INSTITUTO
UNIVERSITÁRIO
DE LISBOA

Predicting the minimum point in a bear market using macroeconomic variables

Guilherme Teixeira Bica

Master in Economics

Supervisor:
PhD Sofia de Sousa Vale, Vice Recetor,
ISCTE-IUL

October, 2023

BUSINESS
SCHOOL

Department of Economics

Predicting the minimum point in a bear market using macroeconomic variables

Guilherme Teixeira Bica

Master in Economics

Supervisor:
PhD Sofia de Sousa Vale, Vice Recetor,
ISCTE-IUL

October, 2023

To my family, your sacrifices and unwavering faith in me made this achievement possible.
To my girlfriend, your determined belief in me and constant push for me to pursue this master's degree made this accomplishment a reality.
To my friends, your camaraderie and playful spirit made this journey much more enjoyable.
To my thesis supervisor, Professor Sofia Vale, your invaluable guidance, and relentless support made completing this thesis a journey of growth and knowledge.

To what’s coming.

Table of contents

Table of Figures

Table of Tables

Table of Equations

Resumo

Os mercados em queda, caracterizados por quedas prolongadas nos preços das ações, apresentam desafios significativos para investidores, gestores de carteira e decisores políticos. Prever o ponto mais baixo de um mercado em queda é fundamental para a mitigação e gestão de risco e para as respostas políticas durante períodos de distúrbios financeiros. Tanto quanto foi possível encontrar, este estudo é pioneiro na tentativa de prever o ponto mais baixo dos mercados em queda usando modelos VAR, uma ferramenta estatística amplamente utilizada em macroeconomia e finanças. A pesquisa concentra-se em cinco períodos – os mercados em queda de 1982, 1989, 2003, 2009 e 2022. As variáveis macroeconómicas utilizadas neste estudo para prever o ponto mais baixo do mercado de ações são o PIB, a Taxa de Juro, a Inflação, a Produção Industrial, a Curva de Rendimento, a Taxa de Câmbio e a Oferta Monetária. As conclusões do estudo, baseadas no desempenho do modelo VAR, revelam relações estatisticamente significativas entre as variáveis económicas e o desempenho do mercado de ações durante os mercados de baixa. No entanto, as métricas de desempenho utilizadas no estudo, como o *MAE, RMSE, MAPE, DA* e *Theil's U*, bem como os valores previstos, apontam para a conclusão de que este modelo não deve ser utilizado para prever o ponto mais baixo de um mercado em queda. O modelo não é capaz de ser fiável e consistente nas previsões, uma vez que não consegue captar a complexa dinâmica do mercado acionista.

Classificação JEL:

G01 Crises Financeiras
F47 Previsão e Simulação: Modelos e Aplicações

Palavras-chave:

Mercados em Queda; Mercados Financeiros; Modelo VAR; Previsões; Ponto mais baixo de um mercado em queda; Mercado Acionista dos EUA;

Abstract

Bear markets, characterized by prolonged stock price declines, pose significant challenges for investors, portfolio managers, and policymakers. Predicting the lowest point of a bear market is critical for risk mitigation and policy responses during financial distress. As far as it was possible to find, this study pioneers in attempting to predict the bottom of bear markets using VAR models, a statistical tool prevalent in macroeconomics and finance. The research focuses on five periods—the 1982, 1989, 2003, 2009 and 2022 bear markets. The macroeconomic variables that were used in this study to predict the stock market nadir are GDP, Interest Rate, Inflation, Industrial Production, Yield Curve, Exchange Rate and Money Supply. The study's findings, rooted in VAR model performance, reveal statistically significant relationships between economic variables and stock market performance during bear markets. However, the performance metrics used in the study, MAE, RMSE, MAPE, DA and Theil's U and the forecasted values, point towards the conclusion that this model should not be used to predict the bottom of a bear market. The model is not capable of being reliable and consistent in its forecasts as it fails to capture the complex dynamics of the stock market.

JEL Classification Code:

G01 Financial Crises
F47 Forecasting and Simulation: Models and Applications

Keywords:

Bear markets; Financial Markets; VAR Model; Forecasting; Bottom of Bear Market; US Stock Market;

Introduction

The intricate and ever-evolving world of financial markets has for a long time captivated the attention of scholars, investors, policymakers, and the general public. Among the financial markets, the U.S. stock market stands as a dynamic barometer of economic health and investor sentiment as mentioned in Nyberg (2013). The stock market's performance is not only a reflection of individual corporate fortunes but also a mirror of macroeconomic conditions, global geopolitical events, and systemic forces that influence investor behaviour Chen (2009). Within this expansive and complex landscape, bear markets emerge as distinct phases of market behaviour, characterized by prolonged declines in stock prices, often coupled with economic recessions, financial crises, and a profound shift in investor sentiment Barsky and De Long (1990).

The ability to predict the lowest point, or trough, of a bear market, is a formidable challenge with vast implications for investors, portfolio managers, and policymakers. Such predictions are invaluable in guiding investment decisions, constructing risk mitigation strategies, and crafting effective policy responses during periods of financial distress. Accurate bear market trough predictions can potentially avert substantial financial losses for investors and offer a blueprint for policy measures that can stabilize financial markets during crises Chen (2009).

While the study of financial markets and bear markets, in particular, is not a novel endeavour, the approach undertaken in this research is both pioneering and distinct. Prior research has predominantly concentrated on retrospective analyses, often rooted in fundamental indicators such as earnings, interest rates, and economic data. Examples of these are the works of Nyberg (2013), Altinbas and Biskin (2015) and Vogiazas and Alexiou (2017). These studies typically aim at either projecting the onset of bear markets or analysing their aftermath. Other than that, the existing literature has also looked much more at the stock market in general or analysed bull markets. Examples of these are the works of Mookerjee & Yu (1997), South Korea by Gong and Mariano (1997), Alexakis and Niarchos (2000), Paul and Mallik (2004) and Dionisio et al. (2005). Remarkably, forecasting the precise nadir within a bear market has remained an understudied and underappreciated facet of existing financial literature. Predicting the bottom of a bear market is essential from an economic standpoint as it can help mitigate the potential negative impacts of a prolonged market downturn, such as wealth erosion, reduced consumer confidence, and increased political pressure on government and regulatory authorities to address the economic challenges. This research strives to bridge this gap and is committed to exploring an uncharted domain in bear market prediction. The contemporary global landscape is marked by a series of unprecedented events and uncertainties. The COVID-19 pandemic, the ongoing Ukraine-Russia conflict, oil price shocks, and other geopolitical and economic disruptions have created an environment characterized by heightened unpredictability and volatility in financial markets. In this context, research aimed at predicting bear market troughs has never been more relevant.

This study leverages the Vector Autoregressive (VAR) model, a well-established statistical methodology with applications in various domains, including macroeconomics and finance. The novel aspect of this research, however, lies in the adaptation of VAR models to the complexities of predicting bear market troughs. The research utilizes a VAR model, known for its advantages in analysing multiple interrelated variables, handling endogeneity and making it suitable for forecasting. This dynamic modelling approach captures complex relationships among macroeconomic variables, enhancing predictive power and offering a comprehensive analysis. The model's use is underscored by its prior application in relevant research, exemplified by Gong and Mariano (1997) and Vogiazas and Alexiou (2017). The contribution of this work to the existing literature is the fact that this work uses quarterly data. This is by design to be able to identify wider bear markets and to match the frequency of the economic cycles. Moreover, as far as it was possible to be found, this is an original argument that aims to forecast a particular point in the stock market cycle. The model developed will run five times, one per identified bear market. The definition of bear market adopted in this work is the decrease quarter-over-quarter or quarter-over-quarter-over-quarter of 15%. After the bear markets identification, the data will be cut five periods prior to the bottom of the bear market and the model will forecast the following 10 periods. The total period covered by this study is from Q3 1976 until Q3 2022. The model will make its predictions using the macroeconomic variables GDP, Interest Rate, Inflation, Money Supply, Industrial Production and Yield Curve. The stock market chosen was the United States stock market, represented by the S&P 500. The reader can see each of these variables in the appendix plotted.

It is essential to underscore the innovation in this research by emphasizing that, within the field of financial research, no discernible precedent exists for systematically investigating and forecasting this particular phase of the market cycle. The combination of this model with these macroeconomic variables proved to be insufficient to predict the bottom of the bear markets identified. This thesis is divided into six main chapters, the first being the introduction. In the second chapter, the reader will find a review of the current literature. After comes the Methodology chapter where the techniques used in this research are discussed. Following that comes the Results and Discussion where each bear market identified is analysed using the techniques described in the methodology. Afterwards, the reader will find a conclusions chapter.

Literature Review

The attempt to predict the stock market has been a topic of interest for researchers for a long time. The consensus in the literature evolved into two main arguments, the first that the stock market can be predicted using macroeconomic variables was led by Fama and Schwert (1977) and Sharpe (1964). The second theory defends that the stock market cannot be predicted, led by the work of Bachelier (1900). One of the first and most influential examples of this to the literature, and the building block of the first former argument is the work of Sharpe in 1964. Sharpe (1964) provided his mathematical formula, the Capital Asset Pricing Model (CAPM) to calculate the expected return of an asset based on its beta, or sensitivity to market risk, which provided the foundation for modern portfolio theory and asset pricing models. Although Sharpe did not necessarily try to predict the stock market, his work was significant in understanding the relationship between risk, volatility, and financial markets.

After Sharpe and building on the argument that the stock market can be predicted, Fama and Schwert (1977) examined the relationship between inflation and various asset classes, including stocks, bonds, and real estate. The period studied was from 1926 and 1975 and used United States data. The findings of their paper indicated that inflation had a negative impact on real stock returns due to stocks being more exposed to changes in earnings and interest rates, both of which are influenced by inflation. Similar work was done for other countries such as Singapore by Mookerjee & Yu (1997), South Korea by Gong and Mariano (1997), Greece by Alexakis and Niarchos (2000), Australia by Paul and Mallik (2004) and Portugal written by Dionisio et al. (2005). The period studied ranged between 1975 and 1992, 1974 and 1994, 1987 and 1997, 1984 and 2001 and 1987 and 2002 respectively. All these papers use the same macroeconomic variables: industrial production, inflation, money supply and exchange rate. For the case of Singapore, Mookerjee and Yu's (1997) findings suggest that industrial production and exchange rate had a significant impact on the stock prices of the Singaporean stock market, while money supply and inflation had no impact. To achieve these results the authors used econometric methods such as cointegration and error correction modelling.

The findings in the other economies are quite similar to these. Gong and Mariano (1997) when studying South Korea found that industrial production, inflation and the exchange rate had a significant impact on stock market returns in the country. In particular, the authors found that an increase in industrial production leads to an increase in stock returns and that a decrease in inflation or a depreciation of the exchange rate leads to a decrease in stock returns. There is also evidence of a long-run equilibrium between these three macroeconomic variables and stock returns. To achieve these results, the authors used cointegration and VAR models. For the Greek economy, Niarchos and Alexakis (2000) used regression analysis and Granger causality tests and found that an increase in industrial production and a decrease in inflation would result in an increase in stock market returns. They did not find a significant relationship between money supply, exchange rates, and stock market returns, however. In the case of Australia, the authors performed Granger causality tests to investigate the relationship between the variables. The conclusion of this study resembles the ones of the Greek economy, in that the authors find that a decrease in inflation or an increase in industrial production would lead to an increase in stock market returns. Comparably, Paul and Mallik' (2004) did not find any significant relationship between interest rates and exchange rates, and stock market returns. Finally, for the Portuguese example, Dionisio et al. (2005) used regression analysis. The paper suggests that industrial production and money supply had a significant positive impact on the stock market index in Portugal, while inflation and interest rates had a significant negative impact. They also found a significant positive relationship between the stock market index and exchange rates, although the effect was smaller than that of industrial production and money supply.

