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A model of privately funded public research

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Abstract
We present a simple model of research, in which it is possible to compare the cost of private versus public research activity. The principal aim is to find conditions under which private firms might decide to provide funding, partially or fully, for research carried out by public research organizations. Since the underlying research problem is given, differences in cost depend mainly on the contracts between the firm or the public organization on one side and the researchers employed in the project on the other, together with the incentives which are implied by these contracts. We find that under suitable assumptions it is advantageous for the private firm to outsource its research to a public organization. The free access for the general public to research results obtained by public research organizations may however be an obstacle to outsourcing, unless some public funding is forthcoming.

Keywords Innovation · Research · Public-private cooperation · Contracts

JEL Classification D81 · D82 · D83 · O32

1 Introduction

Collaborations and interactions between industry and universities/public organizations have been pointed out as a way of (a) fostering innovation (Tseng et al. 2020), (b) enhanding the potential to commercialize academic and intellectual property (Badia et al. 2020), (c) increase firms' sales and growth performance (Belderbos et al. 2004) and, lastly, of accelerating the development of risky innovations (Mansfield 1998). Bringing together the forces of private and public activities has a special appeal, particularly when a private firm or a public organisation engages in risky

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innovative projects that involve a considerable amount of research activity and where cost can overrun the potential gains.

Whilst there is considerable research exploring the motivations behind the collaboration between a private firm and a public organization (e.g., Freitas et al. 2013; Stenbacka and Tombak 2020; O'Dwyer et al. 2022), assessing the impact of the researchers' contracts on the likelihood of pursuing research activities involving private and public institutions remains limited. Our primary focus is on research activities where the private firm partially or fully supports research performed in public universities and research institutions. An example is the Structural Genomics Consortium (SGC) in the area of medicine (see e.g. Bountra et al. 2017).[1] An alternative example is the mining and minerals sector in South Africa (Mitra and Genc 2019).[2]

For our analysis, we develop a model of research where the researchers perform repeated trials, modelled as trials with a given success probability. The trials can be carried out by a single researcher or by several researchers working in a team.

The cost of each trial is a formal representation of the effort the individual researcher engages in whilst performing the trial, and if the researcher is hired by a firm, it may be avoided by skipping the trial. It is assumed that the action of carrying out the trial is observable only to the individual researcher, giving rise to an information problem in the case where the researcher is hired by a firm or a public organization.

To deal with the incentive problem arising from the asymmetry of information, we analyse different contractual schemes available to the private and the public institution. In the context of a private firm conducting research using hired employees, we consider first a wage-plus-bonus scheme, where wages are supplemented by a bonus paid to the researcher obtaining the first successful trial outcome; and secondly, a wage-only system where the researchers are subject to random monitoring, and where researchers observed not to be carrying out the trial as expected will incur the payment of a fine. Each of these alternatives may be preferred; by researchers, by the firm or by both, depending on parameter values which in their turn influence

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[1] The SGC is a partnership comprising, among others, six universities from different countries, such as Oxford University in the United Kingdom and University of Toronto in Canada. This partnership is a non-profit organization that conducts pre-clinical research with the aim of accelerating the discovery of new risky innovations, less likely to be conducted by the private sector only. The SGC is funded by governments and by private industry (currently 17 organizations, including 9 pharmaceutical firms). Its labs amount to approximately 200 researchers and other staff. The core foundation of the SGC is that a private firm can contract with the SGC for university researchers to conduct the research that is of interest to the private firm, with the condition that if certain pre-defined criteria are met then all outputs delivered by SGC are released into the public domain and none of the results (delivered by the SGC) can be patent protected. The SGC participates in the pre-clinical stages and exits thereafter. Further development of the medicine takes place in the private firm that alone brings it to market. The private firm pays an amount to the SGC that covers the cost of research. In return, it allows the private sector to have access to a (larger) group of public university researchers.

[2] Mitra and Genc (2019) discuss the partnership between one of the world's largest gold mining companies and a university in South Africa, involving the education, research and innovation of the mining industry as a whole, but also to support the company's own activities and arrangements. According to Mitra and Genc (2019), the company pays two million South African Rand per year, presumably covering the cost of research (and education).

the length of the trial and the effective success probability, taking into account that shirking may occur.

The public research organization may use the same wage-only contract as the private firm, or a salary contract with long-term employment, where the employed researchers receive a fixed payment for participating in the entire research project, but with deduction of a penalty of considerable size in the case of not performing as agreed; that is if caught in shirking at any trial round during the project. If the penalty is large enough, and the researchers are risk averse, then the equilibrium level of shirking may turn out to be much lower under the salary contract than it would be in the case of the wage-only contract. The consequence will be a higher effective success probability in each trial, and this, in its turn, may lower the cost of research conducted in the public organization.

Altogether, if only the wage-plus-bonus and wage-only contract schemes are available to the private firm, then we show that, under suitable assumptions, the firm prefers not to subcontract with the public organization, but to do the research itself. However, if the private firm can subcontract with a public organization under a contract where it covers the cost of the public organization and the public organization uses the salary scheme, then it may be advantageous for the private firm to outsource its research to a public organization. We also show that if outsourcing of research to a public organization is based on the provision that the results can be used by everyone, then the private firm may lose additional earning opportunities otherwise available and consequently, the public alternative may be unattractive even when it is cost-saving. In turns out that even in cases where public research is preferable from a societal point of view, it may be necessary to support its use by private firms by reductions in the cost reimbursement.

The present paper relates to different strands of literature. The interactions between a university or public organization and the private sector and in particular, the incentives and gains from those interactions have been studied by e.g. Bercovitz and Feldman (2007), Bishop et al. (2011), and Audretsch et al. (2012). A specific form of cooperation between private and public institutions, namely public-private partnerships,[3] has been treated at length, cf. e.g. Hart (2003), Bennett and Iossa (2006), and Iossa and Martimort (2015). Several authors study the advantages for firms having universities as research partners, such as access to highly qualified researchers (e.g. Belderbos et al. 2004; Veugelers and Cassiman 2005). Other papers such as Aghion et al. (2008) and Lacetera (2009) argue that researchers in academia prefer to control their own research strategy and therefore, they will demand a wage premium to give up their freedom.

Earlier work on the development of an uncertain research project includes a study by Scherer (1966). Here, it is shown that there is a trade-off between expected time to complete a project and project cost. An extension of the model to Cournot

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[3] This type of arrangement differs from the type of collaboration modelled in this paper. Whilst public-private partnerships represent a mechanism for governments to procure and implement infrastructure and/or services through the private sector, the focus in this paper is on the specific setting of private subcontracting with a public organization for the development of a risky innovation.

competition is given in Scherer (1967). We have chosen to formulate the research problem as one consisting of repeated trials. There is a considerable literature dealing with series of trials, usually featured as a bandit problem, and dynamic noisy learning. The study of strategic experimentation was introduced by Bolton and Harris (1999) (see also Keller et al. 2005; Keller and Rady 2010; Marlats and Ménager 2021). Bergemann and Hege (1998, 2005) and Hörner and Samuelson (2013) study the effect on experimentation when learning is subject to moral hazard, and Halac et al. (2016) show how asymmetric information affects the bonus to be paid to induce effort.[4]

The literature focused on free-riding in groups can be traced back to Olson (1965), Alchian and Demsetz (1972), and Holmström (1982). Bonatti and Hörner (2011) study dynamic moral hazard in a team developing a risky public good and how free-riding influences effort exerted by the agents. Bonatti and Hörner (2017) analyse the effect of receiving bad news during the experimentation. In this setup, they show that information and monitoring can increase optimism and lead to stronger experimentation incentives.

The present paper contributes to the above mentioned strands of literature. Our starting point is a framework similar to a bandit problem, where the risky action in our case is the aim of developing a risky innovation with a small probability of success. This resembles the setup discussed in Bonatti and Hörner (2011, 2017), but we have added a public organization with goals different from those of the private firm, though with the same technology. In particular, a researcher in a public organization is no better and no worse than a researcher in a private firm. We use contract theory to study how to provide incentives for researchers to provide effort, cf. Aghion et al. (2008), and make it advantageous for the private firm to interact with a public organization.

