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Measuring and managing the Value-at-Risk of a stocks and bonds portfolio

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Master in Finance

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PhD, António Manuel Rodrigues Guerra Barbosa, Assistant Professor, ISCTE-IUL

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BUSINESS SCHOOL

Department of Finance

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Resumo

O Economic Capital (EC) pode ser definido como capital em risco resultante de atividades de investimento e é medido pela métrica de risco mais geralmente utilizada: Value-at-Risk (VaR). Para um certo valor máximo pré-definido para o EC, é relativamente a este valor que estratégias de gestão de risco podem ser formuladas. Este estudo estima e gere o VaR de uma carteira composta por ações e obrigações dos mercados Americanos e Europeus de modo a que este não ultrapasse o máximo pré-definido. Dado a variedade de modelos VaR disponíveis, para concluir qual o modelo que oferece as estimativas VaR mais precisas para a carteira utilizada, são considerados 16 modelos diferentes e a sua performance é analisada através de backtest. Usando o modelo que demonstrou melhor desempenho, o VaR da carteira é medido diariamente e gerido através de uma estratégia de cobertura aplicada à exposição em ações pelo período de um ano. A métrica de desempenho Return on Risk-Adjusted Capital (RORAC) é utilizada para analisar o resultado da estratégia de cobertura implementada. Os resultados mostram que a estratégia de cobertura teve sucesso em limitar o valor máximo do VaR e em prevenir perdas maiores.

Palavras Chave: Economic Capital, Value-at-Risk, Backtest, Cobertura, Return on Risk-Adjusted Capital
Classificação JEL: C10, G32

Abstract

The Economic Capital (EC) can be defined as the the capital at risk that derives from investment activities and it is measured by the industry standard risk measurement metric: Value-at-Risk (VaR). Given a pre-defined maximum value for the EC, it is around this target that risk management decisions can be formulated. This work measures and manages the VaR of a portfolio comprised of equities and bonds from the U.S. and European markets such that it does not surpass the pre-defined target. Given the variety of VaR models available, to conclude on which model provides the most accurate measurements for the portfolio in this work, 16 different models are computed and their performance is assessed through a backtest. Using the best performing model, the VaR of the portfolio is measured daily and managed through an equity exposure hedging strategy for a period of one year. To assess the results of the strategy, the Return on Risk-Adjusted Capital (RORAC) performance metric is used. Results show that the equity exposure hedging strategy was successful in limiting the maximum VaR and in safeguarding against further losses.

Keywords: Economic Capital, Value-at-Risk, Backtest, Hedging, Return on Risk-Adjusted Capital
JEL Classification: C10, G32

Contents

List of Abbreviations

BCP - Berkowitz, Christoffersen and Pelletier
DAX - Deutscher Aktienindex
EC - Economic Capital
EUR - Euro
EWMA - Exponential Weighted Moving Average
PV - Present Value
PV01 - Present Value of a Basis Point
QR - Quantile Regression
RORAC - Return on Risk-Adjusted Capital
RM - RiskMetrics
S&P500 - Standard & Poor's 500
SGSt - Skewed Generalized Student-t
UC - Unconditional Coverage
U.S. - United States
USD - United States Dollar
VaR - Value-at-Risk

CHAPTER 1

INTRODUCTION

For financial institutions, managing risk is not only of interest for shareholders, but also a legal obligation. Given the systemic impact financial institutions have in the financial system and economy, through the introduction of the Basel Accords in the late-1980s and subsequent extensions (Shakdwipee and Mehta, 2017), financial regulators sought to strengthen minimum capital requirements with appropriate risk monitoring and reporting regulations to protect financial stability.

While there are several sources of risk, this work deals with market risk, which can be seen as the uncertainty regarding the future value of financial assets that derives from market price movements. But before risk can be managed, it must be measured. The Value-at-Risk (VaR) is the industry standard market risk measurement metric, and as a statistical measure, it can be defined as the maximum expected loss we are confident will not be exceeded for a given significance level and over a given future time horizon (Alexander, 2009). In terms of financial institutions and the management of their internal financial activities, the Economic Capital (EC) refers to the capital at risk that derives from their investment activities and it is measured by the VaR. Therefore, numerically, the EC is equal to the VaR (Jorion, 2007), and given a pre-defined maximum value for the EC, it is around this benchmark that risk management strategies can be formulated.

This work aims to measure and manage the VaR of a portfolio composed of equities and bonds from the U.S. and European markets during the time period of one year from 30 September 2021 to 30 September 2022 such that it does not surpass the pre-defined maximum value for the EC. As the end goal, we analyze the return achieved for the year versus the return that would have been achieved if we did not manage the VaR.

As we first need to measure the VaR, the question of which model should be chosen for this task arises. Following efforts from financial institutions and regulators, the need for a standard metric for market risk measurement was first met in the mid-1990s, where the RiskMetrics VaR model was developed by J.P. Morgan and Reuters (1996). Although the pioneer, the answer may not lie here, and therefore we explore four classes of VaR models in this work: RiskMetrics (RM), Skewed Generalized Student-t (SGSt), Historical and Quantile Regression (QR) and in total 16 different models are computed. Through a Backtest, using historical data, we compute historical VaR estimates for each model using the portfolio composition at 30 September 2021. Time wise, the series of historical VaR estimates for the Backtest spans over 6 years from 1 October 2015 to 30 September 2021 and we assess each model's performance through the Unconditional Coverage (UC) test (Kupiec, 1995) and the Berkowitz, Christoffersen and Pelletier test (Berkowitz et al., 2011).

Using the best performing model and for one year, the VaR of the portfolio is measured and managed daily through an equity exposure hedging strategy. Based on historical ranges, we define the daily EC to a maximum of €1.5 million, which corresponds to close to 1.6% of the portfolio value at 30 September 2021. Having set this target, the daily VaR estimate must not exceed this value and we explore how limiting the VaR by using the defined equity exposure hedging strategy impacts the one year return of the portfolio. For comparison purposes, we define the portfolio with the strategy implemented as the Hedged portfolio and the same portfolio but without the strategy implemented as the Unhedged portfolio.

Results show that the strategy was successful in limiting the VaR to a maximum of €1.5 million throughout the year, whereas if it was not managed, this threshold would have been surpassed during a significant portion of the year starting from February 2022 and reaching a maximum of €2.3 million in May 2022. When comparing the returns achieved for the Hedged and Unhedged portfolios, using the RORAC performance metric, we conclude that while the return for the year was negative in both scenarios, the Hedged portfolio achieved a less negative return while incurring less risk. This means that during the year under analysis, limiting the VaR of the portfolio safeguarded against further losses.

This work is organized as follows: Chapter 2 covers the relevant literature; Chapter 3 presents the data, time frame and portfolio details; Chapter 4 lays the roadmap ahead and dives into the applied methodology; Chapter 5 reveals the results of the Backtest and model selection; Chapter 6 dives into the equity exposure hedging strategy and its impact on portfolio returns; Chapter 7 summarises the results of this work.

CHAPTER 2

LITERATURE REVIEW

History shows that distress in financial institutions can have severe negative impacts on not only financial markets but also on the real economy (Hoggarth et al. (2002) and Dell'Ariccia et al. (2008)), with the great financial crisis of 2008 as a famous example (Baur, 2012). Considering many of these institutions are private, the idea of regulation in order to protect financial stability inevitably arises (Dow, 1996). Through the introduction of the Basel Accords in 1988 and subsequent extensions (Shakdwipee and Mehta, 2017), financial regulators sought to strengthen minimum capital requirements through risk monitoring and reporting regulations.

While risk can be generally categorized into market, credit and operational (Allen et al., 2004), this work deals with market risk. It can be seen as a measurement of the uncertainty regarding the future value of financial assets that derives from market price movements (Alexander, 2009). But before risk can be managed, it must be measured. Even though it has its limitations (Krause, 2003), the VaR is the industry standard market risk measurement metric, and as a statistical measure, it can be defined as the maximum expected loss we are confident will not be exceeded for a given confidence level and over a given future time horizon (Alexander, 2009).

From a regulatory standpoint, the minimum capital requirements are defined as regulatory risk capital (Alexander, 2009) and are calculated based on methods proposed by regulators (Bank for International Settlements, 2023). However, as long as regulatory risk capital requirements are met following the Basel Accords, from an internal management standpoint financial institutions are free to allocate capital to whichever activities they see fit, and in whichever quantities they see fit (Alexander, 2009). For a financial institution, the EC can be defined as the capital at risk that derives from its investment activities (Porteous and Tapadar, 2005) and they will distribute parts of the EC of the whole firm through its various specific activities using a top-down approach. To maintain financial stability at the firm level, the capital at risk resulting from those specific activities must not surpass the EC limit allocated to them. As the EC is measured by the VaR, numerically the EC is equal to the VaR (Jorion, 2007) and given a pre-defined maximum value for the EC, it is around this benchmark that risk management strategies can be formulated.

The need for a standard metric for market risk measurement arose during the mid-1990s following an effort from both financial regulators and international banks where the RiskMetrics (RM) VaR model was developed by J.P. Morgan and Reuters (1996). With the adoption of the VaR as the official measure of market risk in the Basel II Accords and subsequent iterations, the introduction of RiskMetrics was the pivotal step in the adoption of economic capital-based metrics for measuring minimum capital requirements (Allen et al., 2004).

The RM VaR model falls into the parametric category of models. It can also be referred to as Parametric Normal VaR as it assumes that returns follow a normal distribution over time (J.P. Morgan and Reuters, 1996). This assumption goes against the empirical data on financial returns that generally show both negative skewness and excess kurtosis as shown by Fama (1965) and Peiro (1994) which are characteristics of a non-normal distribution. This detail becomes relevant because when picturing a return distribution, extreme losses are present in the left tail of the distribution, and the lower the significance level chosen for the VaR, the deeper we look into the tail. It follows that if the normal distribution does not reflect the real distribution of returns, the RM VaR estimate may not accurately capture the risk incurred, potentially making the RM model unfitting. When estimating the VaR at the 5% significance level, or equivalently, at the 95% confidence level, we are 95% confident that the future loss will not exceed the VaR, and considering this configuration is widely adopted for the RM model, in search for potential flaws in the model Pafka and Kondor (2001) investigate how it performs for significance levels lower than 5%. With this in mind, using 4 years of data from the 30 stocks composing the Dow Jones Industrial Average index, the authors compute the RM VaR for both the 5% and 1% significance levels. Results show that for lower significance levels such as 1%, where we look deeper into the left tail than for the 5% significance level, the impact of the non-normality of returns becomes evident, which increases the possibility that the VaR will be more underestimated the higher the confidence level chosen.

A promising candidate to tackle the non-normality of returns and RM model shortcomings is the Skewed Generalized Student-t (SGSt) distribution (Theodossiou, 1998). Built upon the classical Student-t introduced by McDonald and Newey (1988), it allows for flexibility in the shape of the tails and central region. The SGSt VaR also assumes a parametric distribution for the returns but tries to fit the shape of the distribution to the empirical returns, which then allows to account for the fat tails often observed in financial returns distributions. Lin and Shen (2006) study the SGSt VaR when compared to the RM VaR using daily data for the S&P500, NASDAQ, DAX and FTSE100 indices with a sample size of 3 years. As expected for equity returns, the normality assumption was strongly rejected for all indices using the Jarque-Bera test. The authors proceed to estimate the VaR for all indices for signifiance levels ranging from 55% to 0.1%. Results show that the for the 55% significance level the RM model produces satisfactory results, however, for lower levels the performance starts to deteriorate. This does not happen to the SGSt VaR, as its performance remains robust as we decrease the significance level. The authors conclude that the SGSt VaR is capable of producing more accurate VaR estimates for lower significance levels and is therefore a promising alternative in the realm of the parametric VaR models.

Instead of imposing a structure to the returns through a parametric approach as it is done in the RM and SGSt models, a much simpler approach is available: use the empirical distribution of the returns. This allows for the empirical skewness and kurtosis of the returns distribution to be accurately captured and is the essence of the Historical Simulation VaR, or simply Historical VaR. However, its flexibility does not come without drawbacks. As it fully relies on the empirical sample of past returns, the choice of sample size is a subjective but critical component to the Historical VaR. As explored by Pritsker (2006), a larger sample size widens the diversity of outcomes but, at the same time, as we increase the sample size, the less we reflect the current market volatility conditions. To tackle this issue, a series of papers by Barone-Adesi et al. (1998) and Boudoukh et al. (1998) propose a refinement to the classic methodology by attributing more weight to recent observations, thus moving from a sample where all observations have the same relevance to a sample where recent observations are more meaningful than distant past ones. Hull and White (1998) propose another refinement in which a volatility adjustment methodology is applied where past returns are adjusted such that their magnitude reflects current market volatility levels. In other words, the magnitude of a past return during different market volatility conditions is adjusted in order to reflect current market volatility conditions. Using approximately 9 years of daily data from 12 different exchange rates and 5 different stock indices, Hull and White (1998) compare their methodology with the standard Historical VaR with no adjustment to the weight of the observations and with the Boudoukh et al. (1998) weight adjustment methodology. As expected, when comparing to the standard historical VaR, the volatility adjustment methodology produced more satisfactory results, and when comparing with the Boudoukh et al. (1998) methodology, Hull and White's proposed volatility adjustment produces better results especially for the 1% significance level.

Still in the non-parametric world, as the VaR can be defined as a conditional quantile (Xiao et al., 2015), quantile regressions as introducted by Koenker and Bassett Jr (1978) come as an additional alternative to estimating the VaR. The Quantile Regression (QR) VaR methodology shares similarities with the Historical VaR as it relies on the empirical returns instead of assuming the returns follow a certain parametric distribution. Additionally, one of its biggest advantages is the flexibility it brings for the choice of explanatory variables. Steen et al. (2015) evaluate the performance of the RM and Historical VaR models versus the QR VaR using close to 20 years of daily data for future contracts of 19 different commodities. The finding of the authors corroborate the findings of other studies for the performance of the RM model: it is capable of producing satisfactory results for the 5% significance level, and in this case does it for most commodities but as expected, when the 1% significance level is applied, the performance is no longer as satisfactory. The Historical VaR performed better than RM overall but was still outclassed by the QR VaR for all significance levels. Even though the estimates of the QR VaR are dependent on how the model is configured (i.e. on the explanatory variables chosen, and these are subjective), it has the ability to produce more robust results than RM and equivalent or better results than the Historical VaR. For commodities specifically, it is a promising model.

However, each portfolio is a case on its own and it is likely that the same VaR model for two different portfolios will produce drastically different results, as explored by Alexander (2009). This invokes the need for model performance testing methodologies to assess which model is the best fit for our specific portfolio. To do this, a Backtest procedure is commonly adopted to assess model performance where statistical tests are performed using a historical sample of portfolio returns and a historical sample of corresponding VaR estimates. The choice of which statistical tests to use in the Backtest is up to the user, and despite the considerable literature regarding the possible alternatives (Zhang and Nadarajah, 2018), there is no standard set in stone. A common Backtest performance metric is the number of exceedances which can be defined as an event where the actual loss was greater than the VaR estimate. For this work we adopt the classical UC test (Kupiec, 1995) which, for the period and model under analysis, assesses the number of exceedances. To complement the UC test, we additionally adopt the BCP test (Berkowitz et al., 2011) which, for the occasions where we observe exceedances, checks whether there is autocorrelation between them. In other words, it checks whether the exceedances are independent from each other or occur in clusters and checking for clusters is important as it reveals the model's capacity to adapt to rapidly changing market conditions.

When measuring the VaR, for the cases where it surpasses its pre-defined maximum value, risk management decisions are warranted. As a consequence of adjusting the portfolio exposures through risk management strategies to meet the EC quotas, it comes that the risk profile of the adjusted portfolio differs from the risk profile the portfolio would have had without these adjustments. As Longley-Cook (1998) points, a monetary gain when there is substantial risk is not worth as much as the same monetary gain when there is lower risk. Using the concepts we explored so far, a return achieved with a substantial VaR, and therefore EC, is not as valuable as the same return achieved with a lower VaR, and therefore lower EC. Considering this, the need for a risk-adjusted performance metric arises. The RORAC (Matten, 1996) comes as a solution to our problem as it gives the ratio between returns and the corresponding risk incurred to achieve them. Formally, it is a ratio that relates a non-adjusted return to a risk-adjusted capital base (Matten, 1996). Using the concepts we explored so far it gives the ratio of the returns achieved to the EC (VaR) and it allows for a fair comparison between returns of portfolios with different risk profiles.

CHAPTER 3

DATA AND PORTFOLIO COMPOSITION

The portfolio studied in this work is composed of part equities and part bonds. The various positions in these two asset classes are from the United States (U.S.) and European markets and going forward, we define the Euro (EUR) as our local currency and the U.S. Dollar (USD) as the foreign currency.

The equity portion of the portfolio is composed by twenty-two positions in stocks, long and short, from both the U.S. and European markets. Daily adjusted closing prices in local currency for each stock as well as for the USD/EUR exchange rate were downloaded from YahooFinance (`https://finance.yahoo.com`).

The fixed income portion consists of five fixed coupon government bonds maturing at different dates, with different coupon payments and coupon payment dates. These bonds were issued in two different markets: U.S. and Germany. The characteristics of each bond were obtained from Bloomberg and daily interest rate data was sourced from the Federal Reserve Economic Data website for the USD (`https://www.federalreserve.gov/datadownload/Choose.aspx?rel=H15`) and from the European Central Bank website for the EUR (`https://sdw.ecb.europa.eu/browseSelection.do?node=9689726`). Time wise, we work with daily data for a period of twelve years from 30 September 2010 to 30 September 2022.

Tables 1 and 2 below present the components and details of the portfolio studied in this work.