Barsky and De Long (1990) defined the bull market as a period when the stock market experiences sustained gains of 30% or more and the bear market as a period of sustained losses of 30% or more. The paper suggests that there is a relationship between bull and bear market volatility and macroeconomic variables such as inflation or corporate profits. Estrella and Mishkin's (1996) paper also argues that the stock market can be predicted by using the yield curve, the difference between short-term and long-term government bonds. This research finds that the yield curve inversion is a very reliable predictor of future recessions. The paper proposes that the yield curve can serve as a valuable tool for investors and policymakers alike to predict future economic conditions, including the probability of an upcoming recession. This point is particularly important because the minimum point of a bear market can be the starting point of a bull market.

One important concept introduced by Stock and Watson (2002) is the use of diffusion indexes as a tool for forecasting macroeconomic variables. This method involves building a weighted average of several individual economic indicators, such as employment, retail sales, or industrial production. Stock and Watson (2002) showed that diffusion indexes can accurately forecast macroeconomic variables such as GDP or inflation that can be used to forecast the stock market. Chen (2009) examined the predictive power of variables such as GDP, inflation and yield curve in predicting bear markets in the stock market, an important study for investors and policymakers. The author used various statistical methods such as probit models and signal extraction techniques. The conclusion of this paper states that no single variable can reliably predict changes in the business cycle, however, a combination of indicators may provide useful insights for policymakers, analysts and market participants.

Nyberg (2013) demonstrated that it is possible to predict the stock market using macroeconomic variables. In his work, Nyberg uses a dynamic binary time-series model to predict both bear and bull stock markets. He attempts to predict the United States stock market using four main macroeconomic variables: dividend yield, corporate profits to GDP ratio, default spread and term spread. This model performs very well in predicting both the start and the end of bull and bear markets. The model has a high out-of-sample predictive accuracy and produces better forecasts than other models that do not incorporate binary states. The study suggests that the most important variable in predicting bear markets is default spread and in predicting bull markets is term spread. Tramontana et al. (2013) found in their research that it is possible to predict turning points in the market. They extended a previous work of Huang and Day (1993) and included additional market conditions to investigate the impact of these conditions on market dynamics. The model developed was able to generate realistic fluctuations in both stock prices and trading volume.

Altinbas and Biskin (2015) aimed to identify the most influential macroeconomic factors on stock market returns in their research using feature selection algorithms. The authors used data on 14 different macroeconomic indicators and applied four feature selection algorithms to identify the most important predictors of stock market returns. They found that the most important macroeconomic indicators that can best predict stock market returns are the consumer price index (CPI), the exchange rate, and the industrial production index. The authors studied the Turkish market between January 2001 and December 2014 and concluded that these macroeconomic indicators provide more accurate predictions of stock market returns than the full set of macroeconomic indicators.

The interest in being able to predict market trends does not limit itself to the stock market. Vogiazas and Alexiou (2017) studied the housing market for seven advanced economies: Japan, Germany, France, Canada, Australia, United Kingdom and United States. In this study, the authors employed econometric models such as panel data regressions and Markov switching autoregressive models in order to investigate the relationship between macroeconomic variables and housing prices. The macroeconomic variables used for this study, given the particular type of assets, were the interest rates, income, population growth and credit. The study concludes that housing prices are positively related to income, population growth and credit and negatively related to interest rates. This point is also backed by the literature consensus.

As mentioned previously, the other wave in the literature argues and attempts to demonstrate that macroeconomic variables cannot predict the stock market. Some academics believe that the stock market moves as a random walk. This theory, of course, was initially presented by Bachelier (1900) in his paper "The Theory of Speculation". This was a ground-breaking mathematical model that analysed the behaviour of stock prices and laid the foundation for the argument that states that the stock market cannot be predicted. This would imply that the stock market cannot be predicted using macroeconomic variables, let alone the minimum point in a bear market, as this thesis attempts to predict. Bachelier's key point was that stock prices do not follow a predictable pattern, rather they fluctuate randomly much like the movement of particles in a gas. To do this he employed mathematical methods from probability theory, such as the normal distribution, to model the random fluctuations of stock prices over time. This idea was met with much scepticism in the literature at the time, but it ultimately proved to be highly influential and important for modern finance theory, including for the efficient market hypothesis, which suggests that stock prices reflect all available information and are impossible to consistently beat through active trading.

This exact hypothesis was tested by Lee (1992) who used data from six different stock markets, United States, Japan, United Kingdom, Canada, Germany and France to examine whether stock prices follow a random walk process. To do this the author applied several statistical tests such as the augmented Dickey-Fuller test and the Phillips-Perron test. Contrary to what could be expected, not all stock markets had the same results. The study found that the United States and Japanese stock markets followed a random walk, however, the British, Canadian, French and German did not. Lee does also note that these results were sensitive to the specific statistical test used. For this reason, this academic paper could not conclude that stock markets follow random walks.

However, the main paper that defends this theory is the one made by Fama (1995), who argues that if stocks follow a random walk, it would be impossible to consistently predict the stock market or to generate out-of-normal returns. In order to understand whether or not stocks follow a random walk, Fama analysed a large dataset of stock prices spanning several decades and applied statistical tests including the runs test and the autocorrelation test to evaluate the randomness of the movements in the stock prices. The findings of this study provide strong support for the argument that the stock market follows a random walk. The study finds that stock prices exhibit no significant serial correlation, meaning that past price movements do not predict future price movements. The author also notes that all the apparent patterns in the movements of stock prices can be explained by random chance rather than any underlying systematic behaviour. This study provides an important support to the efficient market hypothesis and suggests that stock prices are inherently unpredictable, or, in other words, this paper suggests that stock prices cannot be predicted by macroeconomic variables. This, of course, is of increased difficulty when the goal is to predict the bottom of a bear market, something this argument would fully disagree with and see as impossible to accomplish.

Despite the support Fama and other authors provide for the efficient market hypothesis, there are multiple critics of this theory. Malkiel (2003) reviewed the various critics made to the theory including the idea that markets are not always efficient due to the presence of behavioural bias and informational asymmetries. The author acknowledges the validity of these criticisms, but he argues they do not completely discredit the theory. This paper concludes that the efficient market hypothesis remains a useful framework for understanding financial markets and that investors should not rely solely on the belief that markets are inefficient to try to beat the market.

This hypothesis was also tested by Agwuegbo et al. (2010) when the authors used monthly data from 2000 to 2009 to investigate whether the stock market prices of Nigeria followed a random walk. To do this, the authors performed statistical tests such as the runs test, the autocorrelation test and tested for the presence of structural breaks in the series. The study found evidence that the stock market prices in the Nigerian market, meaning it would be impossible to predict future stock prices based on previous prices. However, the study also found that there was evidence of structural breaks in the series, suggesting that the stock market prices in Nigeria are influenced by economic and political events. This can mean that for this market it would be possible to predict it using macroeconomic variables even if stock markets follow random walks.

Throughout time the literature on the prediction of the stock market has evolved into two main arguments. The first stated that the stock market could be predicted using macroeconomic variables, while the other believed that the stock market could not be predicted. Many examples of academic articles were able to fundamentally contribute to the first argument, and the wide literature includes smaller economies such as Portugal, Greece or Singapura and larger economies such as the United States, Australia, or South Korea. The literature is so vast that it does not even limit itself to the boundaries of the stock market. Rather, there are also examples of academic papers studying other markets such as the housing market fluctuations. The second main argument is that the stock market cannot be predicted because it follows random walks. This is an argument that dates to 1900 and is still relevant today. However, even if this argument is correct and stock prices do follow random walks, there is evidence of examples where there existed structural breaks meaning the data was affected by external variables such as macroeconomic variables.

Despite the immense amount of academic research on the topic, to the best of the research conducted, there is no academic article attempting to predict the minimum point of a bear market using macroeconomic variables. There are examples of papers that tried to predict the maximum point of a bull market, in the housing market, for instance, but never for the minimum point of a bear market. This point is quite surprising because if a competent model is developed it could mean a dramatic increase in returns for market participants. Moreover, the model can also provide important information for policymakers. It does, however, make the work increase in difficulty and interest as it is an original and untested idea.

Methodology

This section aims to provide a comprehensive and rigorous investigation of the methods used to study the question at hand. The study design, sampling technique, data collection methods, and data analysis procedures are described here. The purpose of this chapter is to provide a clear understanding of how the research was conducted and to ensure that the findings are reliable and valid. The chapter begins with an overview of the research design, followed by a description of the variable's selection process. After that follows a discussion of the data collection methods employed and concludes with a brief overview of the data analysis procedures.

As previously explained, the main goal of this research is to compute a model that can consistently predict the bottom of a bear market using macroeconomic variables. Given that, it becomes clear that the null hypothesis is inexistence of statistical significance in the model produced, while the alternative is that there is statistical significance thus concluding that it is possible to predict the lowest point of a bear market. The study was conducted in three phases. The first was the identification of relevant macroeconomic variables that have been shown to be significant predictors of the nadir of the bear market. In the second phase, a time series analysis will be conducted using historical data to assess the statistical significance of these macroeconomic variables in forecasting the end of a bear market employing a VAR model. Finally, in the third phase, the VAR model was computed, the data was tested for various statistical checks and the model was tested against various performance tests.

$$H_0 = It\ is\ not\ possible\ to\ predict\ the\ bottom\ of\ a\ bear\ market$$
$$H_a = It\ is\ possible\ to\ predict\ the\ bottom\ of\ a\ bear\ market$$

Data

The main task after defining clearly what the null hypothesis and the alternative is to decide what the dependent variable is going to be. The variable studied in this thesis will be the United States stock market. This is the case because it is the biggest stock market in the world currently with a market capitalization of 40.7 trillion USD, well above the second in the list, China, with only 12.2 trillion USD of market capitalization according to data from the World Bank from 2020 (World Bank, 2020). In this research, the United States stock market will be represented by the famous Standard & Poor's 500 index (S&P 500). This is a common practice in the literature when studying the United States stock market. The index tracks the performance of 500 large-cap United States companies. The companies included in the index represent a broad cross-section of the United States economy including many of the largest and most influential companies in the world. As a result, changes in the S&P 500 can provide insights into the overall health and direction of the United States stock market. Moreover, the S&P 500 is a market-weighted index, which means that companies with a larger market capitalization, the total value of their outstanding shares, have a greater impact on the index's performance. Finally, this index being the representant of the United States stock market is quite interesting for this study because it is a widely followed index by investors, analysts, policymakers and other market participants around the world, which means it can serve as a common reference point for discussions and analysis of the United States stock market. The S&P 500 data comes from a Bloomberg Terminal. This is one of the most used data sources in the literature due to the wide range of data for stock market-related topics, therefore, it was chosen to be the source for this data point. The data was extracted from the Bloomberg Terminal in quarterly frequency and ranges from the first quarter of 1973 to the third quarter of 2022.

The following step is to define the macroeconomic variables to be used in this research to predict the bottom of the bear market. The different variables used in the literature for this topic were interest rates, inflation, industrial production, money supply, exchange rate, yield curve, employment, retail sales, gross domestic product growth, dividend yield, corporate profits, default spread, term spread, population growth and credit. However, the main variables that were used in multiple studies were interest rate, inflation, industrial production, money supply, exchange rate, the yield curve and GDP. These are not only the most used variables but also, according to many of the authors mentioned previously in the literature review such as Estrella and Mishkin (1996), Mookerjee & Yu (1997), Gong and Mariano (1997), Alexakis and Niarchos (2000), Paul and Mallik (2004), Dionisio et al. (2005), Stock and Watson (2002) or Chen (2009) and other authors, the most influential variables on the stock market and that can be translated to a higher accuracy to the model developed. For these reasons, these are the variables that will be used in this research. Fortunately, all these variables are publicly available in trusted databases. GDP, interest rate, inflation, industrial production, money supply, exchange rate and yield curve, were sourced from the United States Federal Reserve St. Louis database (FRED).

