

Simple Valuation of Compounded Deferred Tax Assets using a Binomial Algorithm

Abstract

Deferred tax asset (DTA) is a tax/accounting concept that refers to an asset that may be used to reduce future tax liabilities of the holder. In a company's balance, it usually refers to situations where it has either overpaid taxes, paid taxes in advance or has carry-over of losses (the latter being the most common situation). In fact, accounting and tax losses may be used to shield future profits from taxation, through tax loss carry-forwards. The purpose of this chapter is exactly to propose a precise and conceptually sound approach to value DTAs. For that purpose, making use of an adapted binomial CRR (Cox, Ross and Rubinstein) algorithm, we derive a precise way to value DTAs. This way, the DTAs are valued in a similar way of the Binomial Options Pricing Model (Cox et al., 1979), and the subjectivity of its evaluation is greatly reduced. We will see that with the proposed evaluation techniques, the DTA's expected value will be much lower than the values normally used in today's practice, and the bank's financial analysis will lead to much more sound and realistic results.

Keywords: Valuation, Deferred tax asset, accounting, balance sheet, binomial

Introduction

The tax return of a company is based on its accounting financial statements. To provide comparable information, financial statements are prepared according to the International Financial Reporting Standards (IFRS), issued by the International Accounting Standards Board (IASB). The IASB was formed in 2001 to replace the International Accounting Standards Committee that issued International Accounting Standards (IAS). Since the previously issued IASs remain effective, we have that the main body of standards that are used worldwide by several countries are comprised of IFRSs and IASs. The companies' income, depicted by the IFRSs and IASs (referred to simply by the Generally Accepted Accounting Principles GAAP) are their accounting profits, but these may be (and are) different than the taxable profit, since the taxable profit is calculated as a function of the tax law inherent to each country. The number of factors that lead to differences between tax and accounting returns is huge and varies from country to country. One of those factors is of relevance to the present work – the deferral of taxes.

The relationship of DTAs with the creditworthiness of a company as already deserved some work from academic community. The effects of book-tax differences on a firm's credit risk were analysed in (Crabtree & Maher 2009), (Ayers et al. 2010), (Edwards 2011) and (Gallemore 2011); all agreeing that great amounts of deferred taxes were associated with higher risks and lower earnings quality, resulting in a decline of creditworthiness. Additionally, studies of the impact of DTAs on credit ratings led to the conclusion that deferred tax positions are substantial for many firms (between 5% and 10% of all assets according to (Poterba et al 2011)).

Under normal circumstances, a company's DTAs usually originate in the carry-over of losses (though it can also arise from overpaying some taxes). The corresponding rights are registered in the balance sheet as assets, although in (Amir and Sougiannis 1999) it also argued that DTA may have implications for the perception of the firm as a going concern (dubbed as the information effect), since if the DTA

arose from past operating losses, future losses would be likely to incur; this means that future liabilities could be more than likely, and thus such “assets” should be regarded with great suspicion).

These assets may be hard to value, since they are time-limited and may never be used at all. Their value is contingent on the future earnings of the company, and they can be used to shield these future profits from taxation – IAS 12 states that “a DTA should be recognised for all deductible temporary differences, to the extent that it is probable that taxable profit will be available against which the deductible temporary difference can be utilised”. Since corporate income taxation works on an annual basis, the shielding opportunities occur once a year. This is equivalent to saying that we are faced with a compound European option (or an annual Bermuda option) that might be exercised until the last year in which the law will permit shielding, or until the DTAs value has been completely depleted by its use. Consistent with this line of thinking, there is an ongoing debate regarding the appropriateness of including DTA in the banks’ regulatory capital calculation, since by doing so we are assuming its “full” worth; something that is clearly misleading.

Another aspect that must be taken into consideration is that a company can have several DTAs, each with different expiry dates; in this situation it will naturally use the DTA’s with the nearest expiry date first. Thus being, the probability of a certain DTA being used is dictated by all DTAs with shorter expiration dates being used or expired, and that the company generates enough profit to cover the DTA. Such scenario may seem hard to quantify at first, but as we will show, it is rather simple via the use of a binomial algorithm.

Focus on the banking sector

On the special case of the banking sector, banks are required to maintain certain levels of regulatory capital to provide a buffer against potential future losses (Kim and Santomero 1998),(Ryan 2007), (Baesens and van Gestel 2009). In many countries (Kara 2016), banks are allowed to count a portion (or all) of their DTAs towards regulatory capital requirements (since the adoption of SFAS¹ No. 109 in 1992 – specifically the establishment of valuation allowances). Throughout the recent financial crisis (2008-2013), major media outlets routinely drew attention to the banks’ DTA positions, classifying them as tenuous contributions towards regulatory capital. In (Reilly 2009), it was noted that tier 1 capital ratios contained “fluff” – mentioning DTA as the primary culprit, calling in an “airy asset”. The Basel Committee on Banking Supervision specifically targeted the removal of DTAs as a potential method for improving the ability of regulatory capital to protect banks from losses². At the same time, the banking industry has pushed for the opposite; namely a greater inclusion of DTA in the regulatory capital calculation, in an attempt to “ease” the amount of (real) regulatory capital. Gallemore (2011) investigates the credit risk associated with the deferred tax asset component of bank regulatory capital. He hypothesizes that banks that have a larger proportion of regulatory capital composed of deferred tax assets will be more likely to fail. He employs a hazard model to test a sample of commercial banks and finds that the proportion of regulatory capital composed of deferred tax assets is positively associated with the risk of bank failure during the recent financial crisis. Gallemore (2011) attributes his findings to the fact that the benefits of deferred tax assets cannot be realized unless banks generate positive taxable income in the future.

