6th International Conference
APLIMAT 2007
Faculty of Mechanical Engineering - Slovak University of Technology in Bratislava
Session: Modeling and Simulation

USING A COURNOT-NASH MODEL TO STUDY THE CONTRIBUTIONS OF COOPERATION FOR FISHING STOCKS RECOVER AND FOR FISHERS' RENTS

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Abstract. In our study we implement a model to explain the recover of stocks of a common resource pool in the fisheries area. Our aim is to model the behavior of two Fishing Companies through a model supported in Cournot model and in Nash principles of equilibrium. Our Cournot-Nash model is supported in the competing behavior of agents. Afterwards, we present an adjusted model to study the results of cooperation between agents.
We can see that cooperation brings up procedures that allow conservation of species and stocks recover. We conclude that agents' cooperation brings the possibility to get good results for species preservation and good rents for fishers.

Key words. Cournot-Nash Model, Nash Equilibrium.

Mathematics Subject Classification: Primary 91B76, 91A80; Secondary 90B50.

1 Introduction

Optimal management of fishing resources is often studied through a game theory technical framework. In our work, we aim to study sardine fisheries by using economic theory of Cournot oligopoly and Nash equilibrium, combining both approaches.

We have initially studied the Cournot model, as traditionally it has been used in economics. Then, we have adjusted it to integrate Nash equilibrium that is usually used in theory of games. We have considered yet an original variable for this kind of models (fishing effort), in order to study the efficiency of the inputs used in fisheries.

2 A Cournot-Nash Model: The Basic Model

The usual model (an oligopoly model) allows us to determine simultaneously the quantities produced by a few number of companies.

When we consider two fishing agents, catching fish (considered as a homogeneous product) and selling it in the same market, they will try to maximize their own profits (their own rewards in the game). This profit is given by the difference between sales and costs. Each reward function (or profit function) is given by:

$$\pi_1(q_1,q_2) = RT_1 - CT_1 \tag{1}$$

$$\pi_2(q_1,q_2) = RT_2 - CT_2 \tag{2}$$

In these equations πi, RT_i and CT_i are respectively profit, total sales and total cost for player i (i= 1, 2). Sales function of each one of both players results from applying a price to the quantity sold by an agent and it is given by:

$$RT_1 = p(q)q_1 = [A - b(q_1 + q_2)]q_1 = Aq_1 - bq_1^2 - bq_1q_2 \tag{3}$$

$$RT_2 = p(q)q_2 = [A - b(q_1 + q_2)]q_2 = Aq_2 - bq_1q_2 - bq_2^2 \tag{4}$$

and p(q) is the market price as a function of quantities q. The quantity q is the total quantity produced and sold in the market (i.e., $q = q_1 + q_2$). A and b are constant values and q_1 e q_2 are the quantities produced and sold respectively by player 1 and by player 2.

Players costs are

$$CT_1 = cq_1 \tag{5}$$

$$CT_2 = cq_2 \tag{6}$$

and c is a constant (c>0). We consider here that both players have the same costs (and consequently c has the same value for both players). However, this proceeding is not a necessary requirement for the analysis.

$$\pi_1 = Aq_1 - bq_1^2 - bq_1q_2 - cq_1 \tag{7}$$

and

$$\pi_2 = Aq_2 - bq_1q_2 - bq_2^2 - cq_2 \tag{8}$$

Equations 7 and 8 allow us to maximize profits. We can do so through the following equations:

$$\frac{\partial \pi_1}{\partial q_1} = A - 2bq_1 - bq_2 - c = 0 \tag{9}$$

$$\frac{\partial \pi_2}{\partial q_2} = A - bq_1 - 2bq_2 - c = 0 \tag{10}$$

From equations 9 and 10 we must have q_1 e q_2 such as

$$q_1 = \frac{A - bq_2^e - c}{2b} = f(q_2) \tag{11}$$

and

$$q_2 = \frac{A - bq_1^e - c}{2b} = f(q_1) \tag{12}$$

and q_i^e represents the quantity produced by player i, which is his expected production.

Those are the quantities that allow us to maximize the profit for each one of both players, given the expected production for each other's competitor in the market. As this is a simultaneous game, each player makes its own decision about production but it does not know which will be the decision of his competitor in the market. In each one's process of decision, each player will just use the expected value for the production of his competitor in the market. The equations 11 and 12 will be precisely the reaction functions for player 1 and player 2. Each one of them will choose the quantity that maximizes his own profits (given the expected production for his competitor). In fact, the quantity that player i will produce will correspond to the best reaction that this player expects will be the choice made by his competitor.