Using FRED as a main database provides several advantages such as data reliability for it being managed by the Federal Reserve St. Louis, data depth due to the vast amount of variables it covers and data accessibility as it is a free user-friendly database with various tools to read and analyse datapoints. As these are different variables that track different data points, they all have different data ranges that were extracted. Industrial Production (code in FRED: INDPRO) ranges from the first quarter of 1919 until the third quarter of 2022. Industrial Production was extracted with quarterly frequency, seasonally adjusted as an index with 2017 being equal to 100. Inflation (code in FRED: CPALTT01USM657N) ranges from the first quarter of 1960 to the third quarter of 2022. This variable was extracted with quarterly frequency as an index with 1975 Q1 being equal to 100. Interest rate (code in FRED: DGS10) was extracted with data ranging from the first quarter of 1962 up until the third quarter of 2022. It was extracted in quarterly frequency as a percentage. Data for GDP (code in FRED: GDPC1) starts in the first quarter of 1947 and ends in the third quarter of 2022. This variable was extracted as Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate. Money supply (code in FRED: M2SL) was extracted with dates ranging between the first quarter of 1959 and the third quarter of 2022. Money Supply was extracted in Billions of dollars with quarterly data frequency. The yield curve (code in FRED: T10Y2Y) ranges between the third quarter of 1976 and the third quarter of 2022. It was extracted in quarterly frequency as a percentage. Finally, with a considerably smaller range, the exchange rate (code in FRED: DEXUSEU) ranges between the first quarter of 1999 and the third quarter of 2022. This data point was extracted in quarterly frequency as the dollars to one euro amount. The below table, Table 1, summarizes this information.

Variable	**FRED Code**	**Date Range**
Industrial Production	INDPRO	Q1 1919 – Q3 2022
Inflation	CPALTT01USM657N	Q1 1960 – Q3 2022
Interest Rate	DGS10	Q1 1962 – Q3 2022
GDP	GDPC1	Q1 1947 – Q3 2022
Money Supply	M2SL	Q1 1959 – Q3 2022
Yield Curve	T10Y2Y2	Q3 1976 – Q3 2022
Exchange Rate	DEXUSEU	Q1 1999 – Q3 2022

Table 1 - Data Information.

All these variables were extracted with a quarterly frequency. When, by default, the variables were not quarterly, the conversion to quarterly was done directly in the FRED database using the "average" as an aggregation method. In order to define a clear starting point and given that all the variables except the exchange rate are available from the third quarter of 1976, that will be the starting point of this research. The endpoint will be the third quarter of 2022 as all data was extracted up until that point, with the most updated values when the extraction occurred. The use of quarterly data, in data points more frequent than quarterly, is because most variables in this study are by default quarterly, including the dependent variable. Adjusting all variables to a quarterly frequency will avoid making interpolation mistakes. This can be an option in this particular research because the number of observations that will be analysed is very high. Moreover, having this quarterly view will make it so that it is possible to analyse the long-term bear markets, the ones that resist the day-to-day emotional sentiments of investors and other market participants.

Model

The model employed in this research was a Vector Autoregression Model (VAR Model). The utilization of a VAR model in this research can provide several advantages. VAR models offer a multivariate analysis approach, allowing for the simultaneous examination of multiple variables and capturing their interrelationships. This is particularly important given the complex interdependencies often observed among macroeconomic variables. One key advantage of VAR models is their ability to handle endogeneity. In the context of predicting the minimum point of a bear market, where various economic factors interact and mutually influence each other, the VAR framework accommodates contemporaneous interactions among variables. This dynamic modelling approach captures the intricate relationships and provides a comprehensive understanding of the bear market prediction. VAR models are well-suited for forecasting purposes, making them suitable for predicting the minimum point of a bear market. Leveraging historical data, VAR models are capable of generating forecasts for future values of the variables. By incorporating relevant macroeconomic variables, researchers can capture the drivers of bear market dynamics and produce reliable predictions. The inclusion of macroeconomic variables enhances the predictive power of the model and allows for a more comprehensive analysis. Using this model is also quite important as it was used by many researchers in the past. Prime examples of this are Gong and Mariano (1997) or Vogiazas and Alexiou (2017). The formula for the VAR model used in this research can be found below.

$$S\&P_t = c + \phi_1 S\&P_{t-1} + \phi_2 S\&P_{t-2} + \ldots + \phi_\rho S\&P_{t-\rho} + \beta_1 IR_t + \beta_2 CPI_t + \beta_3 IndPro_t + \beta_4 MS_t + \beta_5 YC_t + \beta_6 GDP_t + \beta_7 Forex_t\, Z_t + \varepsilon_t$$

Equation 1 - VAR Model equation

In this equation, c is the constant term (intercept). $S\&P_t$ is the S&P 500 at time t. *'IR'* represents Interest Rate, *'CPI'* stands for Inflation, *'IndPro'* denotes Industrial Production, *'MS'* represents Money Supply, *'YC'* is the Yield Curve, *'GDP'* stands for the GDP, and *'Forex'* means the Exchange Rate. ε_t is the error at time t. $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$ and β_7 are the coefficients for the respective macroeconomic variables. $\phi_1, \phi_2, \ldots, \phi_\rho$ are the autoregressive coefficients for the lagged values of SP500. Finally, Z_t represents the dummy variable at time *t*. This is used because the data range for this variable is much smaller as described before. With this method we can always include the Exchange Rate in the equation while adapting for it. The below table, Table 2, addresses this point. The only variable that is lagged in this model is the S&P 500. This is because, if the macroeconomic variables do explain the movements of the stock market, then it would have explained the past movements as well. If they do not influence the movement of the stock market, then including them or not in the formula would make no difference. The below table,
Table 3, summarizes the information explained in this paragraph.

***t* Value**	***Z* Value**
$< \mathbf{1999}$	0
$\geq \mathbf{1999}$	1

Table 2 - Dummy Variable Value.

Variable	Description
$\boldsymbol{c}$	Constant term (intercept) in the equation
$\boldsymbol{S\&P_t}$	S&P 500 at time t
$\boldsymbol{IR_t}$	Interest Rate at time *t.*
$\boldsymbol{CPI_t}$	Inflation at time *t.*
$\boldsymbol{IndPro_t}$	Industrial Production at time *t.*
$\boldsymbol{MS_t}$	Money Supply at time *t.*
$\boldsymbol{YC_t}$	Yield Curve at time *t.*
$\boldsymbol{GDP_t}$	GDP at time *t.*
$\boldsymbol{Forex_t}$	Exchange Rate at time *t.*
$\boldsymbol{\varepsilon_t}$	Error at time *t.*
$\boldsymbol{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6}$ and $\boldsymbol{\beta_7}$	Coefficients for macroeconomic variables.
$\boldsymbol{\phi_1, \phi_2, \dots, \phi_\rho}$	Autoregressive coefficients for lagged values of S&P 500
$\boldsymbol{Z_t}$	Dummy variable at time *t.*

Table 3 - Variables Description.

There is a list of conditions that need to be met in order to make sure the VAR model in use is effective, and these conditions can be verified by doing a battery of diagnostic tests on the data used to ascertain the fulfilment of these conditions. The tests on the data used for this VAR model were stationarity, heteroscedasticity, exogeneity and multicollinearity. By subjecting the VAR model to these comprehensive examinations, it is the robustness and validity of the model's outcomes are ensured. All these tests were made using the programming language R. For the stationarity it was employed the Augmented Dickey-Fuller (ADF). This widely used test enables to evaluate the presence of unit roots or non-stationarity in the variables.

For investigating heteroscedasticity, the test used was the Breush-Pagan-Godfrey test. This test enables the data to be tested for the presence of heteroscedasticity, which indicates unequal variances in the error terms. By identifying heteroscedasticity, it is possible to guarantee the appropriateness of the model and the reliability of the results it provides. According to the literature consensus, in the case of VAR models usually there is no heteroscedasticity, but there can be conditional heteroscedasticity. This is also highlighted in the works of Vogiazas and Alexiou (2017). After conducting this test, it was clear that there was presence of conditional heteroscedasticity. The method used to address it was to use ARCH (Autoregressive Conditional Heteroskedasticity) which is a specialized model designed to handle conditional heteroscedasticity. ARCH models allow the variance to change over time based on past squared values of the series. This is also discussed in the work of Vogiazas and Alexiou (2017), but it was most famously considered in the work of Engle (1982).

To evaluate exogeneity the chosen test was the Durbin-Wu-Hausman test. This test plays an important role in assessing the absence of correlation between the error terms (residuals) of the model and the independent variables. It is very important to test for exogeneity as it can verify that the VAR model accurately captures the true causal relationships between the variables and mitigates potential issues of endogeneity. Finally, to examine multicollinearity, the process employed was a Correlation Matrix of the variables. This analysis allowed us to assess the presence of high correlation among the independent variables, which can lead to unstable parameter estimates and challenges in interpreting the results. By conducting these rigorous tests on the data used for stationarity (Augmented Dickey-Fuller), heteroscedasticity (Breush-Pagan-Godfrey), exogeneity (Durbin-Wu-Hausman) and multicollinearity (Correlation Matrix), it was now clear that the stringent requirements of the VAR modelling framework were upheld. This comprehensive evaluation of the model ensured its reliability, validity, and ability to provide meaningful insights and predictions. These are also tests that have been performed in the past literature, as evidenced by the works of Vogiazas and Alexiou (2017) and Nyberg (2013). The results of each test are described in the following chapter, Results and Discussion.

Bear Market

After the tests on the data are defined, it is also important to define what a “bear market” is. This is a very important concept given that what this work is trying to accomplish is to develop a model that accurately predicts the bottom of a bear market. This is, unfortunately, not yet a widely accepted concept in the literature with different authors providing different definitions for it. As seen during the literature review, some researchers use as a metric the sustained drop of 30% as evidenced by Barsky and De Long (1990). However, the big difference between this research and Barsky and De Long’s is that this research uses monthly data while the present study uses quarterly frequency of data. If this definition is considered for the bear market, it means that looking at the data for quarter-over-quarter decreases of 30% or quarter-over-quarter-over-quarter decreases of 30% there is only one bear market identified. Running the created model for this one bear market would not provide conclusive evidence that the stock market's nadir can or cannot be predicted using macroeconomic variables.

This happens because many stock market bear markets do not show in this frequency. The first COVID-19 crisis, for instance, created a rapid decrease and subsequential rapid increase in the stock market. While the fall was bigger than 30%, the quarterly frequency does not allow for this bear market to appear as a bear market. To address this limitation and increase the number of observed bear markets, this study adopted a modified definition, classifying a bear market as a drop of 10% quarter-over-quarter or a fall of 10% quarter-over-quarter-over-quarter. This is only logical and consistent with Barsky and De Long's (1990) bear market definition just adapted to quarterly frequency. The quarter-over-quarter-over-quarter is an important addition and is included in order to increase the possible amount of bear markets identified in the data for the range being used. Moreover, it is also important to have this because the variables that will be used to forecast the bottom of the bear market are macroeconomic variables and the definition of a recession in a country is a drop in GDP in two consecutive quarters or quarter-over-quarter-over-quarter. This is also a similar definition to what was adopted by authors studying a similar topic such as Barsky and De Long (1990) in their research. Using this methodology, it is possible to identify five bear markets in the data being used. Therefore, the VAR model developed will run a total of five times, one per identified bear market. A summary of this can be found in the below table, Table 4. To perform this analysis, the data will be cut five periods before the bear market date. This way, it will be possible to isolate the bear markets identified. The model will then forecast the following 10 periods based on the information provided using the formula discussed previously.

Bear Market Dates
Q2 1982
Q1 1989
Q1 2003
Q1 2009
Q2 2022

Table 4 - Bear Markets Dates

The 1982 bear market was primarily driven by a combination of high inflation, recession, and high interest rates in the United States. The Federal Reserve's tight monetary policy under Paul Volcker, aimed at curbing inflation, exacerbated the economic downturn. Fluctuations in oil prices due to geopolitical events, such as the Iranian Revolution and the Iran-Iraq War, added to the uncertainty. Unemployment rose, impacting consumer spending and business investments. The 1989 stock market decline can be attributed to a variety of factors. It was a result of a weakening U.S. economy, which was facing the challenges of a savings and loan crisis and a banking sector burdened by bad loans. There was also a degree of uncertainty due to concerns about the ongoing U.S. and Japan trade tensions. Furthermore, the Federal Reserve was adjusting interest rates in response to these economic conditions.