The paper is organized as follows. In Sect. 2, we introduce the basic model of research and then consider the case where a private firm or enterprise is in charge of the research process, employing a team of researchers. The firm maximizes expected profit subject to overall cost of setting up research and the payments to the researchers, and we compare two different types of contract, namely the wage-and-bonus and the wage-only contracts. In the following section, we consider the public research organization, which takes on a research task on behalf of and financed by an outside private firm. We again consider two types of contract, one of them being the wage-only contract whilst the other is a salary contract. In Sect. 4, we investigate conditions under which it may be advantageous for the private firm to let the public organization do the research. In Sect. 5, we consider the question of openness of research and show how this may influence the comparisons made in previous sections. Section 6 contains some concluding comments.

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[4] See also (Scherer 2011). A related strand of literature is concerned with search games, cf. e.g. Fershtman and Rubinstein (1997), Chatterjee and Evans (2004), Erat and Krishnan (2012) Konrad (2014), Matros and Smirnov (2016), and von Essen et al. (2020).

There are two appendices at the end of the paper. Appendix 1 contains proofs of the propositions stated in the text. In Appendix 2, we discuss how the results are modified if future gains and costs are discounted at a nonzero rate.

2 The private researching firm

In this section, we introduce the technology of research and its implementation in a private research enterprise. We assume that an innovation may be obtained as a result of repeated trials with a fixed, presumably small, probability p of success. The performance of a trial entails a cost c, which may be interpreted as a cost of the effort needed for carrying out the trial. Successful research, which here is a sequence of trials, of which the last one is a success, will result in the payment of a prize v. To allow for possible risk aversion, we assume that researchers assess monetary net gains in accordance with a von Neumann-Morgenstern utility function u, which is twice differentiable, nondecreasing and concave and satisfies $u(0) = 0$.

As a description of research activity, this simple model is of course not pretending to represent the whole picture. But even if it appears as primitive; as a story of the search for minerals in a larger piece of land split into small plots examined sequentially, it comes close to describing what is happening in small developing companies in the pharmaceutical or the chemical industry, cf. e.g. Kaitin (2010), Gay (2014), Zahariev (2014). Here the search for new products with preassigned properties goes through a long series of largely unsuccessful trials with varying combinations of ingredients. When, if at all, a success is achieved, the resulting insights are transferred to other, typically much larger, companies which take care of the trial and marketing phases. The price received by the innovator will depend on future market conditions and the level of completeness of the product delivered, and since this part of the process is left out of our considerations, we just assume that it amounts to v.

Since trials are repeated, possibly a large number of times, there is a time dimension involved, and it might be appropriate to use discounting of gains and costs which occur after several repetitions. In order to obtain as simple an exposition as possible we have chosen to refrain from discounting in the main part of the paper, commenting on the effects of assuming a discount rate different from 0. We do this in Appendix 2.

If a trial, say the kth, is successful, the series of trials is brought to an end. Otherwise, after k trials without success, a further $(k+1)$th trial may be carried out. We allow for the possibility that the researcher may reassess the success probability in view of the observed k failures, so that the success probability takes a new value $p^{(0,k)}$ (the notation indicates that there were 0 successes in k trials). A formal derivation of this revision of beliefs is given in Sect. 5, but at present we take it as given. The researcher will perform a new trial only if expected net gain is nonnegative, that is if

$$p^{(0,k)}u(v-(k+1)c) + (1-p^{(0,k)})u(-(k+1)c) \geq u(-kc). \tag{1}$$

Using the notation

$$\triangle u_x(h) = u(x+h) - u(x)$$

for the change in utility at x caused by a displacement h, we may rewrite (1) as

$$\frac{p^{(0,k)}}{1-p^{(0,k)}} \geq -\frac{\triangle u_{-kc}(-c)}{\triangle u_{-kc}(v-c)}. \tag{2}$$

Here the right-hand side of (2) represents the marginal rate of substitution between (net) gain and cost, and by concavity of u, it is non-decreasing in the length k of the trial history. If perceived success probability decreases as k becomes larger, the condition in (2) determines a maximal length of the trial.

In what follows, we shall assume that research is carried out by teams consisting of m researchers, assumed to be identical with respect to utilities, beliefs and cost. In this case, rounds of m trials are performed simultaneously, and the results are observed after each round. At each round of trials, the success probability of the team is

$$p_m = 1 - (1-p)^m,$$

with the success probability of each individual team member being p_m/m. If $m > 1$, we need to specify the structure of trial costs per round as

$$c = c_0(m) + mc_1,$$

where $c_0(m)$ is a setup cost while c_1 represents the cost of each individual researcher. It seems reasonable to assume that c_0 is a convex function of the team size m, so that larger teams get increasingly costly. We omit the explicit reference to m in the notation for c_0.

So far, we have described the technology of research, without an institutional and contractual framework. We consider the situation where this technology is operated by a private firm employing m of researchers to perform several rounds of trials. The private firm has the objective of maximizing expected profits.

Since the firm is the entity undertaking the research, the prize accrues to the firm and not the researchers, who must be endowed with suitable incentives to carry out the trials. We consider two alternative types of contract, namely (a) a wage payment plus a bonus, and (b) a wage payment only, but under monitoring by the firm.

(a) Contracts with wage payment and bonus (wage-plus-bonus). In this case, the researchers are offered a contract with wage w per trial performed. The setup cost c_0 is paid by the firm while the trial cost c_1 must be paid for by the researcher. In order to prevent free-riding by researchers receiving the wage but not carrying out the trial, thus saving the trial cost c_1, a share $\lambda \in [0,1]$ of the prize is for payment to the researchers in case of success.

The expected net return to the firm after n rounds of trials with m researchers is

$$\Pi_n^\lambda = \sum_{k=1}^{n}((1-\lambda)v - k(c_0+mw))(1-p_m)^{k-1}p_m - n(c_0+mw)(1-p_m)^n, \tag{3}$$

where summation is performed over all instances of $k-1$ rounds of failures followed by one round with a success, for $k = 1, \dots, n$, with the addition of the case where also the last, nth, round has only failures. Using Lemma 1 in Appendix 1, we rewrite this as

$$\Pi_n^\lambda = \left((1-\lambda)v - \frac{c_0 + mw}{p_m}\right)\bigl(1 - (1-p_m)^n\bigr). \tag{4}$$

As above we are interested in conditions for proceeding with the trials after k rounds of failures. For the firm, another round must yield non-zero expected profit,

$$p_m^{(0,k)}((1-\lambda)v - (c_0 + mw)) - \bigl(1 - p_m^{(0,k)}\bigr)(c_0 + mw) \geq 0,$$

where $p_m^{(0,k)}$ is the perceived probability of success after k rounds of failure, and this can be written as

$$\frac{p_m^{(0,k)}}{1 - p_m^{(0,k)}} \geq \frac{c_0 + mw}{(1-\lambda)v - (c_0 + mw)}. \tag{5}$$

For the researchers receiving a fixed wage per trial, the relevant decision does not pertain to termination of the series of trials, but to their active participation. The incentive provided by the share λ depends on the way in which it is administered: If all members of the team receive an equal part, the payment to the individual researcher is $\lambda v/m$ and the perceived probability of getting this payment in the trial round after k rounds of failures is $p_m^{(0,k)}$, then participation in another round of trials yields a utility payoff of

$$p_m^{(0,k)} u\Bigl(\frac{\lambda v}{m} - (k+1)(w - c_1)\Bigr) + \bigl(1 - p_m^{(0,k)}\bigr) u(-(k+1)(w - c_1)),$$

whereas non-participation, i.e. shirking, means that the researcher loses the chance of a share in the prize, but saves the trial cost, giving the utility payoff $u((k+1)w - kc_1)$. It follows that participation is chosen as long as

$$\frac{p_m^{(0,k)}}{1 - p_m^{(0,k)}} \geq -\frac{\triangle u_{(k+1)w-kc_1}(-c_1)}{\triangle u_{(k+1)w-kc_1}\left(\frac{\lambda}{m}v - c_1\right)}. \tag{6}$$

Expected utility of a researcher employed by the firm performing n rounds of trials is

$$U_n^\lambda = \sum_{k=1}^{n} (1-p_m)^{k-1} p_m u\Bigl(\frac{\lambda v}{m} + k(w - c_1)\Bigr) + (1-p_m)^n u\bigl(n(w - c_1)\bigr). \tag{7}$$

Instead of paying a bonus to all the researchers, the firm might pay only those who actually achieve a successful outcome. The payment will then be higher, but it will occur only with probability p. The average bonus, taking into account that success may be obtained by other researchers as well, is

$$p\sum_{i=0}^{m-1}\binom{m-1}{i}p^i(1-p)^{m-1-i}\frac{\lambda v}{i+1}=\frac{\lambda v}{m},$$

where the equality sign is a consequence of Lemma 2 in Appendix 1. If researchers are risk averse or even risk neutral, the more risky prospect of payment only to the successful researchers will not be preferred, as average payment is the same in the two alternatives.