A Nash equilibrium is a solution in which the players strategies represent the best answers one to each other, reciprocally. So, in this model, $q_1 = q_1^e$ and $q_2 = q_2^e$. This means that the quantity that player 1 has produced represents the quantity that player 2 expected player 1 would produce. This is the strategy followed by player 1. The same procedure happens for its competitor player 2. So, we'll have:

$$q_1^* = \frac{A - c}{3b} \qquad (13$$

$$q_2^* = \frac{A - c}{3b} \tag{14}$$

So, q_1^* and q_2^* are the values that represent the Nash equilibrium. In this solution, no one of the players has any incentives to change his own solution, because these ones are the best strategies, each one for each player. Both players follow their best strategies that correspond to the best responses to the strategy followed by his competitor in the market. This is the Cournot-Nash equilibrium for both players.

For more than two players, we can use the same philosophy for the analysis. If there is a large number of players, Cournot model represent a competitive model where we have n homogeneous products and n players in the market. We'll have the following relationships:

$$RT_i = p(q)q_i = Aq_i - bq_i^2 - q_i b \sum_{j \neq i}^{n} q_j \tag{15}$$

and p(q) is given by $p(q) = A - b\sum_{1}^{n} q_i$ (we can see that $\sum_{1}^{n} q_i$ represents the total production for all POs that depends on market price) and $\sum_{j \neq i}^{n} q_j$ represents the sum of production for all players, excluding player i. Given the usual hypothesis that $C_i = cq_i$, we'll have:

$$\pi_i = Aq_i - bq_i^2 - q_i b\sum_{j\neq i}^{n} q_j - cq_i \tag{16}$$

and

$$\frac{\partial \pi_i}{\partial q_i} = A - 2bq_i - b\sum_{j\neq i}^{n} q_j - c \tag{17}$$

As products are homogeneous and marginal costs of each one of all players are the same and equal to c, we'll have the same share of the market for each one of all players. All of them produce the same quantity. As a result, we'll have $q_i = q_j$ for all j and $\sum_{j\neq i}^{n} q_j = (n-1)q_i$, because each one of all players produce the same quantity. So we have now:

$$\frac{\partial \pi_i}{\partial q_i} = A - 2bq_i - b(n-1)q_i - c = 0 \tag{18}$$

or yet

$$\frac{\partial \pi_i}{\partial q_i} = A - b(n+1)q_i - c = 0 \tag{19}$$

and so,

$$q_i^* = \frac{A-c}{b(n+1)} \tag{20}$$

This is the same result we got for two players (n=2). Besides, total quantity is given by

$$q = nq_i^* = \frac{n(A-c)}{b(n+1)} = \left[\frac{A-c}{b}\right]\left[\frac{n}{n+1}\right] \tag{21}$$

If n comes close to infinity, this last expression comes close to 1. This means that when there are a large number of players in the market, we'll have $q = \frac{A-c}{b}$. We also know that in a competitive market, each player will produce until the market price comes to the level of marginal cost. This fact explains the reasons why we have similar results through the Cournot model and through a model of a complete competitive market.

If there is cooperation between players (n = 2), we'll have a collusion solution. Now, we'll have the sum of both profit functions to get the total profit for the collusion. So, we have now:

$$\pi_T = \pi_1 + \pi_2 \tag{22}$$

Our aim is to maximize the aggregate profit:

$$\begin{aligned} \frac{\partial \pi_T}{\partial q_1} &= 0 \\ \frac{\partial \pi_T}{\partial q_2} &= 0 \end{aligned} \tag{23}$$

We'll have now a solution for players collusion. The optimal quantities result from both players collusion. These quantities will assure major profits for the collusion. The consequent results will show that the final global profit is expected to be better than the old situation and one player or both may improve their own situation. This means that, in an extreme situation, one player may rest in a worse situation, although the global situation is always better than the solution without cooperation. So, sometimes it is possible to negotiate and to agree side payments to transfer benefits to the agents (if any) which situation got worse than before. Besides, we can expect that prices will increase and quantities will decrease under cooperation.

