PRICING FOR SCARCITY

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Abstract

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Abstract

In many areas where water is not abundant, water pricing schedules contain significant nonlinearities. Existing pricing literature establishes that efficient schedules will depend on demand and supply characteristics. However, most empirical studies show that actual pricing schemes have little to do with theoretical efficiency results. In particular, there are very few models recommending increasing blocks, whereas we present evidence that this type of tariff structure is abundantly used. Water managers often defend increasing blocks, both as a means to benefit smaller users and as a way to signal scarcity.

Naturally, in the presence of water scarcity the true cost of water increases due to the emergence of a scarcity cost. In this paper, we incorporate the scarcity cost associated with insufficient water availability into the optimal tariff design in several different models. We show that when both demand and costs respond to climate factors, increasing marginal prices may come about as a combined result of scarcity and customer heterogeneity under specific conditions.

We also investigate the effect that rising water scarcity in the long run can have on the steady-state amount of capital invested in water storage and supply infrastructures and obtain some results that are consistent with the static models.

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Keywords: water pricing; nonlinear pricing; increasing block tariffs; water scarcity.

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1 Introduction[1]

In many areas where water is not abundant, water pricing schedules contain significant nonlinearities. When adequate distribution networks exist, utilities tend to be local natural monopolies, consumers cannot choose multiple connections and resale is tricky. Thus it is easy, and often politically expedient, for utilities to undertake extensive price discrimination, both for distinct types of consumers (residential, industrial, agricultural, and so on) and for different levels of consumption within each consumer type. Many utilities use two-part tariffs, with fixed meter charges and a constant unit price, or multipart tariffs, which combine fixed charges and increasing or, less often, decreasing blocks. Occasionally, seasonal price variations are employed to reflect changes in water availability throughout the year. Less common is the imposition of a scarcity surcharge during drought periods, regardless of the season. In extreme droughts water rationing is generally preferred.

This paper presents some relevant characteristics of existing water tariffs (Section 2), focusing on Portuguese tariffs for the residential sector. As expected, tariffs are usually composed by both a meter charge and a volumetric price, but the latter almost always consists of increasing block tariffs (IBT). More surprisingly, considering the well-known significant seasonal differences in water availability in the country, seasonal surcharges or seasonal price variations are not common in Portuguese water tariffs. Moreover, the few that do exist seem to be uncorrelated with regional characteristics in terms of seasonal water scarcity. It should also be emphasized that many utilities incorporate a number of further complications in their water rate calculations, enabling us to say that complexity is definitely the prevailing feature of water tariffs in Portugal. For other countries, the trend towards increasing blocks is also present, as noted in several publications.

It seems that the reasons why most water managers continue to defend increasing blocks are their ability to benefit smaller users and their potential role in signalling scarcity. Although, in the presence of water scarcity, the true cost of water increases due to the emergence of a scarcity cost, it is unclear whether increasing block tariffs are the best way to make consumers

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understand and respond to water scarcity situations, especially when the resulting tariffs are very complex.

In contrast, most results found in the literature on efficient tariff design do not generally recommend increasing price schedules. Only part of the abundant literature on water pricing provides efficiency results, since most studies either compare the properties of different possible price schemes, estimate water demand, or point out the difficulties in implementing more efficient pricing rules. Section 3 summarizes the main efficiency results, indicating justifications for increasing block rates whenever they appear, and noting that none of them is directly related to scarcity.

Current analysis of this issue is specially relevant considering that the Water Framework Directive requires that by 2010 (art.9, n.1) pricing policies in the European Union's member states not only recover the costs of the resource (including enviromental and scarcity costs) but also provide adequate incentives for consumers to use water efficiently, contributing to the attainment of environmental quality targets. In particular, the problem of water scarcity is now recognized by the European Commission as an increasingly relevant one in the face of the increased frequency of extreme climate events that may occur because of climate change, as can be seen in a recent Communication that was issued on the topic (EC (2007)).

This paper proposes different models of efficient and second-best nonlinear prices under scarcity constraints, and concludes that, when both demand and costs respond to climate factors, increasing marginal prices may come about as a combined result of scarcity and customer heterogeneity under specific conditions, even if nonlinear pricing is a consequence of customer heterogeneity and not of water scarcity. Finally, we use a dynamic model to analyze the simultaneous decision on pricing and investment by a public utility and to investigate the effect that rising water scarcity, brought about for instance by global warming, can have on the steady-state amount of capital invested in water storage and supply infrastructures, and conclude that some results are similar to the ones from the previous static models.

2 Existing water tariffs

In 2005, the Portuguese National Water Institute (INAG) released results for the National Survey on Water and Wastewaster Systems for 2002 (recently updated for 2005). While previous surveys had focused only on the water and sewage infrastructures, this one began a systematic gathering of economic information. The INSAAR database contains economic data on the management model followed by water utilities, on investments for the period 1987-2005, and on costs, revenues, prices and quantities of water delivered (to customers or to other water utilities) for the years 1998, 2000, 2002 and 2005. This section provides a brief description of economic data for the year 2005, focusing on the domestic water supply component[2].

The data indicates that 97.5% of water supply tariffs in Portugal are composed of a fixed charge and a volumetric rate. The fixed charge is dependent on the diameter of the pipe. All the 278 water utilities responsible for public water supply at the municipal level and which provided information on tariffs have volumetric rates in their tariffs. Moreover, all but three of the them apply IBT (a few self-supplying organizations and tourist resorts also practice flat rate volumetric prices). The average number of blocks is 5, but it can be as high as 30 in some extreme cases. The majority of utilities using block tariffs charges the volume within each block. Nevertheless, 16.5% of them use a different way to calculate the final tariff, by charging all volume at the price of the last block reached by metered consumption in the period[3]. This causes the marginal price faced by the consumer to have significant peaks at the block limits. In this pricing system, the first cubic meter within a block can cost a consumer several times the price of the previous and the next unit, something that will hardly be clear to the average consumer from the information in the water bill.

The popularity of increasing block tariffs is not a Portuguese peculiarity. Hoffmann, Worthington and Higgs (2006) mentions "the trend in most OECD economies towards metering, increasing block prices and reduced

[2] Because the INSAAR database suffers from a strong presence of missing values, additional data has been requested by the authors directly to the water utilities to fill in the information gaps. The statistics reported in this paper already reflect such data collection and improvement.