The stock market in 2003 experienced a challenging and tumultuous period. It was marked by the aftermath of the bursting dot-com bubble, corporate accounting scandals (notably the Enron scandal) and a global economic slowdown. These factors, along with concerns about the potential for military conflict in the Middle East in the aftermath of the September 11 tragedy, contributed to a bear market in which stock prices significantly declined. In 2009, the stock market faced the depths of the global financial crisis, which had its roots in the collapse of the housing market and the ensuing subprime mortgage crisis. This financial turmoil led to a severe bear market that saw a substantial decline in stock prices. The crisis triggered a series of events, including bank failures, government bailouts, and a credit crunch, which sent shockwaves through the global financial system. In contrast, the 2022 stock market experienced a bear market for different reasons. It grappled with a unique set of challenges, with some key contributing factors being the ongoing global supply chain disruptions, rising inflation rates, and concerns about the pace of economic recovery in the wake of the COVID-19 pandemic. These elements, combined with uncertainty surrounding central bank policies and geopolitical tensions, created an environment in which investors became more risk-averse, leading to a decline in stock prices.

Performance Tests

One important point after the creation and development of the model is to test how it performs against statistical performance tests. Testing the performance of a model is also done using the programming language R. The performance tests employed for this model were the R-Squared, the Mean Absolute Error (MAE), the Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), Directional Accuracy (DA) and Theil's U statistic. These performance tests will compare the real values of the 10 periods and the forecasted 10 periods.

R-squared, a widely used metric, measures the proportion of the variance in the dependent variable (stock market returns) that can be explained by the independent variables (lagged stock market returns and macroeconomic variables). A higher R-squared value suggests that a larger portion of the variability in the stock market can be accounted for by the model's inputs. The formula below, Equation 2, is the formula to calculate the R Squared where R^2 is the coefficient of determination, *RSS* is the sum of squares of residuals, and *TSS* is the total sum of squares. Mean Absolute Error (MAE) calculates the average absolute difference between the predicted values and the actual values. It provides an understanding of the average magnitude of the forecast errors, with lower MAE indicating better accuracy. Equation 3 is the formula to calculate MAE where y_i is the prediction, x_i is the real value, and *n* is the number of data points. Root Mean Square Error (RMSE) measures the average magnitude of the squared forecast errors, giving more weight to larger errors. It indicates the overall difference between the predicted and actual values, with lower RMSE values indicating better accuracy. Equation 4 is the formula to calculate the RMSE where N is the number of data points, ex_i is the prediction, x_i is the real value.

$$R^2 = 1 - \frac{RSS}{TSS}$$

Equation 2 - R Squared formula.

$$MAE = \frac{\sum_{i=1}^{n} |y_i - x_i|}{n}$$

Equation 3 - MAE formula

$$RMSD = \sqrt{\frac{\sum_{i=1}^{n}(x_i - ex_i)^2|}{N}}$$

Equation 4 - RMSD formula

Mean Absolute Percentage Error (MAPE) calculates the average percentage difference between the predicted and actual values. MAPE offers insights into the relative error of the model's predictions. Lower values indicate better accuracy. Equation 5 represents the formula to calculate the MAPE where n is the number of predictions, F_t is the prediction, A_t is the real value. Directional Accuracy (DA) assesses the model's ability to predict the correct direction of the stock market movement (e.g., increase or decrease). It measures the proportion of correct directional predictions, indicating the model's effectiveness in capturing the general trend. Equation 6 is the formula for DA where *CF* is the number of correct forecasts and *NF* is the total number of forecasts. Additionally, Theil's U is a statistical measure used to assess the forecast accuracy, considering both bias and variance in the predictions. It provides valuable insights into the overall performance of a forecasting model, considering the relative contributions of systematic and unsystematic errors. Equation 7 is the formula for Theil's U where *A* is the change in actual values and *P* is the change in forecasted values. By employing these comprehensive performance tests, the VAR model is evaluated in the various aspects of accuracy, directional prediction ability and others. These tests provide a thorough assessment of the model's performance in predicting the stock market.

$$M = \frac{1}{n}\sum_{t=1}^{n} \left| \frac{A_t - F_t}{A_t} \right|$$

Equation 5 - MAPE formula

$$DA = \frac{CF}{NF}$$

Equation 6 - DA Formula

$$U = \frac{\left[\frac{1}{n}\sum_{i=n}^{n}(A_i - P_i)^2\right]^{\frac{1}{2}}}{\left[\frac{1}{n}\sum_{i=n}^{n} A_i^2\right]^{\frac{1}{2}} + \left[\frac{1}{n}\sum_{i=n}^{n} P_i^2\right]^{\frac{1}{2}}}$$

Equation 7 - Theil's U formula

In this chapter, a comprehensive methodology has been presented for conducting the research and addressing the objectives outlined in this thesis. Through a systematic approach, the data collection process, and implementation of experimental procedures have been carefully described. Additionally, the various analytical techniques employed, including statistical analyses, have been discussed to ensure a rigorous investigation of the research question. Building upon this solid foundation, the subsequent chapter, titled "Results and Discussion", will delve into the application of the methods described previously. The obtained results will be presented and analysed, aiming to draw meaningful insights and conclusions from the work developed.

Results & Discussion

As previously mentioned, this thesis aims to predict the bottom of a bear market. In the previous section, it was possible to identify five bear markets when considering a drop of 10% quarter-over-quarter or a drop of 10% quarter-over-quarter-over-quarter. In the model that was developed in R Studio, it was set to return the predictions for the following 10 periods. Given that the goal of the thesis is to predict the lowest point in the bear market, the data inserted into the model will be cut up until 5 periods prior to the bear market bottom. With this view for each bear market, it is going to be possible to tell if the model is consistently being able to predict the nadir of the stock market. The first investigation will be performed on the bear market of April 1982.

The model was run five times, once per bear market. Please find a detailed description of each result for each of the bear markets.

1982 Bear Market

As mentioned in the methodology section, the formula used for this VAR model uses lagged values of the dependent variable, the S&P 500. For this, it is important to find what must be the optimal value for this lag. This value was obtained by minimizing the information criteria. All the information criteria pointed out to have a lag of one in this bear market. For this bear market, the data ranged from the third quarter of 1976 and the first quarter of 1981. The adjusted R-squared of the VAR model is 0.9175 and the p-value is 8.532e-06. This indicates that there is a strong statistically significant relationship between the studied variables. These are positive results that indicate that the VAR model used is well-specified and captures meaningful relationships between the data. After doing the Augmented Dickey-Fuller it was clear that none of these variables are stationary as all the p-values were greater than the significance level of 0.05.

Following that, the Ganger causality test was performed. All the variables, except Money Supply and Industrial Production, Granger cause S&P 500 for this data range. For this reason, the VAR model was adapted to deal with Industrial Production and Money Supply as exogenous variables and the rest as endogenous. Taking now a look at the Correlation Matrix of Residuals, it is noticeable that although no variable is close to one, no variable is close to zero either when compared with the S&P 500. This test corroborates the previous when the correlation of the stock market and Money Supply is the lowest in the matrix with only 13.57% The variable closest to one or minus one is inflation with 96.60%. Using the VAR model to forecast the following 10 periods, it is clear that the model is not capable of capturing the bear market. A chart with this information can be seen in the appendix, in Figure 9. However, the real values never go beyond the lower and upper lines of the forecast.

Looking at the performance metrics, the MAE is very high with 502.1153 meaning that on average, the model's predictions have an absolute error of approximately 502.1153 units when compared to the actual observed values. A similar situation happens with the RMSE. The value of 608.62 represents the square root of the average squared errors between the model's predictions and the actual data. It would be desirable to have a lower MAE and RMSE. For the MAPE, the model outputs a result of 38%. This means that, on average, the model's predictions have an absolute percentage error of approximately 38 % when compared to the actual observed values. This is a positive value as the smallest possible output is the desired outcome for this model. The Directional Accuracy outputted a value of 2.89. This means that the model can correctly predict 2.89 times out of 10 the direction of change in the data. The model's ability to predict the correct direction of change is relatively low with this value, as it is less than 50%. A higher value would indicate better performance in correctly predicting the direction of change. The Theil's U resulted in a value of 2.30. Similarly, to the previous performance tests, this is a high value when the desirable outcome would be a smaller one.

A model with high values of Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE), low Directional Accuracy (DA), and a high Theil's U Statistic is generally not a suitable choice for predicting the stock market. These metrics collectively indicate that the model's predictions have a significant degree of error, both in terms of magnitude and direction. Moreover, looking at the graph, the model is not capable of acknowledging the bear market and for that reason, it also fails to find the bottom of it. This indicates that the model needs improving for this data range. In the following segment, a detailed examination and analysis of the outcomes and implications of the 1985 bear market will be conducted.

1989 Bear Market

The analysis period for the 1989 bear market was covered from the third quarter of 1976 to the third quarter of 1987, which is five quarters prior to the lowest point in the dataset. After evaluating various lag options, the VAR model with a lag of five quarters emerged as the most suitable choice based on minimized information criteria. This VAR model, applied to the 1989 bear market, produced an adjusted R-squared value of 0.9604 and an exceptionally low p-value of 2.2e-16. These results indicate a robust and highly statistically significant relationship among the included variables. These findings underscore the appropriateness of the VAR model in capturing meaningful relationships within the data. Following this, an Augmented Dickey-Fuller test was conducted. It revealed that all variables for this period are non-stationary as all have p-values bigger than the 0.05 significance level. Subsequently, a Granger causality test was carried out. All variables, except for the Yield Curve and Interest Rate, had a causal influence on fluctuations in the S&P 500 during the specified period. As a result, the VAR model was adjusted to treat the Yield Curve and Interest Rate as exogenous variables, while the remaining variables were regarded as endogenous.

Shifting our focus to the Correlation Matrix of Residuals, it is relevant to note that none of the variables exhibit correlations close to one in comparison to the S&P 500. However, Industrial Production, Money Supply and GDP showed correlations close to zero with 0.05000, -0.0673 and -0.0153 respectively. When employing the VAR model to forecast the following ten periods, it becomes evident that the model struggles to accurately capture the dynamics of the 1989 bear market, as depicted in the accompanying appendix, in Figure 10. The forecast displays significant volatility, with values oscillating between extremes, approaching zero only to revert to higher values, much higher at times than the actual values. The positive note is that indeed the forecast does predict the correct bottom of the bear market and that is the goal of the model.

Looking at the model's performance metrics, it is observable that the model exhibits a very high Mean Absolute Error (MAE) of 3803.64. Similarly, the Root Mean Squared Error (RMSE) of 7353.61 indicates a substantial degree of error between the model's predictions and the actual data. Regarding the Mean Absolute Percentage Error (MAPE), the model registers a value of 1155.35. The Directional Accuracy metric yields a value of 2.11, suggesting that the model correctly predicts the direction of market movement approximately 2.11 times out of 10.

Finally, Theil's U Statistic generates a value of 230.7336. These poor performance metrics results were expected when looking at the charts and the values it yielded with the high volatility.

In conclusion, our analysis of the 1989 bear market period spanning from the third quarter of 1976 to the third quarter of 1987 reveals several key insights. The VAR model with a lag of five quarters demonstrated its suitability, supported by a high R-squared value and an exceptionally low p-value, signifying a robust and statistically significant relationship among the included variables. However, it's important to note that despite its strengths, the VAR model struggled to accurately capture the dynamics of the bear market, showing significant volatility in its forecasts. The model's performance metrics, including high Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE), suggest notable discrepancies between predictions and actual data. Additionally, the Directional Accuracy metric indicates room for improvement in correctly predicting market direction. Theil's U Statistic further highlights the model's limitations.

2003 Bear Market

The period analysed for the 2003 bear market spanned between the third quarter of 1976 and the third quarter of 2001. This timeframe captures the dynamics leading up to the bear market, culminating in the aftermath of the dot-com bubble burst. The VAR model with a lag of eight emerged as the most suitable choice, given the minimized information criteria. The VAR model for the 2003 bear market yields an adjusted R-squared value of 0.9916, indicating an exceptionally strong explanatory power, and an extraordinarily low p-value of less than 2.2e-16. These findings signify a robust and statistically significant relationship between the included variables, which already include Exchange Rate at this stage but with a low number of observations. Such results underscore the appropriateness of the VAR model in capturing meaningful relationships within the data.

