INSTITUTO
UNIVERSITÁRIO
DE LISBOA

Carbon Hedging

Diogo Filipe Sousa Vieira

Master in Finance

Supervisor:
PhD, José Carlos Gonçalves Dias, Full Professor,
ISCTE-IUL

July 2023

Department of Finance

Carbon Hedging

Diogo Filipe Sousa Vieira

Master in Finance

Supervisor:
PhD, José Carlos Gonçalves Dias, Full Professor,
ISCTE-IUL

July 2023

Sumário

As alterações climáticas parecem ser o principal tema de debate entre políticos, empreendedores e comuns. É inegável o impacto negativo deste evento sobre qualquer agente económico e, portanto, são necessárias medidas adequadas para combater esta ameaça. A comercialização de licenças de carbono é uma potencial arma no arsenal da Comissão Europeia para combater as alterações climáticas, no entanto, no ponto de vista empresarial, este esquema representa um risco adicional para a geração de lucros, pelo que deve ser minimizado. A proteção contra o risco das licenças de carbono, contudo, poderá comprometer a sua eficácia na limitação das emissões de gases de efeito estufa. Para entender o impacto da cobertura do risco das licenças de carbono, esta tese analisa como funcionam e como são avaliadas as estratégias que utilizam contratos de futuros, de forwards e de opções vanilla, tanto de estilo europeu quanto de americano. A quantificação dos contratos de futuros e de forwards é feita através de não-arbitragem, as opções europeias são avaliadas aplicando indução retroativa usando simulações de Monte Carlo e as opções americanas utilizando o método Mínimos-Quadrados de Monte Carlo. Depois de entender as estratégias, uma série de regressões lineares de mínimos-quadrados são utilizadas para determinar como estas impactam as emissões de gases efeito de estufa. O estudo conclui que as estratégias de cobertura com custos negativos e/ou custos muito baixos poderão contribuir de forma positiva para o aumento das emissões de gases de efeito estufa.

Palavras-Chave: Licenças de Carbono, EU ETS, Estratégias de Cobertura, Monte Carlo; Opções, Opções Americanas, LSM

Classificação JEL: G12, C58

Abstract

Climate change seems to be the core subject of debate among politicians, companies and everyday people. Its increasing impact on every economic agent is undeniable, and important measures must be taken by each. Carbon trading is a reliable weapon on the European Commission's arsenal to fight against climate change. Nevertheless, from the firm's perspective, this scheme represents an additional risk to their profits, hence it needs to be contained. However, hedging against the European carbon allowances might jeopardize its effectiveness in reducing greenhouse gas emissions. To understand the impact of carbon hedging, this thesis analyses how strategies that utilize futures and forwards contracts and vanilla options, both European and American-style, work and how they are priced. Pricing futures and forwards is done via no-arbitrage, European options are priced using backward induction using Monte Carlo simulations, and American options are priced using the Least-Squares Monte Carlo method. After understanding the strategies, a series of Least-Squares linear regressions are used to determine how they impact GHG emissions. The study concludes that hedging strategies with negative costs and/or very low costs do in fact contribute positively to the increase of greenhouse gas emissions.

Keywords: Carbon Permits, EU ETS, EU Allowances, Greenhouse Gas, Hedging Strategies, Monte Carlo, Options, American Options, LSM

JEL Classification: G12, C58

Contents

List of Figures

List of Tables

Glossary of Acronyms

ATM	At the money
BSM	Black, Scholes and Merton
CEV	Constant Elasticity of Variance
DCC-GARCH	Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroscedasticity
ETF	Exchange Traded Fund
ETS	Emissions Trading Scheme
EU	European Union
EUA	European Union Allowances
EWMA	Exponentially Weighted Moving Average
GBM	Geometric Brownian motion
GHG	Greenhouse Gases
LSM	Least-Squares Monte Carlo
NDC	National Determined Contributions
OLS	Ordinary Least Squares
OTC	Over-the-counter
OTM	Out of the money
SRF	Short-Range Forward
SV	Stochastic Volatility
WCP	Writing Covered Put

1 Introduction

We are on the fast track to approaching the climate "deadline" set by the Paris Agreement in 2016 and its difficulty is ever-growing. This is "perhaps the greatest challenge humankind has ever faced" (IEA, 2021). Setting the limit of 2 ºC, preferably, 1.5 ºC relative to the Pre-Industrial Era is the main aspiration set by the United Nations. Signed by 194 Parties (193 States plus the European Union), it requires all its members' cooperation and devotion to check each step of this ambitious goal (United Nations, n.d.). In order to reach it, each member must determine their National Determined Contributions (NDCs) to tackle one of the most important stages: Carbon Neutrality by 2050 (United Nations Climate Change, n.d.). There is a considerable list of phases to overcome to tackle the climate crisis, just as switching to renewable green sources of energy, and increasing the production process efficiency, among many others. However, the one that will be focused on this thesis is putting a price on carbon emissions.

The most commonly known form of pricing externalities to everyday people is through taxes. In the environmental context, taxes increase the costs of the economic agents who consume/produce any type of product, mainly electricity, with a significant carbon print, from fossil fuels (Green, 2008). By adding indirectly the price of carbon through taxes, companies and people will be incentivized to change their behaviour and transition to greener and low-carbon technologies (Green, 2008), which eventually can help to improve wasteful consumption and prevent further damage to the environment (European Commission, n.d.).

It is, however, possible to take a more direct approach other than taxes. Instead of letting the government set a tax level, it is possible to let the free market give a price tag to carbon emissions. Based on this idea, the European Union created the Emissions Trading Scheme (EU ETS) in which governments distribute a fixed amount of "carbon permits" or "carbon allowances" (Capoor and Ambrosi, 2009). This means that whenever a company exceeds its allowance, it can purchase more permits from another company that has an exceeding number of permits. This scheme is known as "Cap and Trade". The fundamental theory is that as the supply of permits decreases, their price will increase, forcing companies to adopt a cleaner production process instead of trading for new permits (Gilbertson et al., 2009). Even though the impact of these EU ETS is thought to be positive from an environmentalist point of view, "companies wish to make profits and to avoid excessive risks" (Green, 2008, p.70). Therefore, it is only logical that the first step they will take is to try to minimize the impact the ETS will have on them, hence there is a need to cover their positions. Although this new "commodity" is not actually tangible, it is traded as any other commodity (Gilbertson et al., 2009), therefore, minimizing its overhead should be done as one.

Although it is expected for companies to try to minimize any risks they might be exposed to, for the carbon allowances case, the European Commission must also consider the potential effects that such behaviour might incite on its design purpose. If by issuing carbon allowances the main objective is to incentivize the reduction of air pollution, then by mitigating their risks by using hedging strategies, companies will be overlapping the implemented regulatory instruments. The consequence could be a negative impact on the EU ETS's effectiveness in culminating its main goal (Chevallier et al., 2011).

As a Dissertation, this thesis will approach this problem by analysing the main commodity hedging strategies using financial products, such as forwards, futures, European and American vanilla options, and strategies using a combination of options. Furthermore, an empirical analysis using a series of Ordinary Least-Squares (OLS) regressions will be done in order to understand how hedging techniques might affect greenhouse gas emissions.

While the pricing of futures and forwards is relatively simple and swift, pricing options, especially American options, is not as straightforward. There are quite a few different methodologies regarding the evaluation of both European and American vanilla options. For instance, the Binomial/Trinomial models, finite difference schemes and Monte Carlo simulations are more commonly used to price American-styled options. The Black, Scholes and Merton Model (BSM) (Black and Scholes, 1973 and Merton, 1973) and the Constant Elasticity of Variance Model (CEV) (Cox and Ross, 1976), are more commonly used to price European-styled options.). In this thesis, the intent is to use and understand two different pricing methodologies to the ones mentioned above. For European-styled options strategies, pricing will be done via Monte Carlo Simulation. For American-styled options strategies, the Least-Squares Monte Carlo approach (LSM) (Longstaff and Schwartz, 2001) methodology will be used.

2 Literature Review

As mentioned, this dissertation will focus on the study of hedging strategies from the companies' perspective against the rise in carbon permit prices. Needless to say, research regarding multiple topics is required in order to better understand the methodological approach that will be taken in this dissertation as well as to enable a more coherent practical study. Therefore, it is reasonable to divide the literature review into the following sections: The Underlying Asset, Hedging and Pricing.

2.1 The Underlying Asset

There has been quite an amount of research regarding the use of carbon permits ever since it was implemented in 2005, either debating in favour or against their use. Regardless, all the written material relating to this subject does a reasonable job explaining what and how the European Union's Emissions Trading Scheme works.

Convery et al. (2009) explain in detail the implementation of the European Union's Emissions Trading Scheme and focuses on the first trading phase (2005 to 2007), between all 4 phases: the second trading phase (2008-2012); third phase (2013-2020); and fourth phase (2021-2030) (European Commission, n.d.). The goal is to give a better understanding to the common reader about the impact of capping and pricing GHG (Greenhouse Gases) emissions.

According to Convery et al. (2009), the EU ETS drew its inspiration from Dales (1968), who believed that if the market can feasibly establish policies, then any regulatory entity should take advantage of that, to apply said policies efficiently. Successively, the authors argue that the rise of the EU ETS was founded on two failures: The European Commission's failure to fight against the Kyoto Protocol regarding the inclusion of trading as a flexible instrument against climate change and the European Commission's failure to introduce a generalized carbon tax to the EU during the 1990s. The first of these events culminated in the EU embracement of the idea of a "cap and trade" methodology despite having fought against it previously. The second event, which resulted from member states not being interested in losing their national tax system autonomy, made the EU ETS the only centralized feasible way to directly fight climate change.

Gilbertson et al. (2009) approach the Carbon Trading subject firstly by analysing how the Emissions Trading Scheme was implemented. According to the authors, there are two types of carbon trading: "cap and trade" and "carbon offsetting". Nevertheless, for the sake of efficiency, it will solely be focused on “cap and trade” trading. The first of the two consists of “governments or intergovernmental bodies like the European Commission distribute licenses to pollute to major industries.”

In practical terms, the EU ETS allows for the possibility of a company being able to either trade the permits with another company that can “make equivalent changes more cheaply” or directly reduce their emissions (Gilbertson et al., 2009). The foundation of this premise is that, as the supply of permits decreases, their prices will subsequently increase, henceforth companies will cease to be able to purchase new permits at a competitive price since it will begin to be more affordable to decrease the emissions instead. This way, the GHG emission will be “caped” by setting a legal limit to the emission levels which can be lowered by governmental entities over time, restricting even further air pollution.

Green (2008) provides a practical example in the electricity industry, where there are only two main fuel materials for electric production: coal and natural gas. In his example, when electricity produced via natural gas (which is less pollutant than coal, but more expensive than coal), the carbon allowances will have to rise to offset this price difference, hence making natural gas-powered stations more competitive and, consequently, less carbon will be created in order to produce the same amount of electricity.

However, Gilbertson et al. (2009) argue that the “trade” component of the “cap and trade" scheme contributes only to a greater manoeuvre around the pollution problem since companies with an exceeding number of allowances make a profit out of them and companies who used all theirs, are able to save money by purchasing more instead of investing in less pollutant production. For this reason, the authors defend that this scheme fails its main purpose.

Similarly, Green (2008) argues that a better alternative to price this externality would be the utilization of green taxes. The author claims that by using this method, the variable cost of electricity is only raised by a fixed amount in contrast to what happens by using carbon permits, which depends mostly on the market conditions. According to Baldursson and Von der Fehr (2004), “Given a certain difference in efficiency between taxes and quotas, the introduction of uncertainty and risk aversion will, in general, tilt the balance of regulation by taxes” (Baldursson and Von der Fehr, 2004, p.703)

Due to the considerable amount of literature arguing against the effectiveness of the EU ETS, it might be safe to assume that the system was indeed not the most effective. For this reason, Brink and Vollebergh (2016) suggested an adjustment in the capping method as well as other options to reform the design of the EU ETS itself. For instance, its regulation made that the allowances allocated freely to EU companies were given in a fixed amount while its price was given by supply and demand. This obviously creates the possibility that the available allowances are higher than the demand for them, hence dragging the EU allowances prices down (Brink and Vollebergh, 2016). The EUA prices might also have been affected by the EU's legislation regarding green energy and energy saving, which contributed to the reduction of the carbon footprint of the whole area, and, consequently, the demand for the allowances was affected (Brink and Vollebergh, 2016).

An abundance of permits and a low price will not encourage companies to invest in greener technologies and further reduce their greenhouse gas emissions, making the net zero goals established for 2050 impossible to achieve (Grubb, 2012). In order to respond to this problem, Brink and Vollebergh (2016) suggested simply tightening the cap. Combining the cap-and-trade system with the establishment of minimum and maximum of allowances prices would be preferable to the usage of one instrument when marginal abatement costs are uncertain. (Roberts and Spence, 1976).

2.2 Hedging

There are many alternatives when it comes to covering a position on an underlying asset. For instance, a very unorthodox way to hedge an underlying asset could be by using another asset with a negative correlation. This method is studied by Saeed et al. (2020) in an environmental context. The authors analyse the ability to hedge green assets, such as green bonds, renewable energy stocks and/or green Exchange Traded Funds (ETFs), against "dirty" assets, such as crude oil or other energy ETFs.

As implied previously, Saeed et al. (2020) first measure the conditional correlation of each asset through a Dynamic Conditional correlation Generalized Autoregressive Conditional Heteroscedasticity (DCC-GARCH) model and then proceed with a dynamic portfolio analysis in order to measure the hedging effectiveness (Saeed et al., 2020). Based on this analysis it is concluded that there is a strong negative correlation between green assets, especially green bonds, and dirty energy stocks/energy ETFs.

Interestingly, "the hedging effectiveness of clean energy stocks is superior to that of green bonds, especially for energy ETF" (Saeed et al., 2020, p.14). This is justified by its larger ability to nullify somewhat dirty energy investments when compared to green bonds. These results are indeed optimistic in the sense that they emphasise the relevancy of green investments as a reliable approach to cover dirty investments, promoting their investment to more market participants as well as increasing policy makers' "arsenal" in fighting climate change.

On the other hand, a more conventional way to hedge a spot position on an underlying asset is hedging using financial derivatives such as futures, forwards, or even financial options. However, option trading, even for hedging purposes used to be seen as gambling (Cox and Rubinstein, 1985), which resulted in tight governmental regulations, postponing the introduction of financial options into regulated markets (Sakong et al., 1993). Nonetheless, as options increased in popularity, their utilization for hedging increased, which was a trend seen among farmers who used financial markets to hedge their risks (Sakong et al., 1993).

Hedging with financial derivatives is the approach that this dissertation is going to study as well as the approach that Taušer and Čajka (2014) when analysing hedging strategies from a wheat farmer's perspective. The holder of the long position, in the case of Taušer and Čajka's study (2014), is the farmer who harvests wheat and needs to protect himself against price decreases, and it is necessary to come up with effective strategies to do so.