[3] An additional 1.4% combine both calculation procedures in the tariff schedule applying one or the other according to the block of consumption reached.

subsidies for residential water supply", as reported by Dalhuisen, Groot and Nijkamp (2001) to the European Commission in 2001. The OECD itself not only reports the growing use of IBT by stating that "there is evidence that the use of such tariffs [IBT] is increasing" (OECD (2003*a*)), but also seems to support their use by saying that "there seem to be clear potential benefits from increasing block tariff structure" (OECD (2003*a*)). Bartoszczuk and Nakamori (2004) point out that "the strong tradition of low tariffs for households and increasing block rates is present in Belgium, Italy, Greece, Portugal, Spain and US". With the Belgian exception, we find very similar climate conditions in these countries (or parts of them, given the size of the US). The use of IBT in these and other countries is also well documented in several OECD reports (OECD (2006), OECD (2003*a*), OECD (2003*b*) , OECD (1999*a*), OECD (1999*b*) or OECD (1999*c*)). One of the advantages of IBT, pointed out by several authors and also in the OECD reports, is related with affordability for poorer households. Nonetheless, it should be noted that in Portugal water expenses fall below 1% of average disposable income (Roseta-Palma, Monteiro, Meireles, Mestre and Sugahara (2006)). Furthermore, the affordability argument cannot explain the use of a large number of blocks.

One feature we would expect to see in Portuguese water tariffs given the variable weather conditions, which include significant seasonal weather differences, namely in rainfall, and the existence of drought-prone regions, is seasonal surcharges. However, no more than 3% of water utilities use such tools in their water tariffs. Moreover, their location seems unrelated to the water availability problems in the country, with most of them being located in the wetter regions of the coastal northwest of the country. The few seasonal surcharges we do find are in place during the summer months and typically raise the price of the higher blocks between 30%-50%.

It is clear that simplicity is not a prevalent feature of Portuguese water tariffs. The calculation process of the IBT (volume charged within each block or at the price of the last block) can be mixed in some utilities, depending on the consumption block. Tariffs can combine blocks with flat (nonvolumetric) fees within some blocks with volumetric rates for others. Specific formulas are sometimes applied within the blocks to find the unit price. Water availability charges that are fixed within each block, but variable among blocks, are sometimes levied and added to the price. Some utilities practice social tariffs for disadvantaged households or, apart from the usual tariff differentiation by customer class, propose special contracts with different prices to various types of specific consumers (from farmers, factories or services to schools, sporting clubs or nonprofit organizations, to name a few). Furthermore, additional complications can be found in wastewater price schedules.

Finally, it should be noted that the 87% value for the national cost recovery level for water supply falls below 100% (considering only financial costs), and the situation is even worse for wastewater drainage and treatment services with a value of 57% (INAG/MAOTDR (2007)). This can be explained by the fact that some utilities do not charge for wastewater at all, while others make the payment dependent of variables such as apartment area; number of inhabitants/beds/rooms, real estate value of the house or building or taxable income. The majority of wastewater utilities levy at least some of their charges based on water consumption levels, so that both payments are part of the water bill.

A more detailed analysis of the costs and revenues of the Portuguese water supply and wastewater industry can be found in Monteiro (2007). Monteiro and Roseta-Palma (2007) present an in-depth description of the existing tariff structures, including all customer classes.

3 Efficient water pricing literature

In this section, we review the literature on water pricing, focusing on the results dealing with nonlinear pricing, scarcity and seasonal rates[4]. Several important issues are not specific to the water sector: marginal cost pricing, capacity constraints, resource scarcity, revenue requirements or nonlinear pricing are significant in the more general framework of regulated public utilities, as is clear from books like Brown and Sibley (1986) and Wilson (1993). However, such issues appear in this sector combined with some of its peculiarities, such as the large capital investments which turn suppliers into local natural monopolies, the seasonal and stochastic variability of the resource it aims to supply and the essential value of the good for its

[4] A more detailed survey can be found in Monteiro (2005).

consumers.

The first question to be addressed in the water pricing literature was the incompatibility between the marginal pricing recommendation from microeconomics and the average cost pricing practice in the water industry. Although cost recovery is an important goal, so that average costs are clearly paramount in the utilities' actual rate setting, the idea, stressed many times by economists, is that more attention needs to be paid to marginal costs. A water user will decide whether or not to consume an additional unit by comparing the benefit associated to that unit with its price (which may or may not be the same as average price, depending on the rate structure). Therefore, in the absence of external effects, social net benefits should be maximized when the price per unit is equal to the marginal cost of supplying the water. This literature dates back to the 60's (Hirshleifer, de Haven and Milliman (1960)), but despite the overwhelming evidence in favour of marginal cost pricing as a more efficient pricing tool, the discussion has not fully subsided. Briand (2006), for example, uses a dynamic computable general equilibrium model to question the application of average cost pricing in Senegal. Moreover, there are more efficient ways of achieving a balanced budget than average cost pricing. For example, two-part tariffs can separate the recovery of fixed and variable costs through fixed charges and volumetric rates on water consumption. Second-best Ramsey pricing can, as shown in the following sections, differentiate price according to the customers' price-elasticities of demand, charging higher tariffs to customer types that respond less to price changes. This technique allows the utility to recover costs while sacrificing as little welfare as possible.

Since marginal cost pricing does not ensure that the water utility will break even, as average-cost pricing does, the harmonisation of efficiency with the balancing of the utility's budget has been the subject of much attention. Collinge (1992), Kim (1995), Griffin (2001) and Schuck and Green (2002) have all dealt with this question. While Collinge (1992) works out a way to return excess profit to consumers through tradable discount coupons (arguing his method does not require the utility to gather information on water demand), Kim (1995) relies on Ramsey second-best pricing to ensure that a two product utility producing residential and non-residential water collects enough revenue to meet its costs. Schuck and Green (2002) also base their analysis on a Ramsey pricing rule, while Griffin (2001) proposes a threshold on water consumption to be added to a two-part tariff, generating credits to the consumer below the threshold (as in Collinge (1992), the aim is to return excess profits).

The importance of price differentiation, according to the type of customer or the season of the year, is another question that is covered in the literature. Temporal price variation in particular has been analysed by several authors, who have pointed out the advantages of having intra-annual price changes to reflect differences in marginal costs, with the aim of enhancing efficiency (an early example is Gysi and Loucks (1971)). A more recent paper is Schuck and Green (2002), which presents a supply-based water pricing model (where price changes with water availability). It uses a conjunctive use system for farming with stochastic surface water flows and combines it with second-best (Ramsey) water pricing. It considers the possibility of recharging the aquifer with excessive surface water in bountiful years, although not without a cost. The authors use simulation techniques to test their model on a Californian water district using land, water and energy, and conclude that a supply-based pricing policy reduces the use of these three resources in periods of drought.