In line with the previous bear market analysis, an Augmented Dickey-Fuller test was carried out, indicating that all variables under examination exhibit non-stationarity. After that, a Granger causality test was executed to delve into causal associations. In this context, it was determined Interest Rate, Inflation, Yield Curve and GDP do not have a Granger-causal impact on fluctuations in the S&P 500 during the specified period while the remainder do.

Consequently, the VAR model was adjusted to treat Interest Rate, Inflation, Yield Curve and GDP as exogenous variables, while the remaining variables were classified as endogenous. When looking at the Correlation Matrix of Residuals, it is clear that only Interest Rate and Inflation display correlations close to zero with 0.0259 and 0.0825 respectively when compared with the S&P 500.

Utilizing the VAR model to predict the following ten periods unveils its incapacity to precisely grasp the intricacies of the 2003 bear market. In the graph in the appendix, Figure 11, the contrast between the forecasted values and the real values is evident. However, with this dataset, the VAR model forecasts the fall to zero on period number 6 and the continuation at that value for the remainder of the forecasted periods. That is an unrealistic perspective, even under the abnormal circumstances of a bear market. Moreover, it is worth noting that due to this steep drop to zero, the real values of the stock market do not stay within the upper and lower bounds of the forecast confidence interval. The previous comments become even more evident when looking at the performance metrics. For example, the MAE exhibits a value of 923.9641. Similarly, the RMSE of 1097.903 suggests a substantial degree of disparity between the model's forecasts and the actual data. Concerning the MAPE, the model registers a value of 90.52393%. The Directional Accuracy metric returns a value of 2.11. Finally, Theil's U Statistic yields a value of 10.84383.

Once again, for the 2003 bear market, this does not appear to be a suitable model to predict the bottom of the bear market. The VAR model does yield a very positive R-squared and p-value, but the forecasted values reaching zero is a big red flag for the realism in this model and the underwhelming performance metrics with the MAE, RMSE, MAPE, DA, and Theil's U Statistic, suggesting its limited suitability for forecasting stock market behaviour during the 2003 bear market. These metrics collectively point to significant errors in both magnitude and direction within the model's predictions. Therefore, the model falls short of accurately capturing the dynamics of the 2003 bear market and its nadir.

2009 Bear Market

The period analysed for the 2009 bear market spanned between the third quarter of 1976 and the third quarter of 2007, five quarters before the lowest point in our data. The VAR model with a lag of nine emerged as the most suitable, given the minimized information criteria. The

VAR model for the 2009 bear market yields an adjusted R-squared value of 0.9874 and an extraordinarily low p-value of 2.2e-16. These findings signify a robust and statistically significant relationship between the included variables that, at this stage, already include the Exchange Rate. Such results underscore the appropriateness of the VAR model in capturing meaningful relationships within the data. Subsequently, an Augmented Dickey-Fuller test was performed, revealing that all variables under consideration are non-stationary, as evidenced by p-values exceeding the 0.05 significance threshold. Following this, a Granger causality test was conducted. In this context, all variables, except for the Yield Curve and GDP, Granger caused variations in the S&P 500 during the specified timeframe. Consequently, the VAR model was adapted to treat the Yield Curve and GDP as exogenous variables, while the remainder were deemed endogenous.

Turning attention to the Correlation Matrix of Residuals, it is noteworthy that none of the variables exhibit a correlation close to zero or one when compared to the S&P 500. Specifically, the correlation between the stock market and Inflation is the closest to zero, standing at 7.77%, while the Yield Curve demonstrates the highest correlation at 34%. Employing the VAR model to forecast the subsequent ten periods reveals its inability to accurately capture the dynamics of the 2009 bear market, as illustrated in the accompanying appendix, Figure 12. Unlike the previous examples, this time the real values do go beyond the upper and lower bounds of the forecast. However, the confidence interval in this example is also lower than previous examples. It is also noteworthy the direction of the trend comes back to the confidence interval at the end of the period.

When assessing the performance metrics, the model exhibits a relatively high Mean Absolute Error (MAE) of 485.0585. Similarly, the Root Mean Squared Error (RMSE) at 591.9568 suggests a considerable degree of error between the model's predictions and the actual data. In terms of the Mean Absolute Percentage Error (MAPE), the model returns a value of 46.26%, indicating that, on average, the model's predictions exhibit an absolute percentage error of approximately 46.26% relative to the actual observations. The Directional Accuracy metric yields a value of 2.22, suggesting that the model correctly predicts the direction of market movement approximately 2.22 times out of 10. Although this value falls below the 50% mark, it provides valuable insights into the model's performance in correctly discerning the direction of change. Finally, Theil's U Statistic yields a value of 3.14, which is notably high. In an ideal scenario, a smaller value would be preferable for this metric.

In summary, although the very positive R-squared and p-value for the VAR model, the model yielded an elevated MAE, RMSE, MAPE, low DA, and a high Theil's U Statistic is generally not well-suited for predicting stock market behaviour. These metrics collectively indicate that the model's predictions exhibit significant errors in terms of magnitude and direction. Furthermore, the model fails to accurately identify the dynamics of the 2009 bear market and its bottom. This highlights the need for refinement and improvement of the model within the context of the 2009 stock market scenario.

2022 Bear Market

The period analysed for the 2022 bear market spanned between the third quarter of 1976 and the first quarter of 2021. The VAR model with a lag of four emerged as the most suitable choice, given the minimized information criteria. The VAR model for the 2022 bear market yields an adjusted R-squared value of 0.9936, indicating an exceptionally strong explanatory power, and an extraordinarily low p-value of 2.2e-16. These findings indicate a robust and statistically significant relationship between the included variables.

In line with the previous bear market analysis, an Augmented Dickey-Fuller test was carried out, indicating that all variables under examination exhibit non-stationarity. After that, a Granger causality test was executed to delve into causal associations. In this context, it was determined that the Yield Curve does not have a Granger-causal impact on fluctuations in the S&P 500 during the specified period, while the remainder do. Consequently, the VAR model was adjusted to treat the Yield Curve as an exogenous variable, while the remaining variables were classified as endogenous. When looking at the Correlation Matrix of Residuals, it is clear that only the Yield Curve does not granger cause the S&P 500, displaying a correlation close to zero with -0.09794999. This is also the value in the entire matrix that is the closest to zero,

For this particular bear market, it is important to note that predicting 10 periods would leave three periods short of real data as all the information for the stock market was extracted up until Q3 2022 and the VAR would predict until Q2 2023. For that reason and given that it is already known the values for the last three periods that were unavailable at the time of the extraction, the research will use the values of 3824.14, 4124.51 and 4455.59 for the Q4 2022, Q1 2023 and Q2 2023 respectively. Otherwise, the performance tests would not be able to accurately compare real values with forecasted values.

Utilizing the VAR model to predict the following ten periods unveils its incapacity to precisely grasp the intricacies of the 2022 bear market. In the graph in the appendix, Figure 13, the contrast between the forecasted values and the real values is evident. The general direction of the forecasted values is up while the real values do experience a fall. This means that in fact, the model was not capable of predicting the bear market, and due to that, not capable of predicting the bottom of the 2022 bear market. Looking at the performance tests, the Mean Absolute Error (MAE) exhibits a value of 1420.973. Similarly, the Root Mean Squared Error (RMSE) of 2051.26 suggests a substantial degree of disparity between the model's forecasts and the actual data. Concerning the Mean Absolute Percentage Error (MAPE), the model registers a value of 33.7463%. The Directional Accuracy metric returns a value of 1.777778. Finally, Theil's U Statistic yields a value of 8.197098.

Once again, for the 2022 bear market, this VAR model does not appear to be a suitable model to predict the bottom of the bear market. The VAR model does yield a very positive R-squared and p-value, but the forecasted values reaching zero is a big red flag for the realism in this model and the underwhelming performance metrics with the MAE, RMSE, MAPE, DA, and Theil's U Statistic, suggesting its limited suitability for forecasting stock market behaviour during the 2022 bear market. These metrics collectively point to significant errors in both magnitude and direction within the model's predictions. Therefore, the model falls short of accurately capturing the dynamics of the 2022 bear market and its nadir.

Conclusion and Recommendations

Based on the detailed analysis of the five bear markets conducted in this thesis, it is evident that the VAR model employed in this study presents several strengths and limitations in predicting the bottom of bear markets. This will be the chapter where the conclusions of the thesis will be taken, as well as recommendations for future studies will be addressed.

In the case of the 1982 bear market, the VAR model exhibited a strong statistical relationship among the included variables, as indicated by a high adjusted R-squared and a low p-value. However, despite these positive statistical indicators, the model struggled to accurately predict the nadir of the market, as evidenced by high Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and a low Directional Accuracy (DA). Additionally, the unrealistic forecasted values raised concerns about the model's applicability to this dataset. Moving on to the 1989 bear market, the VAR model displayed a robust statistical relationship among variables and successfully predicted the market's bottom in technical terms. Nevertheless, it exhibited high MAE and RMSE, suggesting significant errors in magnitude, and its Directional Accuracy remained suboptimal. The model's performance metrics pointed to limitations despite its ability to forecast the bottom. Moreover, the fact that the forecast had the stock market go to zero and back to higher values 5 times does not provide much safety to its ability to predict the bottom of the bear market, even if the first time it goes to zero happens in the same period as the lowest point of this bear market.

The analysis of the 2003 bear market revealed that the VAR model excelled in terms of statistical significance, with a high adjusted R-squared and low p-value. However, just like the previously analysed examples, the model's forecasted values became unrealistic, and its performance metrics, including MAE and RMSE, indicated substantial discrepancies. In this example, the model is not capable of predicting the nadir of the bear market of the dot com bubble. The examination of the 2009 bear market highlighted again the model's strong statistical foundation but its inability to accurately predict market dynamics and its lowest point. High MAE, RMSE, and MAPE values pointed to significant errors, while the Directional Accuracy and Theil's U Statistic underscored its limitations in predicting market behaviour. This model is, according to these statistical performance tests not suitable for this task. Lastly, looking at the 2022 bear market, the VAR model once again yielded a high R-squared and a low p-value, but the predictability ability of the model was insufficient as the model was not capable of predicting the bear market nor the nadir of the bear market. The performance tests for this period corroborates this point.

In conclusion, the VAR model demonstrated a strong statistical relationship among variables in each bear market. It was an initial very positive sign. However, it consistently struggled to accurately predict the nadir of the stock market with the exception of 1989 when it predicts that the stock market goes to zero and back multiple times, a highly unlikely scenario. High error metrics, unrealistic forecasts, and inconsistent directional accuracy indicate that the model requires refinement and improvement to be a reliable tool for predicting bear market bottoms. Further research and adjustments to the model's parameters may enhance its performance and make it a valuable tool for investors and financial analysts in the future.

The previous chapters have provided a comprehensive analysis of the VAR model's performance in predicting the bottom of bear markets for various historical periods. While this thesis has shed light on the strengths and limitations of the model, it also opens avenues for further research and suggests areas where improvements can be made.

One potential avenue for future research is the inclusion of additional variables that may influence stock market behaviour during bear markets. While this thesis focused on a set of macroeconomic indicators, incorporating sentiment analysis from news articles, social media data, or geopolitical factors could enhance the model's predictive accuracy. Sentiment analysis tools and alternative data sources could provide valuable insights into market sentiment, which often plays a crucial role in bear market dynamics. Extending the analysis to different timeframes may also yield valuable insights. This thesis covered the period between the third quarter of 1976 and the third quarter of 2022. Future research could explore different timeframes or consider rolling windows to capture evolving market dynamics and account for structural changes in the financial markets. Moreover, this thesis focused on quarterly data on a data point, the stock market, that has the possibility to be observed at a much smaller scale with observations possible to be obtained at the second. This simple fact has prevented the discovery of many bear markets. Taking the example of the 2020 COVID-19 crisis, it affected the stock market in a very strong way at first, but the stock market rapidly rolled back. This rollback was so steep that it made the S&P 500 variable end the year higher than where it started. Doing this research with more observations under the same database would increase the amount of bear markets to analyse that are not visible at the quarterly frequency. While VAR models have their merits, incorporating advanced machine learning techniques such as deep learning, recurrent neural networks (RNNs), or machine learning ensembles may offer improved predictive capabilities. These techniques can capture complex nonlinear relationships and patterns that may be missed by traditional linear models like VAR.

