A resectorization of fire brigades in the north of Portugal

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Abstract. Sectorization consists of grouping the basic units of a large territory to deal with a complex problem involving different criteria. Resectorization rearranges a current sectorization avoiding substantial changes, given a set of conditions. The paper considers the case of the distribution of geographic areas of fire brigades in the north of Portugal so that they can protect and rescue the population surrounding the fire stations. Starting from a current sectorization, assuming the geographic and population characteristics of the areas and the fire brigades' response capacity, we provide an optimized resectorization considering two objectives: to reduce the rescue time by maximizing the compactness criterion, and to avoid overload situations by maximizing the equilibrium criterion. The solution method is based on the Non-dominated Sorting Genetic Algorithm (NSGA-II). Finally, computational results are presented and discussed.

Keywords: sectorization, resectorization, fire brigades, multi-objective problems

1 Introduction

In Portugal, fire brigades are the core of civil protection. They have several missions, such as extinguishing fires and pre-hospital emergencies. Prompt response to emergency incidents is primordial since delays in the rescue can have tragic consequences. However, increasing urban development presents an increase in demand and a challenge to the emergency response mechanisms. Fire brigades must protect and rescue the population in the areas surrounding their fire stations. This paper intends to contribute, to this desideratum, by applying the concept of sectorization to a real case.

Given a large territory or network composed of basic units or indivisible regions, a sectorization consists of grouping the basic units into sectors considering one or more objectives, intending to simplify a problem. Resectorization intends to change a current sectorization by avoiding substantial changes, but responding to a set of conditions. The term resectorization is also known as redistricting, as indicated in the references [5,11].

Resectorization will be applied to the current sectorization of a real situation in northern Portugal. The goal is to create compact and balanced sectors of the fire brigades' operation to decrease rescue time and avoid work overload. For that, we also propose new measures to calculate the compactness and equilibrium of the sectorization. To solve the fire brigades problem, we will employ a popular multi-objective optimization method named Non-dominated Sorting Genetic Algorithm (NSGA-II) [6]. This algorithm has also been used to solve sectorization problems [7,8].

The remainder of the paper is organized as follows. Section 2 presents a review of the relevant literature. Section 3 describes the case study and also the data used. Section 4 explains the solution method. Section 5 presents the results and a brief discussion. Finally, Section 6 puts forward the conclusions.

2 Relevant Literature

This section briefly summarizes some of the literature relevant to sectorization problems.

Fire brigades have been the subject of several studies over time. One of the topics covered is to understand the characteristics and incidence of fires to improve the rescue time and reduce their consequences [4,12].

Sectorization problems have many real-world applications. Political districting is one of the oldest, aiming to divide the territory neutrally and avoid gerrymandering. Bozkaya *et al.* [3] propose a tabu search and adaptive memory heuristic to solve a single multicriteria problem involving contiguity, population equality, compactness and socio-economic homogeneity, using real data from Edmonton. Bação *et al.* [1] use a genetic algorithm to solve the problem of electoral districting.

Regarding resectorization, political redistricting is a common application [9]. It intends to redesign the boundaries of existing legislative districts for electoral purposes. Using a contiguity procedure, one of the used algorithms exchanges population units between a district with a population smaller than the ideal and a district with a larger population.

Moreover, Assis *et al.* [5] applied resectorization to meter reading in power distribution networks and proposed a Greedy Randomized Adaptive Search Procedure (GRASP) to solve a case with real-world customers in Brazil. Vahdani *et al.* [11] proposed a bi-objective optimization model to plan a humanitarian districted logistics network, with several simultaneous decisions concerning emergency facility location-allocation, redistricting and service sharing.

Another field studied is school redistricting, which consists of adjusting the boundaries of schools [2]. In this paper, the authors assign students to schools considering a balance of the schools' socioeconomic compositions and the total travel distance.

3 Case Study

This section describes the resectorization of fire brigades case study. It involves 6 fire brigades and the 175 indivisible regions they serve, situated in the north of Portugal.

