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Forecasting Bitcoin returns volatility using GARCH methods

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Masters in Management

Supervisor:
PhD, Diana Elisabeta Aldea Mendes, Associate Professor
ISCTE-IUL

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BUSINESS
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I dedicate this thesis to my biggest supporter, my sister Luísa,

Aknowledgements

First and foremost, I would like to express my most sincere gratitude to professor Diana Aldea Mendes, without her continuous support and patience this achievement would not be possible.

I want to thank my family, who always had a positive word to say, and especially my parents, for always pushing me into becoming my best version and teaching me the true meaning of hard work and perseverance. To my sister Luísa, who accompanied this process closer than anyone, words will never be enough to thank her for the understanding attitude and for all the calm and serenity she transmitted throughout the toughest times. I will forever be grateful.

I would not be where I am without the support of my friends, the ones that accompanied me for years and the ones I had the priviledge to meet along the way. I am the luckiest for having the right people on my life, who are able to bring a smile to my face even on the days I doubt the most about myself. I will always cherish the priviledge of having a life filled with such good company.

Resumo

Este estudo tem como objetivo investigar a dinâmica do preço e volatilidade da Bitcoin. O primeiro passo da análise consiste em examinar os retornos diários da Bitcoin, identificando uma série temporal estacionária com clusters de volatilidade acentuada. Essas características, a par com (a)simetria e a (não)uniformidade da dispersão identificadas, sugerem a adequação do uso de modelos ARCH/GARCH para a análise estatística.

Para determinar o modelo mais apropriado, são avaliados diversos modelos GARCH, EGARCH e modelos GARCH com variáveis exógenas. A avaliação inclui uma análise cuidadosa dos valores de AIC e BIC e a interpretação dos coeficientes dos parâmetros dos modelos. A significância estatística dos coeficientes confirma o impacto dos retornos passados ao quadrado e das variâncias condicionais na volatilidade atual.

O estudo culmina numa análise detalhada da previsão do Value at Risk, (VaR), sendo que o modelo EGARCH (1,1) com distribuição t-student se destaca como o mais eficaz na captura do VaR dos retornos da Bitcoin, com base no número de quebras identificadas a níveis de confiança de 99% e 95%. A pesquisa destaca a importância de escolher um modelo que esteja alinhado com o perfil de risco e os objetivos de investimento do utilizador. No entanto, também reconhecemos algumas limitações no nosso estudo, como a incapacidade de usar uma variável exógena na previsão da VaR e a necessidade de métodos computacionais mais avançados em futuras investigações.

Abstract

This study aims to investigate the dynamics of Bitcoin's price and volatility. The analysis begins by examining Bitcoin's daily returns, identifying a stationary time series with pronounced volatility clusters. These characteristics, combined with (a)symmetry and (non)uniform dispersion, suggest the suitability of ARCH/GARCH models for statistical analysis.

To determine the most appropriate model, a range of GARCH, EGARCH, and GARCH models with exogenous variable models are evaluated. The assessment includes a careful examination of AIC and BIC values and the interpretation of the coefficients of the model parameters. The statistical significance of coefficients confirms the impact of past squared returns and conditional variances on current volatility.

The study culminates in a detailed analysis of Value at Risk (VaR) forecasting, with the EGARCH (1,1) model with a Student's-t distribution emerging as the most effective in capturing Bitcoin returns' VaR, based on the number of exceedances identified at 99% and 95% confidence levels. The research underscores the importance of choosing a model that aligns with the user's risk profile and investment goals. However, it also acknowledges some limitations, such as the incapacity of using the exogenous variable in VaR forecasting and the potential for more advanced computational methods in future investigations.

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Abreviations

AIC - Akaike Information Criterion

APARCH - Asymmetric Power Generalized Autoregressive Conditional Heteroskedasticity

ARCH - Autoregressive Conditional Heteroskedasticity

ARMA – Autoregressive Moving Average

AVGARCH – Asymmetric Absolute Value Generalized Autoregressive Conditional Heteroskedasticity

BIC – Bayesian Information Criterion

CBOE - Chicago Board Options Exchange

DJIA - Dow Jones Industrial Average

EGARCH – Exponential Generalized Autoregressive Conditional Heteroskedasticity

EPA – Equal Predictive Ability

GARCH – Generalized Autoregressive Conditional Heteroskedasticity

GFS – Global Financial Cycle

GJR-GARCH - Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroskedasticity

GPR – Geopolitical Risks

HAR - Heterogeneous Autoregressive

ICSS - Iterative Sums of Squares

IGARCH – Integrated Generalized Autoregressive Conditional Heteroskedasticity

MIDAS - Mixed Data Sampling

MS-GARCH - Markov Switching Generalized Autoregressive Conditional Heteroskedasticity

PACF - Partial Autocorrelation Function

TGARCH – Threshold Generalized Autoregressive Conditional Heteroskedasticity

VaR – Value at Risk

VIX - Volatility Index

WTI – West Texas Intermediate

CHAPTER 1

Introduction

Financial time series data exhibits distinctive characteristics and stylized facts that reveal the need for advanced modeling techniques to capture the inherent complexity of this kind of times series. This research delves into the realm of volatility, an essential component of financial markets and risk assessment.

In recent years, the landscape of financial markets has witnessed a significant transformation, with the emergence of cryptocurrencies as a novel and dynamic asset class. This surge has been particularly driven by the unparalleled dominance of Bitcoin. Being inherently volatile assets, gaining a comprehensive understanding of the dynamics of cryptocurrency volatility holds paramount significance. This understanding has practical implications, influencing investment strategies, risk management, derivative valuation, and even public and private policy decisions (Aharon et al., 2023). The influence of cryptocurrencies extends beyond their intrinsic characteristics, as they are interconnected with other global financial markets. This interrelation is particularly evident in the context of crude oil, a vital and geopolitically significant resource that holds a dominant position within the energy market (Zhang & Ji, 2019).

From the literature consulted, it is our understanding that major cryptocurrencies exhibit the specific statistical characteristics that provide a rationale for the widespread adoption of Autoregressive Conditional Heteroskedasticity (ARCH) and its more evolved counterpart, Generalized ARCH (GARCH) models and Exponential GARCH (EGARCH) models. For the purpose of this study, we will adopt Bitcoin as the focus of our analysis.

The innovative factor of this thesis relates to the use of exogenous variables to predict the Bitcoin's daily returns volatility and VaR forecast. The exogenous variable taken into account was the also highly volatile asset: crude oil. By analyzing the dynamics of Value at Risk, we seek to improve our understanding of the risk assessment and management of these digital assets, thus contributing to the broader field of financial economics.

In the subsequent sections, we will first introduce the core concepts relevant to depict a theoretical framework on this topic in chapter 2. In chapter 3, we will summarize the empirical findings from studies conducted in the context of financial time series volatility using GARCH models. In chapter 4 we will present and describe the data used to achieve this study and in chapter 5 we will describe the results reached through our data analysis. Finally, in chapter 6, we will elaborate our conclusions and recommendations for further research on this topic.

CHAPTER 2

Theoretical Framework

2.1. Cryptocurrencies

Initially emerging as specialized assets utilized within small online communities, cryptocurrencies have transitioned into the mainstream, capturing the interest of both financial professionals and the public (Bergsli et al., 2022a). Digital currencies play an important role on making electronic payments easier, given that they eliminate the need of a bank (or other third party) intermediation. (Panagiotidis et al., 2022).

Recently, the cryptocurrency market has experienced a fast growth in volume and number of tradable coins, thus capturing the attention of several shareholders such as governments, investors, firms, and the public (Fung et al., 2022). The total market capitalization of the asset class increased significantly, soaring from $295 billion in March 2018 to a staggering $1.5 trillion by March 2021, primarily driven by the unquestionable supremacy of Bitcoin (Fung et al., 2022). Cryptocurrencies share multiple characteristics with precious metals, such as safe haven, hedge, and diversification for risk assets (Corbet et al., 2020). Specifically, the hedging capability of precious metals is usually compared to Bitcoin (Dyhrberg, 2016).

Cryptocurrencies are inherently volatile assets. Unlike other currencies, cryptocurrencies operate independently and are not tied to any national economy (Bergsli et al., 2022). Recent studies have evidence that the volatility generated in cryptocurrency markets transmits to other financial markets worldwide (Hsu et al., 2021; Uzonwanne, 2021). Gaining insight into the dynamics of this asset category volatility is essential for “investment, hedging strategies, and derivative valuation in financial markets as well as for public and private policymaking” (Aharon et al., 2023).

Some of the most relevant specificities of cryptocurrencies compared to other fiat currencies (i.e., any form of currency declared by the government as an acceptable form of transaction) are their higher return and volatility profile (Fung et al., 2023). In fact, several studies found significant excess volatility of Bitcoin relative to the US dollar and the S&P 500 index (Baumöhl, 2019; Charfeddine et al., 2020)

Across the literature, evidence can be found that major coins exhibit non-zero skewness, heavy tails, excess volatility relative to assets like gold and the US dollar, long-memory, and volatility persistence (Fung et al., 2022), which justifies the generalized use of GARCH modelto depict the volatility of Bitcoin and other cryptocurrencies. Aharon et al. (2023), revisits evidence from literature that shows that major cryptocurrencies present an inverse asymmetrical volatility: positive shocks increase price volatility more than negative ones.

Being the most popular cryptocurrency and having a large amount of historical data, we chose Bitcoin as the focus of this study. However, there is evidence that cryptocurrencies are highly correlated (Bergsli et al., 2022), thus some of this study's conclusions can be inferred for other cryptocurrencies.

2.2. Crude oil prices and their predictive capacity

Crude oil is a vital resource with major global geopolitical and economic significance and assumes a dominant place in the energy market (Zhang & Ji, 2019). It is possible to prove that a crude oil shock can affect investors' willingness to hold Bitcoin (Bruno & Shin, 2015). Studying the interactions between these two variables is necessary since it can profoundly impact economic stability and influence investor behavior and policy decisions.

As an energy market, crude oil prices have the power to affect Bitcoin prices, which is also observed for other commodity markets (Okorie & Lin, 2020). The study of the relation between these two asset classes is particularly interesting, given that cryptocurrency mining is a process that consumes significant amounts of energy resources.

2.3. Volatility and Financial Time Series Analysis

Volatility can be defined as the level of variation of the price of an asset over time and is used to quantify the risk of the asset category. It is usually obtained from calculating the annualized standard deviation of the daily, weekly, or monthly variations and presented as a percentage. As volatility measures the deviations about the mean, the values will be closer to the mean, the smaller the volatility value is.

Statistically, a time series is a set of sequential observations made over time. In contrast to linear regression models, in which the observation order does not impact on subsequent analysis, the same cannot be said for time series: the order of the collected data is crucial. Time series are characterized by large data size, presenting high dimensionality, and their continuous update. One must consider time series as a whole since "unlike traditional databases where similarity search is exact match based, similarity search in time series data is typically carried out in an approximate manner" (Fu, 2011).

Volatility cannot be directly observed; thus in order to estimate its behavior, it must be modelled. Volatility estimation models must focus on capturing persistence, mean reversion, the impact of positive and negative shocks, and the influence of exogenous variables. Literature suggests that asset returns time series exhibit peculiar characteristics: an asymmetrical distribution with heavy tails and negative skewness (Durante et al., 2015).

