 2000b; and Dittmann, Maug, & Spalt, 2013). Notwithstanding the simplicity of these traditional options, a burgeoning number of studies—see, for instance, Johnson and Tian (2000a); Ju, Leland, and Senbet (2014) and Bernard, Boyle, and Chen (2016)—advocates the use of non-traditional stock options as a more effective way to induce risk taking (Ju, Leland, & Senbet, 2014) or to create incentives to increase stock price (e.g., Bernard, Boyle, & Chen, 2016). These studies usually ignore dilution (except Ju, Leland, & Senbet, 2014) and do not take into account the potential future increase in the size of the firm's assets. Moreover, they overlook the important fact that stock can be as risky as options in a levered firm (Merton, 1974). Our paper fills this gap in the literature and evaluates the risk incentive effects of regular calls, lookback calls, Asian calls and power call options in the context of a levered firm.

We do not examine the incentives provided by repriciable calls or put options because they might lead to *ex post* wrong incentives (see Ju, Leland, & Senbet, 2014). In addition, we also ignore the risk incentives provided by indexed executive options because they are not, in general, a very efficient form of compensation—see Dittmann, Maug, and Spalt (2013). With respect to regular calls, we show that managers prefer a combination of shares of stock and options in order to obtain a certain level utility. This is in stark contrast to what was concluded by Ju, Leland, and Senbet (2014), who argue that managers always prefer shares of stock in lieu of regular calls because regular call options make their portfolio too risky. Under our framework, both options and stock are modeled as call options and, as a result, the reasoning of Ju, Leland, and Senbet (2014) does not hold. Our results suggest that there exists an optimal number of options and stock that minimizes the costs to the firm and induces higher risk taking, and that this number should be adjusted as the underlying investment technology changes over time (see Core & Guay, 1999; and Athanasakou, Ferreira, & Goh, 2022).

Contrary to the results of Ju, Leland, and Senbet (2014), we also find that lookback calls are not strictly better (in terms of risk incentives and total cost) than regular calls. In fact, our results seem to suggest that regular call options outperform lookback calls in several occasions. Moreover, the argument of Ju, Leland, and Senbet (2014) that lookback calls create stronger risk incentives than regular calls because their delta is always greater than 1 and, hence, greater than that of a regular call (assuming no dividends) is inaccurate. We thus contribute to the literature by shedding light on the ineffectiveness of lookback calls (vis-á-vis regular calls) in inducing higher (and more desirable) risk taking, and on the delta effect of lookback and regular calls. In particular, we show that the delta of a lookback call is not always greater than that of a regular call and that it is never greater than 1. Our results seem to suggest an idea completely opposed to that advocated by Ju, Leland, and Senbet (2014): higher (resp., lower) delta is associated with lower (resp., higher) risk taking. The economic rationale for this is simple: if the delta is high, the executive needs to make less effort to achieve a given level of utility than if the delta were low. As a result, higher delta induces lower risk taking. In this paper, we show that lookback options might be less effective than regular calls when their delta is higher than that of a regular call which, ultimately, contradicts the arguments of Ju, Leland, and Senbet (2014).

We also find that, in general, Asian calls and power calls are more effective than regular calls or lookback calls in inducing higher risk taking. Power options (with an appropriate power coefficient) induce higher risk taking (and can even be more cost-effective) than lookback or regular calls because an increase in firm volatility has a higher impact on the power call than on the regular or lookback call. Note, however, that power options will induce higher risk taking only if the manager is not too risk averse, the reason being that as the manager gets more risk averse, she will become more concerned about the risk of her portfolio and, thereby, might take more conservative investment decisions. Thus, despite their usefulness, it is not always optimal to compensate managers with power calls because they might induce too little or too much risk taking depending on manager's risk aversion and the overall structure of her portfolio. In addition, it is not straightforward (from a practical point of view) to choose the power coefficient that will induce managers to choose riskier (but positive NPV) projects.

Finally, we show that Asian calls are a superior remedy for alleviating the agency costs of deviating from the optimal volatility level, because linking manager's pay to average firm value instead of firm value itself reduces the overall risk of her portfolio. As a result, she is more willing to take investment decisions that optimize firm value. Moreover, Asian options make it less likely for managers to commit fraud by manipulating stock price or taking advantage of inside information since the payoff is based on the average firm value over the life of the option instead of a single date as in the case of power options, regular calls or lookback calls.[2] The opportunity costs from the issuance of new stock options (Bodie, Kaplan, & Merton, 2003) for executives holding underwater options (Sun & Shin, 2014) can also be mitigated by designing compensation plans including Asian options, given that the executive's incentives to maximize firm value may still be intact when the stock price decline is due to shocks beyond the executive's control.

Our paper relates to that of Douglas (2006), but has some important differences. Douglas (2006) studies how leverage and dynamically consistent contracts interact in such a way that minimizes information rents and maximizes firm value. He does not study the specific type of compensation contracts that induces the desired results. In this paper, however, we analyze how options (both traditional and non-traditional) jointly with stocks interact with leverage in order to induce more desirable managerial risk taking. It is assumed in our model that for a given volatility level, the firm value obtained is not the only possible one, but happens to be highest among different investment policies. As a result, our main focus is not in studying the mix of equity, options and debt that maximizes firm value but, instead, the risk level induced by these types of equity-based compensation packages. In contrast to ours, Douglas' model assumes that the manager is risk-neutral with respect to her returns. This assumption is not consistent with empirical evidence that shows that managers are typically risk averse.

The remainder of this work is organized as follows. Section 2 lays out our theoretical framework. Section 3 presents a battery of numerical results to illustrate the effect that the inclusion of debt as well as the consideration of non-traditional stock options in compensation packages have on managerial risk choices. Sections 3.7, 3.8 and 3.9 contain our main contribution to the literature, where we present several

[2] Note that even though the payoff of a lookback call option depends on the minimum firm value during the life of the option, it is still less effective than an Asian call in reducing the management's incentives to fraudulent behaviors. This is because the manager holding a lookback call in her portfolio might still have incentive to increase the terminal stock price so that her payoff is greater.

results contradicting the current literature as well as potential benefits of adoption of Asian options in compensation packages. Section 4 concludes. The analysis concerning the power option as well as other accessory results are relegated to an Internet Appendix and to the Appendix.

2 Theoretical model

In this section, we extend the model proposed by Ju, Leland, and Senbet (2014) in order to incorporate debt in the firm's capital structure.

2.1 Investment technology

Assume a firm financed with debt, equity, and employee stock options, whose time-t value is given by:

$$V_t(\sigma) = D_t(\sigma) + S_t(\sigma) + X_t(\sigma), \tag{1}$$

where σ is the firm volatility chosen by the executive, $D_t(\sigma)$ is the time-t market value of debt, $S_t(\sigma)$ is the time-t market value of stock and $X_t(\sigma)$ is the time-t market value of options. For convenience, and following Ju, Leland, and Senbet (2014), we assume that the firm has one share of stock outstanding with price $S_t(\sigma)$ and one employee stock option outstanding with price $X_t(\sigma)$. We also assume that the option has exercise price K and maturity T. The debt is a zero-coupon bond with face value F and maturity at time T_D. It is noteworthy to mention that although granting restricted stock and stock options to employees does not always result in a cash inflow to the firm, there is a sacrifice of cash that needs to be accounted for and, thereby, it should be reflected in the firm's market value—see Bodie, Kaplan, and Merton (2003). Our reasoning also follows from the fact that according to the rules of the *Financial Accounting Standard Board's Statement of Financial Accounting Standard 123R*, US firms must expense the full cost of their stock options over the life of the option.

The proceeds from the exercise of the options are assumed to be reinvested in the company, thus increasing its size. Note that employee stock options are corporate warrants because exercising the options results in the firm issuing new shares of stock and receiving the strike price. Therefore, when pricing employee stock options, the warrants' analog can be applied.

In this paper, we closely follow Crouhy and Galai (1994) to derive our model. This framework is well known and has been widely applied in the literature—see, for instance, Hanke and Pötzelberger (2002); Handley (2002); Lim and Terry (2003); Koziol (2006); Jarrow and Trautmann (2011); Abínzano and Navas (2013); Anderson and Core (2017) and Glória et al. (2024). The maturity of the options is assumed to be shorter than that of the debt, i.e., $T < T_D$. Thus, events that are expected to occur after the option expires, but before the debt expiration date, can affect the level of volatility to be chosen today by the executive.[3]

It is further assumed that there exists a benchmark firm (with value V') that initially follows an identical investment policy, but is financed entirely by equity. Hence, for $0 \leq t < T$, we have that

$$V'_t(\sigma) = V_t(\sigma). \tag{2}$$

Following Ju, Leland, and Senbet (2014), we define the initial value of the leveraged firm as

$$V_0(\sigma) = V_0 - a\left(\frac{\sigma - \sigma_0}{\sigma_0}\right)^2, \tag{3}$$

where V_0 is the optimal firm value and a measures the costliness of deviating from the optimal volatility level, σ_0.[4] Note that

$$V_0(\sigma) = D_0(\sigma) + S_0(\sigma) + X_0(\sigma) = V_0 - a\left(\frac{\sigma - \sigma_0}{\sigma_0}\right)^2, \tag{4}$$

where the functional forms of $D_t(\sigma)$, $S_t(\sigma)$ and $X_t(\sigma)$ are derived in the subsequent subsections. Equation (4) shows that the choice of σ has impact on both equity, $S_t(\sigma) + X_t(\sigma)$, and debt, $D_t(\sigma)$, which ultimately impacts $V_0(\sigma)$. The firm's first best investment policy is achieved at $\sigma = \sigma_0$, which corresponds to the maximum value of $V_0(\sigma)$. In this case, the firm adopts all positive NPV projects.

2.2 Expected return

Under the physical probability measure $\mathbb{P}$ and conditional to the current (time-t) σ-algebra, the Capital Asset Pricing Model (CAPM) framework implies that

$$\frac{\mu_V(\sigma) - r}{\mu_V(\sigma_0) - r} = \frac{\mathrm{Cov}(\tilde{\mu}_V(\sigma), \tilde{\mu}_m)}{\mathrm{Cov}(\tilde{\mu}_V(\sigma_0), \tilde{\mu}_m)}, \tag{5}$$

where $\mu_V(\sigma)$ is the firm's subjective expected return corresponding to σ, $\tilde{\mu}_m$ is the random return of the market, $\tilde{\mu}_V(\sigma)$ is the (random) return corresponding to σ and r is the risk-free rate. Assuming that the covariance is proportional to the risk level, σ, we get

[3] This assumption precludes the case $T_D = T + \epsilon$, where $\epsilon \in \mathbb{R}_+$ is very small. To see this, notice that as ϵ approaches zero, T_D becomes indistinguishable from T, i.e., in the limit we will have the equality $T_D = T$, which contradicts our assumption that $T_D > T$.