It is thus clear that these DTAs must be correctly valued, and that simply adding them to the balance sheet in full as an asset might contribute to obfuscate the company’s true financial condition – even a situation in which the DTA is fully used before its expiration date, we still must account for the cost of capital. In (Moody’s 2015), DTAs were considered “a low-quality form of assets, and thus a low-quality

¹ Statement of Financial Accounting Standards

² Consultative document of December 2009, entitled “Strengthening the resilience of the banking sector”.

source of capital”, and consequently, Moody’s decided to “limit the contribution of DTAs in its calculations of banks' tangible common equity (TCE)”. As analysed in De Vries (2018), several DTA valuation methods can be used, but they are essentially very subjective, and basically result in a valuation allowance, for which there is no consistent accepted method to calculate – this chapter aims to resolve such shortcomings.

Valuation Techniques

In this chapter we will explore two different valuation techniques based on the CRR binomial algorithm; one that resumes itself to the use of an approximated closed formula (with a binomial tree in the background, as a theoretical basis), and another that effectively uses a binomial tree. The CRR binomial algorithm was chosen due to being the main method used to value options. DTAs are somewhat similar to options with same base differences; and thus this chapter simply tweaks the CRR binomial algorithm to account for the main differences, and to allow for a more precise estimate of the DTA’s value, such as is currently done for stock options.

Both methods comprise a standard set of steps, explained ahead, which basically provides the framework to use the expected future earnings and current DTAs with their associated expiry date.

Valuation of the DTAs must be performed by following some steps. Initially, a vector must be constructed containing the company’s net losses, ordered by the years that each loss is eligible to be used as a DTA. Assuming that N is the maximum number of years that can be accounted for, we have a vector with N positions, where position 1 contains the company’s previous losses whose corresponding DTA can only be accounted for in the following year; position 2 contains the company’s losses whose corresponding DTA can only be accounted for in the next 2 years, and so on until N. Considering such a framework, we constructed a vector $P_Losses\{1:N\}$, compiling all of the company’s negative results eligible for consideration in our deferred tax assets’ calculations. Note that the vector $P_Losses\{1:N\}$, when multiplied by the tax rate (that we assume constant over the years), would yield a vector of $DTA\{1:N\}$, ordered by expiration dates; however, for ease of calculations, we will work with the $P_Losses\{1:N\}$ vector and only multiply by the tax rate at the end of our calculations.

After this first step, we must estimate the company’s performance over the next N years. There are many ways to achieve this, however, for the moment, let us focus on the company’s projected result for the next year (assumed positive), and on the estimated annual volatility σ of the business, so that we can build up a binomial tree with up and down movements (such as performed in the Binomial CRR options pricing model). If we will only consider yearly time frames, we have $\Delta t = 1$, and thus the up (u^+) and down (d^-) movements are given as (from (CRR 1979), with the condition that the variance of the log of the profit is σ^2):

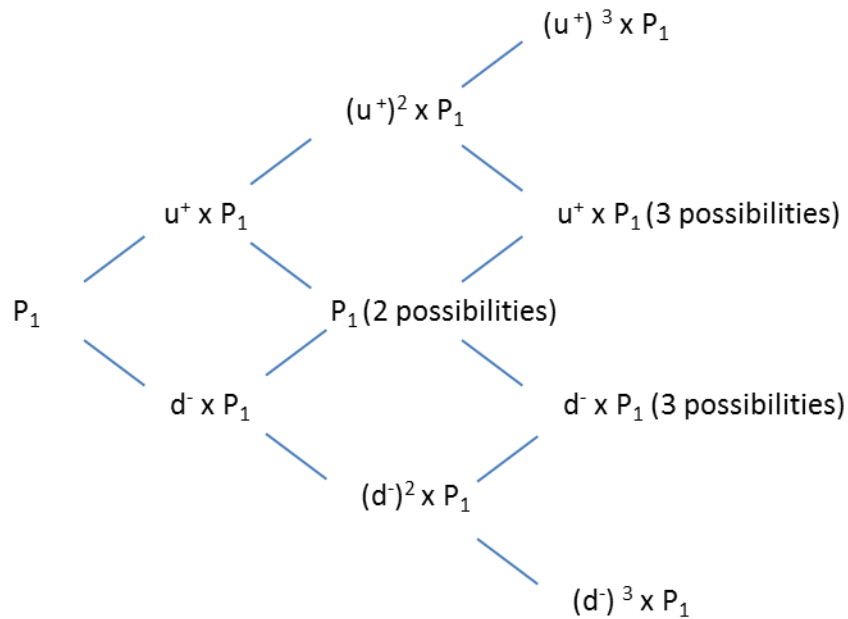
$$u^+ = e^{\sigma\sqrt{\Delta t}} = e^{\sigma} \quad \text{and} \quad d^- = e^{-\sigma\sqrt{\Delta t}} = e^{-\sigma}. \quad \text{Note that } u^+ = \frac{1}{d^-}.$$

Consequently, from an initial estimate of profits P_1 , that is given with a certainty of I_1 ($0 < I_1 \leq 1$), we can project several scenarios with the upward and downward movements of the binomial tree. At first we will only assume projected positive profits throughout the upcoming years, but later we will also allow for a more realistic scenario with negative cash flows.