3 A New Model Considering the Variable Fishing Effort

Cournot model is a model of quantities. In the usual fishing theories, there is a variable representing quantities that has a formal relationship with fishing effort. In fact, there is an obvious relationship between fishing effort and catches of fish. Catches represent the production of fish (q). In our model, we replace this variable (quantities) by another variable, related to that one, what represents precisely the fishing effort (E). Our aim is to analyze if there is a direct relationship between fishing effort and cooperation, in order to study if lower levels for fishing effort induce politics for more protection of species, more sustainable resources and better rents for fishers in the long term, as well. So, the usual equations

$$\pi_i(q_1, q_2) = RT_i - CT_i \tag{24}$$

(i = 1, 2) are replaced in the model by the equations

$$\pi_i(E_1, E_2) = RT_i - CT_i \tag{25}$$

(i = 1, 2). We have q_i as a function of E_i and X. In fact, $q_i = f(E_i, X)$ and represents the produced quantity (i.e, catches), E_i is the fishing effort used by PO_i, X is the biomass level for sardine and πi, RT_i and CT_i are respectively profit, total sales and total cost for PO_i, as before. We'll have now the following functions for sales:

$$RT_1 = p(q)q_1(E) = [A - b(q_1 + q_2)]q_1 = [A - b(f(E_1, X) + f(E_2, X))]f(E_1, X) \tag{26}$$

$$RT_1 = Af(E_1, X) - bf(E_1, X)^2 - bf(E_1, X)f(E_2, X) \tag{27}$$

and

$$RT_2 = p(q)q_2(E) = [A - b(f(E_1, X) + f(E_2, X))]f(E_2, X) \tag{28}$$

$$RT_2 = Af(E_2, X) - bf(E_1, X)f(E_2, X) - bf(E_2, X)^2 \tag{29}$$

Besides, players costs are given by $CT_1 = cE_1$ and $CT_2 = cE_2$, in which c is a constant (c>0). We consider, as before, the same costs (c) for each player. As we have already seen, this proceeding is not a necessary requirement for the analysis.

Now, we'll formalize the model in order to solve the mathematical problem through the following equations:

$$\frac{\partial \pi_1}{\partial E_1} = 0 \tag{30}$$

and

$$\frac{\partial \pi_2}{\partial E_2} = 0 \tag{31}$$

Now, we'll get the optimal values for E_1 and E_2. Additionally, $E_i = E_i^e$ represents the level for fishing effort expected by player j for player i ($i \neq j$, i, j = 1, 2). The solution E_i^* (i=1,2) represents the optimal fishing effort for player i. These levels will maximize individual rents for players, in a competition basis and represent the optimal levels for fishing effort, those that maximize rents for each player, knowing previously the expected level for his competitor's fishing effort in the market. As this is a simultaneous game, each player makes his own decision about its fishing effort that it will apply to the fishery, but it does not know the decision that is made by his rival in the market. So, each company just considers the expected level of fishing effort for his competitor in the market. Consequently, we'll know the reaction functions for each player and, as it was already seen, we will determine the fishing effort level that maximizes rents for each player, given the expected fishing effort for his competitor. The decision of each player about his fishing effort level represents the best response to the decision made by his competitor in the market. Such a decision, made by one player, represents the decision his competitor would expect this player would make about fishing effort. Consequently, without cooperation, the solution for the problem is a Nash equilibrium solution and it is the best solution E_i^*. In this level for fishing effort, no player has any advantages to change his own strategy in the game because this is the best level of fishing effort and it represents his best strategy.

Now, we'll consider that both players cooperate and that they make arrangements in order to get benefits from the collusion between them. Now, both players will maximize aggregate rents, instead maximizing their own rents, individually. That is the goal. In order to see this, consider now

$$\pi_T = \pi_1 + \pi_2 \tag{32}$$

To maximize the aggregate profit we'll have a solution (the collusion solution) from the resolution of the following:

$$\begin{aligned} \frac{\partial \pi_T}{\partial E_1} &= 0 \\ \frac{\partial \pi_T}{\partial E_2} &= 0 \end{aligned} \tag{33}$$

In the solution for this problem, aggregate fishing effort is expected to reach a lower level than the sum of the reached levels for each individual solution. This is consistent with benefits expected for the members of players, because costs of fishing are expected to be lower. As an additional result, it is expected that the market price would be higher and the aggregate rent would be higher, as well. Besides, as the aggregate fishing effort is expected to be lower, we can expect that fishers will control catches of fish as well, and consequently, we'll expect to get a stock's management more compatible with objectives of conservation of species.

4 Analysis and Results

Besides the optimal control (linear) problem usually used to study cooperative and non-cooperative fishing games, we may have an interesting model based on the Cournot Model that allows us to conclude that cooperation brings very interesting results when we intend to preserve species and that cooperation may improve rents for fishers. In fact, the Cournot-Nash model we implemented is a simple model and it has the big advantage of being easily understood by the stakeholders of fishing sector. This model shows the disadvantages of non-cooperation. An extension of the model shows, as well, that cooperation between agents allows better results by improving the global situation and the levels of stocks for fish.

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