Let us define the following parameters:

$R = \{1, ..., n\}$ the set of indivisible regions;
$B = \{1, ..., m\}$ the set of fire brigades;
d_{ji}, the euclidean distance between the center of region $j \in R$ and the brigade $i \in B$;
p_j, the number of inhabitants in region $j \in R$;
q_j, the area of the region $j \in R$;
A_i, the number of ambulances of the brigade $i \in B$;
V_i, the number of fighting vehicles of the brigade $i \in B$;
e_i, the number of firefighters of the brigade $i \in B$;

$$X^E_{ji} = \begin{cases} 1, \text{if the region } j \text{ is assigned to the brigade } i \text{ in the current sectorization} \\ 0, \text{otherwise} \end{cases}$$

Fig. 1: The territory addressed in this study is represented in green, located in the north of Portugal.

The demand of region s_j depends on its number of inhabitants and its area, taking the form

$$s_j = \alpha_1 P_j + \alpha_2 Q_j, \tag{1}$$

where α_1 and α_2 are constants,

$$P_j = \frac{p_j}{\sum_{j \in R} p_j} \tag{2}$$

and

$$Q_j = \frac{q_j}{\sum_{j \in R} q_j}. \tag{3}$$

As the regions have a large population diversity and areas, these values had to be adimensionalized. The proportion of each region to the total was considered to join both values in the same formula.

These two characteristics of the regions, P_j and Q_j, are used to categorize the data better. The constants α_1 and α_2 will be used as relative weights of these two characteristics, adjusted according to specific needs. These values are related to the probability of a rescue event which depends differently on the population and the area of the region. It is assumed that the larger the population, the greater the probability of being necessary ambulances and firefighters. Also, a region with a larger area will probably have more incidents, requiring more vehicles and firefighters.

The capacity of each fire brigade i derives from its number of ambulances, its number of fighting vehicles and its number of firefighters, and is given by

$$c_i = \beta_1 g_1 A_i + \beta_2 g_2 V_i + \beta_3 E_i, \tag{4}$$

where

$$E_i = \frac{e_i}{\sum_{i \in B} e_i}. \tag{5}$$

Constant parameters g_1 and g_2 have been adjusted to reflect the current state of the most overloaded fire brigade in the set B. g_1 is the ratio between the total population currently served by this corporation and its number of ambulances. Similarly, g_2 is the ratio between the total area currently served by this corporation and its number of fighting vehicles.

The constants β_1, β_2 and β_3 represent the weights of each characteristic of the fire brigades, which can be adjusted based on the population's needs.

3.1 Objective Functions

The case study aims to create sectors where service centers are the fire stations and the indivisible regions are the basic units. For this, two objective functions are chosen, related to the criteria: Equilibrium and Compactness.

Let us define the following variables:

$$x_{ji} = \begin{cases} 1, \text{if the region } j \text{ is assigned to the fire brigade } i \text{ in the resectorization} \\ 0, \text{otherwise} \end{cases}$$

Equilibrium:

The idea is to distribute the regions equitably among the fire brigades, being necessary to assume the capacity of each fire brigade, c_i, and the demand of each region, s_j.

The objective is to minimize the standard deviation of the occupancy percentage of fire brigades:

$$\min f_1 = \sqrt{\frac{\sum_{i \in B}(k_i - \bar{k})^2}{m}}, \tag{6}$$

where $k_i = \frac{\sum_{j \in R} s_j x_{ji}}{c_i}$ is the percentage of used capacity of fire brigade i and $\bar{k}$ is the average of k_i.

This objective function was created to obtain an equitable distribution of demands. It is necessary to consider each corporation's capacity to minimize the possibility of overload.

Compactness:

The view is to distribute the regions among the fire brigades such that the rescue time is minimal. For this, we attend to the distance between the fire departments and the regions, d_{ji}, and the regions' needs, s_j.

The objective is to minimize the distance between fire departments and regions, weighted by the demand of each region:

$$\min f_2 = \sum_{j \in R} \sum_{i \in B} d_{ji} s_j x_{ji}. \tag{7}$$

By minimizing the distance between regions and the assigned corporation, we expect to minimize the rescue time. Additionally, the demand is regarded, since a region that needs to be visited more frequently will have greater weight because it will take more time.

3.2 Similarity of the Solutions

The similarity is a measure to evaluate the resemblance, or coincidence, between two solutions. In resectorization, it is important to measure how different is the new solution from the original one. The importance of this measure arises clearly in firefighters' cases to facilitate the adaptation of firefighters to changes.