The analysis of capacity constraints on water supply and the related issued of optimal timing for system expansion is another subject that dates back to the 60's and 70's, when the problem of supplying enough water to meet the needs was mostly seen as a problem of increasing capacity (Riordan (1971), Riley and Scherer (1979), or Manning and Gallagher (1982), are examples of authors dealing with these issues). The problem of water storage is related to the problem of resource variability, resulting either from expected seasonal rainfall variability or from the more uncertain occurrence of longer periods of drought, which can alternate with plentiful rain or even floods.

The scarcity of the resource itself is a more recent concern in the literature. It has accompanied a change in water managers' concerns, from water supply increase to water demand management. Moncur and Pollock (1988) deal with the problem of determining the scarcity rent of water. They consider the case of a water utility with groundwater as its only source, and use a nonrenewable resource efficient extraction model to determine the scarcity value and the efficient path of price in the future. They calculate the scarcity value through the consideration of the future increase in costs originated by the necessity to use costly backstop technologies (such as desalination or trans-basin diversions) to satisfy water demand. They apply their model to Honolulu and find the scarcity value to be approximately twice the current water charge. Elnaboulsi (2001) includes a constraint on the water available which, when binding, allows the determination of the shadow value of water resources to be included in the price. Griffin (2001) demonstrates that the price should include opportunity costs such as the marginal user cost of water (for renewable or non-renewable sources) and the marginal capacity cost. This issue will be developed in the following sections.

Finally, in relation to nonlinear prices, while we can find examples of authors who support the use of increasing block tariffs for water (Gysi and Loucks (1971) is, again, an early example), such support is based on distributional considerations and not on efficiency. Cardadeiro (2005) is a partial exception. He introduces a social benefit of universal access, through the consideration of a positive externality for the first few liters/person/day, due to public health improvements. The existence of only two blocks in the tariff is imposed on the model, as it is argued that such an externality makes sense only for those first few liters. The result, as expected, is that social welfare can be maximized by setting the first block price lower than the second. In another of the rare water pricing models applying nonlinear pricing, Elnaboulsi (2001) develops a model of optimal nonlinear pricing of water and wastewater services, considering the issues of temporal variation, capacity constraints, scarcity and consumer heterogeneity. He concludes that the marginal price should be constant or decreasing, in which case a menu of two-part tariffs can be constructed in such a way that it would be equivalent to offering consumers quantity discounts.

4 Scarcity in a simple model

A simple view of the main aspects of efficiency in water prices is presented by Griffin (2001) and Griffin (2006). His model includes three pricing components: the volumetric (ie. per unit) price, the constant meter charge and the one-off connection charge. The latter is meant to reflect network expansion costs and will not be considered in our model.[5] We focus on the volumetric part of the tariff, not taking into account the two-part tariff case. On the other hand, he assumes a single volumetric price and does not allow for more general nonlinear prices, as neither consumer heterogeneity nor purchase size cost dependency are taken into account. In fact, Griffin (2001) stresses "the inefficiencies of block rate water pricing" (pp. 1339 and 1342).

A static model for different (identified) consumer groups, with a scarcity constraint, shows that the marginal cost pricing rule still holds. Define $B_j(w_j)$ as the increasing and concave monetized benefit of water consumption for consumer group j, with $j = 1, ..., J$ and $C(w)$ as the (convex) water supply costs[6], which depend on the total water supplied, ie. $w = \sum_{j=1}^{J} w_j$. Water availability is limited, with the maximum amount denoted as W. The welfare maximization problem is

$$\begin{gathered} \underset{\{w_j\}}{Max} \quad \sum_{j=1}^{J} B_j(w_j) - C(w) \\ s.t. \sum_{j=1}^{J} w_j \leq W \end{gathered} \tag{1}$$

resulting in first order conditions[7]

$$\frac{dB_j}{dw_j} = \frac{dC}{dw} + \mu \qquad \forall_j \tag{2}$$

$$\sum_{j=1}^{J} w_j \leq W, \quad \mu \geq 0, \quad \mu(W - \sum_{j=1}^{J} w_j) = 0 \tag{3}$$

where μ is the Lagrangean multiplier and it is assumed that all w_j are positive (every consumer requires a minimum amount of water). The efficiency result, expressed in equation (2), indicates that the marginal benefit of water consumption should be equal to marginal costs (including scarcity costs if the constraint is binding). Also, the marginal benefit needs to be the same across consumers, since marginal cost is the same. Finally, with a unit price

[5] Access to water supply networks is nearly universal in Portugal by now, with 92,3% nationwide connection rates and 100% in urban areas IA/MAOTDR (2006).

[6] We do not explicitly consider fixed costs for simplicity, because they do not change the conclusions.

[7] There are no cross effects in demand, ie. $\frac{dB_j}{dw_i} = 0$ for $i \neq j$.

p_j the benefit maximization problem for each consumer is

$$\underset{w_j}{Max} \quad B_j(w_j) - p_j w_j \tag{4}$$

$$\Leftrightarrow \quad \frac{dB_j}{dw_j} = p_j \tag{5}$$

so that the efficient unit price must be the same for all consumers and is given by

$$p = \frac{dC}{dw} + \mu \tag{6}$$

as in Griffin (2006).[8] The lower the W the tighter the constraint, meaning that price should rise to reflect increasing scarcity. However, this rule does not ensure that the water utility's budget is balanced, namely if there are fixed costs or if marginal cost is not constant. Although a fixed meter component could be adjusted to reflect such concerns, the second-best pricing rule is obtained by imposing a break-even constraint such as (7) on problem (1). This is known as Ramsey pricing. Note that $p_j(w_j)$ is now the inverted demand of consumer j.