Using derivatives, the hedger should enter an opposite and proportional position to its spot position (Taušer and Čajka, 2014), hence fulfilling a full-hedge strategy (Haigh and Holt, 2000). The authors make an important point by stating that both futures wheat price and local cash price – where the wheat is physically traded – have the same importance to the farmer since both have a strong positive correlation between them (Taušer and Čajka, 2014). Trading these products enables the possibility of farmers selling their wheat through them via physical settlement, however, since most already have their clients established, it is more practical to hedge their positions instead. This extra flexibility, thus, has increased the popularity of derivatives trading (Irwin and Sanders 2012).

It is important to analyse how hedging with different derivatives will affect the results. For this reason, the authors highlight the resemblances and divergences between the futures and forwards, especially, since these two are two sides of the same coin. For instance, both lock the price of the underlying asset at a future date. However, due to its standardized nature, it is more difficult to find a 100% futures contract that matches all of the desired parameters. On the other hand, both parties negotiate a forward contract, which is celebrated in Over-The-Counter markets, such that it has all the required details of each part, hence being more personalized (Taušer and Čajka, 2014). Furthermore, forward contracts, unlike futures contracts, do not have a physical settlement, they have a cash settlement, where the difference in price between the start and end dates of the contract (Taušer and Čajka, 2014).

Unlike the previous contracts, the option contract does not lock the price of the underlying asset like the futures and forward contracts do. Instead, it provides insurance by offering "a way for investors to protect themselves against adverse price movements in the future while still allowing them to benefit from favourable price movements" (Hull, 2015). Nevertheless, these benefits come with a cost – the option premium – which should be considered by the hedger.

An important remark was made by Böhringer et al. (2008). They give importance to the possibility that overlapping regulatory instruments might impact the efficiency of said instruments on global environment policy. Chevallier et al. (2011) further argue that the usage of financial instruments to hedge EU allowances could sway their impact on reducing GHG production.

2.3 Pricing

Evaluating the hedging strategies and pricing its derivatives will be the main focus of this dissertation, therefore it is needed to reinforce what has been studied and discussed concerning this topic. The strategies composed of European and American vanilla options, as mentioned previously, will be priced via Monte Carlo simulation. Nonetheless, it would be proper to first revise the literature on financial derivatives regarding the underlying asset as well as the necessary methodologies underlying the pricing technique.

2.3.1 Derivatives

A relevant study conducted by Homburg and Wagner (2009) has verified that the European market for emissions trading has seen an increase in liquidity and, consequently, an increase in the volume of futures and options contracts followed in both Over-the-Counter (OTC) markets as well as in Exchange markets. In the OTC market, the relationship between the EUA prices and its forward prices is one of cost of carry (Homburg and Wagner, 2009), which measures all of the costs relating to storage and interest paid to finance the asset (Hull, 2015).

However, the relationship between the spot prices and the futures prices has barely any studies analysing it before the study of Homburg and Wagner (2009). The relationship between any other physical commodity's price and its correspondent futures price is determined by a no-arbitrage condition that takes into account the storage cost of the commodity as well the interest rates, or as the authors put it, the convenience yield (Homburg and Wagner, 2009). This is the case since futures contracts have a physical settlement, therefore the cost of storage must be taken into account. Additionally, the cost of storage can be considered to be stochastic since it depends on storage availability and, consequently, this might increase the level of complexity in the spot and futures prices relationship (Homburg and Wagner, 2009). As mentioned previously, it is not necessarily beneficial to hold EUAs since they are only needed once a year in order to comply with the EU's legislation. The authors develop this premise by explaining that when futures contracts mature before the compliance date, it will serve just as well as holding the commodity itself. On the other hand, if it expires after the compliance date, holding a futures contract becomes futile.

For a commodity such as carbon permits, only interests might be considered as a significant cost of storage. In order to test this hypothesis, Homburg and Wagner (2009) formulated an equation where they compare the theoretical futures price (having as a convenience rate only the risk-free rate) and the observed futures prices. Their empirical evidence showed that, despite that the trial period enabled several arbitrage opportunities, the futures prices and the spot prices of the EU allowances were indeed linked by the cost-of-carry approach just like their forward contracts counterpart.

The goal now is to focus on another financial derivative used to trade carbon allowances, options, which were introduced one year after the implementation of the EU ETS by the European Climate Exchange (ECX) (Chevallier et al., 2011). It was previously analysed the impact of options on the carbon permits futures markets as well as the actual cash market. The hypothesis suggests that introducing options as a tradable derivative into the EUA spot and futures markets might affect their volatility due to the likely impact on the producer's decisions through intertemporal arbitrage (Chevalier et al., 2011; Weaver and Banerjee, 1990) as well as the increase in leverage associated to option pricing and an increase in speculation trading. Furthermore, their decisions might also be influenced by a mix of legitimate information and signals from options traders which might create asymmetric information relating to the underlying asset itself (Back, 1993). Other research, such as that of Mayhew (2000) and Fleming and Ostdiek (1999) also study the impact of the introduction of financial derivatives in their respective spot markets (Mayhew, 2000) in a broader range of markets, such as a variety of equities, commodities and bonds, and of Fleming and Ostdiek (1999) in the crude oil market specifically.

Chevallier et al. (2011) find that options trading may actually have influenced volatility in the EU ETS cash markets between April 2005 and April 2008. They reached this conclusion after computing volatility for the futures market and spot markets via a variety of GARCH models and analysing the volatility results with an econometric model. Furthermore, they also conclude that their Generalized Autoregressive Conditional Heteroscedasticity (GARCH) estimates are "statistically different before and after the introduction of the derivatives market" (Chevallier et al., 2011). However, their collective results suggest that options are not impacting systematically the EUA spot market's stability.

2.3.2 Volatility

An important component to prices its derivative is, of course, the volatility of the underlying asset. To calculate the volatility, first, the natural log daily returns need to be calculated, and, subsequently, its standard deviation. However, the main issue with this volatility methodology is the fact that the standard deviation gives the same weight to an event that took place several years in the past to a recent event. In practical terms, this does not seem very realistic, hence it is better to use more robust volatility models.

The most popular models would be the GARCH (1,1) model, the Exponentially Weighted Moving Average (EWMA) and Stochastic Volatility (SV). The first two of these models are very similar, being able to give additional prominence to more recent and relevant events.

A study conducted by Ding and Meade (2010) concluded that, when testing with real data, the GARCH model was better at forecasting than the SV model. Furthermore, the EWMA was shown to be more accurate than the standard GARCH. Therefore, the EWMA is a reliable option due to its robustness and its more computationally simplistic nature (Ding and Meade, 2010).

2.3.3 Monte Carlo

Pricing European-style vanilla options using a Monte Carlo approach can be especially useful when the underlying asset pays discrete dividends (Boyle, 1977). According to Boyle (1977), this methodology produced quite accurate results together with confidence limits on the estimates. As for American-style vanilla options, Monte Carlo simulations can be even more useful since the "numerical integration approach or the finite difference method would be more efficient" (Boyle, 1977, p.11). Furthermore, it is relevant to add that we are dealing with multiple stochastic factors, and this is one of the key functionalities of the Monte Carlo simulation (Barraquand, 1995).

According to Brandimarte (2006), a few of the main advantages of using Monte Carlo simulations, for either option pricing, simulating hedging strategies or even for estimating Value at Risk, are its flexibility, its ability to consider stochastic volatility and other complicated features of more complex financial products. Its main disability, however, is its computational burden which may limit the number of replications and, therefore, the simulation's robustness. Fortunately, this issue is only a minor inconvenience due to the increasing computational capacity in the past few decades.

Both European and American options have the same starting point: the simulation itself. In this simulation, it is going to be assumed that the underlying asset's path generation follows the geometric Brownian motion (GBM). Brandimarte (2006) codes a function in MATLAB that can emulate the GBM, called *AssetPaths.* With this code, a matrix of sample paths is generated where each row represents one simulation, and each column represents a new time step.

2.3.4 Least Squares Monte Carlo

The general idea was that computing the price of an American option would be impossible using simulation techniques from a computational point of view. The implementation of an optimal early exercise time imposed a problem: this exercise strategy had to be computed recursively, nonetheless, whenever simulation techniques are applied to price the option at "any time along any of the paths there is only one future path", making the usage of this values biased (Stentoft, 2004a).

Longstaff and Schwartz (2001) proposed a "simple yet powerful" (Longstaff and Schwartz, 2001, p.114) approach for approximating the price of American options via Monte Carlo Simulation. In their article, they propose using an Ordinary Least-Squares regression to determine the optimal exercise time during the lifetime of the option. Their main focus is not the simulation itself, but the conditional expectation function which determines when the option should be exercised. In this function, the authors only include paths for which the option is in the money. Due to this specification, the model will be able to increase its efficiency since it only targets relevant paths for exercising the option and reduces the amount of computational power needed.

In order to test this model, they proceeded to price a vanilla American put option, followed by an American–Bermuda–Asian Option and a few other derivatives and compared the results to other methodologies so as to test its robustness. The authors conclude that the ability to evaluate American options unravels a wider range of applications that simulation techniques may have, particularly when evaluating multiple factors markets as well as implementing more advanced models.

A few other studies following Longstaff and Schwartz's came to test the robustness of the LSM technique. One of such study is conducted by Moreno and Navas (2003). In their study, the authors analyse how different polynomial basis functions impact the robustness of the LSM by testing each function on different American options. For the American Put, the authors obtain very similar results regardless of the functions used and the few differences occurred due to numerical errors. Although basis functions with up to five terms increase the option price, with a higher number of terms, the option price does not necessarily increase, as a matter of fact, in some cases, the price decreased and then increased. Another important note to take from this study is that the standard errors generated in these tests are rather low, both in- and out-of-sample tests. The culmination of these results led the authors to conclude that the LSM technique is indeed very robust when pricing American put options.

Stentoft (2004b) published his study that analysed how the Least Squares Monte Carlo methodology converged to the true price. Stentoft (2004b) proves this by allowing the number of terms in the regression to tend to infinity as well as the number of simulated paths used. The theorem used in his study emphasized the influence of the number of repressors, M, and the number of simulated paths, N tending to infinity to obtain convergence, and it also gave optimal rates to select the number of regression terms concerning the number of paths. Furthermore, the higher this optimal rate is, the smoother the conditional expectation function should be.

3 Methodology

In order to price the derivatives, all the parameters need to be obtained. First and foremost, the daily historical data on the permits' prices were extracted from January 1st, 2013 - the year of the 3rd phase of EU ETS and the year of major modifications on the program (Bagchi and Velten, 2014) - until January 2nd, 2023 from Sendeco2's website. From here, the spot price (€83.18) at that date will be used, as well as the necessary data to obtain the volatility of this commodity.

Other important data necessary for this study are the interest rates. A one-year period will be considered for the calculation of the futures prices, and a 6-month period will be considered for options prices. Hence the one-year and the six-month continuously compounded euro interest rates on January 2nd, 2023 will be considered (2.4878% and 2.1572% respectively). This data was retrieved from the European Central Bank's website itself.

To convert the rates needed into continuously compounded rates, the bellow equation was used:

$$r_c = ln\ (1 + r_m), \tag{3.1}$$

where:

- r_c is the continuously compounding rate;
- r_m is the periodically compounded rate m.

3.1 Exponentially Weighted Moving Average

The EWMA is a volatility approach that aims to capture events in the past of the underlying asset and attribute a given weight according to the amount of time that passed, i.e. the older the event, the less impact it will have on the underlying asset's behaviour. The EWMA is a specific case of the more popular GARCH, being more simplistic. In this model, λ represents the relationship between the most recent events and the oldest during the life of the spot asset. RiskMetrics assign a λ value of 0.94, but it can also be estimated via the Maximum Likelihood method.

The EWMA can be expressed by:

$$\sigma_t^2 = (1 - \lambda)u_{t-1}^2 + \lambda\sigma_{t-1}^2, \tag{3.12}$$

where:

- σ_t^2 is the variance calculated at time t;

- λ is the smoothing factor;
- u_{t-1}^2 are the squared compounding returns given at $t-1$.

3.1.1 Maximum Likelihood Estimation Method

For this dissertation, the EWMA variance will be computed by using an estimate via MLE (maximum likelihood estimation) method, rather than using the RiskMetrics assigned value of 0.94 (Zumbach, 2006).

To estimate the λ for the EWMA model, λ will have as its starting point 0.94. Subsequently, a λ will be fitted to satisfy the below expression:

$$max \sum_{i=1}^{m}[-ln\,(\sigma_t^2) - \frac{u_t^2}{\sigma_t^2}],$$

where,

- m is the number of observations.

3.2 Hedging Strategies Pricing

3.2.1 No Arbitrage Pricing

To compute the theoretical price of a 1-year futures contract on the carbon permits, it is necessary to consider all the input variables needed. As with any underlying asset, the main components are the spot price, time to maturity and interest rate. Since carbon permits are considered to be traded as a commodity, it is implied that, upon expiration, there will be a physical delivery of the underlying commodity, hence, it should be taken into consideration storage and transportation costs. This would be the case for any other commodity, nonetheless, it is intuitive that this commodity has no obvious physical form (Homburg and Wagner, 2009) and so, no physical delivery will take place.

Similarly, the futures' counterpart, forward contracts, usually settle with cash, where the difference in prices between the start and maturity date is delivered. Therefore, no physical delivery of the underlying commodity is needed. This means that the theoretical price for futures and forward contracts, with the same characteristics, will be the same.

The futures price of the European Union Allowances will be given by:

$$F(t,T) = S_t \times e^{r_c \times (T-t)}, \tag{3.3}$$

where:

- S_t is the spot price at time t;
- r_c is the continuously compounding interest rate between time t and time T.

Moreover, it will also be relevant to define the payoff of a futures contract at maturity to evaluate, in practical terms, the hedging strategy:

$$CF_{fut}(T,\phi) = N \times \phi S_t - \phi F(t,T), \qquad (3.4)$$

where:

- N is the contract size;
- S_t is the spot price of the EUA spot price at time t;
- $\phi = 1$ for the long position and $\phi = -1$ for the short position.

3.2.2 Monte Carlo Simulation

In order to price financial options, European-style and American-style, the Monte Carlo approach will be used. The paths of this simulation are assumed to follow a geometric Brownian motion. Brandimarte (2006) describes the Monte Carlo simulating geometric Brownian motion:

$$S_{t+\delta t} = S_t e^{(\upsilon\delta t + \sigma\sqrt{\delta t}\epsilon)}, \qquad (3.5)$$

where:

- S_t is the spot price at time t;
- δt is a time step;
- $\epsilon \sim N(0,1)$ is a random standard normal variable.

Brandimarte (2006) also wrote a MATLAB code able to apply equation (3.5) straightforwardly. The input variables are the spot price of the underlying asset (SO), the drift adjustment (mu), the volatility of the underlying asset (sigma), the time in years (T), the number of time steps (NSteps) and the number of replications (NRepl). The simulation will be performed for 365 time-steps and 1,000,000 paths.

