Valuation of Deferred Tax Assets using a Closed Form Solution

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Abstract

Deferred tax asset (DTA) is a tax/accounting concept that refers to an asset that may be used to reduce future tax liabilities of the holder. It usually refers to situations where a company has either overpaid taxes, paid taxes in advance or has carry-over of losses (the latter being the most common situation). DTAs are thus contingent claims, whose underlying assets are the company's future profits. Consequently, the correct approach to value such rights implies necessarily, the use of a contingent claim valuation framework. The purpose of this chapter is exactly to propose a precise and conceptually sound mathematical approach to value DTAs, considering future projections of earnings and rates, alongside the DTA's legal time limit. We will see that with the proposed evaluation techniques, the DTA's expected value will be much lower than the values normally used in today's practice, and the company's financial analysis will lead to much more sound and realistic results.

Keywords: Valuation, Deferred tax asset, accountancy, balance sheet, binomial

I - Introduction - the deferral of taxes

There have been many attempts to reach a conformity about the way income tax is treated, that is, to uniformize tax rates and regulations across international entities, but the complexity of this topic has raised some issues and critics; Hanlon, et al (2005) and Atwood, et al (2010) stated that earnings persistence and the association between current earnings and future cash flows are lower when the level of required book-tax conformity is higher. The potential benefits would include lower compliance costs for reporting income and the potential lowering of incentives to mislead the IRS (Internal Revenue Service) and capital markets (basically deterring entities from engaging into tax shelters and schemes).

The tax return of a company is based on its accounting financial statements. To provide comparable information, financial statements are prepared according to the International Financial Reporting Standards (IFRS), issued by the International Accounting Standards Board (IASB). The IASB was formed in 2001 to replace the International Accounting Standards Committee that issued International Accounting Standards (IAS). Since the previously issued IASs remain effective, we have that the main body of standards that are used worldwide by several countries are comprised of IFRSs and IASs. The companies' income, depicted by the IFRSs and IASs (refereed to simply by the Generally Accepted Accounting Principles GAAP) are their accounting profits, but these may be (and are) different from the taxable profit, since the taxable profit is calculated as a function of the tax law inherent to each country. The number of factors that lead to differences between tax and accounting returns is huge and varies from country to country. One of those factors is of relevance to the present work – the deferral of taxes.

Remove DTAs from the Balance Sheet?

Laux (2013) conducted a study to analyse the relationship between the information content of financial statements and the net deferred taxes account. Naturally, as we evaluate deferred taxes, we may find both deferred tax assets and deferred tax liabilities; the difference will result in net deferred taxes (we will henceforth refer to these net deferred taxes simply as deferred tax assets, or DTAs). The main conclusion was that the exclusion of DTAs from the results helped access the main differences from the different company's performance. This is highly related to the cost/benefit of disclosing information on DTAs since that the cost of acquiring and utilizing this information seems to nullify the benefits. Also, on the same topic, Burgstahler, et al (2002), concluded that in some occasions, managers tend to manipulate the net deferred tax asset account to increase earnings and avoid losses. This possible manipulation is also something that should be kept in mind when evaluating balance sheets where such accounts are present.

The problem of accounting DTAs on a present value basis is that under the actual rules adopted by FASB (Financial Accounting Standards Board), the deferred tax accounts are, in many cases, unlikely to reverse in the foreseeable future, since companies seem to be able to defer taxes indefinitely (Ron Colley et al, 2007). These authors address this statement in the study "Deferred Taxes in the Context of the Unit Problem" where they remove the deferred tax assets from the balance sheets. The authors state that income taxation is an aggregate phenomenon and that an aggregate approach is required, making use of the flow-through accounting method. The main argument states that, if we see taxation as a transaction between the private/ public sectors and the governmental authority, then this method would result in an equality of the tax provision and the cash outflow for a certain period, therefore eliminating deferred tax assets and liabilities. The idea of removing deferred taxes from the balance sheet has been supported by other authors like Chaney (1989) and Ketz (2010), that argue that deferred tax accounting is too complex, too expensive and too inconsistent with the US GAAP.

Valuation and Accounting of DTAs

The valuation and accounting of DTAs is the topic that must be discussed and clarified. The most important thing to notice is that deferred tax assets add value to the balance sheet since they represent the net present value of the future tax benefits (it is important to note that classical accounting relations only hold when the DTA value is indeed adjusted to its net present value (Eli Amir et al, 1997)). To determine the best way to account for deferred taxes, Amir and his peers conducted some research where they introduced net deferred taxes as a completely distinct category of assets, using the market value of equity per share as the dependent value. Amir and his team found that the valuation coefficient on deferred tax liabilities from depreciation and amortization was close to zero; also, deferred taxes from restructuring charges had valuation coefficients larger than other deferred tax components. They also concluded that the net realizable value of deferred taxes from losses and credits carried forward were negatively correlated with stock prices. In the end they concluded that even though these types of assets are very different in nature from the rest of the assets in the balance sheet, they should nonetheless be accounted for (with some subjective adjustments) in a way like any other asset or liability.

DTAs and European options

DTAs may be hard to value, since they are time-limited and may never be used at all. Their value is contingent on the future earnings of the company, and they can be used to shield these future profits from taxation – IAS 12 states that "a DTA should be recognised for all deductible temporary differences, to the extent that it is probable that taxable profit will be available against which the deductible temporary difference can be utilised". Since corporate income taxation works on an annual basis, the shielding opportunities occur once a year. This is equivalent to saying that we are faced with a compound European option (or an annual Bermuda option) that might be exercised until the last year in which the law will permit shielding, or until the DTAs value has been completely depleted by its use. Consistent with this line of thinking, there is an ongoing debate regarding the appropriateness of including DTA in the banks' regulatory capital calculation, since by doing so we are assuming its "full" worth; something that is clearly misleading.

Focus on Bank's DTAs

On the special case of the banking sector, banks are required to maintain certain levels of regulatory capital to provide a buffer against potential future losses (Kim & Santomero, 1998), (Ryan 2007), (Baesens & van Gestel, 2009). In many countries (Kara 2016), banks can count a portion (or all) of their DTAs towards regulatory capital requirements (since the adoption of SFAS[1] No. 109 in 1992 – specifically the establishment of valuation allowances).

Under normal circumstances, a bank's DTAs usually originates in the carry-over of losses (though it can also arise from overpaying some taxes). The corresponding rights are registered in the balance sheet as assets, although in Amir and Sougiannis (1999) it also argued that DTA may have implications for the perception of the firm as a going concern (dubbed as the information effect), since if the DTA arose from past operating losses, future losses would be likely to incur; this means that future liabilities could be more than likely, and thus such "assets" should be regarded with great suspicion).

Throughout the recent financial crisis (2008-2013), major media outlets routinely drew attention to the banks' DTA positions, classifying them as tenuous contributions towards regulatory capital. In Reilly (2009), it was noted that tier 1 capital ratios contained "fluff" – mentioning DTA as the primary culprit, calling in an "airy asset". The Basel Committee on Banking Supervision specifically targeted the removal of DTAs as a potential method for improving the ability of regulatory capital to protect banks from losses[2]. At the same time, the banking industry has pushed for the opposite; namely a greater inclusion of DTA in the regulatory capital calculation, to "ease" the amount of (real) regulatory capital.

Gallemore (2011) investigated the credit risk associated with the deferred tax asset component of bank regulatory capital. He hypothesized that banks that have a larger proportion of regulatory capital composed of deferred tax assets were more likely to fail. He employed a hazard model to test a sample of commercial banks and found that the proportion of regulatory capital composed of deferred tax assets was positively associated with the risk of bank failure during the recent financial crisis. Gallemore (2011) attributed his

[1] Statement of Financial Accounting Standards
[2] Consultative document of December 2009, entitled "Strengthening the resilience of the banking sector".

findings to the fact that the benefits of deferred tax assets couldn't be realized unless banks generated positive taxable income in the future.

High DTAs = Low creditworthiness

The relationship of DTAs with the creditworthiness of a company has already deserved some work from academic community. The effects of book-tax differences on a firm's credit risk were analysed in Crabtree & Maher (2009), Ayers et al (2010), Edwards (2011) and Gallemore (2011); all agreeing that great amounts of deferred taxes were associated with higher risks and lower earnings quality, resulting in a decline of creditworthiness. Additionally, studies of the impact of DTAs on credit ratings led to the conclusion that deferred tax positions are substantial for many firms (between 5% and 10% of all assets according to Poterba et al (2011).

How to value?

It is thus clear that the DTAs must be correctly valued, and that simply adding them to a bank's or company's balance sheet in full as an asset might contribute to obfuscate the institution's true financial condition – even a situation in which the DTA is fully used before its expiration date, we still must account for the cost of capital. Moodys (2015) reported DTAs were considered "a low-quality form of assets, and thus a low-quality source of capital", and consequently, Moody's decided to "limit the contribution of DTAs in its calculations of company's tangible common equity (TCE)". As analysed in De Vries (2018), several DTA valuation methods can be used, but they are essentially very subjective, and basically result in a valuation allowance, for which there is no consistent accepted method to calculate – this chapter aims to resolve such shortcomings, by solving for the expected value of the DTA, in the sense of calculating exactly which amounts are expected to be discounted as tax payments, and when.