$$\sum_{j=1}^{J} p_j(w_j)w_j - C(w) = 0 \tag{7}$$

Using equation (5), the welfare maximizing prices will now be given by

$$\frac{p_j - \left(\frac{dC}{dw} + \frac{\mu}{1+\lambda}\right)}{p_j} = \frac{\lambda}{1+\lambda}\frac{1}{\xi_j\left(w_j^*\right)} \tag{8}$$

where ξ_j is the absolute value of the price elasticity of j's demand and λ is the Lagrange multiplier of (7). This is a version of the so-called Inverse Elasticity Rule, which states that the mark-up of prices over marginal cost will be inversely related to the demand elasticity, so that consumers with lower demand elasticities will pay higher prices and vice-versa. The only new term is $\frac{\mu}{1+\lambda}$, which reflects the scarcity cost. It adds to the price faced by the consumer the opportunity cost of using a scarce resource, but it does not affect the shape of the price schedule. Nonlinear prices may arise in this model because of heterogeneity in the consumers' preferences (different price-elasticities), not because of scarcity. Nonlinear prices would be

[8] The same result can be obtained with the more complicated pricing formula from Griffin (2001). In that case the bill paid by each consumer is given by $Bill_j = M + p(w_j - \overline{w})$, where M is the meter charge and $\overline{w}$ is a budget-balancing parameter.

increasing if the price-elasticities decrease with higher optimal consumption choices and decreasing otherwise. It should be noted that if the scarcity cost is defined as a tax which the supplier collects but does not keep, along the lines of what is already done in some European countries, the model will have to be changed accordingly. This is particularly important when several suppliers share available water, since none of them will adequately provide for external scarcity costs.

5 Scarcity with a distribution of consumer types

In this section a more complete model is presented, explicitly characterizing demand behavior through the definition of a continuum of consumer types. Model development is based on Brown and Sibley (1986) as well as Elnaboulsi (2001). A new parameter, θ, is introduced to reflect differences in consumer tastes, which can encompass a number of variables, including income, family size, or housing. A consumer with tastes given by θ will now enjoy net benefits of $B(w,\theta) - P(w)$, where $P(w)$ is the total payment for water consumption. It is assumed that $B(0,\theta) = 0$ and that high values of θ imply higher consumption benefits ($\frac{\partial B}{\partial \theta} > 0$, $\frac{\partial^2 B}{\partial \theta \partial w} > 0$). The distribution of θ throughout the consumer population is described by a distribution function $G(\theta)$ and the associated density function $g(\theta)$. Maximum and minimum values for the taste parameter are represented by $\overline{\theta}$ and $\underline{\theta}$, respectively, so that $G(\overline{\theta}) = 1$ and $G(\underline{\theta}) = 0$.

The first order condition of each consumer's net benefit maximization is

$$\frac{\partial B(w,\theta)}{\partial w} = \frac{dP}{dw} \equiv p_m \tag{9}$$

which is similar to condition (5) except the right-hand side represents the slope of the total payment function, i.e. the marginal price p_m. The only restriction to the shape of $P(w)$ is that, if concave, it must be less so than the benefit function to ensure that the decision is indeed a maximizing one. Using the consumer's choice, $w(\theta)$, the value function is

$$V(\theta) = B(w(\theta),\theta) - P(w(\theta)) \tag{10}$$

To find the properties of the optimal payment function with a scarcity restriction, or rather the second best function given the break-even constraint, the following problem can be solved

$$\begin{array}{ll} \underset{w(\theta)}{Max} & \int_{\underline{\theta}}^{\overline{\theta}} V(\theta)g(\theta)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} \left[P\left(w(\theta)\right) - C(w(\theta))\right] g(\theta)d\theta \\ s.t. & \begin{array}{c} \int_{\underline{\theta}}^{\overline{\theta}} \left[P\left(w(\theta)\right) - C(w(\theta))\right] g(\theta)d\theta = 0 \\ \int_{\underline{\theta}}^{\overline{\theta}} w(\theta)g(\theta)d\theta \leq W \end{array} \end{array} \tag{11}$$

where the first component of the objective function represents consumer surplus aggregating all consumer types, and the second component is profit. Some manipulations yield a more tractable version of the problem. Substituing $P\left(w\left(\theta\right)\right)$ using equation (10), noting that $G(\theta) - 1 = \int g(\theta)d\theta$ and using the envelope theorem to see that $\frac{\partial V}{\partial \theta} = \frac{\partial B}{\partial \theta}$, consumer surplus can be rewritten using integration by parts

$$\int_{\underline{\theta}}^{\overline{\theta}} V(\theta)g(\theta)d\theta = V(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial B}{\partial \theta}(1 - G(\theta))d\theta \tag{12}$$

and the Lagrangean that must be maximized is

$$\begin{aligned} \mathcal{L} &= V(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial B}{\partial \theta}(1 - G(\theta))d\theta + (1+\lambda)\int_{\underline{\theta}}^{\overline{\theta}} \left(B\left(w\left(\theta\right), \theta\right) - V(\theta) - C(w(\theta)\right) g(\theta)d\theta \\ &\quad + \mu\left(W - \int_{\underline{\theta}}^{\overline{\theta}} w(\theta)g(\theta)d\theta\right) &(13) \\ &= -\lambda V(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} (1+\lambda)\left(B\left(w\left(\theta\right), \theta\right) - C(w(\theta)\right) g(\theta) - \lambda \frac{\partial B}{\partial \theta}(1 - G(\theta))d\theta \\ &\quad + \mu\left(W - \int_{\underline{\theta}}^{\overline{\theta}} w(\theta)g(\theta)d\theta\right) &(14) \end{aligned}$$

For the case where $V(\underline{\theta}) = 0$, which is the most relevant, the consumer with the lowest taste parameter value has no net benefit and the first order condition for each θ is

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial w(\theta)} &= 0 &(15) \\ &= (1+\lambda)\left(\frac{\partial B}{\partial w} - \frac{\partial C}{\partial w}\right) g(\theta) - \lambda \frac{\partial^2 B}{\partial w \partial \theta}(1 - G(\theta)) - \mu g(\theta) = 0 \end{aligned}$$

Using equation (9), a mark-up condition similar to the one from the previous model (equation (8)) can be derived:

$$\frac{p_m - \left(\frac{\partial C}{\partial w} + \frac{\mu}{1+\lambda}\right)}{p_m} = \frac{\lambda}{1+\lambda}\frac{1}{\xi(w,\theta)} \tag{16}$$

where $\xi(w,\theta)$ represents the absolute value of the elasticity in each incremental market (see Appendix A). As expected, the same conclusions as in the discrete case apply to this model regarding the role of customer heterogeneity (here represented by different θ) in generating nonlinear prices, while the scarcity cost does not affect the price schedule shape, but only its level.