In the context of financial time series modeling, it is assumed that returns have a deterministic component (including seasonality, trends, and specific relationships with other variables) and a stochastic component (that captures the unpredictable part of a variable's behavior). The stochastic component is usually modeled according to statistical methods and is typically associated with volatility.

The most relevant stylized facts (i.e., the common behavioral patterns across different markets) identified throughout the study of the stochastic component of the volatility of financial assets are non-normality, limited evidence of short-term predictability in returns, and strong evidence of predictability in squared returns (Guo, 2022). Across the literature, there is evidence that major cryptocurrencies exhibit non-zero skewness, heavy tails, excess volatility relative to assets like gold and the US dollar, long-memory, and volatility persistence (Fung et al., 2022), which justifies the generalized use of GARCH models to depict these currencies volatility.

2.4. ARCH/GARCH models

Prior to Autoregressive Conditional Heteroskedasticity (ARCH) models, the existing econometric models assumed "a constant one-period forecasting" (Engle, 1982). The ARCH model introduced by Engle was built upon "serial uncorrelated processes with non-constant variances conditional on the past, but non-constant unconditional variances" (Engle 1982). The first application of this model goal was to estimate the volatility of United Kingdom inflation. The author demonstrated that the variance of inflation shocks was not constant but dependent on past shocks. For return series that follow the stationary process (as in eq.1), when the aim is to define the conditional variance, the underlying processes of the considered shocks (Et) to the returns must be specified (McAleer, 2014). Recurring to the ARCH framework, the conditional variance can be defined as a constant unconditional variance and a function of past errors and is specified in the following way:

$$r_t = \mu + \varepsilon_t \, , \varepsilon_t = \; z_t \sigma_t$$

Eq (1)

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2$$

Eq (2)

Where r_t represents the return series, μ the mean and ε_t the error term. To ensure that the conditional variance is not negative, we must assume as parametric constraints $\omega > 0$ and $\alpha_i \geq 0$, with ω being the constant term and α_i representing the shock. In financial modeling, the constant term represents the average level of volatility that the model assumes when we do not take into account other variables. The ARCH model is symmetric, which means that positive and negative shocks of the same size will impact the conditional variance equally in magnitude.

A few years later, in 1986, Bollerslev suggested improving Engle's work and introduced the Generalized ARCH (GARCH) model. The first application of this model focused on predicting the conditional variances of US quarterly inflation rates from 1948 to 1983, and it was proven a success mainly due to the model's "ability to succinctly capture volatility clustering in financial rates of returns" (Bollerslev, 2023) and thus allowing for more flexibility than the traditional ARCH. This is achieved by including lagged conditional variances in the model equation, as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

Eq (3)

This model adopts the parametric restrictions that $\omega > 0;\ \alpha_i > 0\ \forall\ i;\ \beta_j \geq 0\ \forall\ j$; to ensure that the conditional variance is always non-negative and $\alpha_i + \beta_j < 1\ \forall\ i, j$ to guarantee that the covariance process is stationary. As ARCH, the GARCH model is symmetric and presents the same response for positive and negative shocks.

2.5. EGARCH

Financial data commonly displays asymmetric volatility, i.e., positive and negative shocks affect volatility differently. Nelson (1991) proposed EGARCH models to account for asymmetric volatility specifically. An EGARCH model can be described as follows:

$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i(|e_{t-i}| - \sqrt{2/\pi}) + \sum_{j=1} \gamma e_{t-j} + \sum_{k=1}^{q} \beta_k \ln \sigma_{t-q}^2$$

Eq (4)

Where $\ln \sigma_t^2$ is the natural algorithm of the conditional variance at time t, ω is the constant term, e_{t-j} represents the past squared returns at time t-j and $|e_{t-1}|$ its absolute value. For a better understanding of the equation, we can look at the expressions one step at a time:

$$\sum_{i=1}^{p} \alpha_i(|e_{t-i}| - \sqrt{2/\pi})$$

Eq (4.1)

$$\sum_{j=1} \gamma e_{t-j}$$

Eq (4.2)

$$\sum_{k=1}^{q} \beta_k \ln \sigma_{t-q}^2$$

Eq (4.3)

The term represented in equation (4.1) takes into consideration the asymmetric effect of past return shocks on volatility. The parameter α represents the coefficients for past squared returns (e_{t-1}). The absolute value of e_{t-1} ensures that both positive and negative shocks are being considered. The 2nd part of the equation (Eq. (4.2)) deals with the impact of past conditional variances on the current volatility and coefficient γ measures how past conditional variances affect the current volatility. Finally, through equation (4.39), the model controls how past logarithms of conditional variances influence the current volatility.

2.6. Value at Risk Forecasting

In times of extreme uncertainty, such as the 2008 global financial crisis, they revealed the importance of improving the quality of forecasts of risk measures. Firstly, introduced in the nineties, this risk measure arose to quantify exchange risk. In the banking sector, Basel

Committee on Banking Supervision has determined VaR to be the best approach to decide on minimum capital requirements for market risk (Thavaneswaran et al., 2020).

Value at Risk (VaR) is one of the most recognized and used financial risk measures, and it elaborates an estimate of the potential loss in value of a portfolio of assets over a given period at a specified confidence level (Owusu Junior et al., 2022). VaR correct computation is crucial for the estimation of other quantile-based risk measures. Its construction requires a quantile estimate of the far-left tail of the unconditional returns distribution (Braione & Scholtes, 2016).

In this sense, VaR can be defined as the loss in a traded asset portfolio, such that there is a given probability (p) of losses equal or superior to the VaR in the specified transaction period and a probability of (1-p) of losses that are less than the VaR. Thus, VaR can be obtained by

$$P[Q \leq -VaR(p)] = p$$

Eq (5)

VaR consists of three parameters: the confidence level, time horizon and a given value. The confidence level (that in this study will be calculated at 95% and 99%) determines the level of certainty associated with the risk measurement. The higher the confidence levels, the greater the losses will be. For example, when we consider a 99% VaR, we expect that for each 100 VaR observation at least once, the financial investment loss will be greater than the one estimated recurring to VaR. The time horizon stems for the time interval under analysis; the greater the horizon, the greater the loss. Finally, the given value is the monetary value that is at risk for loss during the time horizon at the proposed confidence level.

There are various methods for calculating VaR, but the most frequently used ones are the parametric approach, the historical simulation and the Monte Carlo Simulation. The parametric approach is built upon the premise that asset returns follow a certain probability distribution (such as normal or t-student distribution) that estimates VaR based on statistical properties (like mean and standard deviation). The historical simulation method recurs to the historical data price of the asset, calculating the potential losses over the selected time horizon in the past, using the historical losses to compute the future potential losses. The Monte Carlo approach follows a method that generates many possible scenarios for the asset price and calculates VaR through the distribution of simulated returns. In our study, the method selected to conduct the VaR forecasting of Bitcoin returns volatility.

It is relevant to note that in the case of having actual losses in the portfolio that exceed the calculated value in the estimated loss, there is a break in VaR. Nevertheless, if the actual observed portfolio loss is above the estimated only a few times, it does not necessarily mean that the estimated VaR was a failure. When this happens, it is crucial to understand the frequency of these failures.

CHAPTER 3

Literature Review

In the following chapter, we will delve into the body of recent research on using ARCH/GARCH frameworks to model the volatility of Bitcoin and other cryptocurrencies and several volatile financial assets.

The study led by Aharon et al. (2023) aimed to demonstrate the importance of considering structural breaks in volatility forecasting by using asymmetrical volatility models with endogenously detected structural breaks. The authors analyse the asymmetric volatility behavior of five cryptocurrencies: Bitcoin, Ethereum, Dogecoin, Ripple, and Monero, using as data the daily prices for the five currencies from May 2013 to April 2022. Structural breaks were detected using the modified iterative sums of squares (ICSS) algorithm. The results of this study show that there is a decrease in volatility persistence after incorporating structural breaks into asymmetric GARCH models. In fact, after considering structural breaks, the asymmetric behavior of all the considered cryptocurrencies increased (due to the increase in the value of the asymmetric term).

Wei et al., (2023) used the GARCH-MIDAS model with the incorporation of cryptocurrency policy and price uncertainty (and other frequently used uncertainty measures) to compare the in-sample impacts and out-of-sample predictability of these uncertainty variables on the volatility forecast of gold and silver futures markets. The period considered was from January 2014 to May 2022. The authors used a GARCH-MIDAS model since this kind of model allows for different variable data frequencies (daily precious metals returns versus monthly uncertainty indices), with the further extension to a GARCH-MIDAS-X model, adding to the model an exogenous low-frequency impactor. The in-sample results of this study demonstrate that cryptocurrency uncertainty significantly impacts the volatilities of precious metal futures markets, and the out-of-sample results further confirm the predictive ability of cryptocurrency uncertainty on volatility forecasting of the precious metal market.

Research conducted by Fung et al. (2022) acknowledges the rapid changes occurring in the cryptocurrency market and records the financial properties of assets, as well as identifies the most suitable models for describing volatility across a wide range of coins. Various previous studies disclosed the inefficiencies of the cryptocurrency market, yet, efficiency varies across time and individual coins. To address the literature gap, the authors analyzed data from 254 cryptocurrencies organized by traded volume and usage categories (e.g., Finance, Blockchain, Technology) and reported their adherence to stylized empirical facts of asset returns. The data collected refers to the period between March 2019 and March 2021. The authors modeled volatility using eight ARMA-GARCH models (Standard GARCH; TGARCH; AVGARCH; APARG; EGARCH; IGARCH; GJR-GARCH) and six distributions for the error term, resulting in 48 distinct specifications per coin. The authors examined the out-of-sample 1%-VaR forecasts for each model specification using standard back-testing procedures. As expected, the authors concluded that GARCH models accounting for the common features of cryptocurrency's return behavior (long memory, conditional heteroskedasticity, heavy tails, and negative leverage effects) show evidence of the best goodness-of-fit properties. It is found that the TGARCH family presents the best goodness-of-fit for about one-third of the considered coins. The study also suggests that cryptocurrencies are well described by Student's t error distributions, with these GARCH specifications accounting for 80% of the coins. On the contrary, the GARCH normal and skewed normal specifications are never chosen.

Panagiotidis et al. (2022) recured to Markov Switching GARCH models with the aim of identifying structural changes in the volatility of the cryptocurrencies without a previous specification at the time of the change. MS-GARCH models incorporate regime switching dynamics, i.e., changes in the market regimes, that affect volatility. The main conclusions reached were that when we look at the data tested, it seems that MSGARCH models fit the data better for over half of the cryptocurrencies studied. Additionally, when predictions are made using MSGARCH models, they turn out to present a higher degree of accuracy more than 60% times. For a given number of regimes and a given conditional distribution, the EGARCH model provides worse results than GARCH and TGARCH models, both in- and out-of-sample. The analysis of two asymmetric models (EGARCH and TGARCH) evidence the presence of negative leverage effect in most of the considered cryptocurrencies, indicating that positive past returns have a greater impact on the volatility of cryptocurrencies compared to the negative past returns, as found in other authors work (Fung et al., 2022).

