S&P 500 options term structure

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Departamento de Finanças

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Resumo

Esta dissertação deriva a estrutura temporal do VIX e estuda o seu comportamento e também a sua relação com os retornos do índice S&P 500 .

A estrutura temporal do VIX frequentemente apresenta um declive positivo, mostrando um declive negativo ocasionalmente. Há uma relação clara entre declives negativos e condições económicas menos favoráveis, sendo declives negativos mais comuns em alturas mais economicamente *stressantes*. O aspeto da curva está também relacionado com os retornos do SPX, verificando-se retornos mais elevados em semanas que apresentem declive positivo.

O formato e dados da estrutura temporal do VIX são frequentemente usados por investidores como uma maneira de verificar a volatilidade esperada pelo mercado.

Keywords: Opções sobre o S&P 500, Estrutura Temporal, VIX

Códigos JEL: G11, G13

Abstract

This dissertation derives the S&P 500 term structure and studies its behaviour and relation with S&P 500 returns.

The VIX term structure is normally sloping upwards, showing a downward slope only occasionally. There is a clear relation between more stable market conditions and an upward sloping VIX term structure, with the term structure showing downward slope during more stressful, less favorable market conditions. The shape of the VIX term structure is also linked to posterior SPX returns, with higher returns being verified when the term structure shape is sloping upwards.

The shape and data portrayed by implied volatility term structures are used often by investors as a way to check the market's expectation of volatility and to deduct about the state of the stock market.

Keywords: S&P 500 Options, Term Structure, VIX

JEL codes: G11, G13

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CHAPTER 1

Introduction

The S&P 500 Index (SPX) is a stock market index that tracks the 500 biggest companies listed on stock exchanges in the United States. It is one of the most followed indices by investors and, therefore, is one of the indices with the most money invested in assets tied to the performance of the index.

In 1993, the Chicago Board Options Exchange (CBOE) introduced the CBOE Volatility Index (VIX), the most widely followed index of market volatility, which is an estimate of the S&P 500 return volatility over the next 30 days derived from S&P 500 option prices.

Given the growing popularity of the VIX, the CBOE launched VIX futures and options in 2004 and 2006 respectively, and have since become some of the most actively traded contracts on the CBOE.

Normally, at any given time, there are no SPX options trading in the market with exactly 30 days to maturity. Therefore, the VIX is actually calculated using an interpolation between two nearby maturities.

The VIX term structure is the relationship between VIX values and maturity dates. Is depicted by connecting the VIX values of different SPX options expiration dates. Much like the interest-rate yield curve, it is displayed on a graph where the X-axis represents expiration dates, and the Y-axis represents the VIX values.

Bakshi, Crosby, Gao, and Xue (2021) study the behaviour of the VIX futures term structure. This dissertation follows closely their analysis, with the main difference that we study the VIX term structure and not the VIX futures term structure.

Throughout this thesis we try to answer the following questions:

(1) What is the normal shape of the VIX term structure?
(2) What is the behavior of the VIX term structure under different economic states?
(3) Is there a relation between the shape of the VIX term structure and posterior SPX returns?

In order to attempt to answer the above questions, this dissertation is organized as follows: Chapter 2 provides a summary of the VIX, explaining its origins and evolution over time, and a review of the paper by Bakshi et al. (2021), addressing the facts discovered and the methodology used by the authors as well as the difference between the VIX futures curve and the VIX term structure, and finally the methodology used to compute the volatility index; In Chapter 3 we employ the methods used by the aforementioned authors to our data set in order to study the VIX term structure, how often is it upward sloping and under which economic circumstances as well as if the VIX term structure is a good way to assess the stock market returns; finally Chapter 4 concludes.

CHAPTER 2

Literature Review

2.1. The VIX

Introduced in 1993 by the Chicago Board Exchange (CBOE), the CBOE Volatility Index (VIX) originally measured the 30-day expected volatility, implied by eight near-the-money S&P 100 options from the two nearest maturities. Originally, the VIX was fundamentally an average of the Black-Scholes implied volatilities.

In late 2003, the CBOE updated the method of computing the VIX using a model-free approach, which is based on the theoretical work of Britten-Jones and Neuberger (2000), Carr and Madan (1998), and Demeterfi, Derman, Kamal, and Zou (1999). The new VIX still measures the 30-day implied volatility, but is now computed using S&P 500 index (SPX) options, using all available strikes, from the two maturities between more than 23 days and less than 37 days, the near-term and next-term maturities, respectively. Once the near-term option's maturity is in 23 days or less, the next-term option shifts to near-term and a new option takes the next-term place.

The new VIX also has a more understandable economic meaning compared to the old VIX, as it is the price of a linear portfolio of options with this being one of the reasons that motivated the change, according to Carr and Wu (2006).

Until October 2014, only SPX monthly options (expiring on the third Friday of each month) were used to compute the VIX. Since the 6th of October, 2014, the CBOE proceeded to use weekly SPX options (expiring every Friday with the exception of the third Friday of each month) in the calculation of the VIX. This change was made due to weekly SPX options volume increasing and representing, at the time, one third of all SPX options traded. The change also allowed the VIX calculation to be more precise.

In March 2004, few months after the switch to the new VIX, the first derivatives on the VIX were released, starting with VIX futures, and VIX options coming in 2006. VIX derivatives were the first derivatives traded in an exchange that allowed traders and investors to directly trade market volatility.

The VIX it is also known as the "fear gauge". The higher the value, the more perceived stress on the market, with values above 20 being considered high VIX and indicating periods of higher volatility, while values below 20 considered low VIX value and suggesting low risk environment.

Nowadays, there are other indices similar to the VIX, that measure the implied volatility for different maturities, for example the 9-day VIX (VIX9D), or the 3-month VIX (VIX3M).

2.2. VIX term structure and the VIX futures curve

The VIX term structure is a term structure that, for a given date, portrays the expected stock market's volatility implied by the SPX options prices until the maturity of the SPX options. The VIX term structure is displayed on a graph and made by linking together VIX values for different maturities. The Y-axis represents the VIX value computed using the "VIX Methodology" and the X-axis represents the time, more specifically, the expiration dates of the SPX options available in the market.

The VIX portrayed in the term structure is calculated using the same methodology as the VIX, with the main difference being that the VIX displays 30-day implied volatility, while the VIX term structure depicts a set of values gauging volatility from the given date until each maturity. The VIX term structure is very used by market analysts, investors and traders as a way to quickly learn the volatility expected by the market for different time periods.

The VIX futures curve is also commonly referred to as the VIX term structure, although there are some key differences between the two. Firstly, the information used in these term structures is different. The VIX futures curve is made connecting prices of VIX futures contracts with different expiration dates. Secondly, the VIX futures curve is read as the volatility expected for 30 days after the expiry date of each future contract portrayed in the curve, while the VIX term structure depicts a set of values for the expected volatility between the present date and each expiration date of the term structure.