6 Scarcity in demand, cost, and availability

The previous sections have shown that scarcity, represented as a quantity constraint, has a direct effect that can be seen as an increase in real marginal cost, so that even when coupled with a budget balancing restriction it cannot in itself explain a preference for increasing rates. In order to evaluate other effects of scarcity in a more general sense, this section introduces into the previous models exogenous weather factors, ϕ, which affect water availability as well as consumer benefits and supply costs. It is assumed that a higher value of ϕ means hotter and drier weather, implying that $\frac{\partial B_j}{\partial \phi} > 0$, $\frac{\partial^2 B_j}{\partial w_j \partial \phi} > 0$ (water demand increases, for example due to irrigation or swimming pools), $\frac{\partial C}{\partial \phi} > 0$, $\frac{\partial^2 C}{\partial w \partial \phi} > 0$ (supply costs are higher due to extra pumping or treatment costs), and $\frac{dW}{d\phi} < 0$ (less available water).

Introducing these factors into the models from sections 4 and 5 does not change the fundamental result for the second-best price schedule, expressed by the inverse elasticity rule. The first-order conditions for the discrete and the continuous cases become:

$$\frac{p_j - \left[\frac{\partial C(w^*,\phi)}{\partial w^*} + \frac{\mu}{1+\lambda}\right]}{p_j} = \frac{\lambda}{1+\lambda}\frac{1}{\xi_j\left(w_j^*,\phi\right)} \tag{17}$$

$$\frac{p_m - \left(\frac{\partial C(w^*,\phi)}{\partial w^*} + \frac{\mu}{1+\lambda}\right)}{p_m} = \frac{\lambda}{1+\lambda}\frac{1}{\xi(w^*,\theta,\phi)} \tag{18}$$

Nonlinear pricing is still a consequence of consumer heterogeneity and not of scarcity considerations. However, the shape of the resulting price schedule may now be affected by the influence of the exogenous weather factor on the price-elasticities for the different consumer types.

6.1 Impact of scarcity on the shape of the price schedule

As noted earlier, the marginal unit price and the mark-up for each consumer type or market increment depend inversely on its price-elasticity of demand. Nonlinear prices would be increasing if the price-elasticities decrease with higher optimal consumption choices and decreasing otherwise. We can investigate the conditions under which the resulting price schedule is increasing, constant or decreasing and how they are affected by the weather parameter. The partial derivatives of the elasticity with respect to the optimal level of water consumption are, for the discrete and the continuous model, respectively:

$$\frac{\partial \xi_j\left(w_j^*,\phi\right)}{\partial w_j^*}=-\frac{\left[\frac{\partial^2 B_j\left(w_j^*,\phi\right)}{\partial w_j^{*2}}\right]^2 w_j^*-\frac{\partial B_j\left(w_j^*,\phi\right)}{\partial w_j^*}\left[\frac{\partial^3 B_j\left(w_j^*,\phi\right)}{\partial w_j^{*3}}w_j^*+\frac{\partial^2 B_j\left(w_j^*,\phi\right)}{\partial w_j^{*2}}\right]}{\left[\frac{\partial^2 B_j\left(w_j^*,\phi\right)}{\partial w_j^{*2}}w_j^*\right]^2} \tag{19}$$

$$\frac{\partial \xi(w^*,\theta,\phi)}{\partial w^*}=-\frac{\left[\frac{\partial^2 B(w^*,\theta,\phi)}{\partial w^{*2}}\right]^2 w^*-\frac{\partial B(w^*,\theta,\phi)}{\partial w^*}\left[\frac{d^3 B(w^*,\theta,\phi)}{dw^{*3}}w^*+\frac{\partial^2 B(w^*,\theta,\phi)}{\partial w^{*2}}\right]}{\left[\frac{d^2 B(w^*,\theta,\phi)}{dw^{*2}}w^*\right]^2} \tag{20}$$

The price schedule will be increasing, constant or decreasing according to whether $\dfrac{\partial \xi}{\partial w^*}$ is negative, null or positive. The conditions for each case are described below (because the result is the same for the discrete and the continuous models we only present them once in a general form).

In order for elasticity to stay the same regardless of consumption, implying that efficient unit price will be constant, the following condition is necessary and sufficient:

$$\frac{\partial \xi(w^*,p_m)}{\partial w^*}=0 \Leftrightarrow \frac{\frac{\partial B}{\partial w^*}\left[\frac{\partial^3 B}{\partial w^{*3}}w^*+\frac{\partial^2 B}{\partial w^{*2}}\right]}{\left[\frac{\partial^2 B}{\partial w^{*2}}\right]^2 w^*}=1 \tag{21}$$

Likewise, for $\frac{\partial \xi}{\partial w^*} < 0$ the expression on the right-hand side of equation (21) must be smaller than 1 and for $\frac{\partial \xi}{\partial w^*} > 0$ it must be greater than 1. It can be shown that the sign of $\frac{\partial^3 B}{\partial w^{*3}}$, which reflects the curvature of the demand function, plays a very important role in determining the shape of the resulting price schedule. In particular, $\frac{\partial^3 B}{\partial w^{*3}} < 0$ is a sufficient condition for IBT to be efficient. Additionally, to verify the impact of the weather parameter on the price schedule we just have to differentiate the expression from (21) in relation to ϕ. We omit the lenghty resulting expression and present only sufficient conditions for the result to be negative, i.e., for the influence of the weather variable on the price schedule to reinforce the case for IBT.

$$\frac{\partial^3 B}{\partial w^{*3}} < 0 \tag{22}$$

$$\frac{\partial^3 B}{\partial w^{*2} \partial \phi} > 0 \tag{23}$$

$$\frac{\partial^4 B}{\partial w^{*3} \partial \phi} < 0 \tag{24}$$

Condition (22) means that the demand function would have to be strictly concave. Condition (23) implies that the demand function's negative slope would have to become less steep as temperature and dryness increase. Finally, condition (24) requires the demand function to become more concave as temperature and dryness increase. Why do these conditions favour the adoption of IBT in hotter and drier regions or time periods? They seem to create a framework where willingness to pay for water consumption increases more with temperature in high demand consumers than in those with low demand profiles, decreasing the difference in marginal valuation of the initial consumptions and the more extravagant ones. This is consistent with the fact that low demand residential consumers have a mainly indoor water use which does not vary much with weather conditions, whereas high demand residential consumers include those with gardens to sprinkle or swimming pools to fill in the summer, therefore showing a more variable demand pattern.