Over the years, several studies (Kang & Yoon, 2013; Lu et al., 2014; Wu & Shieh, 2007) evidenced that the market risk of commodity futures contracts time series presented characteristics commonly observed in spot price time series (non-normality, non-zero skewness and volatility clustering), thus it makes sense to apply GARCH models to estimate their Value at Risk (VaR). Following the same approach, Guo (2022) applied GARCH models for the risk management of Bitcoin futures, using close price daily data from December 18, 2017 to December 4, 2020. One of the key findings of this study was that heavy-tailed distributions perform better than the normal distribution predicting the Bitcoin futures returns and that, by comparing the three used distributions (Student's t, Non Inverse Gaussian (NIG) and Normal distribution) "the NIG distribution has the best in-sample performance" (Guo, 2022). The author considers that "it is more important to introduce a heavy-tailed distribution than select a type of GARCH models" (Z. Y. Guo, 2022).

To comprehend and forecast the price volatility of Bitcoin, Bergsli et al. (2022) considered two types of models: heterogeneous autoregressive (HAR) models and generalized autoregressive conditional heteroskedasticity (GARCH) models. The data used was the daily price of Bitcoin from 1-Jan-14 to 19-Sep-18, providing a total of 1720 observations. GARCH models were estimated using daily returns data, and HAR models were estimated using the realized variance calculated from high-frequency data. The effectiveness of the models is assessed by the authors using realized variance as a stand-in for genuine variance. The retracted conclusions state that EGARCH and APARCH exhibit greater performance among the employed GARCH models. The authorsconsider that Heterogeneous Autoregressive models (HAR) perform better than even the best performing GARCH models.

Jara & Piña (2023) used a combined approach to assess the efficacy of FX interventions. The authors apply this methodology to analyze five FX interventions observed in Chile since the adoption of a fully flexible exchange rate regime in September 1999, including the intervention in 2019. It used a Markov-Switching GARCH model to calculate the likelihood of the exchange rate volatility to be in a high or low state, to assess if it is possible and effective to consider FX interventions for stabilizing exchange rate volatility. Additionally, the authors use a high-frequency local projection setting, taking various domestic and international financial factors into account as control variables so that it can determine the impact of FX interventions on exchange rate volatility and on the volatility states and their persistence over time. According to the authors, the most effective model to study Chile's exchange rate volatility is a switching regime model built on an ARMA (0,1)-EGARCH(1,1) model characterized by a persistent low-volatility state.

Venter & Maré (2022) research was focused on the suitability of a GARCH option pricing model applied to price discovery in volatility index option markets (Chicago Board Options Exchange (CBOE) Volatility Index (VIX)). This study used futures price data obtained from CBOE and included the following delivery dates: 19-Aug-20, 16-Sep-20, 21-Oct-20, and 18-Nov-20. However, the author remarks that it is important to utilize error distributions that integrate skewness and kurtosis to accurately model the risk-neutral dynamics. The results of this study corroborate those of earlier studies, showing that the GARCH option pricing model can be a valuable instrument for analyzing the pricing dynamics of volatility index options.

Notably, the authors stressed the importance of including error distributions that take skewness and kurtosis into account to accurately describe risk-neutral dynamics. This emphasizes the need to consider different error distributions.

Segnon et al. (2023) contributed to the literature by studying the impact of geopolitical risks in forecasting future stock market volatility by resorting to an autoregressive Markov-switching GARCH mixed-data-sampling (AR-MSGARCH-MIDAS) framework, considering Dow Jones Industrial Average (DJIA) prices and monthly GPR observations from January 1900 until 31 March 2022 (around 33,000 daily and 1467 monthly observations). The model adopted considers nonlinearities, volatility regime shifts, and the impact of geopolitical risks and macroeconomic variables, separating volatility into short-term components (day-to-day liquidity concerns) and long-term components (monthly observations on geopolitical risk and other macroeconomic variables). For the out-of-sample analysis, the authors employed equal predictive ability (EPA) tests, constructed model confidence sets, and used forecast-encompassing tests for model selection. The authors discovered that non-Markov-switching models benefit from the addition of GPRs when predicting the volatility of the US stock market one to four months in advance, resulting in considerable accuracy gains. On the other hand, the impact of geopolitical concerns on predicting one-month-ahead stock market volatility is minimal when using superior Markov-switching MIDAS prediction models that more effectively account for nonstationary features and volatility regime changes. Overall, the study's findings demonstrated that the effect of geopolitical risk will vary depending on how well the model can account for regime changes and nonstationary characteristics of the data.

To understand the volatility of crude oil prices Zhao, (2022) addresses the multifaceted challenge of comprehending both short-term and long-term factors influencing oil price fluctuations and assessing the varying impacts of these factors on different types of oil, such as WTI and Brent crude oil. Employing a Mixed Data Sampling (MIDAS) framework, which accommodates high and low-frequency data integration, the study delves into four distinct categories of influencing factors: commodity attributes, financial factors, geopolitical events, and alternative energy. A Lasso (Least Absolute Shrinkage and Selection Operator) approach is used to address multicollinearity issues in the model. Notably, it identifies that the impact of these factors can evolve over time, emphasizing the need for a dynamic perspective on oil price volatility. Furthermore, the study introduces the innovative use of the Lasso approach to mitigate multicollinearity issues in the model. The results reveal that incorporating geopolitical risk, economic policy uncertainty, and alternative energy dynamics significantly enhances the prediction of oil price volatility, particularly in a Markov-switching MIDAS model, shedding light on the nuanced nature of oil market dynamics and presenting a valuable tool for investors and policymakers alike.

The study led by Guo et al. (2023) assesses the impact of climate policy uncertainty (CPU) and climate-related disasters on natural gas futures price volatility, aiming to understand and quantify these risks in the energy market context. To address this issue, the authors employed the GARCH-MIDAS model, selecting daily natural gas futures as a proxy for energy futures due to its global energy significance and lower carbon emissions. Monthly climate policy uncertainty indices represent transition risk, while climate-related disasters (measured through their monthly frequencies) serve as indicators of physical risk. The data sample period was from January 1, 1991, to July 29, 2022. It is concluded that the GARCH-MIDAS model is well-suited to manage monthly CPU indices and disaster frequencies in conjunction with daily natural gas futures prices, facilitating a comprehensive analysis of climate risk factors in the market. The empirical results reflect that CPU and disaster frequency significantly impact the secular component of natural gas futures price volatility, evidencing good tracking power on in-sample volatility. However, regarding predictive scenarios, only disaster frequency could increase the accuracy of volatility predictions.

Liang et al., (2022) study, contributed to the literature by introducing extreme weather factors within the GARCH-MIDAS framework for the predictability of natural gas market volatility. Additionally, an extended GARCH-MIDAS-ES model is introduced to account for the impact of extreme shocks (ES). The empirical findings prove that including weather indicators in predictive models enhance their performance in forecasting natural gas market volatility and that certain extreme weather events provide valuable information for improved prediction.

The study led by Salisu et al. (2022) explores the predictive role of the global financial cycle (GFCy) on crude oil returns volatility, resorting to a GARCH-MIDAS framework. This model is used due to the simultaneous work on different frequency variables: predicting high-frequency daily oil market volatility using low-frequency global predictors, specifically the monthly GFCy index. The study compares the GARCH-MIDAS-GFCy model’s predictive performance with the conventional GARCH-MIDAS-RV model, assessing their robustness to varying risk aversion and leverage ratios. The authors found that the GARCH-MIDAS-GFCy model consistently outperforms the conventional GARCH-MIDAS-RV model across various measures of oil market volatility and forecasting horizons. What’s significant is that this outperformance remains robust even when considering different levels of risk aversion and leverage ratios. These results underscore the substantial predictive value and economic importance of the global financial cycle in forecasting energy market volatility. This suggests that leveraging a common factor that influences global asset prices, as captured by the GFC, proves highly effective in forecasting low-frequency energy market returns volatility.

Aware of the complexity of achieving an accurate streamflow prediction and its importance in the contexts of water resource management, flood control, and infrastructure development, Wang et al. (2023) developed an ARIMA-MS-GARCH model, which considers both structural breaks and regime changes in streamflow time series to improve prediction accuracy. In terms of methodology, the authors identified structural breaks using the Bai and Perron test, distinguishing different fluctuation states and inferring future streamflow fluctuations based on calculated regime change probabilities. The findings of this study reveal that when structural breaks and regime changes are considered, the accuracy of streamflow prediction significantly increases.

CHAPTER 4

Data

4.1. Bitcoin

The aim of this study is to construct a GARCH model that forecasts and estimates the volatility and VaR of Bitcoin. It is under our consideration to study the daily volatility of Bitcoin, and we will take the closing price to compute the daily return of Bitcoin, i.e. the percentual change registered in Bitcoin closing prices.

The Bitcoin data was collected from Yahoo Finance and it is publicly available. The sample period considered was from 1 January 2017 to 30 June 2023, generating a sample of 2369 observations. This time interval was selected since it includes the 2018 financial crisis, the COVID-19 pandemic and the Russia-Ukraine conflict.

Figure 1: Evolution of Bitcoin daily closing prices (USD) between 01/01/2017 and 30/06/2023

From the observation of Figure 1, we can conclude that the evolution of Bitcoin closing prices for the selected period was not steady. Throughout 2017 the closing price increased consistently, reaching 20.000 USD in December. In 2018 it can be observed a downward trend, as a consequence of the financial crisis observed during this year, with a year-end closing price below 5.000 USD. During 2019 and the biggest part of 2020 Bitcoin price progressively increased, facing some fluctuations. For example, the generalized financial insecurity felt upon the emergence of the COVID-19 pandemic. On the transition from 2020 to 2021 Bitcoin prices registered a remarkable growth, surpassing the closing price of 60.000 USD in April 2021, reaching its all-time high of approximately 68.000 USD by the end of the year. Around the first quarter of 2022 the closing price of Bitcoin decreased considerably, mirroring the effect of the Russia-Ukraine conflict.

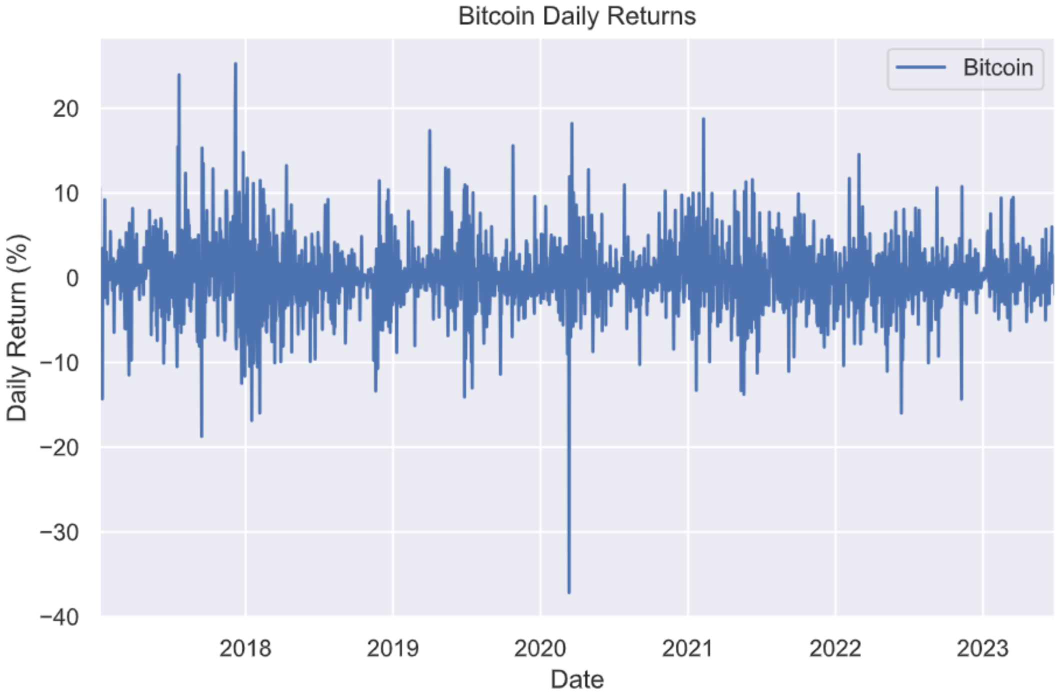

Figure 2: Evolution of Bitcoin daily returns between 01/01/2017 and 30/06/2023

Focusing on the evolution of the daily returns of Bitcoin (Figure 2), the time series that we aim to study, we can easily observe that although the time series faces some fluctuations, it is largely stationary and presents volatility clusters i.e., the data is organized in groups of observations with high variances followed by other groups of high variances and the same applies for groups of low volatility tend to cluster together as well.