[4] Note that σ_0 is not exactly the same as the benchmark volatility level considered in Ju, Leland, and Senbet (2014) because, with the introduction of debt, equity itself is a call option and, hence, a positive function of σ. We thank the anonymous referee for pointing this out.

$$\mu_V(\sigma) = r + \frac{\sigma}{\sigma_0}(\mu_V(\sigma_0) - r). \tag{6}$$

The main motivation for this specification lies in the fact that, unlike the Black and Scholes (1973) framework where investors can dynamically hedge their option positions, risk averse executives are, usually, not allowed to sell and hedge their options.

2.3 Firm value dynamics

The unleveraged firm value, for a given volatility level σ, is modeled (under the physical probability measure) as a time-homogeneous diffusion process solving the stochastic differential equation

$$\frac{dV'_t(\sigma)}{V'_t(\sigma)} = \mu_{V'}(\sigma)dt + \sigma dB_t^{V'}, \tag{7}$$

where $V'_0(\sigma)$ is given by Eq. (3), $\mu_{V'}(\sigma)$ is given by Eq. (6) and $\{B_t^{V'}, t \geq 0\}$ is a standard Brownian motion. It is noteworthy to emphasize that $\mu_{V'}$ arises from the fact that executives cannot trade their options and are restricted from taking actions such as short-selling company securities or hedging company stock risk. At the same time, the value of executive's holdings in other companies is assumed to follow another diffusion process given by

$$\frac{dO_t}{O_t} = \mu_O dt + \sigma_O dB_t^O, \tag{8}$$

where $\{B_t^O, t \geq 0\}$ is another standard Brownian motion correlated with $\{B_t^{V'}, t \geq 0\}$, i.e.,

$d\left\langle B_t^{V'}, B_t^O \right\rangle = \rho dt$. When valuing $S_t(\sigma)$ and $X_t(\sigma)$ (i.e., the market value of firm contingent claims), $\mu_{V'}(\sigma)$ is replaced by the risk-free rate, i.e., a change of measure is made from the original physical measure to the risk neutral measure that takes as numeraire the money-market account. The terminal values, under the physical measure, are given by

$$V'_T(\sigma) = V'_0(\sigma)e^{(\mu_{V'}(\sigma)-\frac{\sigma^2}{2})T+\sigma B_T^{V'}}, \tag{9}$$

$$O_T = O_0 e^{(\mu_O - \frac{\sigma_O^2}{2})T+\sigma_O B_T^O}. \tag{10}$$

2.4 The executive's terminal wealth

Following Ju, Leland, and Senbet (2014), we assume that the executive has risk-free investment I, holdings of shares of other companies O_0, N_S shares of company stock, and N_X call options with strike price K and maturity T in her portfolio. In order to obtain the executive's wealth, we need to take into account the terminal value of her company holdings composed of shares of stock and call options. We recall that the proceeds from options exercise are reinvested in the company, hence increasing its size. Therefore, if the N_X options are not exercised, at maturity of the debt the value of the levered firm, V_{T_D}, will be equal to the value of the unlevered firm, V'_{T_D}. On the other hand, if the N_X options are exercised, the amount $N_X K$ received from the exercise of the options is reinvested and the value of the levered firm at T_D becomes $V_{T_D} = V'_{T_D}\left(1 + \frac{N_X K}{V'_T}\right)$. Thus, $N_X K / V'_T$ simply measures the scale expansion of the firm's assets.

As shown by Crouhy and Galai (1994), the option should be exercised only if the post-expiration value of the diluted share is greater than the strike price K. This is essentially driven by the fact that the exercise of the options, which results in a scale expansion of the firm, may reduce the probability of default and, consequently, increase the value of debt, which, in turn, causes a reduction in the share price. The post-expiration value of a share of stock, $S_T(\sigma)$, can be written as follows

$$S_T(\sigma) = \begin{cases} V'_T(\sigma) - D_T^{NX}(\sigma) \equiv S_T^{NX}(\sigma) & \text{if options are not exercised} \\ \dfrac{V'_T(\sigma) + N_X K - D_T^X(\sigma)}{1 + N_X} \equiv S_T^X(\sigma) & \text{if options are exercised} \end{cases}, \tag{11}$$

where $D_T^X(\sigma)$ and $S_T^X(\sigma)$, or $D_T^{NX}(\sigma)$ and $S_T^{NX}(\sigma)$, represent the value of debt and of a share of stock at time T if options are exercised, and if options are not exercised, respectively. Since $S_T^X(\sigma)$ is an increasing function of $V'_T(\sigma)$, we can find a value of the firm, $\bar{V}'_T$, such that $S_T^X(\bar{V}'_T) = K$.

The time-T_D value of a share of stock, S_{T_D}, is thus given by

$$S_{T_D}(\sigma) = \begin{cases} \max\left(V'_{T_D}(\sigma) - F, 0\right) & \text{if options were not exercised at time } T \\ \dfrac{\max\left[V'_{T_D}(\sigma)\left(1 + N_X K / V'_T(\sigma)\right) - F, 0\right]}{1 + N_X} & \text{if options were exercised at time } T \end{cases}, \tag{12}$$

where $V'_T(\sigma)$ is given by Eq. (9) and V'_{T_D} is given by

$$V'_{T_D} = V'_T(\sigma) e^{(r - \frac{\sigma^2}{2})(T_D - T) + \sigma(\tilde{B}_{T_D}^{V'} - \tilde{B}_T^{V'})},$$

with $\{\tilde{B}_t^{V'}, t \geq 0\}$ being another standard Brownian motion. Note the change from the physical probability measure to the risk neutral probability measure in the last equation. This is due to the fact that after the maturity of the option, the executives are allowed to sell their shares of stock.

Hence, and following Merton (1974), we can also model the time-T value of the firm's equity as an option on $V_T(\sigma)$ with strike price F and time to maturity $T_D - T$, i.e.,[5]

$$S_T(\sigma) = \begin{cases} c_T(V'_T(\sigma), F, T_D) & \text{if} \quad V'_T(\sigma) \leq \bar{V}'_T \\ \dfrac{c_T(V'_T(\sigma) + N_X K, F, T_D)}{1 + N_X} & \text{if} \quad V'_T(\sigma) > \bar{V}'_T \end{cases}, \tag{13}$$

where $c_t(A_t(\sigma), K, T)$ denotes the time-t value of a call option on $A(\sigma)$, with strike K and maturity at time T.

The time-T value of the executive company holdings is given by $N_S S_T(\sigma) + N_X X_T(\sigma)$, where $X_T(\sigma)$ represents the terminal payoff of the option granted to the executive. The executive's terminal wealth is obtained as

$$W_T = Ie^{rT} + O_T + N_S S_T(\sigma) + N_X X_T(\sigma), \tag{14}$$

which is dependent on the payoff structure of the chosen compensation scheme.

2.5 Regular call options

In this subsection, we assume that the executive has N_X regular (or plain-vanilla) call options in her portfolio. Therefore, the time-T value of the executive company holdings is given by

$$N_S S_T(\sigma) + N_X (S^X_T(\sigma) - K)^+, \tag{15}$$

and the executive terminal wealth follows from Eq. (14):

$$W_T = Ie^{rT} + O_T + N_S S_T(\sigma) + N_X (S^X_T(\sigma) - K)^+. \tag{16}$$

The initial market values of the firm's individual claims (stock and options) are necessary to compute the total cost to the firm. In this sense, we assume that there exists an equivalent martingale measure $\mathbb{Q}$ under which the discounted firm value is a martingale. Thus, we can write

$$\frac{dV'_t(\sigma)}{V'_t(\sigma)} = rdt + \sigma d\tilde{B}^{V'}_t, \tag{17}$$

and, using Itô's lemma,

$$\begin{aligned} V'_T(\sigma) &= V'_t(\sigma) e^{(r - \frac{\sigma^2}{2})(T-t) + \sigma(\tilde{B}^{V'}_T - \tilde{B}^{V'}_t)} \\ &\stackrel{d}{=} V'_t(\sigma) e^{(r - \frac{\sigma^2}{2})(T-t) + \sigma y}, \end{aligned} \tag{18}$$

[5] Note that Eq. (13) can also be obtained through Eq. (11) and the put-call parity.

where $y \sim \mathcal{N}(0, T-t)$.[6] Following Crouhy and Galai (1994), we can value the firm's share of stock at any time t ($< T$) by discounting the expected value of its time-T price—given in Eq. (13)—at the risk-free discount rate r:

$$\begin{aligned} S_t(\sigma) &= e^{-r(T-t)}\mathbb{E}_{\mathbb{Q}}\Bigg[c_T(V'_T(\sigma), F, T_D)\mathbb{1}_{\{V'_T(\sigma)\le \bar{V}'_T\}} \\ &\quad + \frac{c_T(V'_T(\sigma)+N_X K, F, T_D)}{1+N_X}\mathbb{1}_{\{V'_T(\sigma)>\bar{V}'_T\}}\Bigg|\mathcal{F}_t\Bigg] \\ &= \frac{e^{-r(T-t)}}{\sqrt{2\pi(T-t)}}\Bigg(\int_{-\infty}^{\bar{y}} c_T(V'_T(\sigma), F, T_D)e^{-\frac{y^2}{2(T-t)}}dy \\ &\quad + \int_{\bar{y}}^{\infty}\frac{c_T(V'_T(\sigma)+N_X K, F, T_D)}{1+N_X}e^{-\frac{y^2}{2(T-t)}}dy\Bigg), \end{aligned} \tag{19}$$

where $\mathbb{E}_{\mathbb{Q}}[R|\mathcal{F}_t]$ denotes the (time-t) expected value of the random variable R, conditional on time-t σ-algebra $\mathcal{F}_t$ and computed under the equivalent martingale measure $\mathbb{Q}$, $\mathbb{1}_{\{B\}}$ is the indicator function of event B, and

$$\bar{y} := \frac{\ln(\bar{V}'_T/V'_t(\sigma)) - (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}.$$