To develop more on the VIX futures curve, Bakshi, Crosby, Gao and Xue (2021) conducted an analysis on the VIX futures curve, using daily settlement prices of VIX futures, from April 25, 2006 to December 31, 2019, a total of 3447 daily observations. Futures price at time t for the n-th futures contract is denoted by $F_t^{(n)}$ and in order to study the futures curve, the slopes were computed as follows:

$$\text{Slope}_t^{(n)} = \log\left(\frac{F_t^{(n)}}{F_t^{(1)}}\right), \text{ for } n = 2, \ldots, 6. \tag{2.1}$$

Evaluating how often $\text{Slope}_t^{(6)}$ is positive, Bakshi et al. (2021, page 3) have found that the VIX futures curve is in contango 82% of the time, meaning that near-term VIX futures contracts are priced cheaper than next-term contracts in 82% of the daily entries. This evidence implies that the VIX futures curve portrays backwardation, when near-term VIX futures contracts are priced higher than next-term ones, only on occasion. Another finding was on the tapering of the VIX futures curve, as the average value of $\log\left(\frac{F_t^{(n)}}{F_t^{(n-1)}}\right)$ decreases as n takes on larger values.

To reinforce the contango characteristic, the following daily regression was computed:

$$\log\left(F_t^{(n)}\right) = \alpha_t + \beta_t \log\left(\mathcal{T}_t^{(n)}\right) + \epsilon_t^{(n)}, \text{ for } n = 1, \ldots, 6 \tag{2.2}$$

where $\mathcal{T}_t^{(n)}$ is the time to maturity of the n-th VIX futures contract. The authors concluded that the hypothesis where β_t is equal to 0 is rejected, thus rejecting the VIX futures curve being flat.

Examining the slopes of the VIX futures curve in conjunction with daily VIX values, Bakshi, Crosby, Gao and Xue (2021, page 4) concluded that the VIX futures curve portrays backwardation when during stressful economic states. It was found that, when the daily VIX is at 18 or lower, the percentage of slopes being positive is higher than 91%, across all slopes, as well as the average value of each slope decreasing with higher VIX values.

Doing the same analysis but with market state being daily S&P 500 index returns, it was concluded that the VIX futures curve presents backwardation when the S&P 500 index returns are more accentuated, in either direction. Average value of the slopes is negative when the returns' variation is higher than 3.5% and the percentage of positive slopes is below 13% when daily SPX returns have large variations.

These methods of analysis will be applied to our data set to study the VIX term structure.

2.3. VIX Methodology

Following CBOE (2021, Equation (1)), the VIX calculation at time t for the SPX options with maturity at time T is given by the square root of

$$S^2\left(t,T\right) = \frac{2}{T-t}\sum_{i=-L}^{M}\frac{\Delta K_i}{K_i^2}\,e^{r_t\times(T-t)}\,O_t(X_t,K_i,T) - \frac{1}{T-t}\left[\frac{F(t,T)}{K_0}-1\right]^2, \qquad (2.3)$$

where r_t denotes the time t risk-free rate with maturity T, X_t is the SPX level at time t, $F(t,T)$ is the time t SPX forward price with maturity at time T, K_i is the i-th available strike price,

$$O_t\left(X_t,K_i,T\right) := \begin{cases} p_t\left(X_t,K_i,T\right)\times\mathbb{1}_{\{K_0>K_i\}} + c_t\left(X_t,K_i,T\right)\times\mathbb{1}_{\{K_0<K_i\}} <= i\neq 0 \\ \\ \frac{c_t(X_t,K_0,T)+p_t(X_t,K_0,T)}{2} \qquad <= i=0 \end{cases} \qquad (2.4)$$

where $c_t(X_t,K_i,T)$ and $p_t(X_t,K_i,T)$ are, respectively, the time t prices of European-style calls and puts on the SPX with strike K_i and maturity at time T, K_0 is the first strike below $F(t,T)$ whereas K_{-L} and K_{-M} are, respectively, the lowest and highest strike prices contained in the strike range available in the market with a non-zero bid, and

$$\Delta K_i := \begin{cases} K_M - K_{M-1} & <= i = M \\ \\ \frac{K_{i+1}-K_{i-1}}{2} & <= -L < i < M \\ \\ K_{-L+1} - K_{-L} & <= i = -L \end{cases} . \tag{2.5}$$

The algorithm to implement equations (2.3) through (2.5) can be summarize in the following steps:

(1) Determine $F(t,T)$ computed through put-call parity and using the strike price at which the absolute difference between the call and put prices is smallest;
(2) Determine K_0, the strike price immediately below $F(t,T)$;
(3) Select out-of-the-money (OTM) put options with strike prices below K_0 that have bid prices higher than zero;
(4) Select OTM call options with strike prices above K_0 that have bid prices higher than zero;
(5) Compute the options mid-prices (average bid-ask prices) for each selected strike;
(6) For the strike K_0, use the average of the put and call options prices.

Implementing the above algorithm to all the available SPX options maturities at time t, gives the VIX term structure at time t.

To compute the VIX with constant 30 day maturity we select two available SPX options maturities, T_1 and T_2, such as $T_1 - t > 23$ days and $T_1 - t < 37$ days. Using the VIX values for the maturities T_1 and T_2, the VIX with constant 30 day maturity is computed using linear interpolation with the following expression:

$$VIX_t = 100 \times \sqrt{\frac{1}{T-t}\left[\frac{S^2(t,T_1)\,(T_1-t)\,(T_2-T) + S^2(t,T_2)\,(T_2-t)\,(T-T_1)}{T_2-T_1}\right]}, \tag{2.6}$$

where T_1 and T_2 denote the two available maturity dates, and T denotes the interpolated maturity date such that $T - t$ is 30 days.

CHAPTER 3

Empirical analysis

In this chapter we will replicate the analysis made by Bakshi et al. (2021) in order to study the behaviour of the VIX and the VIX term structure.

3.1. Data Description

The SPX options data was retrieved from the CBOE, which provides historical bid and ask prices of options based on quotes at 3:45 p.m.[1] The sample starts in April 25, 2006 and ends in April 30, 2019. Only data for the standard SPX monthly maturities were used in the analyses. In selecting the sample, we restrict the analysis of the term structure to the nearest six available maturities at a given trading day t. After removing half-trading days from the sample, there are a total of 3217 days in our sample.

The interest rate curve was obtained from the U.S. Department of the Treasury. To estimate the risk-free interest rate for a given maturity T, we apply a cubic spline to the available interest rates.

The data set was verified and validated by comparing 150 randomly selected term structures obtained with the term structures that CBOE provides on their website.[2] It is worth to note that CBOE's website does not provide the term structure for 3:45 p.m. exactly nor do they provide term structures at the exact same time every day. The closest times available are near 3:44:50 p.m. for the majority of the days.

The algorithm to implement equations (2.3) through (2.5) and all the numerical analyses were made using MatLab R2022a.

3.2. Analysing the VIX

The biggest benefit of an index is the ability to compare its current level to its own past levels, allowing to visualise the evolution of the index and to understand how it evolved over time.

Figure 1 portrays the daily S&P 500 index levels alongside with daily VIX levels, through April 25, 2006 until April 30, 2019. It is possible to observe that the VIX reached a record level of 80.3 on Thursday, November 20, 2008. This record-high value coincides with one of the lowest S&P 500 index values registered during the 2008 financial crisis.

An interesting observation is that VIX often spikes when the S&P 500 index decreases. This phenomenon is easily observed during the 2008 financial crisis, where high VIX values were the norm and extreme peaks happened with some regularity. As we can see

[1] All timestamps are US Eastern Time.