II - DTA Mathematical Model

Let us consider a DTA with official book value $D_{\max}$ and a lifespan of T (in years). The effective (realistic) value of such DTA is contingent on future profits and shall always be (equal or) lower than $D_{\max}$. The effective value of the considered DTA, D, can be represented as:

$$D = \sum_{t=1}^{T} \frac{R_t^{\,+} - R_{t+1}^{\,+}}{\prod_{j=1}^{t} \left(1+\tau_j\right)}, \tag{1}$$

where τ_j is the interest yield in year *j*, R_t is the remaining book value DTA in the beginning of year *t* defined as

$$R_t = D_{\max} - \sum_{i=1}^{t-1} u_i^+ , \tag{2}$$

u_i is the profit in year *i* multiplied by taxes (basically, it is the tax payment that is discounted from the DTA) and $(\cdot)^+$ denotes the operation $x^+ = \max\{x,0\}$. Both the yearly profits and yields are assumed to be independently distributed random variables (RVs). Then, the following the objective will be to find the expected value of *D* which can be expressed as:

$$\bar{D} = \sum_{t=1}^{T} \mathrm{E}\left[1 \Big/ \prod_{j=1}^{t} (1+\tau_j) \right] \left(\mathrm{E}\left[R_t^+ \right] - \mathrm{E}\left[R_{t+1}^+ \right] \right). \tag{3}$$

Computing $\bar{D}$ basically requires finding expressions for the expected value of the interest yield weight $\mathrm{E}\left[1 \Big/ \prod_{j=1}^{t} (1+\tau_j) \right]$ and for the expected value of the positive part of the remaining DTA in the beginning of each year *t*, $\mathrm{E}\left[R_t^+ \right]$.

a) Expected Value of the Interest Yield Weight

In order to compute the expected value $\mathrm{E}\left[1 \Big/ \prod_{j=1}^{t} (1+\tau_j) \right]$, we first write it as

$$\mathrm{E}\left[1 \Big/ \prod_{j=1}^{t} (1+\tau_j) \right] = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \frac{1}{\prod_{j=1}^{t} (1+\tau_j)} p_{\tau_1}(\tau_1) \ldots p_{\tau_t}(\tau_t) \, d\tau_1 \ldots d\tau_t \tag{4}$$

where $p_{\tau_j}(\tau_j)$ is the probability density function (PDF) of the interest yield in year *j*. We will consider that all the τ_j are independently distributed and follow the same PDF but with different means $\bar{\tau}_j$ and standard deviations σ_{τ_j}.

According to the Cox-Ingersoll-Ross (CIR) model (Cox et al. 1985), the interest yield would follow a non-central chi-squared distribution, but in this chapter, due to the reduced time spans of the DTAs and ability to limit the estimated variability of the interest yield, we will assume an uniform distribution for the interest yield in order to provide mathematical tractability (and obtain a closed form solution). So being, we have:

$$p_{\tau_j}(\tau_j) = \begin{cases} \dfrac{1}{2\sqrt{3}\sigma_{\tau_j}}, & \tau_j \in \left[\bar{\tau}_j - \sqrt{3}\sigma_{\tau_j}, \bar{\tau}_j + \sqrt{3}\sigma_{\tau_j} \right] \\ 0, & \text{otherwise} \end{cases} . \tag{5}$$

It is now easy to show that

$$\int_{-\infty}^{+\infty} \frac{1}{(1+\tau_j)} p_{\tau_j}(\tau_j) d\tau_j = \frac{1}{2\sqrt{3}\sigma_{\tau_j}} \log\left(\frac{1+\bar{\tau}_j+\sqrt{3}\sigma_{\tau_j}}{1+\bar{\tau}_j-\sqrt{3}\sigma_{\tau_j}}\right) \tag{6}$$

which results in

$$\mathrm{E}\left[1\middle/\prod_{j=1}^{t}(1+\tau_j)\right] = \prod_{j=1}^{t} \frac{1}{2\sqrt{3}\sigma_{\tau_j}} \log\left(\frac{1+\bar{\tau}_j+\sqrt{3}\sigma_{\tau_j}}{1+\bar{\tau}_j-\sqrt{3}\sigma_{\tau_j}}\right). \tag{7}$$

b) Expected Value of Remaining DTA

According to (3), the computation of $\bar{D}$ also requires the evaluation of the expected value of the positive part of the remaining DTA in the beginning of each year t, i.e., $\mathrm{E}\left[R_t^+\right]$. In order to obtain an expression for $\mathrm{E}\left[R_t^+\right]$, we start with the derivation of the PDF of R_t. First we rewrite (2) as

$$R_t = D_{\max} - U_t, \tag{8}$$

where U_t is the sum of all profits multiplied by taxes up until the year t-1, which is defined as:

$$U_t = \sum_{i=1}^{t-1} u_i^+, \tag{9}$$

comprising a sum of independent, rectified, RVs u_i^+. If each non rectified RV u_i is described by a PDF $p_{u_i}(u_i)$, then u_i^+ follows the associated rectified PDF which is given by:

$$p_{u_i^+}(u_i^+) = (1-p_i)\delta(u_i^+) + p_{u_i}(u_i^+)H(u_i^+) \tag{10}$$

where $\delta(x)$ is the Dirac delta function, $H(x)$ is the unit step function

$$H(x) = \begin{cases} 0, & x \le 0 \\ 1, & x > 0 \end{cases} \tag{11}$$

and p_i is the probability of having positive profit, i.e.,

$$p_i = \int_0^{+\infty} p_{u_i}(u_i)\, du_i \tag{12}$$

Note that when the second term in (10) is normalized by p_i, it represents the truncated PDF associated with $p_{u_i}(u_i)$ which can be written as

$$p_{u_i|u_i>0}(u_i) = p_{u_i}(u_i)H(u_i)/p_i \,. \tag{13}$$

Knowing that the PDF of a sum of independent RVs can be found using the convolution of the individual PDFs (Hogg et al 2004), we can write the PDF of U_t as:

$$p_{U_t}(U_t) = \left(p_{u_1^+} * \ldots * p_{u_{t-1}^+}\right)(U_t)$$
$$= \sum_{k_1=0}^{1} \cdots \sum_{k_{t-1}=0}^{1} p_1^{k_1} \ldots p_t^{k_{t-1}} (1-p_1)^{1-k_1} \ldots (1-p_t)^{1-k_{t-1}} \mathrm{C}_{\{k_i\}_{i=1}^{t-1}}(U_t) \tag{14}$$

where * denotes the convolution operation which, for two functions $f(x)$ and $g(x)$ is defined as

$$(f * g)(x) = \int_{-\infty}^{+\infty} f(v) g(x-v) dv \quad . \tag{15}$$

Therefore, in (14) $\mathrm{C}_{\{k\}_{i=1}^{t-1}}(U_t)$ corresponds to the (t-1)-fold convolution

$$C_{\{k_i\}_{i=1}^{t-1}}(U_t) = \left[\left((1-k_1)\delta + k_1 p_{u_1|u_1>0}\right) * \ldots * \left((1-k_{t-1})\delta + k_{t-1} p_{u_{t-1}|u_{t-1}>0}\right)\right](U_t) . \tag{16}$$

The second form in (14) makes it explicit all the possible outcomes in terms of years with positive profit and with loss during the timeframe in use. Combining (8) with (14) we can express the PDF of R_t as

$$p_{R_t}(R_t) = \sum_{k_1=0}^{1} \cdots \sum_{k_{t-1}=0}^{1} p_1^{k_1} \ldots p_t^{k_{t-1}} (1-p_1)^{1-k_1} \ldots (1-p_t)^{1-k_{t-1}} \mathrm{C}_{\{k_i\}_{i=1}^{t-1}}(D_{\max} - R_t) \tag{17}$$

which allows us to compute $\mathrm{E}\left[R_t^+\right]$ using

$$\mathrm{E}\left[R_t^+\right] = \int_{-\infty}^{+\infty} \max\{R_t, 0\}\, p_{R_t}(R_t)\, dR_t$$
$$= \sum_{k_1=0}^{1} \cdots \sum_{k_{t-1}=0}^{1} p_1^{k_1} \ldots p_t^{k_{t-1}} (1-p_1)^{1-k_1} \ldots (1-p_t)^{1-k_{t-1}} I_{\{k_i\}_{i=1}^{t-1}}(D_{\max}) \tag{18}$$

where we defined the following auxiliary coefficient required for the summation terms

$$I_{\{k_i\}_{i=1}^{t-1}}(D_{\max}) = \int_{0}^{+\infty} R_t\, \mathrm{C}_{\{k_i\}_{i=1}^{t-1}}(D_{\max} - R_t)\, dR_t \quad . \tag{19}$$