To assess the asymmetry of the time series, we obtained the histogram depicted in Figure 3 below. The obtained figure suggests that the time series data follows an almost symmetric distribution. The distribution resembles the shape of a bell, with the concentration of most of the data on the central zone of the histogram, around zero. This suggests that most observations present values close to the mean and the data dispersion is relatively uniform. The observed histogram indicates that the data follows a leptokurtic bell-shaped distribution.

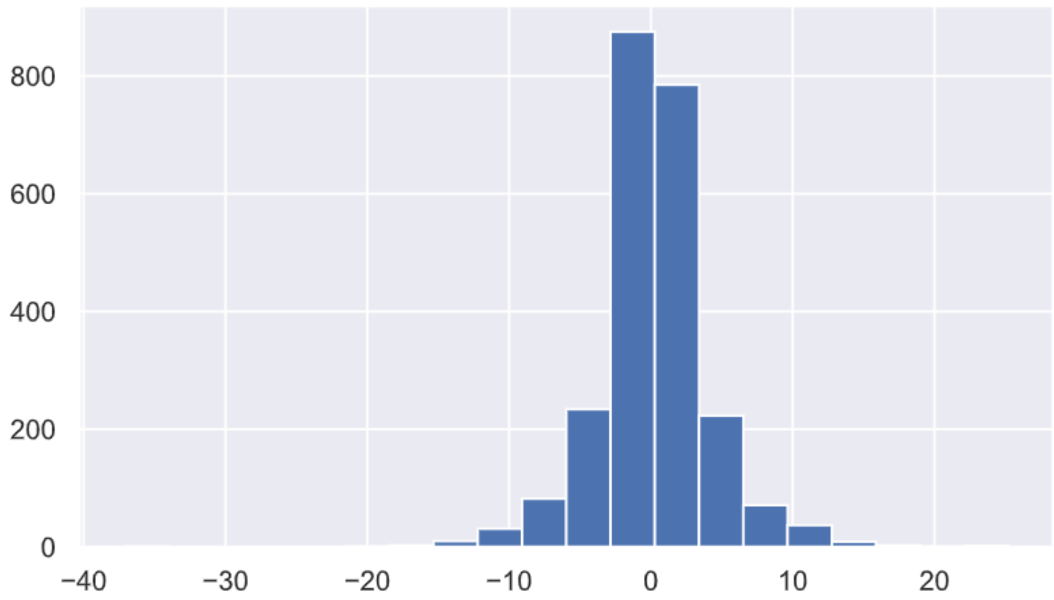

Figure 3: Bitcoin Daily Returns Histogram

To gain an insight into the autocorrelation of the data, we generated a partial autocorrelation function (PACF) for the Bitcoin returns, considering a lag of 20. The obtained plot (Figure 4) suggests that the autocorrelation coefficients are close to zero, which indicates that at these lags, there is little evidence of substantial autocorrelation (conditional mean).

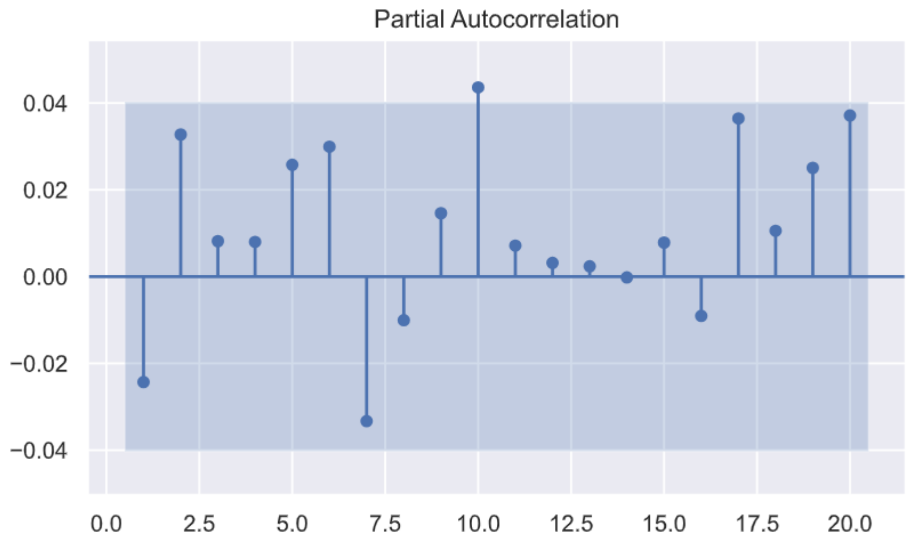

Figure 4: Partial autocorrelation of the Bitcoin Returns

Given that the focus of our study is to address the volatility associated with Bitcoin Returns, we plotted the squared Bitcoin returns PACF (Figure 5), which exhibits the presence of points that extend beyond the confidence interval at lags 1, 3, 4, 7 and 16, evidencing that there is significant autocorrelation at these specific lags (conditional variance). This indicates that past squared returns of these lags influence the current squared returns, highlighting the importance of accounting for temporal dependencies in volatility. In this sense, we conclude that adopting GARCH models for understanding and forecasting the volatility dynamics of Bitcoin returns and estimating financial risk is appropriate.

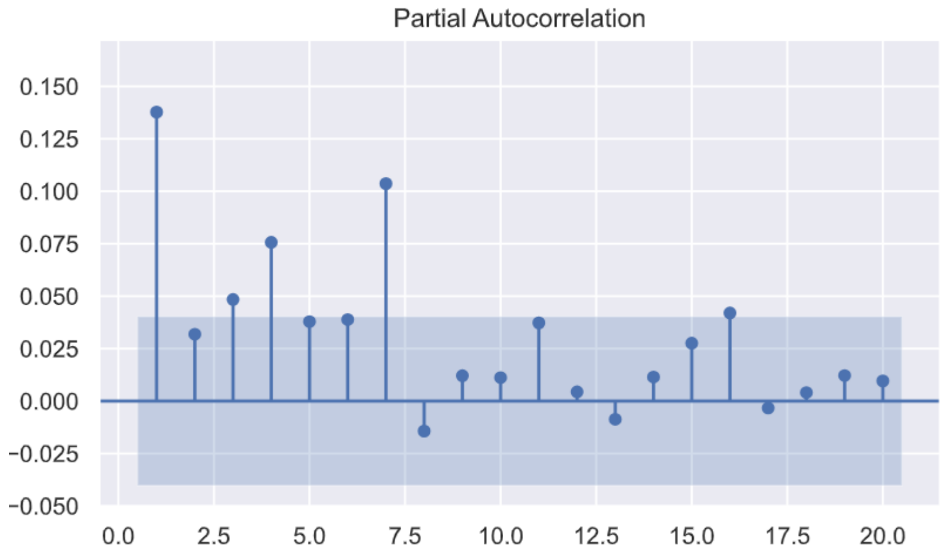

Figure 5: Partial autocorrelation of the squared Bitcoin Returns

4.2. Crude Oil

For some of the models developed, we will use Brent crude oil prices as an exogenous variable. We will treat the data like we did for Bitcoin, by computing the daily return of crude oil using the Brent crude oil daily closing prices obtained from Yahoo Finance. We will consider the same period (1 January 2017 to 30 June 2023) and the sample size is 2369 as well.

Crude oil is a volatile asset and its price can fluctuate due to a variety of factors: geopolitical events (such as territorial disputes), supply and demand dynamics, new energy policies, among others. During the considered period, crude oil prices witnessed the most significant increases during periods of considerable uncertainty. For example, the prices evidence a high degree of volatility during 2018, and in this year was marked by geopolitical tensions like the US withdrawal from the Iran nuclear deal, which contributed to supply concerns, that were reverted by the end of the year, since it was registered a downwards tendency. The prices in 2019 remained approximately constant, recording small fluctuations, and it was at the time that the COVID-19 pandemic arose, that the greatest fall during the considered period is observed. This is justified by the lockdowns and travel restrictions impact on global supply. As the pandemic began to improve, with travel restrictions being lifted and the distribution of the COVID-19 vaccine, the crude oil prices regained some value. In 2021, the Suez Canal blockage also created tension in the crude oil market. Throughout 2022 and 2023 the crude oil prices present a high volatility, mainly due to the Russia-Ukraine conflict and inflation pressures.

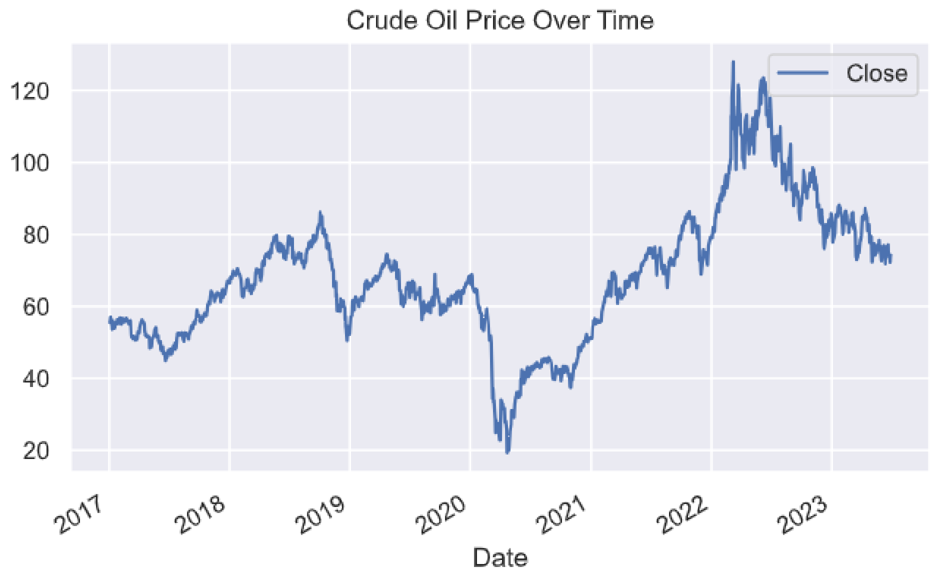

Figure 6: Evolution of Crude Oil closing prices (USD) between 01/01/2017 and 30/06/2023

As for Bitcoin prices, we computed the crude oil daily returns and obtained a largely stationary time series that also presents volatility clusters, as can be observed in Figure 7.