3.2.3 European-style Vanilla Option

Once price paths have been simulated, computing the European option is straightforward. From the paths generated at maturity, the payoffs of the option must be computed.

$$f_\phi = (\phi K - \phi S_T, 0)\,, \qquad (3.6)$$

where:

- S_T is the spot price at maturity T;
- K is the strike price;
- $\phi = 1$ for the vanilla put and $\phi = -1$ for the vanilla call.

The next step is to simply discount the average payoff to the starting date t. Since EUA is a commodity with no physical delivery, the risk-free interest rate will be the only factor to consider. For the price of a European vanilla option, we have:

$$p_t(S, K, T, \phi) = \frac{\sum_i f_\phi(S,K)}{n} e^{-r_c(T-t)}, \tag{3.7}$$

where:

- p_t is the price of the option;
- $f_\phi(S, K)$ is the vector of the payoff of the option at maturity;
- n is the number of paths.

3.2.4 American-style Vanilla Options

This type of option is more complex to price due to its ability to be exercised at any time until maturity.

To price this option, the Monte Carlo simulation will be used as well with a similar methodology mentioned previously, however, unlike its European counterparts, which can only be exercised at maturity, pinpointing the optimal exercise period is crucial for its pricing.

The approach from Longstaff and Schwartz (2001) will be the key to overcoming this issue. The Least-Squares Monte Carlo uses a simple regression model based on the simulation itself to determine the optimal exercise periods, however, only paths that are In the Money (ITM), i.e. with a positive payoff, will be considered (Longstaff and Schwartz, 2001).

According to Longstaff and Schwartz (2001), the continuation value of an American option is given by:

$$(\omega;\ t_k) = E_Q[\sum_{j=k+1}^{K} C(\omega, t_j; t_K, T)\ e^{-\int_{t_k}^{t_j} r(w,s)\, ds)}\ |\ F_{t_k}], \tag{3.8}$$

where:

- $C(\omega, t_j; t_K, T)$ represents the cash flows paths conditional on the non-early exercise before time t as well as the option holder following an optimal exercise strategy;
- $r(w, s)$ is the risk-free discount rate.

However, equation (3.8) can be represented "as a linear combination of a countable set of $F_{t_{k-1}}$ measurable basis functions" (Longstaff and Schwartz, 2001, p.122):

$$F(\omega;\ t_{k-1}) = \sum_{j=0}^{\infty} a_j\ L_j(X), \tag{3.9}$$

where:

- a_j coefficients are constant;

and:

$$L_j(X) = e^{-\frac{X}{2}} \frac{e^X}{j!} \frac{d^j}{dX^j}(X^j e^{-X}), \tag{3.10}$$

where:

- X is the value of the underlying asset.

Once the optimal exercise period is defined for each path, the following steps resemble the European-style counterpart, where the cash flows are discounted back and averaged out.

3.2.5 Protective Call

This will be the first hedging strategy using options that will be discussed. This strategy uses a long call option position with strike K to hedge a short position on an underlying asset against price increases. Its profit structure is as follows:

$$\begin{aligned} f_{pc} &= -S_T + S_0 - c_0, \qquad S_T < K, \\ f_{pc} &= -K + S_0 - c_0, \qquad S_T \geq K, \end{aligned} \tag{3.11}$$

- S_0 is the spot price time t=0;
- c_0 is the call option's premium.

3.2.6 Writing Covered Put

This strategy utilizes a short spot position to hedge a short position on an underlying asset against small increases in price. Its profit structure is as follows:

$$\begin{aligned} f_{wp} &= -K + S_0 + p_0, \qquad S_T < K, \\ f_{wp} &= -S_T + S_0 + p_0, \qquad S_T \geq K, \end{aligned} \tag{3.12}$$

where:

- p_0 is the put option's premium.

3.2.7 Short-Range Forward

The short-range forward uses a combination of options with different strike prices to hedge a short position on an underlying asset. It is composed by a short put position with a strike price K_1 and a long call position with a strike price K_2. Its profit structure is as follows:

$$\begin{aligned} f_{sf} &= -K_1 + S_0 - c_0 + p_0, \qquad S_T < K_1, \\ f_{sf} &= -S_T + S_0 - c_0 + p_0, \qquad K_1 \leq S_T < K_2, \\ f_{sf} &= -K_2 + S_0 - c_0 + p_0, \qquad S_T \geq K_2, \end{aligned} \tag{3.13}$$

3.3 Multiple Linear Regression

In the following section, a series of multiple linear regressions will be performed to analyse potential relationships between a dependent variable and several independent variables. For this empirical study, the variables will have quarterly frequency between 2010 and 2022, since there only exists quarterly data for the GHG emissions for this time period. The variables that will be used are the GHG emissions, as already stated (from Eurostat); the EUA prices, averaging their prices per quarter; the prices of the derivatives calculated from each EUA price; and ETFs which replicate the performance of green technology companies (iShares Global Clean Energy ETF (ICLN)) and fossil fuel related companies (iShares Global Energy ETF (IXC)), both retrieved from Yahoo Finance.

Multiple linear regression is given by:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_p X_{ip} + \varepsilon_i, \tag{3.14}$$

where:

- i represents each observation with $i = 1, \ldots, n$;
- n is the number of observations;
- p is the number of independent variables;
- X_{ip} represent each independent variable;
- Y_i represents the dependent variable;
- ε_i is the normally distributed error of the regression model.

The statistical information provided by the regressions relating the relationships between the dependent and independent variables will be generated using the OLS estimator.

4 Analysis and Discussion of Results

In this section, various hedging strategies will be studied. Since the strategies will be analysed from the companies' perspective, they will be considered hedging against the increase of the carbon allowances' prices. Furthermore, the companies do not possess any carbon permit at the time of the hedging strategy creation, consequently, they are said to be in the short position of the spot asset.

4.1 Hedging with Futures Contracts

The first strategy that will be analysed will be hedging using futures contracts. By using this financial contract, companies are able to determine at which price they will trade a given asset at a predetermined time in the future. Due to the standardized nature of the contract, both parties can trade among themselves with confidence since the exchange provides a guarantee that the contract will be honoured (Hull, 2015).

However, the arguably best aspect of this contract is the lack of cash flows when the contract is first celebrated between the counterparties, meaning that the long position only has to pay the agreed price at the delivery date without any kind of premiums underlying the deal to the other counterparty.

In this scenario, the company will enter the long position in a one-year futures contract on the EUA in order to fix its price regardless of future fluctuations. According to Hull (2015), this is called a long hedge, since the company knows it will eventually have to purchase the EUA in the foreseeable future but feels that locking the price in advance might be a safer approach. Furthermore, since the company needs to buy 50,000 units of EUAs but will only need to spend the money in the future, this will be considered a short asset position.

Let today be January the 2nd 2023 (the first trading day of the year). The company forecasts that its emissions for this year will exceed the carbon allowances given by the EU, so, the company will need to purchase 50,000 units of EUAs in order to meet the EU's carbon cap and avoid penalty fees. According to the European Energy Exchange (EEX), the volume of the futures contracts traded is 1,000 units of EUA. The spot price today is 83.18 euros per ton of carbon and the continuously compounded risk-free euro rate is 2.4878%. Since there is no way of knowing what the price will be when their carbon requirements need to be met, the company has decided to lock, as of that day, the price they will buy the carbon permits. Using equation (3.3), the no-arbitrage futures price for one unit of EUA will be:

$$F(0{,}1) = 83.18\ \times e^{0.024878\times(1-0)}$$

$$F(0{,}1) = 85.28.$$

In practice, the company is now obliged to pay the counterparty this fixed price for each carbon permit they will need to fulfil their carbon requirements, and the counterparty is obliged to deliver the requested number of carbon permits at the agreed delivery date.

If, at the delivery date, the price of the EU allowances is higher than 85.28 euros, then the hedging strategy was successful since it allowed the company to save the difference on this transaction. Let's suppose that, at maturity, the carbon permit's price is 90 euros per ton of carbon. Then, using equation (3), the payoff will be:

$$CF_{fut}(1{,}1) = 50\ \times 1{,}000 \times (90 - 85.28)$$

$$CF_{fut}(1{,}1) = €236{,}000.$$

This means that, in this scenario, the company would spend 4,264,000 euros on carbon permits instead of 4,500,000 euros if it have bought directly in the spot market, saving in the process 236,000 euros.

However, the opposite scenario is also a possibility. If the price of the spot price went down, let's say to 80 euros, the company would regret purchasing the futures contracts instead of buying directly on the spot market. In this scenario time, the payoff would be:

$$CF_{fut}(1{,}1) = 50\ \times 1{,}000 \times (80 - 85.28)$$

$$CF_{fut}(1{,}1) = -€264{,}000.$$

4.2 Hedging with Forward Contracts

As seen in the chapters above, forwards contracts and futures contracts have similar characteristics, including even the theoretical pricing formula. However, one of the main differences is the place where these contracts are traded. Since forward contracts are negotiated directly between the counterparties and, no other entity participates in the deal, then there are no guarantees that the contract will be honoured by either party. Furthermore, this also enables a more tailored contract, better fitting the needs of each counterparty (Taušer and Čajka, 2014).

Another important difference between these two contracts is the settlement, with forward contracts usually having a cash settlement rather than a physical settlement. Although it is already established that, despite a true delivery on the carbon permits, the delivery does not incur any storage or transportation costs, in practical terms, this way of settling the contract is still relevant.

To illustrate, suppose that the company contacts an investment back to enter a forward position with the same characteristics suggested in the scenario depicted with futures, i.e. the price of the forward contract would cost 85.28 euros per ton of carbon and the contract size will be 50,000 carbon permits. At maturity, the bank pays the difference between the agreed forward price and the price on the spot market. From here, the company just needs to purchase the EUAs from the spot market.

Just like in the example used previously, the payoff if the spot price at maturity is 90 euros per ton of carbon will be €236,000 which the company will use as a "discount" to buy the permits. However, if the spot price decreases to 80 euros per ton of carbon instead, the company would add to their expenses €264,000.

4.3 Hedging with European-Style Options

Using options as hedging tools grants the company more flexibility. Unlike its previously mentioned counterparts, this type of financial product gives the option, not the obligation, to the buyer of the contract, to exercise its right to buy/sell (call/put) a given asset with a predetermined price (strike price) at maturity.

4.3.1 Protective Call

The "option" feature of the contract makes it very attractive since, if the price of the underlying asset is not favourable for the long position, then the contract would not be exercised, and the asset could be bought in the spot market instead. However, this flexibility comes at a cost, referred to as the option's premium. In practice, the company is buying insurance in case the EUA price increases and pays a fair premium.

It is important to reinforce that, according to Homburg and Wagner, (2009) it is not necessary to hold the EUAs before the compliance date, henceforth it is reasonable to trade options on the futures contracts rather than the underlying asset itself. According to the EEX, if the option's life ends in the money, the futures contract underlying the option will be psychically delivered.

Let today's EUA futures price be 85.28€ as it was calculated in the previous example. The company will enter a long position into a call futures option, which gives the right to buy a long futures contract on the EUAs, maturing in 6 months. The strike price will be set at 83 euros.

Since the company needs 50,000 units of EUAs, it will need to trade 50 option contracts, since the underlying futures contract only corresponds to 1,000 units of the underlying asset. The 6-month risk-free rate is at 2.1572% and the volatility of the 1Y futures is 1.793%.

To price this European call option, a Monte Carlo simulation was performed to simulate 1,000,000 possible futures prices at maturity. Furthermore, the average of the option's payoff was taken and discounted back to the start of the contract. Using the parameters stated above, the price of this European call option is 3.1715 euros. For reference, if the pricing methodology used was the BSM, the price of this European-style call option with the same parameters, would be 3.1708 euros, which is very close to the price obtained via Monte Carlo simulation.

With this in mind, the strategy's diagram will be:

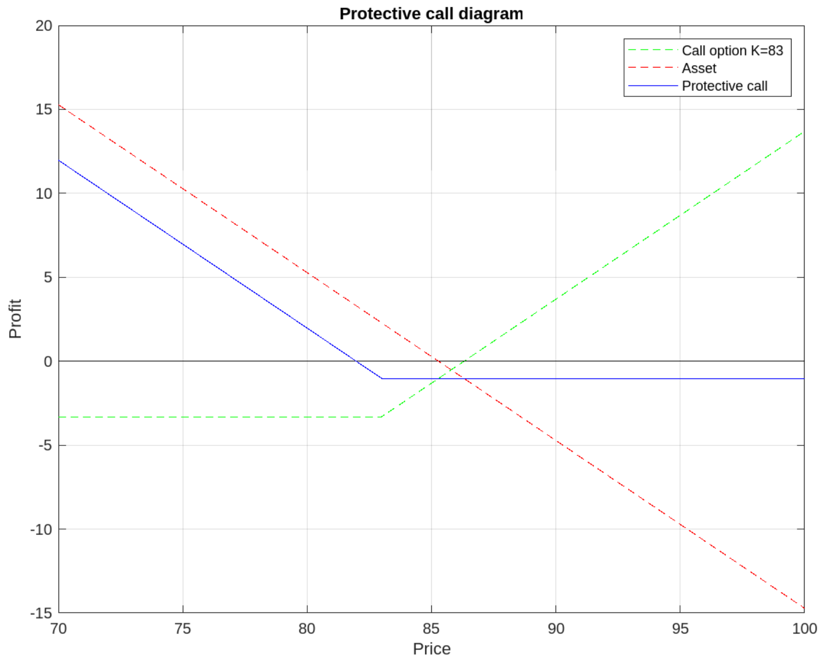

Figure 4.3.1- Protective Call Diagram

In the above graph, the protective call diagram resembles a put option rather than a call option. This can be explained by the combination of the positions taken on the underlying asset and the call option.

	$S_T < K$	$S_T \geq K$
Short asset	$-S_T$	$-S_T$
Long call	0	$S_T - K$
Initial cash flow	$S_0 - c_0$	

Protective call (profit)	$= -S_T + 85.28 - 3.1715$ $= 82.1085 - S_T$	$= -83 + 85.28 - 3.1715$ $= -0.8915$

Table 4.3.1- Protective Call Payoff Structure

This protective call's profit can be summarised by max($82.1085 - S_T$, -0.8915) which is very similar to the equation (5) when $\phi = 1$. This means that if the EUAs futures price at maturity ends up being higher than the predetermined strike price of 83 euros per ton of carbon, then the company's jeopardy will not exceed 0.8915 euros per carbon permit futures purchased by the company.

Let's assume that at maturity the futures price is 87 euros per ton of carbon. The company, at the start of the contract, purchases the necessary call options with a strike price of 83 euros per ton of carbon. In total, the cost of the contracts is 158,575 euros. Furthermore, since the company needs to buy 50,000 units of EUAs but will only need to spend the money in the future, this is assumed to be the same as receiving 4,264,000 euros.

By fixing the price of each carbon allowance futures at 83 euros per ton of carbon, the company will not be spending more than the initial 158,575 euros plus the 4,150,000 euros at maturity, when the futures would be purchased, which would be deducted from the 4,264,000 euros the company had reserved for the transaction. In other words, the company would limit their loss to 44,575 euros, which represents, as mentioned above, 0.8915 euros per carbon permit futures.