Refining the VAR model used in this thesis is essential to improve its predictive accuracy. This could involve exploring alternative lag structures, considering different model specifications, or optimizing hyperparameters. A more comprehensive evaluation of lag selection methods and model validation techniques should be undertaken. Addressing the issue of unrealistic forecasted values is crucial. Future research should explore techniques to mitigate this problem, such as introducing bounds or constraints on forecasted values or employing alternative time series models that handle extreme events more effectively. A scenario forecast can also be used to maximize the reliability of the model. Exploring feature engineering techniques to identify key economic indicators or other variables that have a more significant impact on bear market bottoms can lead to a more parsimonious and interpretable model. Taking into account external factors such as central bank policies, geopolitical events, and global economic trends in the model may improve its predictive capabilities as currently the model is only taking in information as is from the FRED database for the selected variables. These external factors often have a substantial influence on stock market behaviour during bear markets. Enhancing the interpretability of the model's results can provide more actionable insights for investors and policymakers. More than that, integrating real-time data into the model can make it more responsive to changing market conditions and improve its timeliness in predicting bear market bottoms.

In summary, this thesis has laid the foundation for understanding the strengths and limitations of using a VAR model to predict bear market bottoms. However, there is ample room for further research and improvements in this area. The recommendations outlined above provide a roadmap for future studies aimed at developing more accurate and reliable tools for predicting bear market bottoms, which can be of significant value to investors, financial analysts, and policymakers.

In this thesis, it was possible to analyse an extensive literature review on the matter of forecasting the stock market with the use of macroeconomic variables and VAR models.

Following that, the reader was able to understand the context in which this thesis was built. In the Methodology chapter, it was discussed what would the most appropriate techniques be for a model that aims to forecast such a complex variable as this one that was attempting to predict the stock market bottom. Then the chapter with the results and discussion discussed the outcome obtained for the various stock markets identified and the forecasts that the VAR model using the methods and techniques ascribed earlier outputs. In this final chapter of the thesis, it was presented a culmination of the research. In this chapter, it was concluded that the model as is does not provide practical implications for the various stakeholders, but the relevance of further research still exists. This chapter also sheds light on the recommendations for future studies.

References

Agwuegbo, S. O. N., Adewole, A. P., & Maduegbuna, A. N. (2010). A Random Walk Model for Stock Market Prices. *Journal of Mathematics and Statistics*, *6*(3), 342–346.

Altinbas, H., & Biskin, O. T. (2015). Selecting Macroeconomic Influencers on Stock Markets by Using Feature Selection Algorithms. *Procedia Economics and Finance*, *30*, 22–29. https://doi.org/10.1016/s2212-5671(15)01251-4

Bachelier, L. (1900). *The Theory of Speculation*.

Barsky, B. R., & Bradford De Long, J. (1990). *Bull and Bear Markets in the Twentieth Century*.

Chen, S. S. (2009). Predicting the bear stock market: Macroeconomic variables as leading indicators. *Journal of Banking and Finance*, *33*(2), 211–223. https://doi.org/10.1016/j.jbankfin.2008.07.013

Dionisio, A., Mendes, D., Vidigal Da Silva, J., & Menezes, R. (2005). *Analysis of the Relationship Between Stock Market Prices and Macroeconomic and Financial Factors: A Linear Approach*. http://www.eia.doe.gov/oil_gas/petroleum/info_glance/prices.html

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica, 50(4).

Estrella, A., & Mishkin, F. S. (1996). *The Yield Curve as a Predictor of U.S. Recessions* (Vol. 2, Issue 7).

Fama, E. F. (1995). *Random Walks in Stock Market Prices*.

Fama, E. F., & Schwert, G. W. (1977). Asset Returns and Inflation. In *Journal of Financial Economics* (Vol. 5). North-Holland Publishing Company.

Gong, F., & Mariano, R. S. (1997). Stock Market Returns and Economic Fundamentals in an Emerging Market: The Case of Korea. In *Financial Engineering and the Japanese Markets* (Vol. 4). Kluwer Academic Publishers.

Lee, U. (1992). *Do Stock Prices Follow Random Walk? Some International Evidence*.

Malkiel, B. G. (2003). *The Efficient Market Hypothesis and Its Critics*.

Mookerjee, R., & Yu, Q. (1997). Macroeconomic variables and stock prices in a small open economy: The case of Singapore. In *Pacific-Basin Finance Journal* (Vol. 5).

Niarchos, N. A., & Alexakis, C. A. (2000). The Predictive Power of Macroeconomic Variables on Stock Market Returns. The Case of the Athens Stock Exchange. In *Πανεπιστήμιο Πειραιώς / «SPOUDAI»* (Vol. 50, Issue 2).

Paul, S., & Mallik', G. (2004). *Macroeconomic Factors and Bank and Finance Stock Prices: The Australian Experience* (Issue 1).

Sharpe, W. F. (1964). Capital Asset Prices: A Theory Of Market Equilibrium Under Conditions Of Risk. *The Journal of Finance*, *19*(3), 425–442. https://doi.org/10.1111/j.1540-6261.1964.tb02865.x

Stock, J. H., & Watson, M. W. (2002). *Macroeconomic Forecasting Using Diffusion Indexes*.

Tramontana, F., Westerhoff, F., & Gardini, L. (2013). The bull and bear market model of Huang and day: Some extensions and new results. *Journal of Economic Dynamics and Control*, *37*(11), 2351–2370. https://doi.org/10.1016/j.jedc.2013.06.005

Vogiazas, S., & Alexiou, C. (2017). Determinants of Housing Prices and Bubble Detection: Evidence from Seven Advanced Economies. *Atlantic Economic Journal*, *45*(1), 119–131. https://doi.org/10.1007/s11293-017-9531-0

Weihong Huang, & Richard Day. (1993). Chaotically switching bear and bull markets: the derivation of stock price distributions from behavior rules. *Nonlinear Dynamics and Evolutionary Economics*, 169–182.

World Bank. (2020). Retrieved from https://data.worldbank.org/indicator/CM.MKT.LCAP.CD?end=2020&most_recent_value_desc=true&start=1975.

Appendix A

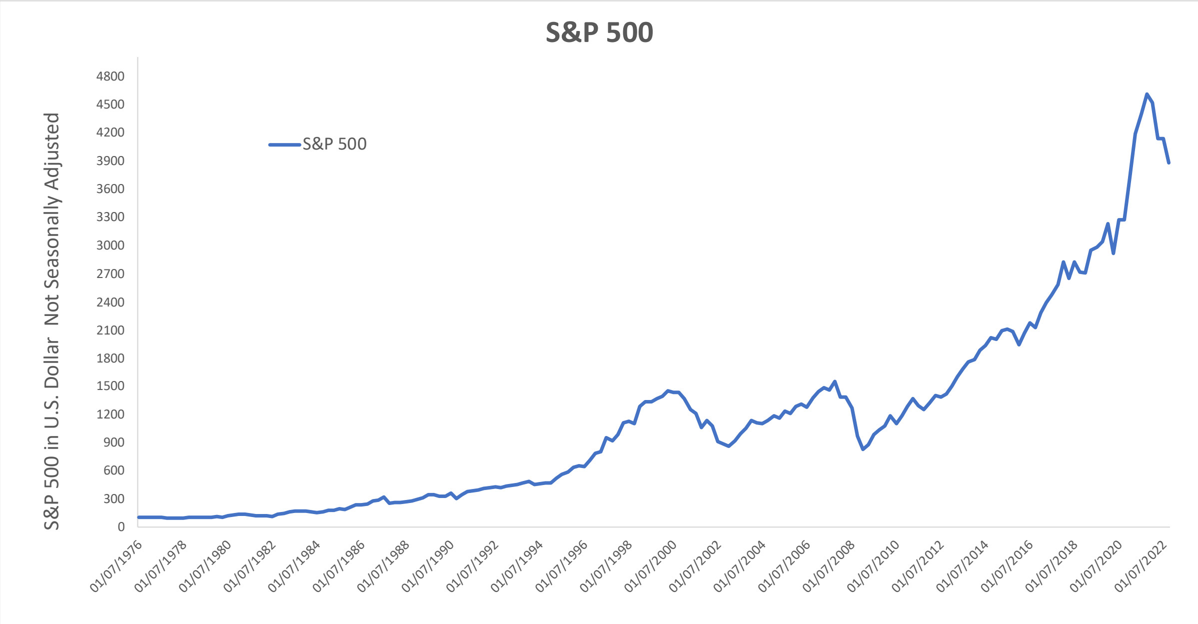

Figure 1 - S&P 500 Graphical Representation. Data from Bloomberg.

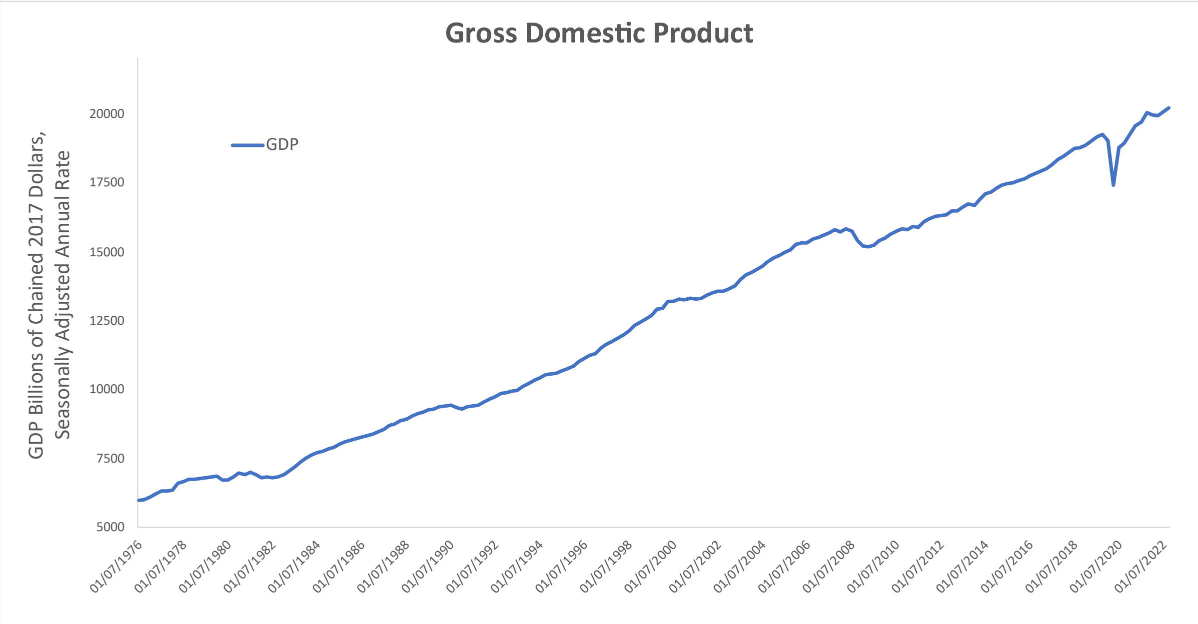

Figure 2 - GDP Graphical Representation. Data from FRED.