Method 1A – simple mean value and positive profits

In this method, we are only interested on the binomial tree’s mean values at each step for valuation, in order to obtain an easy and simple estimate. Looking at the binomial tree, we can infer that the mean values (accounting for the combinations of the inner values; $\binom{n}{k}$) for each year are given by

(after some manipulation) $P_1 \times \left(\frac{1+(u^+)^2}{2u^+}\right)^{t-1}$ (note that this mean increases every year, since $\frac{u^++d^-}{2} > 1$). We can therefore create a vector with the mean projections of future profits multiplied by the risk neutral probability³ (this method will only yield positive numbers, since the initial number is assumed positive, and all others are found by applying positive multipliers; other methods that could yield negative numbers must convert the negative numbers into zeros and calculate the resultant mean values with these zeros instead, as will be done in method 2 further along) for the next N years as $F_Profits\{1:N\}=(P_1, P_1 \times \left(\frac{1+(u^+)^2}{2u^+}\right), \dots, P_1 \times \left(\frac{1+(u^+)^2}{2u^+}\right)^{N-1})$.



Mean: $P_1 \quad P_1 \times \frac{1+(u^+)^2}{2u^+} \quad P_1 \times \left(\frac{1+(u^+)^2}{2u^+}\right)^2 \quad P_1 \times \left(\frac{1+(u^+)^2}{2u^+}\right)^3$

Figure 1 – Binomial projection of profits

With the two vectors of previous losses and future profits; $P_Losses\{1:N\}$ and $F_Profits\{1:N\}$, we can now apply the valuation method for the deferred tax assets. Note that, basically, what is at stake here is to “spread” the accumulated losses over the projected earnings of the company for the following years.

³ All profits mentioned in this chapter are assumed to be multiplied by the risk-neutral probability, so that they can be discounted using the risk-free rate

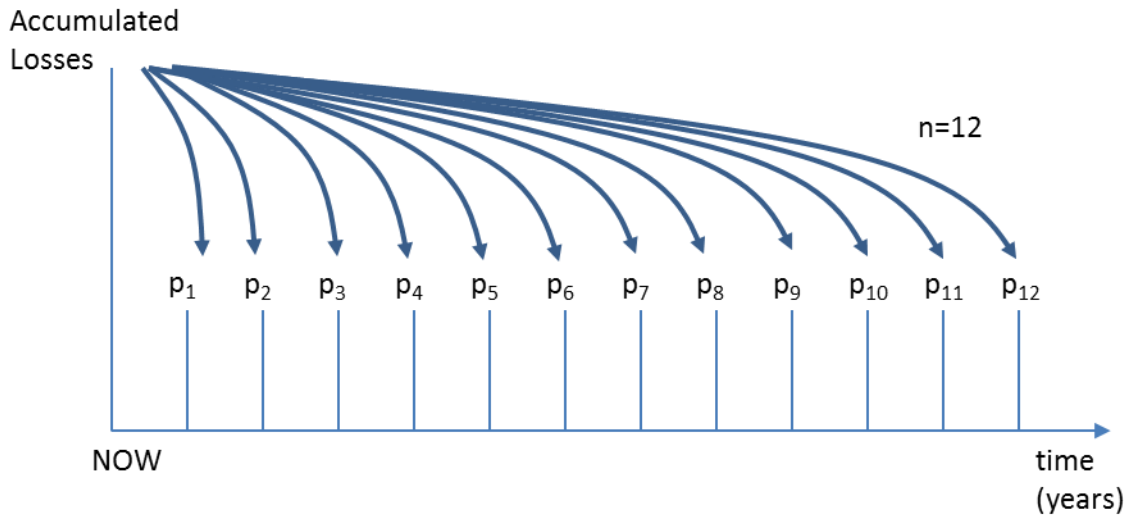


Figure 2 – Spreading the carry-over of losses through the following years

We will now create a new vector, LimitedLosses{1:N}, from the vector P_Losses{1:N}, that estimates the maximum amount of losses/DTAs that will actually be used when calculating the DTAs' value. This new vector will match the company's losses to the projected gains, and will be constructed as follows:

Running through all positions of P_Losses{1:N}, starting from position 1 to N, we will have to match the corresponding positions to F_Profits{1:N}. P_Losses{1} will be compared to F_Profits{1}, and LimitedLosses{1}=Minimum(P_Losses{1}; F_Profits{1}). The next positions will have to consider the sum of the profits up until their year, discounted by what was already attributed by the preceding years. So being, for position (year) 2, we have:

$$\text{LimitedLosses}\{2\} = \text{Minimum}(P_Losses\{2\}; \text{Sum}(F_Profits\{1,2\}) - \text{LimitedLosses}\{1\}).$$

And for year $1 < n \leq N$ we have

$$\text{LimitedLosses}\{n\} = \text{Minimum}(P_Losses\{n\}; \text{Sum}(F_Profits\{1,n\}) - \text{Sum}(\text{LimitedLosses}\{1,n-1\})).$$

Next we will create a final vector, that will only accommodate the projected profits used to offset to losses, out of the projected mean profits. This vector is easy to build, and basically is similar to the vector F_Profits{1:N}, but with a slight modification; the vector's sum must equal the sum of vector LimitedLosses{1:N} by means of zeroing and reducing the final positions of the vector F_Profits. This operation can be done iteratively, and the base code for calculating each position is given in the Appendix.