Asset	Ticker/ ISIN	Original Currency	No. Shares/ Face Value	Price	Value (EUR)	Allocation (%)
Apple Inc.	AAPL	USD	16 083	140.71	1 950 593	2.02
AMD, Inc.	AMD	USD	19 299	102.90	1 711 724	1.78
Amazon.com, Inc.	AMZN	USD	11 696	164.25	1 655 941	1.72
Caterpillar Inc.	CAT	USD	12 866	187.78	2 082 450	2.16
Disney	DIS	USD	14 621	169.17	2 131 904	2.21
JPMorgan & Co.	JPM	USD	11 258	159.14	1 544 261	1.60
Coca-Cola	KO	USD	48 248	50.93	2 117 977	2.20
Merck, Inc.	MRK	USD	29 241	72.56	1 828 935	1.90
Tesla, Inc.	TSLA	USD	10 234	258.49	2 280 299	2.37
Visa Inc.	V	USD	10 965	221.18	2 090 529	2.17
Walmart Inc.	WMT	USD	17 545	137.19	2 074 633	2.15
adidas AG	ADS.DE	EUR	7 711	266.83	2 057 581	2.13
Allianz SE	ALV.DE	EUR	11 016	184.99	2 037 808	2.11
BASF SE	BAS.DE	EUR	28 642	61.41	1 758 841	1.82
Bayer	BAYN.DE	EUR	44 064	45.51	2 005 371	2.08
Deutsche Bank	DBK.DE	EUR	198 288	10.81	2 142 586	2.22
Deutsche Post	DPW.DE	EUR	39 658	52.03	2 063 224	2.14
Deutsche Telekom	DTE.DE	EUR	88 128	16.76	1 477 212	1.53
EDP, S.A.	EDP.LS	EUR	440 640	4.35	1 917 244	1.99
Siemens	SIE.DE	EUR	15 422	138.00	2 128 234	2.21
Sonae, SGPS, S.A.	SON.LS	EUR	2 203 200	0.86	1 902 305	1.97
Volkswagen AG	VOW3.DE	EUR	(13 219)	183.64	-2 427 617	-2.52
Total Equity			–	–	**38 532 035**	**39.97**
US Treasury 2027	US912810PS15	USD	11 377 740	107.54%	12 236 051	12.69
US Treasury 2028	US912810PV44	USD	10 343 400	103.85%	10 741 736	11.14
German Bund 2028	DE0001135069	EUR	9 000 000	107.43%	9 668 697	10.03
German Bund 2027	DE0001102424	EUR	12 000 000	105.87%	12 704 899	13.18
German Bund 2025	DE0001102374	EUR	12 000 000	104.36%	12 522 719	12.99
Total Bonds			–	–	**57 874 102**	**60.03**
Total Portfolio			–	–	**96 406 137**	**100.00**

Table 1. Portfolio composition at 30 September 2021. This table showcases the assets that comprise the portfolio used in this work as well as the amount invested in each one converted from USD to EUR where appropriate. The exchange rate as at 30 September 2021 is 0.8640. Figures may not sum to total due to rounding.

Bonds	ISIN	Maturity	Original Currency	Coupon per Year	Coupon (%)	Face Value (EUR)	Fair Value (EUR)
US Treasury 2027	US912810PS15	15/01/2027	USD	2	2.375	11 377 740	12 236 051
US Treasury 2028	US912810PV44	15/01/2028	USD	2	1.750	10 343 400	10 741 736
German Bund 2028	DE0001135069	01/04/2028	EUR	1	0.650	9 000 000	9 668 697
German Bund 2027	DE0001102424	15/08/2027	EUR	1	0.500	12 000 000	12 704 899
German Bund 2025	DE0001102374	15/05/2025	EUR	1	0.500	12 000 000	12 522 719

Table 2. Bond Characteristics. The Fair Value of the bond is the sum of the PV of all its future cash flows discounted to 30 September 2021 and converted to EUR where appropriate. The exchange rate as at 30 September 2021 is 0.8640. We obtain the future cash flows by Equations 2 and 3 and calculate their PV by Equation 4.

CHAPTER 4

METHODOLOGY

The goal of this work is to measure and manage the VaR of a portfolio over a one year period going forward from today such that it does not surpass the pre-defined Economic Capital. We take today as 30 September 2021.

Since there are various VaR models to choose from, as well as many possible variants of the same model, a crucial question arises: which model should we adopt for the one year period going forward considering the particularities of the portfolio as of today? The correct model cannot be determined with certainty without testing different possibilities and assessing their accuracy. This assessment is called Backtesting, and it is this process that will give us the answer to the question posed above.

Using historical data and today's portfolio composition, for each VaR model under analysis, we will compute a historical series of daily VaR estimates as if the portfolio we have today existed in the past and its VaR was estimated in real time. Time wise, the series spans over 6 years from 1 October 2015 to 30 September 2021. We will explore four different classes of models: RiskMetrics, SGSt VaR, Historical VaR and QR VaR and within these four, we also explore different variations of each one so that we have a wide range of candidates to analyze and choose from. In the Backtest, through specific statistical tests, we analyze the performance of each model and conclude if the model is a correct fit for our portfolio going forward or not.

Although different from each other, all four classes of models share one crucial input for their computation: the portfolio volatility. This chapter goes through the methodology and steps required to calculate portfolio returns, model portfolio volatility and compute each VaR model. In Chapter 5 we cover the Backtest and the statistical tests to assess model performance, as well as their results, and choose the model we will apply for the one year period going forward in the VaR Management portion of this work presented in Chapter 6.

4.1. Risk Factor Exposure Mapping

We refer to the sources of risk as risk factors, which can be seen as the variables directly influencing the value of the assets that compose the portfolio. As such, the first step is to identify these variables and quantify the exposure to each of them through risk factor mapping, where we map each portfolio position to an equivalent exposure in terms of risk to the corresponding risk factor. The risk factors each position is mapped into depends on the type of asset and for different assets, different risk factors are needed. It follows that to estimate the VaR in EUR, which is our local currency, all exposures have to be quantified in EUR. To achieve this, we convert each exposure mapped from foreign to local currency using the appropriate exchange rate on the date of the Backtest, which in our case is 30 September 2021.

In the subsections below we explore the exposure mapping methodology for each asset class present in our portfolio. Table 3 at the end of this section presents the exposures to each risk factor that result from the risk factor exposure mapping.

4.1.1. Equity

The value of an investment in a stock depends on the number of shares and the market price of the respective stock. Therefore, the risk factor for each stock in our portfolio is the respective change in its market price.

Regarding the risk factor mapping, the exposure to the change in price of each stock consists in the amount of capital invested in it, and for each stock, we obtain the amount of capital invested by multiplying the number of shares by the respective market price:

$$S_{i_t} = N_{i_t} \times P_{i_t} \times FX_t, \tag{1}$$

where FX_t is the exchange rate at 30 September 2021 used to convert the exposure from foreign to local currency.

4.1.2. Bonds

A bond is a financial instrument that consists of a stream of cash flows. Its fair value derives from its future cash flows discounted to present time, and so the main risk factor for its value is the interest rate used to discount the future cash flows. If the interest rate increases (decreases) the bond value decreases (increases). As interest rates are not static and may vary along changes in market conditions, it then becomes imperative from a risk measurement and management standpoint to quantify the sensitivity of bond positions to changes in interest rates.

For a fixed coupon bond, its collection of future cash flows consists in the coupon payments throughout the life of the bond plus the final payback payment. The coupon paid is given by:

$$Coupon = N \times \frac{c_n}{n}, \tag{2}$$

where N is the monetary amount invested in the bond (also referred to as face value), c_n is the annual coupon rate (annual interest paid by the bond each year until the maturity date) and n is the coupon frequency (the number of times per year the coupon payments are made).

The final payback payment that occurs at the maturity of the bond includes the redemption of the face value and is given by:

$$C_m = N \times \left(1 + \frac{c_n}{n}\right) \tag{3}$$

Let C_T denote a future cash flow, T the time in years from now until the date of the cash flow (also referred to as maturity) and r_T the continuously compounding interest rate for the time period from now until T. It follows that the present value (PV) of cash flow C_T is:

$$PV_{C_T,r_T} = C_T \times e^{-r_T \times T} \tag{4}$$

The present value of a basis point ($PV01$) measures the PV sensitivity of a given cash flow to a one basis point decrease in the interest rate r_T. It is approximated by a first-order Taylor expansion as:

$$\begin{aligned} PV01_{C_T,r_T} &\approx \frac{\partial PV_{C_T,r_T}}{\partial\, r_T} \times (-0.01\%) \\ &= T \times PV_{C_T,r_T} \times 0.01\% \end{aligned} \tag{5}$$

where T is the maturity of the cash flow.

It follows that the first-order approximation to the change in the PV of a cash flow, which is its P&L, can be written as a function of the $PV01$ as:

$$\Delta PV_{C_T,r_T} = -PV01_{C_T} \times \frac{\Delta r_T}{0.01\%}, \tag{6}$$

where $\frac{\Delta r_T}{0.01\%}$ is the absolute change in interest rate converted to basis points.

Given the above, for cash flow C_T, its P&L depends on the sensitivity of the cash flow to a one basis point increase in the interest rate ($-PV01_{C_T}$) and on the actual change in interest rate that occurred (Δr_T). If the change in interest rate is positive (negative) the

P&L is negative (positive). In the case of a bond with n cash flows, its P&L is the sum of the n P&Ls given by applying Equation 6 to each cash flow.

However, a bond with n cash flows will have n different interest rates as risk factors (one for each cash flow maturity), and in a portfolio with a large number of bonds, the problem eventually becomes intractable. Another hurdle arises arises when a future cash flow occurs at a date for which there is no data available for the corresponding interest rate r_T, and without an appropriate interest rate, the cash flow cannot be correctly discounted to present time to then compute its $PV01$.

To tackle these hurdles, and as Alexander (2008) suggests, we adopt a vertices mapping approach where we map cash flows with non-standard maturity to a set of standard maturity interest rates for which data is available. In the cash flow mapping process, a vertex is a standard maturity to which there is available interest rate data. The mapping approach we adopt going forward is the PV+PV01 invariant mapping, which consists in preserving the PV and $PV01$ of the original cash flow of non-standard maturity.

Let PV_{C_T} denote the PV of the original cash flow with maturity T, and T_1 and T_2 the standard maturity vertices directly below and above T, respectively, to which there is available interest rate data. Let x_{T_1} and x_{T_2} denote the proportions of PV_{C_T} mapped into vertex T_1 and T_2, respectively. We preserve the PV of the original cash flow through the following condition:

$$\underbrace{x_{T_1}}_{PV\ mapped\ to\ vertex\ T_1} + \underbrace{x_{T_2}}_{PV\ mapped\ to\ vertex\ T_2} = \underbrace{PV_{C_T}}_{PV\ original\ cash\ flow} \tag{7}$$

For the $PV01$ invariant mapping, the sum of the $PV01$ of the mapped cash flows equals the $PV01$ of the original cash flow. This ensures that the sum of the P&L of the two mapped cash flows matches the P&L of the original cash flow following a parallel shift of one basis point in the interest rate curve[1]. We preserve the $PV01$ of the original cash flow through the following condition:

$$T_1 x_{T_1} + T_2 x_{T_2} = T PV_{C_T} \tag{8}$$

Finally, we simultaneously preserve the PV and $PV01$ by joining Equations 7 and 8. We obtain the values for x_{T_1} and x_{T_2} that satisfy both conditions simultaneously as:

$$x_{T_1} = \frac{T_2 - T}{T_2 - T_1} \times PV_{C_T} \tag{9}$$

and

$$x_{T_2} = \left(1 - \frac{T_2 - T}{T_2 - T_1}\right) \times PV_{C_T} \tag{10}$$

[1]When the interest rate curve shifts by one basis point it means all spot rates shift by 0.01%.

We repeat the PV+PV01 process for every cash flow and for every bond in our portfolio, and in terms of exposure, each mapped standard maturity cash flow is subject to changes in its respective standard maturity interest rate, which is given by its $PV01$.

Recalling Equation 6, in terms of risk factor mapping, each standard maturity interest rate (vertex) is a risk factor and the exposure to each each risk factor is the sum of all $PV01$ mapped to that maturity multiplied by -1. Finally, for bonds from the U.S. market, since their cash flows are denominated in USD, we convert the $-PV01$ exposures to EUR using the exchange rate at 30 September 2021.

4.1.3. Currency

Since our local currency is the EUR and the portfolio in analysis has positions in assets from foreign markets, these positions are not only exposed to the risk factors of the respective asset, but also to the exchange rate from foreign to local currency. In our case, assets in USD are also exposed to the USD/EUR exchange rate. In terms of mapping, the exposure to the USD/EUR risk factor is the total amount of capital invested in assets denominated in USD, for both equity and bonds, converted to EUR. We once again do this conversion using the exchange rate at 30 September 2021.

4.1.4. Portfolio Exposures

Table 3 below presents the risk factors and their corresponding mapped exposure at 30 September 2021. For Equity, we obtain the exposures shown in the table below through the methodology described in subsection 1.1 of this chapter. For Bonds, the risk factors are the standard maturity interest rates[2] and the exposure to each standard maturity interest rate is the sum of all $-PV01$ mapped to that maturity. We obtain the $-PV01$ by the methodology described in subsection 1.2. For Currency, we obtain the exposure to the USDEUR exchange rate by the methodology described in subsection 1.3.

Equity		**Bonds**		**Currency**	
Risk Factor	**Exposure (EUR)**	**Risk Factor**	**Exposure (EUR)**	**Risk Factor**	**Exposure (EUR)**
AAPL	1 950 593	USD3M	-4.70	USDEUR	44 447 033
AMD	1 711 724	USD6M	-6.58		
AMZN	1 655 941	USD1Y	-33.78		
CAT	2 082 450	USD2Y	-89.69		
DIS	2 131 904	USD3Y	-198.86		
JPM	1 544 261	USD5Y	-6 660.54		
KO	2 117 977	USD7Y	-5 521.62		
MRK	1 828 935	USD10Y	–		
TSLA	2 280 299	EUR3M	–		
V	2 090 529	EUR6M	-5.93		
WMT	2 074 633	EUR1Y	-12.05		
ADS.DE	2 057 581	EUR2Y	-36.21		
ALV.DE	2 037 808	EUR3Y	-2 611.19		
BAS.DE	1 758 841	EUR5Y	-6 667.93		
BAYN.DE	2 005 371	EUR7Y	-8 705.40		
DBK.DE	2 142 586	EUR10Y	–		
DPW.DE	2 063 224				
DTE.DE	1 477 212				
EDP.LS	1 917 244				
SIE.DE	2 128 234				
SON.LS	1 902 305				
VOW3.DE	-2 427 617				

Table 3. Risk factor exposures map in EUR at 30 September 2021. These exposures are used to calculate historical portfolio returns through Equation 14 for the Backtest.

[2]Each standard maturity interest rate is labeled as a composite of "Currency" + "Maturity". For example, the 3-year USD interest rate is labeled as USD3Y.

4.2. Returns

For the stocks in our portfolio, and keeping the capital invested (M_{Stock}) constant, the P&L is given by the change in market price of its risk factor, which is the stock price, as:

$$P\&L_{Stock_t} = M_{Stock} \times \left(\frac{P_t}{P_{t-1}} - 1 \right) \tag{11}$$

For an investment in a bond, each cash flow is exposed to a different interest rate and therefore changes in that respective interest rate. For the bonds in our portfolio, keeping the capital invested and $-PV01$s constant, using Equation 6 the total P&L corresponds to the sum of the P&Ls of all mapped cash flows as:

$$P\&L_{Bonds_t} = \sum_{i=1}^{n} -PV01_{T_i} \times \frac{\Delta r_{T_i}}{0.01\%} \tag{12}$$

An investment in an asset denominated in foreign currency generates exposure to not only the risk factor of that specific asset directly but also to the exchange rate between foreign and local currency indirectly. In other words, by investing in stocks and bonds denominated in foreign currency we are generating an exposure to the exchange rate between foreign and local currency equivalent to the amount invested ($M_{Currency}$). As the risk factor is the exchange rate between foreign and local currency (FX), the P&L deriving from the exposure to foreign currency is given by:

$$P\&L_{Currency_t} = M_{Currency} \times \left(\frac{FX_t}{FX_{t-1}} - 1 \right) \tag{13}$$

With a series of daily stock prices, spot interest rates and currency exchange rates, using the equations above and the mapped exposures to each risk factor at 30 September 2021 presented in Table 3, we compute the historical time series of daily P&Ls for the portfolio composition at 30 September 2021 in vector form as:

$$P\&L_{Portfolio_t} = \begin{bmatrix} M_{Stock_i} \\ \vdots \\ -PV01_{T_i} \\ \vdots \\ M_{Currency} \end{bmatrix}^T \times \begin{bmatrix} \left(\frac{P_{i_t}}{P_{i_{t-1}}} - 1 \right) \\ \vdots \\ \frac{\Delta r_{T_i}}{0.01\%} \\ \vdots \\ \left(\frac{FX_t}{FX_{t-1}} - 1 \right) \end{bmatrix}, \tag{14}$$

where the first vector transposed remains constant and corresponds to the exposures to each risk factor and the second vector corresponds to the change in the respective risk factor.

We transform the portfolio P&L into a percentage return as:

$$R_t(\%) = \frac{P\&L_{Portfolio_t}}{Portfolio\,Value}, \tag{15}$$

where the denominator is the portfolio value at 30 September 2021 (see Table 1).

4.3. Volatility

We recall the beginning of this chapter where we touch on the two crucial components needed in order to successfully complete the Backtest process: portfolio returns to use in the back tests and portfolio volatility modeling in order to be able to compute the VaR models we sought to test. In section 1 of this chapter we dived into how to identify the appropriate risk factors for each asset class and how to quantify our portfolio's sensitivity to them. Having done this, using the portfolio exposures we mapped for the portfolio composition on 30 September 2021 (see Table 3) and keeping them constant, we simulated historical returns using Equations 14 and 15. We are then left with a series of historical returns for our current portfolio composition.

The next step that unables us to compute the VaR models we mentioned previously is volatility modelling. Volatility of returns, σ, corresponds to the standard deviation of the returns, and the simplest way of computing it is by selecting a historical sample of past returns and calculating its standard deviation. However, with this methodology we are implying that every observation in the sample has the same weight, regardless of how recent or not the observation is. This method may be unfitting as σ is equally influenced by observations that are far into the past, and with minimal relation to the present, as it is by recent observations.

Since VaR is a forward-looking measure, the distant past may not be of relevance, and so the Exponential Weighted Moving Average (EWMA) volatility model aims to tackle this issue. The model attributes higher weights to most recent observations, hence reflecting current market conditions. The constant responsible for this weight attribution is λ and while it can vary between 0 and 1, the lower the λ, the more weighted is attributed to recent observations. While the choice of λ is subjective, following the findings in the RiskMetrics technical document produced by J.P. Morgan and Reuters (1996), a λ of 0.94 proved to be the best general fit when dealing with daily returns and so we will adopt this value going forward.

From the historical daily returns, we estimate the EWMA variance recursively as:

$$\hat{\sigma}_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\hat{\sigma}_{t-1}^2, \tag{16}$$

where $\hat{\sigma}_t^2$ is the variance estimated for day t on day $t-1$, r_{t-1} is the return observed on day $t-1$ and $\lambda \in (0,1)$ is the smoothing factor.

4.4. Value-at-Risk Models

The VaR can be defined as the maximum expected loss over a future time horizon h and for a given significance level of α. Going forward we adopt a significance level of $\alpha = 1\%$, or equivalently, a confidence level of $(1 - \alpha)$ and a future time horizon of 1 day ($h = 1$). This means that with while holding the current portfolio, we are 99% confident that over the next day the observed loss will not exceed the VaR estimate.

Formally, the h-day $100\alpha\%$ VaR ($VaR_{h,\alpha}$) is minus the α-quantile of the h-day return distribution. As Alexander (2009) points out, for any $0 < \alpha < 1$, the α-quantile of the h-day distribution of a continuous random variable X is a real number x_α such that:

$$P(X < x_\alpha) = \alpha \tag{17}$$

If the distribution function of X is known, then the α-quantile (x_α) for any given value of α is given by:

$$x_\alpha = F^{-1}(\alpha), \tag{18}$$

where F^{-1} is the inverse cumulative distribution function of X.