The value of the option at time t, with $t < T$, is given by

$$X_t(\sigma) = \frac{e^{-r(T-t)}}{\sqrt{2\pi(T-t)}}\int_{\bar{y}}^{\infty}\left(\frac{c_T(V'_T(\sigma)+N_X K, F, T_D)}{1+N_X} - K\right)e^{-\frac{y^2}{2(T-t)}}dy. \tag{20}$$

Therefore, the total cost to the firm, at time 0, is obtained as

$$TC = a\left(\frac{\sigma-\sigma_0}{\sigma_0}\right)^2 + N_S S_0(\sigma) + N_X X_0(\sigma). \tag{21}$$

2.6 Lookback calls

We now assume that instead of regular calls, the executive holds N_X floating-strike lookback call options in her portfolio. Given that lookback calls are always in-the-money, at maturity the executive will pay $N_X S_T^{min}(\sigma)$ to the firm, where $S_T^{min}(\sigma)$ will be defined shortly. First, recall that the time-T value of the executive company holdings is given by

[6] The notation $X \sim \mathcal{N}(\mu, \sigma^2)$ is meant to indicate that the random variable X possesses a univariate normal law with mean $\mu \in \mathbb{R}$ and variance $\sigma^2 \in \mathbb{R}_+$.

$$N_S\left(\frac{c_T(V'_T(\sigma)+N_X S_T^{min}(\sigma),F,T_D)}{1+N_X}\right) \\ +N_X\left(\frac{c_T(V'_T(\sigma)+N_X S_T^{min}(\sigma),F,T_D)}{1+N_X}-S_T^{min}(\sigma)\right). \tag{22}$$

Define $f(x)=c_T(x,F,T_D)/(1+N_X)$. In this paper, we define $S_T^{min}(\sigma)$ to be

$$S_T^{min}(\sigma)=\inf_{0\le u\le T} f(V'_u(\sigma))=\frac{c_T(V_T'^{min}(\sigma),F,T_D)}{1+N_X}, \tag{23}$$

where $V_T'^{min} := \inf_{0<u\le T}(V'_u)$ is the minimum firm value during the life of the option. Thus, at time T, the executive pays $S_T^{min}(\sigma)$ to the firm and receives one share of stock, $S_T(\sigma)$, worth $c_T(V'_T(\sigma)+N_X S_T^{min}(\sigma),F,T_D)/(1+N_X)$. Given the convexity of f, $S_T(\sigma)$ is greater than $S_T^{min}(\sigma)$ almost surely.

Assuming now that $V'_T=V_0(\sigma)e^{Z_T}$, where Z_T is a drifted Brownian motion given by $Z_T=(r-\frac{\sigma^2}{2})T+\sigma\tilde{B}_T^{V'}$, the initial value of the firm's share of stock is equal to

$$S_0(\sigma)=e^{-rT}\mathbb{E}_{\mathbb{Q}}\left[\frac{c_T(V'_T(\sigma)+N_X S_T^{min}(\sigma),F,T_D)}{1+N_X}\Big|\mathcal{F}_0\right] \\ =e^{-rT}\int_{-\infty}^{\infty}\int_{-\infty}^{\min(z,0)}\frac{c_T(V'_0(\sigma)e^z+N_X S_T^{min}(\sigma),F,T_D)}{1+N_X}f(z,m)dmdz, \tag{24}$$

where $f(z, m)$ is the joint density of the Brownian motion z and its minimum m (see, for instance, Campolieti & Makarov (2014), Section 10.4) given by

$$f(z,m)=\frac{2(z-2m)}{(T-t)\sigma^2\sqrt{2\pi\sigma^2(T-t)}}e^{\frac{\alpha z}{\sigma^2}-\frac{\alpha^2(T-t)}{2\sigma^2}-\frac{1}{2}\left(\frac{z-2m}{\sigma\sqrt{T-t}}\right)^2}, \tag{25}$$

with $\alpha=r-\sigma^2/2$. The time-0 value of the option, under the risk-neutral probability measure $\mathbb{Q}$, is given by

$$X_0(\sigma)=e^{-rT}\int_{\infty}^{\infty}\int_{-\infty}^{\min(z,0)}\left(\frac{c_T(V_0(\sigma)e^z+N_X S_T^{min}(\sigma),F,T_D)}{1+N_X}-S_T^{min}(\sigma)\right) \\ \times f(z,m)dmdz. \tag{26}$$

2.7 Asian calls

For simplicity, we use V'_T and $V'_T(\sigma)$ interchangeably. The Asian option has payoff $(\hat{V}'_T-K)^+$, where $\hat{V}'_T$ denotes the geometric average of the firm value from time zero to time T.[7] For the sake of convenience, we use the limiting case where the firm value is continuously monitored and the geometric average is defined as

$$
\begin{aligned}
\hat{V}'_T &:= e^{\frac{1}{T}\int_0^T \ln V'_t dt} \\
&= e^{\log V'_0(\sigma)+(\mu_{V'}-\frac{\sigma^2}{2})\frac{T}{2}+\frac{\sigma}{T}\int_0^T B_t^{V'} dt} \\
&\overset{d}{=} V'_0(\sigma)e^{\frac{1}{2}(\mu_{V'}-\frac{\sigma^2}{2})T+\frac{\sigma}{\sqrt{3}}\sqrt{T}x},
\end{aligned}
\tag{27}
$$

where $x \sim \mathcal{N}(0,1)$. The last line of Eq. (27) follows from Itô's lemma and the fact that $\int_0^t B_s^{V'} ds = \int_0^t (t-s)dB_s^{V'} \sim \mathcal{N}\left(0, \frac{t^3}{3}\right)$—see, for instance, Shreve (2004, page 149). Thus, it is easy to see that the average firm value follows, under the physical measure, a geometric Brownian motion but with different drift and volatility parameters (Kemna & Vorst, 1990):

$$
\frac{d\hat{V}'_t}{\hat{V}'_t} = \frac{1}{2}\left(\mu_{V'}(\sigma) - \frac{1}{6}\sigma^2\right)dt + \frac{\sigma}{\sqrt{3}} d\hat{B}_t^{V'},
\tag{28}
$$

where $\{\hat{B}_t^{V'}, t \geq 0\}$ is another standard Brownian motion. Note that $\hat{V}'_0(\sigma) = V'_0(\sigma)$ by definition. Thus, the initial firm value is equal to the value of an asset tracking the average firm value, but the tracking asset has lower volatility and grows at a lower rate than the firm value does: $\frac{1}{2}\left(\mu_V(\sigma) - \frac{1}{6}\sigma^2\right) < \mu_V(\sigma)$. This has important implications in the risk incentives provided by Asian options when compared with other types of options, as we discuss in the next section. The value of executive company holdings is thus defined as

$$
N_S S_T(\sigma) + N_X\left(\hat{V}'_T - K\right)^+.
\tag{29}
$$

As in the previous subsections, the risk neutral value of S_t $(t < T)$ is given by

$$
\begin{aligned}
S_t(\sigma) = \frac{e^{-r(T-t)}}{2\pi\sqrt{T-t}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &\Bigg(c_T(V'_T(\sigma), F, T_D)\mathbb{1}_{\{\hat{V}'_T \leq K\}} \\
&+ \frac{c_T(V'_T(\sigma) + N_X K, F, T_D)}{1+N_X}\mathbb{1}_{\{\hat{V}'_T > K\}}\Bigg) e^{-\frac{1}{2}\left(\frac{y^2}{T-t}+x^2\right)} dx dy,
\end{aligned}
\tag{30}
$$

where $V'_T(\sigma)$ and $\hat{V}'_T$ are given by Eqs. (18) and (27), respectively. A closed-form solution for the time-t price of an Asian call option with geometric averaging is available in the literature (Kemna & Vorst, 1990):

$$
X_t(\sigma) = V'_t(\sigma)e^{-\hat{q}(T-t)}\mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2),
\tag{31}
$$

[7] We consider geometric average in lieu of arithmetic average because the former penalizes *mean preserving spreads* while the latter does not. As a result, a better alignment between managers and shareholders is achieved since they both prefer steady growth to volatile swings.

where $\hat{q} = \frac{1}{2}\left(r + \frac{1}{6}\sigma^2\right)$,

$$\begin{aligned} \hat{\sigma} &= \frac{\sigma}{\sqrt{3}}, \\ d_1 &= \frac{\log(V_t'(\sigma)/K) + (r - \hat{q} + \frac{1}{2}\hat{\sigma}^2)(T-t)}{\hat{\sigma}\sqrt{T-t}}, \qquad \text{and} \end{aligned} \tag{32}$$

$$d_2 = d_1 - \hat{\sigma}\sqrt{T-t}.$$

2.8 Optimal corporate risk policy

Consistent with prior research, we assume that the executive is risk averse and has constant relative risk aversion specified by the power utility function

$$U(W_T) = \begin{cases} \dfrac{W_T^{1-\Lambda}}{1-\Lambda}, & \text{if } \Lambda \neq 1 \\ \ln W_T, & \text{otherwise} \end{cases}, \tag{33}$$

where $\Lambda > 0$ is a measure of risk aversion, usually called the *coefficient of relative risk aversion*: a larger Λ indicates a higher degree of risk aversion. The ultimate goal of the executive is to choose a volatility level σ that maximizes her expected utility

$$\max_{\sigma} \mathbb{E}_{\mathbb{P}}\left[U\left(Ie^{rT} + O_T + N_S S_T(\sigma) + N_X X_T(\sigma)\right) \middle| \mathcal{F}_0 \right], \tag{34}$$

where $S_T(\sigma)$ is defined in the last subsections and O_T is given by Eq. (10).

There are a few points that are noteworthy to emphasize from Eq. (34). First, if the financing decision has no effect on the total value of the firm, lower leverage is associated with higher expected utility. Our framework nests the Ju, Leland, and Senbet (2014) model as special case when $F = 0$. Second, the volatility level σ affects $S_T(\sigma)$ because more volatile returns increase the value of equity holders' call option, which reduces the value of debt. Therefore, the interests of debt and equity conflict. Equity holders prefer higher firm volatility, which raises the value of their long call; debtholders prefer lower firm volatility, which increases the value of their short call (Anderson & Core, 2017). In this sense, as long as the increase in firm volatility is associated with a more than sufficient increase in expected return to compensate for the increase in uncertainty, the executive will endeavor to increase firm volatility in order to increase the value of her equity stake and the value of her call option on equity.