After the above procedure, the vector UsedAmount {1:N} might either be equal to F_Profits{1:N} (if the company has great amounts of losses to recoup), or be a vector with its final value smaller than the corresponding position of the F_Profits vector, and have zeros fill the last positions. It is this vector, UsedAmount {1:N}, that we must discount at a convenient discount rate (we will use the risk-free discount rate, since we are using profits multiplied by the risk-neutral probability) in order to estimate the amount of previous losses in today's value that will be used to cover the projected future profits.

$$\text{Recoup_amount} = \sum_{i=1}^N \frac{\text{UsedAmount}\{i\}}{(1 + r_f)^i}$$

In the special case of UsedAmount {1:N}=F_Profits{1:N}, we have simply to calculate an annuity, and as such

$$Recoup_amount = \frac{P_1}{r_f - g} \left[1 - \left(\frac{1 + g}{1 + r_f} \right)^N \right]$$

With

$$g = \left(\frac{1 + u^2}{2u} \right) - 1$$

The final result, the effective value of the DTA (Deferred Tax Assets) is obtained by multiplying the obtained amount at today's value by the tax level T^4 and the probability attributed to P_1 , namely I_1 . The deferred tax asset for this scenario would thus be

$$Value_DTA_1 = Recoup_amount \times T \times I_1$$

If $I_1 \neq 1$, then we would run all S scenarios to cover all probabilities (sum of all " I "s=1) and

$$Value_DTA = \sum_{i=1}^S Value_DTA_i$$

As a final note, we can calculate the maximum and minimum amount of deferred tax assets by filling the $F_Profits$ vector with the maximum/ minimum projected profit instead of the mean profit (in the special case of $UsedAmount\{1:N\}=F_Profits\{1:N\}$, this would mean a $g=u-1$ or $g=d-1$ respectively).

Examples for Method 1A

Taking as an example a situation where the company tax is 20%, the discount rate r_f is 5%, the profit projection for the upcoming year is 50.000€ and $\sigma=0,8$ (we will assume this value unless stated otherwise⁵), as seen in Table 1.

Table 1 – Initial conditions

Tax	20%
Discount Rate	5%
Profit projection for 1st year	50.000€
sigma	0,8

With the above conditions in mind, we have to arrange our previous losses vector by years left to expiry. In our case (Table 2), we have that losses of 40.000€ have a recoup expiration date of 1 year, losses of 120.000€ have a recoup expiration date of 2 years and so on.

Table 2 – Losses, Projected Profits and Valuation of DTA, example 1 (in €), method 1A

Years Left to discount	0	1	2	3	4	5
Losses		40000	120000	50000	100000	30000
Mean Projected Profits		50000	66871,747	89436,61	119615,7	159978,2
Limited Losses		40000	76871,747	50000	100000	30000

⁴ The tax level is also a stochastic variable and could be treated as one; though for simplicity we will assume it as a constant in this chapter

⁵ We used a very high σ due to the forthcoming examples, were we will portray possible negative profit projections

Used Amount		50000	66871,747	89436,61	90563,39	0
Wrongfully used DTA	68000,00					
Realistic DTA	52007,83					

The mean projected profits vector is easily built, having in mind the initial projected profit and the its volatility, σ . Having these two vectors, we proceed to calculate the limited losses vector, which is basically a projection of how much of our losses are we really expecting to be covered from the DTA. For previous losses whose corresponding DTA has an expiration date of 1 year from now, we have that the future profits will probably fully cover the losses, and thus account the whole value of 40.000€. For year 2, we have losses of 120.000€, and projected profits of only 66.871,75€. From this, we can already infer that the company will lose the right to use part of these losses as DTA. The effective value it will use will be 10.000€ of losses with expiry date of 2 years that will effectively be used in year 1 (to cover the 10.000€ difference between the 40.000€ losses and 50.000€ profits), added to the 66.871,75€ of profits in year 2. Thus, the total loss from year 2 that will effectively be used is $66.871,75€ + 10.000€ = 76.871,75€$ - the rest will be forfeited. For years 3 to 5, the losses will always be less that the profits, and thus full usage is projected.

The Used Amount vector is the most important vector of all. Basically, it tells us what part of the profits we can effectively deduce from having losses carry-over. In year 1, we have it to be 50.000€ (used by the 40.000€ loss with expiration of 1 year, and 10.000€ with expiration of 2 years). In year two, we will discount the full profits of 66.871,75€ with losses ending in the second year. In the third year, the 89.436,61€ will be covered by losses of 50.000€ ending in year 3, and 39.436,61€ of losses ending in year 4. The remaining losses (year 4 and year 5) amount to 90.563,39€, and this amount will all be used in year 4. Year 5 will have no benefit whatsoever from DTA.

With the Used Amount vector constructed, estimating the final amount is easy; simply discount all values of the used amount vector to present day values and multiply it by the Tax rate. This will yield our final value of DTA's worth – 52.007,83€ in this case. Note that current practices value the DTAs simply as the sum of all previous losses multiplied by the tax rate, which in this case would yield $20\% \times 340.000€ = 68.000€$ (it is always higher than the calculated values, since it assumes that all DTAs are going to be used, without being discounted by any rate at all!).

If we opted for a $\sigma=0,2$, then the values would be those summarized in Table 3 (example 2). From the table we can see that, due to the lower projected profits, we wouldn't be able to use the full amount of DTAs, yielding a lower Limited Losses vector, and thus a lower Used Amount vector – although the wrongfully used DTA in the balance sheet is still the same 68.000€, note that the realistic DTA drops to 41.198,11€.