The similarity measure used in this work is the percentage of regions that stay unchanged, which is given by:

$$\frac{\sum_{j\in R}\sum_{i\in B} X^{E}_{ji} x_{ji}}{|R|} \geq \delta, \qquad \delta \in [0,1]. \tag{8}$$

This measure is used as a constraint. The decision-maker will define the preferred minimum similarity level, δ.

4 Solution Method

The paper follows NSGA-II to resectorize the fire brigades. NSGA-II is one of the most recognized and implemented multi-objective optimization (MOO) methods. As in all MOO methods, NSGA-II classifies all the solutions for each objective simultaneously and separately regarding their performance. These ranks are constructed considering two parameters: (i) set of dominating members, and (ii) domination count. The former counts how many other solutions a solution dominates. The latter counts by how many other solutions a solution is dominated. A solution (say x) dominates another solution (say y) if solution x performs better than solution y for at least one objective while it is not worse than y in any other objectives. This definition helps to build the two sets for each solution. More precisely, the set of dominating members of a solution includes the solutions dominated by that solution. Moreover, the domination count of a solution sums up the number of solutions that dominate that solution. These two sets are constructed for each solution. The solutions with zero domination count are allocated in the first Pareto frontier and considered the best ones. The solutions that are just dominated by the solutions located in the first Pareto frontier are allocated in the second Pareto frontier. This sequence continues until all the solutions are allocated in the Pareto frontiers. The solutions in the same Pareto frontier are regarded equally good. This domination logic ultimately helps keep the best solutions concerning their performances over generations in the objective space.

As in all evolutionary algorithms, NSGA-II starts by initializing the population. The population usually includes a set of solutions that are randomly generated. In the case of resectorization, the solutions must follow a specific order based on the existing solution (i.e. current sectorization) while still allowing for some randomness.

In this work, the "matrix form binary grouping" (MFBG) genetic encoding system [10], suitable for sectorization problems, is used to compose the solutions. MFBG is a binary matrix of size composed of (J × I), where J and I represent the total number of basic units and sectors, respectively. Each row can be considered a binary set representing the sector assignment of the corresponding basic unit.

It is possible to see its format below:

$$M = [x_{ji}]_{J \times I}$$

where

$$x_{ji} = \begin{cases} 1 & \text{if basic unit } j \text{ is assigned to sector } i \\ 0 & \text{otherwise} \end{cases}$$

The solutions included in the population are evaluated according to their performance on the two objectives specified in Section 3.1 according to their Pareto dominance in the objective space.

NSGA-II seeks the objective space over generations to find superior solutions. An entire generation (i.e. iteration) consists of some steps, namely, selection, crossover, and mutation, until the stopping criterion occurs.

Selection picks the parent solutions necessary for the crossover. We used tournament selection. In this method, two solutions are randomly selected from the population and evaluated according to their performances for each objective. If there is domination between the solutions, we keep the dominant solution as a parent. If the two solutions are in the same Pareto frontier, since they are not comparable, we observed an NSGA-II specific concept, called Crowding Distance (CD), to select the parent solution. CD represents the concentration of the solutions in the Pareto frontier, such that a solution with a lower value of CD is more crowded by other solutions. Therefore, solutions with higher CD are assumed to be better and more representative of the population to increase diversity.

The selected parent solutions are mated to create off-spring solutions during the crossover. In the current work, we used multi-point crossover, which selects random rows from the parent solutions and switches them to form two off-spring solutions. This crossover method does not require a crossover probability due to the design of the MFBG genetic encoding system.

Finally, the mutation operator is just implemented on the off-springs in a given probability to increase the diversity in the population. The mutation occurs with a certain probability imposed on the population. The algorithm decides row by row if a mutation happens, i.e., if a point will be assigned to another sector or not.

All the solutions are evaluated regarding their similarity to the existing solution using the equation presented in Section 3.2 The solutions with a higher difference than the desired similarity level are eliminated from the population. Then, new ones are generated for each deleted solution to keep the population size constant over generations.

The process is described in the Algorithm.

The parameters, namely population size and mutation probability were 100 and 0.03, respectively. Finally, the number of generations was 500 and used as a stopping criterion.

