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Real Interest Rate Parity between Brazil and the United States (2001-2022)

Diogo Antunes de Siqueira Costa

Master in Economics

Supervisor:
Ph.D. Diptes Chandrakante Prabhudas Bhimjee, Invited Assistant Professor, ISCTE Business School, ISCTE-IUL

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Department of Economics

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À Mozi, com todo meu amor

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Abstract

The present Dissertation aims to verify the validity of the Real Interest Rate Parity (RIRP) condition between Brazil and the United States. Real interest rates are at the core of modern macroeconomics and monetary policy analysis. Based on data from the Central Bank of Brazil (Banco Central do Brasil, BCB) and the United States Federal Reserve from 2001 to 2022, we evaluate the Brazilian capital market integration with the rest of the world as the real interest rate equalization is a crucial indicator in order to achieve the said integration. Assuming no restrictions on arbitrage in goods and financial markets and rational expectations, the RIRP condition suggests that real interest rates tend to equalize across countries. Despite the intuitive appeal of the RIRP, empirical evidence is inconclusive, with quite mixed findings and results in the academic literature. Using unit root analysis, we assess the convergence of real interest rates in Brazil to the international benchmark (the United States real interest rate), thus confirming the RIRP hypothesis. More specifically, we conclude that the stationarity of real interest rate differentials must be evaluated under the prism of multiple structural breaks. Furthermore, if real interest rates in Brazil depend on factors beyond the scope of domestic monetary policymakers, monetary policy independence (at least in its conventional form) is limited.

Keywords: Monetary Policy; Interest Parity, Real Interest Rates; Structural Breaks.

JEL Classification: E0; E43.

Resumo

A presente Dissertação tem como objetivo verificar a validade da condição de Paridade da Taxa de Juros Real (*Real Interest Rate Parity* - RIRP) entre Brasil e Estados Unidos. As taxas de juros reais estão no centro da macroeconomia moderna e da análise da política monetária. Com base em dados do Banco Central do Brasil (BCB) e do *Federal Reserve* dos Estados Unidos de 2001 a 2022, avaliamos a integração do mercado de capitais brasileiro com o resto do mundo, pois a equalização da taxa de juros real é um indicador crucial para conseguir a referida integração. Assumindo ausência de restrições à arbitragem em bens e mercados financeiros e expectativas racionais, a condição RIRP sugere que as taxas de juros reais tendem a se igualar entre os países. Apesar do apelo intuitivo da RIRP, a evidência empírica é inconclusiva, com achados e resultados bastante variados na literatura acadêmica. Utilizando a análise de raiz unitária, avaliamos a convergência das taxas de juros reais no Brasil para o referencial internacional (taxa de juros real dos Estados Unidos), confirmando assim a hipótese da RIRP. Mais especificamente, concluímos que a estacionariedade dos diferenciais das taxas de juros reais deve ser avaliada sob o prisma de múltiplas quebras estruturais. Além disso, se as taxas de juros reais no Brasil dependem de fatores além do escopo dos formuladores de política monetária doméstica, a independência da política monetária (pelo menos em sua forma convencional) é limitada.

Palavras-chave: Política Monetária; Paridade de Juros, Taxas de Juros Reais; Quebras Estruturais.

Classificação JEL: E0; E43.

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CHAPTER 1

Introduction

The present Dissertation aims to verify the validity of the Real Interest Rate Parity (RIRP) condition between Brazil and the United States. Real interest rates are at the core of modern macroeconomics and monetary policy analysis (Woodford, 2003, Walsh, 2017). Additionally, the validity of the RIRP is also an essential issue for policymakers. Equality of real interest rates across countries implies that the effective control of the domestic monetary authority over real interest rates is limited by the degree to which monetary policy can influence the world real interest rate (if it can be influenced at all by a small open economy). Even though Brazil is an unusually closed economy as measured by trade penetration, it might be deemed a small open economy (Areosa & Medeiros, 2007). To better understand the Brazilian financial integration with the rest of the world, we propose the following research question: Do Brazilian real interest rates converge to the international benchmark? The convergence of Brazilian interest rates towards the United States benchmark can provide important information to the monetary policy debate. Evidence of the RIRP between Brazil and the United States would reveal a high degree of market integration. Additionally, a reversion tendency to a positive differential would highlight the importance of the risk premium for an emerging market such as Brazil, an economy closely related to that of the United States.

Concerning the loss of power over the monetary policy linked to the RIRP hypothesis, we recognize that central banks increasingly resort to non-conventional tools, which provide a higher degree of flexibility and autonomy. Still, it is crucial to assess the financial market integration between Brazil and the United States in monetary policy efficacy. According to Rey (2015), one of the determinants of the global financial cycle is monetary policy in the center country. If "*capital is freely mobile, the global financial cycle constrains national monetary policies regardless of the exchange rate regime*" (Rey, 2015:p.1). The idea of the trilemma is notorious in international macroeconomics and finance. If capital flows are free in a financially integrated world, it is possible to have independent monetary policies only by having a floating exchange rate. That is, floating exchange rates enable monetary policy independence. Rey (2015) postulates that the global financial cycle transformed the trilemma into a dilemma: independent monetary policies are possible if and only if capital flows are controlled. In that sense, the 'impossible trinity' would become an 'irreconcilable duo'. In this extreme financial integration context, studying the RIRP hypothesis between Brazil and the United States can contribute to the discussion about monetary policy independence. This is highly relevant not only given the significant weight of these two giant economies to the American Continent, but also to the global economy.

The Uncovered Interest Rate Parity (UIRP) in foreign exchange markets proposes that the difference between domestic and foreign interest rates is equal to the expected change in the exchange rate. In other words, the UIRP conditions assert that if domestic and foreign bonds are perfect substitutes, with perfect capital mobility, two bonds can only pay different interest rates if agents expect a compensating movement in the exchange rate (Rogoff, 2002)[1]. This parity, along with the purchasing power parity (PPP)[2], would equalize real rates of return.

Uncovered interest rate parity

$$i_{t+1} = i^{*} + E_t(e_{t+i} - e_t) \qquad (1)$$

i_{t+1}: $domestic\ nominal\ interest\ rate\ (one\ year\ ahead)$
i^{*}: $foreign\ nominal\ interest\ rate\ (exogenous)$
$E_t(e_{t+i} - e_t)$: $expected\ rate\ of\ change\ of\ the\ exchange\ rate$
e: $logarithm\ of\ the\ exchange\ rate\ (home\ currency\ price\ of\ foreign\ currency)$

One central issue concerning the RIRP is the calculation of real interest rates. Given that the real interest rate is the nominal interest rate minus the expected inflation[3],

$$r_t = i_t - E_t(\pi_{t+i}) \qquad (2)$$

the measurement of inflation expectations could prompt the use of econometric techniques[4]. However, available forecasts for the examined economies (the United States and Brazil) simplify this step.

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[1] The Covered Interest Rate Parity (CIRP) postulates that it is impossible to earn a profit by borrowing in one currency and lending in another, while fully covering the foreign exchange risk.

[2] An extensive discussion of purchasing power parity is beyond the scope of this Dissertation. For now, the simplest statement of PPP is that the common currency price of an identical bundle of goods is equalized across different economies, at least from a theoretical perspective.

[3] Because the real rate is the return expected at the beginning of the period, it is also frequently referred to as the *ex-ante* real rate. This terminology is used to differentiate it from the *ex-post* real rate, or the actual real return from holding the bond for one period.