Table 3 - Losses, Projected Profits and Valuation of DTA, example 2 (in €), method 1A

Years Left to discount	0	1	2	3	4	5
Losses		40000	120000	50000	100000	30000
Mean Projected Profits		50000	51003,338	52026,81	53070,82	54135,78

Limited Losses		40000	61003,338	50000	55097,63	30000
Used Amount		50000	51003,338	52026,81	53070,82	30000
Wrongfully used DTA	68000,00					
Realistic DTA	41198,11					

Another example could be the following, with $\sigma=0,8$, a different set of DTAs and expiry dates (Table 4, example 3):

Table 4 – Losses, Projected Profits and Valuation of DTA, example 3 (in €), method 1A

Years Left to discount	0	1	2	3	4	5
Losses		0	120000	0	100000	300000
Mean Projected Profits		50000	66871,75	89436,61	119615,7	159978,2
Limited Losses		0	116871,7	0	100000	269030,4
Used Amount		50000	66871,75	89436,61	119615,7	159978,2
Wrongfully used DTA	104000					
Realistic DTA	81857,51					

In this third example, we keep the same profit projections as in the first, but now the previous losses vector is different; it has some zero value positions, and some high values, especially for the loss whose corresponding DTA has an expiry date of 5 years from now. Note how the limited losses is constructed, with losses whose corresponding DTA have an expiry date of 2 years covering both the first and second year (forfeiting the rest) and notice how the losses whose corresponding DTA have expiry dates of 4 and 5 years cover the whole of the amounts of years 3 to 5. Thus being, all of the projected profits are covered, and effectively the used amount is equal to the profits vector, amounting to a value of 81.857,51€. The wrongfully used value of DTAs, obtained by summing all the previous losses and multiplying by the tax level would equal 104.000€!

In this case, the final recoup amount is thus calculated by discounting all of the used amount (or profits vector⁶), which in this special case is an annuity

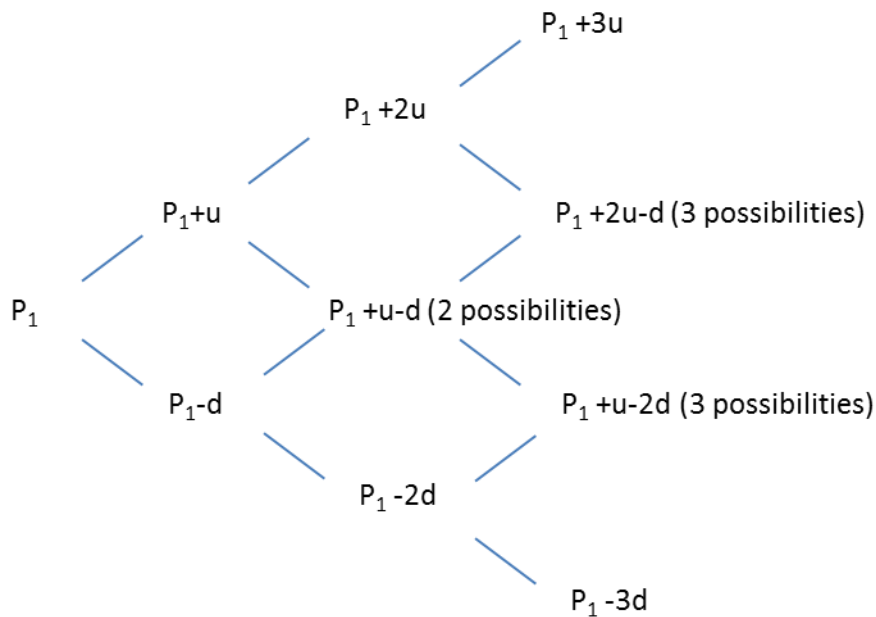
$$Recoup_amount = \frac{P_1}{r_f - g} \left[1 - \left(\frac{1+g}{1+r_f} \right)^N \right]$$

The example above considers that we are sure of yielding a profit of 50.000€ in one year's time – this may not always be the case - If we say that there is a 80% chance of the company yielding a profit of 50.000€ in one year's time with a sigma of 0.8, and 20% chance of yielding losses, we can multiply our final amount by 80% - this is what is done in valuation allowance, but by calculating the used amounts and discounting them correctly, we have a much better estimation. We can also study several scenarios and give a probability value to each, and make a final calculation based on the relative weights of each result.

Method 1B – simple mean value with possibility of future losses (negative profits)

⁶ Again, remember that the profit vector is adjusted by the risk-neutral probability.

In method 1A we assumed only outcomes involving positive profits; that was done merely for simplicity sake. In this method we adjust the model to accommodate for the likelihood of sustaining negative profits (if the company has deferred tax assets to use its because it has incurred in losses before, being therefore likely it will incur on losses again). The consideration of this framework compels us to drift from the multiplicative factors used in the CRR, and use them only to set the initial deviations from the initial projected profit. These deviations will remain fixed for the whole of the tree, allowing for negative profits (the CRR model is built for stock prices that never go below 0) – of course that, for what the DTA is concerned, the negative profits should be converted to 0. Considering $u = P_1 \times (u^+ - 1)$ and $d = P_1 \times (1 - d^-)$, and looking at the binomial tree, we can infer that the mean values (accounting for the combinations of the inner values; $\binom{n}{k}$) for each year are given by $P_1 + (t - 1) \left(\frac{u-d}{2}\right)$, as long as the whole tree boasts positive values. If the lower portion of the tree crosses the zero threshold, then the mean profit value for consideration should be higher, since the negative values should be considered as zeros for calculating the mean – from this we can state that the mean values without any correction (assuming all values are positive, even if they aren't) will be equal or smaller than when we correct for the negative values (and thus have a lower bound on the DTA valuation, as we will see later).