The α-quantile value obtained (x_α) translates into the maximum loss we expect to be exceeded with α probability. As we refer to the VaR as a loss, its value comes represented in absolute terms:

$$VaR_{h,\alpha} = -F^{-1}(\alpha) = -x_\alpha \tag{19}$$

Figure 1 below illustrates the VaR estimate for a given return distribution.

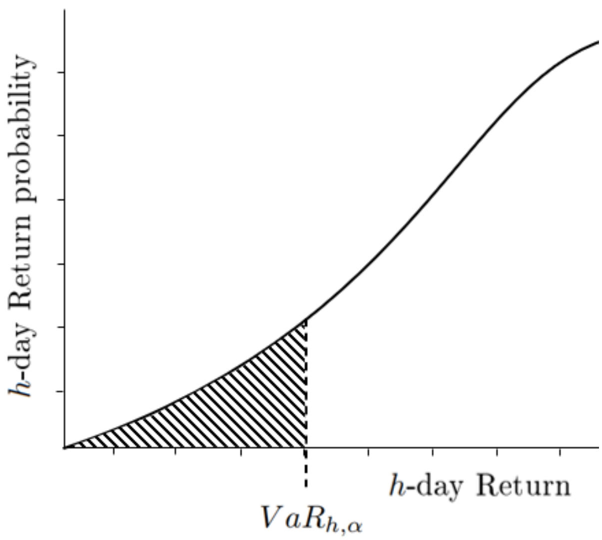

Figure 1. VaR illustration. The curve represents the h-day return distribution. The coloured area under the curve represents the α cumulative probability.

In the following subsections we dive into the methodology necessary to compute the four classes of VaR models mentioned in the first page of this chapter.

4.4.1. RiskMetrics VaR

Let X denote a continuous random variable which represents portfolio returns. The core characteristic of the RiskMetrics VaR model is that it assumes returns follow a normal distribution, that is, $X_h = N \sim (\mu_h, \sigma_h)$, where μ_h and σ_h are the estimated mean and standard deviation, respectively.

We recall Equation 19 and, considering we are dealing with a normal distribution, it follows that:

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha) \times \sigma_h - \mu_h, \tag{20}$$

where $\Phi^{-1}(\alpha)$ denotes the α-quantile of the standard normal distribution.

With regards to μ_h, Alexander (2009) suggests using $\mu_h = 0$ for small time horizons, and since this work deals with daily data and therefore daily VaR estimates ($h = 1$), this then becomes a reasonable assumption. From here, we simplify the equation above and compute the h-day $100\alpha\%$ RiskMetrics VaR as:

$$VaR_{h,\alpha} = -\Phi^{-1}(\alpha) \times \sigma_h, \tag{21}$$

where σ_h is estimated using the EWMA volatility model through Equation 16.

4.4.2. Skewed Generalized Student-t (SGSt) VaR

As mentioned in Chapter 2, the distribution of financial asset returns often departs from the normal distribution by exhibiting fatter tails. Because of this, the normal distribution can underestimate the probability of large negative returns. It then follows that, by assuming a normal distribution, there is a high possibility the VaR will be underestimated for low significance levels (e.g. 1%).

Figure 2 below illustrates the normal distribution VaR underestimation problem described above.

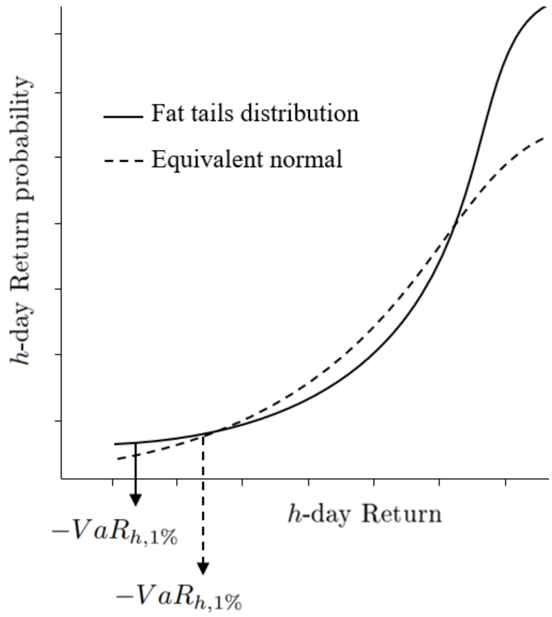

Figure 2. Normal VaR underestimation. We observe that for higher confidence levels such as 99% (which means a VaR significance level (α) of 1%), starting from the left, the fat tailed distribution accumulates probability faster than the equivalent normal. This means the 1% worst return is a worse return than the equivalent normal, which translates into a higher VaR.

In its shape, the standardized SGSt distribution (Theodossiou, 1998) aims to capture the deviations away from normality and its density function $T_{0,1,\lambda,p,q}$ depends on the λ, p and q parameters: $\lambda \in (-1, 1)$ determines the skewness (if $\lambda = 0$ the distribution is symmetric, if $\lambda > 0$ or $\lambda < 0$ the distribution is positively or negatively skewed, respectively), $p > 0$ controls and shape of the central region of the distribution and $q > 0$ controls the shape of the tail region of the distribution. To estimate these parameters, we use the maximum likelihood method such that the estimated parameters make the SGSt distribution resemble the real return distribution of our portfolio as much as possible. To make the model reflective of current market conditions, we re-estimate the parameters every trading month and compute four different SGSt VaR series where what differs between each one is the size of the rolling sample of portfolio returns used for the parametrization: the variants use 250, 500, 750 and 1000 daily observations.

Formally, we compute the h-day $100a\%$ SGSt VaR as:

$$VaR_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) \times \sigma_h - \mu_h, \tag{22}$$

where $T^{-1}_{0,1,\lambda,p,q}(\alpha)$ denotes the α-quantile of the standard SGSt distribution. Equivalently to the RiskMetrics VaR described in the previous subsection, we adopt $\mu_h = 0$ and estimate σ_h by the EWMA volatility model and we simplifty the above equation to:

$$VaR_{h,\alpha} = -T^{-1}_{0,1,\lambda,p,q}(\alpha) \times \sigma_h \tag{23}$$

4.4.3. Volatility-Adjusted Historical VaR

In the RM and SGSt VaR models explored thus far, we impose that the portfolio returns follow a certain defined distribution, which can prove itself unrealistic or computationally complex as in the case of the SGSt VaR. Moving away from the parametric world, the Historical VaR brings a simpler approach to the table: use the empirical distribution of returns directly and rely on the α-quantile of this distribution to estimate the VaR.

We estimate the Historical VaR through the following steps: choose the sample size n, compute h-day past empirical returns by keeping the current portfolio exposure to the risk factors constant for the sample period (this way we simulate the returns the current portfolio would have had in past empirical market conditions), sort the returns from worst to best, start accumulating probability from the worst return upwards (each observation has a probability of $\frac{1}{n}$) and finally, the VaR is given by minus the return with α cumulative probability.

As mentioned in Chapter 2, the choice of sample size is crucial as a larger sample size widens the diversity of returns but, at the same time, as we increase the sample size the less we reflect the current market conditions. This is the main problem with the simple Historical VaR as each observation has the same weight, therefore the current volatility of returns has the same impact as the volatility of the oldest returns in the sample. The volatility-adjusted Historical VaR proposed by Hull and White (1998) aims to solve this shortcoming by, while still giving the same weight to every observation, adjusting the volatility of the entire series of returns. This way, the entire sample reflects the current market conditions. To do this, we first obtain a series of volatility estimates ($\hat{\sigma}_t$) and then adjust the series of returns as:

$$\hat{r}_t = \frac{r_t}{\hat{\sigma}_t}\hat{\sigma}_T, \tag{24}$$

where $\hat{r}_t$ is the adjusted return, T is the VaR measurement date and $t < T$. We define this model as the T volatility-adjusted Historical VaR.

However, as we estimate volatility ($\hat{\sigma}_t$) through the EWMA model, and recalling Equation 16, as r_t becomes known we can estimate the volatility for day $t+1$ ($\hat{\sigma}_{t+1}$). As such, instead of using the volatility estimate for day t that was computed at the end of day $t-1$ (and therefore not using date t information), we can further develop the volatility adjustment process by:

$$\hat{r}_t = \frac{r_t}{\hat{\sigma}_{t+1}}\hat{\sigma}_{T+1}, \tag{25}$$

and we define this model as the $T+1$ volatility-adjusted Historical VaR.

Recalling the importance of sample size for the Historical VaR in particular, for both the methods described in Equations 24 and 25, we compute four different VaR series, where what differs between each one is the sample size of volatility-adjusted returns: the variants use 250, 500, 750 and 1000 daily observations.

Formally, we compute the h-day $100\alpha\%$ volatility-adjusted Historical VaR as minus the α-quantile of the sample of volatility-adjusted returns.

4.4.4. Quantile Regression VaR

Still in the non-parametric world, as the VaR is minus the α-quantile of a series of portfolio returns, we can estimate the VaR through a quantile regression where the dependent variable is the portfolio return and using some explanatory variables of our choosing.

Formally, the α-quantile (q_α) regression VaR can be estimated as:

$$VaR_\alpha \equiv -q_{\alpha,y} = -(\hat{a} + \hat{b}x_i), \tag{26}$$

where y represents the portfolio return and $\hat{a}$ and $\hat{b}$ the estimated parameters of the α-quantile regression of y onto x. It follows that the parameters of the α-quantile regression of y onto x can be determined through a minimization problem (Koenker and Bassett Jr, 1978) as:

$$(\hat{a}, \hat{b}) = \arg\min_{a,b} \sum_{i=1}^{n} [y_i - (a + bx_i)] \, (\alpha - I_{y_i-(a+bx_i)<0}), \tag{27}$$

where $I_{y_i-(a+bx_i)<0}$ is an indicator function of event:

$$I_{y_i-(a+bx_i)<0} = \begin{cases} 1, & \text{if } y_i - (a + bx_i) < 0 \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

The parameters are estimated using a sample size of 1000 daily observations and, similarly to the SGSt VaR, they are re-estimated every trading month.

We compute a series of daily VaR estimations for three different QR VaR specifications, and defining the dependent variable y as the portfolio return, the first specification is given by:

$$y_t = b \times \sigma_t + \varepsilon_t, \tag{29}$$

where the explanatory variable σ_t is the volatility estimate for day t computed using the EWMA volatility model.

While also using σ_t as an explanatory variable, the second and third specifications use an additional explanatory variable of $\sigma_{5d,t}$ and $\sigma_{20d,t}$, respectively, which are the average of the volatility of the last 5 and 20 trading days. The second specification is given by:

$$y_t = b \times \sigma_t + c \times \sigma_{5d,t} + \varepsilon_t, \tag{30}$$

and the third specification is given by:

$$y_t = b \times \sigma_t + c \times \sigma_{20d,t} + \varepsilon_t, \tag{31}$$

Formally, we compute the h-day $100\alpha\%$ QR VaR as:

$$VaR_{\alpha,t} = -(\hat{b} \times x_t + \hat{c} \times z_t), \tag{32}$$

where x_t and z_t are the explanatory variables.

For ease of presentation going forward, we assign a number to each VaR model computed. Table 4 below presents the number assigned to each model and its respective description.

Model No.	Description
1	RiskMetrics
2	SGSt, rolling sample of 250 obs.
3	SGSt, rolling sample of 500 obs.
4	SGSt, rolling sample of 750 obs.
5	SGSt, rolling sample of 1000 obs.
6	T volatility-adjusted Historical, rolling sample of 250 obs.
7	T volatility-adjusted Historical, rolling sample of 500 obs.
8	T volatility-adjusted Historical, rolling sample of 750 obs.
9	T volatility-adjusted Historical, rolling sample of 1000 obs.
10	$T+1$ volatility-adjusted Historical, rolling sample of 250 obs.
11	$T+1$ volatility-adjusted Historical, rolling sample of 500 obs.
12	$T+1$ volatility-adjusted Historical, rolling sample of 750 obs.
13	$T+1$ volatility-adjusted Historical, rolling sample of 1000 obs.
14	Quantile Regression with EWMA volatility as the explanatory variable, rolling sample of 1000 obs.
15	Quantile Regression with EWMA volatility and 5-day rolling average of EWMA volatility as the explanatory variables, rolling sample of 1000 obs.
16	Quantile Regression with EWMA volatility and 20-day rolling average of EWMA volatility as the explanatory variables, rolling sample of 1000 obs.

Table 4. Model number and respective description. All models use the EWMA volatility model with $\lambda = 0.94$ for their volatility estimates. Models 6 to 9 use the methodology given by Equation 24 and models 10 to 13 use the methodology given by Equation 25. The rolling sample on the SGSt and Quantile Regression models are the sample of returns used to estimate the model parameters.

CHAPTER 5

BACKTEST AND MODEL SELECTION

Through Chapter 4 we dived into the methodology necessary to compute the four classes of VaR models we want to test for our portfolio. Overall, we computed 16 different models and for each one we computed a series of daily historical VaR estimates spanning over 6 years from 1 October 2015 to 30 September 2021 and we define this test period as the global period. The next step is to assess the performance of each model and choose the best one to apply to the one year period going forward.

The main performance metric is the number of exceedances, and as we are dealing with daily data, we refer to an exceedance as an event where the actual return for the day is worse than the VaR estimate for the same day. To evaluate these events we adopt two inference tests: the UC (Kupiec, 1995) and BCP (Berkowitz et al., 2011), where the first evaluates the number of exceedances and the second evaluates the autocorrelation between exceedances.

Even though the two tests assess the performance of a model from different points of view, our decision will be primarily supported by the results of the UC test, while the BCP helps us differentiate between models that have similar UC test performance. This is because even when a model has a low number of exceedances, if these are consecutive or occur within few days of each other, the model can fail the BCP test (depending on the lag of the test). On the other hand, if a model has a high number of exceedances, if they are not consecutive or occur within few days of each other, the model can pass the BCP test (depending on the lag of the test).

Timewise, we apply the UC and BCP tests to the global period as well as for each year within this time interval when relevant. While it is useful to assess and compare the performance of each model for specific time periods (for example, if we want to assess how a model performs during certain market conditions or expect certain past market conditions to repeat themselves in the near future), the decision will be mainly based on the results for the global period as this offers more homogeneous results.

Sections 1 and 2 present the methodology for each test. Section 3 presents the results of the Backtest and the selected model for the one year period going forward.

5.1. Unconditional Coverage Test

For the UC test, a model is said to be well specified when the number of exceedances is in-line with the significance level α of the VaR model (Alexander, 2009). Recalling the definition of VaR as the worst loss we are $1 - \alpha$ confident will not be exceeded, it comes that there is a α chance that the loss will in fact be worse than the VaR. For example, for a sample of 1000 daily VaR estimates, if the VaR is estimated at the 99% confidence level (and therefore the significance level $\alpha = 1\%$), we expect that the α worst observed loss will be exceeded $1000 \times \alpha = 10$ times. This translates into 10 exceedances, or equivalently, an exceedance rate of $\frac{1000}{10} = 1\% = \alpha$.

Formally, for a sample of n observations, for each observation we identifty an exceedance through an indicator function as:

$$I_{\alpha,t} = \begin{cases} 1, & \text{if } r_t < -VaR_{1,\alpha,t} \\ 0, & \text{otherwise} \end{cases}, \tag{33}$$

where r_t is the return observed for day t and $VaR_{1,\alpha,t}$ is the VaR estimated for day t. We are left with a series of n observations where each observation is either 1 or 0, depending on the indicator function above.

We test if the null hypothesis that the indicator function, which is assumed to follow an i.i.d. Bernoulli process, has a probability equal to the significance level α of the VaR model (Alexander, 2009). Formally, the null and alternative hypothesis for the UC test are:

$$H_0 : \pi_{obs} = \pi_{exp} \equiv \alpha$$
$$H_1 : \pi_{obs} \neq \pi_{exp},$$

where π_{obs} and π_{exp} are the observed and expected exceedance rates, respectively,

The test statistic is:

$$LR_{UC} = \left(\frac{\pi_{exp}}{\pi_{obs}}\right)^{n_1} \left(\frac{1-\pi_{exp}}{1-\pi_{obs}}\right)^{n_0}, \tag{34}$$

where n_1 and $n_0 = n - n_1$ are the number of exceedances and non-exceedances, respectively.

The distribution of the test statistic under the null hypothesis ($\pi_{obs} = \pi_{exp}$) follows a chi-squared distribution with one degree of freedom: $-2\ln(LR_{UC}) \sim \chi_1^2$.

Recalling the beginning of the section, we can deem the model well specified if the null hypothesis described above is not rejected at the 95% confidence level. In other words, it means that according to the test the exceedance rate is within the expected value of α.

5.2. BCP Test

For the BCP test, a model is said to be well specified when the exceedances are independent from each other (Berkowitz et al., 2011), which means we cannot predict when the next exceedance will happen based on an exceedance that has already occured. In other words, the autocorrelation in exceedances is 0 at all lags. For example, in the case where we estimate the VaR daily, in a volatile week where an exceedance is observed, the next trading days may also present exceedances whether these are consecutive or not, thus presenting an exceedance clustering event, and therefore signaling that the model may not be adapting fast enough to recent volatility increases. The BCP test aims to capture these events.

Formally, the null and alternative hypothesis for the BCP test are:

$$
\begin{aligned}
&H_0 : \hat{\rho}_k = 0, \forall k \in \{1, \ldots, K\} \\
&H_1 : \exists k \in \{1, \ldots, K\} \text{ s.t. } \hat{\rho}_k \neq 0,
\end{aligned}
$$

where $\hat{\rho}_k$ is the lag k autocorrelation of the series of n observations where each observation is either 1 or 0, given by the indicator function described in Equation 33 and K is the maximum autocorrelation lag considered in the test.

The test statistic is given by:

$$
BCP\left(K\right) = n\left(n+2\right)\sum_{k=1}^{K}\frac{\hat{\rho}_k^2}{n-k}, \tag{35}
$$

where n is the sample size, or number of observations, of the test.

The distribution of the test statistic under the null hypothesis ($\hat{\rho}_k = 0$) follows a chi-squared distribution with K degrees of freedom: $BCP\left(K\right) \sim \chi^2_K$. We are free to choose the lag K, but before we do that its important to be aware of the properties of a larger or smaller K. A larger K offers insight into higher order autocorrelations, but because the test statistic of the null hypothesis follows a chi-squared distribution with K degrees of freedom, as we increase K we increase its critical value, which in turn makes the null hypothesis harder to reject. On the other hand, if we choose a smaller K, we increase the sensitivity of the test but at the same time we ignore the autocorrelations with lags $> K$. Aware of these properties, we will compute the BCP test for $K = 1$ and go up to $K = 5$, which means we will test for autocorrelation up to the 5^{th} lag while still keeping the sensitivity of the test at an acceptable level.