On the opposite spectrum, if the price of the futures contract is at 80 euros at maturity, the company will instead purchase the underlying on the futures market and since that at the start of the contract the company did not spend any amount on purchasing futures contracts and only spent 158,575 euros, the company would be saving 105,425 euros on the EUA futures contracts, or 2.1085 euros per futures contract.

4.3.2 *Writing Covered Put*

This strategy uses a put option to hedge a short asset position. Just as mentioned above, a put option gives the right to the buyer to sell the underlying asset at a predefined price. This also means that the short position of this contract is obliged to buy the asset at that price, having as a reward the option's premium. Therefore, the company can still obtain the underlying futures contracts and receive a small discount for doing so. This strategy is more conventionally used to hedge a short put position against price decreases, but it can also be used to hedge a short asset position against small increases in price.

Let the market conditions remain the same as in the previous examples. To hedge the short position on the EUA futures, the company will enter a short put option contract with a strike price of 84 euros. The premium of this put option is 0.008058 euros. Just like the protective call above, this put option was calculated using a Monte Carlo simulation. Since this is a put option, the only difference is the payoff structure, using equation (5) with $\phi = 1$. As a reference, the value obtained for a put option using the same market characterising with the BSM model is 0.008057 euros. The diagram composed of this hedging strategy is the following:

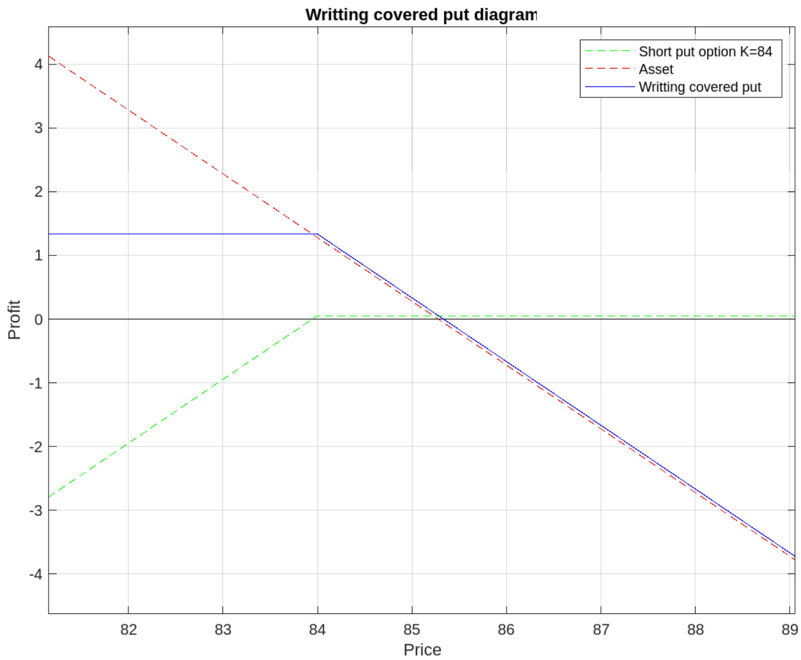

Figure 4.3.2 - Writing Covered Put Diagram.

Although a short put option is used in this strategy, the diagram resembles a short call option. This can be explained, just as before, by Table 2:

	$\boldsymbol{S_T < K}$	$\boldsymbol{S_T \geq K}$
Short asset	$-S_T$	$-S_T$
Short put	$-(K - S_T)$	0
Initial cash flow	$S_0 + p_0$	
W.C.P. (profit)	$= -K + 85.28 + 0.0081$ $= 1.2881$	$= -S_T + 85.28 + 0.0081$ $= -S_T + 85.2881$

Table 4.3.2 - Writing Covered Put Payoff Structure.

This table explains that by combining a short asset position with a short put position, the result is a maximum savings of 64,405 euros when the option is out of the money (OTM) and an unlimited minimum profit. This makes this strategy more useful when the company is expecting the price of the underlying to increase slightly.

From a practical perspective, if, at maturity, the EUA futures price rises to 85 euros, the strategy would be able to cover this increase. The premium received at the start of the contract, which, for 50,000 futures contracts, would be 405 euros, and, the 4,264,000 euros could have been spent directly in the futures market, would still be able to save 14,405 euros. Nevertheless, if the futures price, at maturity, were to be any higher than 85.2881 euros, then the company would not be taking losses.

If, however, the put option is exercised by the long position, the company is obliged to purchase the futures contracts at 84 euros per carbon ton, which is still lower than the current spot price of 85.28 euros per carbon ton. Considering the premium received at the start of the contract, the company would have saved 64,405 euros.

4.3.3 Short-Range Forward

The short-range forward is a hybrid between the writing covered put and the protective call. The option positions underlying the short-range forward, however, will have different strikes than the precedents. This strategy is used when the price of the underlying is expected to have a limited increase in the future. While the protective call can limit the losses without sacrificing the upside potential, the writing covered put limits that upside potential and mitigates some of the prior's cost.

The lower the strike price of the long call, the more protection it will provide, but that also implies a higher cost. On the other hand, the higher the strike price of the short put, the more it limits the upside potential of the protective call, hence reducing further the cost of the strategy.

In this scenario, the company will enter a long call position with a strike price of 87 euros and a short put position with a strike price of 84 euros, which is the same put option used in the WCP strategy above. The cost of the call, using the same variables used in previous examples, is 0.1487 euros and the premium received from the short put position is 0.0081 euros. Similarly, to the protective call and the WCP computed above, this call option was also calculated using a Monte Carlo simulation, and then discounting back its payoff at maturity to the start of the contract. Using the BSM model as a reference, the call option with K=87 would be 0.1489 euros.

The bellow diagram is generated:

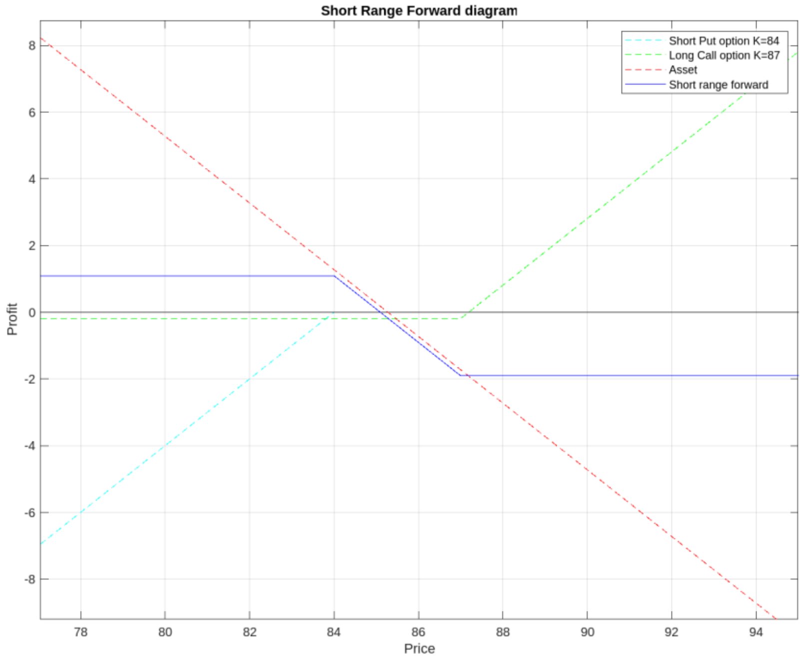

Figure 4.3.3 - Short-Range Forward Diagram

As seen in the figure below, the short-range forward does resemble both strategies. Nevertheless, its profit structure further emphasises their similarities:

	$S_T < K_1$	$K_1 \leq S_T \leq K_2$	$S_T > K_2$
Short asset	$-S_T$	$-S_T$	$-S_T$
Short put $K_1 = 84$	$-(K_1 - S_T)$	0	0
Long call $K_2 = 87$	0	0	$S_T - K_2$
Initial cash flow		$S_0 + p_0 - c_0$	
S.R.F. (profit)	$= -84 + 85.28 + 0.0081 - 0.1487 = 1.1394$	$= -S_T + 85.28 + 0.0081 - 0.1487 = 85.1394 - S_T$	$= -87 + 85.28 + 0.0081 - 0.1487 = -1.8606$

Table 4.3.3 - Short-Range Forward Profit Structure

In a scenario where the price of the underlying futures contract, until the strategy's maturity, increases to 90 euros, the company's expense would not exceed 87 euros per futures contract. The cash flow received at the start of the contract, 4,256,970 euros, would be able to soften the expense generated by purchasing the contracts at 87 euros per carbon ton, limiting it to 93,030 euros.

Unlike the protective call strategy which, if the futures price is lower than the strike price, enables the company to have virtually unlimited savings, this strategy will only allow a maximum amount of potential savings, depending on how much the futures price decreases. If, for instance, the cash futures price at maturity, is 80 euros per carbon ton, the company would only save 56,970 euros. Since the company is obliged to purchase futures contracts at 84 euros if the futures price is any lower than that, the put option would be exercised by the long position, therefore, similar writing covered put strategy, the company would buy the futures contracts at a discount, saving 56,970 euros.

4.4 Hedging with American-Style Options

American options can offer the hedger extra flexibility when hedging against price fluctuations. Just like the European counterparts, American options allow the long position holder to buy or sell (call/put) a given asset at a fixed price. However, the main difference between these two products is when it can be exercised. A European option can only be settled at the end of the contract period, but the American option can be exercised at any time until maturity. This means that the holder of the long position can exercise the option whenever its payoff is most beneficial.

4.4.1 American Protective Call

The ability to early exercise an option, however, brings more complexity, in theory, when pricing the product, and in practice, to determine when is optimal to exercise. The early exercise can generate scenarios that might make the long position regret the early exercise. For instance, an American call with a strike price of 83 euros is exercised early when the underlying's price is 87 euros. The payoff at this point would be 4 euros. However, at maturity, the spot price would decrease to 75 euros. In this situation, the holder of the long positions would have benefited more by buying the underlying asset directly rather than through an option at 80 euros, hence regretting its early exercise. For this reason, there is no inceptive on exercising an American call on an underlying asset which does not have any dividend yield early, hence, theoretically, its price should be equal to a European call option.

To confirm this idea, an American call option will be priced using the same input variables as in the European protective call above. Pricing the American option using a Monte Carlo Simulation, uses a similar methodology to the European-style counterpart. However, instead of considering every single path, only paths the mature ITM are considered to determine the continuation and intrinsic values more viably. The next steps require the setup of the least-squares regression variables which will define if the option is exercised immediately or if it will continue to the next period. The independent variable *X*, just like in the Longstaff and Schwartz (2001) example, is composed of the underlying asset's price at in ITM paths at a given time t.

The dependent variable Y is denoted by the discounted cash flows that were received in the following time step if the option is not exercised in time t. Now Y is regressed relatively to X and X^2 for each time period. Having the continuation and intrinsic values at each time step, the final step is to average all the discounted cash flows.

The price of the European call option with a strike price of 83 euros is 3.1708 euros. By proceeding as mentioned above, the price of the American counterpart is 3.1711 euros. Note that the discrepancy between the two prices is not due to the nature of the products, but due to they being computed via Monte Carlo simulation, therefore, every time each product is priced, a new simulation is generated and for this reason, there will be small differences. Nonetheless, if these computations were done enough times, the values would converge.

4.4.2 American Writing Covered Put

An American put option holder does suffer so much from the same fear of regret seen when holding an American call. Since a put option gives the right to sell at a given price and not buy, when the option is deep ITM, delaying its exercise might lead to a decrease in the present value of the underlying's price, hence decreasing its potential payoff. If the option is deep in the money, the possibility of it finishing OTM still might make the long position holder regret exercising early, nevertheless, if an option is deep ITM, it is very unlikely to finish OTM.

Let an American put option with a strike price of 84 euros be exercised when the underlying's asset was 75 euros. In this scenario, the long position gained a payoff of 9 euros. If, at maturity, the underlying asset's price increases to 80, then delaying the early exercise would decrease the option's payoff to only 4 euros. On the other hand, although unlikely, if the price increased to 85 euros, the long position holder would have regretted selling the underlying asset at 84 euros, since it could have been sold at 85 euros in the spot market.

Unlike the American call, the American put truly gives more flexibility to the long position since it is more unlikely to regret an early exercise, therefore, this is reflected in the option's premium. By using the same methodology used when calculating the American call, the price of an American put with a strike price of 84 euros, and all the other parameters already used in prior examples, would be 0.01053 euros. This is a higher premium when compared with the European counterpart (0.008058 euros), which is expected.

Since the writing covered put uses a short put position to hedge small increases in price, the extra flexibility would not necessarily benefit the hedger besides the increase on the received premium. In fact, if the option is exercised, it most likely would generate a potentially worse outcome than it would if the option was European style. In other words, if the option was exercised, it implies that the price of the underlying asset is below the strike price and therefore, the short put position would not be able to purchase the underlying asset at a lower price than it could have if it bought it on the futures market directly. Since the long position is looking to maximize its payoff, this means that the short position's payoff could be affected negatively.

4.4.3 American Short-Range Forward

As it was already established in the American Protective call strategy, an American call is as useful as a vanilla European call. This means that an American SRF would not be completely American since it would be composed of a European call and an American put options.

The base concept of this strategy is still the same as in the SRF seen above: use a long call position to limit possible losses in case of a price increase of the underlying asset, and use a short put position to limit the upside gains in case it is not expected any major price fluctuations, hence decreasing the overall cost of the strategy.

Since the call used in this strategy will be the same European call discussed previously (with K=87, and premium 0.1487 euros), the focus will be on the short put position which is American style. Just like the American WCP, and the European SRF, the strike price of this put is 84 euros for this option, meaning that its price will be 0.01053 euros. The overall cost of this strategy would then be 0.13817 euros per EUA futures. When compared with the European-style SRF, there is a small reduction of 0.00243 euros in the strategy's initial cost. This is, of course, justified by the increased premium earned from the American short put. Unlike the American writing covered put where there is still a somewhat plausible possibility of regretting an early exercise, in this scenario, the put option ending OTM at maturity does not concern the company as much since they are expecting no significant price fluctuations.

4.5 The Impact of Hedging EUA on GHG Emissions

Although it is understandably logical that companies will attempt to mitigate their exposure to the EU carbon allowances, legislators must not overlook the potential impact of their hedging strategies. As Chevallier et al. (2011) put it, such overlap in the regulatory instruments might impact the EUA prices negatively, hence affecting its efficiency in reducing GHG emissions.

From here, the idea that hedging against carbon permits' price increases might have a negative influence on GHG emissions becomes the main hypothesis of this dissertation.