[4] For example, Magonis & Tsopanakis (2013) measure expected inflation calculating forecasts from a Markov Switching model. Specifically, they use 12-step ahead forecasts from a Markov switching model. The choice for this method has driven from the fact that the resulting Markov switching estimates have the advantage that they incorporate the process of agents' learning when making the forecasts. In the authors words, this would constitute "*other methods based on forecasts from AR models quite naïve*".

If the PPP and UIRP hold, there will be a convergence of real interest rates, that is:

$$r_t = r_t^* \quad (3)$$

r_t: $domestic\ real\ interest\ rate$

r_t^*: $real\ interest\ rate\ of\ the\ benchmark\ economy$

In other words, the RIRP is the corollary of two important relationships, the UIRP and PPP. Suppose an economy is small and both classical parity relations hold even in the short run. In that case, monetary policy cannot influence the *ex-ante* real rate of interest, and an essential tool for the monetary authority would be lost.

Two primary methodologies are applied in the recent empirical work to study the RIRP. On the one hand, several studies examine the existence of real interest rate parity looking into the stationarity properties of real interest rate differentials with unit root and stationarity tests. On the other hand, a methodological framework that has also been used is based on the comovement between real interest rates. To be more specific, assuming the general form:

$$r_t = \alpha + \beta r_t^* + \varepsilon_t \quad (4)$$

between real interest rates, cointegration techniques have been applied to examine whether the relation holds. The strict form of RIRP requires the parity condition, $\beta = 1$.

The remainder of this Dissertation is organized as follows: Chapter 2 presents the most relevant literature review; Chapter 3 describes the data used in this research; Chapter 4 discusses the methodology applied for the empirical analysis; Chapter 5 is dedicated to presenting and critically discussing the empirical findings. Finally, Chapter 6 concludes.

CHAPTER 2

Literature Review

Real interest rates are at the core of modern macroeconomics and monetary policy analysis (Woodford, 2003, Walsh, 2017). Empirical validation of the RIRP between Brazil and the United States would reveal a high degree of market integration. Additionally, a reversion tendency to a positive differential would highlight the importance of the risk premium for an emerging market such as Brazil. Several studies have tested this parity hypothesis since the pioneering papers of Cumby & Obstfeld (1982) and Mishkin (1984). The results, however, are somewhat mixed, with no conclusive answer regarding the existence of real interest rate parity in the empirical literature.

Cumby & Obstfeld (1982) find significant evidence against the two classical parity conditions that link prices and nominal interest rates internationally (UIRP and PPP) and on their corollary, the RIRP. The authors argue that whether the failure of the parity relations would give monetary autonomy to small open economies is an entirely different question.

Likewise, the empirical evidence gathered by Mishkin (1984) strongly rejected the hypothesis of real rate equality and the joint hypotheses of uncovered interest parity and *ex-ante* relative PPP. The author highlights that the results do not imply the existence of irrationality or unexploited profit opportunities. Risk premiums in the forward exchange market can exist, differing across countries, and varying over time with risk-averse economic agents. Furthermore, transaction costs, the non-substitutability of different countries' goods, and problems with prices indices can imply the violation of PPP conditions even when there are no unexploited profit opportunities.

Chung & Crowder (2004) specify a set of sufficient parity conditions for real interest rates to be equalized internationally. Using multivariate unit root tests, which have significantly greater power than univariate alternatives, they demonstrate that these sufficient conditions were not satisfied for five industrialized nations from 1960 to 1996. The results suggest that no single violation can explain the failure of RIRP in all cases. However, the Fisher relation[5] is the least likely to violate the RIRP equilibrium, whereas uncovered interest rate parity (UIRP) seems the most frequent relationship that does not hold. The results are consistent with a nonstationary risk premium in the foreign exchange market.

[5] The Fisher relation is the relationship between nominal and real interest rates under inflation. The real interest rate is the difference between the nominal interest rate and inflation rate. According to the Fisher hypothesis, the nominal interest rate moves together with the expected inflation rate, leaving *ex-ante* real rates unaltered.

Ferreira & León-Ledesma (2007) present empirical evidence on the real interest parity hypothesis for a set of emerging and developed countries[6]. This was done by carrying out unit-root tests on the *ex-post* real interest differentials concerning the United States and Germany. The results support the hypothesis of a rapid reversion towards a zero differential for developed countries and a positive one for emerging markets, which should also be the case for Brazil. Mean reversion is typically faster for emerging market economies.

Arghyrou et al. (2009) test for real interest rate convergence in the European Union 25-member area from 1996 to 2005. Using the two-break test proposed by Lee & Strazicich (2003) on real interest rate differentials, the authors find evidence of convergence for the majority of the countries. However, they highlight that this process is gradual and subject to structural breaks. Furthermore, convergence is rejected for a non-negligible number of countries.

Magonis & Tsopanakis (2013) perform an econometric research of the *ex-ante* version of the RIRP, examining whether the real interest rates of 17 countries of the Organization for Economic Co-operation and Development (OECD) co-move in the long run with the United States real interest rate. Additionally, they employ the test proposed by Carrion-i-Silvestre & Sansó (2006) to identify a potential structural break in the cointegrating equations under investigation. Apart from examining individual countries, they use the Westerlund (2007) panel cointegration test to explore the sample as a panel. The results provide evidence in favor of the RIRP. For ten economies out of 17, the cointegration of real interest rates holds for the whole period. Four countries present findings that suggest a break in the cointegrating relation, while there is no evidence for three economies included in the sample. Taking advantage of the increased test power using panel data, the null of no cointegration is strongly rejected, suggesting that the RIRP hypothesis is valid.

In an innovative study, Chang & Su (2015) incorporate RIRP fulfillment testing to allow multiple sharp breaks and smooth transitions. Most of the existing literature deals with structural breaks only as sharp drifts. The authors test for RIRP in the Group of Seven countries using *ex-ante* and *ex-post* definitions of real interest rates. The empirical results present substantial evidence in favor of the RIRP theory.

Sekioua (2008) emphasizes a strong criticism of the recent literature on real interest rate differentials: most studies are only concerned with whether those processes contain a unit root. The author stresses that rejecting the unit root hypothesis is not necessarily evidence in favor

[6] Argentina, Brazil, Chile, Mexico, and Turkey are the small open economies of emerging markets included in the sample.

of the RIRP. Since unit root tests can reject the non-stationarity hypothesis even if the process is still persistent, it is relevant to measure how long it takes for the deviations to die out.

The empirical evidence on the stochastic properties of real interest rates is mixed as well. Rose (1988) examines orders of integration of inflation rates and nominal interest rates and finds that the nominal interest rate contains a unit root while the inflation rate is stationary. The author concludes that any linear combination of the stationary inflation rate and non-stationary nominal interest rate should be non-stationary.

Otherwise, some studies find evidence that real interest rates follow a stationary process. Lai (1997), based on fractional integration analysis, reaches empirical results that strongly support that *ex-ante* real interest rates display mean reversion, but in a particular manner not captured by the usual stationary processes. This shows that, even though empirical works can demonstrate the persistence of real interest rates, there is still the issue related to the form this persistence should take.