High demand residential consumers are also usually associated with higher income levels (reflected in θ in our model) which means that water expenses can weigh very little on their budget. In this context, relative water demand rigidity between high and low demand users may increase, with high income and high demand users being more willing and able to afford the ever more scarce water as temperature increases. In the presence of a Ramsey pricing policy (with price levels inversely related with price-elasticities of demand) this would mean that the tariff schedule would tend towards IBT as temperature increases and a bigger share of the water utility's revenues would be generated by high demand consumers. This may be an explanation for the fact that IBT's are more frequent in countries with hotter and drier climate, as it is in Europe where we find them mainly in the Mediterranean countries. Further research in water demand estimation that explicitly takes into account both climate variables and price structures could shed some light on whether the conditions presented above actually hold.

6.2 Impact of scarcity on water consumption

We now evaluate the impacts of scarcity in a two-consumer version of the simplest model from section 4 (with and without the budget balancing constraint). The welfare maximization problem when no budget balancing constraint is imposed becomes:

$$\begin{aligned} \underset{\{w_1,w_2\}}{Max} \quad & \sum_{j=1}^{2} B_j(w_j,\phi) - C(w,\phi) \\ & s.t. \sum_{j=1}^{2} w_j \leq W(\phi) \end{aligned} \tag{25}$$

As before, marginal benefit must be equal for both consumers, so that the marginal price must be the same, and the effects of the weather on costs and on scarcity aren't consumer-specific, so there is no scarcity related reason to use increasing marginal prices.

This may no longer be the case when a breakeven constraint is imposed on the model, resulting the inverse elasticity rule presented in equation (17). If both the physical and the financial constraints are binding, the first-order conditions provide a solution for $w_1^*(\phi)$, $w_2^*(\phi)$ and $\mu^*(\phi)$, which can be used for comparative static analysis of ϕ. The main results for the case without the budget balancing constraint can be summarized as follows:

- the sign for $\frac{d\mu^*}{d\phi}$ is undetermined, but will be positive if we assume that the marginal benefit of consumption increases more with drier weather conditions than the marginal cost of water supply (excluding the opportunity cost of the resource): $\frac{\partial^2 B_j}{\partial w_j \partial \phi} > \frac{\partial^2 C}{\partial w \partial \phi}, \forall_j$.

- $\frac{dw_j^*}{d\phi}$ is negative for both consumers, as expected, only in the case of homogeneous consumers. If the marginal benefit functions and the way they respond to weather conditions ($\frac{\partial^2 B_j}{\partial w_j \partial \phi}$) differ, then the sign becomes undetermined, specially for the type whose demand increases more with the increase in temperature. If the consumer types differ enough it may become efficient to have one type of consumers (those whose willingness to pay increases more with temperature increases and the resulting scarcity) increasing their water consumption during the drier periods at the cost of the water savings of the one whose marginal benefits change less. This conclusion can be interpreted in terms of high vs low demand consumer types as we have done so far or in terms of different customer classes (residential customers, farmers, factories, ...) where some customer class increases consumption during the summer months (for example, agricultural irrigation). The necessary and sufficient condition for consumer type 1 to increase its optimal consumption with temperature increases is:

$$\frac{dW}{d\phi} > \frac{\dfrac{\partial^2 B_1}{\partial w_1 \partial \phi} - \dfrac{\partial^2 B_2}{\partial w_2 \partial \phi}}{\dfrac{\partial^2 B_2}{\partial w_2^2}} \tag{26}$$

The conclusion is rather different for the case with Ramsey pricing. Assuming heterogeneous types, $\frac{dw_j^*}{d\phi}$ is always negative. No consumer class increases consumption in scarcity times no matter how valuable the water is to them. This is because, with Ramsey pricing, the greater willingness to pay from one consumer type will be reflected in a less elastic water demand. This is taken into account in the water utility optimization problem which assigns the group's optimal consumption a higher price (thus balancing the utility's budget with second-best efficiency). The quantity demanded by the group falls accordingly, so that in this context the higher valuation of water in a scarcity situation does not provoke higher consumption, like it did in the case without the financial constraint, where the group which valued water the most could, in some cases (through the utility's pricing decisions), "lead" the other to save water so it can consume more.

7 Dynamic analysis of scarcity

The previous models' inclusion of weather/scarcity impacts not only on water availability, but also on benefit and cost functions, can be carried over to a dynamic setting that enables us to study the long run effects of climate change on water resources, namely on the amount of necessary investment on water supply, treatment and storage infrastructure. We adapt a dynamic model by Brock and Dechert (1985) for the public utility pricing and investment decisions so it is consistent with the characteristics of our previous static models. We consider, that in the long-run, water scarcity can be dealt with through the combination of water demand management (through marginal cost pricing or Ramsey pricing) and investment in water infrastructure. For example, seasonal water inflow variability can be dealt with through dam construction to stabilize the amount of available water supply, thus allowing average yearly water availability to increase. Or alternative sources, other than surface water, can be explored, like groundwater pumping or seawater desalination. The main novelty in the dynamic model is the introduction of a water availability production function depending positively on capital invested in water supply infrastructure and negatively on the weather variable.

Let t denote the time period, K_t the capital invested in water withdrawal, treatment, storage and distribution infrastructure and W_t be determined by the water production function:

$$W_t = f(K_t, \phi_t) \tag{27}$$

where $\frac{\partial W}{\partial K} > 0$, and $\frac{\partial W}{\partial \phi} < 0$ as before.

Capital can be built upon by investment in infrastructure, I_t, and it will depreciate at rate δ, so that its evolution through time is given by:

$$\dot{K} = I - \delta K$$

Following Brock and Dechert (1985), we assume the total investment cost in period t to be given by $I_t + c(I_t)$ (price of capital is normalized to 1), where $c(I_t)$ represents installation costs and $\frac{\partial c(I_t)}{\partial I_t} > 0, \frac{\partial^2 c(I_t)}{\partial I_t^2} > 0$. Furthermore, we denote by $BL(w_t, \phi_t) = B(w_t, \phi_t) - C(w_t, \phi_t)$ the social net benefit from water consumption. The assumptions made in previous sections about the benefit and cost functions apply.