Also note that since the exercise of the options decreases the value of a share of stock and increases the value of debt, the manager might try to drive the volatility down so that her option is not exercised and the value of her equity stake is greater. However, the decrease in volatility also affects (more specifically, decreases) the equity value. Moreover, given that the manager is risk averse, she will simultaneously prefer a less volatile distribution of returns. The optimal σ is, thus, the result of the interaction among all these effects. Third, σ affects the expected utility more directly through its effect on $V_0(\sigma)$.

3 Numerical results

This section presents the numerical analysis. For this purpose, we adopt the parameters configuration of Ju, Leland, and Senbet (2014), but augmented by T_D and F. More specifically, we consider the following base values: $a = 50$, $V_0 = 100$, $r = 5\%$, $\sigma_0 = 0.38$, $\mu_V(\sigma_0) - r = 7\%$, $\sigma_O = 0.2$, $\rho = 0.2$, $\mu_O = 12\%$, $\Lambda = 2$, $NCW_0 = 0.32$, $f_{NC} = 0.8$, $T = 5$, $N_S = 0.32\%$, $N_X = 0.38\%$, $F = 60$, $K = V_0(\sigma)$ and $T_D = 7$. NCW_0 denotes the executive's initial non-company wealth and f_{NC} is the fraction of NCW_0 invested in other companies. Table 1 illustrates the incentive effects of regular calls. For completeness, we also report the results for executives in an unlevered firm in an Internet Appendix.[8]

The volatility chosen is now much higher than that of an unlevered firm for all the parameter constellations, suggesting that debt helps to reduce the agency costs of deviating from the optimal volatility level. The reason is simple. Recall that $V_t(\sigma) = X_t(\sigma) + S_t(\sigma) + D_t(\sigma)$. Therefore, keeping the firm value constant, the higher the leverage $(D_t(\sigma))$, the lower the value of equity $(X_t(\sigma) + S_t(\sigma))$. As a result, a higher volatility is required to maximize $S_t(\sigma)$ and to ensure that the option has a non-negligible probability of finishing in-the-money.

Thus, debt induces some risk-shifting incentives for managers. Nonetheless, up to a certain point, risk shifting incentives do not hurt bondholders since the manager is taking actions to optimize the overall value of the firm. Some caution is needed however in this analysis. Implicit in our assumptions is the fact that $V_0(\sigma)$ is not the only possible firm value for the risk level σ, but happens to be the highest among different investment policies (Ju, Leland, & Senbet, 2014). If the manager adopts any one of many possible investment policies that result in lower firm values for the same risk level, the risk-shifting incentives can, in fact, be hurtful to bondholders. In this connection, tying manager's compensation to the firm's performance is important in order to reduce these problems. Our results are not consistent with those of Kim, Patro, and Pereira (2017), who argue that high leverage is likely to dampen the impact of risk-increasing incentives provided to the manager, but agree with those of Coles, Daniel, and Naveen (2006); Dong, Wang, and Xie (2010) and Chava and Purnanandam (2010).

[8] We recall that these results (collected in Appendixes B and C) are obtained using the unlevered case considered in Ju, Leland, and Senbet (2014). Eventual tiny differences are justified by the fact that the codes in Ju, Leland, and Senbet (2014) were written in Fortran and called IMSL Fortran library routines for doing the integration and minimization, while our codes were written in MATLAB and called the corresponding built-in functions for doing integration and minimization.

Table 1 Risk effects of compensation contracts with regular calls in a levered firm

	σ	$V_0(\sigma)$	VC	TC	$\mathbb{E}_{\mathbb{P}}[U(W_T)]$	$10^3 \frac{\partial \mathbb{E}_{\mathbb{P}}[U]}{\partial V}$	10^3PPS
Base	0.323	98.888	19.809	1.382	−1.3054	6.517	3.825
$\Lambda = 0$	0.582	85.936	32.843	14.382	1.4276	12.030	12.030
$\Lambda = 4$	0.222	91.367	9.471	8.831	−1.4656	23.756	3.298
$a = 10$	0.253	98.875	13.936	1.366	−1.2916	7.355	4.409
$a = 30$	0.302	98.730	17.988	1.531	−1.3013	6.775	4.001
$a = 70$	0.336	99.040	20.854	1.236	−1.3076	6.372	3.727
$a = 90$	0.343	99.161	21.531	1.120	−1.3091	6.280	3.665
$N_X = 0.0\%$	0.319	98.724	19.497	1.470	−1.3730	6.425	3.408
$N_X = 0.2\%$	0.323	98.878	19.818	1.357	−1.3323	6.475	3.648
$N_X = 0.5\%$	0.323	98.872	19.746	1.421	−1.2908	6.543	3.927
$N_X = 1.0\%$	0.320	98.753	19.368	1.634	−1.2465	6.625	4.264
$N_S = 0.0\%$	0.450	98.286	29.602	1.826	−1.7378	2.575	0.853
$N_S = 0.2\%$	0.350	99.678	22.223	0.531	−1.4334	5.536	2.694
$N_S = 0.5\%$	0.298	97.667	17.324	2.694	−1.1586	7.450	5.550
$N_S = 1.0\%$	0.260	94.977	13.397	5.625	−0.9009	8.550	10.535
$f_{NC} = 0.0$	0.327	99.021	20.139	1.251	−1.4340	7.610	3.701
$f_{NC} = 0.5$	0.327	99.036	20.178	1.237	−1.3286	6.619	3.750
$f_{NC} = 1.0$	0.319	98.698	19.364	1.569	−1.3040	6.633	3.901
$K = 0.5V_0(\sigma)$	0.301	97.861	33.149	2.455	−1.2352	7.171	4.700
$K = 0.8V_0(\sigma)$	0.317	98.622	24.741	1.665	−1.2841	6.695	4.060
$K = 1.2V_0(\sigma)$	0.327	99.043	15.292	1.211	−1.3209	6.407	3.672
$K = 1.5V_0(\sigma)$	0.331	99.155	9.198	1.077	−1.3371	6.321	3.536
$NCW0 = 0.2$	0.297	97.600	17.208	2.653	−1.8126	12.011	3.656
$NCW0 = 0.5$	0.349	99.671	22.193	0.614	−0.9317	3.527	4.063
$NCW0 = 1.0$	0.389	99.972	25.515	0.331	−0.5295	1.290	4.600
$F = 0$	0.270	95.848	32.349	4.581	−0.9812	4.707	4.889
$F = 10$	0.275	96.158	29.175	4.237	−1.0329	5.173	4.849
$F = 30$	0.284	96.807	23.783	3.527	−1.1437	6.089	4.655
$F = 50$	0.309	98.264	20.873	2.024	−1.2279	6.270	4.158
$F = 80$	0.358	99.834	18.653	0.411	−1.3599	5.988	3.238
$F = 95$	0.381	100.000	17.768	0.231	−1.4113	5.724	2.874
$F = 115$	0.408	99.721	16.723	0.493	−1.4316	5.168	2.522

Column 1 represents the value of a specific parameter, keeping the remaining parameters fixed at their base case values. Columns 2–8 report the volatility chosen, the current firm value, the market value of one regular call, the total cost to the firm, the expected utility of terminal wealth, the partial derivative of the expected utility with respect to the initial firm value, and PPS defined as the partial derivative of the manager's certainty equivalent with respect to the initial firm value, i.e., PPS $= \frac{\partial U^{-1}(\mathbb{E}_{\mathbb{P}}[U])}{\partial V}$, respectively

3.1 Effect of risk aversion

As in the case of an unlevered firm (see Ju, Leland, & Senbet, 2014), the risk neutral (i.e., $\Lambda = 0$) manager chooses a volatility level (58.2%) higher than the firm

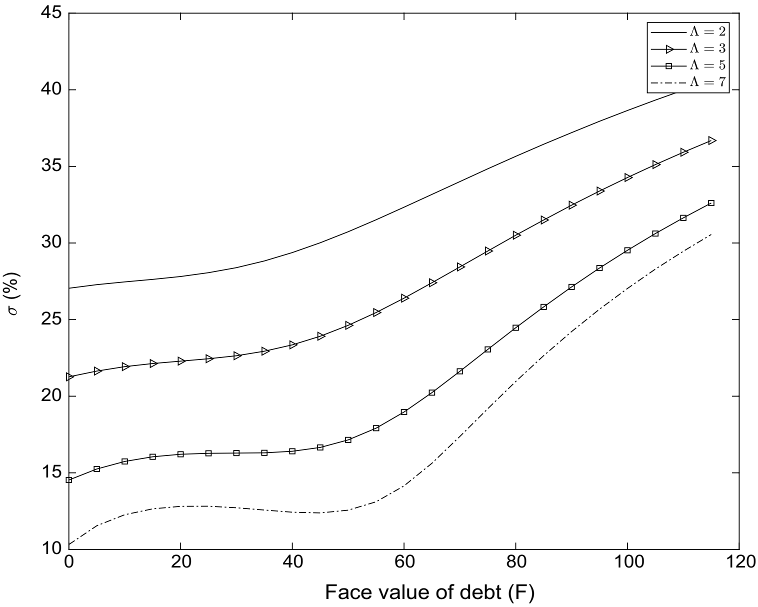

Fig. 1 Effect of leverage in managerial risk choices. The figure plots the volatility level (σ) chosen by the executive as a function of the face value of debt (F) and for different coefficients of relative risk aversion (Λ)

maximizing one (38.0%). This is already well understood in the literature and has a simple explanation: although $V_0(\sigma)$ reaches its maximum value at σ_0, the marginal expected utility is still positive for a risk neutral manager. As a result, the appreciation in option value is greater than the decrease in firm value for σ near σ_0, which induces managers to take a risk higher than the firm maximizing one. Conversely, as Λ increases, the manager becomes more risk averse and ends up adopting safer investments. To further illustrate the effect of leverage in managerial risk choices, Fig. 1 depicts the volatility chosen by the risk averse manager assuming different values for the parameter Λ.