5.3. Backtest Results and Model Selection

We recall the global period of 6 years from 1 October 2015 to 30 September 2021 for which we computed daily VaR estimates for every model. Using the UC and BCP tests presented earlier in this chapter, in this section we assess the performance of each model for both the global period and annual sub-periods when relevant and conclude with the choice of the best model for our current portfolio composition.

This global period totals $n = 1566$ observations, and given the VaR models computed used a significance level of $\alpha = 1\%$, from a UC test point of view we expect a well specified model to have close to $1566 \times 1\% \approx 16$ exceedances.

Traditionally, we reject the null hypothesis when the p-value of the test statistic is below 5%. As such, it follows that a model is accepted by the UC or BCP tests when their null hypothesis is not rejected, that is, if the p-value is above 5%.

Table 5 below presents the number of exceedances, the exceedance rate and p-value of the UC test for all models for the global period.

Model Class	**Model No.**	**Global Period**		
		No. of Exceedances	**Exc. Rate (%)**	**p-value (%)**
RM	1	32	2.04	0.03
SGSt	**2**	**20**	**1.28**	**29.06**
	3	**21**	**1.34**	**19.74**
	4	**21**	**1.34**	**19.74**
	5	**22**	**1.40**	**12.91**
Hist	**6**	**16**	**1.02**	**93.14**
	7	**13**	**0.83**	**48.64**
	8	**18**	**1.15**	**56.16**
	9	**17**	**1.09**	**73.71**
	10	32	2.04	0.03
	11	27	1.72	0.09
	12	30	1.92	0.12
	13	29	1.85	0.25
QR	**14**	**20**	**1.28**	**29.06**
	15	**21**	**1.34**	**19.74**
	16	**21**	**1.34**	**19.74**

Table 5. UC test results for the global period. The rows in bold denote the models that pass the UC test. A model passes the UC test when its corresponding p-value is above 5%. See Table 4 for the description of each model.

We observe from Table 5 that, not surprisingly, we clearly reject the RiskMetrics model represented by model number 1. Its high number of exceedances points to its main flaw of assuming a normal distribution for the returns, which is not the case for our portfolio for the global period (see Appendix A) as its returns display negative skewness, meaning the distribution is asymmetric to the left, as well as excess kurtosis (kurtosis > 3), which translates into fatter distribution tails than the normal distribution.

However, surprisingly, the UC test rejects all the $T+1$ volatility-adjusted Historical models represented by model numbers 10 to 13. Within the accepted, the T volatility-adjusted Historical models represented by model numbers 6 to 9 clearly outperform the other accepted models with a number of exceedances much closer to the expected ($1566 \times 1\% \approx 16$), and within these, the T volatility-adjusted Historical with a rolling sample of 250 observations represented by model 6 comes as the winner for the UC test, closely followed by the T volatility-adjusted Historical with a rolling sample of 1000 observations represented by model 9. For the analysis going forward we exclude the models that were rejected by the UC test in Table 5.

Table 6 below presents the p-values of the BCP test for the global period for the models that passed the UC test. Appendix B.1 presents the BCP test results for the global period and for all models.

Model Class	**Model No.**	***p*-value (%)**				
		Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**
SGSt	2	**60.81**	**76.86**	0.00	0.00	0.00
	3	**59.01**	**74.79**	0.00	0.00	0.00
	4	**59.01**	**74.79**	0.00	0.00	0.00
	5	**57.22**	**72.67**	0.00	0.00	0.00
Hist	**6**	**68.24**	**84.57**	**19.39**	**29.94**	**40.95**
	7	**74.00**	**89.57**	**5.17**	**9.72**	**15.84**
	8	**64.49**	**80.85**	0.10	0.23	0.49
	9	**66.36**	**82.74**	**25.40**	**37.19**	**48.64**
QR	14	**60.81**	**76.86**	0.00	0.00	0.00
	15	**59.01**	**74.79**	0.98	1.99	3.53
	16	**59.01**	**74.79**	0.98	1.99	3.53

Table 6. BCP test results for the global period. The values in bold denote the p-values higher than 5%. The model numbers in bold denote the models that pass the BCP test for the five lags. A model passes the BCP test when its corresponding p-value is above 5%. See Table 4 for the description of each model.

We observe from Table 6 that although all models pass the BCP tests for lags 1 and 2, from lag 3 onward most models are clearly rejected. An encouraging result is that models 6 and 9, which for the UC test presented the best results, also present the best results for the BCP test for all lags.

Through the results obtained by the UC and BCP tests in Tables 5 and 6, respectively, we can confidently narrow our choice to models 6 and 9. To help us form a decision, a deeper analysis that we can perform is to look into the annual sub-periods and assess the performance of each one.

Table 7 presents the exceedance rate (%) and p-value (%) of the UC test for models 6 and 9 for the annual sub-periods comprising the global period. Appendix B.2 presents the UC test results for the annual sub-periods for all models.

Model No.	2021-2020		2020-2019		2019-2018		2018-2017		2017-2016		2016-2015	
	Exc.	**p-value**	**Exc.**	**p-value**	**Exc.**	**p-value**	**Exc.**	**p-value**	**Exc.**	**p-value**	**Exc.**	**p-value**
6	0.77	69.23	1.91	18.91	1.15	81.27	0.77	69.67	0.77	69.67	0.76	68.79
9	0.77	69.23	1.91	18.91	1.15	81.27	0.77	69.67	0.77	69.67	1.15	81.76

Table 7. UC test results for the annual sub-periods. A model passes the UC test when its corresponding p-value is higher than 5%.

We observe that both models behave well and very similarly, with the only difference being for the sub-period of 2016-2015 where model 9 shows an exceedance rate slightly higher than model 6 due to having one more exceedance, but still well within the acceptance range.

One last assessement we can make is to look into the annual sub-periods once again but this time for the BCP test. Tables 8 and 9 below present the p-values of the BCP test for the annual sub-periods and for all five lags tested for models 6 and 9, respectively.

Sub-periods	p-value (%)				
	Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**
2021-2020	89.98	88.42	99.72	99.95	99.99
2020-2019	75.05	91.27	0.75	1.70	3.31
2019-2018	84.96	96.45	99.07	99.74	99.93
2018-2017	89.96	98.41	99.72	99.95	99.99
2017-2016	89.96	98.41	99.72	99.95	99.99
2016-2015	90.00	98.42	99.72	99.95	99.99

Table 8. Model number 6 BCP test results for the annual sub-periods. A model passes the BCP test for a certain lag when its corresponding p-value is above 5%.

Sub-periods	p-value (%)				
	Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**
2021-2020	89.98	98.42	99.72	99.95	99.99
2020-2019	75.05	90.33	2.59	5.24	9.14
2019-2018	84.96	96.45	99.07	99.74	99.93
2018-2017	89.96	98.41	99.72	99.95	99.99
2017-2016	89.96	98.41	99.72	99.95	99.99
2016-2015	84.99	98.46	99.08	99.75	99.93

Table 9. Model number 9 BCP test results for the annual sub-periods. A model passes the BCP test for a certain lag when its corresponding p-value is higher than 5%.

We observe that both models perform very similarly for all sub-periods. Additionally, both models drop the ball on lag 3 for the 2020-2019 sub-period, which is not surprising given it was a year with unusually high levels of volatility.

Despite the analysis done, the choice between both models continues to not be obvious. While model 6 presents a higher UC test p-value for the global period, this advantage comes from the 2016-2015 sub-period, where it presented 1 fewer exceedance than model 9. This is not significant considering what we observed in Table 5 where models 6 and 9 presented exceedances of 16 and 17, respectively. Considering this, one can choose either model, but we will opt for model 9, the T volatility-adjusted Historical VaR with a rolling sample of 1000 observations, because it offers a wider range of potential scenarios due to its higher sample size, and thus, not being limited to what happened only the last year (which is the case when we use 250 observations).

Figure 3 below presents the daily VaR estimates for model 9 and the portfolio's daily P&L over the global period of the Backtest.

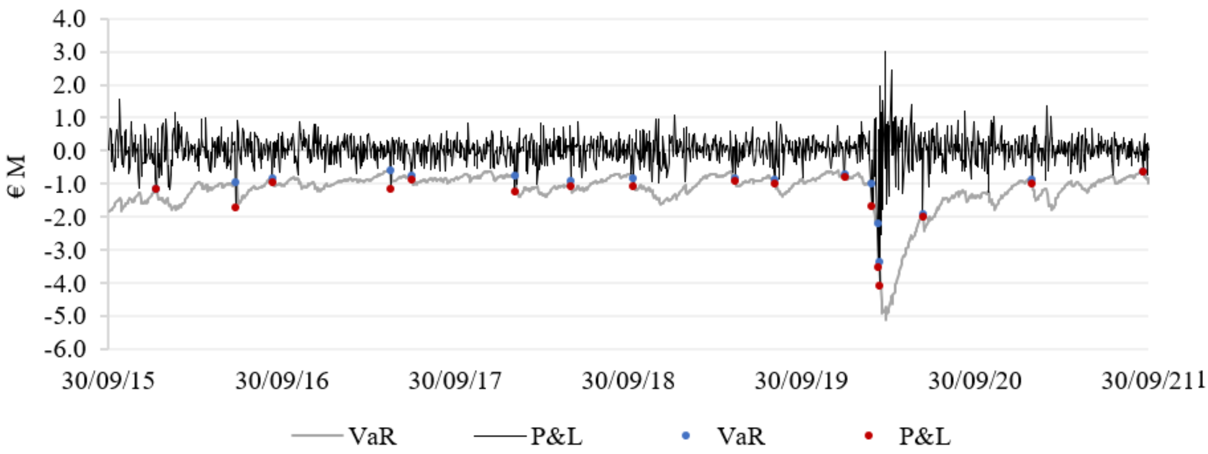

Figure 3. Historical VaR model number 9 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

We observe that for the majority of the global period, the exceedances happened with a considerable amount of days between them. However, during the year of 2020 there are several exceedances occurring within a low number of days. We dive deeper with Table 10 below which presents the exceedance details for the global period for model 9.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
20/09/2021	-0.63	-0.63	-0.01	0.94
27/01/2021	-0.86	-0.99	-0.13	15.62
11/06/2020	-1.92	-1.99	-0.07	3.67
12/03/2020	-3.34	-4.06	-0.72	21.62
09/03/2020	-2.18	-3.50	-1.32	60.80
24/02/2020	-0.97	-1.65	-0.68	69.49
30/12/2019	-0.70	-0.78	-0.08	12.07
05/08/2019	-0.86	-0.97	-0.10	12.03
13/05/2019	-0.83	-0.90	-0.07	8.04
10/10/2018	-0.82	-1.08	-0.26	31.14
31/05/2018	-0.90	-1.06	-0.16	17.84
02/02/2018	-0.75	-1.21	-0.46	62.16
29/06/2017	-0.73	-0.85	-0.13	17.55
17/05/2017	-0.60	-1.15	-0.55	92.84
09/09/2016	-0.84	-0.95	-0.11	12.73
24/06/2016	-0.96	-1.69	-0.73	76.48
08/01/2016	-1.15	-1.16	-0.01	0.91
Average			-0.33	30.35

Table 10. Model 9 global period exceedance details. The date of the exceedances and the days between them add additional detail to the BCP test results.

Its interesting to observe that in fact 3 days separate the exceedances that occured in 12/03/2020 and 09/03/2020, which was correctly captured by the BCP test and showcased in its lower result for Lag 3 presented for the year of 2020 in Table 9.

Appendices B.3.1 to B.3.15 present this analysis for all models which sheds a light on each model's success or failure in passing the Backtest. Additionally, it sheds a light on why some models passed the UC test but failed the BCP test at Lag 3 (for example models 2 and 14.)

CHAPTER 6

VALUE-AT-RISK MANAGEMENT

We recall the definition of EC as the capital at risk resulting from the portfolio's activity during a given time horizon and for a given significance level. This capital at risk is measured by the VaR, which means that the EC and VaR are numerically equal. It follows that if we define a target for the EC of our portfolio, it is around this target that risk management strategies can be formulated.

Going forward we measure the VaR with model 9 chosen in Chapter 5, that is: T volatility-adjusted Historical VaR with a rolling sample of 1000 observations. For this model, we observed in Figure 3 from Chapter 5 that for the Backtest period the VaR ranged mostly between €1 million and €2 million, with the exception being part of the year of 2020 which went well beyond this interval. Additionally, the 30 September 2021 VaR estimate was close to €1 million, which translates into close to 1.0% of the portfolio value. As such, for the one year going forward, we will aim to be within this historical range and go further by setting the daily EC to a maximum of €1.5 million. This means that the capital at risk resulting from holding our portfolio during the next day, which is measured by the 1-day VaR, must be at most €1.5 million.

In order to comply with this target, the process we adopt is as follows: today, right before market close, we estimate the VaR for tomorrow using today's current portfolio composition and, if the estimate is above €1.5 million, we implement a given strategy into the portfolio composition such that the new VaR estimate with the new portfolio composition does not exceed the defined limit of €1.5 million. Tomorrow, at the same time, we repeat the process for the next day and we do this daily for the time period of one year from 30 September 2021 to 30 September 2022.

However, the question of what strategy to implement inevitably arises. If the goal is to reduce risk, we can either alter existing portfolio positions or hedge existing exposures within our portfolio. We will opt for a hedging strategy to reduce risk and to achieve this we first need to assess the contribution of each risk factor to the VaR of the portfolio so that we can formulate a hedging strategy. This is achieved by performing a marginal VaR decomposition through which we obtain the proportion of the portfolio's VaR that can be attributed to each risk factor. In other words, if the VaR is the whole cake, the marginal decomposition gives us the slices that comprise the cake and their respective magnitude.

Section 1 presents the marginal VaR decomposition methodology and resulting strategy. Section 2 presents the results of the strategy for the one year period.

6.1. VaR Decomposition and Management Strategy

Let ∇ denote the gradient vector which lists the sensitivities of the portfolio's VaR to small changes in the exposure to each of the n risk factors away from the current values Θ. Let S be the decomposition vector which lists the current exposures to each risk factor θ_i according to the type of decomposition we aim to perform. We obtain the first sensitivity entry for the gradient vector ∇ with the following process: fix a small number ε (e.g. €1), perturb only the first risk factor exposure θ_1 by ε as:

$$\Theta_1 = \begin{bmatrix} \theta_1 + \varepsilon \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}, \tag{36}$$

obtain the time series of returns for the perturbed portfolio Θ_1, compute the VaR (VaR_{Θ_1}) and the first sensitivity entry for the gradient vector ∇ is given by $\frac{VaR_{\Theta_1} - VaR_{\Theta}}{\varepsilon}$. We repeat this process for each θ_i until we have the sensitivity of all n risk factors.

Finally, we obtain the marginal VaR as:

$$MarginalVaR = S^T \nabla \tag{37}$$

The decomposition performed is up to the user and in our case we will first decompose the VaR estimate by asset class and for the first day it surpasses €1.5 million: 17 February 2022. This allows us to assess how much each asset class contributes to the VaR of the portfolio. Table 11 below presents the marginal VaR decomposition by asset class.

Description	**Asset Class**			**Total**
	Equity	**FX**	**Bonds**	
VaR (%)	1.69	0.01	-0.10	**1.60**
VaR (EUR)	1 656 676	13 848	-101 579	**1 568 946**
VaR Breakdown (%)	105.59	0.88	-6.47	**100.00**

Table 11. VaR decomposition by asset class. This table showcases how much each asset class is contributing to the VaR estimate for 17 February 2022.

We observe that the equity exposure contributes to a higher VaR, the foreign currency exchange rate exposure contributes slightly to a higher VaR and the bonds exposure contribute to a lower VaR. Given this, we dive deeper and for equity and bonds we decompose their risk into assets from the U.S. and European markets, respectively.

Table 12 below presents the VaR decomposition by asset class and market.

Description	Equity			Bonds			Total
	U.S.	European	FX	U.S.	European	FX	
VaR (%)	1.11	0.58	0.01	-0.03	-0.07	0.01	**1.60**
VaR (EUR)	1 086 850	569 826	7 068	-29 907	-71 672	6 779	**1 568 946**
VaR Breakdown (%)	69.27	36.32	0.45	-1.91	-4.57	0.43	**100.00**

Table 12. VaR decomposition by asset class and market. This showcases how much stocks and bonds from the U.S. and European markets contribute to the VaR estimate for 17 February 2022.

Given the above, a possible strategy to decrease the VaR is to hedge the equity exposure for both U.S. and European stocks. We can achieve this by adding short positions in broad stock indices such as the S&P500 and DAX for the U.S. and European stocks, respectively, to our portfolio through exchange traded funds that are commonly available nowadays. We will trade these exchange traded funds in the European market such that we do not incur additional foreign currency exchange rate exposure and we will aim to tackle both equity exposures simultaneously.

Whenever the VaR estimate for the next day is above €1.5 million, we take a short position in each index and the size of each position is defined such that the two following conditions are met: the VaR of the portfolio is at most €1.5 million and the ratio between the marginal VaR for the U.S. and European equities is preserved. For example, looking at Table 12 above we observe that the marginal VaR for the U.S. stocks is approximately double the marginal VaR for the European stocks. As such, the short position we will take in the S&P500 index will also be approximately double the short position we take in the DAX index. We perform the VaR decomposition on the first day of each month and set the ratio for the short positions described above depending on the results of the decomposition.

The VaR is estimated daily and as such the short positions are also adjusted daily. Whenever the short positions are not warranted, meaning the VaR is below €1.5 million without them, the short positions are removed. We repeat this process daily until 30 September 2022.

6.2. VaR Management Results

For comparison purposes, we define the portfolio with the hedging strategy implemented as the Hedged portfolio and the same portfolio but without the hedging strategy implemented as the Unhedged portfolio.

Figure 4 below presents the daily VaR estimates for both the Unhedged and Hedged portfolios for the one year period.