In order to evaluate the integral in (19) we will consider four different cases in terms of number of years with positive profit (i.e. $\#\{i : k_i \neq 0\}$, where # denotes the cardinality of the set): no years with profit, one year with profit, two years with profit and three or more years with profit. A uniform distribution with mean $\overline{u}_i$ and standard deviation σ_{u_i} will be assumed for each yearly profit, u_i, with the PDF expressed as

$$p_{u_i}(u_i) = \begin{cases} \dfrac{1}{b_i - a_i}, & u_i \in [a_i, b_i] \\ 0, & \text{otherwise} \end{cases} \quad , \tag{20}$$

where

$$a_i = \bar{u}_i - \sqrt{3}\sigma_{u_i} \tag{21}$$

and

$$b_i = \bar{u}_i + \sqrt{3}\sigma_{u_i} \,. \tag{22}$$

For this PDF the probability of having positive profit, (12), is simply

$$p_i = \frac{b_i - a_i^{+}}{b_i - a_i} \tag{23}$$

b1) Case of no years with positive profit:

The sequence without positive profit years ($\#\{i : k_i \neq 0\} = \varnothing$) results in a trivial convolution of Dirac delta functions in (16) which is also a Dirac delta function. Therefore (19) results simply in

$$I_{\{k_i\}_{i=1}^{t-1}}(D_{\max}) = D_{\max} \,. \tag{24}$$

b2) Case of only one year, i, with positive profit:

The convolution in (16) is also trivial to compute for the sequences with only one year with positive profit ($\#\{i : k_i \neq 0\} = 1$) as it consists in the convolution of Dirac delta functions with a single truncated (and normalized) PDF obtained from (20), resulting in

$$C_{\{k_i\}_{i=1}^{t-1}}(D_{\max} - R_t) = p_{u_i|u_i>0}(D_{\max} - R_t) \,. \tag{25}$$

Inserting (25) into (19) then gives

$$I_{\{k_i\}_{i=1}^{t-1}}(D_{\max}) = \frac{1}{2\left(b_i - a_i^{+}\right)}\left\{\left[\left(D_{\max} - a_i^{+}\right)^{+}\right]^2 - \left[\left(D_{\max} - b_i\right)^{+}\right]^2\right\} . \tag{26}$$

b3) Case of two years, i and j, with positive profit:

For the sequences with only two positive profit years ($\#\{i : k_i \neq 0\} = 2$), indexed by i and j, (16) simplifies to a convolution of two truncated (and normalized) PDFs obtained from (20), whose resulting expression can be written as

$$C_{\{k_i\}_{i=1}^{t-1}}(D_{\max} - R_t) = \frac{1}{l_i l_j}\begin{cases} D_{\max} - R_t - a_i^{+} - a_j^{+}, & D_{\max} - a_i^{+} - a_j^{+} - \min\{l_i, l_j\} \leq R_t < D_{\max} - a_i^{+} - a_j^{+} \\ \min\{l_i, l_j\}, & D_{\max} - a_i^{+} - a_j^{+} - \max\{l_i, l_j\} \leq R_t < D_{\max} - a_i^{+} - a_j^{+} - \min\{l_i, l_j\} \\ R_t - D_{\max} + b_i + b_j, & D_{\max} - b_i - b_j \leq R_t < D_{\max} - a_i^{+} - a_j^{+} - \max\{l_i, l_j\} \\ 0, & \textit{otherwise} \end{cases} \tag{27}$$

where

$$l_i = b_i - a_i^{+} \,. \tag{28}$$

After inserting (27) into (19) and performing the integral we obtain

$$I_{\{k_i\}_{i=1}^{t-1}}\left(D_{\max}\right)=\frac{1}{l_i l_j}\left[\left(\frac{1}{3}R_t^3+\frac{\left(b_i+b_j-D_{\max}\right)}{2}R_t^2\right)\Bigg|_{\left(D_{\max}-b_i-b_j\right)^+}^{\left(D_{\max}-a_i^+-a_j^+-\max\left\{l_i,l_j\right\}\right)^+}+\right.$$

$$\left.+\left(\frac{\min\left\{l_i,l_j\right\}}{2}R_t^2\right)\Bigg|_{\left(D_{\max}-a_i^+-a_j^+-\max\left\{l_i,l_j\right\}\right)^+}^{\left(D_{\max}-a_i^+-a_j^+-\min\left\{l_i,l_j\right\}\right)^+}+\left(-\frac{1}{3}R_t^3+\frac{\left(D_{\max}-a_i^+-a_j^+\right)}{2}R_t^2\right)\Bigg|_{\left(D_{\max}-a_i^+-a_j^+-\min\left\{l_i,l_j\right\}\right)^+}^{\left(D_{\max}-a_i^+-a_j^+\right)^+}\right] \tag{29}$$

where we adopt the notation $\left(f(x)\right)\Big|_c^d=f(d)-f(c)$.

b4) Case of three or more years with positive profit:

For all the sequences with three or more years with profit ($\#\{i:k_i\neq 0\}\geq 3$), instead of trying to compute all subsequent convolutions we can apply the Central Limit Theorem (CLT) and approximate the sum of the nonzero u_i^+ as a Gaussian distribution with mean $\sum_{i=1}^{t-1}k_i\overline{u_i^+}$ and squared standard deviation $\sum_{i=1}^{t-1}k_i\sigma_{u_i^+}{}^2$ where

$$\sigma_{u_i^+}{}^2=\frac{\left(b_i-a_i^+\right)^2}{12} \tag{30}$$

and

$$\overline{u_i^+}=\frac{a_i^++b_i}{2}. \tag{31}$$

In this case we can write

$$C_{\{k_i\}_{i=1}^{t-1}}\left(D_{\max}-R_t\right)=\frac{1}{\sqrt{2\pi\sigma'^2}}e^{-\frac{\left(R_t-\mu'\right)^2}{2\sigma'^2}} \tag{32}$$

where

$$\sigma'^2=\sum_{i=1}^{t-1}k_i\sigma_{u_i^+}{}^2 \tag{33}$$

and

$$\mu'=D_{\max}-\sum_{i=1}^{t-1}k_i\overline{u_i^+} \tag{34}$$

Performing the integration in (19) then results in

$$I_{\{k_i\}_{i=1}^{t-1}}\left(D_{\max}\right)=\frac{\sigma'}{\sqrt{2\pi}}e^{-\frac{\mu'^2}{2\sigma'^2}}+\frac{1}{2}\mu' erfc\left(-\frac{\mu'}{\sqrt{2\pi}\sigma'}\right) \tag{35}$$

where $erfc(x)$ is the complementary error function defined as

$$erfc(x)=\frac{2}{\sqrt{\pi}}\int_{x}^{+\infty}e^{-\upsilon^2}d\upsilon \tag{36}$$

c) Case of Independent and Identically Distributed Profits

If we assume that the RVs u_i are not only independent but also identically distributed with mean $\bar{u}$, standard deviation σ_u and PDF $p_u(u)$, then most of the previous expressions can be simplified. In this case (18) becomes

$$\mathrm{E}\left[R_t^+\right]=\sum_{k=0}^{t-1}\binom{t-1}{k}p^k(1-p)^{t-1-k}I_k(D_{\max}) \tag{37}$$

where $\binom{t-1}{k}$ denotes number of combinations of *t*-1 elements taken *k* at a time,

$p=\int_0^{+\infty}p_u(u)du$,

$$I_k(D_{\max})=\int_0^{+\infty}R_t\,\mathrm{C}_k(D_{\max}-R_t)dR_t\,, \tag{38}$$

$\mathrm{C}_k(U_t)$ is simply the k-fold convolution of the truncated PDF $p_{u|u>0}(u)$ with itself and *p* is the probability of having positive profit which, for uniformly distributed RVs (23), simplifies to

$$p=\frac{b-a^+}{b-a}\,. \tag{39}$$

Repeating the explicit computation of (38) for the four different cases in terms of years with positive profit we obtain:

$$I_0(D_{\max})=D_{\max} \tag{40}$$

for the sequences of no years with positive profit,

$$I_1(D_{\max})=\frac{1}{2(b-a^+)}\left\{\left[(D_{\max}-a^+)^+\right]^2-\left[(D_{\max}-b)^+\right]^2\right\}, \tag{41}$$

for the sequences with only one year with positive profit,

$$I_2(D_{\max})=\frac{1}{(b-a^+)^2}\left[\frac{2}{3}\left((D_{\max}-b-a^+)^+\right)^3-\frac{(D_{\max}-2b)}{2}\left((D_{\max}-b-a^+)^+\right)^2-\right.$$

$$-\frac{1}{3}\left((D_{\max}-2b)^+\right)^3-\frac{1}{3}\left((D_{\max}-2a^+)^+\right)^3+\frac{(D_{\max}-2b)\left((D_{\max}-2b)^+\right)^2}{2}+$$

$$+\frac{\left(D_{\max}-2a^{+}\right)\left(\left(D_{\max}-2a^{+}\right)^{+}\right)^{2}}{2}-\frac{\left(D_{\max}-2a^{+}\right)}{2}\left(\left(D_{\max}-b-a^{+}\right)^{+}\right)^{2}\Bigg], \tag{42}$$

for the sequences with two years with positive profit and (approximately)

$$I_{k\geq 3}\left(D_{\max}\right)=\frac{\sigma'}{\sqrt{2\pi}}e^{-\frac{\mu'^{2}}{2\sigma'^{2}}}+\frac{1}{2}\mu' erfc\left(-\frac{\mu'}{\sqrt{2\pi}\sigma'}\right) \tag{43}$$

for all the remaining sequences (three or more years with positive profit). In (43), we use $\sigma'^{2}=k\sigma_{u^{+}}{}^{2}$ and $\mu'=D_{\max}-k\overline{u^{+}}$.