[2] https://www.cboe.com/tradable_products/vix/term_structure/.

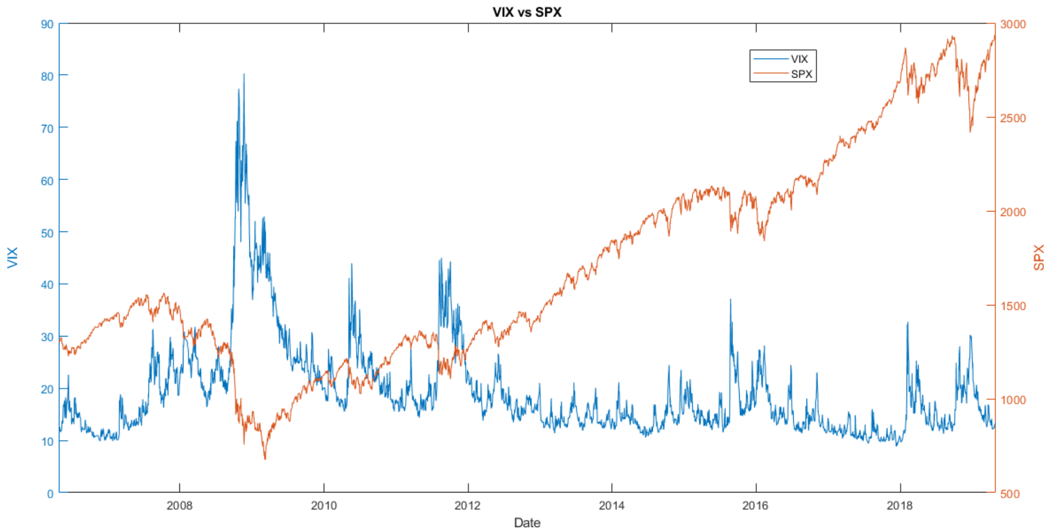

FIGURE 1. VIX and S&P 500 index daily levels, April 25, 2006 - April 30, 2019

in Table 1, the correlation coefficient between daily changes in the VIX and daily SPX returns is -0.83. This inverse relation is highlighted more when the SPX decreases and there is a spike in the VIX, as the VIX tends to take on lower levels and maintain those levels while the S&P 500 index grows. We can observe this between 2013 and late 2015.

This inverse relation between both indices is also observed in late 2011, when the SPX decreased following a downgrade of the U.S. to "AA+"from "AAA"by several credit rating agencies. Due to a "flash crash" in August 2015 it is also possible to observe this inverse relation, as well as in 2018 when the SPX dropped 9% as investors feared tighter monetary policies and the ongoing "trade war"between U.S and China.

Despite this inverse relation between both indices being commonly observed, it does not occur every time. In fact, as observed in Table 1, the S&P 500 index growth is registered in 54.83% of the time considered in our data set and the VIX was up in 45.91% of the observations. The indices had movements of the same nature in 9.85% of the time, meaning that one in every ten days the indices moved in the same direction.

No. of obs.	Correlation Coefficient	SPX Up (%)	VIX Up (%)	SPX and VIX Up (%)
3217	-0.832	54.83	45.91	9.85

TABLE 1. Correlation coefficient between the daily S&P 500 index returns and daily VIX variations, daily occurrences with SPX growth, with VIX growth and with simultaneous growth of both indices.

In addition to studying the relation between the VIX and the S&P 500 index, it is also worth to analyse how the VIX usually behaves and what are usual values for the volatility index.

Year	No. of obs.	Mean	Percentiles						
			5th	10th	25th	50th	75th	90th	95th
2006	162	**13.27**	10.23	10.69	11.56	12.25	14.93	16.96	17.77
2007	247	**17.49**	10.37	11.04	13.08	16.52	21.91	25.38	26.52
2008	240	**32.72**	18.64	19.57	21.56	24.71	41.15	61.79	67.48
2009	245	**31.50**	21.06	22.08	24.45	28.53	39.65	44.98	47.23
2010	251	**22.59**	16.36	17.25	18.36	21.79	25.34	29.85	33.80
2011	251	**24.19**	15.80	16.18	17.49	20.53	31.49	36.43	40.12
2012	246	**17.83**	14.56	15.03	15.91	17.59	19.14	21.59	22.45
2013	249	**14.26**	12.36	12.57	13.05	13.76	15.03	16.70	17.32
2014	247	**14.27**	11.41	11.65	12.44	13.81	15.28	17.22	19.01
2015	250	**18.87**	12.65	12.95	13.97	15.75	18.50	22.74	26.30
2016	250	**15.97**	11.89	12.13	13.15	14.62	17.88	22.52	24.42
2017	249	**11.32**	9.71	9.93	10.40	11.16	11.92	12.80	14.20
2018	248	**16.72**	11.49	11.93	12.91	15.33	19.83	23.33	25.81
2019	82	**15.63**	12.33	12.77	13.44	14.85	17.02	19.58	20.84
2006-2019	3217	**19.18**	10.86	11.75	13.35	16.55	22.03	28.87	37.09

Table 2. Ranges for VIX daily levels, April 25, 2006 - April 30, 2019

Table 2 summarises the ranges of the VIX throughout the time period of our data set. Firstly, the average value for the VIX across the entire time interval considered is 19.18, with a median value of 16.55. The VIX was between 13.35 and 22.03 (a range of 8.68 index points) 50% of the time, between 11.75 and 28.87 (a range of 17.15 index points) 80% of the time, and between 10.86 and 37.09 (a range of 26.23 index points) 90% of the time throughout the data set considered.

Since Table 2 also shows the mean VIX value for every year in our data set, it is possible to understand that the highest mean values were registered in 2008 and 2009, years that refer to the 2008 financial crisis. During 2008, 50% of the time the VIX was between 21.56 and 41.15, a range of 19.59 points, the biggest registered in our data set. It was also in 2008 that was registered the biggest difference between the 5th percentile and the 95th percentile, with a range of 48.84 index points.

The second largest difference was in 2009, with VIX values between 21.06 and 47.23 90% of the time. The third largest range was registered in 2011, also a year with economic troubles, where the 5th and 95th percentile were 15.80 and 40.12, respectively.

In contrast, years without stock market crashes, like 2017, is when the VIX registered the lowest average value, as well as the narrowest difference between 5th and 95th percentile, with only 4.49 index points differentiating the two.

Figure 2 demonstrates the behaviour of the VIX using a histogram. The VIX usually has lower values, specially between 12 and 18 index points. VIX values higher than 30, what is considered a high VIX, are much rarer. This supports the idea that the VIX usually takes on smaller values, considering the S&P 500 index is usually growing. Higher VIX values are less frequent, only occurring when the SPX decreases.