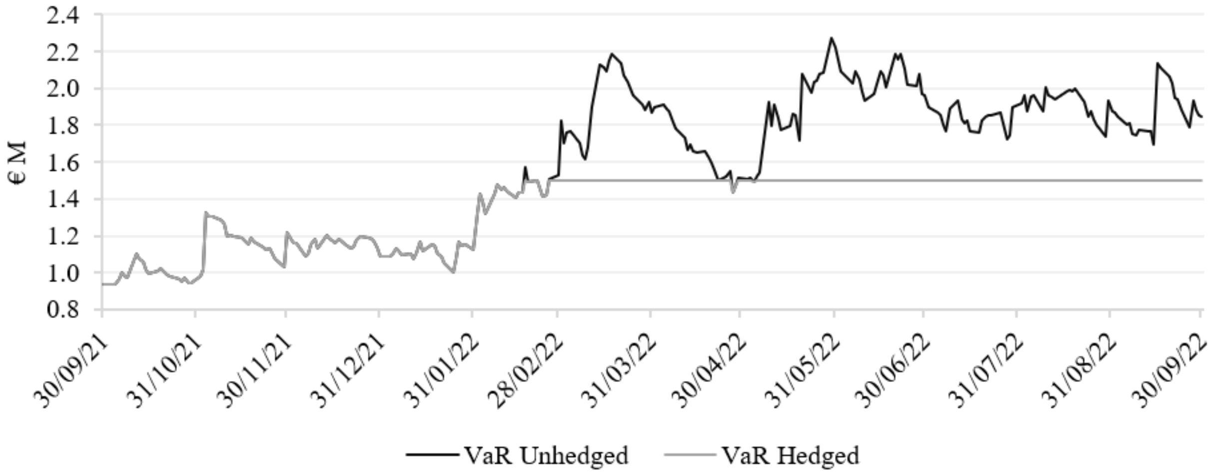

Figure 4. Daily VaR estimates.

Figure 5 below presents the hedge position evolution through the one year period.

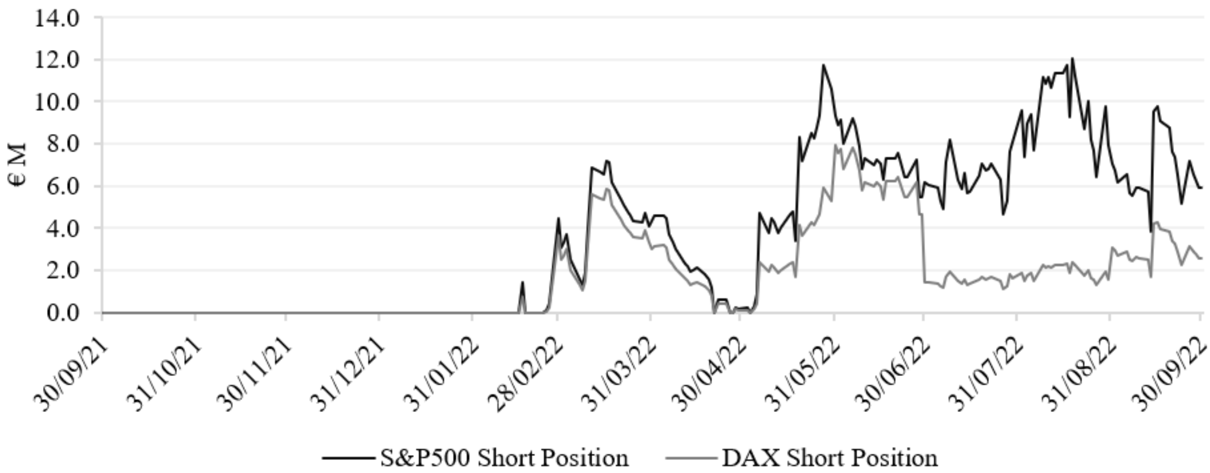

Figure 5. Daily short positions.

We observe from Figures 4 and 5 that whenever the VaR exceeds the target of €1.5 million, a short position in both S&P500 and DAX stock indices is implemented. Because of this, the VaR of the Hedged portfolio is kept at a maximum of €1.5 million throughout the year, whereas for the Unhedged portfolio the VaR goes well beyond this benchmark, reaching a maximum of close to €2.3 million at the end of May 2022.

Figure 6 below presents the daily P&L for both the Unhedged and Hedged portfolios for the one year period. Figures 7 and 8 present the Unhedged and Hedged VaR (shown as a loss) performance for the one year period.

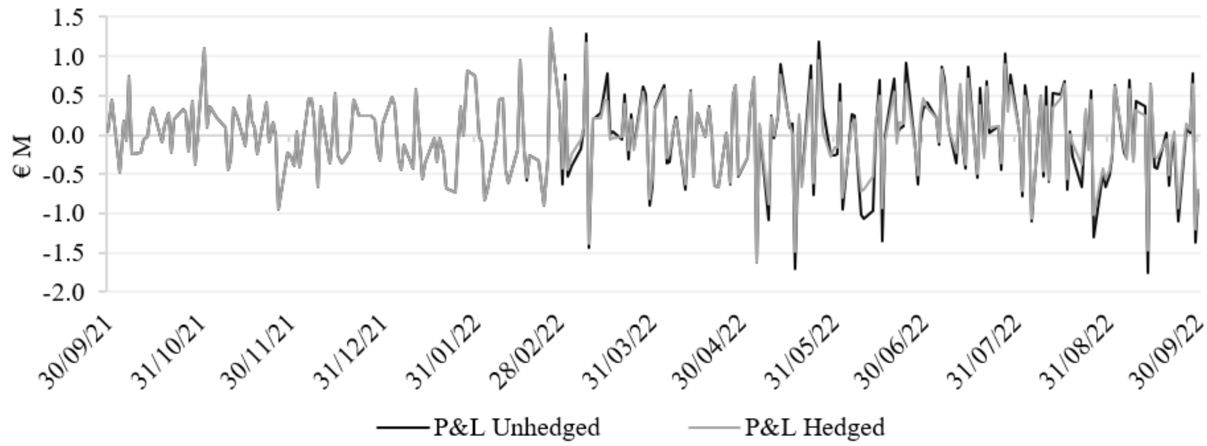

Figure 6. Daily P&L.

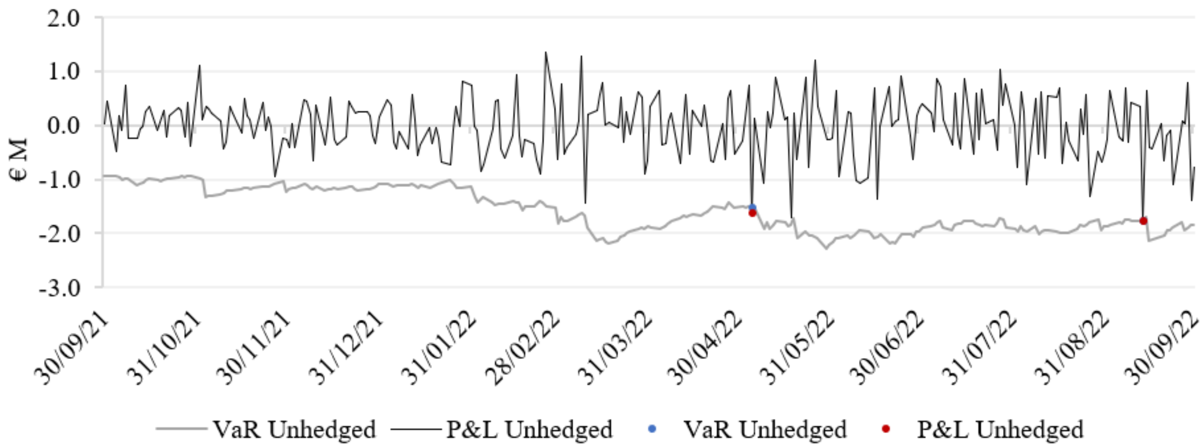

Figure 7. Unhedged VaR performance for the one year period.

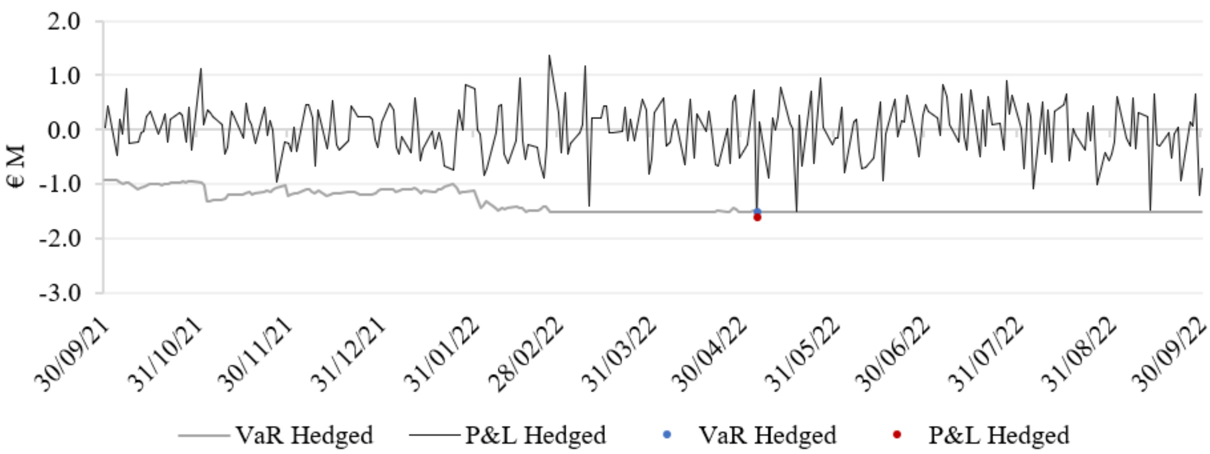

Figure 8. Hedged VaR performance for the one year period.

We observe from Figure 6 how the Unhedged portfolio shows more volatile daily P&Ls than the Hedged portfolio (see Appendix A for the Unhedged and Hedged returns descriptive statistics). This is more evident for periods where the short position is larger, which causes a wider discrepancy between the Unhedged and Hedged portfolios P&Ls. This is especially noticeable in the May 2022 to June 2022 period and onwards.

Additionally, we observe from Figures 7 and 8 that the Unhedged VaR presented two exceedances for the year, while the Hedged VaR presented one. This difference can be attributed to the lower volatility of returns for the Hedged portfolio since if we look at the day of the second exceedance, despite the VaR estimate for the Hedged portfolio being smaller the loss did not surpass it. Nonetheless, the size of the exceedances in both cases was still low.

Finally, Figure 9 below presents the daily cumulative P&L for both the Unhedged and Hedged portfolios for the one year period.

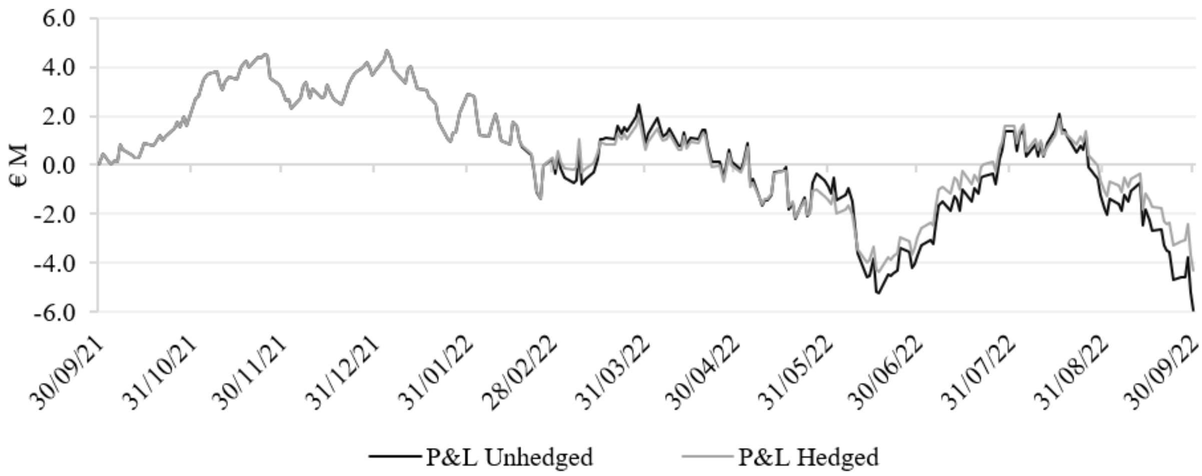

Figure 9. Daily cumulative P&L.

We observe from Figure 9 that both portfolios returned a loss for the year. However, the loss was smaller for the Hedged portfolio, which indicates that the strategy, while decreasing the VaR of the portfolio, was successful in safeguarding against further losses.

Table 13 below presents the returns for the year for both the Unhedged and Hedged portfolios.

Metric	**Portfolio**	
	Unhedged	**Hedged**
P&L (€M)	-5.96	-4.36
Return (%)	-6.15	-5.27

Table 13. One year portfolio returns.

We observe that the Hedged portfolio delivered a smaller loss for the year than the Unhedged portfolio, which is what we observed from Figure 9 before. However, because both portfolios have different risk profiles due to the strategy implemented, it may not be fair to directly compare the returns without accounting for the risk profile of each portfolio. This invokes a need for a risk-adjusted performance metric to level the playing field.

The RORAC (Matten, 1996) relates the return observed with the risk incurred to achieve it. We calculate the RORAC as:

$$RORAC = \frac{P\&L}{EC}, \tag{38}$$

where $P\&L$ is the P&L for the year and EC is the sum of the daily EC (measured by the VaR) throughout the year.

Table 14 below presents the RORAC for the year for the Unhedged and Hedged portfolios.

Metric	**Portfolio**	
	Unhedged	**Hedged**
P&L (€M)	-5.96	-4.36
EC (€M)	414.36	356.16
RORAC (%)	**-1.44**	**-1.23**

Table 14. RORAC for the one year period.

From these results we conclude that the Hedged portfolio incurred losses of smaller magnitude when compared to the Unhedged portfolio while lowering the risk incurred, which for the given market conditions during the year, it is a satisfactory result.

CHAPTER 7

CONCLUSION

The goal of this work was to measure and manage the VaR of our portfolio on a daily basis during a one year period from 30 September 2021 to 30 September 2022 such that it did not surpass a pre-defined maximum value.

The portfolio studied in this work was comprised of part equities and part bonds from both the U.S. and European markets and before we could manage its VaR, we first had to measure it. Given there are several classes of models to measure market risk, the question of which one is the best fit for our specific portfolio composition arose. To answer this question, we computed four classes of VaR models: RiskMetrics, SGSt, Historical and QR. Within these four classes, we computed 16 different models in total such that we had a wide range of candidates to analyze and choose from. The performance of each model was analyzed through a Backtest comprised of the UC and BCP tests.

The Backtest analysis using the portfolio composition at 30 September 2021 presented both expected and unexpected results. As expected, as the VaR was computed at the 1% significance level, the RiskMetrics VaR which assumes returns follow a normal distribution failed the Backtest convincingly, signaling that the returns of our portfolio do not follow a normal distribution and as such the RiskMetrics model is unfit. Still in the parametric category, the SGSt VaR model which aims to tackle the shortcomings of the RiskMetrics model, while not failing the Backtest, did not impress. Outside of the parametric world, the QR VaR passed the Backtest with results similar to the SGSt VaR. Finally, the Historical VaR models with the T volatility adjustment methodology produced the best results, and after a further analysis where the performance for individual annual sub-periods was assessed, we chose the T volatility-adjusted Historical VaR with a rolling sample of 1000 observations model as the best fit for our portfolio.

In order to manage the VaR, and based on historical ranges, the limit chosen was a daily maximum of €1.5 million. Starting from 30 September 2021, and with the model chosen previously, daily VaR estimates were computed and if the VaR estimate for the next day surpassed the pre-defined maximum, a risk management strategy would have to be implemented to decrease the VaR to the maximum value of €1.5 million.

The VaR first surpassed the maximum pre-defined threshold of €1.5 million on 17 February 2022 and in order to assess the sources of risk comprising this exceeding VaR measurement, a VaR decomposition was performed and it revealed that the source of the risk was in the U.S. and European equities. As such, in order to decrease the VaR to a maximum of €1.5 million, an equity exposure hedging strategy was implemented which consisted in simultaneous short positions in the S&P500 and DAX stock market indices according to the proportions revealed by the VaR decomposition. This methodology was carried forward during the rest of the year where the positions were increased, decreased or removed depending on the VaR estimates relative to the pre-defined maximum and for comparison purposes, we defined the portfolio with the strategy implemented as the Hedged portfolio and the same portfolio but without the strategy implemented as the Unhedged portfolio.

Results showed that the hedging strategy managed to keep the VaR at a maximum of €1.5 million throughout the year, whereas if it was not managed, this threshold would have been surpassed during a significant portion of the year starting from February 2022 and reaching a maximum of close to €2.3 million in May 2022. Finally, when comparing the returns achieved for the Hedged and Unhedged portfolios, using the RORAC risk-adjusted performance metric, we conclude that while the return for the year was negative for both portfolios, the Hedged portfolio returned a less negative return while also incurring less risk over the period, which means it safeguarded against further losses when compared to the Unhedged portfolio.

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Appendices

APPENDIX A

Portfolio Returns Descriptive Statistics

In the below table we present the portfolio returns descriptive statistics for the global period and annual sub-periods of the Backtest as well as for the one year period from 2021 to 2022 presented in Chapter 6, this time for both the Unhedged and Hedged portfolios.

Time Period	**Mean**	**Median**	**Max**	**Min**	**StdDev**	**Skew**	**Kurt**
Unhedged Portfolio 2022-2021	0.0226%	0.0092%	1.4322%	-1.8442%	0.5704%	-0.3694	3.3787
Hedged Portfolio 2022-2021	-0.0192%	-0.0046%	1.4305%	-1.6896%	0.5401%	-0.3528	3.5529
2021-2020	0.0336%	0.0262%	1.4121%	-1.2941%	0.3688%	-0.0575	4.3071
2020-2019	0.0486%	0.0934%	3.1147%	-4.2143%	0.7227%	-1.2458	11.7363
2019-2018	0.0352%	0.0587%	1.1140%	-1.1171%	0.3827%	-0.3716	3.5665
2018-2017	0.0152%	0.0279%	0.8973%	-1.2582%	0.3488%	-0.3800	3.9311
2017-2016	0.0317%	0.0608%	0.8912%	-1.1929%	0.3259%	-0.2983	3.3985
2016-2015	0.0318%	0.0618%	1.6216%	-1.7509%	0.4613%	-0.1905	4.0102
Global Backtest Period	0.0332%	0.0607%	3.1147%	-4.2143%	0.4553%	-0.9131	13.7758

Table 15. Portfolio returns descriptive statistics. The Global Backtest Period ranges from 30 September 2021 to 1 October 2015.

APPENDIX B

Backtest Model Performance Details

B.1. BCP test results for all models for the global period.

Model Class	**Model No.**	***p*-value (%)**				
		Lag 1	**Lag 2**	**Lag 3**	**Lag 4**	**Lag 5**
RM	1	40.83	64.59	1.77	1.04	1.66
SGSt	2	60.81	76.86	0.00	0.00	0.00
	3	59.01	74.79	0.00	0.00	0.00
	4	59.01	74.79	0.00	0.00	0.00
	5	57.22	72.67	0.00	0.00	0.01
Hist	6	68.24	84.57	19.39	29.94	40.95
	7	74.00	89.57	5.17	9.72	15.84
	8	64.49	80.85	0.10	0.23	0.49
	9	66.36	82.74	25.40	37.19	48.64
	10	40.83	50.44	1.41	2.87	4.26
	11	48.68	61.63	0.11	0.22	0.41
	12	43.88	62.93	0.70	1.40	2.27
	13	45.45	61.46	0.40	0.80	1.36
QR	14	60.81	76.86	0.00	0.00	0.00
	15	59.01	74.79	0.98	1.99	3.53
	16	59.01	74.79	0.98	1.99	3.53

Table 16. BCP test results for all models for the global period. The Global Period ranges from 30 September 2021 to 1 October 2015. A model passes the BCP test for a certain lag when its corresponding p-value is above 5%.