III - Putting the Mathematical Model to Use

In order to clarify the use of the presented expressions we describe a simple example of the computation of the expected value of a DTA with a lifespan of T=3 years, considering independent and identically distributed uniform RVs for the profits. In this case, according to (3), the intended value is computed using

$$\bar{D}=\sum_{t=1}^{3}\mathrm{E}\left[1\Big/\prod_{j=1}^{t}\left(1+\tau_{j}\right)\right]\left(\mathrm{E}\left[R_{t}^{+}\right]-\mathrm{E}\left[R_{t+1}^{+}\right]\right). \tag{44}$$

where the terms inside the summation are

$$\mathrm{E}\left[R_{1}^{+}\right]=D_{\max}, \tag{45}$$

(obtained from (37) and (40)),

$$\mathrm{E}\left[R_{2}^{+}\right]=\left(\frac{a^{+}-a}{b-a}\right)D_{\max}+\frac{1}{2(b-a)}\left\{\left[\left(D_{\max}-a^{+}\right)^{+}\right]^{2}-\left[\left(D_{\max}-b\right)^{+}\right]^{2}\right\}, \tag{46}$$

(obtained from (37), (40) and (41)),

$$\mathrm{E}\left[R_{3}^{+}\right]=\left(\frac{a^{+}-a}{b-a}\right)^{2}D_{\max}+\frac{\left(a^{+}-a\right)}{\left(b-a\right)^{2}}\left\{\left[\left(D_{\max}-a^{+}\right)^{+}\right]^{2}-\left[\left(D_{\max}-b\right)^{+}\right]^{2}\right\}+\left(\frac{b-a^{+}}{b-a}\right)^{2}I_{2}\left(D_{\max}\right) \tag{47}$$

(obtained from (37), (40), (41) and (42)), and

$$\mathrm{E}\left[R_{4}^{+}\right]=\left(\frac{a^{+}-a}{b-a}\right)^{3}D_{\max}+\frac{3\left(a^{+}-a\right)^{2}}{2\left(b-a\right)^{3}}\left\{\left[\left(D_{\max}-a^{+}\right)^{+}\right]^{2}-\left[\left(D_{\max}-b\right)^{+}\right]^{2}\right\}+$$
$$+3\frac{\left(b-a^{+}\right)^{2}\left(a^{+}-a\right)}{\left(b-a\right)^{3}}I_{2}\left(D_{\max}\right)+\left(\frac{b-a^{+}}{b-a}\right)^{3}\left(\sqrt{\frac{3}{2\pi}}\sigma_{u^{+}}e^{-\frac{\left(D_{\max}-3\overline{u^{+}}\right)^{2}}{6\sigma_{u^{+}}{}^{2}}}+\frac{1}{2}\left(D_{\max}-3\overline{u^{+}}\right)erfc\left(-\frac{D_{\max}-3\overline{u^{+}}}{\sqrt{6\pi}\sigma_{u^{+}}}\right)\right) \tag{48}$$

(obtained from (37), (40), (41), (42) and (43)).
Therefore, the expected value for the DTA is given as:

$$\bar{D} = \sum_{t=1}^{3} \mathrm{E}\left[1 \Big/ \prod_{j=1}^{t}\left(1+\tau_j\right)\right]\left(\mathrm{E}\left[R_t^{+}\right]-\mathrm{E}\left[R_{t+1}^{+}\right]\right) = \sum_{t=1}^{3}\left[\prod_{j=1}^{t}\frac{1}{2\sqrt{3}\sigma_{\tau_j}}\log\left(\frac{1+\bar{\tau}_j+\sqrt{3}\sigma_{\tau_j}}{1+\bar{\tau}_j-\sqrt{3}\sigma_{\tau_j}}\right)\right]\left(\mathrm{E}\left[R_t^{+}\right]-\mathrm{E}\left[R_{t+1}^{+}\right]\right) =$$

$$= \frac{1}{2\sqrt{3}\sigma_{\tau_1}}\log\left(\frac{1+\bar{\tau}_1+\sqrt{3}\sigma_{\tau_1}}{1+\bar{\tau}_1-\sqrt{3}\sigma_{\tau_1}}\right)\left(\mathrm{E}\left[R_1^{+}\right]-\mathrm{E}\left[R_2^{+}\right]\right)+\prod_{j=1}^{2}\left[\frac{1}{2\sqrt{3}\sigma_{\tau_j}}\log\left(\frac{1+\bar{\tau}_j+\sqrt{3}\sigma_{\tau_j}}{1+\bar{\tau}_j-\sqrt{3}\sigma_{\tau_j}}\right)\right]\left(\mathrm{E}\left[R_2^{+}\right]-\mathrm{E}\left[R_3^{+}\right]\right)+$$

$$+\prod_{j=1}^{3}\left[\frac{1}{2\sqrt{3}\sigma_{\tau_j}}\log\left(\frac{1+\bar{\tau}_j+\sqrt{3}\sigma_{\tau_j}}{1+\bar{\tau}_j-\sqrt{3}\sigma_{\tau_j}}\right)\right]\left(\mathrm{E}\left[R_3^{+}\right]-\mathrm{E}\left[R_4^{+}\right]\right)$$

IV - Simulation Results

Monte Carlo simulations were run for 10.000 loops, and the DTA's lifespan was assumed to be 10 years, with book value D_{max}=100 (adimensional). Both the yearly interest yield and each year's mean expected profit multiplied by the tax, u_i, assumed a uniform distribution (Note that u_i =10 is equivalent to a profit of 50 and tax rate of 20%, for instance). In Figure 1 we compare the simulated values to the analytical values obtained by the deduced formulas, to find that they coincide. This test was actually executed for all figures and it was observed that the analytical values always matched the simulated values almost to perfection, proving that the CLT based approximation adopted for 3 or more years with positive profit proved to be very accurate. The curves of Figure 1 represent the cumulated DTA usage (to the present value) at the end of each year[3]; whereas the curves of Figure 2 portray the yearly DTA consumption under the same conditions.

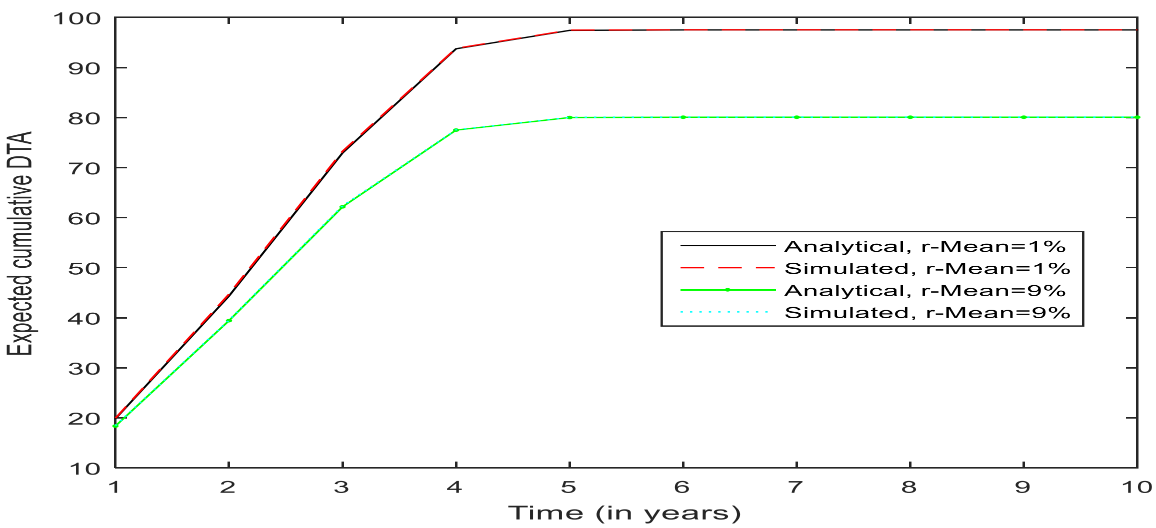

Figure 1- Expected cumulated DTA; increasing mean profits – analytical vs simulated results

[3] Note that only the annual results are simulated, and thus the results could be represented only by points; lines joining the points were chosen in order to improve the readability of the results