As expected, the higher the manager's aversion to risk, the lower the volatility she chooses regardless of the level of leverage. Figure 1 also shows that the relation between leverage and manager's risk choices is non-monotonic (especially for extremely risk averse managers, i.e., $\Lambda = 5$ and $\Lambda = 7$). It appears that moderate values of F (about 50) are the initial points at which leverage starts to induce high risk taking. Below that level, the choice of risk depends on the manager's risk aversion. If the manager is extremely risk averse (e.g., $\Lambda = 7$), increasing leverage induces higher risk taking until a certain point (about 20), but after that level, leverage induces even lower risk taking. On the other hand, if the manager is not too risk averse (e.g., $\Lambda = 2$), she slowly increases volatility until the foregoing threshold (i.e., until about 50). Appendix D of the Internet Appendix presents two figures similar to Fig. 1 but now with $r = 0\%$ and $r = 10\%$, respectively. The results are qualitatively the same.

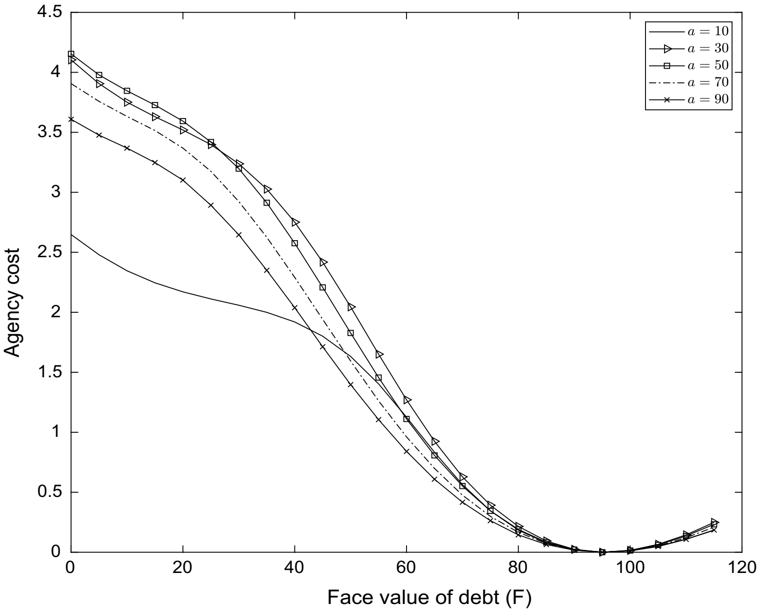

Fig. 2 Effect of leverage in the agency costs of deviating from the optimal volatility level. The agency cost, in the y-axis, is calculated in the following way: $a\left(\frac{\sigma-\sigma_0}{\sigma_0}\right)^2$, where a is the costliness of deviating from the optimal volatility level σ_0, and σ is the volatility chosen by the executive that maximizes her expected utility of terminal wealth under the physical measure $\mathbb{P}$

3.2 Effect of investment technology

Following Ju, Leland, and Senbet (2014), we define the agency cost as the deviation of the firm value, $V_0(\sigma)$, from the optimal firm value, $V_0(\sigma_0)$, that is: $a\left(\frac{\sigma-\sigma_0}{\sigma_0}\right)^2$. From Table 1, the agency costs for $a \in \{10, 30, 50, 70, 90\}$ are 1.117, 1.264, 1.125, 0.939 and 0.853, respectively. Thus, the results are qualitatively similar to the ones reported in Ju, Leland, and Senbet (2014), i.e., for the set of parameters considered, it appears that the agency cost is stronger for moderate values of a. However, the agency cost is now smaller, as expected, and is negatively correlated with leverage until a certain threshold of F. After that threshold, the agency costs tends to increase with leverage. The results indicate that there exists a particular value of F such that the manager will choose the optimal volatility level (0.38) and the firm value will be equal to the optimal firm value. Figure 2 depicts these results.

Figure 2 shows that as leverage increases, the agency costs of deviating from the optimal volatility level decrease until the point where the optimal volatility level $\sigma_0 = 0.38$ is reached. After that level, a further increase in leverage increases the agency costs. Consistent with what was mentioned previously, Fig. 2 shows that the agency cost is higher for moderate values of a, i.e., $a = 30$ and $a = 50$. Appendix E of the Internet Appendix presents two figures similar to Fig. 2 but now with $r = 0\%$ and $r = 10\%$. The results are, in general, similar to the ones presented in

Fig. 2, except that now, lower leverage (if $r = 0\%$) or higher leverage (if $r = 10\%$) is required to achieve the optimal volatility level.

3.3 Effect of increasing the portion of call option, or company shares or non-company shares

Similar to the manager of the unlevered firm, the risk averse manager of the levered firm appears to take lower risk as the call option portion in her portfolio increases. The reason lies in the fact that as the portion of options in her portfolio increases, her portfolio becomes riskier and, hence, she may reduce the risk level of the firm to reduce her portfolio risk. However, unlike the manager of the unlevered firm, the volatility chosen by the risk averse manager of the levered firm does not change much for different portions of call options. The reason is that the introduction of debt in the firm's capital structure induces the manager to take higher risk. Thus, leverage dampens the manager's willingness to decrease firm volatility even when the portion of call options in her portfolio becomes large.

The effect of increasing the company stock component in a manager's portfolio is interesting for several reasons. First, when no company stock is granted to the manager ($N_S = 0.0\%$), she chooses a volatility level, σ, of 0.45, which is above the firm maximizing one. This is because the option effect dominates the risk aversion of the manager. Second, as the portion of company stock in a manager's portfolio increases, she adopts safer investments. Strikingly, she seems to adopt much safer investment policies when the portion of company shares increases than when the portion of call options in her portfolio rises. This is because after a certain degree of leverage, the sensitivity of debt to firm volatility grows more negative and, as this happens, the stock sensitivity to volatility tends to increase in order to offset losses to options with gains against the debt—see Anderson and Core (2017). As a consequence of this, the manager takes more conservative investment decisions in order to reduce her portfolio's risk. Similar to the manager of an unlevered firm, the manager chooses lower risk levels as the portion of her non-company wealth in shares of other companies increases.

3.4 Effect of strike price

Table 1 indicates that even though the resulting σ's are below the firm maximizing one, σ_0, they are substantially higher than that of an unlevered firm for different strike levels. Nonetheless, the volatility chosen by the managers of both firms (i.e., levered and unlevered firms) is positively correlated with the level of the strike price, i.e., the higher the strike price, the higher the risk level the manager takes. This is because a high strike price makes the option (deep) out-of-the-money and, thus, a high volatility is required in order to ensure that the option finishes in-the-money. This is consistent with Tian (2004) who finds that premium options provide higher (systematic) risk incentives than discount options.

3.5 Effect of diversification

As Ju, Leland, and Senbet (2014) put it, if the relative portion of executive's company holdings (stock and call options) is small in the manager's portfolio (i.e., large NCW_0), the manager has incentives to adopt riskier investments in order to maximize her call option payoff, since she is not too worried whether the options will finish out-of-the-money. It is important to note, however, that very high NCW_0 will induce the manager to take risk above the firm maximizing one. The results are more severe in the case of levered firms since, in this case, the managers take higher risk. As for f_{NC}, results in Table 1 suggest that a significant flat fee induces managers to take higher risk, given that lower f_{NC} corresponds to higher portion of investments on risk-free assets.

3.6 Expected utility, utility and pay-performance sensitivities with respect to the firm value

Still considering the results contained in Table 1, column 6 reports the expected utility, column 7 presents the partial derivative of the expected utility with respect to the firm value (utility sensitivity) and column 8 depicts the partial derivative of the certainty equivalent of the manager's wealth with respect to the firm value (pay-performance sensitivity). As expected, the introduction of debt decreases the expected utility, increases utility sensitivity and decreases the pay-performance sensitivity for the set of parameters considered. The reason of the lower utility for a risk averse manager in a levered firm stems directly from the decrease in the value of equity caused by debt issuance. Given that strict alignment of manager compensation with shareholder interest is optimal for all-equity firms and should be lower for levered firms, the utility sensitivity for a risk averse manager in a levered firm will be higher than that of a manager in an unlevered firm, reaching its nadir in the limiting case when $F \to 0$ (i.e., as the firm becomes unlevered). This is due to the concavity of the power utility function. Finally, as debt increases, the pay-performance sensitivity should optimally decline because with larger debt and increased risk-shifting incentives, the management compensation structure plays a larger "precommitment role" and smaller "alignment with shareholders" role (John & John, 1993).

3.7 Minimizing the total cost to the firm

We now examine the optimal mix of stock-based components of the compensation that minimizes the total cost, defined in Eq. (21), while preserving the manager's utility obtained in Table 1. Table 2 contains the numerical results.

Table 2 shows that, for most cases, it is more efficient to use a combination of company shares and regular options to achieve a given level of utility for the manager of a levered firm. This is in stark contrast with the case of the unlevered