Mean: P_1 $P_1 + \frac{u-d}{2}$ $P_1 + 2\left(\frac{u-d}{2}\right)$ $P_1 + 3\left(\frac{u-d}{2}\right)$

Figure 3 – Binomial projection of profits

We can therefore create a vector with the mean projections of future profits for the next N years as $F_Profits\{1:N\}=(P_1, P_1+\left(\frac{u-d}{2}\right), \dots, P_1 + (N - 1) \left(\frac{u-d}{2}\right))$, if $P_1 - (N - 1)3d > 0$, else the mean profits vector must be adjusted for the negative profit projections (using zeros instead of the negative numbers), yielding higher values – it is this correction that effectively increases the complexity of this method when compared to method 1A.

Examples for Method 1B (with possibility of future losses)

Taking the same initial conditions as those presented in Table 1 and the same losses of example 1, we have first to adjust the mean projected profits' vector. In fact, since future losses are admissible, we have to adjust the mean of future profits to treat the negative profits as zeros. To do this, we must check the lowest values of the binomial tree in each iteration and adjust the mean of the iteration at hand if necessary. Starting with 50.000 for the first year (mean=50.000 in this case), we will now analyse the following years, from 2 to 5, in Table 5.

Table 5 – Adjustment of negative profits for Method 1B

	2 ^o year	3 ^o year	4 ^o year	5 ^o year
# of elements	2	4	8	16
Smallest element	P-d	P-2d	P-3d	P-4d
Value of smallest element	22466,45	-5067,1	-32600,7	-60134,2
2^o Smallest element	P+u	P-d+u	P-2d+u	P-3d+u
Value of 2^o smallest element	111277	83743,49	56209,94	28676,39
Mean Projected Profits w/o adjustment	66871,75	83743,49	100615,2	117487
Mean Projected Profits with adjustment	66871,75	85010,27	104690,3	121245,4

From Table 5, we see that the estimated profits of the 3rd, 4th and 5th year had to be adjusted. After this operation being made, the rest is similar to what was done in Table 2, method 1A. The results for Method 1B are now shown in Table 6.

Table 6 – Example 1 using method 1B

Years Left to discount	0	1	2	3	4	5
Losses		40000	120000	50000	100000	30000
Mean Projected Profits		50000	66871,75	85010,27	104690,3	121245,4
Limited Losses		40000	76871,75	50000	100000	30000
Used Amount		50000	66871,75	85010,27	94989,73	0
Wrongfully used DTA	68000,00					
Realistic DTA	51971,41					

Looking at the results of Table 6, we see that the DTA value is similar to that of Table 2, though a bit smaller as expected, due to the possibility of negative profits. In Table 7 we reproduced example 3 (of Table 4), now using method 1B, which also yields a smaller DTA value overall.

Table 7 – Example 2 using method 1B

Years Left to discount	0	1	2	3	4	5
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Losses		0	120000	0	100000	300000
Mean Projected Profits		50000	66871,75	85010,27	104690,3	121245,4
Limited Losses		0	116871,75	0,00	100000	210946
Used Amount		50000	66871,75	85010,27	104690,3	121245,4
Wrongfully used DTA	104000,00					
Realistic DTA	72567,34					

A fourth example can use a total arbitrary Projected Profits Vector (equal for both methods 1A and 1B). In this case, the whole vector is given as input, and the DTA value is calculated accordingly (Table 8).

Table 8 – Losses, Arbitrary Projected Profits and Valuation of DTA, example 4 (in €)

Years Left to discount	0	1	2	3	4	5
Losses		0	120000	0	100000	300000
Mean Projected Profits		20000	0	110000	0	0
Limited Losses		0	20000	0	100000	10000
Used Amount		20000	0	110000	0	0
Wrongfully used DTA	104000,00					
Realistic DTA	22813,95					

Once again, notice the dynamics of the limited losses vector and the final used amount vector (which in this case is the same as the mean projected profits, meaning that the previous losses were able to cover the entire profits). This deterministic algorithm is indeed effective and simple to use, although we can't always predict the profits with absolute certainty – notice that now the final value was 22.813,95€, while the wrongfully used value of DTAs is, as in the previous example, equal to 104.000€.

Method 2A - Binomial tree alternative with positive profits

In this second method, we base our reasoning on a binomial tree of possible outcomes for profits, each with a representing probability. Assuming a binomial state change, the profit will either rise or fall by u^+ or $d^+=1/u^+$ respectively, with probability P_u or P_d . The probability of changing state is calculated by the risk neutral approach (as in the CRR Binomial model), meaning that the previous' year profit will either rise or fall (by the u^+ factor), and being risk neutral, the values of rising or falling multiplied by the respective probabilities and adjusted by the risk-free rate of interest will yield the previous year's profit. Thus, we have that the probability of the profit rising is given by

$$P_u = \frac{(1 + r_f) - 1/u^+}{u^+ - 1/u^+}$$

And the probability of the profit falling is $P_d=1-P_u$.

The state tree will be something like this (Figure 4):

dates), and the rest will be transferred to the next states. If the sum of the PL is under the projected profit, all of the PL will be used for the DTA calculation, and the transferred PL to subsequent states will be 0 (basically we are using the intermediate vectors Limited Losses and Used Amount as in method 1). As in the previous method, the values are then multiplied by the tax rate and added to the DTA value.