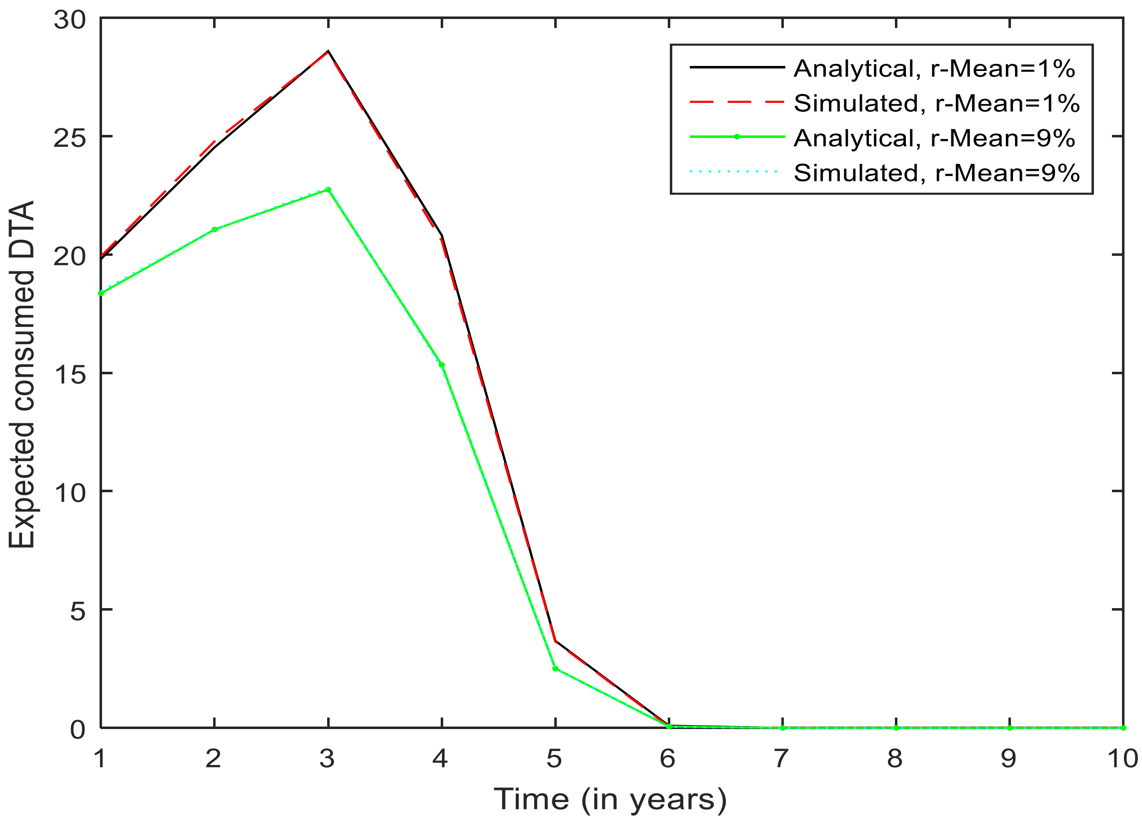

Figure 2- Expected yearly DTA consumption; increasing mean profits – analytical vs simulated results

Figure 3 and Figure 4 present the same simulations present in Figure 1 and Figure 2, but now with only the simulated results. The variable $\overline{u_i}$ was made to increase from 20 (adimensional) with increments of 5 units each year; all having a fixed standard deviation σ_{u_i} of 10. From Figure 3 notice that the initial DTA value is a bit less than 20 (due to the discount factor), climbing up to almost 100 (if there was no discount factor, the cumulated DTA would reach 100). As expected, the higher yield will output the lowest DTA value. Looking at Figure 4, we can see that by year 6 the DTA was all used up, which means that it only took 5 years for these DTA to be fully used, each with different present values due to the different yields.

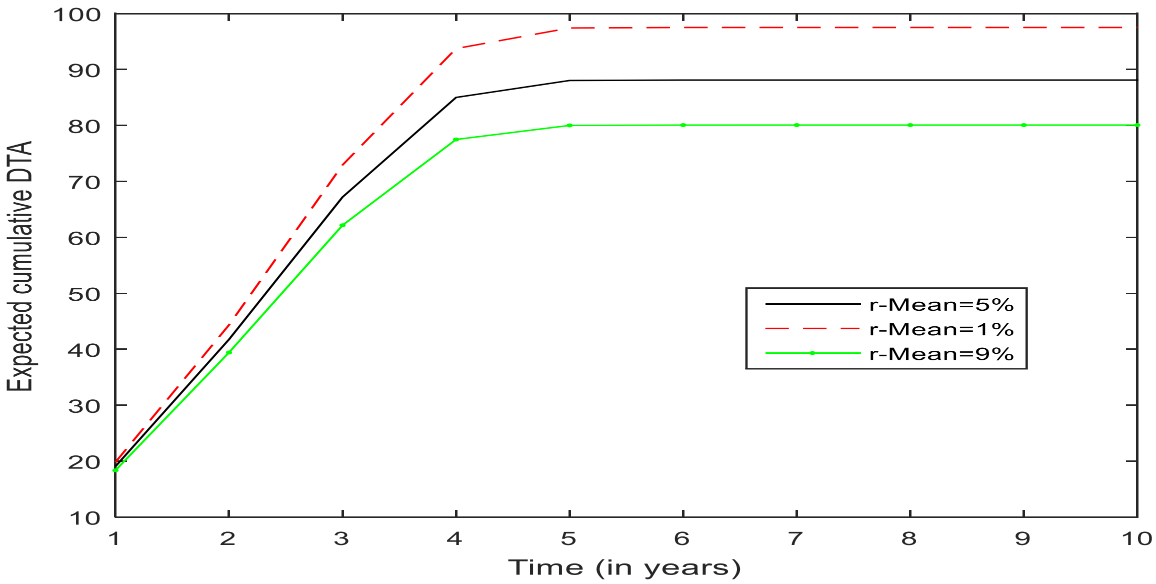

Figure 3- Expected cumulated DTA; increasing mean profits

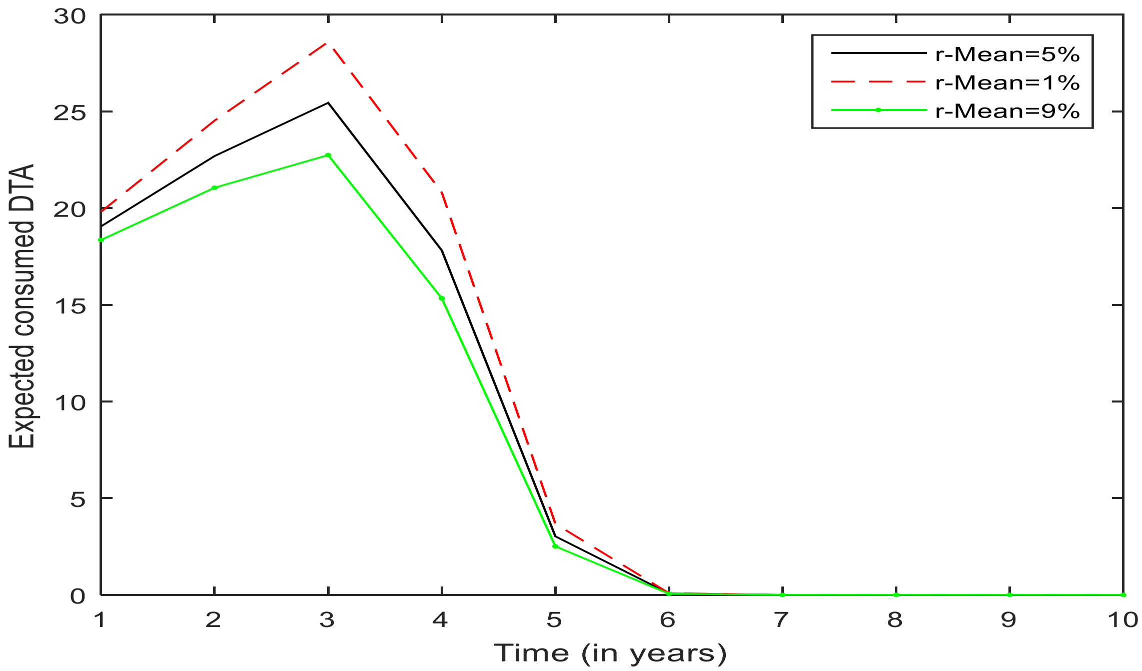

Figure 4- Expected yearly DTA consumption; increasing mean profits

In Figure 5 and Figure 6 we have a similar situation as before, but now with the standard deviation σ_{u_i} starting at 10 and increasing 4 units each year. With this increasing deviation, note that the expected use of the totality of the DTA is deferred to the seventh year (previously it was the fifth year).