The firm is assumed to select the proportion λ of the share offered to researchers and the team size m so as to maximize expected profits (4) under the constraints given by the two stopping rules (5) and (6). The latter express that after a long series of failures, the firm may consider a further round as being unprofitable, or the researcher may choose to save the cost rather than to perform the trial.

For a comparison with other contractual arrangements, it is useful to know which of the two constraints will be decisive for the length of the trial series. We consider what could be considered as the typical case where wages cover the cost of the researchers whereas the share of the final gain going to the firm is larger than that of the successful researcher.

Proposition 1 *Let λ and m be profit maximizing shares and team sizes for the wage-plus-bonus contract. Assume that*

$$\frac{(1-\lambda)}{c_0+mw}\geq\frac{\lambda}{mc_1}. \tag{8}$$

Then the participation constraint of the researcher is binding. Conversely, if the researchers are risk-neutral and their participation constraint is binding, then (8) is satisfied.

The proof of Proposition 1 (and of the propositions to follow) is in Appendix 1. Intuitively, both stopping criteria depend on a ratio between the cost of carrying out a further trial and its possible net gain. Under the assumptions of the proposition, this ratio will be smaller for the firm than for the researcher, so that if $p_m^{(0,k)}$ is decreasing in k, the inequality will be violated for the researcher at an earlier stage than for the firm.

(b) Wage-only contracts. If the researchers receive only a wage payment independent of whether the trial is a success or a failure, it will be necessary to base the incentives on the possibility of losing some or all of the income otherwise promised. We assume that the firm monitors its researchers and fines them if they are caught shirking. Since monitoring is costly, it will be performed by a random selection of researchers, with the crucial parameter being the probability of a researcher being monitored during a given trial, with $0\leq q\leq 1$. For the researcher, shirking from a trial entails a gain of c_1, but if selected for monitoring, a fine to the amount of δ will be subtracted from the wages of the shirking researcher. We assume that the researcher chooses a probability π of shirking, roughly corresponding to a decision about the fraction of trials in which shirking will take place. The monitoring cost to the firm is assumed to be linear in q with a coefficient $\xi>0$.

If the researchers are shirking with probability π at each trial, the effective success probability of a researcher becomes $p_\pi = p(1-\pi)$, and the success probability in a trial round with m researchers will be $p_{m,\pi} = 1-(1-p_\pi)^m$. Now the expected utility of a researcher takes the form

$$U^w(\pi, q, m, n) = \sum_{k=1}^{n}(1-p_{m,\pi})^{k-1}p_{m,\pi}F_k(\pi, q) + (1-p_{m,\pi})^n F_n(\pi, q), \tag{9}$$

with

$$F_k(\pi, q) = \sum_{k_1+k_2+k_3=k}\binom{k}{k_1, k_2, k_3}(q\pi)^{k_1}((1-q)\pi)^{k_2}(1-\pi)^{k_3}u(kw-k_1\delta-k_3c_1) \tag{10}$$

for $k = 1, \dots, n$. In the expression for $F_k(\pi, q)$, k_1, k_2 and k_3 denote the number of instances where the individual has shirked and been detected, shirked without being detected, and not shirked, respectively. Since $U^w(\pi, q, m, n)$ is non-decreasing in n, the researchers do not restrict the number of trials in which to participate, and the length of the series of trial rounds must be determined by the firm. For the firm's choices of q, m and n, the researcher will choose π so that $U^w(\pi, q, m, n)$ is maximal.

For the firm, the situation is different, since both the wage cost w and the monitoring cost ξ of an additional trial should be covered by the additional expected gain. Hence, after k rounds of trials with m researchers participating, an additional round will be performed only if

$$p_{m,\pi}^{(0,k)}\big(v-c_0-m(w+(\xi-\pi\delta))q\big) - \Big(1-p_{m,\pi}^{(0,k)}\Big)(c_0+m(w+(\xi-\pi\delta)q)) \geq 0, \tag{11}$$

where, as before, $p_{m,\pi}^{(0,k)}$ is the probability of success with m researchers shirking with probability π as assessed after k rounds of failure. This can be reformulated as

$$\frac{p_{m,\pi}^{(0,k)}}{1-p_{m,\pi}^{(0,k)}} \geq \frac{c_0+m(w+(\xi-\pi\delta)q)}{v-\big[c_0+m(w+(\xi-\pi\delta)q)\big]}, \tag{12}$$

Let n be the largest value of k for which (12) is satisfied. Then the expected profit of the firm is

$$\begin{aligned}\Pi^w(\pi, q, n, m) = {} & \sum_{k=1}^{n}[v-k(c_0+m(w+(\xi-\pi\delta)q))](1-p_{m,\pi})^{k-1}p_{m,\pi} \\ & - n(c_0+m(w+(\xi-\pi\delta)q))(1-p_{m,\pi})^n.\end{aligned} \tag{13}$$

Using Lemma 1 of Appendix 1, we rewrite this expression as

$$\Pi^w(\pi, q, n, m) = \left(v - \frac{c_0+m(w+(\xi-\pi\delta)q)}{p_{m,\pi}}\right)\big(1-(1-p_{m,\pi})^n\big). \tag{14}$$

The firm chooses the detection probability q, the length of trial n and the team size m so as to maximize $\Pi^w(\pi, q, n, m)$.

The following proposition shows that there is a Nash equilibrium where the researcher chooses a nonzero probability of shirking. The assumption that $\delta > c_1$ seems reasonable, since otherwise shirking, whether detected or not, would be as good as working. We make a slightly stronger assumption.

Proposition 2 *Assume that $\delta > c_1$ and $\delta > \xi$. Then there is a Nash equilibrium $(\hat{\pi}, \hat{q}, \hat{n}, \hat{m})$ with $\hat{\pi} = \xi/\delta$.*

The assumption of the proposition states that the fine is greater than the cost of monitoring the researcher, so that the system of punishing the non-performing researchers hurts the latter more than it hurts the firm. In the following, this condition is assumed to be satisfied, and whenever treating the wage-only contract, we assume that the particular Nash equilibrium of Proposition 2 is realized.

(c) Comparison of the two contracts. Whether the wage-only or the wage-plus-bonus contract is best for the researcher or for the firm will depend on the parameter values. The wage-plus-bonus contract offers an additional source of income, but this income is subject to risk, and we may expect that the attitudes toward risk matter. Even for a risk neutral researcher, the incentive for participation should remain active even after a series of failures, meaning that the potential gain should be quite large compared to the trial cost.

Proposition 3 *Assume that researchers are risk neutral, and that*

$$\frac{c_1}{\frac{c_0}{m} + w} < \frac{\lambda}{1-\lambda}. \tag{15}$$

If $(1-\lambda)p_m^{(0,n_w)} > p_{m,\xi/\delta}^{(0,n_w)}$, then the researcher prefers the wage-plus-bonus contract over the wage-only contract.

The inequality in (15) states that the trial cost of the researcher relative to that of the firm is smaller than the relative gain, so that the researcher would be at least as interested in an extra trial as the firm. The second condition pertains to the perceived success probability without shirking after the n_w trials that would have been performed under the wage-only contract, showing that this perceived probability remains quite high, so that the effective success probability with shirking must be low. Since the conditions of the proposition restrict the share λ both upwards and downwards, we may expect that the wage-plus-bonus contract will be chosen less often than the wage-only contract.

If the assumptions on parameter values stated in Proposition 3 do not hold, in particular if the share λ becomes too large, then the bonus contract becomes unattractive for the firm. We state this in the proposition below.

Proposition 4 *Let λ, m and n be profit maximizing share, team size and trial length for the wage-plus-bonus contract. Assume that*

$$\frac{\lambda p_m v}{mw} \geq 2 \ln 2 \frac{\delta}{\delta - \xi} - 1 \tag{16}$$

Then there is a wage-only contract which is better for the research firm than the wage-plus-bonus contract.