Table 2 Minimizing the total cost with company shares and regular calls

	σ	$V_0(\sigma)$	10^2N_S	VC	10^2N_X	TC	$10^3\frac{\partial \mathbb{E}_{\mathbb{P}}[U]}{\partial V}$	10^3PPS
Base	0.332	99.216	0.238	20.503	1.165	1.169	6.089	3.573
$\Lambda = 0$	0.589	84.832	0.257	32.733	0.470	15.475	12.129	12.129
$\Lambda = 4$	0.223	91.412	0.311	9.479	0.751	8.817	23.639	3.282
$a = 10$	0.292	99.465	0.220	17.222	1.534	0.932	6.520	3.908
$a = 30$	0.318	99.202	0.232	19.307	1.280	1.186	6.239	3.684
$a = 70$	0.341	99.257	0.255	21.241	0.959	1.105	6.059	3.544
$a = 90$	0.347	99.327	0.258	21.786	0.919	1.033	5.996	3.499
$N_X = 0.0\%$	0.346	99.594	0.182	21.752	1.006	0.738	5.575	2.958
$N_X = 0.2\%$	0.338	99.392	0.208	21.019	1.189	0.986	5.846	3.294
$N_X = 0.5\%$	0.330	99.145	0.241	20.269	1.351	1.277	6.139	3.685
$N_X = 1.0\%$	0.321	98.791	0.307	19.425	1.166	1.621	6.564	4.225
$N_S = 0.0\%$	0.382	99.999	0.099	25.056	0.000	0.065	3.637	1.204
$N_S = 0.2\%$	0.359	99.842	0.140	22.912	0.863	0.444	5.106	2.485
$N_S = 0.5\%$	0.307	98.131	0.397	17.967	1.564	2.386	7.098	5.287
$N_S = 1.0\%$	0.267	95.607	0.859	13.903	2.584	5.230	8.368	10.311
$f_{NC} = 0.0$	0.333	99.224	0.256	20.569	0.957	1.130	7.235	3.519
$f_{NC} = 0.5$	0.336	99.316	0.239	20.797	1.135	1.067	6.181	3.502
$f_{NC} = 1.0$	0.329	99.096	0.229	20.139	1.347	1.316	6.152	3.618
$K = 0.5V_0(\sigma)$	0.301	97.858	0.374	33.196	0.221	2.437	7.203	4.721
$K = 0.8V_0(\sigma)$	0.320	98.750	0.275	24.971	0.639	1.576	6.544	3.969
$K = 1.2V_0(\sigma)$	0.341	99.480	0.240	16.349	1.560	0.924	5.854	3.355
$K = 1.5V_0(\sigma)$	0.344	99.558	0.277	10.267	1.238	0.742	5.928	3.316
$NCW0 = 0.2$	0.302	97.917	0.268	17.658	1.047	2.426	11.596	3.529
$NCW0 = 0.5$	0.361	99.879	0.225	23.103	1.029	0.501	3.244	3.737
$NCW0 = 1.0$	0.378	99.999	0.375	24.697	0.173	0.285	1.342	4.787
$F = 0$	0.281	96.596	0.407	33.431	0.000	3.797	4.908	5.098
$F = 10$	0.277	96.346	0.385	29.521	0.061	4.016	5.323	4.990
$F = 30$	0.286	96.912	0.286	23.888	0.616	3.452	5.976	4.569
$F = 50$	0.318	98.671	0.222	21.527	1.316	1.759	5.796	3.844
$F = 80$	0.364	99.906	0.280	19.068	0.709	0.383	5.792	3.132
$F = 95$	0.376	99.993	0.351	17.363	0.174	0.214	5.868	2.946
$F = 115$	0.398	99.893	0.384	15.912	0.000	0.285	5.426	2.647

Column 1 represents the value of a specific parameter, keeping the remaining parameters fixed at their base values, except the number of company shares and the number of regular calls, which are chosen to minimize the total cost while preserving the utility level at each corresponding entry in Table 1. Columns 2–8 report the volatility chosen, the current firm value, the number of shares chosen, the market price of one regular call, the number of regular calls chosen, the total cost to the firm, the partial derivative of the expected utility with respect to the initial firm value, and PPS defined as the partial derivative of the manager's certainty equivalent with respect to the initial firm value, i.e., PPS $= \frac{\partial U^{-1}(\mathbb{E}_{\mathbb{P}}[U])}{\partial V}$, respectively

firm of Ju, Leland, and Senbet (2014), where it is more cost-effective to use only company shares.

3.8 The impact of lookback calls on the investment risk choice

Ju, Leland, and Senbet (2014) argue that lookback calls are more effective than regular calls in reducing the agency costs of deviating from the optimal risk level. According to the authors, this is because unlike regular calls, lookback calls are always in-the-money and thus the manager is willing to take higher and more desirable risk. In this paper, we analyze the incentives provided by lookback calls in a new (and more realistic) setting where the firm is financed with both equity and debt. Table 3 reports the results.

The number of lookback calls is chosen to yield the same utility level as each corresponding entry in Table 1. Our results contrast with those of Ju, Leland, and Senbet (2014), inasmuch as lookback calls are not strictly preferred to regular calls. That is, regular calls are more effective than lookback calls in reducing the agency costs of deviating from the optimal volatility level in several cases. Although the difference is not very pronounced, Table 3 also shows that the total cost to the firm is higher when lookback calls are used in lieu of regular calls in several situations. However, neither lookback calls nor regular calls should be used when the firm is overleveraged or when the manager holds a significant portion of non-company wealth as they induce managers to take excessively risky investments. Results from Tables 1 and 3 also indicate that lookback calls induce higher risk taking than regular calls when leverage, F, is low, while regular calls induce higher risk taking as the firm becomes highly-leveraged.

Given that lookback calls are always in-the-money before maturity, one might think that they should entail more risk taking than regular calls. Interestingly, and as noted by Ju, Leland, and Senbet (2014) for a pure-equity firm, the entries of Tables 1 and 3 corresponding to different Λ's indicate that when regular calls induce too little risk taking ($\Lambda = 4$), lookback calls induce more, and when regular calls induce too much risk taking ($\Lambda = 0$) lookback calls induce less. Ju, Leland, and Senbet (2014) argue that this is because the delta of a lookback call is always greater than that of a regular call. In fact, they argue that the delta of a lookback call is always greater than 1. We contend that the authors are mistaken in this argument, i.e., the delta of a lookback call is never greater than 1 and, similarly, the delta of a lookback call is not always greater than that of a regular call. Appendix A proves that the delta of a lookback call is never greater than 1. Meanwhile, we show that the delta of a lookback call is not always greater than that of a regular call by computing it numerically. Table 4 illustrates the results.[9]

The results show that, for the set of parameters studied in this paper, the delta of the regular call is greater than that of a lookback call except when the regular call is deep-out-of-the-money for a pure-equity firm or out-of-the-money for the levered firm (not reported). These results are in stark contrast with the ones reported by Ju,

[9] We also compute the delta of regular and lookback calls resulting from randomly obtaining the strike price, time to maturity, implied volatility, interest rate and dividend yield 2500 times. In line with what is showed in Table 4, the delta of a regular call is greater than that of a lookback call in most cases (approximately 94% of the time). These results are available upon request.

Table 3 Risk effects of compensation contracts with lookback calls in a levered firm

	σ	$V_0(\sigma)$	VLB	$10^2 N_L$	TC	$10^3 \frac{\partial \mathbb{E}_{\mathbb{P}}[U]}{\partial V}$	10^3PPS
Base	0.322	98.816	47.859	0.066	1.410	6.099	3.579
$\gamma = 0$	0.556	89.303	55.546	0.312	11.070	11.891	11.891
$\gamma = 4$	0.238	92.980	38.735	0.000	7.188	15.917	2.879
$a = 10$	0.228	98.403	40.707	0.063	1.807	6.960	4.172
$a = 30$	0.296	98.540	45.914	0.065	1.680	6.343	3.746
$a = 70$	0.335	99.016	48.907	0.065	1.213	5.969	3.491
$a = 90$	0.343	99.161	49.568	0.065	1.070	5.888	3.436
$N_X = 0.2\%$	0.324	98.910	48.093	0.029	1.299	6.045	3.406
$N_X = 0.5\%$	0.320	98.760	47.723	0.087	1.476	6.127	3.677
$N_X = 1.0\%$	0.316	98.564	47.266	0.155	1.703	6.208	3.995
$N_S = 0.0\%$	0.431	99.096	55.286	0.101	0.959	2.376	0.787
$N_S = 0.2\%$	0.345	99.577	49.936	0.074	0.585	5.132	2.498
$N_S = 0.5\%$	0.298	97.668	45.545	0.058	2.653	7.044	5.247
$N_S = 1.0\%$	0.261	95.085	41.490	0.052	5.489	8.220	10.128
$f_{NC} = 0.0$	0.319	98.725	47.644	0.088	1.511	7.609	3.701
$f_{NC} = 0.5$	0.320	98.742	47.678	0.098	1.499	6.581	3.728
$f_{NC} = 1.0$	0.323	98.884	48.030	0.029	1.325	5.799	3.411
$K = 0.5V_0(\sigma)$	0.315	98.515	47.156	0.174	1.760	6.225	4.080
$K = 0.8V_0(\sigma)$	0.320	98.733	47.659	0.096	1.507	6.139	3.723
$K = 1.2V_0(\sigma)$	0.323	98.872	47.997	0.044	1.344	6.068	3.478
$K = 1.5V_0(\sigma)$	0.324	98.926	48.134	0.023	1.280	6.035	3.376
$NCW_0 = 0.2$	0.295	97.504	45.250	0.059	2.711	11.348	3.454
$NCW_0 = 0.5$	0.347	99.616	50.075	0.069	0.619	3.252	3.746
$NCW_0 = 1.0$	0.385	99.993	52.871	0.064	0.248	1.134	4.044
$F = 0$	0.301	97.857	49.536	0.212	2.561	4.589	4.767
$F = 10$	0.297	97.632	49.028	0.176	2.744	4.925	4.617
$F = 30$	0.294	97.422	48.250	0.118	2.880	5.644	4.315
$F = 50$	0.306	98.115	47.886	0.119	2.150	6.095	4.043
$F = 80$	0.349	99.670	46.331	0.094	0.547	5.986	3.237
$F = 95$	0.372	99.976	44.952	0.088	0.225	5.752	2.888
$F = 115$	0.393	99.944	42.463	0.146	0.265	5.626	2.745

Column 1 represents the value of a specific parameter, keeping the remaining parameters fixed at their base values. Columns 2–8 report the volatility chosen, the current firm value, the market value of one lookback call, the number of lookback calls, which is chosen to yield the same utility level as each corresponding entry in Table 1, the total cost to the firm, the partial derivative of the expected utility with respect to the initial firm value, and the PPS defined as the partial derivative of the manager's certainty equivalent with respect to the initial firm value, i.e., PPS = $\frac{\partial U^{-1}(\mathbb{E}_{\mathbb{P}}[U])}{\partial V}$, respectively

Leland, and Senbet (2014), who argue that risk averse (resp., risk neutral) managers take higher (resp., lower) risk when compensated with lookback options because the delta of a lookback call is always greater than that of a regular call. These puzzling results seem to suggest that lower (resp., higher) option delta induces risk averse

Table 4 Delta of regular calls and lookback calls

$V_0(\sigma)$	K	σ	Δ_R	Δ_L
95.8460	$V_0(\sigma)$	0.2705	0.7629	0.4757
89.5436	$V_0(\sigma)$	0.5538	0.7942	0.7074
92.8501	$0.5V_0(\sigma)$	0.2363	0.9798	0.4404
94.4325	$0.8V_0(\sigma)$	0.2532	0.8684	0.4581
96.7804	$1.2V_0(\sigma)$	0.2836	0.6641	0.4888
97.5166	$1.5V_0(\sigma)$	0.2953	0.5377	0.5004
97.8315	$1.8V_0(\sigma)$	0.3009	0.4342	0.5058
97.9207	$2.0V_0(\sigma)$	0.3025	0.3757	0.5074
97.9383	$2.5V_0(\sigma)$	0.3028	0.2593	0.5077

Column 1 represents the value of an unlevered ($F = 0$) firm when $\Lambda = 0$ (row 2) and when $\Lambda = 2$ (remaining rows). Columns 2 and 3 report the strike price and the firm volatility used in the calculations. The remaining parameters used in the calculation are $T = 5$ and $r = 0.05$. These parameters correspond to the ones used by Ju, Leland, and Senbet (2014). Columns 5 and 6 report the delta of a regular call and that of a lookback call, respectively, both computed numerically through the finite difference method

managers to take higher (resp., lower) risk. The reason for this is that if the delta of the option is low, the executive needs to make more effort to achieve a given level of utility than if the delta were high. Similarly, options with higher delta induce risk neutral managers to take higher risk because they are not too worried that their portfolio becomes too risky. Thus, from Table 4, as the regular call options become very deep-out-of-the-money, they will induce managers to take higher risk than lookback calls. To confirm this result, Table 5 and Fig. 3 illustrate the volatility chosen by the risk averse manager for different strike levels when compensated with regular calls or with lookback calls, both for levered and unlevered firms.