Neely & Rapach (2008) survey the literature on the long-run persistence of real interest rates and gather information about researches that claim empirical evidence favoring real interest rates as: a unit root process (e.g., Rose, 1988; and Mishkin, 1992); a fractionally integrated process (e.g., Tsay, 2000; Phillips, 2005; and Karanasos et al., 2006); a non-linear process (e.g., Million, 2004; and Koustas & Lamarche, 2010); or a mean-reverting covariance stationary process with structural breaks.

In a research about the persistence of inflation, inflation expectations, and real interest rate in Brazil, Silva & Leme (2011) – based on Auto-Regressive Fractionally Integrated (ARFIMA) models and unit root tests with structural breaks – find evidence that the Brazilian inflation can be taken as stationary and mean-reverting, with some degree of persistence. As for inflation expectations, they detect non-stationarity in the fractional integration model due to structural breaks, meaning that they can also be taken as stationary with mean reversion. Finally, the interest rate shows some signs of non-stationarity, which had already been found in the unit root tests. However, they emphasize that it cannot be characterized by a unit root of a pure form, but as a fractionally integrated process with some long memory.

CHAPTER 3

Data

We use monthly data from the Central Bank of Brazil (Banco Central do Brasil, BCB) and the Federal Reserve Bank of Cleveland. We use the 1-year real interest rate estimated by the Federal Reserve Bank of Cleveland for the American real interest rate. For the Brazilian case, we employ the Selic rate[7] and the expected inflation rate for the IPCA[8] (from the Focus – Market Readout[9]). The Selic rate, or 'over Selic', is the Brazilian federal funds rate. It is the weighted average interest rate of the overnight interbank operations - collateralized by federal government securities - carried out at the Special System for Settlement and Custody (Selic). The *Índice de Preços ao Consumidor Amplo* (IPCA), or Extended National Consumer Price Index, is a reference inflation metric in Brazil. Currently, the target population for the IPCA encompasses families with household incomes ranging from 1 to 40 minimum wages, from whatever source, living in some urban regions of Brazil.

We recognize that several different interest rates could be used for the tests. Still, we consider that the difference between the Selic rate in annual terms and the expected inflation over the next 12 months from the Focus – Market Readout provides a reasonable indicator of the *ex-ante* inflation in Brazil. There is no unique variable that rational economic agents can compare to evaluate possible arbitrage opportunities in a globally integrated financial market.

An advantage of using the period from 2001 to 2022 is that after the mid-1990s, Brazil had already liberalized its capital market and had advanced substantially in the trade liberalization process. It is noteworthy that from July 1994, Brazil had a new currency, the Real. As previously discussed, the RIRP is based on the assumptions of frictionless goods and assets markets. The choice to start our analysis from November 2001 was made because the statistics for the Brazilian market expectations are available from then up to the most recent Focus-Market Readout. The most recent data available for our study was from February 2022.

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[7] Available at: www.bcb.gov.br/en/monetarypolicy/selicrate.

[8] Data from BCB´s "Time Series Management System", IPCA - Inflation accumulated over the next 12 months - % change - Average. Available at: https://www3.bcb.gov.br/sgspub/

[9] The Focus – Market Readout summarizes the statistics calculated over market expectations collected until the previous Friday. It is released every Monday. The report brings figures and statistics regarding market expectations for price indices, economic activity, exchange rate and the Selic rate, among others. The expectations are provided by market analysts, not by the BCB itself. The BCB carries out the 'Focus Survey', compiling forecasts of about 140 banks, asset managers, and other institutions (real sector companies, brokers, consultancies, etc.). Available at: www.bcb.gov.br/en/monetarypolicy/marketexpectations.

Essential descriptive statistics are presented in Table 1.

Table 3.1

Descriptive statistics of the time series

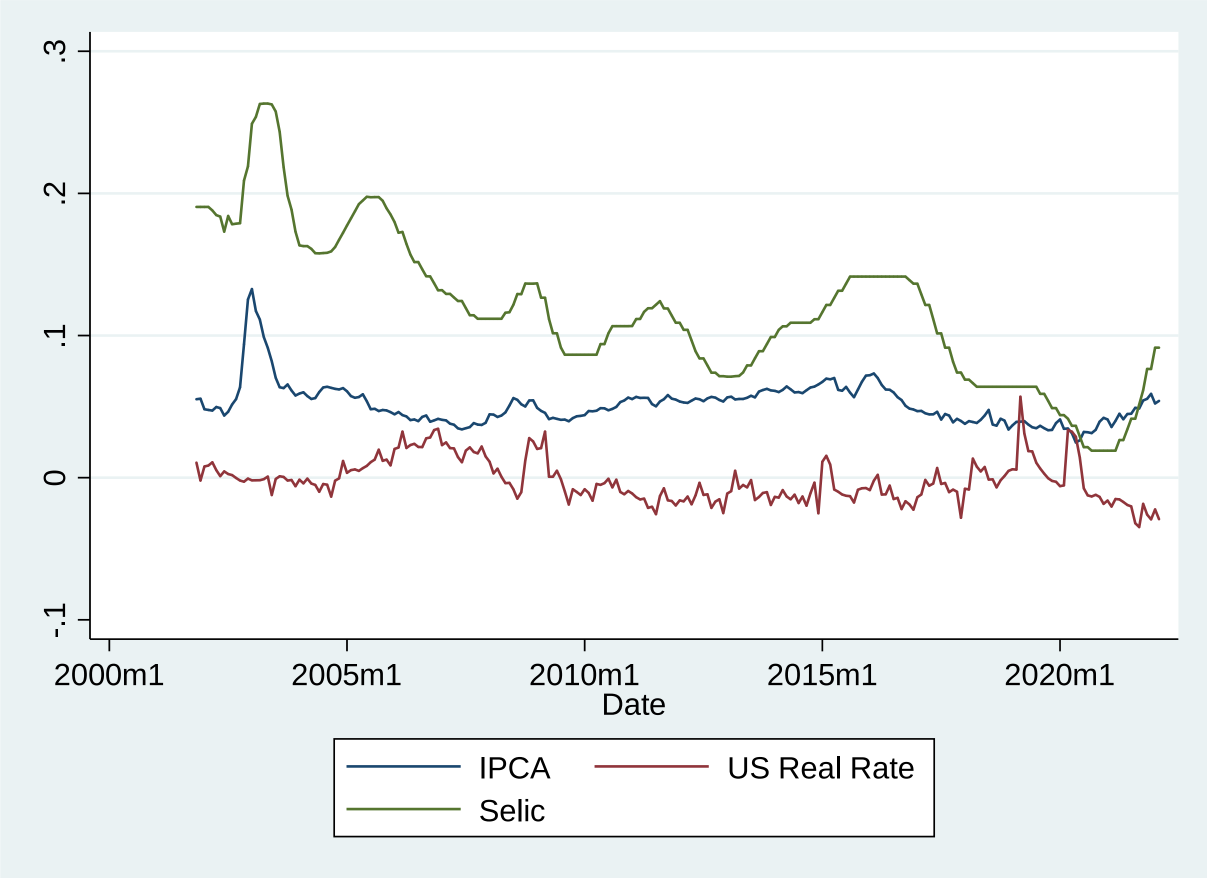
	Selic	IPCA	American real interest rates
Mean	11.73%	5.18%	-0.20%
Minimum	1.90%	2.48%	-3.47%
Maximum	26.32%	13.27%	5.70%
Standard Deviation	5.25%	1.45%	1.49%
Observations	244	244	244

Source: Performed by author in STATA

Figure 3.1 illustrates the three time series included in the sample data for this research. The American real interest rate seems stable throughout the two-decade period, with no clear upward or downward trend. More recently, after the Covid-19 pandemic crisis, which started roughly in February 2020, there seems to be a negative short-run trend. The expected inflation rate for the IPCA in Brazil, except for the spike between 2002 and 2003, looks surprisingly stable, considering that Brazil is an emerging market characterized by political risks and economic fragilities usually absent in developed countries. Lastly, the Brazilian nominal interest rate, Selic, shows a possible negative trend, clearly marked by an alternate upward and downward movement. Starting with the Covid-19 crisis, the Selic line crossed the expected inflation line, beginning an unprecedented period of *ex-ante* negative real interest rates. Still, the situation was reversed by the end of the sample timeline, and Brazilian *ex-ante* real interest rates were again positive.