To understand how hedging strategies might impact the price of their underlying asset and, subsequently, affect greenhouse gas emissions, several least-squares regressions will be done. The first regression will have as its dependent variable the GHG emissions, and as its explanatory variable the EUA prices, to apprehend how the EU ETS is affecting the reduction of GHG emissions. The second regression will have as the dependent variable the GHG emissions and as explanatory variables, each derivative. The derivatives used in the hedging strategies previously analysed, concerning their impact on the price of their underlying The third and last regression, will be similar to the previous, adding two ETFs which replicate the performance of green technology companies (IShares global clean energy) and fossil fuel related companies (SPDR Dirty energy).

4.5.1 Data Description

In this section, all the external databases necessary for this study will be described. Any relevant events among the data will be considered.

4.5.1.1 Greenhouse Gas Emissions

As one of the biggest focuses of today's world, there is plenty of statistical data relating to carbon and other gas emissions, especially in Europe. There is a wide range of manners as to how they are decomposed, such as by economic sectors or countries. Furthermore, most of the data is only available with yearly frequency. Nevertheless, for this study, it is more interesting to view it from a broad perspective, that is, to study emissions from the European Union as a whole, as well as with quarterly frequency.

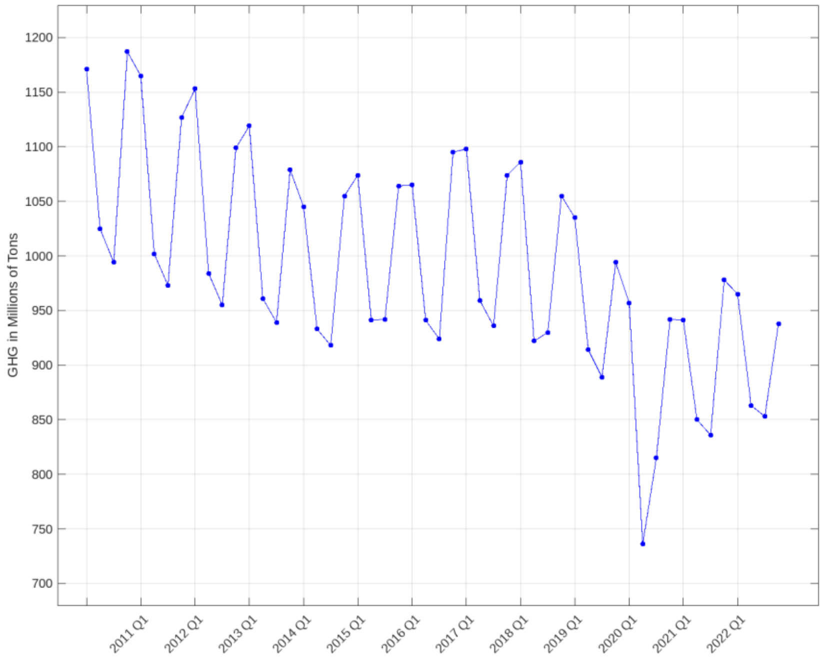

Figure 4.5.1 - Greenhouse Gas Emissions in the European Union

From the first quarter of 2010 until the fourth quarter of 2022, there is a general descending tendency in the tonnage of GHG emitted. There are, of course, a few years outlying the general tendency, namely 2017, where there was a small increase in emissions in that year overall, and 2020, where there was a substantial decrease. The former, according to the European Environmental Agency (2019), is justified by the increase in consumption by the transportation sector, and by the increase of several industrial sectors throughout Europe. The increase in emissions, however, was slightly offset by improvements in clean energy production and efficiency. The latter is simply justified by the pandemic and systematic lockdown, which decreased considerably consumption in multiple sectors, especially transportation (Bhanumati et al., 2022).

One interesting pattern is noticeable among the data, and its explanation is rather intuitive. Every year, the GHG emissions peak in the fourth or first quarters, and plummet in the second or third semester. Its explanation is, naturally, the climate. Since the first and fourth quarters coincide with autumn and winter in Europe, when there is a higher energy consumption for heating, higher emissions are a natural consequence (Tsafos, 2022). In the second and third quarters, there is no need for the same high energy intensity as in the colder months of the year.

4.5.1.2 European Union Carbon Allowances

The EU ETS has had three trading phases, being that, at present, at its fourth phase is already ongoing. For this empirical study, the focus will be on the historical prices between the second phase and the fourth phase (2010-2022), to match the same period of data available on the GHG emissions. Furthermore, due to the GHG emissions frequency, instead of using daily historical data on the prices directly, the quarterly average of the prices will be considered.

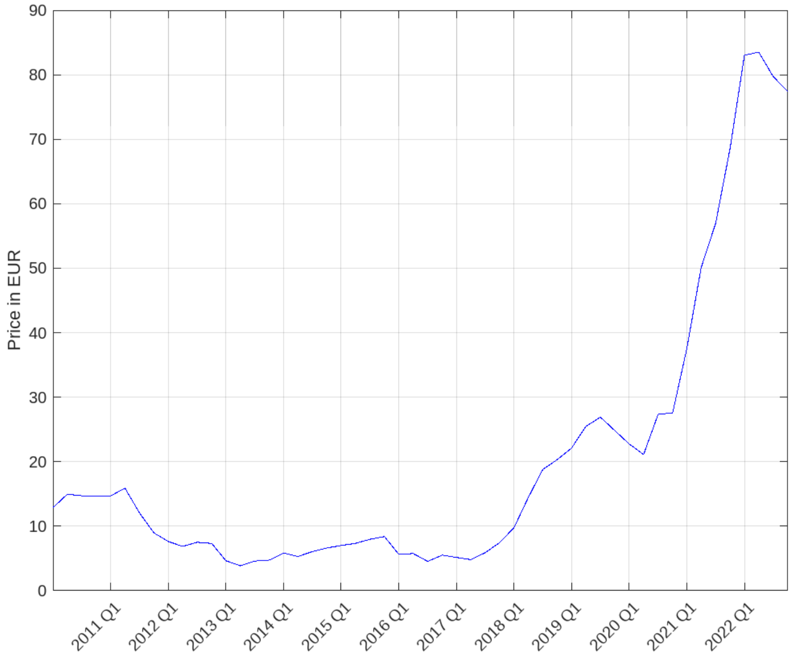

Figure 4.5.2 - European Union Carbon Allowances' Price

The EUA prices have stayed relatively steady between the end of 2012 and 2017. This period coincides with the third trading phase of the carbon permits, where some changes were implemented to the EU ETS. For instance, an EU-wide carbon cap was put in place, replacing multiple national-wide caps and replacing the free-allocation method used at the beginning of previous trading periods with auctions. In the second quarter of 2011, there was a decrease in price to under 10 euros until the end of the year, which was justified by the 2008 economic crisis, which led to a surplus of permits, since there were lower production levels, hence less carbon emissions, affecting the price through the whole of second phase (European Commission, n.d.). In the third quarter of 2017, the prices started to steadily increase, except in 2020, due to the pandemic, and at the end of 2022, due to the energy crisis and subsequent inflation partly caused by the Russian invasion of Ukraine (Branthôme, 2022).

4.5.1.3 iShares Global Energy ETF (IXC)

The iShares Global Energy ETF is not as well geographically diversified as the previous ETF. It has the majority of its holdings in North American companies (70.91%) and European companies (23.13%), such as Exxon Mobil Corporation, Chevron Corporation and Shell Plc. The remaining holdings are divided between Asia-Pacific companies (3.23%) and Latin-American companies (1.73%) (ETF, n.d.). Despite the lack of geographical diversity, most, if not all, of the companies replicated on this ETF have a significant influence on the global energy market and are affected similarly when there are global energy events.

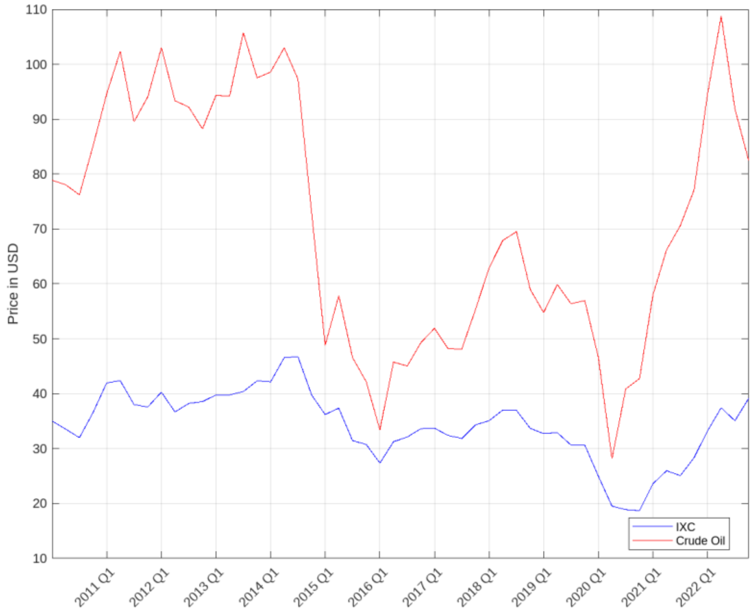

Figure 4.5.3 - iShares Global Energy ETF (IXC) vs. Crude Oil

Since this ETF replicates companies which deal mainly with oil and oil products, it is logical that they are impacted by the oil price. This can be seen when comparing its performance with the oil's performance. Between 2014 and 2016, according to Stocker et al. (2018), the world has seen one of the biggest oil price dips since 1986. The efficiency increase in extracting shale oil in the United States was the major driver of this price dip since it flooded the market with more supply. This was naturally reflected in the stock prices of energy companies.

Following this period, three other major instances occurred: 2018, 2020 and 2023. In 2018, according to Egan (2018), the market shock was partly similar to the 2014 shock. In 2020, the disruption in market demand due to the abrupt stop caused by the pandemic, made a few oil and gas producers see their stock price plummet (Good and Gull, 2021). In 2022, oil prices had a peak over the first semester due to Russia's invasion of Ukraine, and remained volatile subsequently (Smith, 2022).

4.5.1.4 Euro Interest Rates

The 1-year and 6-month Euro interest rates, provided by the European Central Bank, represent the risk-free rates used to calculate both futures prices and the options' premiums, respectively.

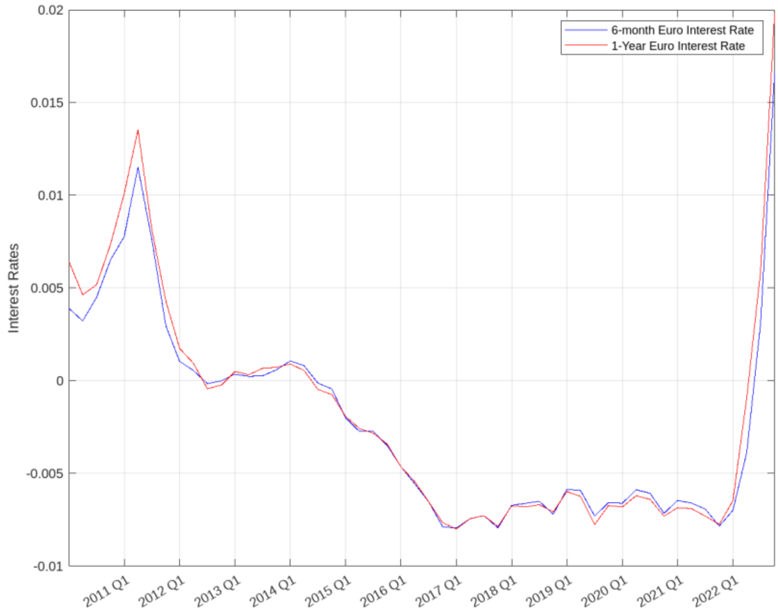

Figure 4.5.4 - Euro Interest Rates

There is a small difference between the two rates throughout this period. Usually, rates with shorter terms are smaller due to the term period. For instance, a bond with a longer period to maturity is expected to pay a higher interest rate than a bond with a shorter period to maturity (FRED, n.d.).

There are two main interest rate hikes, the first one being in 2011 and 2022. The former can be explained by the Eurozone debt crisis which put pressure on the whole area (Stewart, 2011). The 2022 hike is justified by the European Central Bank raising the interest rates in order to mitigate inflation (Sparks, 2022).

4.5.2 Correlation Coefficients

A practical method to analyse how different assets interact with each other is to observe their correlation coefficients.

	GHG	EUA	EUAFut	iSharesD	EProtCall	EWCP	ESFR	AWCP	ASFR
GHG	1.0000								
EUA	-0.4349	1.0000							
EUAFut	-0.4329	0.9999	1.0000						
iSharesD	0.4252	-0.2728	-0.2677	1.0000					
EProtCall	-0.4149	0.9933	0.9946	-0.2263	1.0000				
EWCP	0.2954	-0.7401	-0.7350	0.2659	-0.6889	1.0000			
ESFR	0.1043	-0.0853	-0.0733	0.3422	0.0267	0.5478	1.0000		
AWCP	0.2987	-0.7499	-0.7449	0.2564	-0.7004	0.9991	0.5301	1.0000	
ASFR	0.3055	-0.6754	-0.6677	0.3435	-0.6002	0.9631	0.7292	0.9590	1.0000

Table 4.5.1- Correlation Coefficients

As expected, all the hedging strategies are very closely correlated. However, more complex derivatives are slightly less correlated to their underlying asset, which is expected since there are more variables affecting them.

The Greenhouse Gas emissions, naturally, are positively correlated with the dirty energy ETF.

Another interesting mark regarding the correlation coefficients is the fact that the EU allowances and the GHG emissions are negatively correlated, which means that when the EUA price increases, the GHG emissions decline. A similar intuition also applies to the futures prices. As for the options strategies, more specifically, the protective call, when its premium increases, the GHG emissions decrease.

On the opposite spectrum, the other strategies have mostly low or negative costs to the company, hence, every time the negative costs increase, the GHG emissions increase.

4.5.3 Greenhouse Gas Emissions vs EU Allowances

The first analysis will be done between the Greenhouse Gas emissions and the carbon permit prices, in which the goal will be to better understand the impact that the EUA price fluctuations have on the GHG emissions in the European Union. The fundamental hypothesis behind this first regression is that the EU ETS is contributing to the reduction of GHG emissions in the EU. The regression will estimate coefficients that will define a model line, fitting the model as close as possible to reality using the data available.

By running the regression on MATLAB, the model closely resembles the one given by equation (3.14). MATLAB calculates a series of statistics that elucidate the strength of the model.

```
GHGvsEUA =


Linear regression model:
    y ~ 1 + x1

Estimated Coefficients:
                   Estimate      SE        tStat       pValue  
                   ________    ______    _______    __________

    (Intercept)     1029.2     16.825      61.17    1.1384e-48
    x1             -1.9198     0.5622    -3.4147      0.001275


Number of observations: 52, Error degrees of freedom: 50
Root Mean Squared Error: 89.3
R-squared: 0.189,  Adjusted R-Squared: 0.173
F-statistic vs. constant model: 11.7, p-value = 0.00127
```

Figure 4.5.5 - GHG vs. EUA Regression Output

The coefficient of determination, or R-squared, measures how well the explanatory variables are able to explain the dependent variable, varying between 0 and 1 (Nakagawa, et al., 2017). The R-squared obtained on this regression is 0.189. This value can be interpreted as the explanatory variable being able to explain 18.9% of the dependent variable's movement. The determination coefficient, although theoretically possible, is usually neither zero nor one. Since this is a linear regression, where only one variable is used to explain the movements of another variable, the R-squared is not expected to be very high, however, 0.189 does seem to be a significant portion of the model.