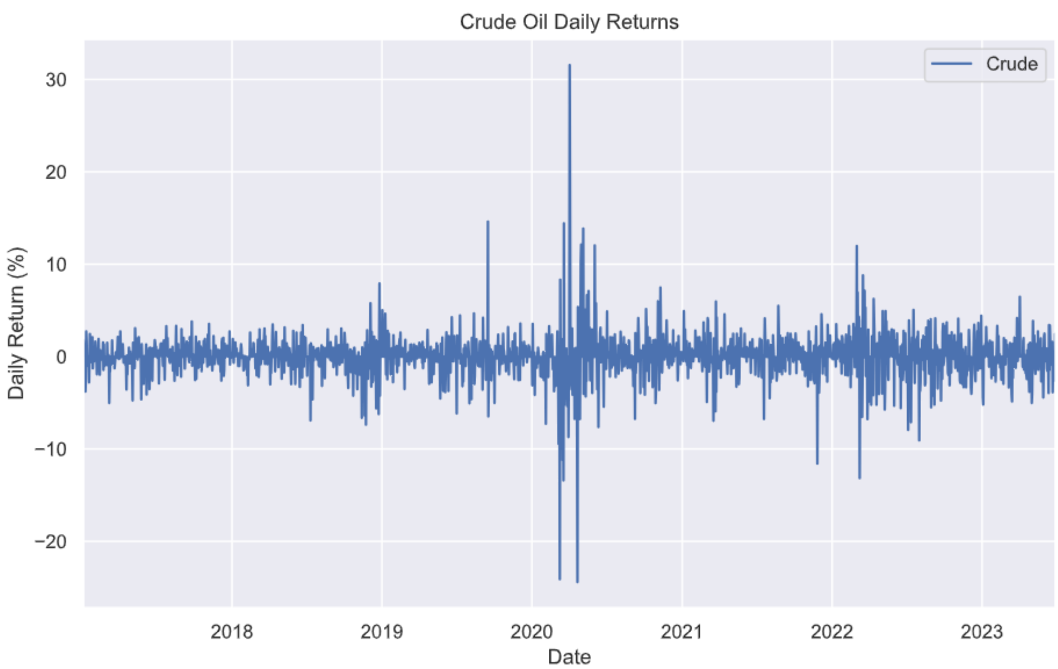

Figure 7: Evolution of Crude Oil daily returns (USD) between 01/01/2017 and 30/06/2023

CHAPTER 5

Results

5.1. GARCH models - without exogenous variables

Given that in the previous chapter, we detected the presence of ARCH effects in this time series, the following step will be to determine the efficiency of applying GARCH models to the considered data. We tested five prediction models for the Bitcoin returns volatility and compared their efficiencies.

In the table below (Table 1), we present a summary of the coefficients obtained in each model:

- Model 1 – GARCH (1,1) with a Student's-t distribution
- Model 2 – GARCH (1,1) with a normal distribution
- Model 3 – GARCH (2,2) with a Student's-t distribution
- Model 4 – ARCH (1) with a Student's-t distribution
- Model 5 – ARCH (3) with a Student's-t distribution

Table 1: AIC and BIC of GARCH models

• **Model**	**AIC**	**BIC**
Model 1 – GARCH (1,1) Student's-t distribution	12460.7	12489.6
Model 2 – GARCH (1,1) normal distribution	12960.0	12983.1
Model 3 – GARCH (2,2) Student's-t distribution	12464.7	12505.1
Model 4 – ARCH (1) Student's-t distribution	12661.8	12684.9
Model 5 – ARCH (3) Student's-t distribution	12605.0	12639.6

The Akaike and Bayesian Information Criterion (AIC and BIC) are model selection criteria considering model complexity and fit goodness. AIC favors models that explain the data well while keeping the number of parameters relatively low. On the other hand, BIC has a stronger penalty for model complexity than AIC, since it includes a term proportional to the logarithm of the sample size. When comparing GARCH models, lower AIC and BIC values indicate a better trade-off between goodness of fit and model complexity. As presented in Table 1, the model that shows the lower AIC and BIC values is Model 1, which represents a GARCH(1,1) model with a Student's-t distribution, followed by Model 2, a GARCH (2,2) with a Student's-t distribution model. In this sense, we can conclude that the best kind of GARCH models for studying the volatility of Bitcoin returns are the ones that follow a t-student distribution, since returns distributions are asymmetric and leptocurtic.

In the table below (Table 2), we present the parameter coefficients for each model. Every model includes an omega and an alpha coefficient at least. The omega estimated coefficient represents the constant volatility term, i.e., the component of the conditional variance that is not dependent on past returns or past conditional variances. The alpha coefficients represent the ARCH features of the model and capture the effects of squared past returns on the conditional variance. Simply, it reflects the volatility sensitivity to return's recent shocks. The beta estimated coefficients are associated with the GARCH term and capture the volatility persistence.

The first conclusion we can draw from the analysis of the p-values is that for each model, every estimated coefficient presents positive p-values, suggesting a direct relationship between past squared returns and

Table 2: P-values of each GARCH model coefficient

Model	**Omega**	**Alpha 1**	**Alpha 2**	**Alpha 3**	**Beta 1**	**Beta 2**
Model 1 – GARCH (1,1) with a Student's-t distribution	0.183	6.448e-08			2.052e-258	
Model 2 – GARCH (1,1) with a normal distribution	2.668e-03	1.512e-04			1.430e-141	
Model 3 – GARCH (2,2) with a	0.205	6.059e-06	2.552e-03		0.755	6.978e-39

Student's-t distribution						
Model 4 – ARCH (1) with a Student's-t distribution	2.603e-08	4.431e-04				
Model 5 – ARCH (3) with a Student's-t distribution	2.650e-09	1.111e-03	3.021e-03	1.093e-03		

As mentioned previously, the model that presents the better goodness of fit according to both AIC and BIC criteria is Model 1. Looking at the p-values of the coefficients associated with Model 1, we can gain the following insights:

- Omega (ω) p-value of 0.183 > 0.05 indicates that the constant volatility term is not statistically significant. This suggests that the constant level of volatility does not significantly contribute to explaining the variation in Bitcoin volatility return. In this sense, we can assume that this model does not penalize the absence of a constant term. This observation aligns with a simpler model focusing primarily on capturing the volatility dynamics based on past returns and conditional variances. In simpler words, the model does not require a constant component to accurately describe the conditional variance.
- Alpha (α) < 0.05 evidence that the impact of past squared returns on current volatility is statistically significant, i.e., extreme recent returns have a meaningful role in driving volatility, reflecting that when a significant price change occurs, the market will immediately respond.
- Beta (β) < 0.05 indicates the statistical significance of past conditional variances. This coefficient captures the persistence of volatility, thus a significant and positive β means that the model correctively accounts for the long-term dependency of volatility, which is crucial for assessing the clustering of Bitcoin returns volatility.

After Model 1, we can conclude that Model 3 (GARCH (2,2) with a t-student distribution) is the second-best option within this set of models, following the AIC and BIC figures. A closer analysis of its coefficient's p-values leads to similar conclusions as for the GARCH (1,1) with a t-student distribution: omega (ω) is not statistically significant since it is greater than 0.05;

alpha coefficient p-values (α1 and α2) and beta p-values (β2) are both smaller than 0.05, indicating that for this model the past returns and past conditional variances have a statistical significance impact.

5.2. EGARCH models

To account for asymmetry, we also tested EGARCH models' suitability to study this time series. Table 3 below summarizes the AIC and BIC function results for each model:

- Model 6 – EGARCH (1,1) with a Student's-t distribution
- Model 7 – EGARCH (2,2) with a normal distribution
- Model 8 – EGARCH (2,2) with a Student's-t distribution
- Model 9 – EGARCH (1) with a Student's-t distribution
- Model 10 – EGARCH (3) with a Student's-t distribution

Table 3: AIC and BIC of EGARCH models

Model	**AIC**	**BIC**
Model 6 – EGARCH (1,1) Student's-t distribution	12434.0	12462.9
Model 7 – EGARCH (1,1) normal distribution	12965.4	12988.4
Model 8 – EGARCH (2,2) Student's-t distribution	12434.5	12474.9
Model 9 – EGARCH (1) t-Student's-t distribution	12667.6	12690.7
Model 10 – EGARCH (3) Student's-t distribution	12629.3	12663.9

Similarly to what was observed for GARCH models, the models that present the best goodness of fit according to both of these criteria are Models 6 and 8, representing respectively an EGARCH (1,1) and EGARCH (2,2) models that follow a Student's-t distribution.

Table 4: P-values of each EGARCH model coefficient

Model	**Omega**	**Alpha 1**	**Alpha 2**	**Alpha 3**	**Beta 1**	**Beta 2**
Model 6 – EGARCH (1,1) Student's-t distribution	2.240e-03	1.717e-14			0.000	
Model 7 – EGARCH (1,1) normal distribution	2.976e-04	2.417e-07			0.000	
Model 8 – EGARCH (2,2) Student's-t distribution	8.160e-03	1.369e-08	0.145		3.662e-11	0.857
Model 9 – EGARCH (1) Student's-t distribution	5.788e-53	3.271e-10				
Model 10 – EGARCH (3) Student's-t distribution	2.688e-41	1.887e-08	2.463e-07	4.514e-04		

Delving into the p-values of the coefficients from the originated EGARCH models, we can conclude that the model with the best goodness of fit (according to AIC and BIC) presents p-values that suggest a strong statistical significance of the coefficients:

- Omega (ω) p-value of 2.240e-03 < 0.05, thus, we can infer that including a constant term in our volatility model is statistically significant.
- Alpha (α) and Beta (β) p-values are both below 0.05, indicating the statistical significance of past squared returns and past conditional variance.

Although presenting suitable AIC and BIC functions, Model 8 will be rejected from our study since its α1 and β2 p-values are greater than 0.05, indicating that including more parameters in the model will most likely not improve and validate the model.

In this sense, along with Model 6, we will select for the forecasting analysis Model 10, an EGARCH (3) with a Student's-t distribution. This model was chosen because it presents the third lowest AIC and BIC, and the p-values of all its coefficients indicate statistical significance.

5.3. GARCH models – with exogenous variables

In this section, we will explore if considering crude oil prices as an exogenous variable increases the model's suitability. We estimated two models:

- Model 11 - GARCH (1,1) with Student's-t distribution (as identified in the previous section as the model with the best performance)
- Model 12 - ARCH (2) with a Student's-t distribution.

Comparing the two models with exogenous variables (Model 11 *vs* Model 12), we can conclude that the model that presents the best performance is Model 11, since it shows AIC and BIC values both smaller than the ones registered for Model 12. This means that, when considering an exogenous variable, including lagged conditional variances and accounting for volatility, persistence translates into a better performing model.

Table 5: Comparison between Model 11 and Model 12

	Model 11	**Model 12**
AIC	12460.7	12625.8
BIC	12488.8	12660.4
Omega	0.195	2.41e-09
Alpha 1	2.052e-07	7.435e-04
Beta 1	2.004e-242	1.399e-03

From Table 5, we can additionally describe the statistical significance of the coefficients of this new model. The results present similarities to the ones obtained in Model 1: the constant term (omega, ω) gives a p-value of 0.195, thus higher than 0.05 and both alpha (α) and beta (β) present p-values below 0.05, indicating that the impact of past squared returns and past conditional variances on current volatility is statistically significant.

5.4. Volatility forecasting

In this section, we will delve into the analysis of the Bitcoin volatility forecasting ability of each of the five models selected from the previous sections:

- Model 1 – GARCH (1,1) with a Student's-t distribution
- Model 3 – GARCH (2,2) with a Student's-t distribution
- Model 6 – EGARCH (1,1) with a Student's-t distribution
- Model 10 - EGARCH (3) with a Student's-t distribution
- Model 11 – GARCH (1,1) with a Student's-t distribution, considering an exogenous variable.