Figure 3.1

United States Real Interest Rates, Selic Rate, and IPCA Inflation Expectations



CHAPTER 4

Methodology

The two main methodologies applied in the most recent empirical work to research the RIRP are based either on the stationarity properties of real interest rate differentials or on the comovement between real interest rates. The former makes use of unit root and stationarity tests, while the latter is performed through cointegration tests. Therefore, in the present Dissertation, unit root tests are applied to verify the stationarity of the time series and the differentials between real interest rates. Considering that the power of unit root tests may decrease if there is a structural change, even if the data is stationary, we also apply structural break tests. In addition, we verify the existence of cointegration between the Brazilian and American real interest rates time series. All tests were performed in STATA 17.

4.1. Stationarity tests

To test for unit root non-stationarity, multiple methods can be used. One of the most common tests is the augmented Dickey-Fuller (ADF) test for a unit root in a variable's autoregressive representation. So, we first test the stationarity of the real interest rate disregarding any possible structural changes or non-linearities in the series.

Using the correct number of lags in conducting an augmented Dickey-Fuller test is essential. Too few lags mean that the regression residuals will not behave like white-noise processes. Therefore, the model will not appropriately capture the actual error process, so the coefficient of interest and its standard error will not be well estimated. Including too many lags will reduce the power of the test to reject the null of a unit root since the increased number of lags means the estimation of additional parameters and the loss of degrees of freedom. The degrees of freedom decrease with the increase in the number of parameters estimated and the reduction in the number of usable observations. One observation is lost for each additional lag included in the autoregression. Consequently, unnecessary lags will reduce the power of the Dickey-Fuller test to detect a unit root (Enders, 2014).

Following the ADF test, we also apply the PP (Phillips and Perron) and KPSS (named after Kwiatkowski, Phillips, Schmidt, and Shin) unit root tests. To strengthen specific results, we apply the modified Dickey-Fuller known as DF-GLS. The test is, essentially, an ADF test in which the time series is transformed via a generalized least squares (GLS) regression before performing the test. Elliott et al. (1996) have shown that the DF-GLS test has significantly greater power than the previous versions of the augmented Dickey-Fuller test.

Because the power of these unit root tests may decrease if there is a structural change, even if the data is stationary, we paid special attention to structural break tests. For evaluating possible structural breaks, we apply the Zivot & Andrews (1992) test, a variation of Perron's structural break test (Perron, 1989) in which an endogenous breakpoint is estimated rather than fixed. Next, we used Bai & Perron (1998) test for multiple structural breaks in time series. To deal with the gap in the empirical literature related to break dates identification and the number of relevant structural changes selection, Kapetanios (2005) presents a unit root test against alternatives with an unspecified number of breaks (up to an exogenously given maximum). Nonetheless, the test has been shown to perform poorly in identifying the correct number of shifts and their dates (Vogelsang & Perron, 1998; Lee & Strazicich, 2001).

As pointed out by Güney et al. (2015), one of the drawbacks of the ADF test is that it has low power if the adjustment to equilibrium is nonlinear. For example, transaction costs may lead to nonlinear dynamics in the real interest rate. Kapetanios et al. (2003), given that conventional unit root tests might lack power when series display nonlinear dynamics, developed a new procedure to allow for nonlinear adjustment. This unit root test procedure, which we also applied, is based on an exponential smooth transition (ESTAR) regression model.

Although the test proposed by Kapetanios et al. (2003) is convenient for testing the stationarity in the case of nonlinear adjustment, it does not address the possibility of structural breaks.

4.2. Cointegration tests

At first, we planned to examine the comovement between the real interest rates of Brazil and the United States through the following well-established tests: (i) Engle and Granger’s two-step method and the (ii) Johansen technique. However, after we gathered evidence of different orders of integration in the time series, we decided to employ the Autoregressive Distributed Lag (ARDL) bounds testing approach for cointegration developed by Pesaran et al. (2001). There are two advantages to doing so: the method can be applied regardless of the order of integration of the variables, I (0) or I (1); and it can be implemented to small or finite sample size.

CHAPTER 5

Empirical Results

5.1. Stationarity Analysis

Unit root tests are tests for checking stationarity in a time series. A time series has stationarity if a change of time origin doesn’t change the shape of the distribution; the presence of unit roots is one cause of non-stationarity.

Figure 5.1

Ex-ante Real Interest Rates in Brazil and the United States

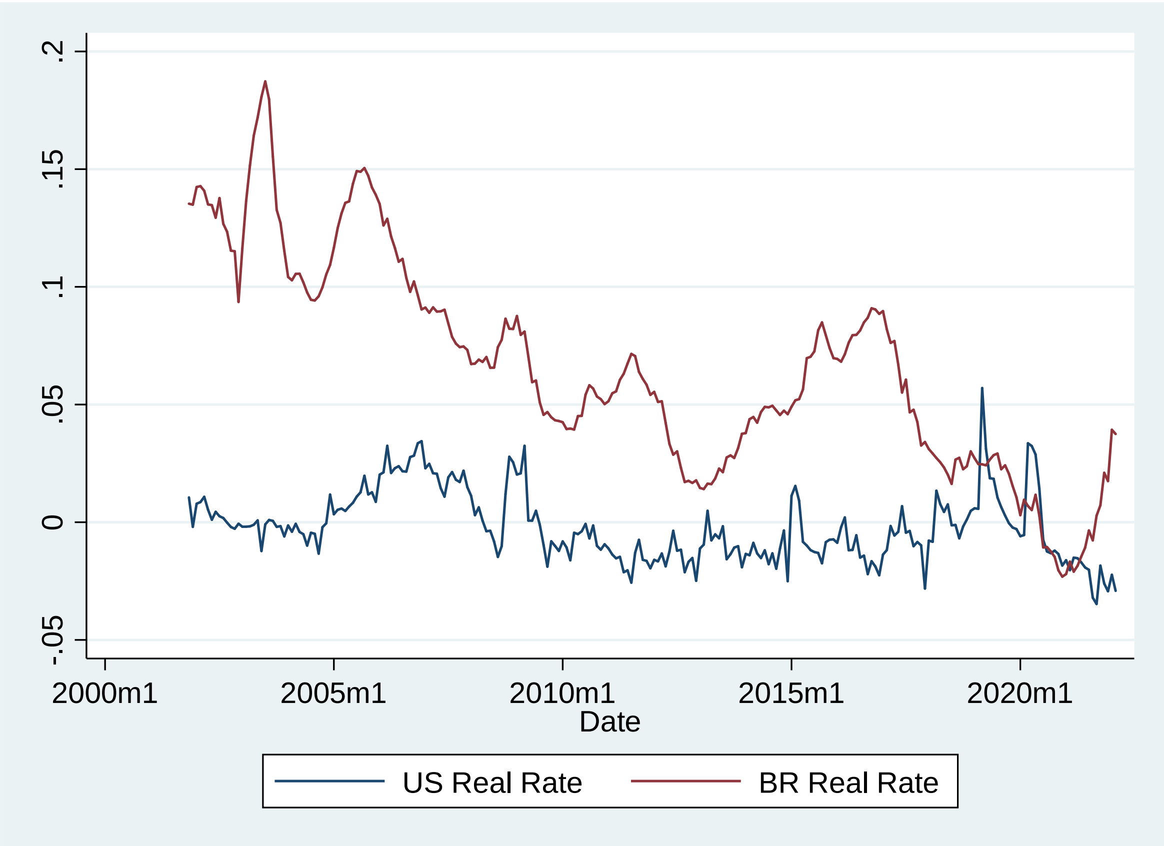
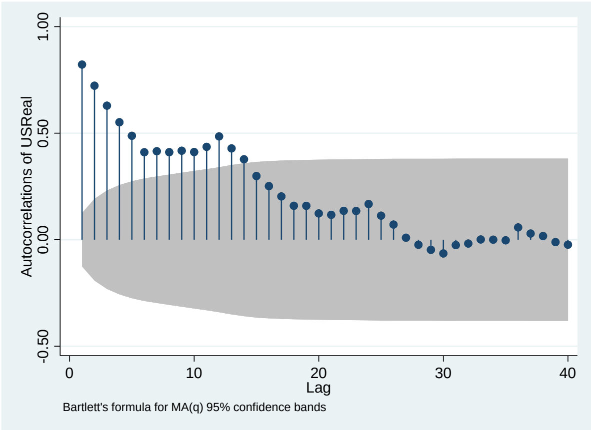