The F-test assesses the relevancy of the model, that is, whether or not the beta coefficients generated by the model are significant or not (Wang, and Cui, 2013). In this situation, the beta coefficients are significant, since the statistical value for this test was 11.7, leading to the small p-value of 0.00127, hence rejecting the null hypothesis for the significance level of 5%.

The β_0 obtained was 1029.2 and the β_1, which in the case of linear regression, determines the slope of the least-squares line, is -1.9198. This means that the line will have a descending slope, indicating that the EUA prices truly hurt the GHG emissions. In other words, every time the price of the EUA increases by one, the companies will need to pay more to continue to emit as much carbon as before, therefore the companies will try to avoid emitting as much carbon as possible, hence GHG emissions will decrease by a factor of 1.9198.

Although the beta coefficients seem to be coherent with the base hypothesis of this regression model, each coefficient needs to be individually tested. To do this, the t-test will be used (Allen, 1997). The t-statistics calculated by MATLAB were 61.17 and -3.4147, for β_0

and β_1 respectively. The p-values from these t-statistics are considerably small, inferior to the 5% significance level, hence rejecting the null hypothesis, meaning that the beta coefficients are definitely significant for the model.

The linear regression for this model will be:

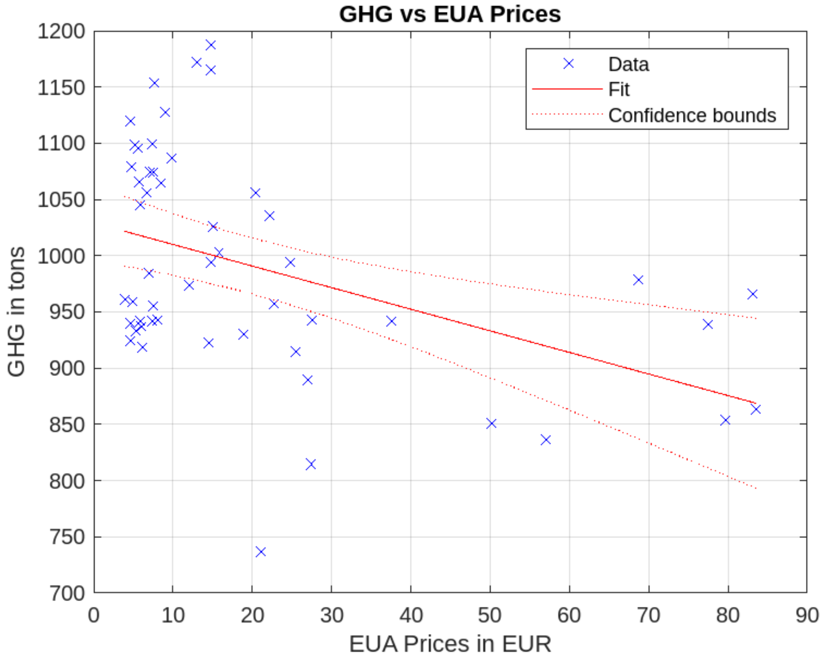

Figure 4.5.6 - GHG vs. EUA Regression Plot

By observing the plot, one might conclude that the model has a problem with heteroscedasticity due to it having many residuals away from the regression line. If the homoscedasticity assumption is refuted, then the beta coefficients may not be reliable (Rosopa et al., 2013). The tests that will be used to determine if the model is indeed homoscedastic or not will be the Breusch-Pagan test (Breusch and Pagan, 1979) and the White test (White, 1980). Both have as their null hypothesis that the model is homoscedastic.

	Breusch-Pagan	**White**
Chi-Square Statistic	0.2187	0
p-value	0.64	0.2226

Table 4.5.2 - Heteroscedasticity Tests

To reject the null hypothesis, the test's p-value must be under the 5% significance level, which is not the case for this model, proving that the coefficients are not biased.

Autocorrelation might also impact the efficiency of the beta coefficients, of the possibility of the residuals being correlated, thus biasing the model (Kenton, 2023). The Durbin-Watson test Durbin and Watson, 1950) is able to determine if the model is significantly autocorrelated.

	Durbin-Watson
DW-Statistic	1.3715
p-value	0.0129

Table 4.5.3 - Durbin-Watson Test for Autocorrelation

According to the test's result, the model has a positive autocorrelation, since the DW statistic is within the range of 0 and 2. However, its value is relatively close to 2, furthermore, the p-value suggests that the model is not autocorrelated for the significance level of 5%, therefore, the model can still be considered efficient.

To add further complexity to the model, the iShares Global Energy ETF is going to be added to the model.

```
GHGvsEUA_Dirty_ETF =


Linear regression model:
    y ~ 1 + x1 + x2

Estimated Coefficients:
                   Estimate      SE       tStat        pValue
                   ________    ______    _______    __________

    (Intercept)     846.45     70.805     11.955    3.8857e-16
    x1             -1.5208     0.5521    -2.7547     0.0082232
    x2              5.1018     1.9261     2.6487      0.010842


Number of observations: 52, Error degrees of freedom: 49
Root Mean Squared Error: 84.4
R-squared: 0.291,  Adjusted R-Squared: 0.262
F-statistic vs. constant model: 10, p-value = 0.000222
```

Figure 4.5.7 - GHG vs EUA plus ETF Regression Plot

According to the F-test, the model is relevant since its p-value is inferior to 5%, hence rejecting the null hypothesis. The beta coefficient related to the ETF is 5.1018, which indicates that the ETF has a positive relationship with GHG emissions. Per every dollar, the price of the IXC increases, and the Greenhouse Gas emissions will increase by a factor of 5.1018. On the other hand, the beta coefficient for the EUA prices decreased slightly, only affecting the GHG emission by a factor of -1.5208. By considering the t-statistics on the MATLAB output, new coefficients can be said to be statistically significant to the model with a 95% confidence level, since both of their p-values are smaller than the significance level of 5%.

The explanatory variables in this model now generate a determination coefficient of 0.291, however, this value cannot be compared to the R-squared generated in the previous output. One downfall of the R-squared is the fact that will always increase every time an explanatory variable is added to the model. Rather, the R-squared adjusted is the correct metric to compare how adding a new variable contributes to the model's efficiency. On the linear regression, the R-square adjusted was 0.173, while the former model generated a new coefficient of 0.262. This means that the iShares Global Energy ETF significantly increased the amount the explanatory variables explain the dependent variable's movements.

This new model does not suffer from heteroscedasticity. Both the White test and Breusch-Pagan tests confirmed this since their p-values were 0.8541 and 0.3796, hence not rejecting the homoscedasticity hypothesis for any significance level. However, the Durbin-Watson test suggests the model suffers from autocorrelation, rejecting the null hypothesis for the 5% significance level. To solve this issue, a simple form transformation to Linear-Log is used, that is, by taking the natural log of the explanatory variables. By doing this, the interpretation of their beta coefficients will change, and the residuals will cease to be correlated to the point of affecting the efficiency of the model.

```
>> GHG_vs_EUA

GHGvsEUA_Dirty_ETF =


Linear regression model:
    y ~ 1 + x1 + x2

Estimated Coefficients:
                   Estimate     SE       tStat      pValue  
                   ________   ______    _______    ________

    (Intercept)     511.42    240.88     2.1231    0.038821
    x1             -31.964    14.625    -2.1856    0.033657
    x2              159.65    63.316     2.5215    0.014989


Number of observations: 52, Error degrees of freedom: 49
Root Mean Squared Error: 85
R-squared: 0.28,  Adjusted R-Squared: 0.251
F-statistic vs. constant model: 9.52, p-value = 0.000321
```

Figure 4.5.8 - Linear-Log GHG vs. EUA plus ETF Regression Plot

The transformation now allows the model to reject the null hypothesis for autocorrelation with a p-value of 0.0382. The beta coefficients changed considerably. These coefficients can be interpreted as variations in percentages. In other words, now, a 1% change in the EUA prices and the ETF prices, will affect the GHG emissions by -0.31964 and 1.5965 million tons of GHG respectively.

4.5.4 Greenhouse Gases vs. Hedging Strategies

The goal of this second regression is to determine how each derivative impacts GHG emissions. The derivatives used in this analysis are European call options, European put options, and American put options. Since the 1-year Futures prices are very closely correlated with the EUA spot prices, and there is only a small difference between the two, the analysis of this derivative will generate very similar results to the ones obtained with the EUA spot prices.

Due to the financial derivatives nature, a model where all the strategies studied above is not possible due to the high correlation between each strategy and the underlying asset, hence generating p-values for the f-test too high to determine any statistical relevancy in a model as such. As an alternative, each strategy will be modelled individually to understand how each affects GHG emissions independently.

		Regression Results							Heteroscedasticity				Autocorrelation	
		F-Test		Estimate	Statistic	p-value	R-squared	R-sq adj	White		Breusch-Pagan		Durbin-Watson	
		Statistic	p-value						Statistic	p-value	Statistic	p-value	Statistic	p-value
Protective call	Beta0	10.4	0.00222	1026.7	60.915	1.4E-48	0.172	0.156	0	0.2041	0.1825	0.6693	1.3455	0.0097
	Beta1			-36.652	-3.2246	0.00222								
European Writting Covered Put	Beta0	4.78	0.0335	1009.1	64.321	9.5E-50	0.0872	0.069	0	0.5409	1.6243	0.2025	1.3449	0.011
	Beta1			497.77	2.1861	0.03352								
European Short Range Forward	Beta0	0.55	0.462	992.07	71.479	5.2E-52	0.0109	-0.0089	0	0.2463	0.2448	0.6208	1.1353	0.00075
	Beta1			273.34	0.74153	0.46184								
American Writting Covered Put	Beta0	4.9	0.0315	1012.7	61.069	1.2E-48	0.0892	0.071	0	0.5415	1.6215	0.2029	1.3452	0.011
	Beta1			498.93	2.2128	0.03151								
American Short Range Forward	Beta0	5.15	0.0276	1012.9	61.62	7.9E-49	0.0933	0.0276	0	0.4674	0.779	0.3774	1.3316	0.0094
	Beta1			701.6	2.2685	0.02765								

Table 4.5.4 - GHG vs Strategies Regressions Output

In Table 4.5.4, all the regression results for each linear regression involving the strategies are shown. According to the F-test, only one out of five strategies is not statistically relevant in describing the GHG emissions' behaviour: the European short-range forward. The p-value of 0.462 is considerably superior to the significance level of 0.05, hence the null hypothesis is very likely to be true.

The protective call regression generated a statistically significant beta of -36.652, having a p-value smaller than 0.05. This beta coefficient implies that per every euro that the call's premium increases, the GHG emissions will decrease by 36.652 million tons. When premiums increase, hedging becomes more expensive, therefore, companies will avoid hedging at all, decreasing the amount of money saved while buying futures contracts, hence decreasing GHG emissions. The strategy has an R-square of 0.172, implying that it has a relatively significant weight in explaining the dependent variable.

The European and American puts behave in a similar fashion and impact greenhouse gas emissions similarly. Their beta coefficients, which are statistically significant according to the t-test, are close to each other, at 497.77 and 498.93 respectively. Both strategies move in the same direction of the GHG emissions, that is, per every euro increased on each put's premium, the GHG emissions will increase by 497.77 and 498.93 million tons respectively. This is due to the fact that the strategies are composed of short-put options. Since the companies receive premiums for shorting the option, they will be incentivized to keep on hedging the futures position, hence saving money in the process, discouraging potential attempts to decrease their emissions. The determination coefficients of each strategy are relatively small, 0.0872 and 0.0892 respectively, implying that the strategies are not able to represent a big portion of the model.

The American short-range forward generates a beta coefficient of 701.6 which is also statistically significant according to the t-test. Although the strategy is a combination of a long call position and a short put position, the call position is considerably OTM, making its premium low. In combination with the premium received from the American short put position, which pays a higher premium than its European counterpart, the company will always receive a premium or pay a very small positive premium for the strategy. Since it is relatively inexpensive to hedge using this strategy and it does not have the same virtual unlimited downside potential as the writing covered put, it stands as an attractive hedging strategy. The R-square of the American short-range forward is still relatively small relative to the one obtained in the protective call regression, nonetheless, it is still slightly higher than the European and American WCPs, implying that it is better able to explain the dependent variable than the WCPs. The Protective Call, however, is still more able to explain the GHG emissions than the remaining strategies.

Relating now to the strength of the models, all were able to pass the tests for heteroscedasticity and autocorrelation, hence making the beta coefficients generated reliable and unbiased.

5 Conclusions

Global warming is a serious threat to all life as we know it and drastic measures need to be taken in order to fight it. Governmental organizations worldwide have taken measures to fight against it while minimizing the impact on the economy. The European Union, for instance, have created a complex trading scheme – the European Union Trading Scheme - in order to let the market regulate and price negative externalities such as greenhouse gas emissions.

Although externalities should be considered when pricing any type of product, this represents an additional expense and risk for any company inside the European Union. To minimize any additional costs, companies are able to hedge against these risks using a variety of financial products: futures contracts, forwards contracts and options contracts.

Using options to hedge against the price increase of the EU carbon allowances, companies have more flexibility due to the contract's nature and due to the possibility of combining more than one option in order to further reduce costs. The options strategies are done over the futures contracts and the strategies are the protective call, writing covered put, European and American, and short-range forward, European and American. To study each strategy and price it, the Monte Carlo approach, and Least Squares Monte Carlo for American options, were used due to their flexible nature and robust results. The analysis allowed for a better understanding of the strategies' behaviour and each best qualities and downfalls.

The protective call allows for mitigating the maximum potential loss created by an unexpected rise in the price of the underlying asset, while not restricting any positive gains in case of a price decrease. However, the protective call comes at a considerable cost, depending on how much ITM the option is at the time of purchase. The European and American writing covered puts to use a short put position to cover small price increases, using the premium received as a discount when purchasing the underlying directly from the market. Furthermore, if the strike price is slightly OTM, although less premium will be received, the company is still able to limit potential losses since it fixed the price at a value lower than the futures price. The European and American short-range forwards use the same fundamental ideas as the protective call and the WCPs, where the call limits the downside potential, but the short put limits the upside potential in exchange for a premium, hence resulting in very low or negative costs to the company.

Following the hedging strategies analysis and quantification, the assessment of their impact on greenhouse gas emissions was done via a series of linear regressions.

Firstly, the impact that EU carbon allowances have on GHG emissions was analysed. The EUA is effectively contributing to the decrease of emissions, having a negative significant impact.

Unexpectedly, the protective call has a negative relationship with greenhouse gases. This result can be explained by the strategy of making hedging against EUA price fluctuations relatively expensive. On the other hand, the other strategies had results more aligned with the original hypothesis. The writing covered puts and the short-range forward contributed positively to the GHG emissions. This can be explained by the strategies generating negative costs and/or very low positive costs, hence encouraging the setup of the hedging strategies and, consequently, neutralizing somewhat the desired effect of the European Union Carbon allowances.