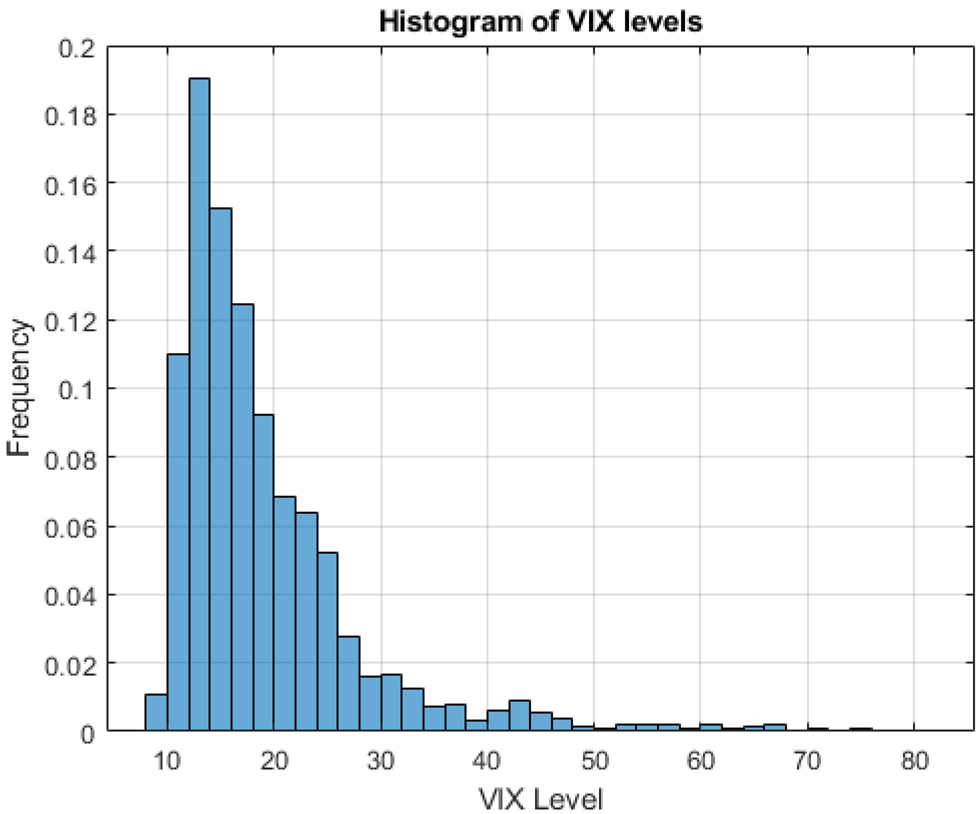

FIGURE 2. Frequency of VIX levels from April 25, 2006 to April 30, 2019. Every interval is of 2 index points.

3.3. VIX term structure

The VIX term structure allows investors, traders and analysts to observe the stock market's expected volatility implied by SPX options. This lets its users compare the market's expectation of volatility to their own expectations.

We will denote the VIX value for the n-th expiration date at time t by $S_t^{(n)}$, $n = 1, \ldots, 6$. Each term structure computed consists in connecting these six VIX values for a given date t.

Table 3 shows the average value of each point of the term structures computed. We can observe that the average value of each point of the term structure tends to grow with larger values of n, indicating that the term structure's values grow with bigger maturities.

An important fact we can also observe in Table 3 is that all values are close to 20, particularly for later expiration dates. This indicates the VIX having a tendency to revert to its mean, as concluded from Table 2 that the VIX average value is around 20 index points.

	$S_t^{(1)}$	$S_t^{(2)}$	$S_t^{(3)}$	$S_t^{(4)}$	$S_t^{(5)}$	$S_t^{(6)}$
Mean	18.91	19.61	20.29	20.76	21.16	21.48

TABLE 3. Average VIX value for each term structure maturity

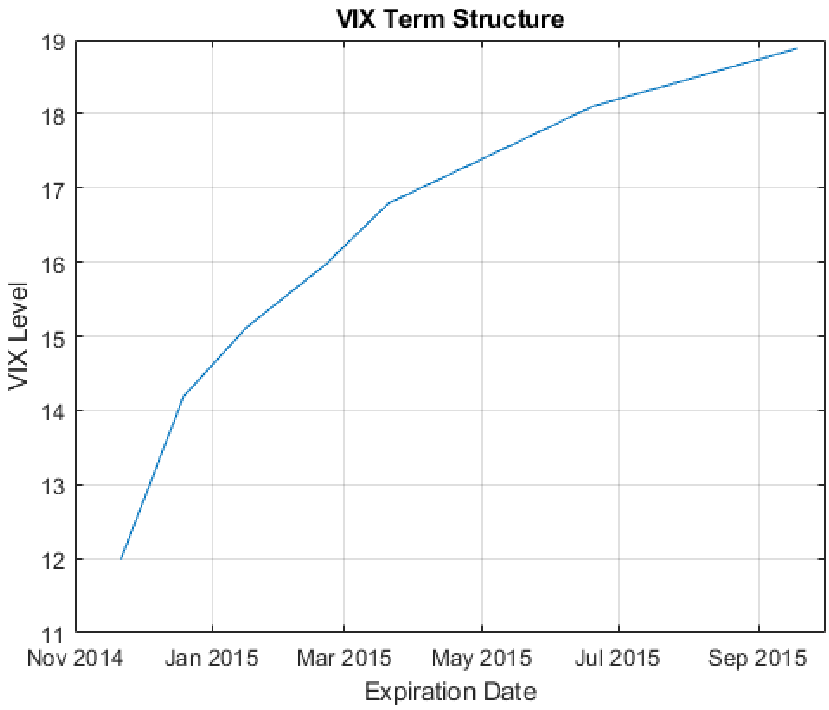

FIGURE 3. VIX term structure at time t = 2014-11-13

The VIX term structure can assume many different aspects, however there are shapes that are more common than others. The following shapes are the most common aspects the term structure takes.

Figure 3 shows a term structure where the VIX level at a certain point is bigger than the point before, $S_t^{(n)} > S_t^{(n-1)}$. This aspect is usually associated with low spot VIX values, that is, when the stock market is stable. Figure 3 is from November 13, 2014, time when the SPX was stable.

Contrastingly, Figure 4 pictures a term structure downward sloping in its entire length. This occurs when spot VIX values are high, translating into less stable market conditions. The term structure pictured is from November 21, 2008, so during the 2008 financial crisis. As we can see, the VIX at the first point is 74.7, an extremely high value.

Another shape possible for the VIX term structure is when the VIX decreases in the short-term but increases in the long term. Term structures like in Figure 5 happen when there is a large spike in the VIX in a very short span of time and the market does not believe that those conditions are going to persist.