We specify that 31st December 2021 is the last observation date for which the model was fitted. In this sense, the target data is the year 2022 and onwards. We extracted the conditional mean and conditional variance of Bitcoin conditional returns for the year 2022 and onwards, removing any missing values from the forecast. We calculated the 1% and 5% quantiles of the distribution specified in the models using the estimated parameters for the last observation in the dataset.

Having the models adequately fitted and validated, we moved on to the Value at Risk (VaR) estimation for the Bitcoin returns in 2022 and 2023. To do so, we started by calculating VaR as the negative of the conditional mean minus the square root of the conditional variance multiplied by the quantiles.

We created a plot of the VaR levels at 99% and 95% confidence levels and defined the colour of scatter points based on whether Bitcoin returns exceed the VaR thresholds. The line depicted in **blue** represents the VaR at a 99% confidence level, and the line in orange represents the VaR at a 95% confidence level. The scatter points represent specific time series values in the selected period. The points are **black** if there is no exceedance, i.e., the actual returns did not fall below the VaR estimate. The **red** scatter points reflect when there is a 1% exceedance and translate into losses worse than expected for 1% of the observations, indicating a high level of risk. Finally, scatter points coloured in **purple** represent time moments when there was a 5% exceedance, meaning that losses were worse than expected 5% of the time, which can suggest a lower but still significant level of risk.

From observing the plots obtained, we concluded that some models do not efficiently capture the Bitcoin returns VaR. This is the case of Model 10 (EGARCH (3)) and Model 11

(GARCH(1,1) following a t-Student distribution and considering crude oil prices as an exogenous variable). We analyzed the exceedances detected for the rest of the models to compare which of them performed the most accurately in the Bitcoin return VaR forecasting.

In Table 6 below, we summarize the number of exceedances observed for each model at the given confidence levels:

Table 6: Number of exceedances observed for each VaR forecast

Model	**99%**	**95%**
Model 1 – GARCH (1,1) with a Student's-t distribution	6	24
Model 3 – GARCH (2,2) with a Student's-t distribution	2	27
Model 6 – EGARCH (1,1) with a t-student distribution	2	24

Comparing the different models for forecasting Bitcoin returns volatility using the parametric approach, we will assume that the model that provides the most reliable estimate of VaR is the one that presents fewer exceedances or breaks. These breaks, represented by the scatter points in red or purple, represent the moments in which the actual loss in Bitcoin returns is superior to the VaR estimate. We can conclude that Model 6 (EGARCH (1,1) with a t-Student distribution) generates the most efficient forecasting of the Bitcoin returns VaR. The parametric estimation of VaR for Bitcoin returns forecasting recuring to this model presents 2 exceedances at a 99% confidence level and 24 at a 95% confidence level, as shown in Figure 8 below.

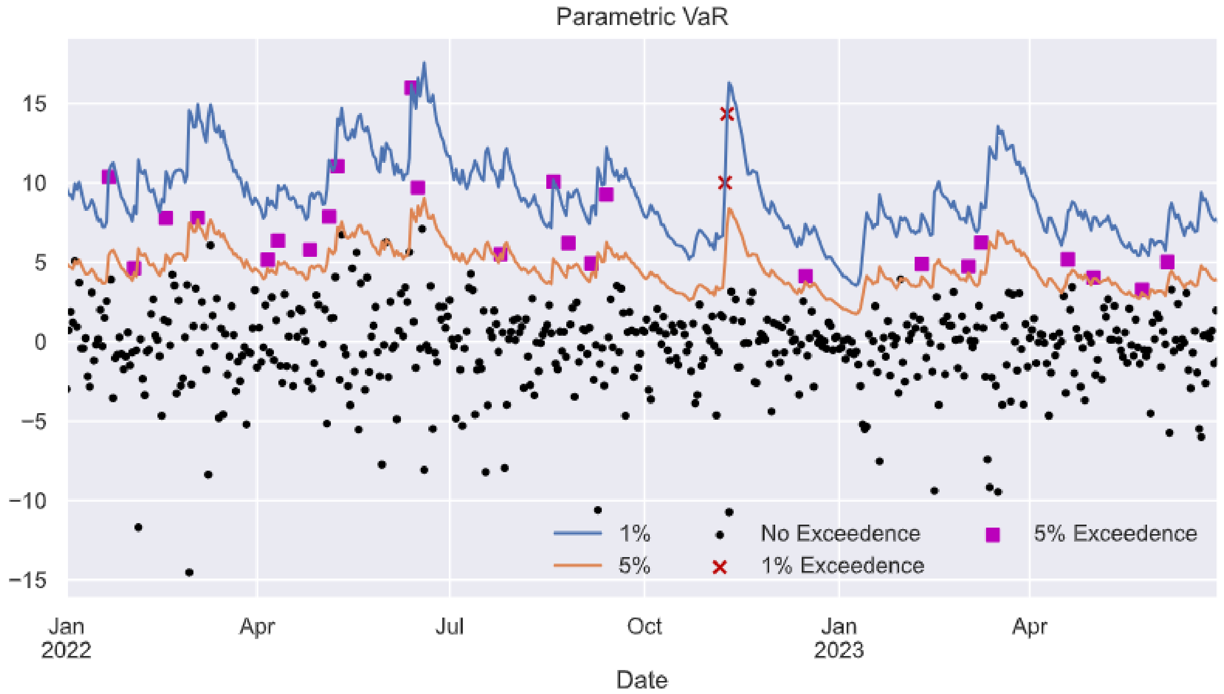

Figure 8: Parametric VaR forecasting – Model 6 (EGARCH(1,1) with a t-student distribution)

The other models produced similar results, with Model 1 evidencing more 1% exceedances and Model 3 presenting more exceedances at a 5% level. A certain level of consistency is observed at the 1% level, given that both Model 3 and Model 6 present the same number of exceedances (2 observations), which means that the models are comparable in terms of capturing more extreme events.

Model 3 has the highest number of breaks at 5% (27 observations against the 24 observations registered for Models 1 and 6); hence, it might be more conservative in estimating risk at a 95% confidence level. A higher number of exceedances means the model possibly overestimates risk at this confidence level. This interpretation has implications for risk management, since more conservative models with more exceedances can result in false alarms. However, a more conservative model leads to a more risk-averse approach, that can translate into missed investment opportunities.

CHAPTER 6

Conclusions

As with other cryptocurrencies and as expected, the evolution of the Bitcoin closing prices was not steady and presented high levels of volatility. When we compute the Bitcoin daily returns, i.e., the percentual change registered in Bitcoin closing prices, we obtain a stationary time series that presents volatility clusters. These characteristics, aligned with the fact that the time series distribution is slightly asymmetric, suggesting a relatively uniform dispersion, were the first indicators that this time series, in statistical terms, is suited to be analyzed recurring to ARCH/GARCH models.

We analyzed the Bitcoin returns, and the squared Bitcoin returns autocorrelation functions (ACF and PACF) for a 20 lag interval. As presumed, the autocorrelation function for Bitcoin returns, at the considered lags, found little evidence of substantial autocorrelation. However, when we consider the squared Bitcoin returns ACF, we observe that there is significant autocorrelation at some lags, indicating that the past squared returns of these lags influence the current squared returns. This led us to solidify the appropriateness of considering GARCH models to study the behavior of Bitcoin's returns volatility.

Delving into the selection of the GARCH model that would be more suited to evaluate the Bitcoin data, we first built models that fall under three different categories: GARCH models without an exogenous variable, EGARCH models without an exogenous variable and GARCH models with an exogenous variable. The first step of this part of our analysis was to focus on the simpler ARCH and GARCH models.

We considered three GARCH models and two ARCH models that differed in number of lags and type of distribution. Taking into account the AIC and BIC values was the first stage to select the models that presented the best goodness of fit. From this assessment, the models that showed the lowest AIC and BIC were Model 1 – GARCH (1,1) and GARCH (2,2), both with a Student's-t distribution. Then, we focused on understanding the p-values of each model, and a first observation was that for every estimated coefficient, there were positive p-values, which indicates a direct relationship between past squared returns and past conditional variances, and the current volatility. The analysis of the coefficients' p-values led us to some conclusions: the omega (ω) coefficient for the GARCH models considered (both following a Student's-t distribution, but at different lags) is not statistically significant since its p-value is greater than

0.05. The alpha and beta coefficients p-values are smaller than 0.05, indicating that - for these models - the past returns and past conditional variances have a statistically significant impact.

Following the simpler models, we also assessed the goodness of fit of five EGARCH models. As was seen with GARCH models, Models 6 and 8—which represent EGARCH (1,1) and EGARCH (2,2) models that follow a Student's-t distribution, respectively—present the best goodness of fit according to both AIC and BIC. When we considered the coefficient associated with each model, we disregarded Model 8 since $\alpha 1$ and $\beta 2$ p-values were superior to 0.05, suggesting that adding more parameters to the model is unlikely to make it better or more reliable. For the forecasting study, we chose Model 10, an EGARCH (3) with a Student's-t distribution, in addition to Model 6, since it has the third-lowest AIC and BIC, and all of its coefficients' p-values showed statistical significance.

Finally, the third category of estimated models considered crude oil prices as an exogenous variable. We find that Model 11 (GARCH (1,1) with a Student's-t distribution) performs better than Model 12 (ARCH (2) with a Student's-t distribution) when we compare two models that include an exogenous variable since it evidences smaller AIC and BIC values. This shows that a more effective model is produced when lagged conditional variances and volatility persistence are considered in the presence of an exogenous variable. Furthermore, Model 11's coefficients are statistically significant; alpha (α) and beta (β) have p-values less than 0.05, demonstrating the importance of past conditional variances and squared returns on current volatility. The p-value for the constant component (omega, ω) is 0.195, which suggests that it is not statistically significant, but the model as a whole is still robust.

At last, we reached a selection of five different GARCH and EGARCH models. One interesting aspect to note is that all models selected follow a Student's-t distribution, which is consistent with the literature consulted: Guo (2022) indicated that it is of greater importance "to introduce a heavy-tailed distribution than select a type of GARCH models" and the study conducted by Fung et al. (2022) concluded that cryptocurrencies are well described by Student's-t error distributions and for 80% of the coins analyzed in this study these were the GARCH specifications chosen, contrarily to GARCH normal and skewed-normal specifications, that the author never selected.

The last step of our study was to evaluate the predictive quality of each model to determine Bitcoin volatility. After eliminating any missing values from the forecast, we retrieved the conditional mean and conditional variance of the Bitcoin returns for the years 2022 and beyond.

Using the predicted parameters for the dataset's final observation, we computed the 1% and 5% quantiles of the distribution given in the models. After assuring the models were properly fitted, we estimated the Value at Risk (VaR) for the Bitcoin returns, creating a plot of the VaR at 99% and 95% confidence levels and colouring the scatter points of the plot based on whether Bitcoin returns exceed the VaR thresholds. From this analysis, we reached some conclusions. Firstly, we assessed that more complex models do not capture efficiently the Bitcoin returns VaR. This is aligned with our expectations. For Model 10 (EGARCH (3)), we are using three lags to model the conditional variance, which affected the volatility estimation and produced an inaccurate forecast. Model 11 (GARCH (1,1) considering crude oil prices) was the model that presented the higher complexity and like Model 10 was unable to capture the Bitcoin returns volatility VaR at a 95% confidence level.