Assuming the resource constraint is binding, so that all the water made available through the water supply infrastructure is consumed, and using r as the appropriate discount rate, the dynamic optimization problem is:

$$\max \int_0^\infty e^{-rt} \{BL(f(K_t, \phi_t), \phi_t) - I_t - c(I_t)\, dt \tag{28}$$

$$s.t. \begin{cases} \dot{K} \equiv I - \delta K \\ K(0) = K_0, \;\; K(\infty) \text{ free} \end{cases} \tag{29}$$

resulting in the autonomous differential equation system:

$$\dot{I} = \frac{(r+\delta)\left(1 + \frac{\partial c(I)}{\partial I}\right) - \frac{\partial BL(K,\phi)}{\partial K}}{\frac{\partial^2 c(I)}{\partial I^2}} \tag{30}$$

$$\dot{K} = I - \delta K \tag{31}$$

whereby the system's steady-state can be described by:

$$\begin{cases} \dot{K} = 0 \\ \dot{I} = 0 \end{cases} \Leftrightarrow \begin{cases} I = \delta K \\ 1 + \frac{\partial c(I)}{\partial I} = \frac{\frac{\partial BL(K,\phi)}{\partial K}}{r+\delta} \end{cases} \tag{32}$$

In a steady-state situation, gross investment merely replaces depreciated capital, and the cost of an additional unit of investment must be equal to the capitalized value of the marginal benefit. It can be shown that the steady state is a saddle point. For every level of current capital, only one investment decision will be located on the stable branches, giving the solution for the investment variable in every time period. If we start from a lower value for K than its steady-state value, than investment should be high initially and it should decrease gradually as we approach the steady-state. If we start from a level of K above the steady-state value, than the investment should be lower than the depreciated capital to allow for the amount of capital invested to decrease. Investment levels should recover as the steady-state is approached.

It should be noted that, since the ϕ value to be considered in the long-run investment decisions should in principle be an average expected value, unexpected and temporary fluctuations in ϕ should not change the investment decisions nor the optimal steady-state level of capital invested. We may then ask the comparative-static question of what impact will an expected permanent increase in ϕ (such as the one that would occur for Mediterranean areas in a global warming context) have. The answer depends on the sign of $\frac{d}{d\phi}\left[\frac{\partial BL(f(K,\phi),\phi)}{\partial K}\right]$, i.e., on the impact of increased temperature and water scarcity on the marginal net benefit of additional units of capital. The steady-state levels of capital and investment would rise with ϕ if $\frac{d}{d\phi}\left[\frac{\partial BL(f(K,\phi),\phi)}{\partial K}\right]$ is positive. Two conditions are sufficient for this to be the case:

$$\frac{\partial^2 B\left(w,\phi\right)}{\partial w \partial \phi} \geq \frac{\partial^2 C\left(w,\phi\right)}{\partial w \partial \phi} \tag{33}$$

$$\frac{\partial^2 f\left(K,\phi\right)}{\partial K \partial \phi} \geq 0 \tag{34}$$

Condition 33 is similar to the one we found in Section 5 for the scarcity cost to increase with temperature. This is expected given that in the dynamic model, water availability can always be increased through investment. Condition 34 requires the marginal productivity of capital not to decrease with water scarcity. If we reverse the signs of the inequalities we have the necessary, albeit not sufficient, conditions for optimal steady-state capital and investment levels to decrease with temperature.

Further research could combine the techniques of nonlinear pricing with optimal control to investigate the long-run properties of nonlinear prices. A description of Ramsey pricing in an isoperimetric problem is presented in Appendix B.

8 Conclusion

We set out to write this paper because of a puzzling question: if increasing block tariffs for water are not recommended in theoretical economic models, why are they so popular in practice? Clearly, having one block where water is charged at a low price (or even a small free allocation) can be justified by the need to ensure universal access to such a vital good. Yet the IBT schemes we found were much more complex than that. Water managers often mention that increasing rates signal scarcity and as such are a useful tool in reducing resource use. We find, after a thorough revision of the literature and an experimentation with different models, that a relatively strong conclusion stands out: the best way to allocate water when scarcity occurs is to raise its price in accordance with its true marginal cost, which includes the scarcity cost. Nonlinear pricing is a consequence of consumer heterogeneity and not of scarcity considerations.

However, the shape of the resulting price schedule may, in specific circumstances, be affected by the influence of the exogenous weather factor on the price-elasticities of the demands for the different consumer types. If high demand consumers' willingness to pay for water rises more with temperature increases relative to low demand consumers than IBT may be more appropriated in countries with hotter and drier climates. This is consistent with the fact that mediterranean European countries are often mentioned in OECD reports to make extensive use of IBT. Other results from our models are: the impact of weather on the scarcity cost depends on the impact that weather has on the marginal net benefit of water consumption; it may be efficient for some consumer types to increase their water consumption in drier periods when marginal cost pricing is followed, but that is not the case in the context of a Ramsey pricing policy. The positive association of the impact of weather on the scarcity cost and on the marginal net benefit of water consumption can be confirmed by introducing dynamic water availability explicitly into the model.

The temporal variability of supply may originate from a regular and expected seasonality or from a more uncertain inter-annual irregularity of water inflows. One possibility for extension of this work is that optimal coping strategies may be different, which can lead us to reconsider the role of capital investments like dam construction in the stabilization of water supply and in the prevention of droughts, namely when compared to demand management tools such as pricing.

There are many other avenues for further research which can now be followed. One is the combination of dynamic water variability with nonlinear pricing techniques. In order to assess the potential of nonlinear prices to promote efficiency in the use of water, to reduce overall water demand, and to recover the costs of water supply, it is also important to consider real water demand profiles. Further work in this area could be directed at testing whether the conditions under which IBT is an efficient policy for drier countries hold. The assertion that IBT are, per se, scarcity signals with the potential to influence consumer behavior even when price elasticities are very low (as they tend to be for water) could also be tested with econometric models. Finally, a comparison between the merits of nonlinear pricing and optimal two-part tariffs regarding the efficiency coupled with a budget constraint in a context of scarcity and consumer heterogeneity could be performed.

9 Appendix A

This Appendix contains the derivation of equation (16). See also (Brown and Sibley (1986, pp.205-6)).