According to the results from Table 5 and Fig. 3, since the delta of lookback call is greater than that of a deep out of the money (resp., near-at-the-money or out-of-the-money) regular calls in the unlevered (resp., levered) firm, managers compensated with lookback calls take slightly less risk than those compensated with regular premium options.

3.9 The impact of Asian calls on the investment risk choice

As discussed in Sect. 2.7, the average value of the firm's assets has lower volatility and grows at a lower rate than the firm's asset value. This has several implications. First, an increase in firm volatility would reduce the expected return of the tracking asset. Second, the average firm value has lower volatility, $(\sigma/\sqrt{3})$, than the firm value, approximately 42.26%. This means that a risk averse manager may prefer to have her incentive pay tied to the average firm value since it makes her portfolio less risky. Table 6 illustrates the numerical results.

Table 5 Risk effects of compensation contracts with regular calls or lookback calls for different strike levels

K	σ	$V_0(\sigma)$	VLB	TC	$\mathbb{E}_{\mathbb{P}}[U(W_T)]\|10^2 N_L$	$10^3 \frac{\partial \mathbb{E}_{\mathbb{P}}[U]}{\partial V}$	10^3PPS
Panel A1: Unlevered firm - Regular call							
$0.5V_0(\sigma)$	0.236	92.850	57.136	7.663	−0.864	4.9053	6.577
$0.8V_0(\sigma)$	0.253	94.433	40.246	6.022	−0.942	4.7989	5.410
$1.2V_0(\sigma)$	0.284	96.780	26.585	3.630	−1.010	4.6607	4.569
$1.5V_0(\sigma)$	0.295	97.517	19.877	2.870	−1.039	4.6460	4.303
$1.8V_0(\sigma)$	0.301	97.831	14.927	2.538	−1.058	4.6609	4.167
$2.0V_0(\sigma)$	0.303	97.921	12.339	2.439	−1.066	4.6770	4.115
Panel A2: Unlevered firm - Lookback call							
$0.5V_0(\sigma)$	0.302	97.886	49.604	2.688	0.529	4.384	5.878
$0.8V_0(\sigma)$	0.302	97.903	49.644	2.560	0.305	4.528	5.105
$1.2V_0(\sigma)$	0.300	97.802	49.417	2.585	0.152	4.627	4.536
$1.5V_0(\sigma)$	0.299	97.713	49.219	2.647	0.095	4.661	4.317
$1.8V_0(\sigma)$	0.297	97.638	49.057	2.705	0.062	4.681	4.185
$2.0V_0(\sigma)$	0.297	97.597	48.969	2.738	0.047	4.690	4.126
Panel B1: Levered firm - Regular call							
$0.5V_0(\sigma)$	0.301	97.861	33.149	2.455	−1.235	7.1714	4.700
$0.8V_0(\sigma)$	0.317	98.622	24.741	1.665	−1.284	6.6954	4.060
$1.2V_0(\sigma)$	0.327	99.043	15.292	1.211	−1.321	6.4075	3.672
$1.5V_0(\sigma)$	0.331	99.155	9.198	1.077	−1.337	6.3207	3.536
$1.8V_0(\sigma)$	0.332	99.187	3.842	1.025	−1.348	6.2859	3.461
$2.0V_0(\sigma)$	0.331	99.185	0.642	1.015	−1.353	6.2785	3.431
Panel B2: Levered firm - Lookback call							
$0.5V_0(\sigma)$	0.315	98.515	47.156	0.174	1.760	6.225	4.080
$0.8V_0(\sigma)$	0.320	98.733	47.659	0.096	1.507	6.139	3.723
$1.2V_0(\sigma)$	0.323	98.872	47.997	0.044	1.344	6.068	3.478
$1.5V_0(\sigma)$	0.324	98.926	48.134	0.023	1.280	6.035	3.376
$1.8V_0(\sigma)$	0.325	98.960	48.219	0.009	1.241	6.013	3.310
$2.0V_0(\sigma)$	0.326	98.975	48.259	0.003	1.222	6.002	3.280

Column 1 reports the strike price, keeping other parameters fixed at their base values. Columns 2 – 8 report the volatility chosen, the current firm value, the market value of one regular or looback call, the total cost to the firm, the expected utility in case of regular call or the number of lookback options that yield the same utility as the regular call, partial derivative of the expected utility with respect to the initial firm value, and the PPS defined as the partial derivative of the manager's certainty equivalent with respect to the initial firm value, i.e., PPS $= \frac{\partial U^{-1}(\mathbb{E}_{\mathbb{P}}[U])}{\partial V}$, respectively

The results indicate that Asian calls are more effective than either regular or lookback calls in inducing managers to take higher risk. These results can be appreciated by using the fact that the average value of the firm's assets has lower volatility than the firm's asset value itself, as discussed earlier. This means that changes in firm volatility have a larger effect on lookback or regular calls than on Asian calls. To put it another way, regular and lookback calls are more sensitive to volatility swings

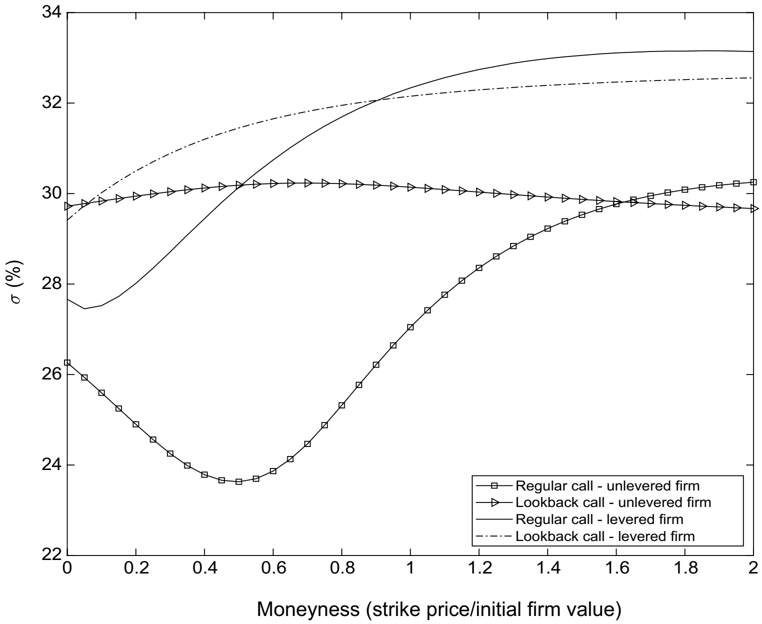

Fig. 3 Risk effects of regular calls or lookback calls in a levered or unlevered firm The figure plots the volatility level (σ) chosen by the executive as a function of the moneyness of her regular or lookback call options, in the context of both levered and unlevered firms

than Asian calls. Consequently, managers compensated with Asian options have higher incentives to increase firm risk because their portfolios are less risky than those of managers compensated with lookback or regular calls. Therefore, when σ is below σ_0, Asian call holders have preference for higher σ since this results in higher firm and option value. The results also indicate that Asian options are more cost-effective than both regular and lookback calls.

Our results provide clear-cut evidence that the adoption of Asian options might have countervailing effects and provide stronger incentives to take risks. Moreover, Asian calls make it less likely for managers to commit fraud by manipulating the firm value or taking advantage of inside information since the payoff is based on the average firm value over the life of the option instead of a single date as in the case of power options, regular calls or lookback calls.

Using compensation data of CEOs at 966 firms from 2007 to 2011, Sun and Shin (2014), drawing on attribution theory, show that firms are likely to issue new options to CEOs with underwater options when the stock price decline is due to exogenous shocks, beyond the CEO's control, namely recessions and low industry performance. It is believed that the issue of new options may recover CEOs incentives to maximize firm value by adopting all positive NPV projects. However, this comes with additional costs to the firm (Bodie, Kaplan, & Merton, 2003). We thus argue that Asian options may be a preferable choice as the terminal payoff is based on the average value instead of the terminal one. More specifically, even when the stock price plunges are due to factors external to the firm, the CEO's incentives to maximize

Table 6 Risk effects of compensation contracts with Asian calls in a levered firm