B.2. UC test results for all models and annual sub-periods.

Model No.	2021-2020		2020-2019		2019-2018		2018-2017		2017-2016		2016-2015	
	Exc.	***p*-value**	**Exc.**	***p*-value**	**Exc.**	***p*-value**	**Exc.**	***p*-value**	**Exc.**	***p*-value**	**Exc.**	***p*-value**
1	2.68	2.38	3.05	0.72	2.30	7.13	1.54	41.87	1.15	80.77	1.53	42.65
2	1.15	81.27	2.29	7.24	1.53	42.26	0.77	69.67	0.77	69.67	1.15	81.76
3	1.53	42.26	2.29	7.24	1.53	42.26	1.15	80.77	0.77	69.67	0.76	68.79
4	1.15	81.27	2.67	2.43	1.53	42.26	1.15	80.77	0.77	69.67	0.76	68.79
5	1.15	81.27	2.67	2.43	1.53	42.26	1.15	80.77	0.77	69.67	1.15	81.76
6	0.77	69.23	1.91	18.91	1.15	81.27	0.77	69.67	0.77	69.67	0.76	68.79
7	0.38	25.22	1.53	42.65	1.15	81.27	0.77	69.67	0.38	25.44	0.76	68.79
8	0.77	69.23	1.91	18.91	1.53	42.26	0.77	69.67	0.77	69.67	1.15	81.76
9	0.77	69.23	1.91	18.91	1.15	81.27	0.77	69.67	0.77	69.67	1.15	81.76
10	1.92	18.68	3.05	0.72	2.30	7.13	1.54	41.87	1.92	18.44	1.53	42.65
11	1.92	18.68	2.67	2.43	1.92	18.68	1.54	41.87	0.77	69.67	1.53	42.65
12	2.30	7.13	3.05	0.72	2.30	7.13	1.15	80.77	1.15	80.77	1.53	42.65
13	2.30	7.13	3.05	0.72	2.30	7.13	1.15	80.77	0.77	69.67	1.53	42.65
14	0.38	25.22	2.67	2.43	1.53	42.26	1.15	80.77	0.77	69.67	1.15	81.76
15	1.15	81.27	2.67	2.43	1.15	81.27	1.15	80.77	0.77	69.67	1.15	81.76
16	1.15	81.27	1.91	18.91	1.53	42.26	1.15	80.77	0.77	69.67	1.53	42.65

Table 17. UC test results for the annual sub-periods for all models. This table presents the exceedance rate (%) and p-value (%) for all models and for the annual sub-periods. A model passes the UC test when its corresponding p-value is above 5%.

B.3. VaR models global period performance and exceedance details.

In the below sections we present the global period performance and exceedance details for all models not presented in the main body of this work. This provides further insight on each model's pass or failure in the Backtest as well as on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.1. RiskMetrics VaR global period performance and exceedance details.

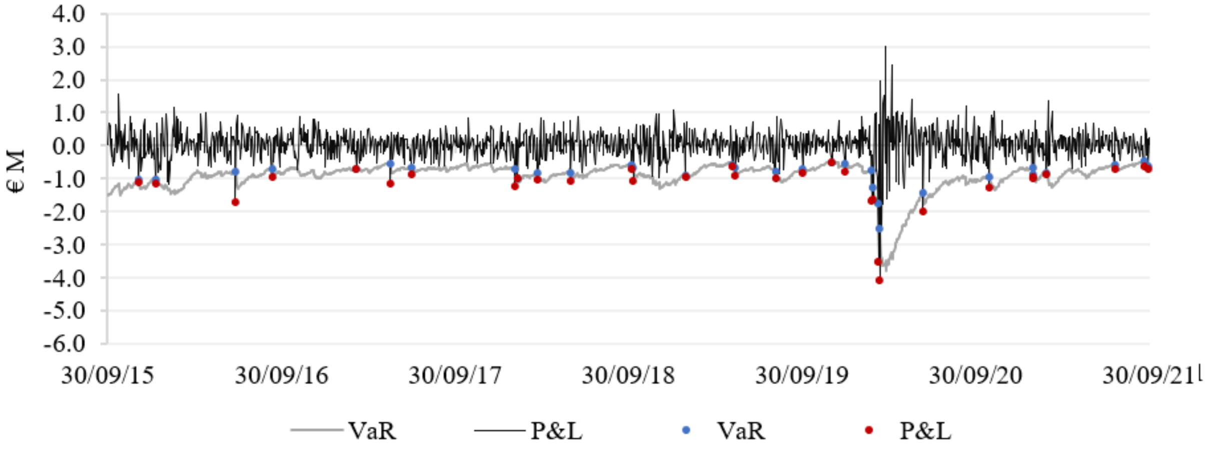

Figure 10. RiskMetrics model number 1 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
28/09/2021	-0.64	-0.72	-0.08	13.00
20/09/2021	-0.48	-0.63	-0.15	31.22
19/07/2021	-0.60	-0.71	-0.10	17.20
25/02/2021	-0.82	-0.88	-0.06	7.87
29/01/2021	-0.85	-0.95	-0.10	11.62
27/01/2021	-0.65	-0.99	-0.34	52.52
28/10/2020	-0.96	-1.25	-0.29	29.96
11/06/2020	-1.42	-1.99	-0.57	39.95
12/03/2020	-2.51	-4.06	-1.55	61.81
09/03/2020	-1.73	-3.50	-1.78	102.79
27/02/2020	-1.25	-1.64	-0.39	30.93
24/02/2020	-0.76	-1.65	-0.89	117.45
30/12/2019	-0.55	-0.78	-0.23	40.93
02/12/2019	-0.50	-0.52	-0.02	3.61
02/10/2019	-0.69	-0.81	-0.12	17.28
05/08/2019	-0.77	-0.97	-0.20	25.46
13/05/2019	-0.65	-0.90	-0.26	39.62
07/05/2019	-0.58	-0.64	-0.06	10.07
28/01/2019	-0.89	-0.93	-0.04	4.90
10/10/2018	-0.68	-1.08	-0.39	57.37
05/10/2018	-0.57	-0.70	-0.13	22.12
31/05/2018	-0.82	-1.06	-0.24	29.18
22/03/2018	-0.84	-1.02	-0.18	21.41
08/02/2018	-0.97	-0.99	-0.02	2.57
02/02/2018	-0.70	-1.21	-0.51	73.34
29/06/2017	-0.66	-0.85	-0.19	28.69
17/05/2017	-0.53	-1.15	-0.62	118.57
06/03/2017	-0.70	-0.70	-0.01	0.92
09/09/2016	-0.70	-0.95	-0.25	35.23
24/06/2016	-0.77	-1.69	-0.92	120.48
08/01/2016	-1.03	-1.16	-0.13	12.42
03/12/2015	-1.04	-1.12	-0.08	7.48
Average			-0.34	37.12
Number of Exceedances				32
Exceedance Rate (%)				2.04

Table 18. RiskMetrics VaR model number 1 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.2. SGSt VaR, rolling sample of 250 obs. global period performance and exceedance details.

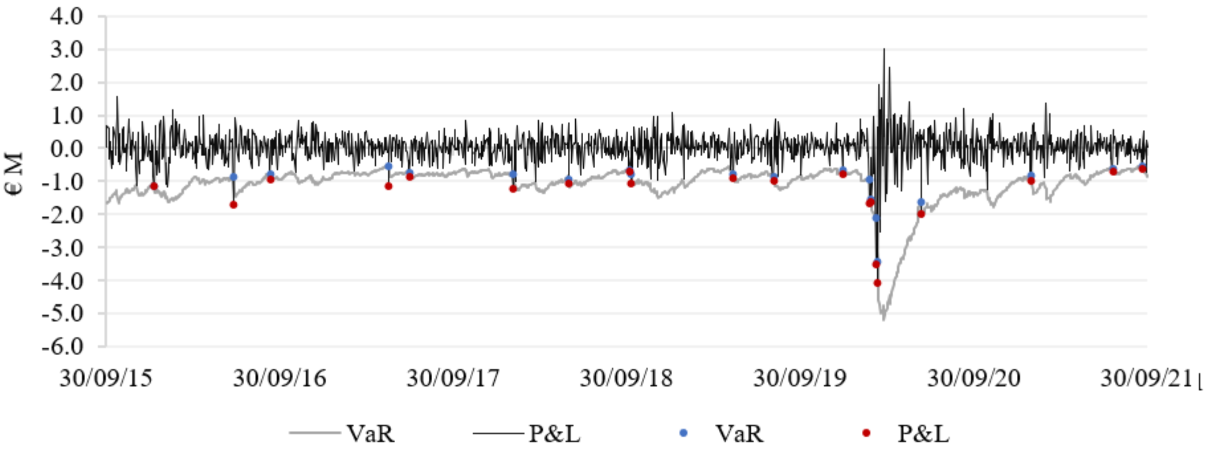

Figure 11. SGSt VaR model number 2 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
20/09/2021	-0.55	-0.63	-0.08	14.93
19/07/2021	-0.63	-0.71	-0.08	12.05
27/01/2021	-0.83	-0.99	-0.16	19.01
11/06/2020	-1.63	-1.99	-0.35	21.64
12/03/2020	-3.44	-4.06	-0.62	18.00
09/03/2020	-2.11	-3.50	-1.39	65.97
27/02/2020	-1.53	-1.64	-0.11	7.16
24/02/2020	-0.93	-1.65	-0.72	77.97
30/12/2019	-0.66	-0.78	-0.12	18.69
05/08/2019	-0.88	-0.97	-0.09	10.08
13/05/2019	-0.77	-0.90	-0.13	16.57
10/10/2018	-0.79	-1.08	-0.28	35.74
05/10/2018	-0.66	-0.70	-0.04	5.33
31/05/2018	-0.96	-1.06	-0.10	9.93
02/02/2018	-0.80	-1.21	-0.41	51.03
29/06/2017	-0.73	-0.85	-0.13	17.53
17/05/2017	-0.56	-1.15	-0.59	106.31
09/09/2016	-0.78	-0.95	-0.17	22.02
24/06/2016	-0.86	-1.69	-0.83	97.40
08/01/2016	-1.15	-1.16	-0.01	1.07
Average			-0.32	31.42
Number of Exceedances				20
Exceedance Rate (%)				1.28

Table 19. SGSt VaR model number 2 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.3. SGSt VaR, rolling sample of 500 obs. global period performance and exceedance details.

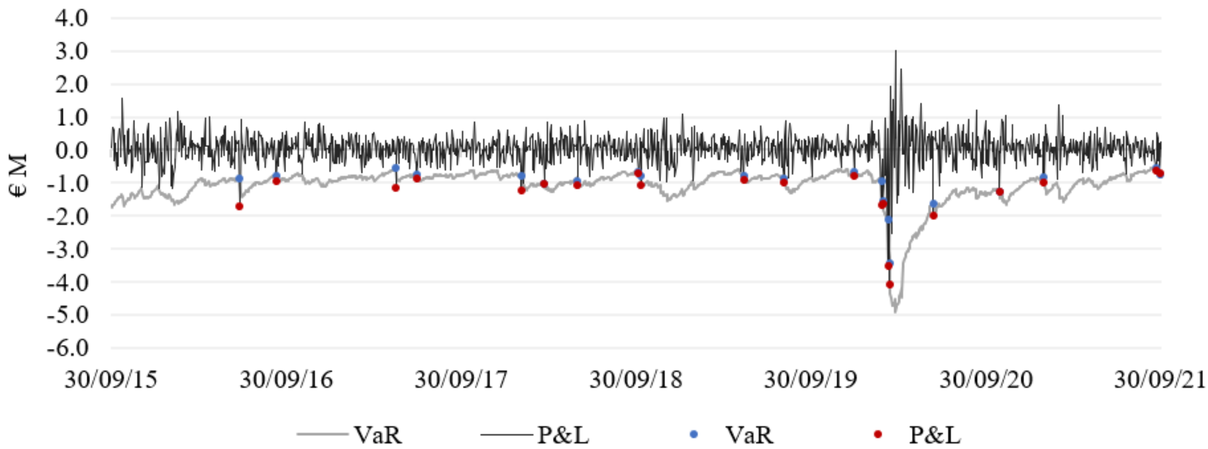

Figure 12. SGSt VaR model number 3 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
28/09/2021	-0.72	-0.72	0.00	0.26
20/09/2021	-0.55	-0.63	-0.08	15.44
27/01/2021	-0.80	-0.99	-0.19	24.46
28/10/2020	-1.19	-1.25	-0.06	4.87
11/06/2020	-1.57	-1.99	-0.41	26.26
12/03/2020	-3.26	-4.06	-0.80	24.56
09/03/2020	-2.06	-3.50	-1.44	69.93
27/02/2020	-1.49	-1.64	-0.15	9.71
24/02/2020	-0.91	-1.65	-0.74	82.22
30/12/2019	-0.66	-0.78	-0.12	17.88
05/08/2019	-0.89	-0.97	-0.07	8.22
13/05/2019	-0.76	-0.90	-0.14	17.99
10/10/2018	-0.78	-1.08	-0.30	37.82
05/10/2018	-0.65	-0.70	-0.05	6.95
31/05/2018	-0.95	-1.06	-0.11	12.07
22/03/2018	-0.96	-1.02	-0.06	6.15
02/02/2018	-0.77	-1.21	-0.45	58.18
29/06/2017	-0.76	-0.85	-0.10	12.60
17/05/2017	-0.60	-1.15	-0.55	92.66
09/09/2016	-0.77	-0.95	-0.18	23.21
24/06/2016	-0.83	-1.69	-0.85	102.35
Average			-0.33	31.13
Number of Exceedances				21
Exceedance Rate (%)				1.34

Table 20. SGSt VaR model number 3 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.4. SGSt VaR, rolling sample of 750 obs. global period performance and exceedance details.

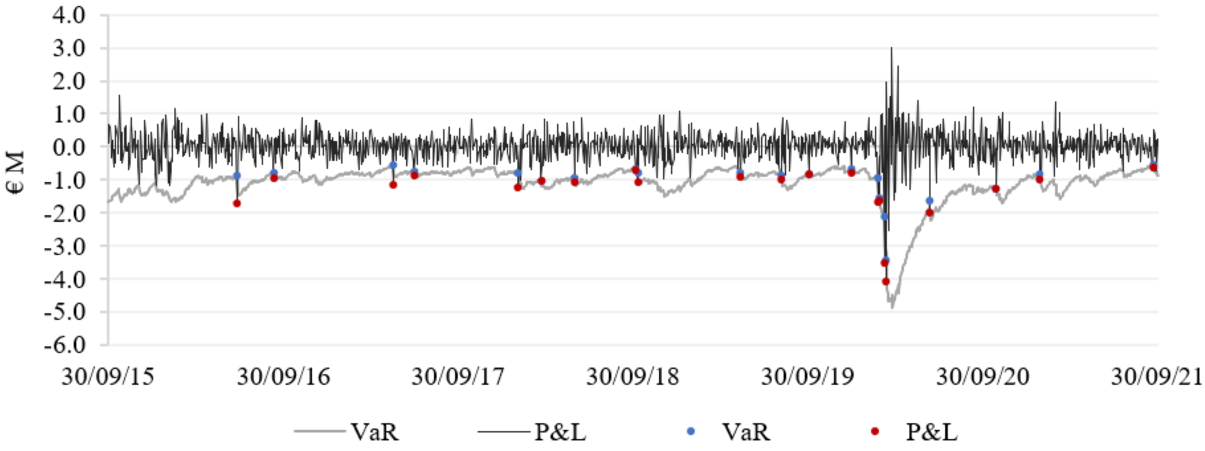

Figure 13. SGSt VaR model number 4 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
20/09/2021	-0.58	-0.63	-0.05	8.22
27/01/2021	-0.82	-0.99	-0.18	21.56
28/10/2020	-1.22	-1.25	-0.03	2.53
11/06/2020	-1.78	-1.99	-0.21	11.67
12/03/2020	-3.24	-4.06	-0.83	25.57
09/03/2020	-2.08	-3.50	-1.43	68.76
27/02/2020	-1.50	-1.64	-0.13	8.96
24/02/2020	-0.91	-1.65	-0.74	80.96
30/12/2019	-0.66	-0.78	-0.12	18.48
02/10/2019	-0.81	-0.81	0.00	0.21
05/08/2019	-0.89	-0.97	-0.08	8.58
13/05/2019	-0.76	-0.90	-0.15	19.25
10/10/2018	-0.79	-1.08	-0.29	36.34
05/10/2018	-0.66	-0.70	-0.04	5.80
31/05/2018	-0.96	-1.06	-0.10	10.38
22/03/2018	-0.98	-1.02	-0.04	3.91
02/02/2018	-0.80	-1.21	-0.41	50.89
29/06/2017	-0.74	-0.85	-0.11	14.74
17/05/2017	-0.59	-1.15	-0.56	96.40
09/09/2016	-0.80	-0.95	-0.15	18.52
24/06/2016	-0.87	-1.69	-0.82	94.59
Average			-0.31	28.87
Number of Exceedances				21
Exceedance Rate (%)				1.34

Table 21. SGSt VaR model number 4 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.5. SGSt VaR, rolling sample of 1000 obs. global period performance and exceedance details.