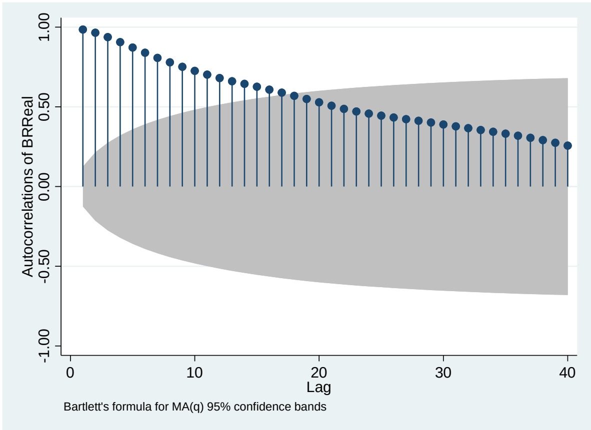
Figure 5.2 portrays the sample's autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of American and Brazilian real interest rates.

Figure 5.2

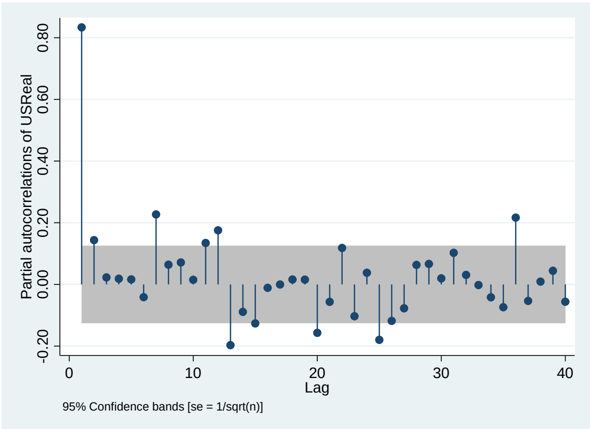
ACF and PACF Plots of Brazil and United States Real Interest Rates



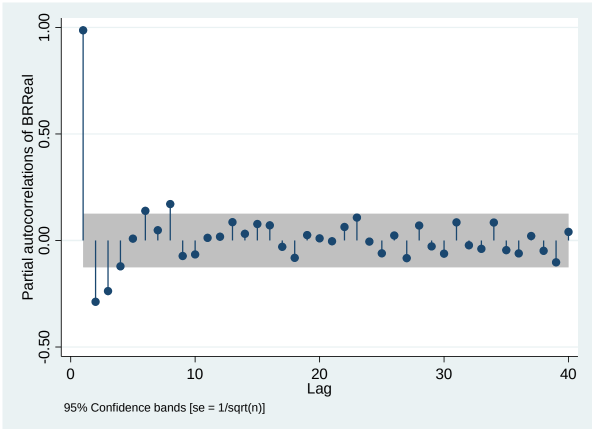
Panel (a): American rates ACF



Panel (b): Brazilian rates ACF



Panel (c): American rates PACF



Panel (d): Brazilian rates PACF

5.1.1. Conventional unit root tests

We start by estimating ADF, PP (Phillips and Perron), and KPSS (Kwiatkowski, Phillips, Schmidt, and Shin) unit root tests. For the selection of the optimal lag order for the ADF test, we observe the results of the final prediction error (FPE), Akaike's information criterion (AIC), Schwarz's Bayesian information criterion (SBIC), and the Hannan and Quinn information criterion (HQIC) lag-order selection statistics. Based on those results (Table 5.1), we select lag order 1 for the United States real rate and lag order 3 for the Brazilian real rate. If a time series follows an AR(1) process, it is not necessary to incorporate the augmented term. But if a time series is generated by a higher-order autoregression, for example AR(3), the augmented terms must be incorporated, and in this case, it would be lag order 2 (one less than the indicated by the lag-order selection statistics). Moreover, we use the AIC to select the lag order because "*AIC is more accurate for monthly data, HQIC works better for quarterly data on samples over 120, and SBIC works fine with any sample size for quarterly data*" (Das, 2019:p.321).

Table 5.1

Lag-order selection statistics, United States and Brazil Real Rates

	LR	FPE	AIC	HQIC	SBIC
United States Real Rates	2 lags	2 lags	2 lags	2 lags	1 lag
Brazil Real Rates	3 lags	4 lags	4 lags	3 lags	3 lags

Source: Performed by author in STATA

The ADF test with one lag (and no trend or drift) for the American real rate shows that the series is stationary. The results are not substantially different if we include a trend or drift (Table 5.2).

Table 5.2

ADF Tests Results, United States Real Rates

	Test Statistic	1%	5%	10%
United States Real Rates (1 lag)	-3.690	-3.463	-2.881	-2.571

Source: Performed by author in STATA

When we select lag order 3 for the ADF, the test with no trend or drift for the Brazilian real rate shows that the series is not stationary. However, if we include a trend or drift, the results indicate that the time series is stationary at the 5% level. More specifically, the constant and the trend term are statistically significant on the model with a trend and drift. Still, the constant is not statistically significant on the model with only a drift (Table 5.3).