Algorithm Pseudocode of NSGA-II

```
Generate N feasible solutions and insert into Population (Pop_size == N)
Evaluation of the solutions according to the selected objectives
Non-Dominated Sorting MinF(.)
Calculate Crowding Distance of each frontier
Generation := 0
while Generation < T do
    while Pop_size < N × 2 do
        Select parents through tournament selection
        Create two off-springs using multi-point crossover in each turn
        Mutate off-springs (for selected P_mut)
        Merge off-springs into the population
        Pop_size := Pop_size + 2
    end while
    for each solution in the current population do
        Evaluation of the solutions according to the selected objectives
        Non-Dominated Sorting MinF(.)
        Calculate Crowding Distance of each frontier
    end for
    while Pop_size ≠ N do
        if N ≥ Pop_size - # solutions in the worst Pareto frontier then
            Remove all solutions located in the worst Pareto frontier.
            Pop_size := Pop_size - # solutions in the worst Pareto frontier
        else
            Remove the worst solution in the worst Pareto frontier regarding the Crowding Distance
            Pop_size := Pop_size − 1
        end if
    end while
    Generation := Generation + 1
    for each solution in the population do
        Evaluation of the solutions according to their similarity
        Delete the solutions with higher differences than the desired similarity level
        Generate feasible solutions and insert into Population to keep Pop_size == N
    end for
    for each solution in the population do
        Evaluation of the solutions according to the selected objectives
        Non-Dominated Sorting MinF(.)
        Calculate Crowding Distance of each frontier
    end for
    Output the results
end while
```

5 Results and Discussion

In this section, we will present the results and a brief discussion.

Based on experience, the constants presented in expressions (1) and (4) were fixed.

Population growth will increase the need for pre-hospital emergencies and, growth in the area of the region may increase the number of forest fires, accidents, etc. The parameters $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$ were respectively defined in expression (1).

It is reported that most of the services provided by firefighters are pre-hospital emergencies. In expression (4), the relative importance of the number of ambulances, the number of other vehicles and the number of firefighters were defined, respectively, $\beta_1 = 0.5$, $\beta_2 = 0.3$ and $\beta_3 = 0.2$.

In Fig. 2, it is possible to see the current sectorization used by the fire brigades. The crosses correspond to the fire brigades stations and the points represent the center of the regions. A different color denotes each sector.

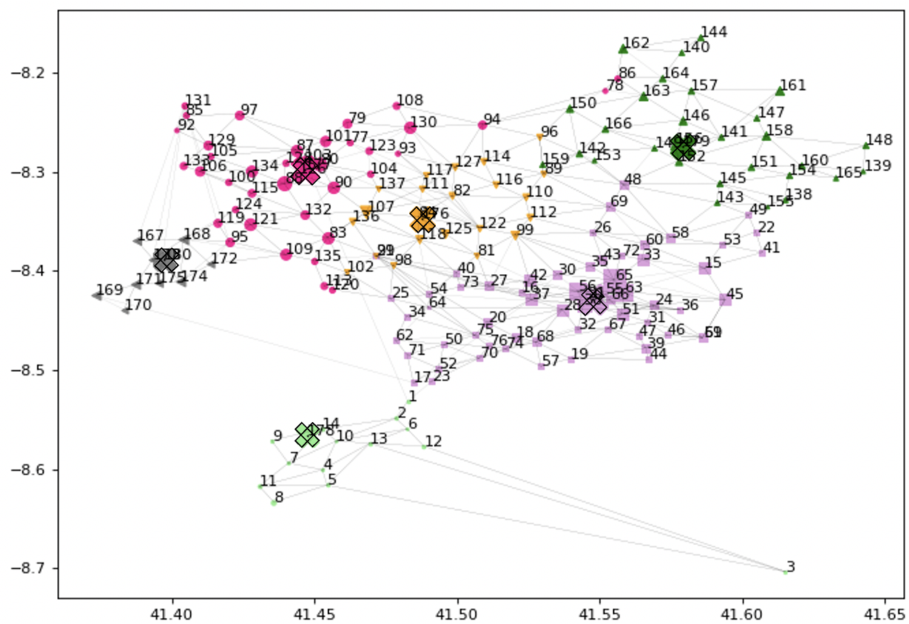

Fig. 2: Current sectorization (Equilibrium=0.284; Compactness=0.0035)

After solving the problem described in Section 3, we obtained several solutions, using the similarity as a restriction, taking 70%.