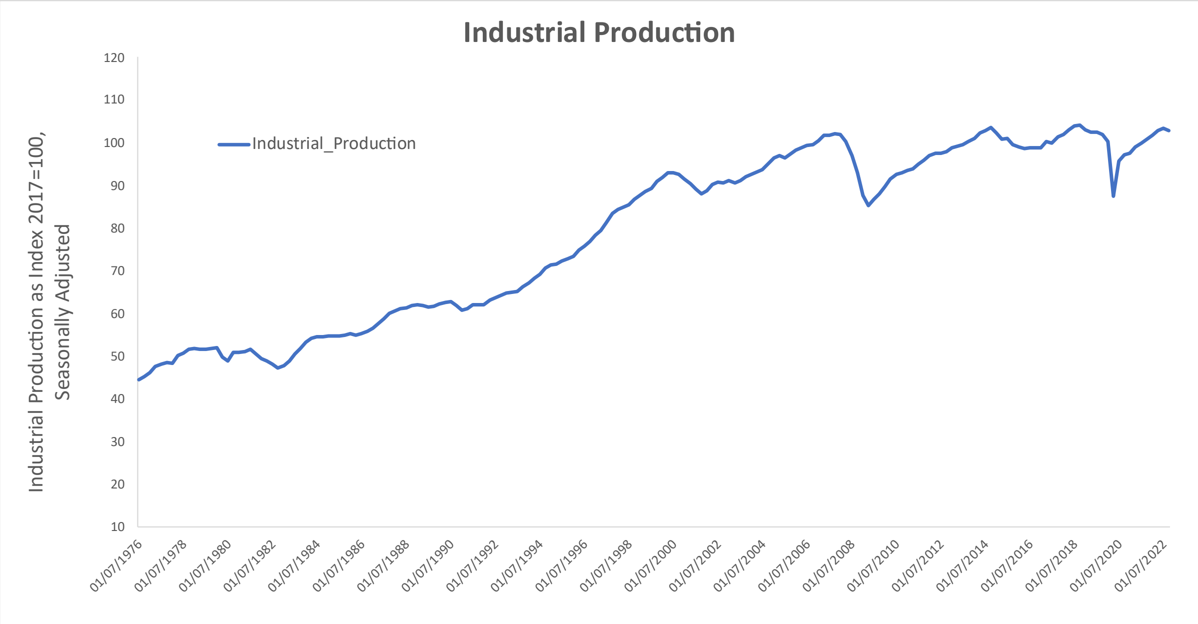

Figure 3 - Industrial Production Graphical Representation. Data from FRED.

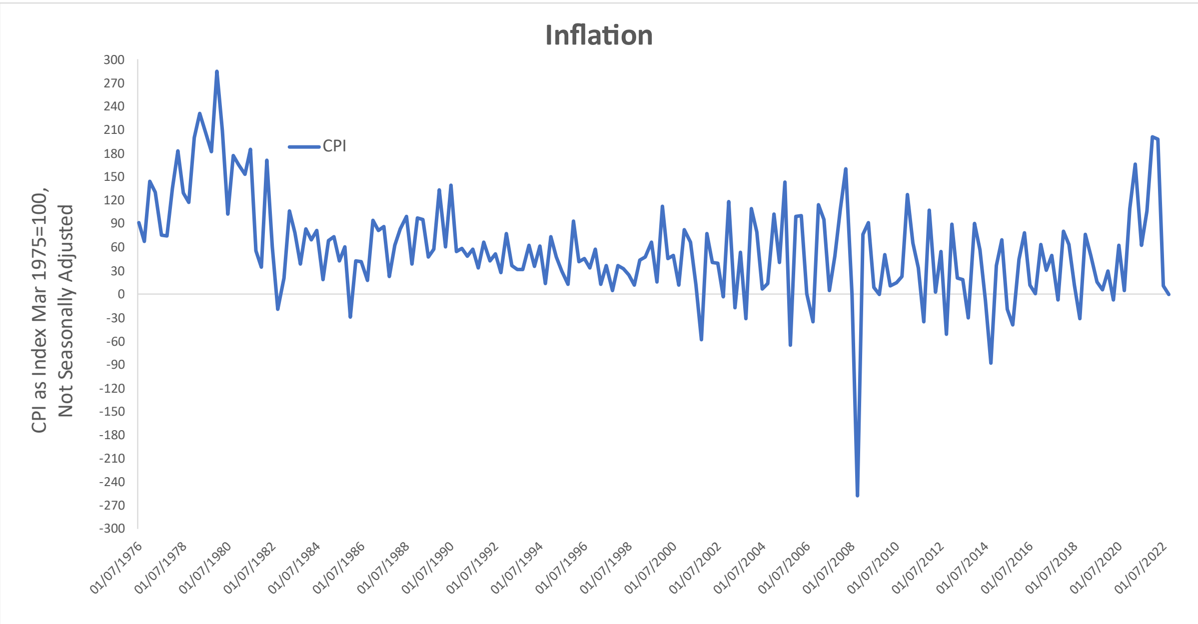

Figure 4 - Inflation Graphical Representation. Data from FRED.

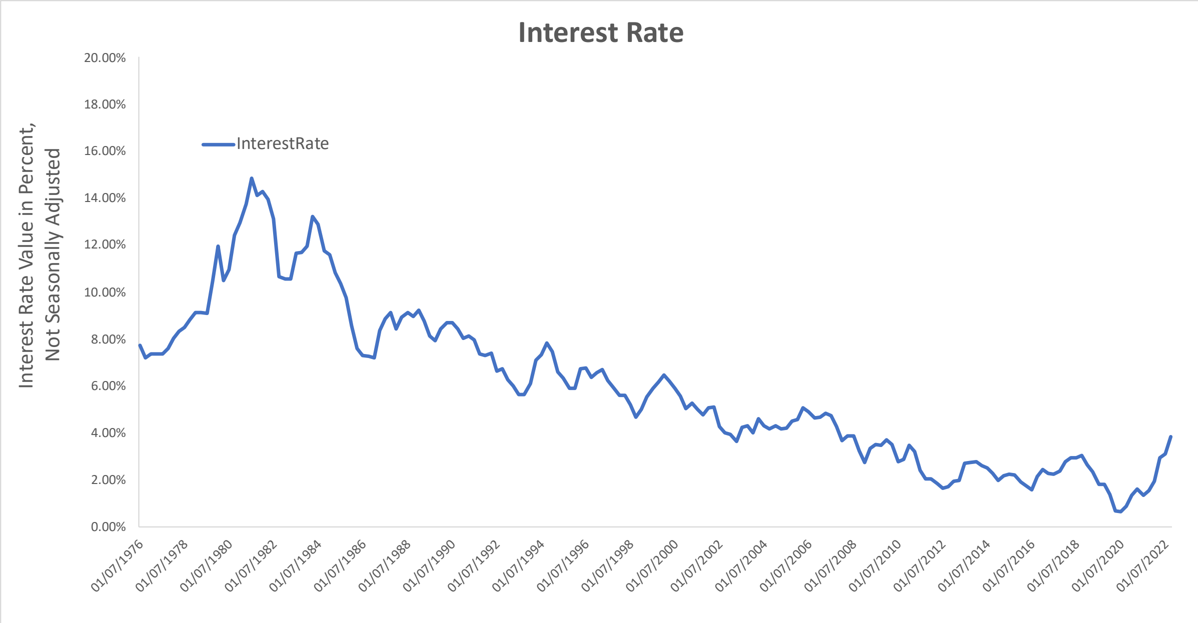

Figure 5 - Interest Rate Graphical Representation. Data from FRED.

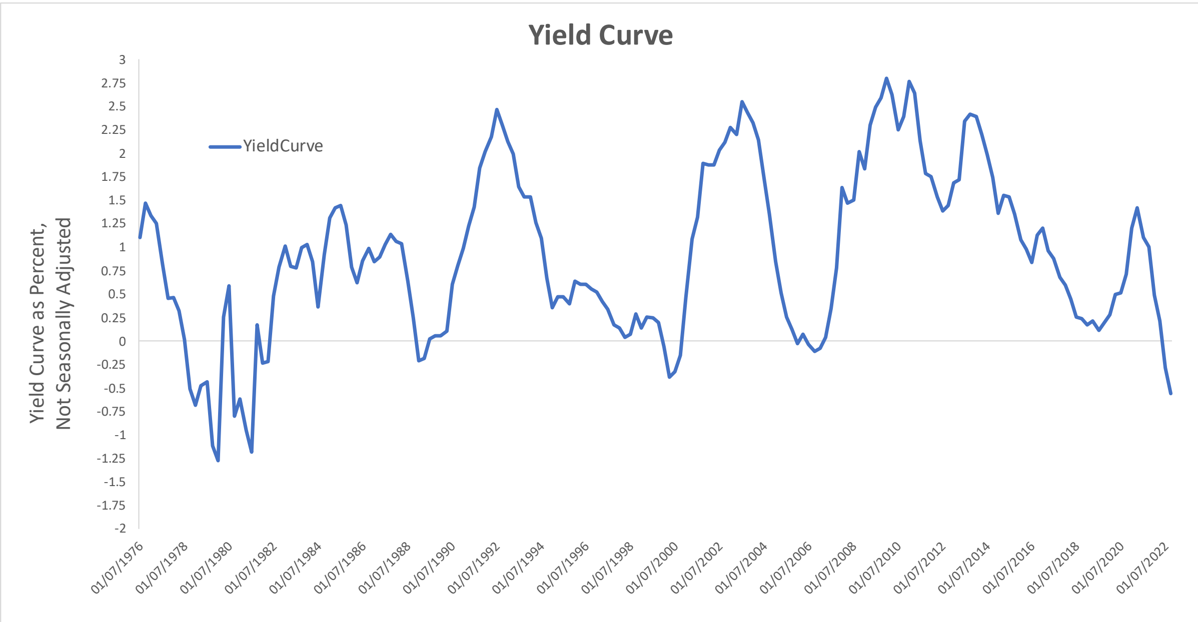

Figure 6 - Yield Curve Graphical Representation. Data from FRED.

Figure 7 - Money Supply Graphical Representation. Data from FRED.

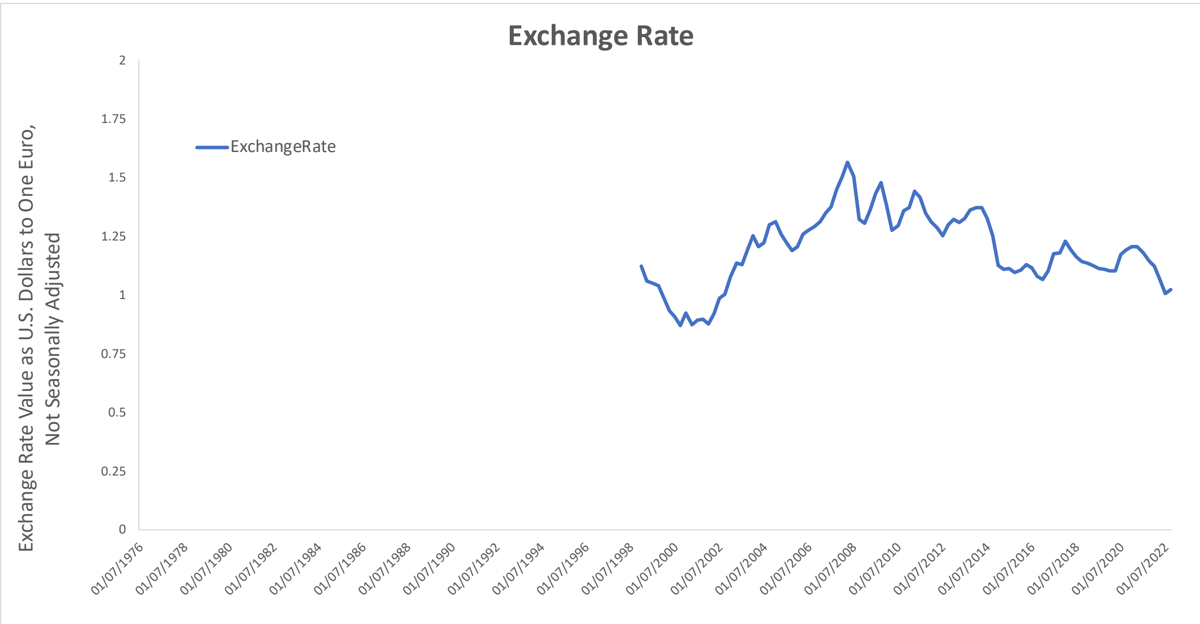

Figure 8 - Exchange Rate Graphical Representation. Data from FRED.