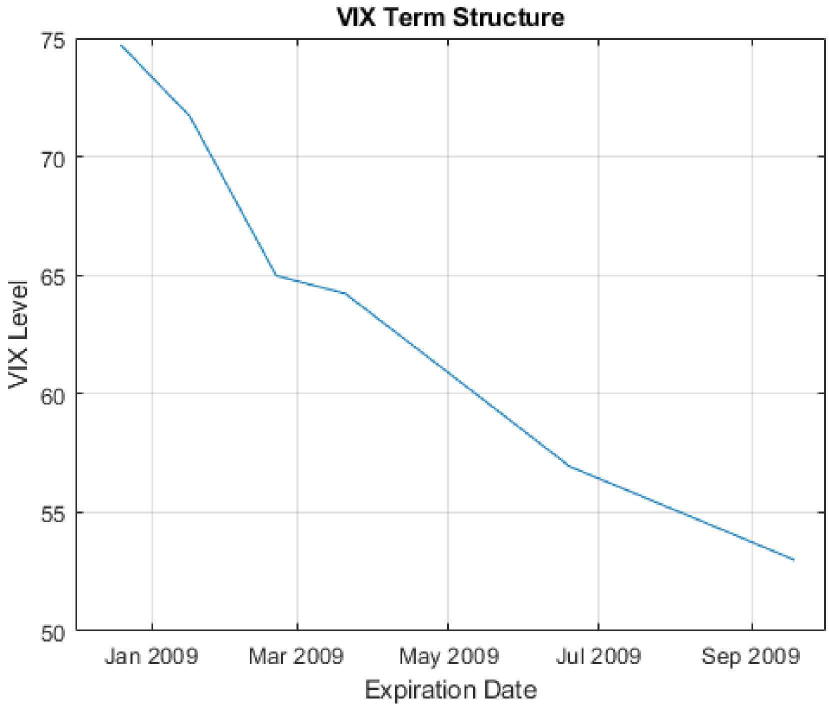

FIGURE 4. VIX term structure at time t = 2008-11-21

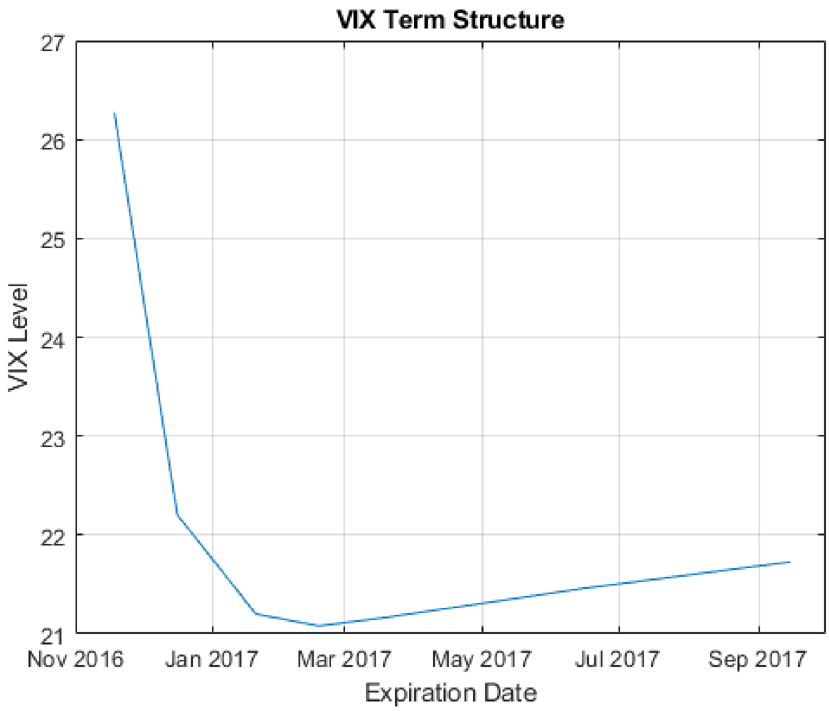

FIGURE 5. VIX Term Structure at time t = 2016-11-04

The reverse can also happen, like in Figure 6, where the VIX first increases but later decreases. These term structures occur when investors judge the market as more volatile in a short term, but don't believe it will persist for a long term.

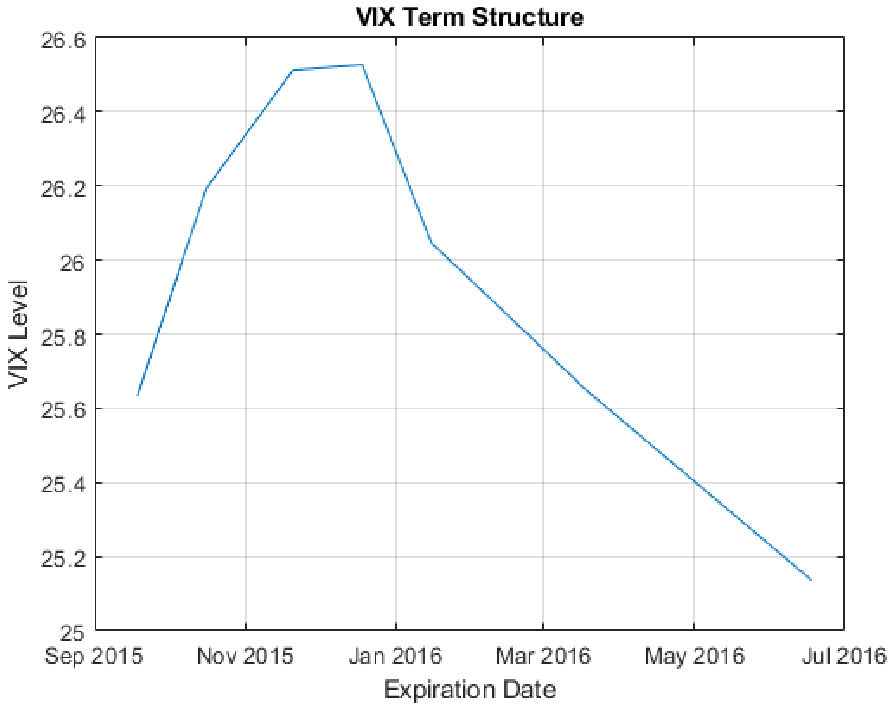

FIGURE 6. VIX term structure at time t = 2015-09-03

3.4. Term structure slope

We compute the slopes of the term structure as in by Bakshi et al. (2021). As we are not working with future contracts on the VIX, there is a slight difference, where instead of using the futures price ($F_t^{(n)}$), we use the term structure's VIX value for each expiration date ($S_t^{(n)}$).

The slopes for the term structure were then computed in the following way:

$$\text{Slope}_t^{(n)} = \log\left(\frac{S_t^{(n)}}{S_t^{(1)}}\right), \text{ for } n = 2, \dots, 6. \tag{3.1}$$

For example, $\log\left(\frac{S_t^{(4)}}{S_t^{(1)}}\right)$ represents the slope, at time t, of the portion of the term structure that starts at the first maturity date and ends at the fourth maturity date.

Table 4 summarizes the findings related to the slopes of the term structure. The mean value of $\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right)$ is 0.056 and 82.22% of those slopes are positive, as indicated by $\mathbb{1}_{\{\text{Slope}_t^{(2)}>0\}}$, while the mean value of $\log\left(\frac{S_t^{(6)}}{S_t^{(1)}}\right)$ is 0.173 and $\mathbb{1}_{\{\text{Slope}_t^{(6)}>0\}}$ is 86.66%. Similar values are also present in the remaining slopes, allowing us to conclude the implication that the VIX term structure is upward sloping for the majority of the time, showcasing downward slope only occasionally.

Another interesting finding that gives substance to the implication aforementioned is the average value being higher for slopes between more distant points of the term structure. It is possible to observe that the average value of $\text{Slope}_t^{(n)}$ is bigger than the average value of $\text{Slope}_t^{(n-1)}$, for $n = 2, \dots, 6$.