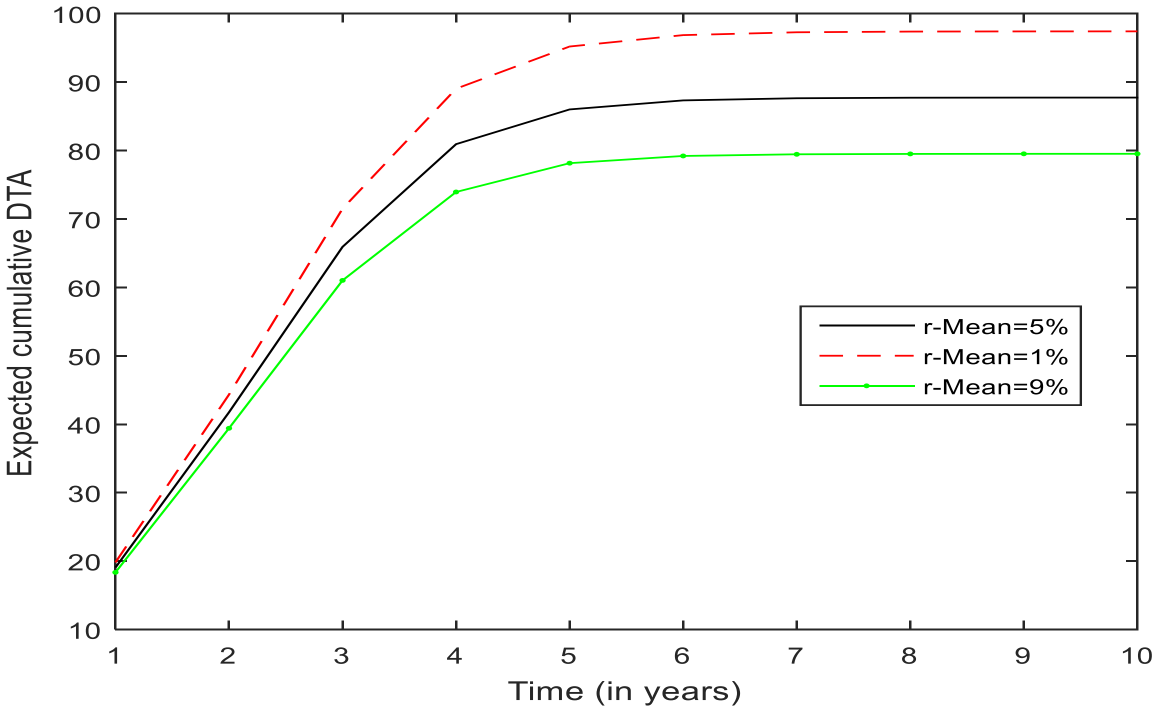

Figure 5- Expected cumulated DTA; increasing mean profits and profit deviation

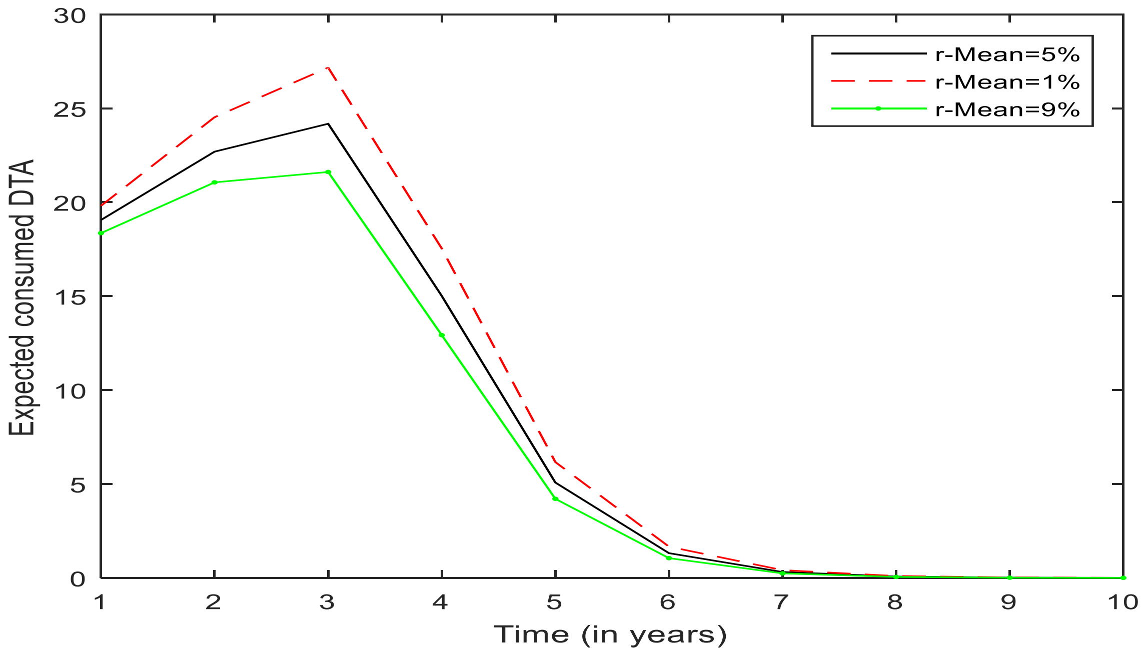

Figure 6- Expected yearly DTA consumption; increasing mean profits and profit deviation

In Figure 7 we have conditions of independent and identical distributions (IID), meaning that both u_i and the interest yield follow the same distribution and have the same mean and variance throughout all the years. Comparing to the previous case, we can see that most of the DTA is used in the fourth year, with just a small remainder being used in the fifth year.

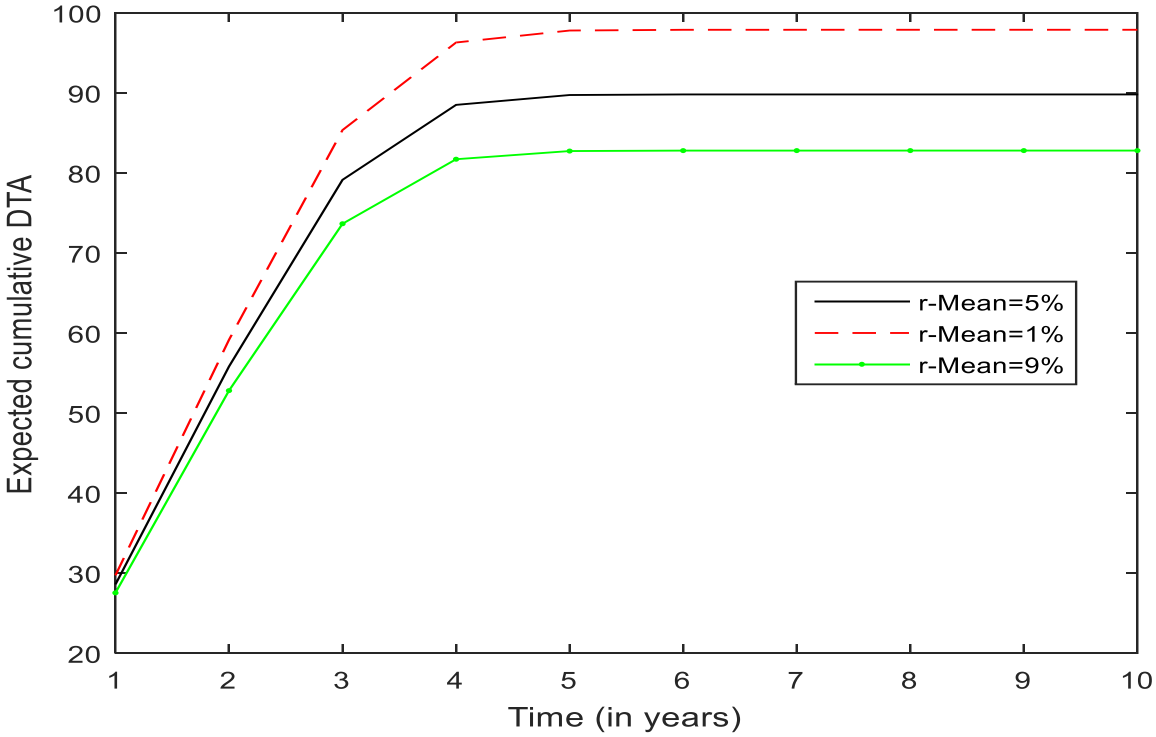

Figure 7- Expected yearly DTA consumption; IID

The rest of the figures explore different combinations. In Figure 8 we have identical means for the u_i and the interest yield, but also have an increasing profit deviation, delaying the DTA's full use to year 7. In Figure 9 we simulated the mean interest yield varying throughout the years (all other component remaining identical), and saw a little loss of value for the DTA compared to the IID of Figure 7, as expected. In Figure 10 we reproduce the scenario of Figure 9 with increasing profit variance, noticing a delay in the time the DTA is fully used up. In Figure 11 we reproduced the scenario of Figure 9, but now also with increasing mean u_i each year, allowing the curves to peak a bit earlier. Finally, in Figure 12 we have varying mean yields and profit alongside a varying standard deviation σ_{u_i} , with some subtle differences from the two previous cases.