Table 5.3

ADF Tests Results, Brazil Real Rates

	Test Statistic	1%	5%	10%
Brazil Real Rates (3 lags)	-2.448	-3.463	-2.881	-2.571
Brazil Real Rates (3 lags, trend and drift)	-3.559	-3.993	-3.431	-3.131
Brazil Real Rates (3 lags, drift)	-2.448	-2.342	-1.651	-1.285

Source: Performed by author in STATA

Following the ADF test, we apply the Phillips-Perron test. An advantage of the PP test is that it is non-parametric, i.e., it does not require selecting the level of serial correlation as in ADF. It instead takes the same estimation scheme as in the DF test, but corrects the statistic to conduct for autocorrelations and heteroscedasticity. The main disadvantage of the PP test is that it is based on asymptotic theory. Therefore, it works well only in large samples. This should not be a problem for the series we analyze, since we have 244 observations. The PP test also shares the disadvantages of ADF tests: sensitivity to structural breaks and poor small sample power. The results are in line with those achieved by the ADF test: the United States real rates are stationary, and Brazilian real rates are not (Table 5.4).

Table 5.4

PP Test Results, United States and Brazil Real Rates

	Test Statistic	1%	5%	10%
United States Real Rates	-4.351	-3.462	-2.880	-2.570
Brazil Real Rates	-1.832	-3.462	-2.880	-2.570

Source: Performed by author in STATA

Next, we apply the KPSS test only to the Brazilian series, given that the United States real rates were shown to be stationary by the ADF and PP tests. KPSS tests are used for testing a null hypothesis that an observable time series is stationary around a deterministic trend (i.e., trend stationary) against the alternative that the series is difference-stationary (Kwiatkowski et al., 1992). At the 1% significance level, the KPSS test shows that Brazilian real rates are not trend stationary (Table 5.5). We can reject the null hypothesis of trend stationarity at least for six lag orders. Since KPSS indicates that the series is not trend stationarity and ADF and PP indicate non-stationarity, we conclude that the series is not trend stationary.

If the time series were stationary around a deterministic trend, it would be necessary to remove it to make it strictly stationary. The detrended series would be checked for stationarity[10]. As previously mentioned, we must remember that ADF and PP tests have low power to distinguish between unit root and near unit root processes (Diebold & Rudebusch, 1991). At this point, it is critical to test whether the series really contains a trend and the best way to estimate the trend. It is inappropriate to difference or detrend a stationary series. Moreover, it is wrong to detrend a unit root process or differentiate a trend stationary process.

Table 5.5

KPSS Test Results, Brazil Real Rates

	1%	2,5%	5%	10%
Lag Order – Test Statistic	0.216	0.176	0.146	0.119
0 – 1.640				
1 – 0.837				
2 – 0.568				
3 – 0.435				
4 – 0.356				
5 – 0.304				

[10] Essentially, there are two ways to eliminate a trend. A trend stationary series can be transformed into a stationary series by removing the deterministic trend. A series with a unit root (difference stationary series) can be transformed into a stationary series by differencing. Using an inappropriate method to eliminate the trend can cause serious problems (Enders, 2014).

	6 – 0.267

Source: Performed by author in STATA

5.1.2. Structural break tests: real interest rate time series

Before analyzing the possibility of the series being trend stationary as indicated by the ADF test with a trend and drift, there is still a possibility that we might be facing a long memory process or a structural break. Perron (1989), in his seminal work, presented the idea that not considering a possible structural break in the data generating process may produce spurious results. Consequently, the power of unit root tests may decrease if there is a structural change, even if the data is indeed stationary. Conventional unit root tests are biased toward a false unit root hypothesis when the data are trend stationary with a structural break. Furthermore, a trend would not be, *a priori*, expected to be present in the data, or, as Kapetanios et al. (2003) argued, "*most reasonable economic theories would predict that real interest rates are trendless with a non-zero mean*". Considering that neither theory nor empirical evidence supports the idea of a trend in real interest rates, we proceed to perform structural break tests to analyze the Brazilian time series.

Subsequently, we test for structural breaks in the time series. The test shows that we can reject the null of no structural break at the 1% significance level. More specifically, the break happened in the second month of 2015 (Table 5.6). It is reasonable to suppose that economic and political changes can engender structural instability in the data generating process. Thus, it is surprising that the estimated break occurred only in the last quarter of the sample period. It was not the first time the Brazilian economy had faced serious challenges during the 20 years of the study. As Reis (2021) observes, "*Brazil between 2011 and 2016 provides the clearest counterpart to the US inflation of the 1970s*" (Reis, 2021:p.29), in the sense the monetary authority unexpectedly and significantly eased policy, rapidly lowering interest rates despite both GDP growth and inflation being above the upper bound of the target.

Before that, in 2002 and 2003, Brazil faced a severe drought, suffered adverse effects from the financial crisis in Argentina, and went through a presidential election that caused unease in the financial markets. These adverse conditions led to a sharp increase in sovereign spreads, inflation, and substantial currency depreciation. However, we must acknowledge that the test excludes the possibility of a structural break too near the beginning or the end of the sample, since it is symmetric trimmed by 15%.

In addition to those periods, we must cite the Great Recession of 2007-2009, a period marked by a general recession in national economies globally. Even though the direct effects on the Brazilian economic growth were not severe (from 2007 to 2009, Brazil´s real GDP growth rates were actually 6,1%, 5,1%, and -0,1%, respectively), international financial markets were indeed challenged by these new macroeconomic conditions associated with the global shock.

During the period in which the structural break was estimated (February 2015), throughout the infamous biennium 2015-2016, Brazil faced an economic crisis alongside a political turmoil that resulted in the impeachment of President Dilma Rousseff. The country's GDP fell by 3,5% in 2015 and 3,3% in 2016. According to data from the state-owned Brazilian Institute of Geography and Statistics (IBGE), this was the worst biennium recession in Brazil's history, surpassing the economic decline of the Great Depression from 1929 to 1930.

Table 5.6

Test for a structural break: Unknown break date, Brazil Real Rates

(Full sample 2001m11-2022m2; Trimmed sample 2004m12-2019m2)

	Statistic	p-value
Brazil Real Rates	187.4515	0.000
Estimated break date 2015m2		

Source: Performed by author in STATA

Given the evidence of a structural break in the Brazilian time series around the biennium, we conclude that the previous results of the ADF test with trend and drift that indicated that the data was trend stationary were not adequate. If the Brazilian real interest rates do not tend to return to a linear trend, we must verify if they follow a difference stationary process (Nelson & Plosser, 1982). In this case, the trend is stochastic and would interact with the structural breaks. Furthermore, given the estimated structural break of 2015, which coincided with a significant economic and political crisis in Brazil, it is reasonable to split the sample in two (2001-2014 and 2015-2022) to assess to stationarity of the two subperiods.

Again, the ADF test with no trend or drift for the Brazilian real rate shows that the series is not stationary in the subperiod 2001-2014. Conversely, if we include a trend or drift, the results indicate that the time series is stationary at the 5% level. More specifically, the constant and trend term are strongly significant, but the drift term alone is not statistically significant (Table 5.8).

The results of the PP test (Table 5.9) are also in line with those achieved by the ADF with no trend or drift test: Brazilian real rates are not stationary in the subperiod. Finally, the KPSS test (Table 5.10) shows that the time series is not trend stationary at the 5% level.