With this study, the hypothesis suggested by Chevallier et al (2011), that overlapping on the regulatory instruments might impact the EUA prices negatively, hence affecting its efficiency in reducing GHG emissions, is therefore confirmed. To ensure that the carbon allowances keep effectively swaying the intended effect on the greenhouse gases, the administrating entities behind the EU Trading Scheme should not disdain and take action on regulating and restricting the usage of these instruments. With this study, the hypothesis suggested by Chevallier et al (2011) that overlap in the regulatory instruments might impact the EUA prices negatively, hence affecting its efficiency in reducing GHG emissions is therefore confirmed.

The empirical study was performed with relatively scarce data, having only 52 observations of Greenhouse gas emissions data in the European Union. For this reason, data on EU carbon allowances and their derivatives had to be restricted to match the quarterly frequency available for the GHG data. This scarcity is justified by the fact that statistics data sources, such as Eurostat, usually publish data with low frequency, such as yearly or quarterly. Once more data on GHG emissions become available, or data with higher frequency, new research could be done using more reliable results on the same matter. Only by studying reliable and plentiful data, the scientific community can find trustworthy solutions to fight Climate Change.

A good suggestion for future research on this matter, depending on data availability, would be to use real market historical futures and options prices rather than using theoretical risk-neutral prices.

References

Allen, M. P. (1997). The t test for the simple regression coefficient. *Understanding Regression Analysis.* 66–70. Springer US. https://doi.org/10.1007/978-0-585-25657-3_14

Back, K. (1993). Asymmetric Information and Options. The Review of Financial Studies, 6(3), 435–472. https://doi.org/10.1093/rfs/5.3.435

Bagchi, C. and Velten, E. (2014). The EU Emissions Trading System: Regulating the Environment in the EU. Climate Policy Info Hub. http://climatepolicyinfohub.eu/eu-emissions-trading-system-introduction

Baldursson, F. M., and Fehr, N.-H. M. von der. (2004). Price volatility and risk exposure: On market-based environmental policy instruments. Journal of Environmental Economics and Management, 48(1), 682–704. https://doi.org/10.1016/j.jeem.2003.06.004

Barraquand, J. (1995). Numerical Valuation of High Dimensional Multivariate European Securities. Management Science, 41(12), 1882–1891. https://doi.org/10.1287/mnsc.41.12.1882

Bhanumati, P. et al (2022). Greenhouse Emissions Rise to Record, Erasing Drop During Pandemic. https://www.imf.org/en/Blogs/Articles/2022/06/30/greenhouse-emissions-rise-to-record-erasing-drop-during-pandemic#:~:text=Emissions%20of%20carbon%20dioxide%20and,mobility%20and%20hampered%20economic%20activity.

Black, F., and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, *81*(3), 637–654. JSTOR. https://doi.org/10.1086/260062

Böhringer, C., Koschel, H., and Moslener, U. (2008). Efficiency losses from overlapping regulation of EU carbon emissions. Journal of Regulatory Economics, 33(3), 299–317. https://doi.org/10.1007/s11149-007-9054-8

Boyle, P. P. (1977). Options: A Monte Carlo approach. *Journal of Financial Economics*, *4*(3), 323–338. https://doi.org/10.1016/0304-405X(77)90005-8

Brandimarte, P. (2006). *Numerical methods in finance and economics: A MATLAB-based introduction* (2nd ed.). John Wiley & Sons.

Branthôme, X. (2022). EU carbon permit prices crash after Russian invasion of Ukraine. https://www.tomatonews.com/en/eu-carbon-permit-prices-crash-after-russian-invasion-of-ukraine_2_1599.html

Breusch, T. S., & Pagan, A. R. (1979). A Simple Test for Heteroscedasticity and Random Coefficient Variation. *Econometrica*, *47*(5), 1287–1294. https://doi.org/10.2307/1911963

Brink, C., Vollebergh, H. R. J., and Werf, E. van der. (2016). Carbon pricing in the EU: Evaluation of different EU ETS reform options. Energy Policy, 97, 603–617. https://doi.org/10.1016/j.enpol.2016.07.023

Capoor, K., and Ambrosi, P. (2009). *State and trends of the carbon market 2009*. http://hdl.handle.net/10986/13403

Chevallier, J., Pen, Y. L., and Sévi, B. (2011). Options introduction and volatility in the EU ETS. *Resource and Energy Economics*, *33*(4), 855–880. https://doi.org/10.1016/j.reseneeco.2011.07.002

Convery, F. J., Perthuis, C., and Ellerman, A. D. (2009). *Pricing carbon. The European Union Emissions Trading Scheme*. Cambridge: Cambridge University Press.

Cox, J. C., and Ross, S. A. (1976). The valuation of options for alternative stochastic processes. *Journal of Financial Economics*, *3*(1), 145–166. https://doi.org/10.1016/0304-405X(76)90023-4

Cox, J. C., and Rubinstein, M. (1985). *Option Markets.* (7th ed.). Englewood Cliffs

Dales, J. H., (1968), Pollution, property and prices. University of Toronto Press. *Canadian Journal of Political Science*, *3*(2), 386-387. https://doi.org/10.1017/S0008423900025282

Ding, J., and Meade, N. (2010). Forecasting accuracy of stochastic volatility, GARCH and EWMA models under different volatility scenarios. *Applied Financial Economics*, *20*(10), 771–783. https://doi.org/10.1080/09603101003636188

Durbin, J., & Watson, G. S. (1950). Testing for Serial Correlation in Least Squares Regression: I. *Biometrika*, *37*(3/4), 409–428. https://doi.org/10.2307/2332391

Egan, M. (2018). The Great Oil Crash of 2018: What's really happening. https://edition.cnn.com/2018/11/21/investing/oil-prices-trump-saudi-arabia/index.html

ETF.com, n.d. IXC iShares Global Energy ETF. https://www.etf.com/IXC

European Central Bank (n.d.). Statistical Data Warehouse. https://sdw.ecb.europa.eu/browseSelection.do?node=9689726

European Commission. (n.d.). Development of EU ETS (2005-2020). https://climate.ec.europa.eu/eu-action/eu-emissions-trading-system-eu-ets/development-eu-ets-2005-2020_en

European Commission. (n.d.). Green Taxation – in support of a more sustainable future. https://taxation-customs.ec.europa.eu/green-taxation-0_en

European Energy Exchange. (n.d.). EU ETS Spot, Futures & Options. https://www.eex.com/en/markets/environmental-markets/eu-ets-spot-futures-options

European Environmental Agency. (2019). Small increase in EU's total greenhouse gas emissions in 2017, with transport emissions up for the fourth consecutive year. https://www.eea.europa.eu/highlights/small-increase-in-eus-total-ghg

Eurostat Statistics Explained. (n.d.). Quarterly greenhouse gas emissions in the EU. https://ec.europa.eu/eurostat/statistics-explained/index.php?title=Quarterly_greenhouse_gas_emissions_in_the_EU

Federal Reserve Bank of ST. Louis (FRED) (n.d.). The Yield Curve. https://research.stlouisfed.org/publications/yield-curve/what-is-the-yield-curve#:~:text=The%20rate%20for%20a%20longer,for%20lending%20for%20longer%20periods.

Fleming, J., and Ostdiek, B. (1999). The impact of energy derivatives on the crude oil market. Energy Economics, 21(2), 135–167. https://ideas.repec.org/a/eee/eneeco/v21y1999i2p135-167.html

Gilbertson, T., Reyes, O., and Lohmann, L. (2009). *Carbon trading: How it works and why it fails* (Vol. 7). Dag Hammarskjöld Foundation Uppsala.

Good, A. and Gull, Z. (2021). S&P 500 posted 2020 gains as COVID-19, oil price crash obliterated energy sector. https://www.spglobal.com/marketintelligence/en/news-insights/latest-news-headlines/s-p-500-posted-2020-gains-as-covid-19-oil-price-crash-obliterated-energy-sector-61975042

Green, R. (2008). Carbon Tax or Carbon Permits: The Impact on Generators' Risks. *The Energy Journal*, *29*(3), 67–89. http://dx.doi.org/10.2307/41323170

Grubb, M. (2012). Strengthening the EU ETS: creating a stable platform for energy sector investment. https://www.researchgate.net/publication/273698191_Strengthening_the_EU_ETS_-_Creating_a_stable_platform_for_EU_energy_sector_investment

Haigh, M. S. and Holt, M. T. (2000). Hedging Multiple Price Uncertainty in International Grain Trade. American Journal of Agricultural Economics, 82(4), 881–896. https://doi.org/10.1111/0002-9092.00088

Homburg, M., U, and Wagner, M. (2009). Futures Price Dynamics of CO2 Emission Allowances: An Empirical Analysis of the Trial Period. *The Journal of Derivatives*, *17*(2), 73–88. https://doi.org/10.3905/JOD.2009.17.2.073

Hull. J. (2015). Options, Futures, and other Derivatives (9^{th} ed.). Pearson.

International Energy Agency (IEA). (2021). Pathway to critical and formidable goal of net-zero emissions by 2050 is narrow but brings huge benefits, according to IEA special report. https://www.iea.org/news/pathway-to-critical-and-formidable-goal-of-net-zero-emissions-by-2050-is-narrow-but-brings-huge-benefits

Irwin, S. H., and Sanders, D. R. (2012). Financialization and Structural Change in Commodity Futures Markets. *Journal of Agricultural and Applied Economics*, *44*(3), 371–396. Cambridge Core. https://doi.org/10.1017/S1074070800000481

JustETF. (n.d.). iShares Global Clean Energy UCITS ETF USD (Dist). https://www.justetf.com/en/etf-profile.html?query=FOSSIL++FUEL&assetClass=class-equity&groupField=index&from=search&isin=IE00B1XNHC34#overview

Kenton, W. (2023). Durbin Watson Test: What It Is in Statistics, With Examples. https://www.investopedia.com/terms/d/durbin-watson-statistic.asp#:~:text=The%20Durbin%2DWatson%20statistic%20will,to%204%20means%20negative%20autocorrelation.

Longstaff, F. A., and Schwartz, E. S. (2001). Valuing American Options by Simulation: A Simple Least-Squares Approach. *The Review of Financial Studies*, *14*(1), 113–147. https://doi.org/10.1093/rfs/14.1.113

Mayhew, S. (2000). The impact of derivatives on cash markets: What have we learned? Working paper.

Merton, R. C. (1973). Theory of Rational Option Pricing. *The Bell Journal of Economics and Management Science*, *4*(1), 141–183. JSTOR. https://doi.org/10.2307/3003143

Moreno, M., and Navas, J. F. (2003). On the Robustness of Least-Squares Monte Carlo (LSM) for Pricing American Derivatives. Review of Derivatives Research, 6(2), 107–128. https://doi.org/10.1023/A:1027340210935

Nakagawa, S., Johnson, P. C., and Schielzeth, H. (2017). The coefficient of determination R 2 and intra-class correlation coefficient from generalized linear mixed-effects models revisited and expanded. *Journal of the Royal Society Interface*, *14*(134). https://doi.org/10.1098/rsif.2017.0213

Roberts, M. J., and Spence, M. (1976). Effluent charges and licenses under uncertainty. Journal of Public Economics, 5(3), 193–208. https://doi.org/10.1016/0047-2727(76)90014-1

Rosopa, P. J., Schaffer, M. M., and Schroeder, A. N. (2013). Managing heteroscedasticity in general linear models. *Psychological Methods*, *18*(3), 335. https://doi.org/10.1037/a0032553

Saeed, T., Bouri, E., and Tran, D. K. (2020). Hedging Strategies of Green Assets against Dirty Energy Assets. Energies,13(12), 3141. https://doi.org/10.3390/en13123141

Sakong, Y., Hayes, D. J., and Hallam, A. (1993). Hedging Production Risk With Options. American Journal of Agricultural Economics, 75(2), 408–415. https://doi.org/10.2307/1242925

Sendeco2, (n.d.). Precios CO2. https://www.sendeco2.com/es/precios-co2

Smith, E. (2022). MARKETS Oil plunge, tech collapse and Fed cuts? Strategist shares possible 2023 market 'surprises'. https://www.cnbc.com/2022/12/07/oil-plunge-tech-collapse-and-fed-cuts-strategist-shares-possible-2023-market-surprises.html

Sparks, C. (2022). ECB Hikes Interest Rates By 50bp To 2.5%. https://businessplus.ie/economy/ecb-hikes-interest-rate-by-50bp-to-2-5/

Stentoft, L. (2004a). Assessing the Least Squares Monte-Carlo Approach to American Option Valuation. Review of Derivatives Research, 7(2), 129–168. https://doi.org/10.1023/B:REDR.0000031176.24759.e6

Stentoft, L. (2004b). Convergence of the Least Squares Monte Carlo Approach to American Option Valuation. Management Science, 50(9), 1193–1203. https://doi.org/10.1287/mnsc.1030.0155

Stewart, H. (2011). European Central Bank raises interest rates to 1.25%. https://www.theguardian.com/business/2011/apr/07/interest-rates-held-again-at-record-low

Stocker et al. (2018). What triggered the oil price plunge of 2014-2016 and why it failed to deliver an economic impetus in eight charts. https://blogs.worldbank.org/developmenttalk/what-triggered-oil-price-plunge-2014-2016-and-why-it-failed-deliver-economic-impetus-eight-charts

Taušer, J., and Čajka, R. (2014). Hedging techniques in commodity risk management. *Agricultural Economics*, *60*(4), 174–182. https://doi.org/10.17221/120/2013-AGRICECON

Tsafos, N. (2022). Europe needs a strategy for seasonal energy balancing. https://www.euractiv.com/section/energy-environment/opinion/europe-needs-a-strategy-for-seasonal-energy-balancing/

United Nations Climate Change. (n.d.). The Paris Agreement. https://unfccc.int/process-and-meetings/the-paris-agreement/the-paris-agreement

United Nations. (n.d.). The Paris Agreement. https://www.un.org/en/climatechange/paris-agreement

Wang, S., and Cui, H. (2013). Generalized F test for high dimensional linear regression coefficients. Journal of Multivariate Analysis, 117, 134–149. https://doi.org/10.1016/j.jmva.2013.02.010

Weaver, R. D., and Banerjee, A. (1990). Does futures trading destabilize cash prices? Evidence for US live beef cattle. *The Journal of Futures Markets (1986-1998)*, *10*(1), 41. https://www.proquest.com/openview/cc3edd1df9a180b45576839c9ab83268/1?pq-origsite=gscholar&cbl=48242

White, H. (1980). *A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity*. *Econometrica*, *48*(4), 817–838. https://doi.org/10.2307/1912934

Yahoo Finance (n.d.). iShares Global Clean Energy ETF (ICLN). https://finance.yahoo.com/quote/ICLN/performance?p=ICLN