We concluded that the model that produces the most efficient forecasting of the Bitcoin returns VaR is Model 6 (EGARCH (1,1) with a Student's-t distribution). The parametric estimation of VaR for Bitcoin returns forecasting using this model generates 2 exceedances at a 99% confidence level and 24 at a 95% confidence level. The two other models considered (Model 1 – GARCH (1,1) with a Student's-t distribution and Model 3 – GARCH (2,2) with a Student's-t distribution) produced similar results. We focused on empirical factors to determine which model forecasts more accurately Bitcoin returns volatility, however, the decision on which kind of model to consider ultimately will depend on the risk profile of the user of the model: More conservative models will produce higher numbers of exceedances, reflecting that the model may be overestimating risk at the given confidence level and can translate in the loss of investment opportunities.

We recognize that these study results reveal some insufficiencies, as we failed to forecast VaR by models that include an exogenous variable, and the model selected for the forecasting presents a simple nature. It would have been interesting to assess if the results were to be different if, instead of using the parametric approach for the VaR forecasting, we used other methods, like the Monte Carlo method and the historical simulation. At this point, the Python library used for the forecasting, as recognized by Kevin Shepard (author of the library) is not properly developed to ensure the correct modeling of GARCH/EGARCH models with exogenous variables. In this sense, it will be key to consider more advanced computational methods for future investigation on this topic.

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Appendix

Constant Mean - GARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	GARCH	**Log-Likelihood:**	-6225.37
Distribution:	Standardized Student's t	**AIC:**	12460.7
Method:	Maximum Likelihood	**BIC:**	12489.6
		No. Observations:	2368
Date:	Wed, Oct 11 2023	**Df Residuals:**	2367
Time:	00:34:56	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.1497	5.052e-02	2.963	3.046e-03	[5.067e-02, 0.249]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.2453	0.184	1.331	0.183	[-0.116, 0.606]
alpha[1]	0.0992	1.835e-02	5.406	6.448e-08	[6.322e-02, 0.135]
beta[1]	0.9008	2.623e-02	34.339	2.052e-258	[0.849, 0.952]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	3.2451	0.192	16.917	3.364e-64	[2.869, 3.621]

Appendix 1: Model 1 - GARCH (1,1) with a Student's-t distribution

Constant Mean - GARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	GARCH	**Log-Likelihood:**	-6475.99
Distribution:	Normal	**AIC:**	12960.0
Method:	Maximum Likelihood	**BIC:**	12983.1
		No. Observations:	2368
Date:	Wed, Oct 11 2023	**Df Residuals:**	2367
Time:	00:36:08	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.2305	7.218e-02	3.194	1.405e-03	[8.904e-02, 0.372]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.9557	0.318	3.004	2.668e-03	[0.332, 1.579]
alpha[1]	0.1040	2.745e-02	3.789	1.512e-04	[5.021e-02, 0.158]
beta[1]	0.8387	3.311e-02	25.332	1.430e-141	[0.774, 0.904]

Appendix 2: Model 2 – GARCH (1,1) with a Normal Distribution

Constant Mean - GARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	GARCH	**Log-Likelihood:**	-6225.33
Distribution:	Standardized Student's t	**AIC:**	12464.7
Method:	Maximum Likelihood	**BIC:**	12505.1
		No. Observations:	2368
Date:	Wed, Oct 11 2023	**Df Residuals:**	2367
Time:	00:36:20	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.1497	5.051e-02	2.963	3.047e-03	[5.066e-02, 0.249]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.4664	0.368	1.267	0.205	[-0.255, 1.188]
alpha[1]	0.0959	2.120e-02	4.524	6.059e-06	[5.436e-02, 0.137]
alpha[2]	0.0919	3.045e-02	3.017	2.552e-03	[3.219e-02, 0.152]
beta[1]	0.0270	8.655e-02	0.312	0.755	[-0.143, 0.197]
beta[2]	0.7852	6.020e-02	13.043	6.978e-39	[0.667, 0.903]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	3.2460	0.198	16.388	2.329e-60	[2.858, 3.634]

Appendix 3: Model 3 – GARCH (2,2) with a Student's-t distribution

Constant Mean - ARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	ARCH	**Log-Likelihood:**	-6326.91
Distribution:	Standardized Student's t	**AIC:**	12661.8
Method:	Maximum Likelihood	**BIC:**	12684.9
		No. Observations:	2368
Date:	Wed, Oct 11 2023	**Df Residuals:**	2367
Time:	00:45:34	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.1861	5.351e-02	3.479	5.038e-04	[8.126e-02, 0.291]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	19.3504	3.476	5.566	2.603e-08	[12.537, 26.164]
alpha[1]	0.4178	0.119	3.513	4.431e-04	[0.185, 0.651]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	2.5780	0.167	15.454	7.091e-54	[2.251, 2.905]

Appendix 4: Model 4 – ARCH (1) with a Student’s-t distribution

Constant Mean - ARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	ARCH	**Log-Likelihood:**	-6296.51
Distribution:	Standardized Student's t	**AIC:**	12605.0
Method:	Maximum Likelihood	**BIC:**	12639.6
		No. Observations:	2368
Date:	Wed, Oct 11 2023	**Df Residuals:**	2367
Time:	00:46:56	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.1828	5.245e-02	3.486	4.905e-04	[8.004e-02, 0.286]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	13.0730	2.196	5.952	2.650e-09	[8.768, 17.378]
alpha[1]	0.3270	0.100	3.261	1.111e-03	[0.130, 0.524]
alpha[2]	0.2434	8.208e-02	2.966	3.021e-03	[8.255e-02, 0.404]
alpha[3]	0.2273	6.962e-02	3.265	1.093e-03	[9.089e-02, 0.364]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	2.6464	0.174	15.200	3.516e-52	[2.305, 2.988]

Appendix 5: Model 5 – ARCH (3) with a t-student distribution

Constant Mean - EGARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	EGARCH	**Log-Likelihood:**	-6212.01
Distribution:	Standardized Student's t	**AIC:**	12434.0
Method:	Maximum Likelihood	**BIC:**	12462.9
		No. Observations:	2368
Date:	Sun, Oct 15 2023	**Df Residuals:**	2367
Time:	17:01:59	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.1211	4.763e-02	2.542	1.103e-02	[2.772e-02, 0.214]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0772	2.526e-02	3.056	2.240e-03	[2.769e-02, 0.127]
alpha[1]	0.2268	2.956e-02	7.670	1.717e-14	[0.169, 0.285]
beta[1]	0.9871	6.781e-03	145.576	0.000	[0.974, 1.000]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	2.9066	0.204	14.277	3.051e-46	[2.508, 3.306]

Appendix 6: Model 6 – EGARCH (1,1) with a Student's-t distribution

Constant Mean - EGARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	EGARCH	**Log-Likelihood:**	-6478.68
Distribution:	Normal	**AIC:**	12965.4
Method:	Maximum Likelihood	**BIC:**	12988.4
		No. Observations:	2368
Date:	Sun, Oct 15 2023	**Df Residuals:**	2367
Time:	17:05:15	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.2199	8.272e-02	2.659	7.835e-03	[5.783e-02, 0.382]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.2263	6.257e-02	3.617	2.976e-04	[0.104, 0.349]
alpha[1]	0.2103	4.073e-02	5.164	2.417e-07	[0.131, 0.290]
beta[1]	0.9229	2.167e-02	42.591	0.000	[0.880, 0.965]

Appendix 7: Model 7 – EGARCH (1,1) with a normal distribution

Constant Mean - EGARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	EGARCH	**Log-Likelihood:**	-6210.25
Distribution:	Standardized Student's t	**AIC:**	12434.5
Method:	Maximum Likelihood	**BIC:**	12474.9
		No. Observations:	2368
Date:	Sun, Oct 15 2023	**Df Residuals:**	2367
Time:	17:06:32	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.1218	4.716e-02	2.582	9.811e-03	[2.935e-02, 0.214]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0681	2.576e-02	2.645	8.160e-03	[1.766e-02, 0.119]
alpha[1]	0.3165	5.574e-02	5.677	1.369e-08	[0.207, 0.426]
alpha[2]	-0.0992	6.800e-02	-1.459	0.145	[-0.232,3.409e-02]
beta[1]	0.9637	0.146	6.617	3.662e-11	[0.678, 1.249]
beta[2]	0.0258	0.143	0.180	0.857	[-0.255, 0.307]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	2.9203	0.209	13.947	3.289e-44	[2.510, 3.331]

Appendix 8: Model 8 – EGARCH (2,2) with a Student's-t distribution

Constant Mean - EARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	EARCH	**Log-Likelihood:**	-6329.82
Distribution:	Standardized Student's t	**AIC:**	12667.6
Method:	Maximum Likelihood	**BIC:**	12690.7
		No. Observations:	2368
Date:	Sun, Oct 15 2023	**Df Residuals:**	2367
Time:	17:09:27	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.1803	6.542e-03	27.556	3.701e-167	[0.167, 0.193]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.3042	0.216	15.318	5.788e-53	[2.881, 3.727]
alpha[1]	0.4648	7.395e-02	6.285	3.271e-10	[0.320, 0.610]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	2.5683	0.168	15.274	1.145e-52	[2.239, 2.898]

Appendix 9: Model 9 – EGARCH (1) with a Student's-t distribution

Constant Mean - EARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	EARCH	**Log-Likelihood:**	-6308.65
Distribution:	Standardized Student's t	**AIC:**	12629.3
Method:	Maximum Likelihood	**BIC:**	12663.9
		No. Observations:	2368
Date:	Sun, Oct 15 2023	**Df Residuals:**	2367
Time:	17:10:25	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.1611	5.280e-02	3.052	2.275e-03	[5.764e-02, 0.265]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.3934	0.252	13.460	2.688e-41	[2.899, 3.888]
alpha[1]	0.4506	8.015e-02	5.622	1.887e-08	[0.294, 0.608]
alpha[2]	0.3350	6.492e-02	5.161	2.463e-07	[0.208, 0.462]
alpha[3]	0.2380	6.785e-02	3.508	4.514e-04	[0.105, 0.371]

Distribution

	coef	std err	t	P>|t|	95.0% Conf. Int.
nu	2.5992	0.176	14.801	1.438e-49	[2.255, 2.943]

Appendix 10: Model 10 - EGARCH (3) with a Student's-t distribution

AR-X - GARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.005
Mean Model:	AR-X	**Adj. R-squared:**	0.004
Vol Model:	GARCH	**Log-Likelihood:**	-6221.11
Distribution:	Standardized Student's t	**AIC:**	12454.2
Method:	Maximum Likelihood	**BIC:**	12488.8
		No. Observations:	2368
Date:	Wed, Oct 11 2023	**Df Residuals:**	2366
Time:	19:20:09	**Df Model:**	2

Mean Model

	coef	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
Const	0.1495	5.036e-02	2.968	2.998e-03	[5.076e-02, 0.248]
Close	0.0767	2.682e-02	2.861	4.227e-03	[2.416e-02, 0.129]

Volatility Model

	coef	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
omega	0.2405	0.186	1.296	0.195	[-0.123, 0.604]
alpha[1]	0.0988	1.902e-02	5.195	2.052e-07	[6.152e-02, 0.136]
beta[1]	0.9012	2.710e-02	33.251	2.004e-242	[0.848, 0.954]