In order to exemplify the whole procedure, consider the initial conditions for the next 3 years depicted in Table 9, where the initial PL vector states that 130.000€ in losses expires in year 1, 70.000€ in year 2 and 80.000€ in year 3:

Table 9 – Initial conditions used to illustrate the binomial method

Tax	20%
Discount Rate	5%
Profit projection for 1st year	100.000€
sigma	0,8
Previous Losses Vector	(130.000; 70.000; 80.000)€

We have the following values for each state:

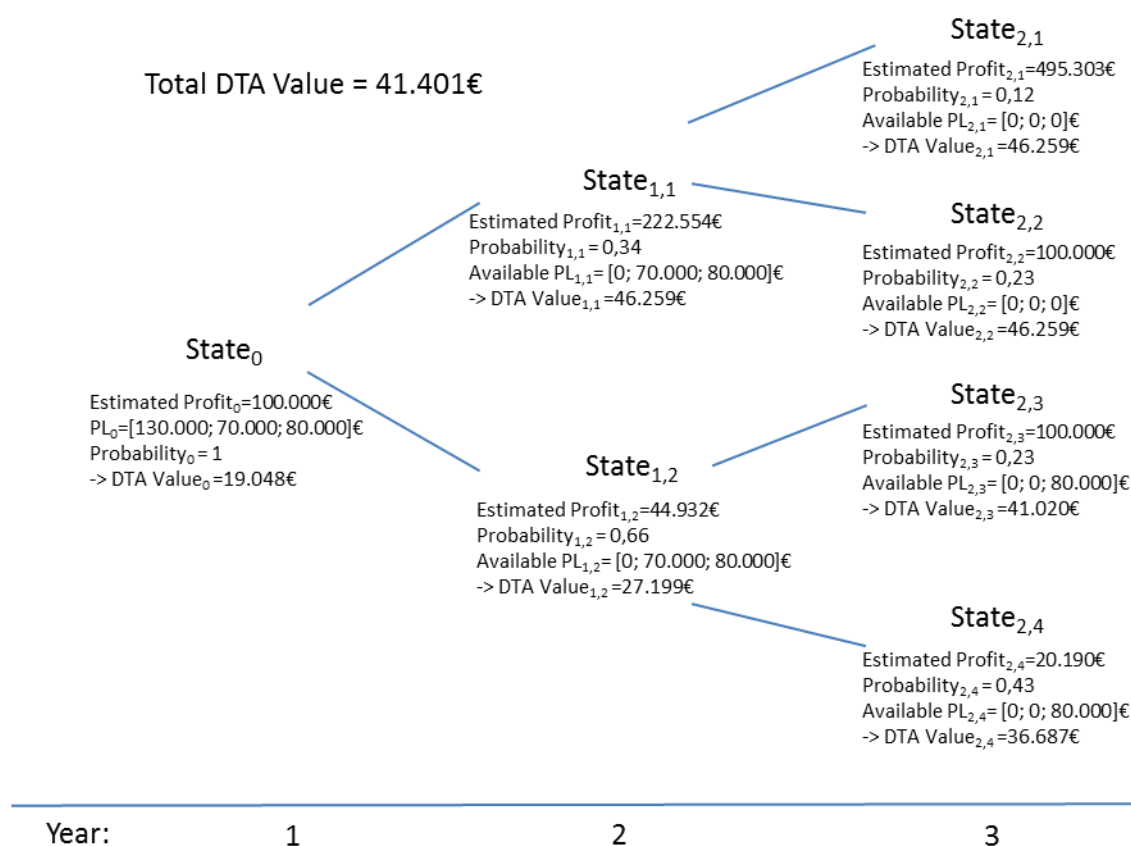


Figure 6 – Example with state evolution in the binomial tree

The total DTA value is obtained by the sum-product of all DTA values of the states in year 3, multiplied by their respective probabilities, yielding 41.401€. Notice that this method will yield a smaller value than method 1A, since method 1A assumes that all states have the same probability and calculates a higher mean value for the estimated profits (notice that this method yields a smaller probability of the

profits rising (34%) than that of the profits falling (66%)). Using method 1A, the total DTA figure would yield 46.118€ (depicted in Table 10), while the value that is inscribed in the balance sheet is 56.000€; clearly significantly more than the values calculated by both methods (35,3% and 21,4% for the realistic values of 41.401€ (Method 2A) and 46.118€ (Method 1A), respectively).

Table 10 – Paralellism of same conditions using method 1A (in €)

Years Left to discount	0	1	2	3
Losses		130000	70000	80000
Mean Projected Profits		100000,00	133743,49	178873,22
Limited Losses		100000	70000	80000
Used Amount		100000,00	133743,49	16256,51
Wrongfully used DTA	56000,00			
Realistic DTA	46118,07			

Method 2B - Binomial tree alternative with possibility of future losses (negative profits)

In this variant, we once again allow for negative profits (which must be converted to zeros afterwards, since we won't be able to use the DTAs in those situations). So being, the only change to method 2A is the estimated profit in each branch, from year 3 onwards. As already performed in method 1B, the profits will either rise or fall by the amount u or d , and thus we can reproduce Figure 6 in Figure 7 with this new methodology:

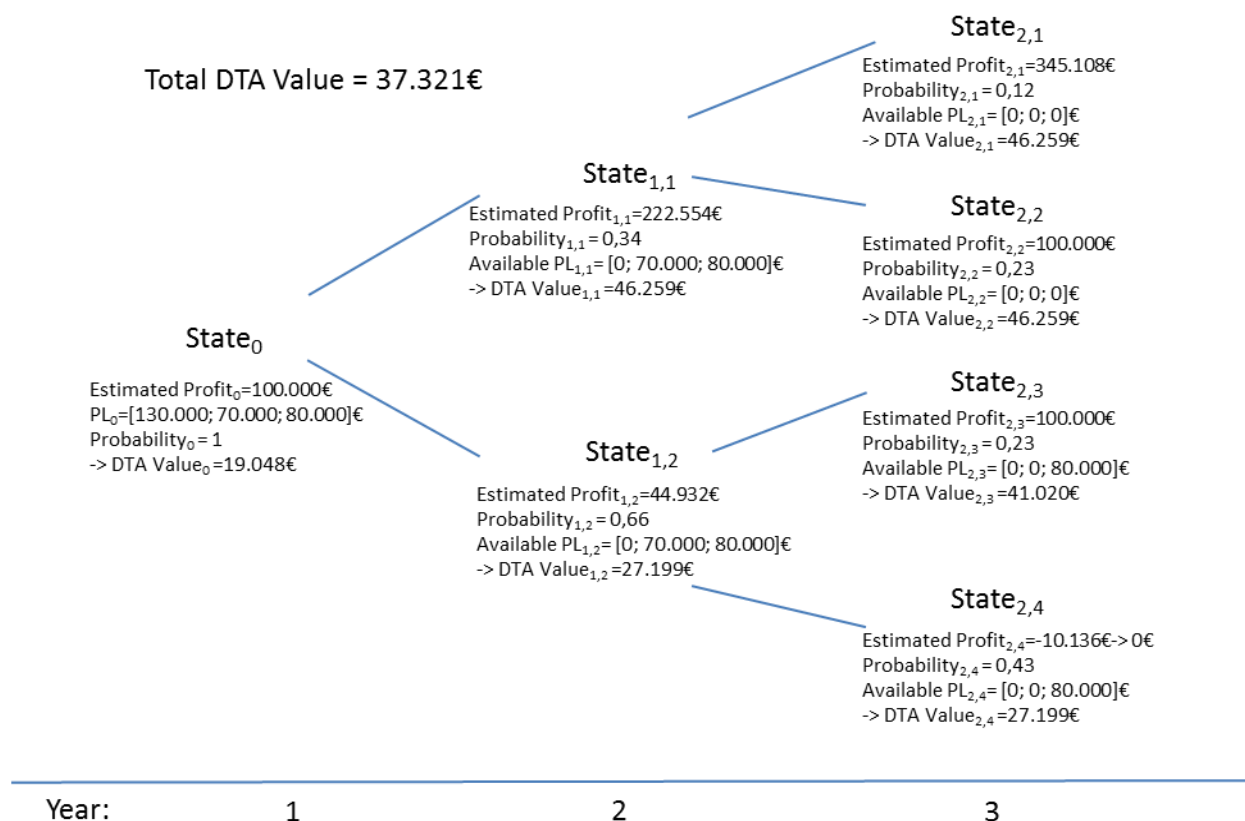


Figure 7 – Example allowing for negative profits, Method 2B

Looking at Figure 7, we can see that indeed state_{2,4} would have a negative profit (-10.136€), and as such was corrected to zero. The DTA value, calculated by multiplying each DTA from year 3 with its corresponding probability (37.321€ from Figure 7) was lower than using method 2A, as expected. Both values are close, and both methods could be employed. If we were to use method 1B (mean projected profit for year 3 calculated as the mean of profits of the third year in Figure 7, yielding 136.277€ in year 3), results would be the same as in Table 10, since the DTAs wouldn't be able to cover the profits in full. These results are shown in Table 11.

Table 11 - Paralellism of same conditions using method 1B (in €)

Years Left to discount	0	1	2	3
Losses		130000	70000	80000
Mean Projected Profits		100000	133743	136277
Limited Losses		100000	70000	80000
Used Amount		100000	133743	16257
Wrongfully used DTA	56000,00			
Realistic DTA	46118,07			

Conclusions

In this chapter we illustrated two simple methods and variants for valuing Deferred Tax Assets (DTAs), via the use of future estimated results and the binomial method. From the results we can see that there are significant differences between the asset value that is inscribed in the balance sheet and the DTA's real value. We also saw that different methods can yield different results, since all methods are contingent on the evolution of future tax earnings, which is the underlying asset used for the DTA's valuation. If companies and banks were to start using either one of these methods (methods 1 are more optimistic and much simpler than methods 2; method 2B is the most realistic and complex method to use) to value their DTAs in the balance sheet, investors would get a much clearer and realistic picture of the company's equity. Probably, the values at stake would also tend to be significantly lower than the value calculates using the current accounting approach (usually valuing the DTA at its maximum potential, which is clearly wrong). Naturally, this over optimistic approach will tend to lead to the underestimation of the company's risk exposure. We must keep in mind that a company that has been originating DTAs due to its poor performance (tax losses), sooner or later, is most likely to face additional DTAs, and will tend to face a non-negligible probability of not being able to "cash-in" its DTAs, rendering them worthless – it is essential that DTA valuation methods use realistic earnings projections, and that these are understood (and approved) by the majority of the shareholders.

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Appendix

Below is the code for filling out the UsedAmount vector:

```
Total_Losses=Sum(LimitedLosses);
UsedAmount=Zeros{1:N};
Intermediate_sum=0;
For i=1 to N
    Intermediate_sum= Intermediate_sum+Profits{i};
    If Intermediate_sum< Total_Losses
        UsedAmount {i}= Profits{i};
    Else
        UsedAmount {i}= Profits{i} - (Intermediate_sum - Total_Losses)
    Exit For (leaving the remaining positions of UsedAmount as zeros, if any)
End
End
```