The assumption of equal wages under the two different incentive schemes may be questioned, since the wage is supplemented by a possible share of the prize in one contract while being the only form of payment in the other. If the result-dependent contract yields a higher expected utility to the researcher than the wage-only contract, one might expect that the researcher would accept a contract of the first type even when the wage part is lower. We shall however keep the assumption of fixed wages, presumably set by market conditions, with further discussion being outside the scope of our model.

In addition, we could have considered an added contractual scheme, involving both a bonus and monitoring. This scheme would be preferable from the researcher perspective, but it would imply, in expectation, greater costs to the private firm vis-à-vis the wage-only contract under monitoring, increasing the incentives for privately funded public research.

3 The public research organization

We now turn to an alternative institutional setup for carrying out research, which we shall refer to as the public research organization. It should be noted at the outset that this organization is not considered to be a government agency pursuing a specific policy, and it could, as well, be a private non-for-profit organization. The research organization may have been set up by a branch of the public sector, with a specific research task and endowed with an appropriate budget, but with the option of obtaining additional funding by performing research for other parties who would reimburse the cost.[5] This option may be used if, over time, the number of research tasks and the staff do not evolve in a proportionate way.

For a research organization of this type, the overall objective would originally have been to carry out specific preassigned research tasks at the lowest possible cost and according to commonly accepted standards of research work. When the preassigned tasks are supplemented by outside research projects, the management will have to consider its objectives with respect to its overall volume of activity, size of staff etc., maintaining the original objectives with respect to quality and cost. Apart from this, there might be restrictions of a practical and/or ethical character on the type of research

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[5] Examples of public research organisations are described e.g. in von Trapp et al. (2016), covering 18 OECD countries. The orientation of public research institutions towards outside funding is described in Coccia (2008).

questions in which the organization may be engaged. For our purposes, we assume that the management can choose how it performs the research once it has accepted the task, and that it can decide upon the size of its staff and the contracts under which research is carried out.

From the point of view of the private firm, the activity terminates if success is obtained, and the firm can turn its attention to other tasks. For a public research organization of the type considered here, its character of being public or semi-public means that its activity should comply with accepted norms of scientific method. In particular, the sequence of trials cannot be terminated in the case of success but it has, also, to fulfil an additional requirement:

Assuming that the published results of the research take the form of a confidence interval for the success probability p, then a preassigned accuracy of the estimation in the form of a given interval length will imply a minimal number ν^0 of trials, see e.g. Grieve and Beal (1991). If m researchers participate in each round, then

$$n^0 = \left\lceil \frac{\nu^0}{m} \right\rceil$$

rounds of trials will be required (here $\lceil a \rceil$ denotes the smallest integer $\geq a$). Performing all n^0 rounds of trials, one may encounter a number of successes between 0 and n^0, and the result of the research, in the form of an estimated interval for p, will be determined accordingly.

As was the case for the private firm, the public organization may select a payment scheme to provide incentives for researcher participation, at least to the extent that it agrees with the general rules for wages in the public sector. It cannot choose the wage-plus-bonus scheme in the version considered in the previous section, as it has no access to the prize gained by successful research. But it may use the wage-only contract, and it has also an alternative option, which is to award long-term contracts, paying salaries to the researchers. We discuss the two contracts in detail below.

(a^*) Wage-only contracts. If the researchers are employed under a wage contract corresponding to that of the private sector and choose shirking probability π, then ex ante expected utility of a researcher can be found for any given number n of trials in the same way as in (9)–(10), but without the possibility of early termination,

$$U_n^w(\pi) = \sum_{k_1+k_2+k_3=n} \binom{n}{k_1, k_2, k_3} (q\pi)^{k_1}((1-q)\pi)^{k_2}(1-\pi)^{k_3} u(nw - k_1\delta - k_3 c_1). \tag{17}$$

For a given announced value of n, the researcher will then choose the value of π in such a way that $U_n^w(\pi)$ is maximal.

If however the researchers choose to shirk with probability π, the number of trial rounds must be increased correspondingly to

$$n(\pi) = \left\lceil \frac{\nu^0}{m(1-\pi)} \right\rceil. \tag{18}$$

The expected cost to the organization of the series of $n(\pi)$ trial rounds is then

$$C_n^w(\pi) = n(\pi)(c_0 + m(w + (\xi - \pi\delta)q)), \tag{19}$$

since the organization must carry the cost of wages and monitoring at each of the mn trials, but is compensated by the expected value of the fines collected from researchers caught in shirking.

As before, when considering the wage-only contract between a private firm and its researchers, we look for a Nash equilibrium in the game where researchers choose shirking probability and the public organization chooses intensity of control q as well as the number of trials necessary to minimize cost while performing the necessary v^0 trials.

Proposition 5 *If* $\delta > \xi$ *and* $\delta > c_1$*, then there is a Nash equilibrium* (π^*, q^*, n^*, m^*) *with* $\pi^* = \xi/\delta, n^* = n(\pi^*)$.

With wage-only contracts, private and public research differ only in two respects; namely (i) the determination of the number of trials to be performed, and (ii) the possibility under private contracting of terminating the series of trials once success is obtained. Both point in the direction of a cost advantage for the private firm. For the public research activity to represent an alternative, also from the point of view of the private firm, we must consider alternative forms of providing incentives to the researchers. The team size will be determined so as to minimize setup costs for the given number of trials.

(b^*) Researchers with salary and long-term employment. So far, we have considered contracts where researchers are engaged on a short-term basis, carrying out a round of trials and receiving a wage for this, possibly supplemented by a bonus. We now consider the situation where researchers are employed by the organization on a long-term basis and paid by salaries to the amount of $s = nw$ for the whole project. By itself, paying the same amount as salaries instead of wages would not have a significant effect, the difference will emerge from the incentives derived from the contract.

We assume that the salary contract covers the series of trials as a whole, so that all details are regulated after the last trial. Researchers caught in shirking at some point during the research period will be fined to the amount D. Here D should be seen as a considerable loss to the employee, incurring, for example, termination of contract, giving rise to a loss of future income as a salaried researcher. Interpreted in the context of an academic institution, what we have called shirking could also be instances of unsatisfactory scientific achievement, and the consequence would be loss of future status as a member of the research staff.

If the researcher chooses to participate with probability π at each trial, then expected utility with n rounds of trials (which again does not depend on the success probability p) is

$$U^s(\pi, q, n, m) = (1 - q\pi)^n \sum_{k=0}^{n} \binom{n}{k} \pi^k (1 - \pi)^{n-k} u(s - (n - k)c_1) + (1 - (1 - q\pi)^n) \sum_{k=0}^{n} \binom{n}{k} \pi^k (1 - \pi)^{n-k} u(s - (n - k)c_1 - D), \tag{20}$$

where $(1 - q\pi)^n$ is the probability that the researcher is never caught in shirking, and $1 - (1 - q\pi)^n$ is the probability that shirking is detected at some time during the series of trials.

On the side of the organization, there is a lower bound $\underline{q} > 0$ for the monitoring activity needed to sustain the contract. The cost to the organization of carrying out n rounds of trials with m researchers is

$$C^s = m\left[s + n\left(\frac{c_0}{m} + \xi q\right) - (1 - (1 - q\pi)^n)D\right], \tag{21}$$

where again $(1 - (1 - q\pi)^n)$ is the probability that the researcher has been reported as shirking at least once during the n trial rounds.

As for the previous types of contract, we look for an equilibrium choice of contract between the public research organization choosing q and its researchers choosing π, and with the number of trial rounds determined by $n = n(\pi)$. As before, team size can be determined once the number of trials is known, and we omit it in the notation for the equilibrium.

Proposition 6 *Assume that*

$$\underline{q}(1 - \underline{q})^{n-1} D \geq c_1. \tag{22}$$

Then the array $\left(0, \underline{q}, n^0\right)$ *is a Nash equilibrium.*

The left-hand side of (22) is the expected fine to be paid if the researcher decides to remain passive in the next trial, and this expected cost exceeds the gain from shirking, which is c_1. On the side of the public organization, it is obvious that if the researchers have chosen $\pi = 0$, then only n^0 trials are needed, and the monitoring cost should be at the minimal level.

For later use, we state the following slightly stronger version of the proposition as a corollary:

Corollary *If* $c_1 \leq \underline{q}D$, *then* $\left(0, \underline{q}\right)$ *is a Nash equilibrium.*

If an equilibrium without shirking exists, the number of trial rounds will be exactly the statistically necessary number n^0. This may contribute to a possible advantage of publicly managed research, and we will exploit the result in the next section. Viewed in this light, it may seem too rigorous to impose a fine as soon as π exceeds 0, but we may interpret π as an unacceptable level of shirking, allowing for some background level of irregularity, not taken into account in the monitoring of the researchers.