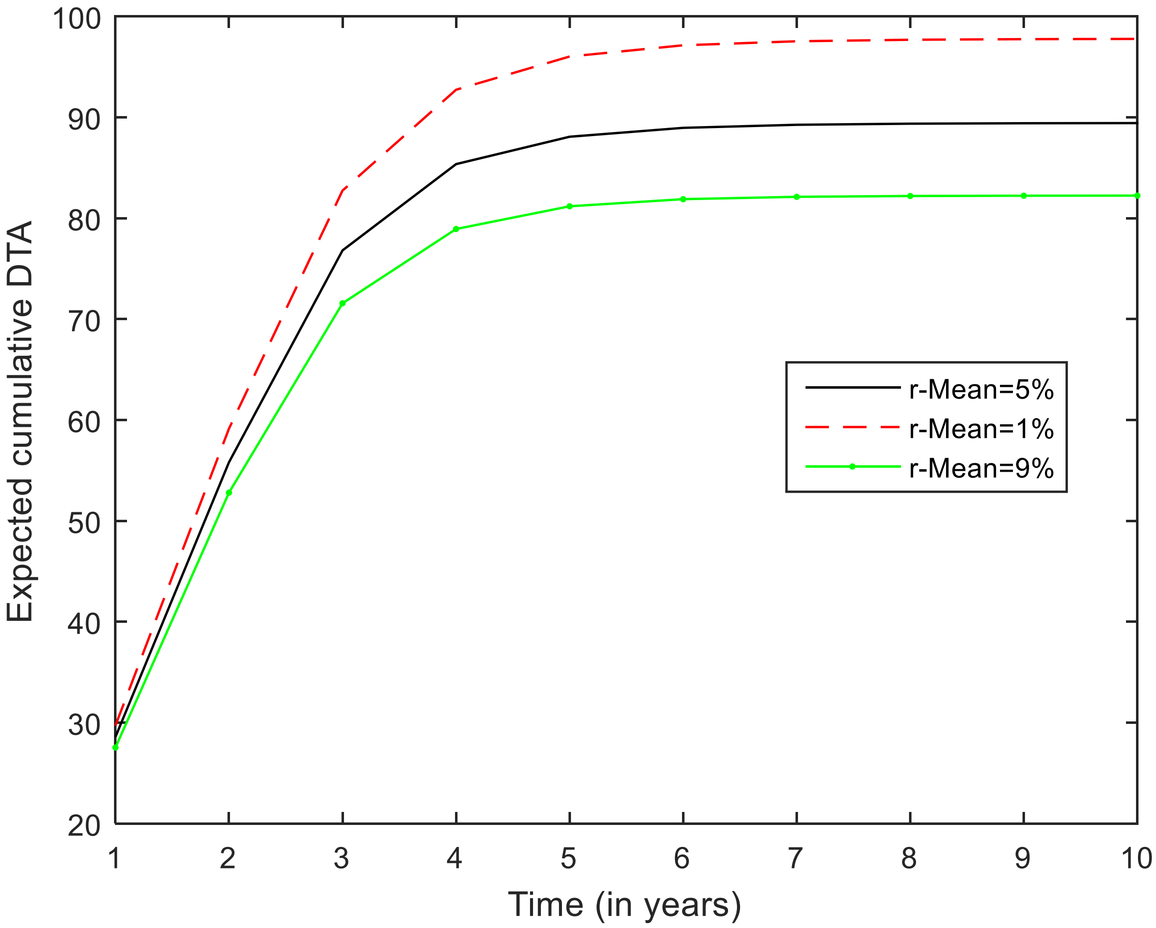

Figure 8- Expected yearly DTA consumption; increasing profit deviation

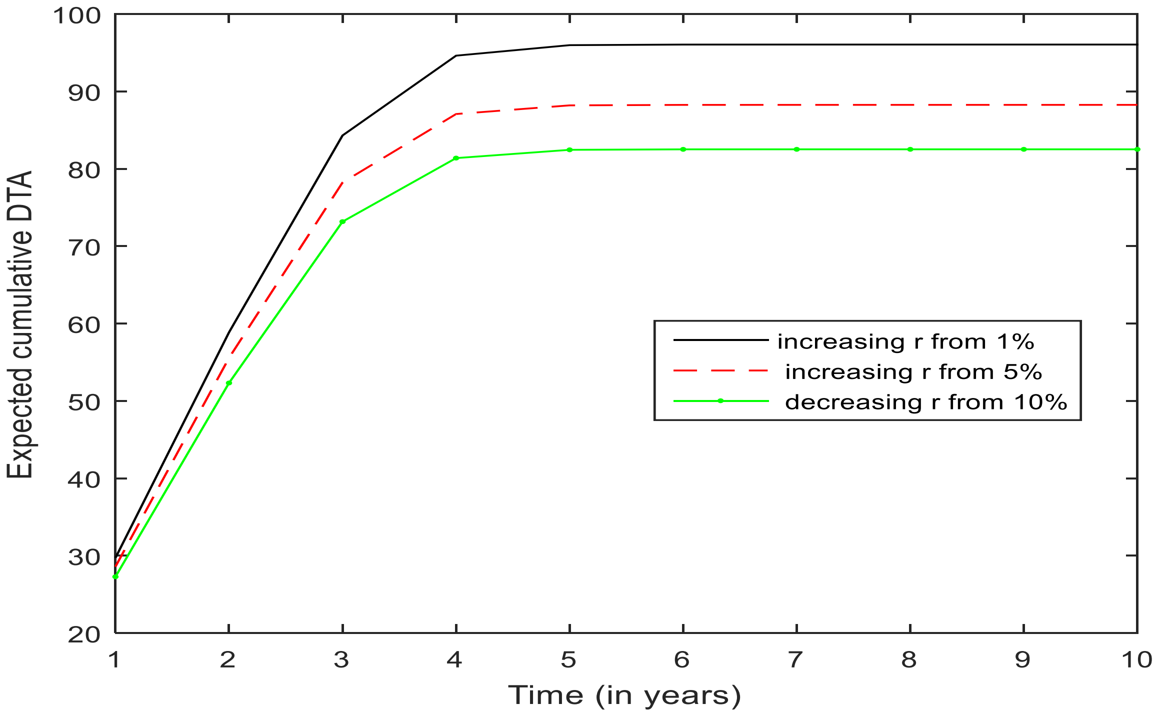

Figure 9- Expected yearly DTA consumption; varying mean yield

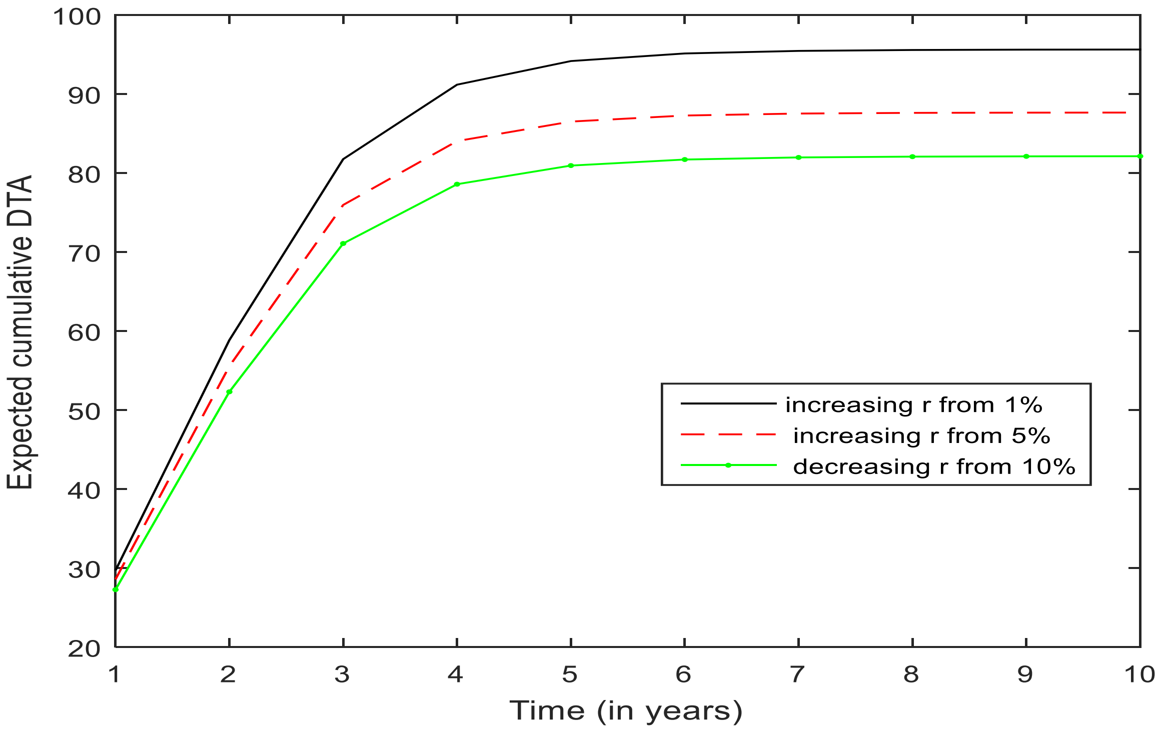

Figure 10- Expected yearly DTA consumption; varying mean yield and increasing profit deviation