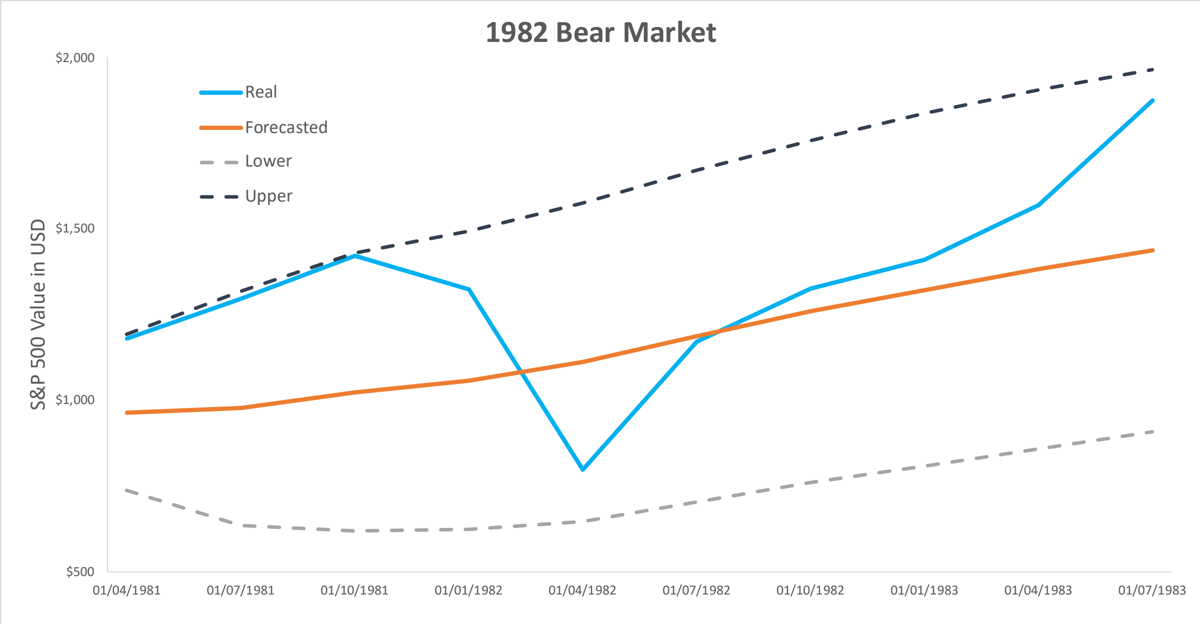

Figure 9 - 1982 Bear Market with Forecasted Values.

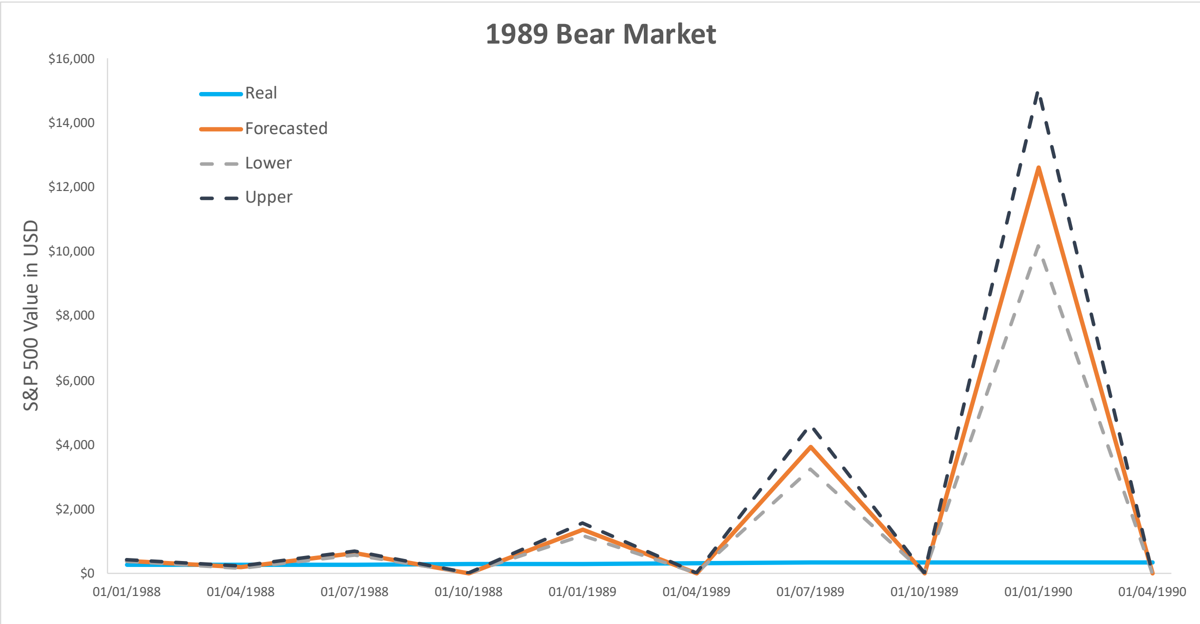

Figure 10 - 1989 Bear Market with Forecasted Values.

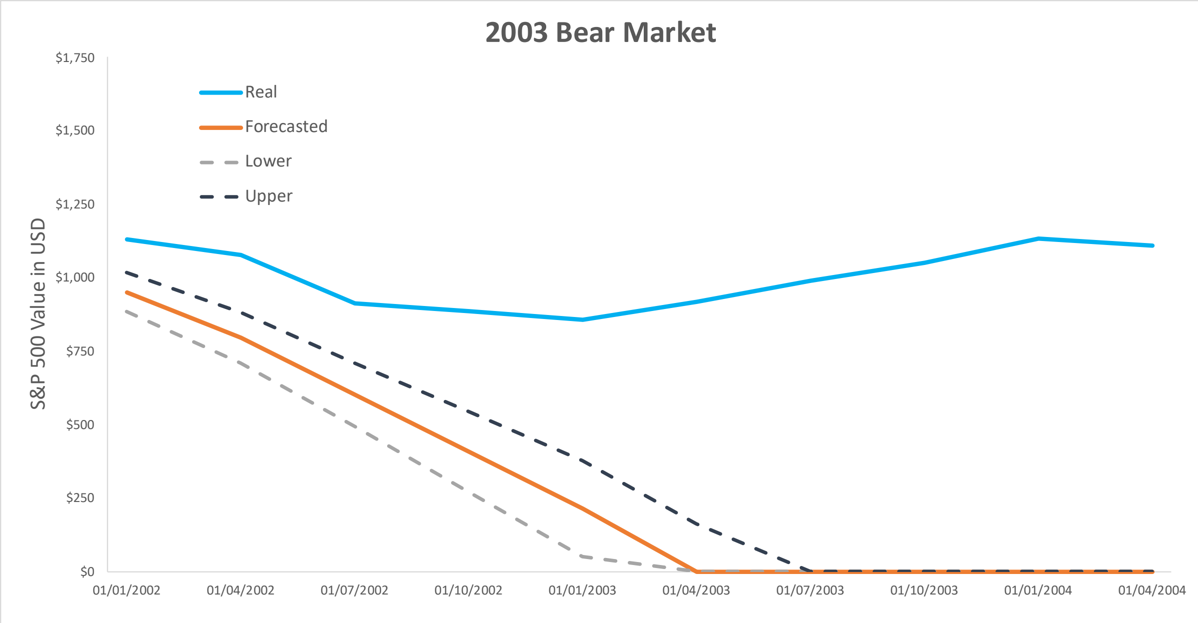

Figure 11 - 2003 Bear Market with Forecasted Values.

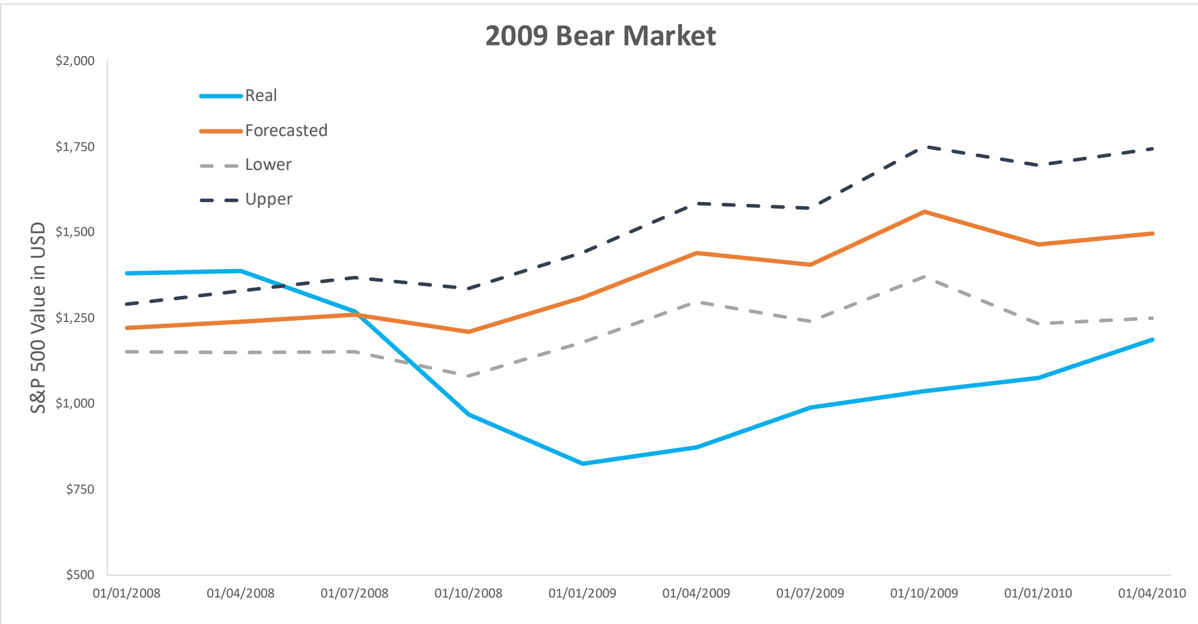

Figure 12 - 2009 Bear Market with Forecasted Values.

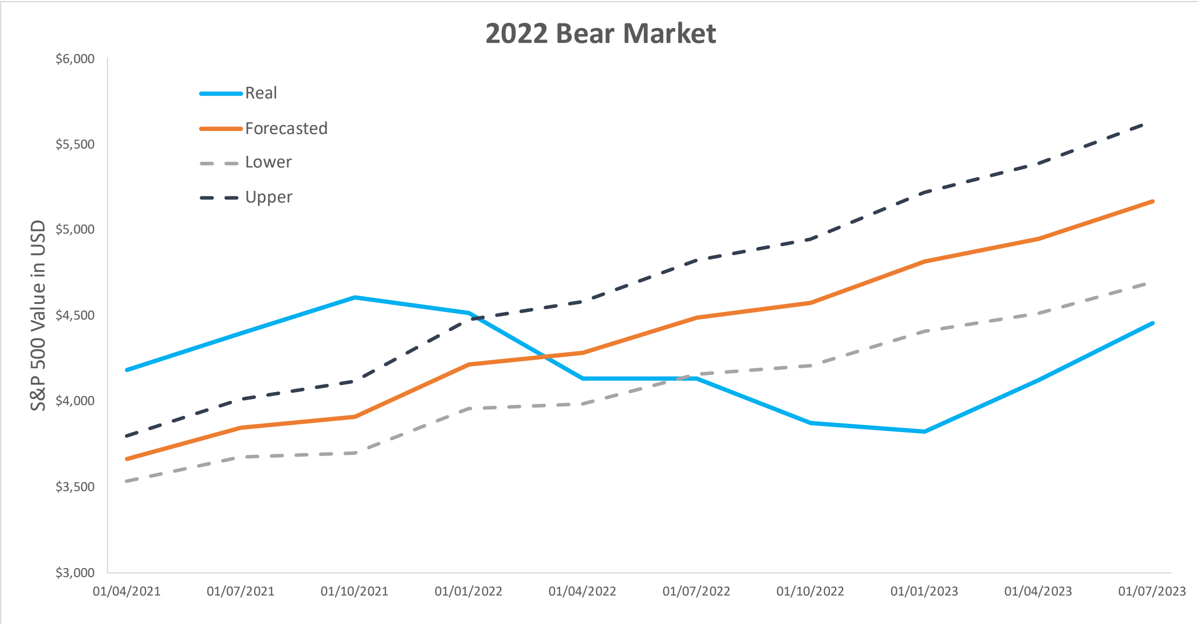

Figure 13 - 2022 Bear Market with Forecasted Values.