(c^*) Comparison of the two contracts. As in the previous section, we compare the two different methods of contracting researchers for the public organization. We assume for this comparison that $s = nw$. This has the consequence that differences in expected utility for the researcher, or in cost to the organization, must originate in the nature of the contract.

For the researcher, the wage contract with shirking results in a longer series of trials and consequently to a larger sum of wage payments, which, however, would be reduced by the levying of fines and which furthermore is subject to risk. For the public organization, the longer series of trials will in the typical case give rise to larger costs. The following proposition states conditions on the parameters for which the salary contract will be preferred by both parties.

Proposition 7 *Let $(\hat{\pi}, \hat{q}, \hat{n}, \hat{m})$ be the equilibrium under the wage contract and $\left(0, \underline{q}, n^0\right)$ the equilibrium under the salary contract.*

(a) *If $\hat{q} \geq \frac{c_1}{\delta}$, a risk averse researcher will prefer the salary contract.*
(b) *If $\underline{q} \leq \frac{\xi}{\delta} w$, the salary contract is less costly than the wage contract for the public organization.*

A comparison of contract types within the public organization, with the given trial setup and a fixed length of trials, is less interesting, in our context, than a comparison with research carried out in private firms, where trials are discontinued after the first occurrence of a successful outcome. This will be the topic of the next section.

4 Private subcontracting with public organization

In the previous sections, we have considered the private and the public research organizations as alternatives, but private firms may choose to subcontract with a public organization, having the latter carry out the research against an agreed payment, which, in this section, is intended to cover the costs of the public organization. The private firm will be entitled to the prize v in case of a success.

Since the public organization will carry out a fixed number of trial rounds, whereas the private research firm will terminate research if a success is obtained, outsourcing of research will make sense for the private firm only in cases where the fixed number of trials is not too far from what would be the optimal number if carrying out the research by itself. In the case where wage-only contracts are used also in public research, and assuming that not only wages, but also the details of monitoring the researchers, are the same, intuition suggests that outsourcing to the public organization would be too costly: Indeed, if carrying out $n^0(\frac{\delta}{\delta-\xi})$ rounds of trials would entail an expected profit greater than $\Pi^w\left(\frac{\xi}{\delta}, \hat{q}, \hat{n}_w, \hat{m}_w\right)$ as stated in (14) above, then this would hold even if trials were discontinued after the first success, contradicting that $\left(\frac{\xi}{\delta}, \hat{q}, \hat{n}_w, \hat{m}_w\right)$ was profit maximizing in the private firm.

This inferiority of public research using wage-only contracts comes as no surprise given the additional constraints on public research, but for completeness we state it here as a proposition.

Proposition 8 *Suppose that both the private firm and the public research organization use wage-only contracts, and that wages and monitoring parameters are identical. Then public subcontracting is unprofitable for the private firm.* □

The situation may however change if the public organization employs its researchers on a long-term basis using salaries instead of per trial remuneration through wages, since this may reduce the extent of shirking and increase the effective probability of success. For the comparisons to follow, we assume that the salary is determined as $s = n^0 w$ so as to avoid that the advantages or disadvantages would be due to outside circumstances, rather than to the internal organization of research.

The following proposition considers a particular case where private research is so unprofitable as not to be initiated at all, whereas a contract with a public research organization may yield a positive expected profit.

Proposition 9 *Assume that*

$$p_{m,\pi^*} < \frac{c_0 + mw}{v}, \quad 1 - (1 - p_m)^{n^0} \geq \frac{n^0(c_0 + m(w + \xi \underline{q}))}{v}. \tag{23}$$

Then private research with wage-only contracts is unprofitable, whereas privately funded public research with the salary contract $(0, \underline{q}, n^0)$ *is profitable to the private firm.*

The cases where public research may be preferential to research in a private firm are not restricted to those where private research is *per se* unprofitable. If the amount paid as salaries equals what would be paid as wages, then an advantage of public over private research must be based on differences in the probability of success, which should be large enough to counterweigh the advantage of opting out after the first success. This is formulated in the following proposition.

Proposition 10 *Assume that the private firm operates under wage-only contracts at the equilibrium* $\left(\frac{\xi}{\delta}, \hat{q}, \hat{m}, \hat{n}_w\right)$, *and that*

$$1 - (1 - p_m)^{n^0} \geq 1 - (1 - p_{m,\xi/\delta})^{\hat{n}_w}, \quad \frac{1 - (1 - p_{m,\xi/\delta})^{\hat{n}_w}}{p_{m,\xi/\delta}} \geq n^0\left(1 + \frac{m\xi \underline{q}}{c_0 + mw}\right). \tag{24}$$

Then outsourcing of research to the public organization operating under salary contracts is advantageous for the private firm.

The assumptions formulated in (24) imply that the desired number of trials in private research should be rather large compared to the fixed number n^0 of trial rounds in public research, mainly due to the level of shirking in private research. The first inequality in (24) states that expected probability of a success in n_w trials with shirking is no greater than expected probability without shirking, even when the number of trials is smaller. In the second inequality, the left-hand side expresses the effective number of trial rounds in private research, taking into account that trials may stop after a success. Hence, the inequality states that even with this option taken into account, private research will need a larger number of trials due to the presence of shirking. The quantity in the bracket on the right-hand side will be close to 1 for small enough values of $\overline{q}$.

A relevant question to ask at this point is what prevents the private firm from performing the research itself with a contract scheme similar to that of the public organization? The answer should be found in the long-term character of the contractual relationships and the nature of the fine which may consist of lost career opportunities, largely outside the scope of our present model. Even if a private firm could mimic the behavior of a public organisation also in this aspect, it is not likely to do so, and therefore it seems reasonable to restrict the option of salaried contracts, as we have done.

5 The externality of public research

An aspect of public research which has not been mentioned so far is that the results of the research should be freely available. This means that research carried out in the public organization differs from research in a private firm, which can exclude outside access to the results. This openness of research in public settings gives rise to an externality which in principle should be taken into account when comparing private research with outsourcing to a public organization. If the private firm is willing to reimburse the cost of carrying out the research to the public organization, rather than doing it itself, it pays for this externality as well, and the cost saving achieved by the public research organization must be large enough to accommodate this payment.

In order to assess the value of this externality, we must extend the model from the specific research problem to the broader context of a family of interrelated research problems, where results obtained in one of them have impacts on the other problems. More specifically, we assume that the research problem considered belongs to a larger family Σ of problems, here to be called *sites,* each of which have a specific success probabilities p_σ, all awaiting to be researched. Carrying out k rounds of trials at a site σ and obtaining r successes will result in a new estimate $p_\sigma^{(r,k)}$ for the success probability of this site, and if sites are similar, the estimates for the other sites may change as well. Intuitively, these new estimates will represent an improvement for those in charge of the research at these sites, and its value should be taken into account when comparing research in a private or a public setting.

To obtain an assessment of the value of information, we adopt a Bayesian approach, assuming that initially the arrays of success probabilities over the sites $\boldsymbol{p} = (p_\sigma)_{\sigma\in\Sigma}$ themselves follow a probability distribution with density function f. We may think of this distribution as representing the common beliefs in society with regard to success probabilities over the sites. For each particular site σ, the beliefs regarding p_σ can then be found as the marginal distribution. We have assumed in the previous sections that decision makers use point expectations for their individual assessment of utility or profits, and this specific value of p_σ may be found as e.g. the expectation or the median. For the comparison results obtained, the specific way of selection is of no importance.

After n^0 trials at the site σ, the records of the trials can be used for updating the initial probability density f. If the success probability is p_σ, and r of the n^0 trials were successful while the remaining were failures, then the probability of this outcome will be $P(r \mid n^0, p_\sigma) = \begin{pmatrix} n^0 \\ r \end{pmatrix} p_\sigma^r (1-p_\sigma)^{n^0-r}$. Bayesian updating of the prior density f gives rise to a posterior density $f^{(r,n^0)}$ with

$$f^{(r,n^0)}(\boldsymbol{p}) = \frac{P(r \mid n^0, p_\sigma) f(\boldsymbol{p})}{P(r \mid n^0)}, \tag{25}$$

where $P(r \mid n^0) = \int P(r \mid n^0, p_\sigma) f(\boldsymbol{p})\, \mathrm{d}\boldsymbol{p}$ is the average probability of r successes in n^0 trials, and the posterior density $f_\sigma^{(r,n^0)}$ of the success probability at site σ can again be found as the marginal distribution of $f^{(r,n^0)}$ at the site σ. The success probabilities $p^{(0,k)}$ after k trials, all resulting in failure, which we used in the previous sections, can similarly be found from the relevant posterior distribution, again after selecting a specific value of $p = p_\sigma$ as a point estimate.