Slope	Mean	$\mathbb{1}_{\{\text{Slope}_t^{(n)}>0\}}$ (%)	SD	Min.	Max.	Percentiles 5th	25th	50th	75th	95th
$\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right)$	**0.056**	**82.22**	0.076	−0.427	0.355	−0.069	0.016	0.059	0.099	0.173
$\log\left(\frac{S_t^{(3)}}{S_t^{(1)}}\right)$	**0.101**	**85.30**	0.111	−0.613	0.463	−0.090	0.038	0.110	0.170	0.269
$\log\left(\frac{S_t^{(4)}}{S_t^{(1)}}\right)$	**0.131**	**85.79**	0.135	−0.667	0.553	−0.096	0.051	0.143	0.216	0.329
$\log\left(\frac{S_t^{(5)}}{S_t^{(1)}}\right)$	**0.154**	**86.66**	0.151	−0.723	0.614	−0.099	0.066	0.171	0.251	0.376
$\log\left(\frac{S_t^{(6)}}{S_t^{(1)}}\right)$	**0.173**	**86.66**	0.166	−0.781	0.661	−0.109	0.074	0.194	0.281	0.411

TABLE 4. Slope of the VIX term structure. The slope is computed as $\log\left(\frac{S_t^{(n)}}{S_t^{(1)}}\right)$, for $n = 2, \ldots, 6$. $\mathbb{1}_{\{\text{Slope}_t^{(n)}>0\}}$ is the fraction (in %) of the observations with positive slopes.

To discard the hypothesis of a flat term structure, we compute the following daily regression:

$$\log(S_t^{(n)}) = \alpha_t + \beta_t \log(\mathcal{T}_t^{(n)}) + \epsilon_t^{(n)}, \text{ for } n = 1, \ldots, 6 \tag{3.2}$$

where $\mathcal{T}_t^{(n)}$ denotes the time to maturity, in days, for the n-th date T of the term structure.

Table 5 summarises the results obtained in the daily regression. The average value of β is 0.0801, allowing us to discard the hypothesis that the term structure is flat. Values for β are statistically significant when the p-value associated is inferior to 0.05.

	Mean	$\mathbb{1}_{\{\beta_t>0\}}$ (%)	SD	Min.	Max.	Percentiles 5th	25th	50th	75th	95th
β	**0.0801**	**86.758**	0.0756	−0.2725	0.2518	−0.0490	0.0335	0.0881	0.1319	0.1916
(t-stat.)	(13.1)									
# p-value < 0.05	2840									
# p-value > 0.05	377									

TABLE 5. Features of β_t from the daily regressions. $\mathcal{T}_t^{(n)}$ denotes the time to maturity, in days, for contract n at time t.

To further study the slope of the term structure, we also computed the slopes between consecutive points in the term structure, that is, in the following way:

$$\text{Slope}_t^{(n)} = \log\left(\frac{S_t^{(n)}}{S_t^{(n-1)}}\right), \text{ for } n = 2, \ldots, 6. \tag{3.3}$$

Table 6 summarises the results obtained from computing all the slopes as defined by Equation 3.3. We can observe that the average value of the slopes computed decrease with bigger values of n, with $\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right)$ having the highest average value with 0.056 and,

Slope	Mean	$\mathbb{1}_{\{\text{Slope}_t^{(n)}>0\}}$ (%)	SD	Min.	Max.	Percentiles 5th	25th	50th	75th	95th
$\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right)$	**0.056**	**82.22**	0.076	−0.427	0.355	−0.069	0.016	0.059	0.099	0.173
$\log\left(\frac{S_t^{(3)}}{S_t^{(2)}}\right)$	**0.045**	**86.57**	0.042	−0.225	0.175	−0.028	0.019	0.050	0.073	0.106
$\log\left(\frac{S_t^{(4)}}{S_t^{(3)}}\right)$	**0.030**	**85.89**	0.030	−0.166	0.101	−0.024	0.013	0.034	0.051	0.073
$\log\left(\frac{S_t^{(5)}}{S_t^{(4)}}\right)$	**0.024**	**87.88**	0.020	−0.067	0.089	−0.012	0.010	0.025	0.038	0.053
$\log\left(\frac{S_t^{(6)}}{S_t^{(5)}}\right)$	**0.018**	**85.73**	0.018	−0.058	0.067	−0.013	0.007	0.021	0.031	0.044

Table 6. Features of slopes between consecutive points in the VIX term structure. The slope is computed as $\log\left(\frac{S_t^{(n)}}{S_t^{(n-1)}}\right)$, for $n = 2, \ldots, 6$. $\mathbb{1}_{\{\text{Slope}_t^{(n)}>0\}}$ is the fraction (in %) of the observations with positive slopes.

contrastingly, $\log\left(\frac{S_t^{(6)}}{S_t^{(5)}}\right)$ having the lowest average value at 0.018 allowing us to confirm the tapering of the VIX term structure throughout its length.

This tapering phenomenon is very easily observed in Figure 3, where the curve starts to flatten more the later is expiration date T.

3.5. Term structure behaviour under different economic states

As seen previously, the VIX has different levels depending on the S&P 500 index level and market behavior in general.

We employ the same analysis made by Bakshi et al. (2021) to better understand how the S&P 500 options term structure behaves during certain economic states. This allows to see how the shape of the term structure changes depending on factors that influence the VIX values.

Table 7 examines the slopes of daily term structures jointly with daily VIX level.

When the VIX is at low levels, such as 15 or less, 99% of the time the VIX term structure is upward sloping for its entire time length, as seen in the slope $\log\left(\frac{S_t^{(6)}}{S_t^{(1)}}\right)$. Regarding the slope until the nearest maturity, the term structure is sloping upwards 95% of the time.

On the contrary, when the VIX is above 30, the number of times the VIX term structure is upward sloping decreases to around 58% in its entire time length, and if considering just the first segment of the term structure, it falls to 45%. This allows us to conclude that it is a lot more likely for the VIX term structure to be upward sloping when the daily VIX level is low.

Regarding intermediate VIX values, such as daily VIX levels between 18 and 30, the term structure shows a likeliness to be upward sloping across all slopes, although the

Slope										
	Lower VIX Limit	< 10	10	13	15	18	22	30	40	> 50
	Upper VIX Limit		13	15	18	22	30	40	50	
	Frequency (days)	34	650	573	635	518	515	153	85	54
Slope	Frequency (%)	1.06	20.21	17.81	19.74	16.10	16.01	4.76	2.64	1.68
$\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right)$	Mean	0.154	0.110	0.083	0.055	0.037	0.022	−0.022	−0.020	−0.083
	$\mathbb{1}_{\{Slope_t^{(2)}>0\}}$ (%)	**100**	**98.77**	**95.46**	**83.78**	**78.57**	**70.68**	**45.75**	**48.24**	**14.81**
$\log\left(\frac{S_t^{(3)}}{S_t^{(1)}}\right)$	Mean	0.256	0.186	0.143	0.106	0.074	0.041	−0.027	−0.028	−0.147
	$\mathbb{1}_{\{Slope_t^{(3)}>0\}}$ (%)	**100**	**98.92**	**98.25**	**90.55**	**84.36**	**70.10**	**53.59**	**54.12**	**5.56**
$\log\left(\frac{S_t^{(4)}}{S_t^{(1)}}\right)$	Mean	0.325	0.238	0.184	0.142	0.100	0.051	−0.028	−0.051	−0.198
	$\mathbb{1}_{\{Slope_t^{(4)}>0\}}$ (%)	**100**	**99.69**	**98.78**	**91.65**	**87.45**	**70.49**	**54.90**	**31.76**	**5.56**
$\log\left(\frac{S_t^{(5)}}{S_t^{(1)}}\right)$	Mean	0.378	0.277	0.214	0.170	0.121	0.062	−0.028	−0.058	−0.227
	$\mathbb{1}_{\{Slope_t^{(5)}>0\}}$ (%)	**100**	**100**	**99.13**	**92.60**	**89.19**	**71.46**	**57.52**	**31.76**	**5.56**
$\log\left(\frac{S_t^{(6)}}{S_t^{(1)}}\right)$	Mean	0.420	0.308	0.239	0.193	0.137	0.069	−0.029	−0.071	−0.256
	$\mathbb{1}_{\{Slope_t^{(6)}>0\}}$ (%)	**100**	**100**	**99.13**	**93.70**	**89.19**	**70.87**	**58.17**	**28.23**	**1.85**