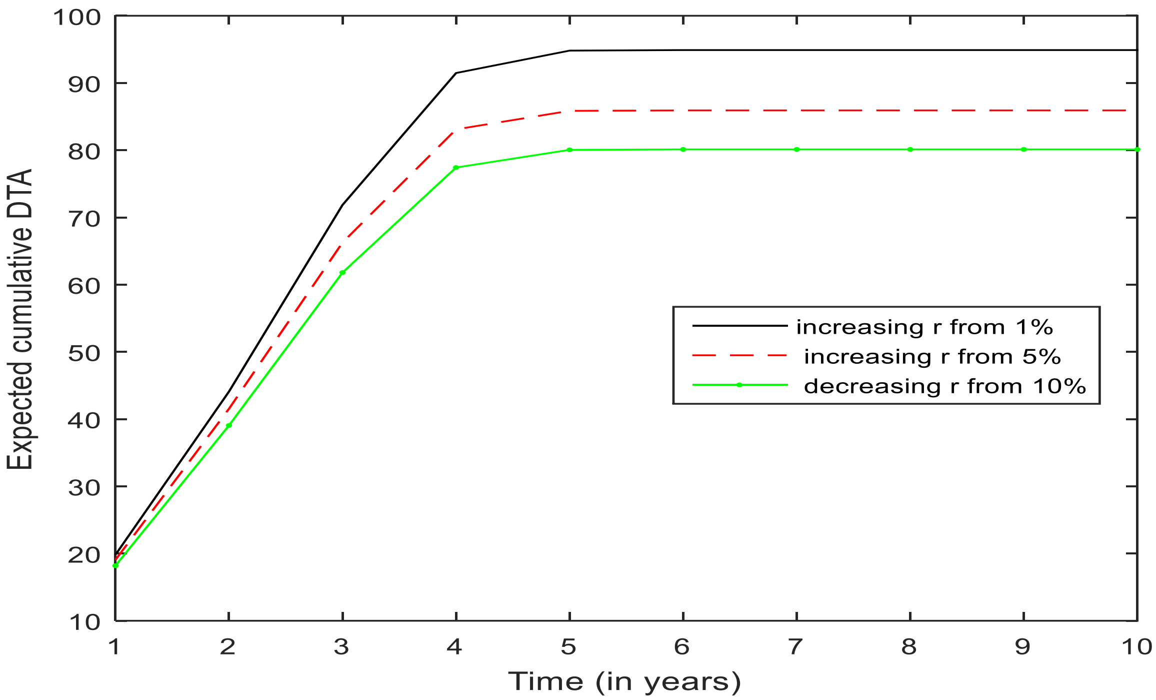

Figure 11- Expected yearly DTA consumption; varying mean yield and increasing profits

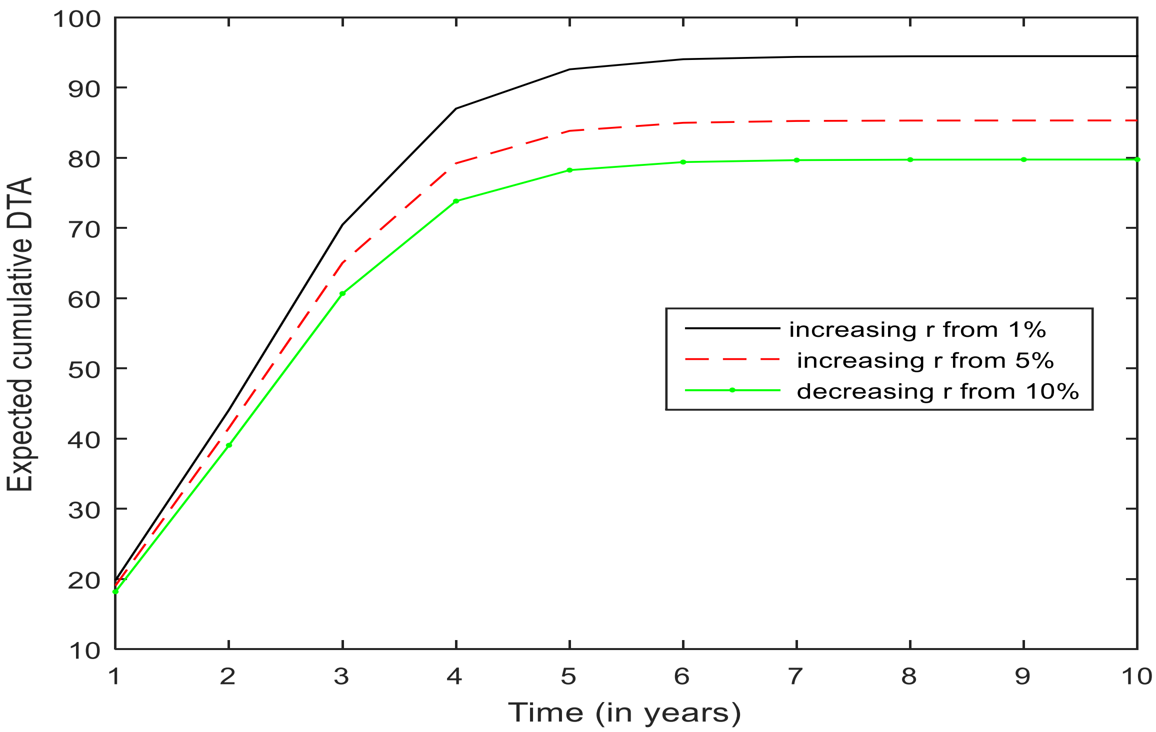

Figure 12- Expected yearly DTA consumption; varying mean yield and increasing mean profit and profit deviation

V - Conclusions

In this work we valued Deferred Tax Assets (DTAs) according to future projected profits, which is the only correct way that they should be valued. Using this valuation technique, the DTA's value on the balance sheet would always be smaller than its nominal value used nowadays and reflect its realistic value, providing all stakeholders with the company's real asset worth, henceforth preventing future (unavoidable) disappointments. Via the projection of future profits and yields using a uniform distribution with associated standard deviations, we account for the most likely scenarios and reach precise deterministic values for the DTAs, allowing the company and its shareholders to possess all necessary information to correctly estimate the company's financial stance and allow for a realistic strategy for the future.

References

Amir, E., Kirschenheiter, M., & Willard, K. (1997). The Valuation of Deferred Taxes. Contemporary Accounting Research, 14(4), 597-622.

Amir, E., and T. Sougiannis. (1999). Analysts‟ interpretation and investors‟ valuation of tax carryforwards. Contemporary Accounting Research 16 (Spring 1999): 1-33.

Atwood, T., Drake, M. S., & Myers, L. A. (2010). Book-tax conformity, earnings persistence and the association between earnings and future cash flows. Journal of Accounting and Economics, 50(1), 111-125.

Burgstahler, D. C., Elliott, W. B., & Hanlon, M. (2002). How Firms Avoid Losses: Evidence of Use of the Net Deferred Tax Asset Account.

Colley, R., Rue, J., Valencia, A. & Volkan, A. (2007), "Deferred Taxes in the Context of the Unit Problem", Journal of Finance and Accountancy

Cox, J.C., J.E. Ingersoll and S.A. Ross (1985). "A Theory of the Term Structure of Interest Rates". Econometrica. 53: 385–407. doi:10.2307/1911242.

Hanlon, M., & Shevlin, T. (2005). Bank-Tax Conformity for Corporate Income: An Introduction to the Issues

Laux, R. C. (2013). The Association between Deferred Tax Assets and Liabilities and Future Tax Payments. SSRN Electronic Journal. Sansing, R. C., & Guenther, D. A. (n.d.). The Valuation Relevance of Reversing Deferred Tax Liabilities.

Ayers, B. C., S. K. Laplante, and S. T. McGuire. (2010). Credit Ratings and Taxes: The Effect of Book-Tax Differences on Ratings Changes. Contemporary Accounting Research 27 (2):359 402.

Baesens, B., and T. van Gestel. (2009). Credit Risk Management. Oxford, Oxfordshire, England: Oxford University Press.

Crabtree, A., and J. J. Maher. (2009). The Influence of Differences in Taxable Income and Book Income on the Bond Credit Market. Journal of the American Taxation Association 31 (1):75-99.

De Vries, Tjeerd, (2018). “Market Consistent Valuation of Deferred Taxes”, MSc thesis, Technical University Delft.

Edwards, A. (2011). Does the Deferred Tax Asset Valuation Allowance Signal Firm Creditworthiness?: Working chapter, University of Toronto.

Gallemore, J. (2011). Deferred Tax Assets and Bank Regulatory Capital: Working chapter, University of North Carolina.

Hogg, Robert V.*; McKean, Joseph W.; Craig, Allen T. (2004).* Introduction to mathematical statistics *(6th ed.). Upper Saddle River, New Jersey: Prentice Hall. p. 692.* ISBN 978-0-13-008507-8. MR 467974.

Kim, D., and A. M. Santomero. (1988). Risk in banking and capital regulation. Journal of Finance 43 (December): 1219-1233.

Moodys Research Report (2015) - https://www.moodys.com/research/Moodys-Reliance-on-global-banks-deferred-tax-assets-poses-potential--PR_340219

Poterba, J. M., N. S. Rao, and J. K. Seidman. (2011). “Deferred tax positions and incentives for corporate behavior around corporate tax changes”. National Tax Journal 64 (1):27-57.

Reilly, D. (2009). Citi, BofA Show Investors Can’t Bank on Capital. Bloomberg (January 28): 1.

Ryan, S. (2007). Financial Instruments and Institutions: Accounting and Disclosure Rules. 2nd ed. John Wilen & Sons, Hoboken, NJ.

Kara, Gazi Ishak (2016). “Bank Capital Regulations around the World: What Explains the Differences?”, Finance and Economics Discussion Series 2016-057. Washington: Board of Governors of the Federal Reserve System, http://dx.doi.org/10.17016/FEDS.2016.057.