For each of the yet unexplored sites $\sigma' \in \Sigma$, let $\Pi^w_{\sigma'}(p_{\sigma'})$ be the equilibrium profit under the wage-only contract given the success probability $p_{\sigma'}$, as found as in Sect. 2. Then expected profit at the site σ' given that the trials at σ had r successes is $\int \Pi^w_{\sigma'}(p_{\sigma'}) f^{(r,n^0)}(\boldsymbol{p})\, \mathrm{d}\boldsymbol{p}$, and the ex ante expected profit (before the trial at σ has been undertaken) is found by averaging over trial outcomes, i.e. as

$$V^1_{\sigma'} = \sum_{r=0}^{n^0} \left[\int \Pi^w_{\sigma'}(p_{\sigma'}) f^{(r,n^0)}(\boldsymbol{p})\, \mathrm{d}\boldsymbol{p} \right] P(r \mid n^0) = \sum_{r=0}^{n^0} \int \Pi^w_{\sigma'}(p_{\sigma'}) P(r \mid n^0, p_\sigma) f(\boldsymbol{p})\, \mathrm{d}\boldsymbol{p}, \tag{26}$$

where we have inserted from (25). Subtracting from $V^1_{\sigma'}$ the expected profit at the site without any trial, which is

$$V^0_{\sigma'} = \int \Pi^w_{\sigma'}(p_{\sigma'}) f(\boldsymbol{p})\, \mathrm{d}\boldsymbol{p}, \tag{27}$$

we obtain the information gain $I_{\sigma'} = V^1_{\sigma'} - V^0_{\sigma'}$. Total information gain I from carrying out the trials at σ can now be assessed as the sum of the gains $I_{\sigma'}$ over all the unexplored sites,

$$I = \sum_{\sigma' \in \Sigma, \sigma' \neq \sigma} I_{\sigma'}. \tag{28}$$

The information gain represents a service provided to society as a whole, and it may be argued that as such it should not be financed by a private firm outsourcing the research to a public organization. From the societal point of view, the information gain in (28) should be subtracted from the amount to be reimbursed by the firm, as is stated in the proposition below.

Proposition 11 *Suppose that research results from the public organization are freely accessible. Under the assumptions of Proposition* 10, *outsourcing of private research to the public research organization will be advantageous for society if*

$$\Pi^w \leq \left[1 - (1-p)^{n^0}\right]v - C^s_{n^0} - I \tag{29}$$

where $I = \sum_{\sigma' \in \Sigma, \sigma' \neq \sigma} \left[V^1_{\sigma'} - V^0_{\sigma'}\right]$ *is the total information gain.*

Clearly, transforming the theoretical insights about the externalities involved into a practical rule for cost reimbursement is close to impossible. Indeed, the monetary value of the information gain would be difficult to assess, since it depends on the initial beliefs about success probabilities over all the relevant sites as well as on the similarity of sites. The main message is therefore that the presence of an information gain poses an additional obstacle for outsourcing, which consequently can take place only if cost savings in public research are substantial, or if the information gain is small or non-existing.

6 Concluding remarks

In the previous sections, we have considered a simple model of research, where an innovation is to be discovered through repeated independent trials. Our purpose has been to investigate conditions on the organisation of the research which may be favorable to public subcontracting. That is, that private firms find it profitable to have the research carried out by a public organisation, rather than doing the research itself. To facilitate the comparison of private and public research, we have deliberately excluded several features which might otherwise be relevant for the problem, such as, for example, the professional or scientific engagement of the researchers and uncertainty with respect to the payoff to be obtained if the innovation is realized. These are important aspects of any process of research or innovation, but abstracting from their possible influence makes it easier to find what may be crucial factors for promoting private financing of public research.

In our framework, the salary contract is available only to the public organization. In practice, contracts of this type may be offered also by nonprofit or even by private research institutions, where the long-term relationship to the employed researchers is considered to be important. If a private firm could mimic the contractual scheme of the public organization, by offering a salary contract to its researchers, then there would be no incentives for the private firm to collaborate with the public organization given that there are no differences in terms of technology.

On the other hand, in our framework, the wage-plus-bonus scheme is only available to the private firm given that the public organization has no direct access to the prize in case of a success. Even though researchers in a public organization may receive some benefits if success occurs (e.g., additional funds to be used for research purposes, reduction in teaching load), it is less common to receive a monetary reward added to their wages.[6] Indeed, survey data from 2013 show that incentive schemes are more prevalent in the private sector than in the public sector (Eurofound 2016). For example, in Norway, roughly 32% of private sector employees were covered by performance reward payment systems, including bonuses, compared to 7% for the public sector employees (Eurofound 2016).

It goes without saying that our specific model of research is too simple to claim any generality of its results, and changing the technology of research, allowing for interdependence of trials both over time and between researchers will improve its realism. The payoff of innovation, which in the model has been given the simplest conceivable form as a fixed sum of money to be paid in the case of a success, would have to be elaborated upon, taking into account the uncertainty connected with this payoff. The choice of incentive schemes considered may be expanded, involving also features from real-world research contracts, such as, for example, milestones and contingent payments for research achievements. What has been shown so far is only a first step, pointing towards a more general theoretical foundation for private funding of public research projects, identifying the cases where private funding is not only a substitute for shrinking government financing, but yields benefits to both parties as well as to society as a whole.

Appendix 1: Proofs of propositions

This section contains the formal proofs of the propositions stated in the text. We begin with a lemma which is used several times.

Lemma 1 *Let* $A, B, p > 0$. *Then for every* $n \in \mathbb{N}$,

$$\sum_{k=1}^{n}(A - kB)(1-p)^{k-1}p - nB(1-p)^n = \left(A - \frac{B}{p}\right)(1 - (1-p)^n). \qquad (30)$$

Proof Write the left-hand side in (30) as

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[6] It should be noted that universities and academics may file patents and obtain future revenues. However, it is still uncommon for academics to be involved in the commercialisation of their research (e.g., see Lissoni et al. 2008).

$$\sum_{k=1}^{n} A(1-p)^{k-1}p - \sum_{k=1}^{n} kB(1-p)^{k-1}p - B(1-p)^n n. \tag{31}$$

The first term in (31) is equal to $A(1-(1-p)^n)$, and the second term can be reduced to

$$\sum_{k=1}^{n} kB(1-p)^{k-1}p = Bp\sum_{k=1}^{n} k(1-p)^{k-1} = Bp\sum_{k=1}^{n} \frac{\mathrm{d}}{\mathrm{d}p}\left(-(1-p)^k\right)$$
$$= Bp\frac{\mathrm{d}}{\mathrm{d}p}\left(-\sum_{k=1}^{n}(1-p)^k\right) = Bp\frac{\mathrm{d}}{\mathrm{d}p}\left(-\frac{1-(1-p)^{n+1}}{p}+1\right) = B\frac{1-(np+1)(1-p)^n}{p}.$$

Inserting in (31) and collecting terms, we get the right-hand side of (30). ☐

Lemma 2 $p\sum_{i=0}^{m-1}\binom{m-1}{i}p^i(1-p)^{m-1-i}\frac{1}{i+1} = \frac{1}{m}$.