Proof. $(1+\lambda)\left(\frac{\partial B}{\partial w}-\frac{\partial C}{\partial w}\right)g(\theta)-\lambda\frac{\partial^2 B}{\partial w\partial\theta}(1-G(\theta))-\mu g(\theta)=0$

since $\frac{\partial B(w,\theta,\phi)}{\partial w}=\frac{dP}{dw}\equiv p_m$

$\Leftrightarrow (1+\lambda)\left(p_m-\frac{\partial C}{\partial w}\right)g(\theta)-\mu g(\theta)=\lambda\frac{\partial^2 B}{\partial w\partial\theta}(1-G(\theta))\Leftrightarrow$

$$\Leftrightarrow \frac{p_m-\left(\frac{\partial C}{\partial w}+\frac{\mu}{1+\lambda}\right)}{p_m}=\frac{\lambda}{1+\lambda}\frac{1}{p_m}\frac{\partial^2 B}{\partial w\partial\theta}\frac{(1-G(\theta))}{g(\theta)}\Leftrightarrow$$

$$\Leftrightarrow \frac{p_m-\left(\frac{\partial C}{\partial w}+\frac{\mu}{1+\lambda}\right)}{p_m}=\frac{\lambda}{1+\lambda}\frac{1}{p_m}\frac{1}{\frac{\partial\underline{\theta}}{\partial p_m}}\frac{(1-G(\theta))}{g(\theta)}\Leftrightarrow$$

where $\underline{\theta}$ indicates the marginal consumer group $\left(\underline{\theta}=\underline{\theta}\left(Q,P\left(Q\right)\right)\right)$

Defining marginal willingness to pay, $\rho\left(w,\theta\right)$, the self-selection condition is $\rho\left(w,\underline{\theta}\right)=p_m$,so that $\dfrac{d\rho}{dp_m}=1\Leftrightarrow\dfrac{\partial\rho}{\partial\underline{\theta}}\dfrac{\partial\underline{\theta}}{\partial p_m}=1\Leftrightarrow\dfrac{\partial\underline{\theta}}{\partial p_m}\rho_\theta=1\Leftrightarrow\dfrac{\partial\underline{\theta}}{\partial p_m}=\dfrac{1}{\rho_\theta}>0$

Since $B_{w\theta}\equiv\dfrac{\partial^2 B\left(w,\theta\right)}{\partial w\partial\theta}\equiv\rho_\theta\equiv\dfrac{\partial\rho\left(w,\theta\right)}{\partial\theta}$, $\dfrac{\partial\underline{\theta}}{\partial p_m}=\dfrac{1}{B_{\theta w}}$

Finally,

$$\Leftrightarrow \frac{p_m-\left(\frac{\partial C}{\partial w}+\frac{\mu}{1+\lambda}\right)}{p_m}=\frac{\lambda}{1+\lambda}\frac{1}{p_m\frac{\partial\underline{\theta}}{\partial p_m}\frac{g\left(\theta\right)}{\left(1-G(\theta)\right)}}\Leftrightarrow$$

$$\Leftrightarrow \frac{p_m - \left(\frac{\partial C}{\partial w} + \frac{\mu}{1+\lambda}\right)}{p_m} = \frac{\lambda}{1+\lambda}\frac{1}{\xi(w, p_m)}$$

which is the condition in the text. $\xi(w, p_m)$ emerges through the following manipulations:

$$\frac{\partial \ln p_m(w)}{\partial p_m(w)} = \frac{1}{p_m(w)}$$

$$\frac{d \ln[1-G(\underline{\theta})]}{dp_m(w)} = \frac{\partial \ln[1-G(\underline{\theta})]}{\partial \ln p_m(w)} \frac{\partial \ln p_m(w)}{\partial p_m(w)} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{[1-G(\underline{\theta})]}\left(-g(\underline{\theta})\frac{\partial \underline{\theta}}{\partial p_m}\right) = \frac{\partial \ln[1-G(\underline{\theta})]}{\partial \ln p_m(w)} * \frac{1}{p_m(w)} \Leftrightarrow$$

$$\Leftrightarrow \frac{d \ln[1-G(\underline{\theta})]}{d \ln p_m(w)} = \frac{-g(\underline{\theta})\frac{\partial \underline{\theta}}{\partial p_m} p_m(w)}{[1-G(\underline{\theta})]} \Leftrightarrow -\frac{\partial \ln[1-G(\underline{\theta})]}{\partial \ln p_m(w)} = \frac{g(\underline{\theta})\frac{\partial \underline{\theta}}{\partial p_m} p_m(w)}{[1-G(\underline{\theta})]}]$$

[note that in general: $\xi_x f(x) = \frac{\partial f(x)}{\partial x}\frac{x}{f(x)} = \frac{\partial \ln f(x)}{\partial \ln x}$] ■

10 Appendix B

This Appendix describes the formulation and the solution to the dynamic water pricing and investment model with a financial constraint. We formulate the problem as an isoperimetric one by adding the following budget balancing constraint to the problem 28:

$$\int_0^\infty e^{-rt}\{D(f(K_t,\phi_t),\phi_t) f(K_t,\phi_t) - C(f(K_t,\phi_t),\phi_t) - I_t - c(I_t)\} dt = 0 \tag{35}$$

We choose to adopt a global constraint for the time horizon to reflect a perfect capital market as in Brock and Dechert (1985). The resulting autonomous differential equation system is:

$$\dot{I} = \frac{(r+\delta)\left[1+\frac{\partial c(I)}{\partial I}\right] - \left[1+\frac{\lambda}{(1+\lambda)} f(K,\phi)\right]\frac{\partial BL(K,\phi)}{\partial K}}{\frac{\partial^2 c(I)}{\partial I^2}} \tag{36}$$

$$\dot{K} = I - \delta K \tag{37}$$

and the steady-state is characterized by:

$$\begin{cases} \dot{K} = 0 \\ \dot{I} = 0 \end{cases} \Leftrightarrow \begin{cases} I = \delta K \\ 1 + \frac{\partial c(I)}{\partial I} = \frac{\left[1+\frac{\lambda}{(1+\lambda)} f(K,\phi)\right] \frac{\partial BL(K,\phi)}{\partial K}}{(r+\delta)} \end{cases} \tag{38}$$

The steady state equilibrium is a stable node if the following expression is greater than 1 or a saddle point if it is less than 1:

$$\frac{-\frac{\partial f(K,\phi)}{\partial K} \frac{\partial BL(K,\phi)}{\partial K}}{\left[1 + \frac{\lambda}{(1+\lambda)} f(K,\phi)\right] \frac{\partial^2 BL(K,\phi)}{\partial K^2}}$$

Comparative statics derivatives are less informative for the isoperimetric problem, but the sign and magnitude of $\frac{d}{d\phi}\left[\frac{\partial BL(f(K,\phi),\phi)}{\partial K}\right]$ is still an important factor in determining whether optimal steady-state leves of capital and investment should rise or fall in a global warming context.

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