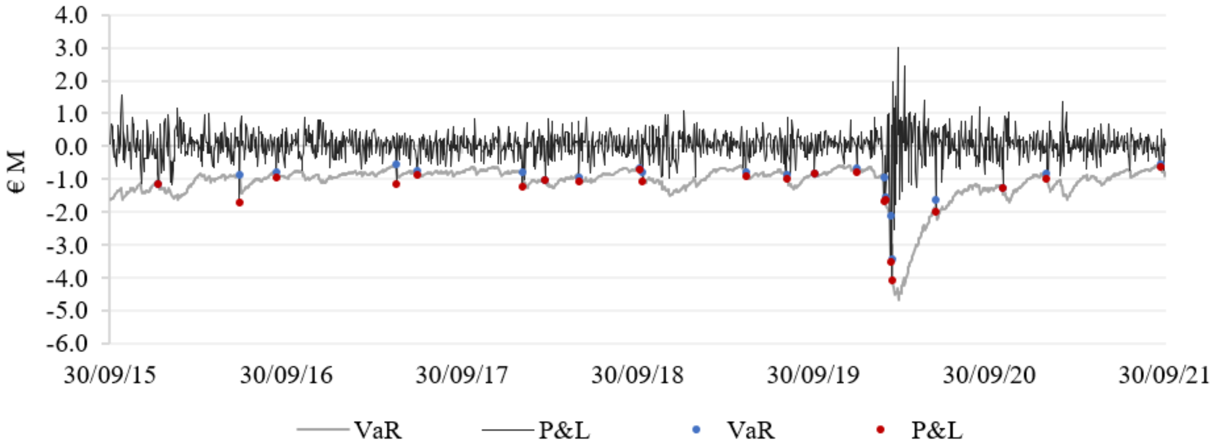

Figure 14. SGSt VaR model number 5 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
20/09/2021	-0.59	-0.63	-0.04	6.84
27/01/2021	-0.82	-0.99	-0.17	21.32
28/10/2020	-1.21	-1.25	-0.04	3.03
11/06/2020	-1.77	-1.99	-0.22	12.41
12/03/2020	-3.11	-4.06	-0.95	30.52
09/03/2020	-2.03	-3.50	-1.48	72.76
27/02/2020	-1.47	-1.64	-0.17	11.54
24/02/2020	-0.89	-1.65	-0.76	85.25
30/12/2019	-0.64	-0.78	-0.14	21.54
02/10/2019	-0.80	-0.81	0.00	0.48
05/08/2019	-0.89	-0.97	-0.08	8.42
13/05/2019	-0.76	-0.90	-0.14	18.14
10/10/2018	-0.79	-1.08	-0.28	35.95
05/10/2018	-0.66	-0.70	-0.04	5.50
31/05/2018	-0.93	-1.06	-0.13	13.50
22/03/2018	-0.96	-1.02	-0.06	5.87
02/02/2018	-0.80	-1.21	-0.41	51.98
29/06/2017	-0.76	-0.85	-0.10	12.75
17/05/2017	-0.60	-1.15	-0.55	92.83
09/09/2016	-0.78	-0.95	-0.17	21.54
24/06/2016	-0.84	-1.69	-0.85	100.78
08/01/2016	-1.12	-1.16	-0.04	3.33
Average			-0.31	28.92
Number of Exceedances				22
Exceedance Rate (%)				1.40

Table 22. SGSt VaR model number 5 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.6. *T* volatility-adjusted Historical VaR, rolling sample of 250 obs. global period performance and exceedance details.

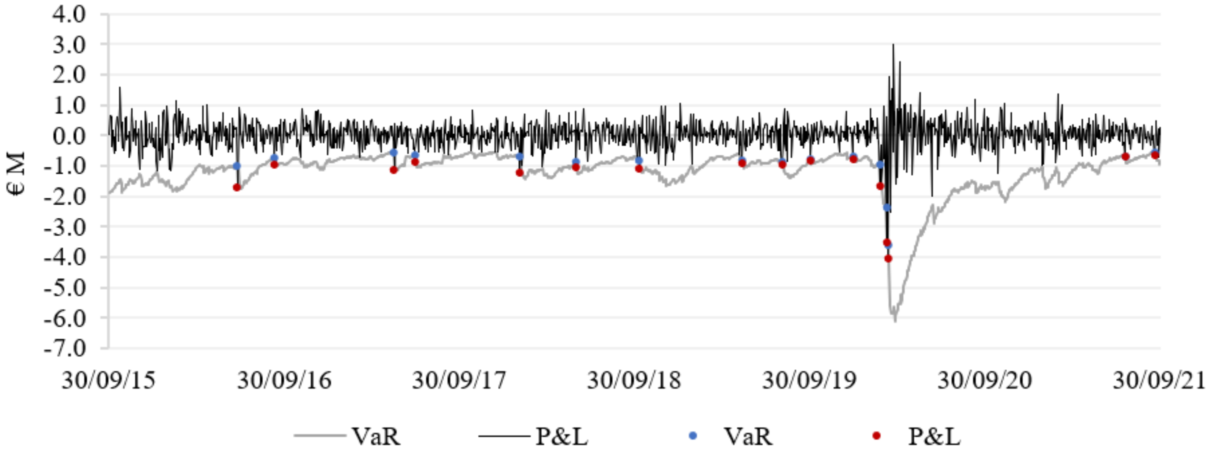

Figure 15. Historical VaR model number 6 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
20/09/2021	-0.56	-0.63	-0.07	12.76
19/07/2021	-0.68	-0.71	-0.03	4.55
12/03/2020	-3.60	-4.06	-0.47	12.99
09/03/2020	-2.36	-3.50	-1.15	48.77
24/02/2020	-0.97	-1.65	-0.68	69.49
30/12/2019	-0.67	-0.78	-0.11	16.70
02/10/2019	-0.78	-0.81	-0.03	4.00
05/08/2019	-0.87	-0.97	-0.10	11.37
13/05/2019	-0.81	-0.90	-0.09	11.54
10/10/2018	-0.82	-1.08	-0.26	31.14
31/05/2018	-0.87	-1.06	-0.19	22.37
02/02/2018	-0.67	-1.21	-0.54	80.62
29/06/2017	-0.65	-0.85	-0.20	30.94
17/05/2017	-0.55	-1.15	-0.60	110.20
09/09/2016	-0.74	-0.95	-0.21	28.46
24/06/2016	-0.99	-1.69	-0.70	70.54
Average			-0.34	35.40
Number of Exceedances				16
Exceedance Rate (%)				1.02

Table 23. Historical VaR model number 6 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.7. *T* volatility-adjusted Historical VaR, rolling sample of 500 obs. global period performance and exceedance details.

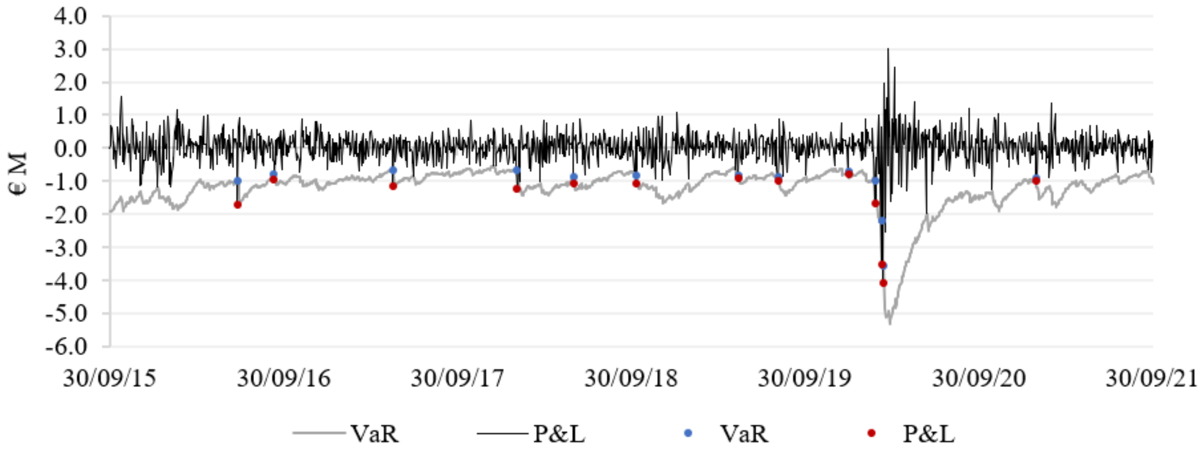

Figure 16. Historical VaR model number 7 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
27/01/2021	-0.92	-0.99	-0.07	8.16
12/03/2020	-3.56	-4.06	-0.50	14.05
09/03/2020	-2.21	-3.50	-1.30	58.65
24/02/2020	-0.97	-1.65	-0.68	69.49
30/12/2019	-0.72	-0.78	-0.07	9.08
05/08/2019	-0.87	-0.97	-0.10	11.37
13/05/2019	-0.81	-0.90	-0.09	11.54
10/10/2018	-0.82	-1.08	-0.26	31.14
31/05/2018	-0.87	-1.06	-0.19	22.37
02/02/2018	-0.67	-1.21	-0.54	80.62
17/05/2017	-0.68	-1.15	-0.47	68.26
09/09/2016	-0.77	-0.95	-0.18	22.82
24/06/2016	-0.99	-1.69	-0.70	70.54
Average			-0.40	36.78
Number of Exceedances				13
Exceedance Rate (%)				0.83

Table 24. HistoricalVaR model number 7 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.8. *T* volatility-adjusted Historical VaR, rolling sample of 750 obs. global period performance and exceedance details.

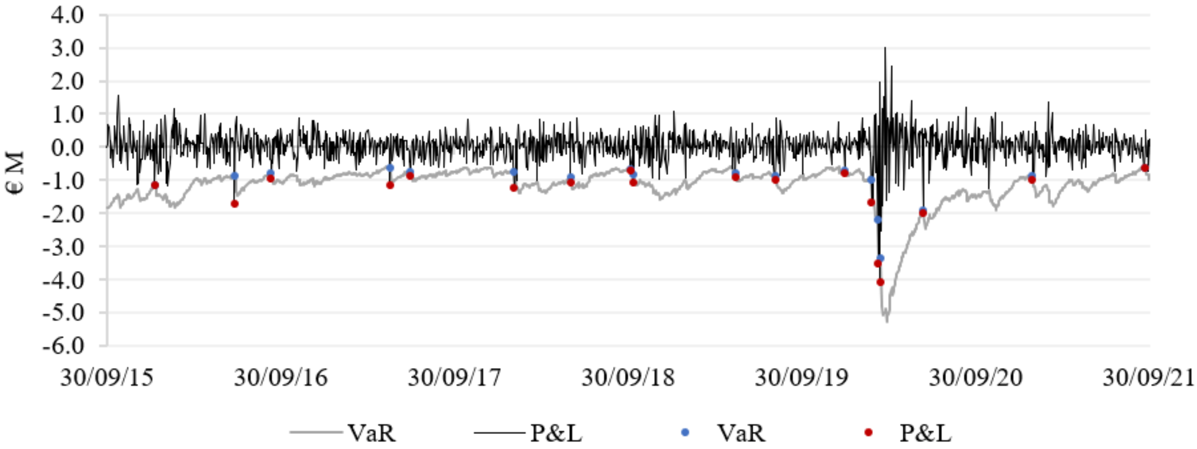

Figure 17. Historical VaR model number 8 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
20/09/2021	-0.63	-0.63	-0.01	0.94
27/01/2021	-0.86	-0.99	-0.13	15.62
11/06/2020	-1.92	-1.99	-0.07	3.67
12/03/2020	-3.34	-4.06	-0.72	21.62
09/03/2020	-2.18	-3.50	-1.32	60.80
24/02/2020	-0.97	-1.65	-0.68	69.49
30/12/2019	-0.70	-0.78	-0.08	12.07
05/08/2019	-0.86	-0.97	-0.10	12.03
13/05/2019	-0.80	-0.90	-0.10	12.20
10/10/2018	-0.82	-1.08	-0.26	31.91
05/10/2018	-0.65	-0.70	-0.05	6.98
31/05/2018	-0.90	-1.06	-0.16	17.84
02/02/2018	-0.75	-1.21	-0.46	62.16
29/06/2017	-0.73	-0.85	-0.13	17.55
17/05/2017	-0.61	-1.15	-0.54	88.71
09/09/2016	-0.77	-0.95	-0.18	22.82
24/06/2016	-0.86	-1.69	-0.82	95.45
08/01/2016	-1.16	-1.16	0.00	0.35
Average			-0.32	30.68
Number of Exceedances				18
Exceedance Rate (%)				1.15

Table 25. Historical VaR model number 8 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.9. $T+1$ volatility-adjusted Historical VaR, rolling sample of 250 obs. global period performance and exceedance details.

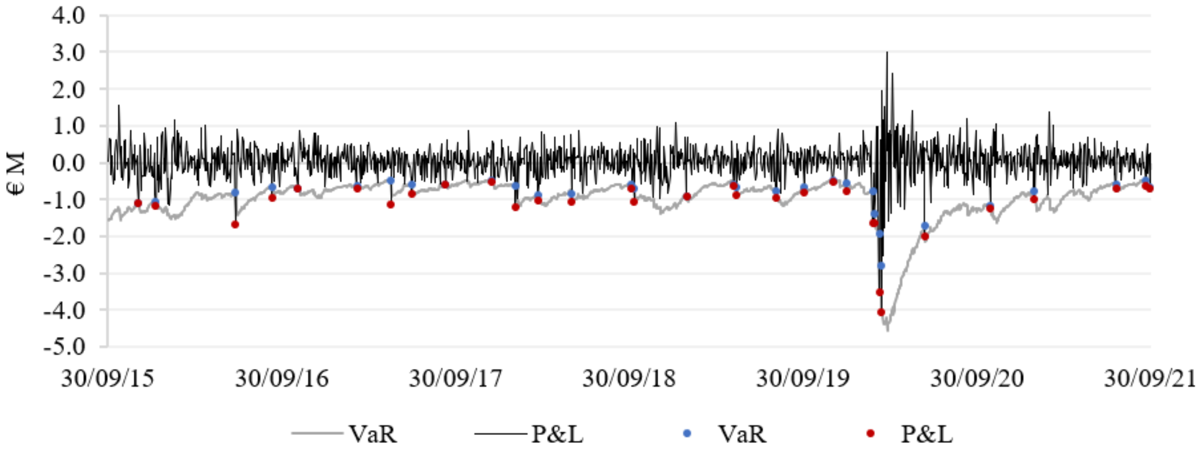

Figure 18. Historical VaR model number 10 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
28/09/2021	-0.68	-0.72	-0.04	6.08
20/09/2021	-0.48	-0.63	-0.15	31.81
19/07/2021	-0.58	-0.71	-0.13	21.76
27/01/2021	-0.79	-0.99	-0.21	26.11
28/10/2020	-1.16	-1.25	-0.09	7.46
11/06/2020	-1.72	-1.99	-0.27	15.72
12/03/2020	-2.81	-4.06	-1.25	44.55
09/03/2020	-1.92	-3.50	-1.58	82.16
27/02/2020	-1.39	-1.64	-0.25	17.61
24/02/2020	-0.79	-1.65	-0.86	108.78
30/12/2019	-0.55	-0.78	-0.23	41.50
02/12/2019	-0.50	-0.52	-0.02	4.03
02/10/2019	-0.66	-0.81	-0.15	23.03
05/08/2019	-0.79	-0.97	-0.18	22.61
13/05/2019	-0.66	-0.90	-0.24	36.44
07/05/2019	-0.60	-0.64	-0.05	7.57
28/01/2019	-0.91	-0.93	-0.02	2.51
10/10/2018	-0.70	-1.08	-0.38	53.79
05/10/2018	-0.58	-0.70	-0.12	19.81
31/05/2018	-0.84	-1.06	-0.22	26.72
22/03/2018	-0.89	-1.02	-0.13	14.69
02/02/2018	-0.63	-1.21	-0.59	93.61
14/12/2017	-0.49	-0.53	-0.03	6.48
08/09/2017	-0.58	-0.61	-0.03	5.08
29/06/2017	-0.59	-0.85	-0.26	43.73
17/05/2017	-0.47	-1.15	-0.68	144.14
06/03/2017	-0.62	-0.70	-0.09	14.24
02/11/2016	-0.69	-0.71	-0.02	2.47
09/09/2016	-0.66	-0.95	-0.29	44.29
24/06/2016	-0.80	-1.69	-0.89	111.02
08/01/2016	-1.08	-1.16	-0.08	7.59
03/12/2015	-1.08	-1.12	-0.03	2.87
Average			-0.30	34.07
Number of Exceedances				32
Exceedance Rate (%)				2.04

Table 26. Historical VaR model number 10 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.10. $T+1$ volatility-adjusted Historical VaR, rolling sample of 500 obs. global period performance and exceedance details.

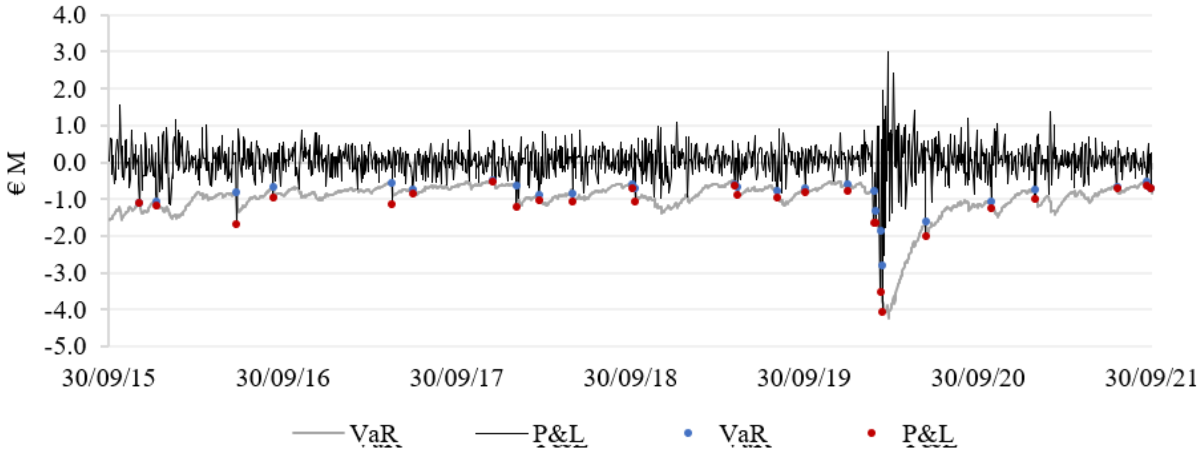

Figure 19. Historical VaR model number 11 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
28/09/2021	-0.71	-0.72	-0.01	0.94
20/09/2021	-0.54	-0.63	-0.09	17.22
19/07/2021	-0.67	-0.71	-0.03	4.70
27/01/2021	-0.72	-0.99	-0.27	36.81
28/10/2020	-1.07	-1.25	-0.18	16.58
11/06/2020	-1.59	-1.99	-0.40	25.02
12/03/2020	-2.80	-4.06	-1.27	45.35
09/03/2020	-1.85	-3.50	-1.65	89.48
27/02/2020	-1.33	-1.64	-0.31	23.38
24/02/2020	-0.79	-1.65	-0.86	108.78
30/12/2019	-0.58	-0.78	-0.20	35.31
02/10/2019	-0.72	-0.81	-0.09	12.60
05/08/2019	-0.79	-0.97	-0.18	22.61
13/05/2019	-0.66	-0.90	-0.24	36.44
07/05/2019	-0.60	-0.64	-0.05	7.57
10/10/2018	-0.70	-1.08	-0.38	53.79
05/10/2018	-0.58	-0.70	-0.12	19.81
31/05/2018	-0.84	-1.06	-0.22	26.72
22/03/2018	-0.89	-1.02	-0.13	14.69
02/02/2018	-0.63	-1.21	-0.59	93.61
14/12/2017	-0.49	-0.53	-0.03	6.48
29/06/2017	-0.72	-0.85	-0.13	18.13
17/05/2017	-0.55	-1.15	-0.60	109.20
09/09/2016	-0.68	-0.95	-0.27	39.79
24/06/2016	-0.80	-1.69	-0.89	111.02
08/01/2016	-1.08	-1.16	-0.08	7.59
03/12/2015	-1.10	-1.12	-0.02	1.85
Average			-0.34	36.05
Number of Exceedances				27
Exceedance Rate (%)				1.72

Table 27. Historical VaR model number 11 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.11. $T + 1$ volatility-adjusted Historical VaR, rolling sample of 750 obs. global period performance and exceedance details.