Proof Define the function $F : \mathbb{R} \to \mathbb{R}$ by $F(x) = p\sum_{i=0}^{m-1}\binom{m-1}{i}p^i(1-p)^{m-1-i}\frac{x^{i+1}}{i+1}$. Then $F(0) = 0$ and $F'(x) = p\sum_{i=0}^{m-1}\binom{m-1}{i}p^i(1-p)^{m-1-i}x^i = p(px+1-p)^{m-1}$, and

$$F(x) = F(0) + \int_0^x p(ps+1-p)^{m-1}\,\mathrm{d}s = \frac{1}{m}(px+1-p)^m.$$

Inserting $x = 1$ we get the result. ☐

Proof of Proposition 1: Assume first that the researcher is risk neutral with $u(x) = x$ for all x. Then we can rewrite (6) as

$$\frac{p_m^{(0,k)}}{1-p_m^{(0,k)}} \geq \frac{c_1}{\frac{\lambda}{m}v - c_1} = \frac{1}{\frac{\lambda v}{mc_1} - 1}. \tag{32}$$

Using the inequality (8) we get that

$$\frac{p_m^{(0,k)}}{1-p_m^{(0,k)}} \geq \frac{1}{\frac{(1-\lambda)v}{c_0+mw} - 1}$$

so that (5) is satisfied, meaning that the binding condition must be (6). Conversely, if (6) is binding, then (5) must be satisfied for any k such that (32) holds, meaning that (6) must be satisfied.

In the general case of a risk averse researcher we notice that

$$-\frac{\triangle u_{(k+1)w-kc_1}(-c_1)}{\triangle u_{(k+1)w-kc_1}\left(\frac{\lambda}{m}v-c_1\right)} \geq \frac{c_1}{\frac{\lambda}{m}v-c_1}$$

since the displacement in the denominator is larger than that of the numerator, so if (6) is satisfied, so is also (5). □

Proof of Proposition 2: First of all we notice that if the researcher chooses $\pi = 0$, then the best reply of the firm is $q = 0$. However, the best reply of the researcher to $q = 0$ is $\pi = 1$, so $\pi = 0$ cannot occur in a Nash equilibrium. Similarly, if $\pi = 1$ and $\delta > c_1$, then $q = 1$ is the best reply for the firm, in which case the researcher should choose $\pi = 0$, so also $\pi = 1$ cannot be a Nash equilibrium strategy.

Since the payoff of the firm is linear in q, the Nash equilibrium value π^0 must be such that $\xi - \pi\delta = 0$, that is $\pi^0 = \xi/\delta$. The array (π^0, q^0, m, n) is a Nash equilibrium if π^0 maximizes $U^w(\pi, q^0, m, n)$ at some $q = q^0$. We show that

$$\frac{\partial}{\partial\pi}U^w(\pi,0,m,n)\big|_{\pi=\pi^0} > 0,\ \frac{\partial}{\partial\pi}U^w(\pi,1,m,n)\big|_{\pi=\pi^0} < 0, \tag{33}$$

so that $U^w(\pi^0, q, m, n)$ must attain a maximum value at some q^0 with $0 < q^0 < 1$.

We begin by considering the quantities $F_k(\pi, q)$, $k = 1, \dots, n_w$. If $q = 0$, the expression for $F_k(\pi, 0)$ reduces to

$$F_k(\pi,0) = \sum_{k_2=0}^{k}\binom{k}{k_2}\pi^{k_2}(1-\pi)^{k-k_2}u(k(w-c_1)+k_2c_1).$$

Here the term $u(k(w - c_1) + (k - k_2)c_1)$ is increasing in k_2, and for the binomial distribution $B(n, \pi)$ we have that the cumulative probabilities $P(h;k,\pi) = \sum_{k_2=0}^{h}\binom{k}{k_2}\pi^{k_2}(1-\pi)^{k-k_2}$ $P(h;k,\pi) = \sum_{k_2=0}^{h}\binom{k}{k_2}\pi^{k_2}(1-\pi)^{k-k_2}$ for $h \leq k$ are decreasing in π. This means that the distribution $B(k, \pi)$ stochastically dominates $B(k, \pi')$ for $\pi > \pi'$, and as $F_k(\pi, 1)$ is the expected utility w.r.t. $B(k, \pi)$ we have that F_k is increasing in π. Moreover, we have that $F_{k'}(\pi, 0) \geq F_k(\pi, 0)$ for $k' > k$, since $B(k', \pi)$ stochastically dominates the distribution which is identical to $B(k', \pi)$ for $k_2 \leq k$ and 0 otherwise, and $u(k(w - c_1) + k_2c_1)$ is increasing in k.

For $q = 1$ we have that

$$F_k(\pi,1) = \sum_{k_1=0}^{k}\binom{k}{k_1}\pi^{k_1}(1-\pi)^{k-k_1}u(k(w-c_1)-k_1(\delta-c_1)).$$

where $u(k(w - c_1) - k_1(\delta - c_1))$ is decreasing in k_1 since $\delta > c_1$, and using again stochastic domination, we conclude that F_k is decreasing in π. Also, arguing as above, we have that $F_k(\pi, 1)$ is nonincreasing in k.

To assess the expected utility, we use that it is expressed in (9) as the expected value of the $F_k(\pi, q)$ with respect to the probability distribution over $\{1, \dots, n\}$ defined by

$$P_\pi(k) = (1 - p_{m,\pi})^{k-1} p_{m,\pi}, k = 1, \dots, n-1, P_\pi(n) = (1 - p_{m,\pi})^n.$$

For each k, $\sum_{h=1}^{k} P_\pi(h) = 1 - (1 - p_{m,\pi})^k$ is decreasing in π, so that $P_{\pi'}$ stochastically dominates P_π when $\pi < \pi'$. It now follows that $U^w\left(\frac{\xi}{\delta}, 0, n, m\right)$ is increasing in π whereas $U^w\left(\frac{\xi}{\delta}, 1, n, m\right)$ is decreasing in π. Since $U^w\left(\frac{\xi}{\delta}, q, m, n\right)$ is C^1 as a function of q, and its derivative is positve at $q = 0$ and negative at $q = 1$, it must attain a maximum at some q^0 in the interval. □

Proof of Proposition 3: Since (15) is satisfied and researchers are risk-neutral, we have by Proposition 1 that the number n_w of trials is determined by the condition (12) for the firm, which can be rewritten as

$$p_{m,\xi/\delta}^{(0,n_w)} v \geq c_0 + mw.$$

Since $(1 - \lambda) p_m^{(0,n_w)} > p_{m,\xi/\delta}^{(0,n_w)}$ we get that

$$p_m^{(0,n_w)} \geq \frac{c_0 + mw}{(1 - \lambda) v},$$

which can be rewritten as (5), and we conclude that the length of trials under wage-plus-bonus is at least n_w.

In each trial round under the wage-only contract, the risk-neutral researcher shirking with probability π gets the wage w plus the expected net gain from shirking, which is

$$(1 - \pi) c_1 - \pi q \delta.$$

If this expected gain differs from 0, then the optimal shirking probability would be either 1 or 0, contradicting the equilibrium with $\hat{\pi} = \xi/\delta$. We conclude that the researcher has net gain w per trial in the wage-only contract, whereas in the wage-plus bonus there is an additional average gain of $p_m \lambda v/m$, so that the latter is preferred. □

Proof of Proposition 4 First of all, we find the team size m' such that the success probability in a trial round is the same for the two contracts,

$$p_{m',\pi} = 1 - (1 - p(1 - \pi))^{m'} = 1 - (1 - p)^m = p_m,$$

which reduces to

$$m' = m \frac{\ln(1 - p)}{\ln(1 - p(1 - \pi))}. \tag{34}$$

The fraction on the right-hand side of (34) is increasing in p, and for $\pi = \frac{\xi}{\delta}$ and $p \leq \frac{1}{2}$ we get that

$$\frac{m'}{m} \leq 2\ln 2\frac{\delta}{\delta - \xi}.$$

To compare expected profits under the optimal wage-plus-bonus contract with a wage-only contract with equilibrium value of π we notice that by (16)

$$\lambda v \geq \left(2\ln 2\frac{\delta}{\delta - \xi} - 1\right)\frac{mw}{p_m} \geq \frac{(m' - m)w}{p_{m,\pi}},$$

so that

$$v - \frac{c_0 + m'w}{p_{m,\pi}} \geq (1 - \lambda)v - \frac{c_0 + mw}{p_m},$$

and it follows from (4) and (14) that $\Pi^w\left(\frac{\xi}{\delta}, \hat{q}, m', n^\lambda\right) > \Pi^\lambda_{n^\lambda}$. □

Proof of Proposition 5: As in the proof of Proposition 2, we notice that if $\pi = \xi/\delta$, then expected cost is independent of q, so that we need only to show that expected utility attains its maximum for some value of q with $0 < q < 1$, and this is done by showing that its derivative is positive at $q = 0$ and negative at $q = 1$. The details of the proof are exactly as in the proof of Proposition 2 and are omitted here. □

Proof of Proposition 6: Write the expression in (20) as

$$U^s(\pi, q, n, m) = G_1(\pi) + G_2(\pi).$$

Assume that the public organization has chosen the monitoring level $\underline{q}$ and number of trial rounds n^0. We consider first the case of risk neutral researchers with $u(x) = x$ for all x. If a researcher contemplates a change from $\pi = 0$ but to some $\pi \neq 0$, then the expected net gain will be

$$G(\pi) = n^0\pi c_1 - (1 - (1 - \underline{q}\pi))^{n^0}D,$$

where the first member is the expected saving due to shirking, and the second is the expected fine given that shirking is detected at least once. The derivative is

$$G'(\pi) = n^0c_1 - n^0\underline{q}(1 - \underline{q}\pi)^{n^0-1}D = n^0(c_1 - \underline{q}(1 - \underline{q}\pi)^{n^0-1}D),$$

and using (22) we have that

$$c_1 \leq \underline{q}(1 - \underline{q})^{n^0-1}D \leq q(1 - \underline{q}\pi)^{n^0-1}D,$$

so that $G'(\pi) \leq 0$ for $0 \leq \pi \leq 1$.