Table 7. Slope of the VIX term structure when market state is the daily VIX value. The slope is computed as $\log\left(\frac{S_t^{(n)}}{S_t^{(1)}}\right)$, for $n = 2, \ldots, 6$. $\mathbb{1}_{\{\mathrm{Slope}_t^{(n)}>0\}}$ is the fraction (in %) of the observations with positive slopes.

average value of the slope being considerably inferior when compared against average slope values for lower daily VIX levels.

If we consider the market state to be the daily S&P 500 index returns, the VIX term structure is upward sloping 94% of the time when daily S&P 500 index returns are between −0.005 and 0.015, as seen in Table 8. It is during this interval that the average slope has the larger value.

When daily S&P 500 index returns are below −0.035, it's rare for the term structure to be sloping upwards, happening in less than 5% of the time, and the average slope is negative. Interestingly, the same is found to happen when daily returns are above 0.035, with only 15% of the term structures computed being upward sloping and also having negative average slopes. The VIX term structure shows a downward slope when there are large movements in the S&P 500 index, independently of the direction of the movement.

Similarly to the finding when the market state is the daily VIX level, the VIX term structure also behaves differently depending on daily returns of the S&P 500 index. This confirms that the term structure being sloping upwards or downwards is linked to current economic states.

Slope										
	Lower Limit	< -0.035	−0.035	−0.025	−0.015	−0.005	0.005	0.015	0.025	> 0.035
	Upper Limit		−0.025	−0.015	−0.005	0.005	0.015	0.025	0.035	
	Frequency (days)	28	41	143	471	1693	654	131	36	20
Slope	Frequency (%)	0.87	1.27	4.45	14.64	52.63	20.33	4.07	1.12	0.62
$\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right)$	Mean	−0.108	−0.048	−0.022	0.026	0.076	0.070	0.024	0.004	−0.045
	$\mathbb{1}_{\{Slope_t^{(2)}>0\}}$ (%)	**10.71**	**29.29**	**41.96**	**71.55**	**89.66**	**90.67**	**73.28**	**55.56**	**25.00**
$\log\left(\frac{S_t^{(3)}}{S_t^{(1)}}\right)$	Mean	−0.165	−0.071	−0.018	0.057	0.132	0.121	0.046	0.021	−0.070
	$\mathbb{1}_{\{Slope_t^{(3)}>0\}}$ (%)	**14.29**	**31.71**	**48.25**	**77.92**	**92.03**	**93.27**	**72.52**	**61.11**	**25.00**
$\log\left(\frac{S_t^{(4)}}{S_t^{(1)}}\right)$	Mean	−0.209	−0.080	−0.017	0.079	0.170	0.155	0.057	0.021	−0.095
	$\mathbb{1}_{\{Slope_t^{(4)}>0\}}$ (%)	**3.57**	**36.59**	**49.65**	**80.25**	**92.14**	**93.73**	**74.05**	**58.33**	**15.00**
$\log\left(\frac{S_t^{(5)}}{S_t^{(1)}}\right)$	Mean	−0.232	−0.085	−0.011	0.097	0.198	0.182	0.069	0.029	−0.106
	$\mathbb{1}_{\{Slope_t^{(5)}>0\}}$ (%)	**3.57**	**36.59**	**53.15**	**81.53**	**92.62**	**94.80**	**74.81**	**61.11**	**15.00**
$\log\left(\frac{S_t^{(6)}}{S_t^{(1)}}\right)$	Mean	−0.256	−0.092	−0.007	0.111	0.221	0.203	0.077	0.029	−0.120
	$\mathbb{1}_{\{Slope_t^{(6)}>0\}}$ (%)	**3.57**	**31.71**	**53.15**	**81.95**	**92.79**	**94.65**	**74.05**	**58.33**	**15.00**

Table 8. Slope of the VIX term structure when market state is the daily S&P 500 index returns. Upper (lower) limit refers to the maximum (minimum) value in the return interval. The slope is computed as $\log\left(\frac{S_t^{(n)}}{S_t^{(1)}}\right)$, for $n = 2, \ldots, 6$. $\mathbb{1}_{\{\mathrm{Slope}_t^{(n)}>0\}}$ is the fraction (in %) of the observations with positive slopes.

3.6. The term structure and weekly S&P 500 index returns

We study the relation between the VIX term structure and SPX returns with the purpose of understanding if the term structure can provide reliable information on future SPX returns.

With this in mind, Table 9 examines the relation between the slopes between consecutive points of the term structure and the weekly S&P 500 index returns realised. Every Tuesday, we computed the slopes in the VIX term structure and the weekly S&P 500 returns realised (in %) during the current week. The SPX values were registered at 3:45 p.m., similarly to the VIX values used, as mentioned previously.

Across all slopes, around 85% of the time the slopes are bigger than 0, meaning they are sloping upwards. The average weekly return when $\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right)$ is positive is 0.144%. When the same slope is negative, the average value of the weekly returns is only 0.090%. This trend holds true to the remaining slopes and in the cases where n is equal to 3 and 6 the average value for the weekly returns even happens to be negative. However, an exception is found in $\log\left(\frac{S_t^{(4)}}{S_t^{(3)}}\right)$, where the average value of the weekly S&P 500 index returns is higher when the slope is negative. Even though the realised weekly S&P 500

					Percentiles		
	No. of obs.	Frequency (%)	Mean	SD	5th	50th	95th
$\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right) > 0$	531	82.71	**0.144**	1.771	−2.914	0.359	2.686
$\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right) < 0$	111	17.29	**0.090**	3.680	−5.957	0.640	5.423
$\log\left(\frac{S_t^{(3)}}{S_t^{(2)}}\right) > 0$	562	87.54	**0.167**	1.890	−3.049	0.389	2.781
$\log\left(\frac{S_t^{(3)}}{S_t^{(2)}}\right) < 0$	80	12.46	**−0.092**	3.804	−5.734	0.434	5.268
$\log\left(\frac{S_t^{(4)}}{S_t^{(3)}}\right) > 0$	558	86.92	**0.133**	1.831	−3.056	0.371	2.714
$\log\left(\frac{S_t^{(4)}}{S_t^{(3)}}\right) < 0$	84	13.08	**0.146**	3.932	−6.162	0.745	5.507
$\log\left(\frac{S_t^{(5)}}{S_t^{(4)}}\right) > 0$	569	88.63	**0.141**	1.920	−3.138	0.378	2.781
$\log\left(\frac{S_t^{(5)}}{S_t^{(4)}}\right) < 0$	73	11.37	**0.085**	3.833	−5.905	0.747	5.466
$\log\left(\frac{S_t^{(6)}}{S_t^{(5)}}\right) > 0$	552	85.98	**0.177**	1.811	−3.036	0.397	2.712
$\log\left(\frac{S_t^{(6)}}{S_t^{(5)}}\right) > 0$	90	14.02	**−0.127**	3.877	−5.978	0.010	5.468

TABLE 9. S&P 500 weekly returns features when the slope between consecutive term structure points is either positive or negative. Weekly SPX return values are shown in percentage. The slope is computed as $\log\left(\frac{S_t^{(n)}}{S_t^{(1)}}\right)$, for $n = 2, \ldots, 6$.

return is in this case higher when the slope is below 0, it is worth to highlight that the standard deviation has also doubled, implying that the volatility was much higher.