Fig. 3 pictures the obtained Pareto Frontier. Equilibrium versus Compactness, in all solutions, with different levels of similarity, are also represented.

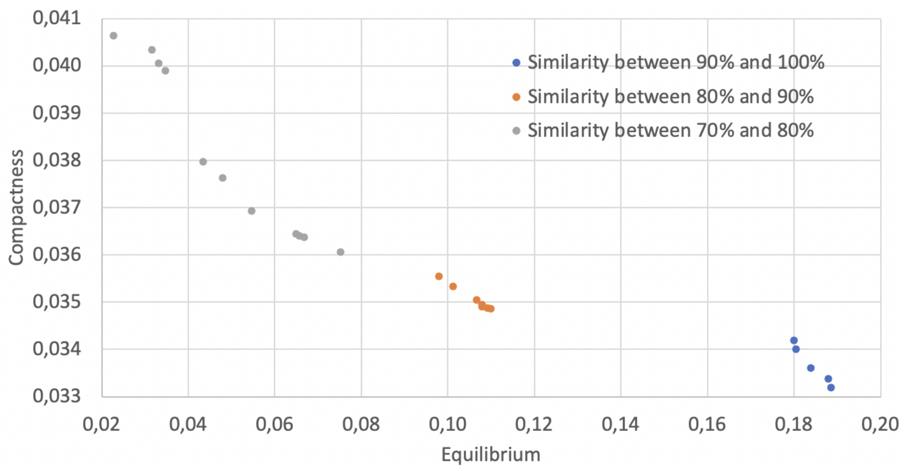

Fig. 3: Pareto Frontier for $\delta = 0.70$

We can see ten solutions for optimized solutions with a similarity between 70% and 80%, five solutions with a similarity between 80% and 90% and five solutions with a similarity between 90% and 100%.

Pareto Frontier permits verifying that the best solutions in terms of equilibrium are presented with a similarity between 70% and 80%. On the other hand, better solutions in compactness are obtained with a similarity between 90% and 100%. In addition, it is possible to see that a small improvement in compactness leads to a large deterioration of the equilibrium values.

It is also possible to see that the equilibrium decreases even for high similarity values (between 70% and 100%). Changing some points to a different sector reduced the equilibrium value substantially. In practice, the regions will be distributed equitably between the fire brigades, reducing the chance of overload.

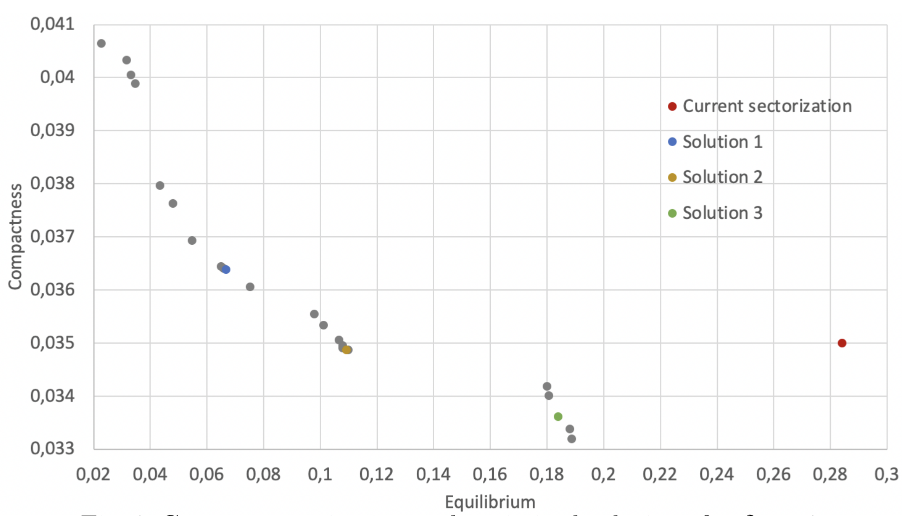

Fig. 4: Current sectorization and optimized solutions for $\delta = 0.70$

In order to visualize the improvements, in Figure 4, we present the equilibrium and compactness values of the current sectorization and the solutions in the Pareto frontier. The solutions in different colors represent the values of objective functions in current sectorization and three optimized solutions with different similarity values that could be suitable (Table 1). All solutions were presented to a person responsible for one of the fire brigades, and these three solutions were pointed out as the most suitable for implementation in the field.