Table 5.7

Lag-order Selection Statistics, Brazil Real Rates (2001-2014)

	LR	FPE	AIC	HQIC	SBIC
Brazil Real Rates	3 lags	3 lags	3 lags	3 lags	3 lags

Source: Performed by author in STATA

Table 5.8

ADF Tests Results, Brazil Real Rates (2001-2014)

	Test Statistic	1%	5%	10%
Brazil Real Rates (2 lags)	-1.997	-3.492	-2.886	-2.576
Brazil Real Rates (2 lags, trend and drift)	-3.818	-4.022	-3.443	-3.143
Brazil Real Rates (2 lags, drift)	-1.997	-2.351	-1.655	-1.287

Source: Performed by author in STATA

Table 5.9

PP Test Results, Brazil Real Rates (2001-2014)

	Test Statistic	1%	5%	10%
Brazil Real Rates	-1.534	-3.491	-2.886	-2.576

Source: Performed by author in STATA

Table 5.10

KPSS Test Results, Brazil Real Rates (2001-2014)

	1%	2,5%	5%	10%
Lag Order – Test Statistic	0.216	0.176	0.146	0.119
0 – 0.524 1 – 0.270 2 – 0.187 3 – 0.146				

Source: Performed by author in STATA

About the 2015-2022 subperiod, the ADF test with no trend or drift for the Brazilian real rate shows that the time series is not stationary (Table 5.12). Unlike in the subperiod 2001-2014, the results also indicate that the series is not stationary if we include a trend or drift. The results of the PP test are also in line with those achieved by the ADF: Brazilian real rates are not stationary in the subperiod (Table 5.13). Finally, the KPSS test shows that the time series is not trend stationary (Table 5.14).

Table 5.11

Lag-order selection statistics, Brazil Real Rates (2015-2022)

	LR	FPE	AIC	HQIC	SBIC
Brazil Real Rates	4 lags	4 lags	4 lags	4 lags	4 lags

Source: Performed by author in STATA

Table 5.12

ADF Tests Results, Brazil Real Rates (2015-2022)

	Test Statistic	1%	5%	10%
Brazil Real Rates (3 lags)	-1.495	-3.530	-2.901	-2.586
Brazil Real Rates (3 lags, trend and drift)	-2.110	-4.071	-3.464	-3.158
Brazil Real Rates (3 lags, drift)	-1.495	-2.373	-1.664	-1.292

Source: Performed by author in STATA

Table 5.13

PP Test Results, Brazil Real Rates (2015-2022)

	Test Statistic	1%	5%	10%
Brazil Real Rates	-0.998	-3.530	-2.901	-2.586

Source: Performed by author in STATA

Table 5.14

KPSS Test Results, Brazil Real Rates (2015-2022)

	1%	2,5%	5%	10%
Lag Order – Test Statistic	0.216	0.176	0.146	0.119
0 – 0.527				
1 – 0.282				
2 – 0.199				
3 – 0.158				

Source: Performed by author in STATA

At this point, we have gathered empirical evidence that Brazilian real interest rates are not stationary in both subperiods indicated by the single structural break test (2001-2014 and 2015-2022).

Continuing with our structural break analysis, we apply Bai and Perron test for multiple breaks on unknown dates (Table 5.15). Considering breaking variables time and the constant term, with a maximum of five possible breaks at a significance level of 5%, the test estimates five break dates in a sequential estimation: February 2005, February 2008, July 2012, February 2016, and February 2019. Break dates are the first date of the subsequent regime.

Table 5.15

Sequential test for multiple breaks at unknown breakpoints (Ditzen et al., 2021)

Bai and Perron Multiple Breaks Test, Brazil Real Rates

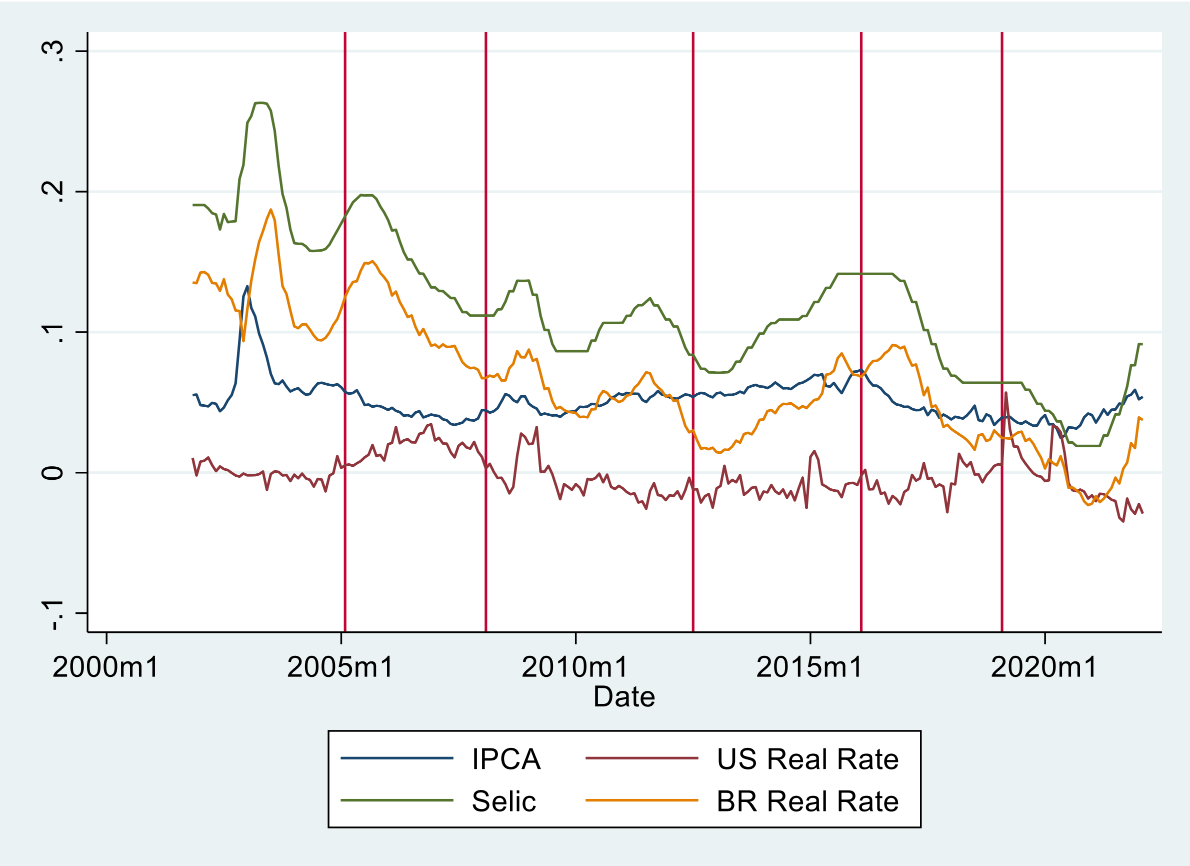
	Test statistic	1% critical value	5% critical value	10% critical value
F (1|0)	93.73	7.68	5.74	4.91
F (2|1)	20.16	8.42	6.47	5.70
F (3|2)	21.50	8.86	7.01	6.14
F (4|3)	28.49	9.34	7.42	6.45
F (5|4)	30.60	9.59	7.64	6.74
	Break dates	95% confidence interval		
	2005M02	2005M01	2005M03	
	2008M02	2008M01	2008M03	
	2012M07	2012M06	2012M08	
	2016M02	2016M01	2016M03	
	2019M02	2019M01	2019M03	

Source: Performed by author in STATA

Considering the additional evidence of structural breaks in the Brazilian time series, it would be possible to split the entire sample into six subperiods. The estimated break dates do not seem to be related to a domestic or international financial crisis. It was not possible to link them to identifiable economic or political shocks (Figure 5.3). A potential explanation for these breaks might be related to the fact that there might be dynamics related to Brazilian financial markets, the study of which are beyond the scope of the present Dissertation.