The results obtained imply that upward sloping VIX term structures are correlated to a growing S&P 500 index.

Table 10 summarises a similar analysis, but only considering weeks where the VIX term structure is strictly upward sloping, meaning all slopes are bigger than 0. Strictly upward sloping weeks make up for 468 weeks out of 642 for a frequency of 72.90%. The average weekly SPX return for those weeks is 0.1695%. This allows us to conclude that a strictly upward sloping term structure indicates a growing stock market.

Table 11 replicates the prior analysis made but only considering weeks where the term structure is strictly sloping downwards. The values obtained contrast immensely with the ones obtained in Table 10. The mean value is only 0.071 and the standard deviation is

			Percentiles		
	Mean	SD	5th	50th	95th
Weekly SPX Returns	**0.169**	1.624	−2.774	0.377	2.622

Table 10. Weekly S&P 500 index returns features for weeks with strictly upward sloping term structure.

the triple when compared to the standard deviation in weekly SPX returns relative to strictly upward sloping term structures.

While a strictly upward sloping term structure indicates a growing financial market, a strictly downward sloping term structure indicates a more volatile, regressing financial market.

			Percentiles		
	Mean	SD	5th	50th	95th
Weekly SPX Returns	**0.071**	4.684	−6.343	1.213	8.120

Table 11. Weekly S&P 500 index returns features for weeks with strictly downward sloping term structure.

Considering the VIX has a different behaviour during more stressful market conditions, we analyse the VIX terms structure and weekly S&P 500 index returns during the 2008 financial crisis. Table 12 restricts the analysis made in Table 9 to the time period correspondent to the 2008 financial crisis.

Firstly, the number of negative slopes is a lot more prominent during this time period than when compared to the values in Table 9.

Secondly, mean weekly SPX return is negative throughout every slope, but there is still a clear difference when the slopes are either positive or negative, with the mean value being considerably smaller when the slope is below 0. The standard deviation of weekly SPX returns is also higher during this time period when compared to Table 9. This indicates how much more volatile the market is during more stressful market states and implies the VIX term structure is a good indicator for such economic states.

	No. of obs.	Frequency (%)	Mean	SD	Percentiles		
					5th	50th	95th
$\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right) > 0$	53	67.09	$-\mathbf{0.285}$	2.523	−4.031	−0.410	4.194
$\log\left(\frac{S_t^{(2)}}{S_t^{(1)}}\right) < 0$	26	32.91	$-\mathbf{0.658}$	5.552	−11.370	−0.163	8.300
$\log\left(\frac{S_t^{(3)}}{S_t^{(2)}}\right) > 0$	57	72.15	$-\mathbf{0.123}$	2.882	−4.025	−0.308	4.866
$\log\left(\frac{S_t^{(3)}}{S_t^{(2)}}\right) < 0$	22	28.75	$-\mathbf{1.144}$	5.443	−11.847	−1.047	8.533
$\log\left(\frac{S_t^{(4)}}{S_t^{(3)}}\right) > 0$	43	55.43	$-\mathbf{0.071}$	2.491	−3.838	−0.308	4.454
$\log\left(\frac{S_t^{(4)}}{S_t^{(3)}}\right) < 0$	36	45.57	$-\mathbf{0.810}$	4.887	−10.614	−0.867	7.322
$\log\left(\frac{S_t^{(5)}}{S_t^{(4)}}\right) > 0$	50	63.29	$-\mathbf{0.286}$	2.912	−4.035	−0.343	4.595
$\log\left(\frac{S_t^{(5)}}{S_t^{(4)}}\right) < 0$	29	36.71	$-\mathbf{0.617}$	4.951	−10.127	−0.860	8.120
$\log\left(\frac{S_t^{(6)}}{S_t^{(5)}}\right) > 0$	42	53.16	$-\mathbf{0.364}$	2.275	−4.019	−0.343	2.944
$\log\left(\frac{S_t^{(6)}}{S_t^{(5)}}\right) < 0$	37	46.84	$-\mathbf{0.458}$	4.976	−10.567	−0.860	7.200

Table 12. S&P 500 weekly returns features for weeks within November 1, 2007 to August 1, 2009, when the slope between consecutive term structure points is either positive or negative. Weekly SPX return values are shown in percentage. The slope is computed as $\log\left(\frac{S_t^{(n)}}{S_t^{(1)}}\right)$, for $n = 2, \ldots, 6$.

CHAPTER 4

Conclusions

In this dissertation, we computed the VIX term structure conveyed by the volatility present in S&P 500 options of various maturities. The "VIX Methodology" was applied in order to obtain expected volatility implied in S&P 500 options for different time lengths. We studied the VIX values obtained, understanding how it behaved over time, how it reacts to different market events and its links to SPX returns. The term structure was derived using a large data set containing the value of the VIX for every contract maturity.

It was found that the VIX average value is 19.18 for our set of observations, with 75% of the observations having a VIX value smaller than 22.03, showing that it is much more common for the volatility index to have a lower level than a high level. There is also a very clear relationship between higher VIX values and the years correspondent to the 2008 financial crisis. A negative correlation was found between the VIX curve and the S&P 500 index curve, with a correlation coefficient of -0.832.

Studying the average value of each point of the term structure, we found that the VIX has a tendency to revert to its mean. The data set considered shows that the S&P 500 options term structure is normally sloping upwards, with 86.66% of the time showing an upward slope.

The behaviour of the VIX term structure changes depending on the economic state. When daily VIX levels are low, the VIX term structure's likelihood of being sloping upwards is a lot higher when compared to high daily VIX level. Assessing the behaviour of the term structure relatively to SPX returns, we conclude that the VIX term structure slopes downward more frequently when daily S&P 500 returns have large variations, in either direction, while showing upward slope when the returns' variation is more reduced.

Overall, the VIX term structure is rarely sloping downward when market state is stable, while showing downward slopes during more stressful market conditions. This trend was also verified during the 2008 financial crisis even though during this time the average weekly SPX return was negative across all slopes.

We also found that positive VIX term structure slopes are linked to, on average, higher SPX returns. SPX weekly returns for weeks when slopes are positive also have a much smaller standard deviation. Strictly upward sloping term structures yielded, on average, a return of 0.169% per week, while strictly downward sloping term structures' average weekly SPX return was just 0.071%.

This allows us to conclude that the shape of the VIX term structure is related to posterior SPX returns.

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