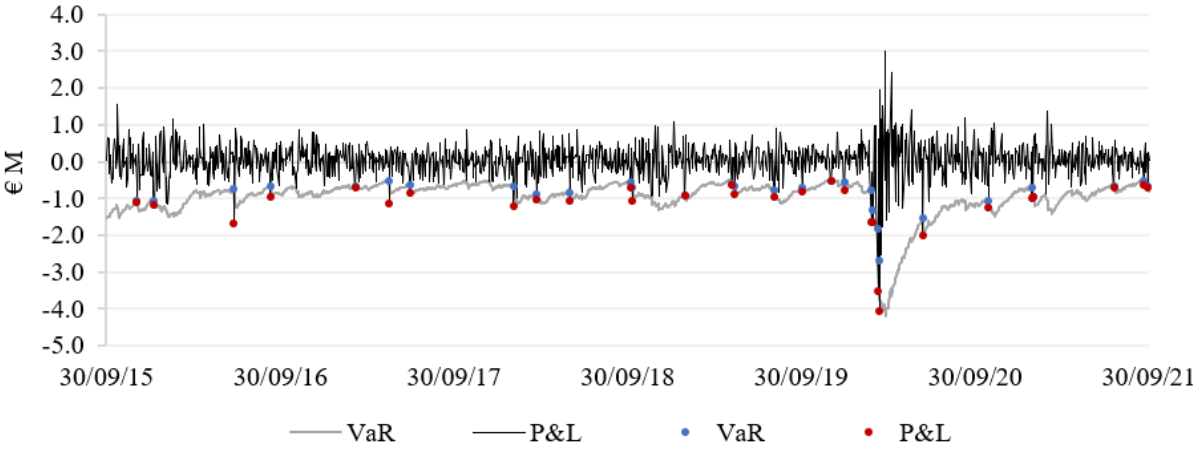

Figure 20. Historical VaR model number 12 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
28/09/2021	-0.68	-0.72	-0.04	5.44
20/09/2021	-0.51	-0.63	-0.12	22.61
19/07/2021	-0.67	-0.71	-0.04	5.28
29/01/2021	-0.95	-0.95	0.00	0.26
27/01/2021	-0.70	-0.99	-0.30	42.51
28/10/2020	-1.07	-1.25	-0.18	16.74
11/06/2020	-1.52	-1.99	-0.47	30.77
12/03/2020	-2.69	-4.06	-1.38	51.20
09/03/2020	-1.83	-3.50	-1.67	91.10
27/02/2020	-1.32	-1.64	-0.31	23.68
24/02/2020	-0.79	-1.65	-0.86	108.78
30/12/2019	-0.57	-0.78	-0.21	37.72
02/12/2019	-0.51	-0.52	-0.01	1.25
02/10/2019	-0.70	-0.81	-0.10	14.61
05/08/2019	-0.79	-0.97	-0.18	23.08
13/05/2019	-0.66	-0.90	-0.24	36.97
07/05/2019	-0.60	-0.64	-0.05	7.57
28/01/2019	-0.91	-0.93	-0.02	2.51
10/10/2018	-0.70	-1.08	-0.38	54.39
05/10/2018	-0.55	-0.70	-0.14	26.24
31/05/2018	-0.86	-1.06	-0.20	23.63
22/03/2018	-0.88	-1.02	-0.14	16.20
02/02/2018	-0.68	-1.21	-0.54	79.18
29/06/2017	-0.64	-0.85	-0.21	33.02
17/05/2017	-0.51	-1.15	-0.64	125.94
06/03/2017	-0.67	-0.70	-0.03	4.33
09/09/2016	-0.68	-0.95	-0.27	39.79
24/06/2016	-0.73	-1.69	-0.96	131.39
08/01/2016	-1.06	-1.16	-0.10	9.24
03/12/2015	-1.07	-1.12	-0.05	4.45
Average			-0.33	35.66
Number of Exceedances				30
Exceedance Rate (%)				1.92

Table 28. Historical VaR model number 12 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.12. $T+1$ volatility-adjusted Historical VaR, rolling sample of 1000 obs. global period performance and exceedance details.

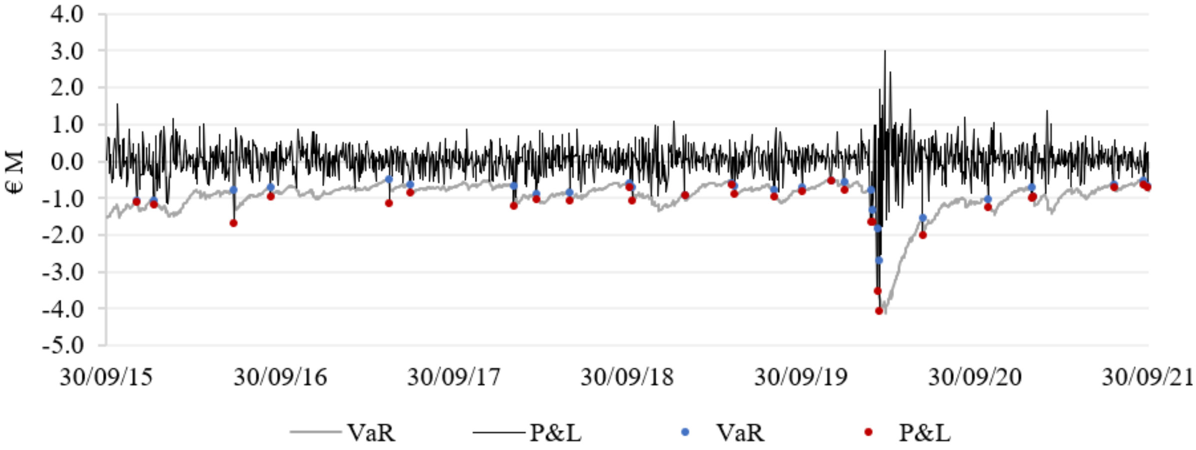

Figure 21. Historical VaR model number 13 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
28/09/2021	-0.68	-0.72	-0.04	5.44
20/09/2021	-0.51	-0.63	-0.12	22.61
19/07/2021	-0.65	-0.71	-0.06	9.51
29/01/2021	-0.95	-0.95	0.00	0.26
27/01/2021	-0.70	-0.99	-0.30	42.51
28/10/2020	-1.03	-1.25	-0.22	21.44
11/06/2020	-1.52	-1.99	-0.47	30.77
12/03/2020	-2.69	-4.06	-1.38	51.20
09/03/2020	-1.83	-3.50	-1.67	91.10
27/02/2020	-1.32	-1.64	-0.31	23.68
24/02/2020	-0.79	-1.65	-0.86	108.78
30/12/2019	-0.57	-0.78	-0.21	37.72
02/12/2019	-0.51	-0.52	-0.01	1.25
02/10/2019	-0.70	-0.81	-0.10	14.61
05/08/2019	-0.79	-0.97	-0.18	23.08
13/05/2019	-0.67	-0.90	-0.23	33.63
07/05/2019	-0.61	-0.64	-0.03	5.35
28/01/2019	-0.93	-0.93	0.00	0.39
10/10/2018	-0.70	-1.08	-0.38	53.79
05/10/2018	-0.58	-0.70	-0.12	19.81
31/05/2018	-0.86	-1.06	-0.20	23.63
22/03/2018	-0.88	-1.02	-0.14	16.20
02/02/2018	-0.68	-1.21	-0.54	79.18
29/06/2017	-0.64	-0.85	-0.21	33.02
17/05/2017	-0.50	-1.15	-0.65	129.39
09/09/2016	-0.72	-0.95	-0.23	31.90
24/06/2016	-0.78	-1.69	-0.91	115.79
08/01/2016	-1.06	-1.16	-0.10	9.64
03/12/2015	-1.06	-1.12	-0.05	4.83
Average			-0.33	35.88
Number of Exceedances				29
Exceedance Rate (%)				1.85

Table 29. Historical VaR model number 13 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.13. Quantile Regression with EWMA volatility as the explanatory variable, rolling sample of 1000 obs. global period performance and exceedance details.

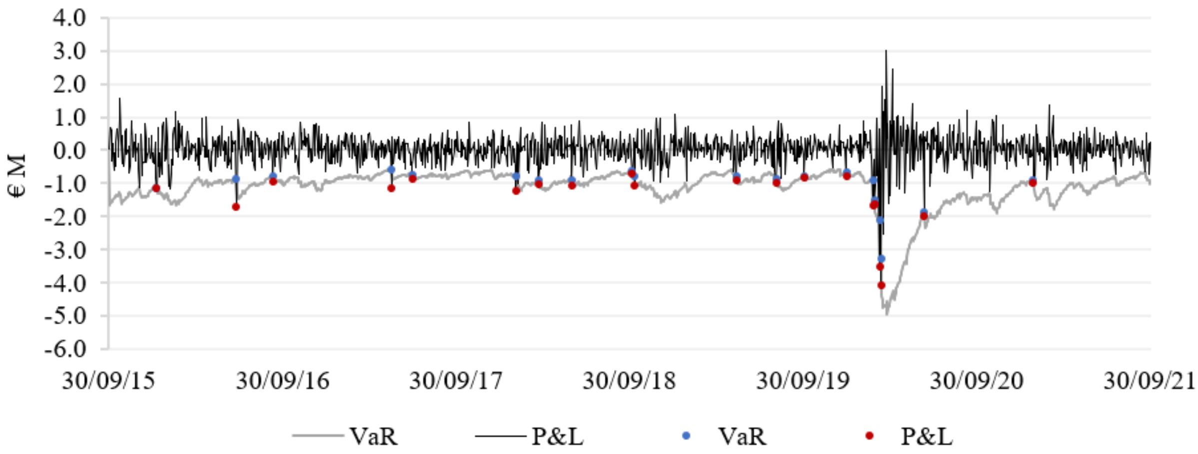

Figure 22. Quantile Regression VaR model number 14 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
27/01/2021	-0.91	-0.99	-0.08	9.24
11/06/2020	-1.86	-1.99	-0.13	6.89
12/03/2020	-3.29	-4.06	-0.78	23.59
09/03/2020	-2.10	-3.50	-1.41	67.02
27/02/2020	-1.52	-1.64	-0.12	7.84
24/02/2020	-0.92	-1.65	-0.73	79.10
30/12/2019	-0.65	-0.78	-0.13	20.17
02/10/2019	-0.77	-0.81	-0.03	4.32
05/08/2019	-0.85	-0.97	-0.12	13.98
13/05/2019	-0.79	-0.90	-0.11	14.33
10/10/2018	-0.77	-1.08	-0.31	39.99
05/10/2018	-0.64	-0.70	-0.06	8.64
31/05/2018	-0.92	-1.06	-0.14	14.91
22/03/2018	-0.92	-1.02	-0.10	10.37
02/02/2018	-0.77	-1.21	-0.44	57.57
29/06/2017	-0.73	-0.85	-0.12	16.98
17/05/2017	-0.57	-1.15	-0.58	103.36
09/09/2016	-0.79	-0.95	-0.16	20.30
24/06/2016	-0.86	-1.69	-0.83	96.13
08/01/2016	-1.14	-1.16	-0.02	2.19
Average			-0.32	30.85
Number of Exceedances				20
Exceedance Rate (%)				1.28

Table 30. Quantile Regression VaR model number 14 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.14. Quantile Regression with EWMA volatility and 5-day rolling average of EWMA volatility as the explanatory variables, rolling sample of 1000 obs.

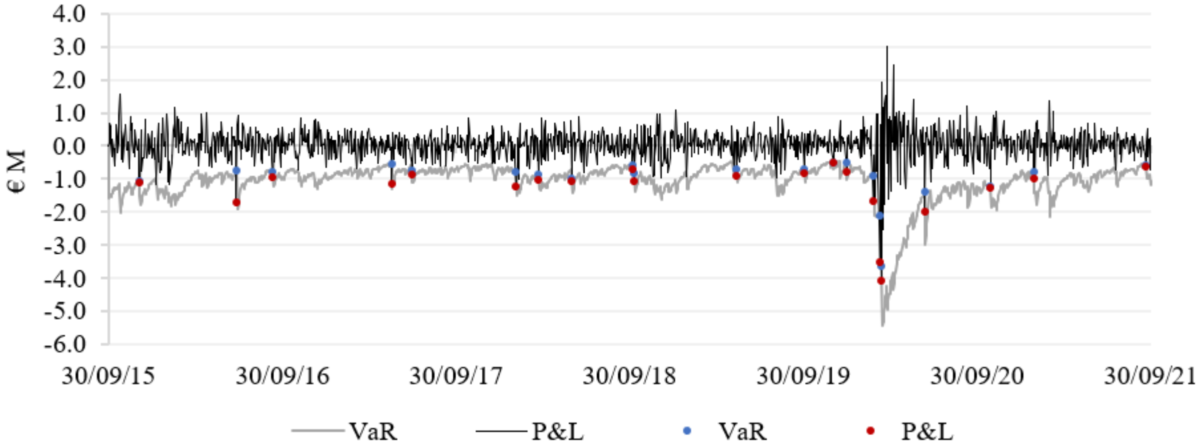

Figure 23. Quantile Regression VaR model number 15 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
20/09/2021	-0.58	-0.63	-0.05	9.26
27/01/2021	-0.77	-0.99	-0.22	29.19
28/10/2020	-1.21	-1.25	-0.04	3.13
11/06/2020	-1.37	-1.99	-0.61	44.61
12/03/2020	-3.63	-4.06	-0.43	11.83
09/03/2020	-2.12	-3.50	-1.38	64.90
24/02/2020	-0.91	-1.65	-0.74	80.98
30/12/2019	-0.52	-0.78	-0.26	49.95
02/12/2019	-0.50	-0.52	-0.02	4.30
02/10/2019	-0.69	-0.81	-0.12	17.91
13/05/2019	-0.71	-0.90	-0.20	27.65
10/10/2018	-0.84	-1.08	-0.24	28.56
05/10/2018	-0.60	-0.70	-0.10	16.68
31/05/2018	-0.99	-1.06	-0.07	6.58
22/03/2018	-0.88	-1.02	-0.14	16.08
02/02/2018	-0.78	-1.21	-0.44	56.17
29/06/2017	-0.76	-0.85	-0.10	12.64
17/05/2017	-0.54	-1.15	-0.61	114.23
09/09/2016	-0.80	-0.95	-0.15	18.58
24/06/2016	-0.76	-1.69	-0.92	121.17
03/12/2015	-1.08	-1.12	-0.03	2.97
Average			-0.33	35.11
Number of Exceedances				21
Exceedance Rate (%)				1.34

Table 31. Quantile Regression VaR model number 15 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.

B.3.15. Quantile Regression with EWMA volatility and 20-day rolling average of EWMA volatility as the explanatory variables, rolling sample of 1000 obs.

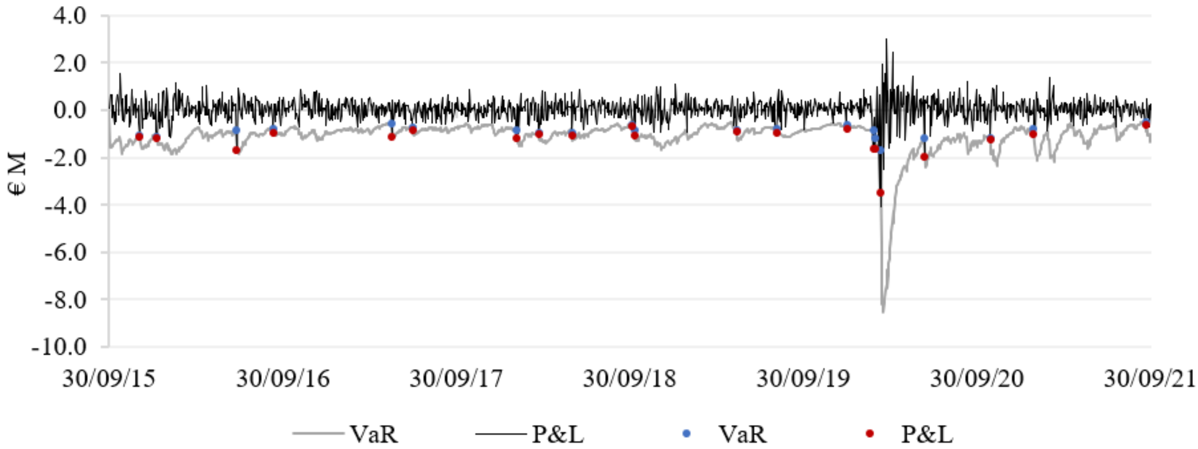

Figure 24. Quantile Regression VaR model number 16 global period performance. The red dots represent the P&L of the days in which the VaR was exceeded. The blue dots represent the VaR estimate for the corresponding days.

Date of Exceedance	VaR (€M)	P&L (€M)	Size of Exceedance (€M)	Size of Exceedance (% of VaR)
20/09/2021	-0.53	-0.63	-0.10	19.32
27/01/2021	-0.78	-0.99	-0.21	26.86
28/10/2020	-1.17	-1.25	-0.08	6.58
11/06/2020	-1.16	-1.99	-0.83	71.00
12/03/2020	-1.66	-3.50	-1.84	110.56
09/03/2020	-1.20	-1.64	-0.43	36.07
24/02/2020	-0.87	-1.65	-0.78	90.27
30/12/2019	-0.62	-0.78	-0.16	25.47
02/12/2019	-0.82	-0.97	-0.15	18.01
02/10/2019	-0.86	-0.90	-0.04	4.24
13/05/2019	-0.85	-1.08	-0.23	26.81
10/10/2018	-0.64	-0.70	-0.06	9.47
05/10/2018	-0.97	-1.06	-0.09	9.10
31/05/2018	-0.94	-1.02	-0.08	8.46
22/03/2018	-0.83	-1.21	-0.38	45.32
02/02/2018	-0.76	-0.85	-0.10	12.79
29/06/2017	-0.56	-1.15	-0.59	105.05
17/05/2017	-0.78	-0.95	-0.17	22.43
09/09/2016	-0.85	-1.69	-0.84	98.52
24/06/2016	-1.14	-1.16	-0.02	1.93
03/12/2015	-1.10	-1.12	-0.02	1.60
Average			-0.34	35.71
Number of Exceedances				21
Exceedance Rate (%)				1.34

Table 32. Quantile Regression VaR model number 16 global period exceedance details. The date of the exceedances and the days between them shed a light on the BCP and UC test results presented in Appendices B.1 and B.2, respectively.