Table 1: Objective functions of the proposed resectorization solutions

Sectorization	**Equilibrium**	**Compactness**	**Similarity**
Current	0.284	0.035	-
Solution 1	0.067	0.036	74.3%
Solution 2	0.109	0.035	80.6%
Solution 3	0.184	0.034	90.3%

In Fig. 5, Fig. 6 and Fig. 7, the three proposed resectorizations are shown.

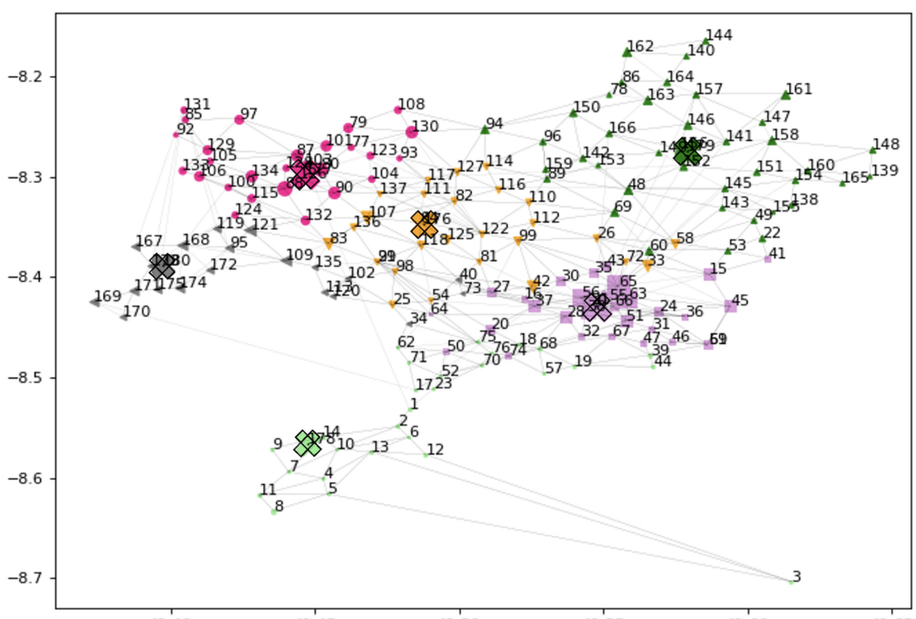

Fig. 5: Solution 1 (Equilibrium=0.067; Compactness=0.036; Similarity=74.3%)

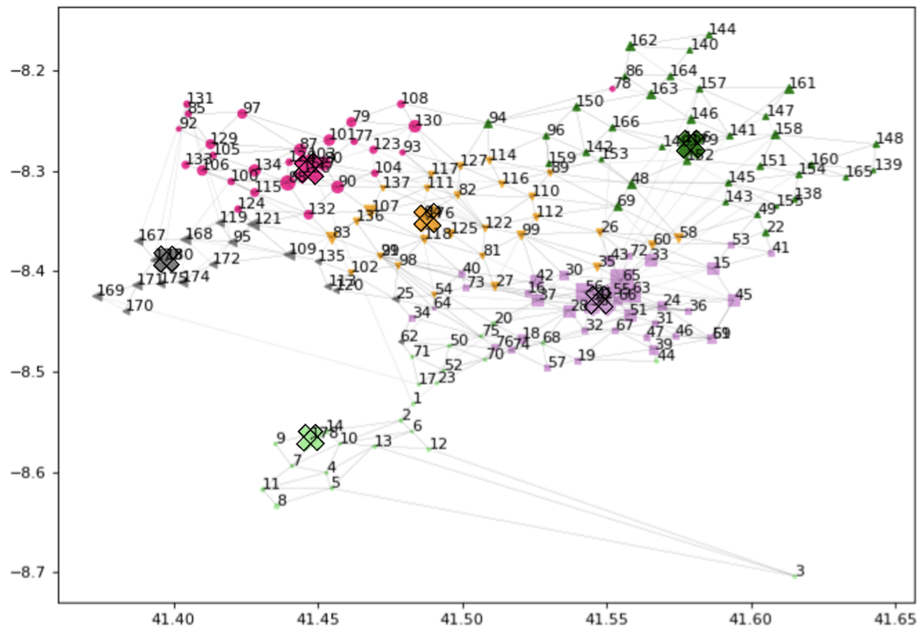

Fig. 6: Solution 2 (Equilibrium=0.109; Compactness=0.035; Similarity=80.6%)