Distribution

	coef	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
nu	3.2660	0.199	16.424	1.288e-60	[2.876, 3.656]

Appendix 11: Model 11 GARCH (1,1) with a Student's-t distribution, considering crude-oil prices as an exogenous variable

AR-X - ARCH Model Results

Dep. Variable:	Close	**R-squared:**	0.005
Mean Model:	AR-X	**Adj. R-squared:**	0.005
Vol Model:	ARCH	**Log-Likelihood:**	-6306.90
Distribution:	Standardized Student's t	**AIC:**	12625.8
Method:	Maximum Likelihood	**BIC:**	12660.4
		No. Observations:	2368
Date:	Wed, Oct 11 2023	**Df Residuals:**	2366
Time:	19:21:47	**Df Model:**	2

Mean Model

	coef	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
Const	0.1716	5.303e-02	3.236	1.210e-03	[6.770e-02, 0.276]
Close	0.0762	2.335e-02	3.263	1.101e-03	[3.043e-02, 0.122]

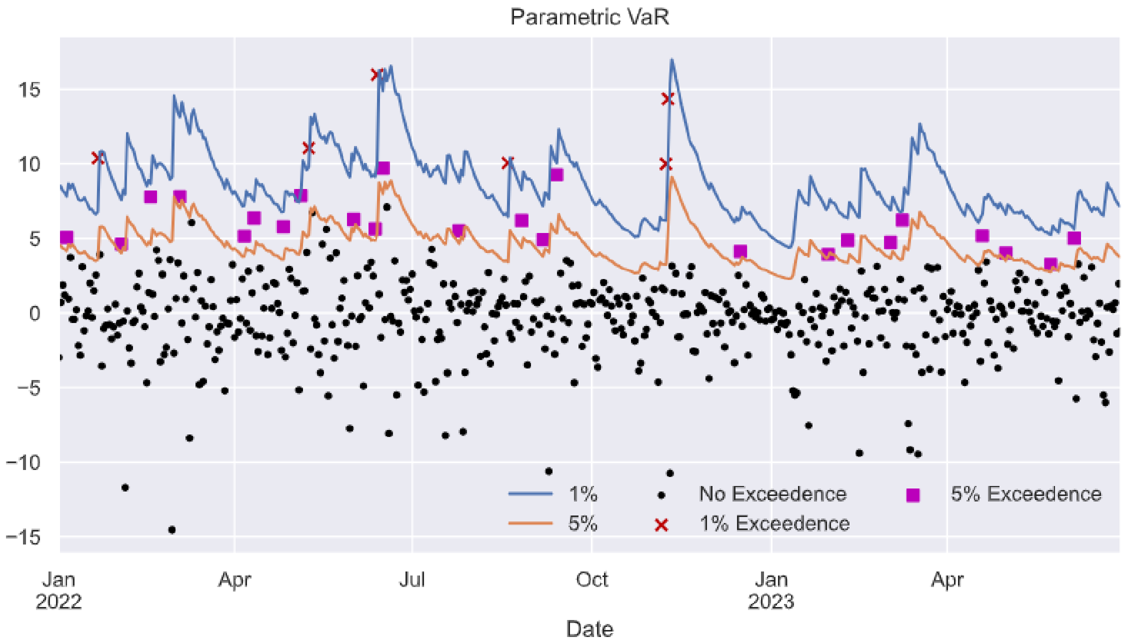
Volatility Model

	coef	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
omega	15.2275	2.554	5.963	2.481e-09	[10.222, 20.233]
alpha[1]	0.3355	9.947e-02	3.373	7.435e-04	[0.141, 0.530]
alpha[2]	0.2892	9.053e-02	3.195	1.399e-03	[0.112, 0.467]

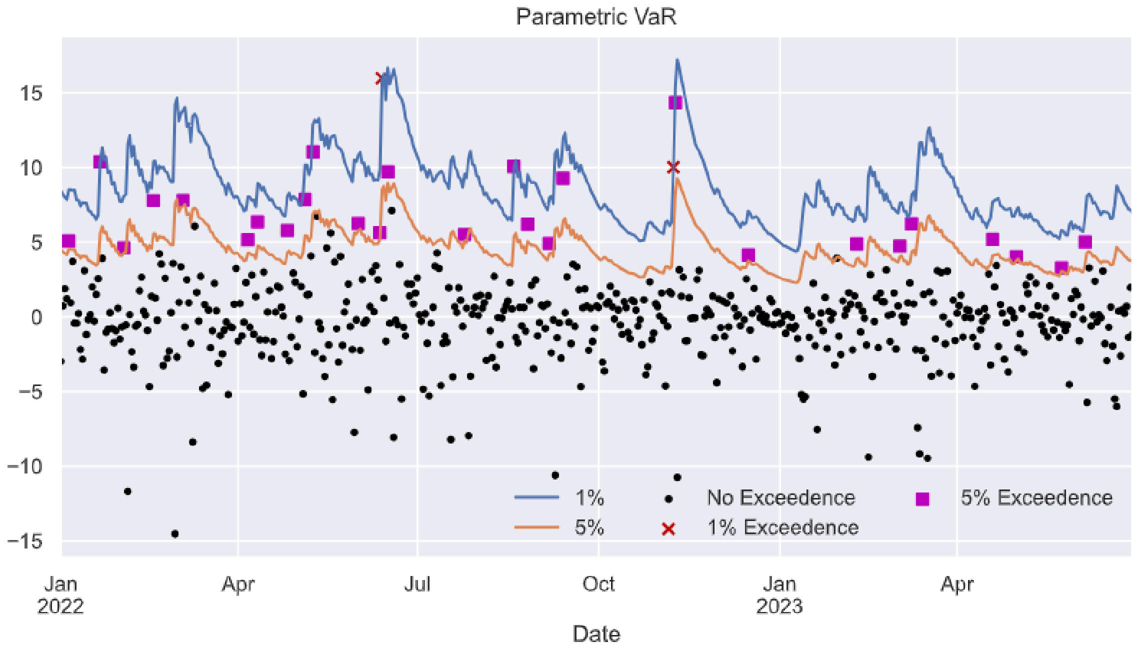
Distribution

	coef	**std err**	**t**	**P>|t|**	**95.0% Conf. Int.**
nu	2.6462	0.176	15.029	4.769e-51	[2.301, 2.991]

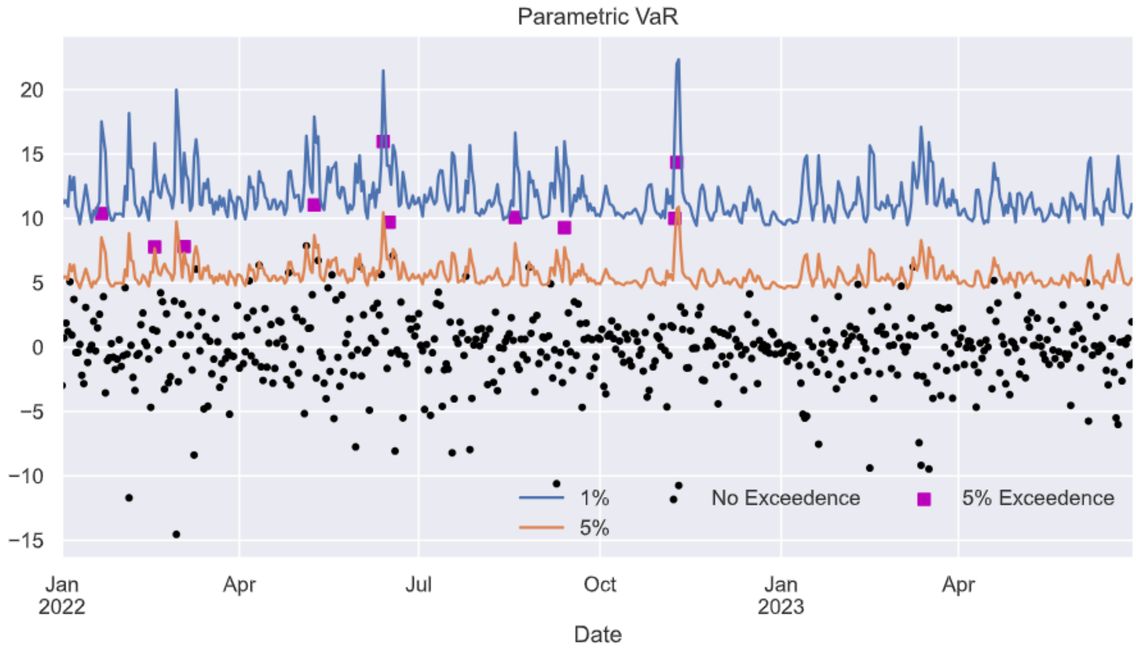
Appendix 12: Model 12 ARCH (2) with a Student's-t distribution, considering crude oil prices as an exogenous variable



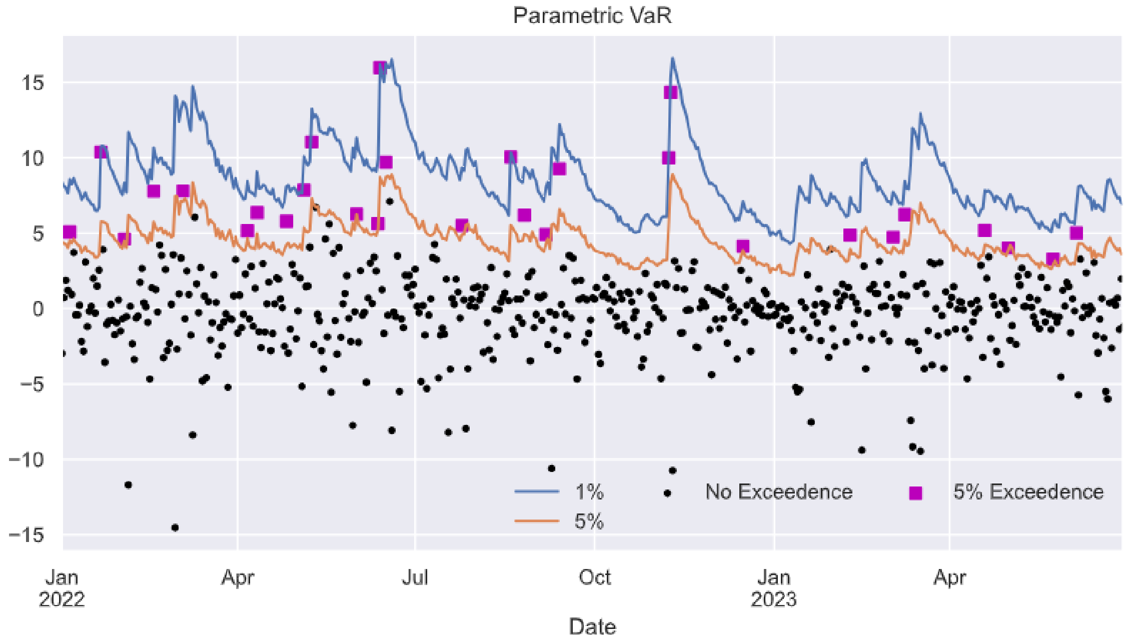
Appendix 13: VaR forecasting, using Model 1



Appendix 14: VAR forecasting, using Model 3



Appendix 15: VaR forecasting, using Model 10



Appendix 16: VaR forecasting, using Model 11