	σ	$V_0(\sigma)$	VA	$10^2 N_A$	TC	$10^3 \frac{\partial \mathbb{E}_{\mathbb{P}}[U]}{\partial V}$	10^3PPS
Base	0.345	99.568	17.429	0.098	0.835	6.209	3.644
$\gamma = 0$	0.537	91.481	19.535	0.918	8.978	10.693	10.693
$\gamma = 4$	0.230	92.217	13.359	0.019	8.201	22.881	3.177
$a = 10$	0.280	99.310	15.718	0.111	1.130	6.777	4.062
$a = 30$	0.330	99.472	17.043	0.103	0.938	6.340	3.744
$a = 70$	0.353	99.639	17.635	0.096	0.759	6.139	3.590
$a = 90$	0.358	99.692	17.764	0.095	0.704	6.095	3.556
$N_X = 0.2\%$	0.336	99.341	17.188	0.056	1.057	6.281	3.539
$N_X = 0.5\%$	0.348	99.653	17.532	0.124	0.753	6.174	3.706
$N_X = 1.0\%$	0.356	99.805	17.749	0.211	0.613	6.078	3.912
$N_S = 0.0\%$	0.419	99.460	19.101	0.344	0.606	2.065	0.684
$N_S = 0.2\%$	0.378	99.998	18.281	0.126	0.257	5.290	2.575
$N_S = 0.5\%$	0.311	98.329	16.372	0.076	2.300	7.151	5.327
$N_S = 1.0\%$	0.265	95.393	14.667	0.048	5.893	8.315	10.245
$f_{NC} = 0.0$	0.350	99.683	17.572	0.106	0.719	7.292	3.546
$f_{NC} = 0.5$	0.349	99.673	17.558	0.105	0.729	6.333	3.588
$f_{NC} = 1.0$	0.339	99.430	17.277	0.092	0.974	6.289	3.699
$K = 0.5V_0(\sigma)$	0.337	99.355	45.314	0.094	1.213	6.339	4.155
$K = 0.8V_0(\sigma)$	0.339	99.410	26.234	0.091	1.063	6.290	3.814
$K = 1.2V_0(\sigma)$	0.348	99.640	11.382	0.111	0.712	6.159	3.530
$K = 1.5V_0(\sigma)$	0.352	99.730	5.993	0.140	0.586	6.093	3.408
$NCW_0 = 0.2$	0.314	98.491	16.487	0.070	1.915	11.347	3.454
$NCW_0 = 0.5$	0.371	99.969	18.113	0.133	0.430	3.410	3.929
$NCW_0 = 1.0$	0.407	99.750	18.888	0.203	0.647	1.273	4.540
$F = 0$	0.327	99.034	16.909	0.346	1.612	4.723	4.906
$F = 10$	0.321	98.813	16.727	0.283	1.793	5.080	4.762
$F = 30$	0.316	98.578	16.550	0.185	1.945	5.805	4.438
$F = 50$	0.340	99.438	17.286	0.175	1.008	6.101	4.047
$F = 80$	0.377	99.997	18.269	0.118	0.352	6.030	3.261
$F = 95$	0.395	99.918	18.667	0.102	0.395	5.903	2.964
$F = 115$	0.419	99.483	19.087	0.138	0.803	5.666	2.764

Column 1 represents the value of a specific parameter, keeping the remaining parameters fixed at their base values. Columns 2-8 report the volatility chosen, the current firm value, the market value of one Asian call, the number of Asian calls, which is chosen to yield the same utility level as each corresponding entry in Table 1, the total cost to the firm, the partial derivative of the expected utility with respect to the initial firm value, and the PPS defined as the partial derivative of the manager's certainty equivalent with respect to the initial firm value, i.e., PPS $= \frac{\partial U^{-1}(\mathbb{E}_{\mathbb{P}}[U])}{\partial V}$, respectively

firm value may still be intact given the fact that she knows that her option's payoff depends on the firm's performance during the entire life of the option instead of a single point in time. This precludes the need of issuing new options to the executive. Hence, the ultimate message of the present paper is that the board of directors should consider the adoption of Asian options when designing executives compensation plans.

4 Concluding remarks

This paper is concerned with the risk incentives provided by traditional and non-traditional stock options in a more realistic setting where the firm is financed with both equity (stock and options) and debt. We show that a moderate degree of leverage and equity-based compensation can be effective in incentivizing risk averse managers to adopt risky but positive NPV projects. These results seem to suggest that some degree of risk-shifting does necessarily hurt bondholders since the manager is endeavoring to optimize the firm value. Contrary to what was concluded by Ju, Leland, and Senbet (2014) for an unlevered firm, we find that managers usually prefer a combination of company shares and regular call options as part of their compensation package in order to obtain a certain level of utility.

Unlike the case of a pure-equity firm where lookback calls are strictly preferred to regular calls, we show that regular calls outperform lookback calls in several cases. These results indicate that lookback calls are not as effective as they were deemed to be in the literature. We argue that Asian calls are a superior remedy to alleviate the agency costs of deviating from the optimal volatility level. Asian calls are also more cost-effective than regular and other nonstandard options. As opposed to what has been asserted in the literature (see (Ju, Leland, & Senbet, 2014)), we document that the delta of a lookback call is not always greater than that of the regular call. This conclusion has important implications on the interpretation of the role of delta in inducing managers to optimize firm value. In particular, we show that, *ceteris paribus*, when the delta of the option is low (resp., high), managers have greater (resp., lower) incentives to change the firm's risk towards the optimal level.

Appendix A. Delta of a lookback call

We prove that the delta of a lookback call is not greater than 1, in opposition to what was argued by Ju, Leland, and Senbet (2014). As usual, we assume no dividends and that the underlying asset price dynamics follows, under the risk neutral measure $\mathbb{Q}$, the geometric Brownian motion

$$\frac{dS_t}{S_t} = r dt + \sigma dB_t, \tag{A.1}$$

where r is the risk-free interest rate, $\sigma \in \mathbb{R}_+$ is a constant and $\{B_t, t \geq 0\}$ is a standard Brownian motion. Let $m_t^T = \inf_{t \leq u \leq T} (S_u)$ be the minimum price during the time-interval $[t, T]$. The terminal payoff of a lookback call is $S_T - m_0^T$. The price of a lookback call at time t is obtained as the discounted risk neutral expectation of its terminal payoff:

$$C_{LC}(S_t) = e^{-r(T-t)}\mathbb{E}_{\mathbb{Q}}\left[S_T - m_0^T \middle| \mathcal{F}_t\right] = S_t - e^{-r(T-t)}\mathbb{E}_{\mathbb{Q}}\left[\min(m_0^t, m_t^T)\middle|\mathcal{F}_t\right]. \tag{A.2}$$

Let $f(x)$ and $F(x)$ be the risk neutral density and cumulative density, respectively, of the minimum price during $[t, T]$ for the process (A.1). Thus, we can write

$$C_{LC}(S_t) = S_t - e^{-r(T-t)}\int_0^{m_0^t} xf(x)dx - e^{-r(T-t)}m_0^t\int_{m_0^t}^{S_t} f(x)dx. \tag{A.3}$$

Now, if we change the price at time t from S_t to $S_t + \epsilon S_t$, for $\epsilon \in \mathbb{R}_+$, the terminal price will be $(1+\epsilon)S_T$ and the minimum during $[t, T]$ will be $(1+\epsilon)m_t^T$. We thus have that the lookback call price is

$$\begin{aligned} C_{LC}(S_t + \epsilon S_t) &= S_t(1+\epsilon) - e^{-r(T-t)}\mathbb{E}_{\mathbb{Q}}\left[\min\left(m_0^t, m_t^T(1+\epsilon)\right)\middle|\mathcal{F}_t\right] \\ &= S_t(1+\epsilon) - (1+\epsilon)e^{-r(T-t)}\mathbb{E}_{\mathbb{Q}}\left[\min\left(\frac{m_0^t}{1+\epsilon}, m_t^T\right)\middle|\mathcal{F}_t\right], \end{aligned} \tag{A.4}$$

i.e.,

$$\begin{aligned} C_{LC}(S_t + \epsilon S_t) = S_t(1+\epsilon) &- (1+\epsilon)e^{-r(T-t)}\int_0^{\frac{m_0^t}{1+\epsilon}} xf(x)dx \\ &- (1+\epsilon)e^{-r(T-t)}m_0^t\int_{\frac{m_0^t}{1+\epsilon}}^{S_t} f(x)dx. \end{aligned} \tag{A.5}$$

From Eqs. (A.3) and (A.5), the change of the lookback call price is given by

$$\begin{aligned} C_{LC}(S_t + \epsilon S_t) - C_{LC}(S_t) = \epsilon S_t &+ e^{-r(T-t)}\int_{\frac{m_0^t}{1+\epsilon}}^{m_0^t} xf(x)dx - \epsilon e^{-r(T-t)}\int_0^{\frac{m_0^t}{1+\epsilon}} xf(x)dx \\ &- e^{-r(T-t)}m_0^t\int_{\frac{m_0^t}{1+\epsilon}}^{m_0^t} f(x)dx - \epsilon e^{-r(T-t)}m_0^t\int_{\frac{m_0^t}{1+\epsilon}}^{S_t} f(x)dx. \end{aligned} \tag{A.6}$$

The delta of the lookback call is thus equal to

$$
\begin{aligned}
\Delta &= \lim_{\epsilon\to 0}\frac{C_{LC}(S_t+\epsilon S_t)-C_{LC}(S_t)}{\epsilon S_t}\\
&= 1+\lim_{\epsilon\to 0}\frac{e^{-r(T-t)}}{\epsilon S_t}\int_{\frac{m_0^t}{1+\epsilon}}^{m_0^t}(x-m_0^t)f(x)dx-\lim_{\epsilon\to 0}\frac{e^{-r(T-t)}}{S_t}\int_0^{\frac{m_0^t}{1+\epsilon}}xf(x)dx\\
&\quad-\frac{e^{-r(T-t)}}{S_t}m_0^t\lim_{\epsilon\to 0}\int_{\frac{m_0^t}{1+\epsilon}}^{S_t}f(x)dx\\
&= 1+\frac{e^{-r(T-t)}}{S_t}\lim_{\epsilon\to 0}\left[\frac{\left(\frac{m_0^t}{1+\epsilon}-m_0^t\right)f\left(\frac{m_0^t}{1+\epsilon}\right)m_0^t}{(1+\epsilon)^2}\right]\\
&\quad-\frac{e^{-r(T-t)}}{S_t}\left[m_0^tF(m_0^t)-\int_0^{m_0^t}F(x)dx+m_0^tF(S_t)-m_0^tF(m_0^t)\right]\\
&= 1-\frac{e^{-r(T-t)}}{S_t}\left[m_0^tF(m_0^t)-\int_0^{m_0^t}F(x)dx+m_0^t\big(F(S_t)-F(m_0^t)\big)\right].
\end{aligned}
\tag{A.7}
$$

Given that $m_0^tF(m_0^t)=\int_0^{m_0^t}F(m_0^t)dx\geq\int_0^{m_0^t}F(x)dx$ for all $x\in[0,m_0^t]$ and $F(S_t)\geq F(m_0^t)$ (because $F(.)$ is non-decreasing and $S_t\geq m_0^t$), $\Delta<1$. □

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Declarations

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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