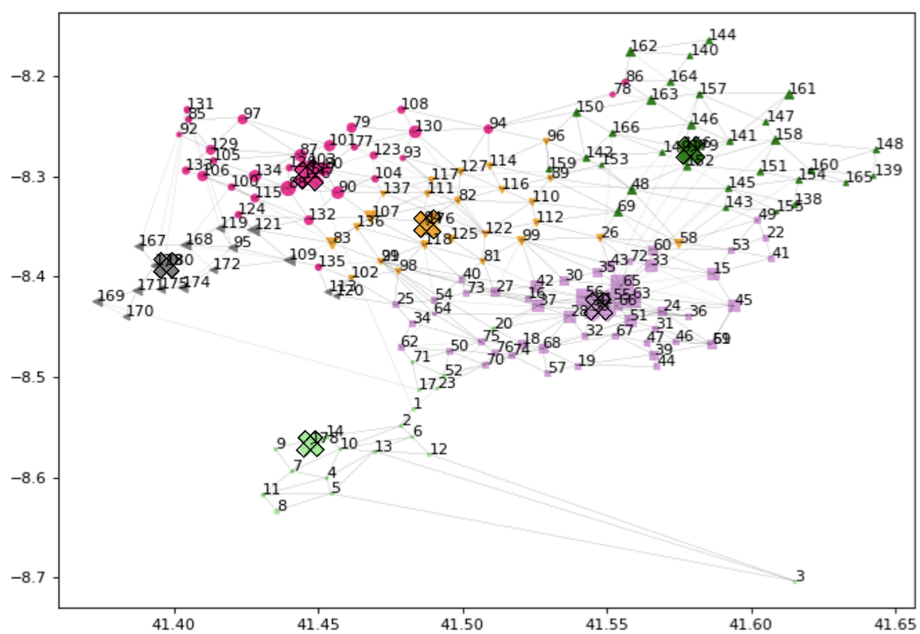

Fig. 7: Solution 3 (Equilibrium=0.184; Compactness=0.034; Similarity=90.3%)

First of all, the solutions presented have better values in terms of equilibrium when compared to the current sectorization, and the lowest equilibrium value is obtained for similarity of 74.3%.

Furthermore, all solutions presented compactness very similar to the current sectorization. However, we can observe a trade-off between compactness and equilibrium, which means better compactness values imply the worst equilibrium values, which follows directly from the concept of nondominated solutions.

6 Conclusions

This study focused on applying sectorization to a real case of fire brigades in the north of Portugal. The purpose was to improve the existing sectorization of the fire brigades, to minimize rescue time and the possibility of overload. For that, we found a way to quantify the fire brigades' response capacity and the needs of the regions.

In order to minimize the rescue time, it was assumed that the new sectors needed to be compact. We proposed a new measure for compactness, which takes into account the distance between regions and fire departments and how often the regions need to be visited. Minimizing the distance between regions and the assigned fire brigade will minimize the rescue time, but it is also important to contemplate the demand. We assume that a region that needs to be visited more frequently would take longer to save and therefore to this region was given a higher weight. The new sectors should also be balanced to minimize the possibility of overload. Therefore, a new metric for equilibrium was proposed, where the used capacity of fire brigades is regarded.

The solutions to the correspondent problems were obtained with NSGA-II.

We observed that the current sectorization of the fire brigades could be improved; the sectors are compacted but not balanced. We verified an inequality in the occupancy rates of the different fire brigades. We have shown that maintaining good compactness can greatly improve the equilibrium between corporations, minimizing the risk of overload. Besides, three optimized solutions were proposed to reduce rescue time with better use of the available resources.

In conclusion, applying the concept of resectorization can contribute to decision-making in the management of fire brigades and the resolution of the related multi-objective problems.

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