We conclude that the risk-neutral researcher will not prefer to change from the safe prospect of getting s with no shirking to a risky prospect of shirking with some $\pi > 0$. It follows then that a risk averse researcher *a fortiori* will prefer the safe prospect with $\pi = 0$. For the organization, the choice of $q = \underline{q}$ is clearly optimal given that $\pi = 0$. □

Proof of Corollary: Writing $\triangle u(x) = u(s - nc_1 + x) - u(s - nc_1)$ for $x \in \mathbb{R}$, the assumption in Proposition 6 can be reformulated as

$$\triangle u(c_1) \leq q_0\big(- \triangle u_{s-nc_1}(-D)\big). \tag{35}$$

We have by concavity of u that

$$\frac{\triangle u_{s-nc_1}(c_1)}{c_1} \leq -\frac{\triangle u_{s-nc_1}(-D)}{D},$$

so that

$$-\frac{\triangle u_{s-nc_1}(c_1)}{\triangle u_{s-nc_1}(-D)} \leq \frac{c_1}{D} \leq q_0,$$

and it follows that (35) is satisfied. □

Proof of Proposition 7: (a) If $\hat{q} \geq \frac{c_1}{\delta}$, then at each trial round the expected gain from shirking, $\frac{\xi}{\delta}c_1$, is no greater than the expected loss arising from being caught, which is $\frac{\xi}{\delta}\hat{q}\delta$, so that the risk averse researcher will prefer the salary contract.

(b) The salary contract will result in a cost saving if the cost of monitoring is smaller than the cost of the additional rounds of trials, that is if

$$n^0 m\underline{q} \leq n^0 \frac{\xi}{\delta}(c_0 + mw)$$

and this inequality follows since $\underline{q} \leq \frac{\xi}{\delta}w$. □

Proof of Proposition 9: The first inequality in (23) gives us that private research will not be undertaken, and from the second inequality we get that expected gain $\left[1 - (1 - p_m)^{n^0}\right]v$ exceeds the cost of carrying out n^0 trials in the public organization. □

Proof of Proposition 10: From the second inequality in (24) we get that

$$(c_0 + mw)\frac{1 - (1 - p_{m,\xi/\delta})^{\hat{n}_w}}{p_{m,\xi/\delta}} \geq (c_0 + mw)n^0\left(1 + \frac{m\xi\underline{q}}{c_0 + mw}\right) = n^0 c_0 + mn^0 w + n^0 m\xi\underline{q}, \tag{36}$$

and using the first inequality in (24) together with (36), we get

$$
\begin{aligned}
\Pi_{n_w} &= v\left[1-(1-p_{m,\xi/\delta})^{n_w}\right] - (c_0 + mw)\left[\frac{1-(1-p_{m,\xi/\delta})^{n_w}}{p_{m,\xi/\delta}}\right] \\
&\le v\left[1-(1-p_m)^{n^0}\right] - (ms + n^0(c_0 + m\xi\underline{q})).
\end{aligned}
$$

We conclude that outsourcing is at least as profitable as private in-house research.

$\square$

Appendix B: Discounting

It has been assumed throughout that trials are instantaneous, so that discounting of prize as well as cost items is unnecessary. This is of course a simplification, research is time-consuming, in some cases to an extent that results are expected only after several years. Assume now that a trial has the duration of one unit of time, and that future payments are discounted by a factor $\beta < 1$ per time unit.

If a private firm takes on research using m researchers on a wage-plus-bonus contract, then expected profit after n trial rounds takes the form

$$
\begin{aligned}
\Pi^\lambda(\pi, q, n, m) = \sum_{k=1}^{n}\left[\beta^k(1-\lambda)v - \left(\sum_{h=1}^{k}\beta^h\right)(c_0 + mw)\right](1-p_{m,\pi})^{k-1}p_{m,\pi} \\
- \left(\sum_{k=1}^{n}\beta^k\right)(c_0 + mw)(1-p_{m,\pi})^n
\end{aligned}
$$

which does not differ much from (3) but cannot be reduced to a simpler form as in (4). For the expected utility of the researcher, the expression corresponding to (7) is

$$
\begin{aligned}
U_n^\lambda = \sum_{k=1}^{n}(1-p_m)^{k-1}p_m\left[\sum_{h=1}^{k-1}\beta^h u(w-c_1) + \beta^k u\left(\frac{\lambda v}{m} + (w-c_1)\right)\right] \\
+ (1-p_m)^n\left(\sum_{h=1}^{n}\beta^h u(w-c_1)\right),
\end{aligned}
$$

somewhat more complicated since utility must be discounted as well.

In the case where the firm chooses the wage-only contract, expected profit over n rounds of trials is

$$
\begin{aligned}
\Pi^w(\pi, q, n, m) = \sum_{k=1}^{n}\left[\beta^k v - \left(\sum_{h=1}^{k}\beta^h\right)(c_0 + m(w + (\xi - \pi\delta)q))\right](1-p_{m,\pi})^{k-1}p_{m,\pi} \\
- \left(\sum_{k=1}^{n}\beta^k\right)(c_0 + m(w + (\xi - \pi\delta)q))(1-p_{m,\pi})^n.
\end{aligned}
$$

For the expected utility over this series of trials, (9) remains unchanged, but the quantities $F_k(\pi, q)$ take the form

$$F_k(\pi, q) = \sum_{h=1}^{k} \beta^h \left[\pi q u(w-\delta) + \pi(1-q)u(w) + (1-\pi)u(w-c_1)\right],$$

The reasoning behind Proposition 2 is not upset by the presence of discounting, so that the existence of a Nash equilibrium $(\hat{\pi}, \hat{q}, \hat{n}, \hat{m})$ with $\hat{\pi} = \xi/\delta$ carries over to the present situation, although the exact value of $\hat{q}$ and $\hat{n}$ may have changed.

As the bonus is received at the (succesful) end of the trial, its effects as an incentive diminish together with β, so that the conclusions in Sect. 3 are reinforced, in the sense that wage-only contracts will be chosen over wage-plus-bonus contracts for a rather large span of parameter values. For comparison of private and public research, we look at the salary contract with discounting. Here the cost over n rounds of trial with m researchers becomes

$$C^s = m\left[\beta^n s + \left(\sum_{h=1}^{k} \beta^k\right)\left(\frac{c_0}{m} + \xi q\right) - (1-(1-q\pi)^n)\beta^n D\right],$$

where it is assumed that salaries as well as fines are paid at the end of the series of trials. The expected utility of the researcher will be

$$\begin{aligned} U^s(\pi, q, n, m) = {} & \sum_{k=1}^{n-1} \beta^k \big(\pi u(w) + (1-\pi)u(w-c_1)\big) \\ & + \beta^n \Big[(1-q\pi)^{n-1})\big(q\pi u(w-D) + (1-q)\pi u(w) + (1-\pi)u(w-c_1)\big) \\ & + \big((1-(1-q\pi)^{n-1})(\pi(w-D) + (1-\pi)(w-c_1-D))\big)\Big]. \end{aligned}$$

Since the payment of the fine is more heavily discounted than outlays avoided during early rounds of trials, the advantage of the salary contract over the wage-only contract is diminished. As a consequence, it must be expected that private outsourcing of research to public organizations will be more advantageous for short-term than for long-term research projects.

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