Yahoo Finance (n.d.). iShares Global Energy ETF (IXC). https://finance.yahoo.com/quote/IXC/history?period1=1262304000&period2=1672358400&interval=1d&filter=history&frequency=1d&includeAdjustedClose=true

Zumbach, G. (2006). *The RiskMetrics 2006 methodology*. 63. https://dx.doi.org/10.2139/ssrn.1420185

Annexes

Annex A – Complementary Tables

Table A.1

EWMA Lambda via Maximum Likelihood Method

EWMA MLE	
Lambda	0.9004
1-Lambda	0.0996
a+b	1.00
Sum of MLE	15550.96

Note*. The Maximum Likelihood Method used to estimate the best fit Lambda for the EUA price variance was calculated using Microsoft Excel's solver application.*

Table A.2

Regression Data

Dates	GHG	EUA	Futures Prices	European Protective call	European WCP	European SRF	American WCP	American SRF	iShares dirty energy	EWMA STD	6M Rates	1Y Rates
2010 Qtr1	1171.0000	12.9019	12.9853	0.6734	0.0058	0.0048	0.0124	-0.0102	35.0367	0.0208	0.0039	0.0064
2010 Qtr2	1025.0000	14.9542	15.0237	0.7746	0.0043	0.0040	0.0116	-0.0102	33.5443	0.0191	0.0032	0.0046
2010 Qtr3	994.0000	14.7079	14.7843	0.7710	0.0011	0.0026	0.0082	-0.0079	31.9900	0.0154	0.0045	0.0052
2010 Qtr4	1187.0000	14.7160	14.8248	0.7871	0.0001	0.0009	0.0071	-0.0071	36.6152	0.0116	0.0065	0.0074
2011 Qtr1	1165.0000	14.7002	14.8492	0.7970	0.0000	0.0004	0.0070	-0.0070	41.9558	0.0098	0.0078	0.0101
2011 Qtr2	1002.0000	15.8923	16.1088	0.8932	0.0001	0.0041	0.0072	-0.0070	42.3749	0.0132	0.0115	0.0135
2011 Qtr3	973.0000	12.0483	12.1468	0.6513	0.0106	0.0147	0.0185	-0.0094	38.0087	0.0267	0.0076	0.0081
2011 Qtr4	1127.0000	8.9436	8.9821	0.4624	0.0186	0.0068	0.0267	-0.0151	37.5767	0.0325	0.0030	0.0043
2012 Qtr1	1153.0000	7.6151	7.6284	0.3885	0.0303	0.0038	0.0379	-0.0190	40.2834	0.0411	0.0010	0.0017
2012 Qtr2	984.0000	6.8576	6.8641	0.3460	0.0145	0.0020	0.0228	-0.0157	36.6744	0.0319	0.0006	0.0009
2012 Qtr3	955.0000	7.5103	7.5071	0.3749	0.0071	0.0004	0.0141	-0.0120	38.2265	0.0240	-0.0001	-0.0004
2012 Qtr4	1099.0000	7.3209	7.3193	0.3662	0.0130	0.0003	0.0193	-0.0143	38.5692	0.0290	0.0000	-0.0002
2013 Qtr1	1119.0000	4.6389	4.6412	0.2422	0.0377	0.0011	0.0448	-0.0178	39.7635	0.0579	0.0004	0.0005
2013 Qtr2	961.0000	3.8509	3.8522	0.2118	0.0491	0.0024	0.0561	-0.0166	39.7841	0.0772	0.0002	0.0003
2013 Qtr3	939.0000	4.5915	4.5946	0.2304	0.0080	0.0009	0.0153	-0.0119	40.3933	0.0294	0.0003	0.0007
2013 Qtr4	1079.0000	4.7417	4.7452	0.2389	0.0104	0.0008	0.0178	-0.0131	42.3442	0.0314	0.0006	0.0007
2014 Qtr1	1045.0000	5.8343	5.8396	0.2955	0.0130	0.0014	0.0200	-0.0138	42.1403	0.0318	0.0011	0.0009
2014 Qtr2	933.0000	5.2970	5.2999	0.2691	0.0199	0.0019	0.0267	-0.0150	46.5459	0.0397	0.0008	0.0006
2014 Qtr3	918.0000	6.0532	6.0504	0.3022	0.0020	0.0001	0.0092	-0.0089	46.7452	0.0183	-0.0001	-0.0005
2014 Qtr4	1055.0000	6.6114	6.6065	0.3289	0.0019	0.0000	0.0091	-0.0089	39.7441	0.0175	-0.0004	-0.0007
2015 Qtr1	1074.0000	7.0008	6.9874	0.3429	0.0080	-0.0015	0.0149	-0.0132	36.2080	0.0241	-0.0020	-0.0019
2015 Qtr2	941.0000	7.3294	7.3106	0.3561	0.0013	-0.0006	0.0082	-0.0082	37.3637	0.0152	-0.0027	-0.0026
2015 Qtr3	942.0000	7.9617	7.9393	0.3867	0.0002	-0.0001	0.0072	-0.0072	31.4561	0.0110	-0.0027	-0.0028
2015 Qtr4	1064.0000	8.3875	8.3589	0.4041	0.0001	-0.0001	0.0071	-0.0071	30.7475	0.0098	-0.0035	-0.0034
2016 Qtr1	1065.0000	5.6208	5.5948	0.2679	0.0146	-0.0041	0.0211	-0.0169	27.3615	0.0312	-0.0046	-0.0046
2016 Qtr2	941.0000	5.7517	5.7205	0.2713	0.0143	-0.0051	0.0209	-0.0176	31.2483	0.0300	-0.0056	-0.0054
2016 Qtr3	924.0000	4.5406	4.5111	0.2125	0.0145	-0.0053	0.0210	-0.0170	32.1075	0.0332	-0.0065	-0.0065
2016 Qtr4	1095.0000	5.5078	5.4658	0.2555	0.0259	-0.0087	0.0330	-0.0242	33.6289	0.0397	-0.0079	-0.0077
2017 Qtr1	1098.0000	5.1534	5.1124	0.2394	0.0279	-0.0099	0.0332	-0.0232	33.7287	0.0417	-0.0079	-0.0080
2017 Qtr2	959.0000	4.8138	4.7781	0.2224	0.0122	-0.0057	0.0184	-0.0161	32.3860	0.0294	-0.0074	-0.0074
2017 Qtr3	936.0000	5.8963	5.8536	0.2725	0.0069	-0.0040	0.0139	-0.0133	31.8475	0.0224	-0.0073	-0.0073
2017 Qtr4	1074.0000	7.4657	7.4073	0.3424	0.0089	-0.0056	0.0164	-0.0157	34.3410	0.0221	-0.0079	-0.0079
2018 Qtr1	1086.0000	9.7652	9.6996	0.4539	0.0135	-0.0070	0.0200	-0.0185	35.0726	0.0238	-0.0067	-0.0067
2018 Qtr2	922.0000	14.4561	14.3581	0.6727	0.0239	-0.0121	0.0293	-0.0260	37.0113	0.0252	-0.0066	-0.0068
Dates	GHG	EUA	Futures Prices	European Protective call	European WCP	European SRF	American WCP	American SRF	iShares dirty energy	EWMA STD	6M Rates	1Y Rates
2018 Qtr3	930.0000	18.8072	18.6821	0.8775	0.0336	-0.0159	0.0398	-0.0346	36.9703	0.0259	-0.0065	-0.0067
2018 Qtr4	1055.0000	20.3373	20.1942	0.9504	0.0904	-0.0275	0.0999	-0.0684	33.7003	0.0392	-0.0072	-0.0071
2019 Qtr1	1035.0000	22.0932	21.9616	1.0410	0.0696	-0.0232	0.0748	-0.0546	32.7143	0.0333	-0.0059	-0.0060
2019 Qtr2	914.0000	25.4765	25.3181	1.1960	0.0378	-0.0166	0.0472	-0.0414	32.9032	0.0250	-0.0059	-0.0062
2019 Qtr3	889.0000	26.9011	26.6935	1.2423	0.0332	-0.0200	0.0392	-0.0365	30.6455	0.0224	-0.0073	-0.0077
2019 Qtr4	994.0000	24.8091	24.6428	1.1549	0.0287	-0.0164	0.0363	-0.0336	30.6591	0.0224	-0.0066	-0.0067
2020 Qtr1	957.0000	22.7417	22.5875	1.0588	0.0427	-0.0196	0.0497	-0.0426	24.9605	0.0267	-0.0066	-0.0068
2020 Qtr2	736.0000	21.1140	20.9837	0.9995	0.0907	-0.0230	0.0997	-0.0652	19.5152	0.0390	-0.0059	-0.0062
2020 Qtr3	815.0000	27.3498	27.1756	1.2831	0.0784	-0.0304	0.0799	-0.0608	18.8866	0.0313	-0.0061	-0.0064
2020 Qtr4	942.0000	27.5592	27.3589	1.2754	0.0545	-0.0265	0.0601	-0.0511	18.6878	0.0270	-0.0071	-0.0073
2021 Qtr1	941.0000	37.5613	37.3052	1.7500	0.0549	-0.0276	0.0633	-0.0561	23.5941	0.0244	-0.0064	-0.0068
2021 Qtr2	850.0000	50.1176	49.7733	2.3325	0.0484	-0.0250	0.0599	-0.0552	25.9719	0.0219	-0.0066	-0.0069
2021 Qtr3	836.0000	57.0011	56.5866	2.6422	0.0589	-0.0346	0.0672	-0.0624	25.0777	0.0218	-0.0069	-0.0073
2021 Qtr4	978.0000	68.7041	68.1746	3.1591	0.1839	-0.0906	0.1845	-0.1503	28.3506	0.0297	-0.0078	-0.0077
2022 Qtr1	965.0000	83.0270	82.4923	3.8800	0.3462	-0.1172	0.3565	-0.2411	33.2248	0.0374	-0.0070	-0.0065
2022 Qtr2	863.0000	83.5184	83.4415	4.0238	0.1749	-0.0395	0.1922	-0.1404	37.4408	0.0297	-0.0039	-0.0009
2022 Qtr3	853.0000	79.7128	80.1893	4.1304	0.0804	0.0338	0.0856	-0.0487	35.0675	0.0252	0.0031	0.0060
2022 Qtr4	938.0000	77.4257	78.9767	4.5705	0.0313	0.2015	0.0462	0.0372	39.1448	0.0255	0.0166	0.0198

Note*. The derivatives in this table were calculated via MATLAB based on the quarterly EUA prices, respective rates and standard deviation.*

Annex B – MATLAB Codes

Figure B.1

Bradimarte's Monte Carlo Simulation MATLAB code

AssetPathsGBM.m

/MATLAB Drive/Euro Monte carlo/AssetPathsGBM.m

```
%Create a random stock price paths function
function SPaths=AssetPathsGBM(SO,mu,sigma,T,NSteps,NRepl)

%creates a matrix of zeros with the rows being the number of
% simulations pretended and the columns being the time steps considered
SPaths =zeros(NRepl, 1+NSteps);

% gives the value of S0(initial stock price considered)
% to the first column of the matrix.The starting pointy for the Monte-Carlo simulation
SPaths(:,1)= SO;

dt =T/NSteps;

%divides the total time T for the number of time steps
%considered in the simulation thus generating a "delta t"
nudt=(mu-0.5*sigma^2)*dt;

sidt= sigma*sqrt(dt);

% This loop calculates the simulation of the price
for i=1:NRepl

    for j=1: NSteps

% This loop calculates the simulation of the price
  % using a Geometric Brownian Motion for every column. Once it finishes for all the columns in a
  % determined row it has another loop so it can do exactly the same thing for the next row
  SPaths(i,j+1)=SPaths(i,j)*exp(nudt+sidt*randn);


    end
end
```

Figure B.2

European-style Option MATLAB code

MONTECARLO_EURO_OPTION.m

/MATLAB Drive/Euro Monte carlo/MONTECARLO_EURO_OPTION.m

```
%European vanilla option calculated via monte carlo simulation
%inputs:
    %S0: spot price of the underlying asset at time 0
    %K: Strike price
    %mu: Expected rate of return, given by the risk-free rate
    %sigma: underlying asste's volatility
    %T: Time until maturity of the option
    %phi: Determines if the option is a call (-1) or a put (1)
    %NSteps: Number of time steps in the simulation
    %NRepl: Number of simulations
%Output:
    %Price: Price of the european vanilla option


function Price= MONTECARLO_EURO_OPTION(SO,mu,sigma,T,K,phi,NSteps,NRepl)
SPaths=AssetPathsGBM(SO,mu,sigma,T,NSteps,NRepl);

%Payoff at maturity T

ST = SPaths(:, end);
payoff = max(phi*K-phi*ST, 0);

%Discount the payoff to time 0
discount_factor=exp(-mu*T);
Price= discount_factor*mean(payoff);

end
```

Figure B.3

Brandimarte's LSM American-style option MATLAB code (Adapted)

ExampleLSM1.m

/MATLAB Drive/LSM/ExampleLSM1.m

```
function price =ExampleLSM1(S0,mu,sigma,T,K,phi,NSteps,NRepl)

%paths
Spaths=AssetPathsGBM(S0,mu,sigma,T,NSteps,NRepl);
Spaths(:,1)=[];

dt=T/NSteps;

discountvector= exp(-mu*dt*(1:NSteps)');
alpha = zeros(3,1);

%payoff if exercised at maturity
Cashflows = max(0,phi*K-phi*Spaths(:,NSteps));

ExerciseTime=NSteps*ones(NRepl,1);

for step=NSteps-1:-1:1
    if phi == 1
       InMoney=find(Spaths(:,step)<K);
    else
        InMoney=find(Spaths(:,step)>K);
    end

    XData=Spaths(InMoney,step);

    %independent Variables
    RegrMat=[ones(length(XData),1), XData,XData.^2] ;
    YData=Cashflows(InMoney).*discountvector(ExerciseTime(InMoney)-step);

    %Independent Variables
    RegrMat=[ones(length(XData),1), XData,XData.^2] ;

    %Dependent variable
    YData=Cashflows(InMoney).*discountvector(ExerciseTime(InMoney)-step);

    %Regression with no ANOVA analysis
    alpha=RegrMat\YData;

    %value if early exercise
    IntrinsicValue= phi*K-phi*XData;

    %value if continuation
    Continuationvalue= RegrMat*alpha;

    %How many times the option is exercised right away
    Index=find(IntrinsicValue>Continuationvalue);

    Exercisepaths=InMoney(Index);

    Cashflows(Exercisepaths) = IntrinsicValue(Index);
    ExerciseTime(Exercisepaths)=step;

end

price=max(phi*K - phi*S0, mean(Cashflows.*discountvector(ExerciseTime)) );
```

Table B.4

Futures prices MATLAB code

Futures.m

/MATLAB Drive/Regression/Futures.m

```
data = EUA;

Futures = zeros(size(data));
Riskfree=Rate1Y;

for i = 1:length(data)
    Futures(i) = data(i)*exp(Riskfree(i)*1);
end
```