Figure 5.3

Ex-ante Real Interest Rates in Brazil and the United States and 5 Structural Breaks

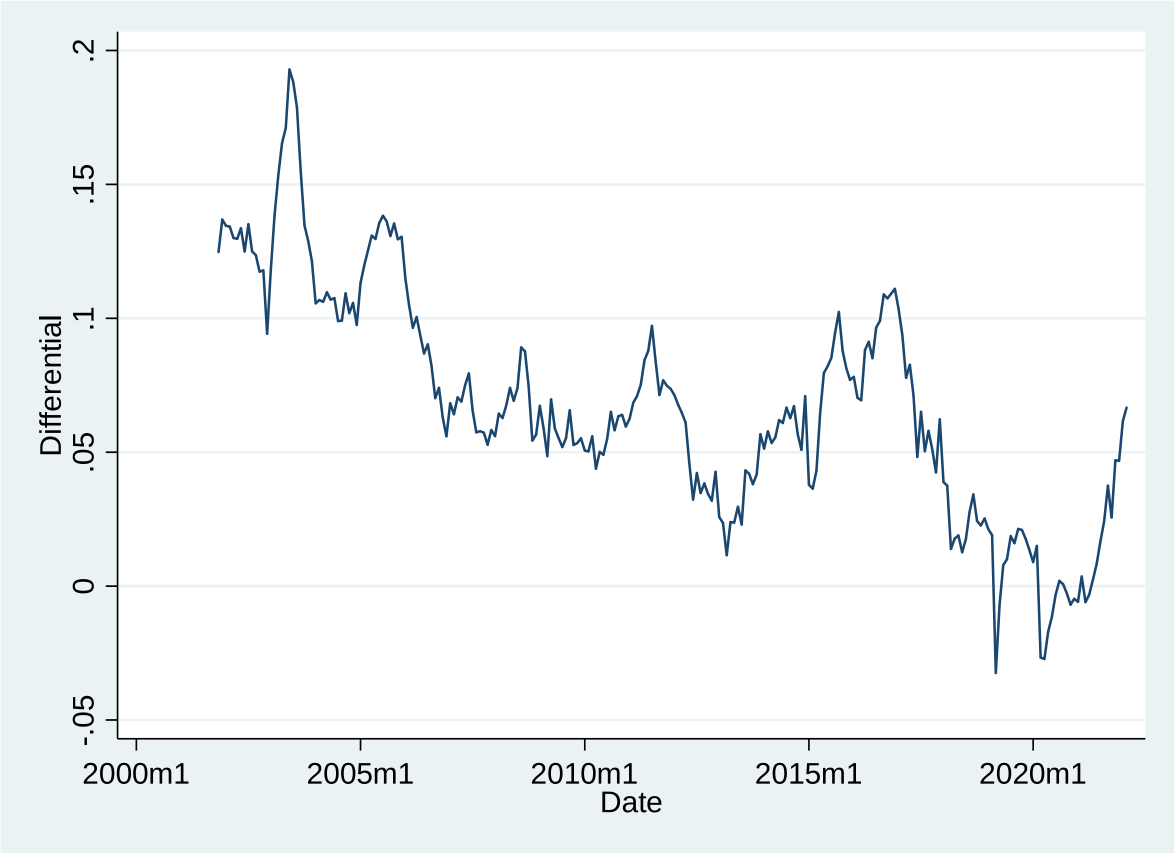


5.1.3. Structural break tests: differentials

Given the empirical evidence collected so far about the stationarity of American real interest rates during the entire period under examination and the non-stationarity of the Brazilian rates in the same interval, but with signs of multiple structural breaks along the way, we aim to unveil whether Brazil experienced *ex-ante* real interest rate stationary differentials in relation to the United States. Therefore, we carry out the same set of unit root tests to characterize the dynamic behavior of those differentials.

Figure 5.4

Real Interest Rates Differentials Between Brazil and the United States



The ADF test with no trend or drift and the test with trend and drift for the differentials show that the series is not stationary. However, if we include just a drift, the results indicate that the time series is stationary at the 5% level. But on the model with a drift, the constant is not statistically significant (Table 5.17).

Table 5.16

Lag-order Selection Statistics, Real Interest Rates Differentials

	LR	FPE	AIC	HQIC	SBIC
Differentials	1 lag	1 lag	1 lag	1 lag	1 lag

Source: Performed by author in STATA

Table 5.17

ADF Tests Results, Real Interest Rates Differentials

	Test Statistic	1%	5%	10%
Differentials (0 lag)	-2.167	-3.462	-2.880	-2.570
Differentials (0 lag, trend and drift)	-2.698	-3.992	-3.431	-3.131
Differentials (0 lag, drift)	-2.167	-2.342	-1.651	-1.285

Source: Performed by author in STATA

According to the PP test, the real interest rate differentials are not stationary (Table 5.18). And according to the KPSS test, they are not trend stationary (Table 5.19).

Table 5.18

PP Test Results, Real Interest Rates Differentials

	Test Statistic	1%	5%	10%
Differentials	-2.206	-3.462	-2.880	-2.570

Source: Performed by author in STATA

Table 5.19

KPSS Test Results, Real Interest Rates Differentials

	1%	2,5%	5%	10%
Lag Order – Test Statistic	0.216	0.176	0.146	0.119
0 – 1.240 1 – 0.649 2 – 0.447 3 – 0.345 4 – 0.285 5 – 0.245 6 – 0.216				

Source: Performed by author in STATA

Subsequently, we test for structural breaks in the time series. According to the Zivot-Andrews test, a single structural break could not be identified with statistical significance (Table 5.20).

Table 5.20

Zivot-Andrews Test Results, Real Interest Rates Differentials

(allowing for break in both intercept and trend)

	Test Statistic	1%	5%	10%
Differentials	-3.466 at 2013m8	-5.57	-5.08	-4.82

Source: Performed by author in STATA

Unlike the previous test, when applying the Bai and Perron test for multiple breaks in unknown dates, five break dates are estimated at 5% statistical significance. Considering breaking variables time and the constant term, with a maximum of five possible breaks, the test estimates the following break dates: April 2006, April 2009, April 2012, February 2016, and February 2019 (Table 5.21).

Table 5.21

Sequential test for multiple breaks at unknown breakpoints (Ditzen et al., 2021)

Bai and Perron Multiple Breaks Test, Real Interest Rate Differentials

	Test statistic	1% critical value	5% critical value	10% critical value
F (1|0)	49.99	7.68	5.74	4.91
F (2|1)	38.13	8.42	6.47	5.70
F (3|2)	30.89	8.86	7.01	6.14
F (4|3)	42.65	9.34	7.42	6.45
F (5|4)	20.88	9.59	7.64	6.74
	Break dates	95% confidence interval		
	2006M04	2006M03	2006M05	
	2009M04	2009M03	2009M05	
	2012M04	2012M03	2012M05	
	2016M02	2016M01	2016M03	
	2019M02	2019M01	2019M03	

Source: Performed by author in STATA

Given the five estimated break dates, we can divide the original sample into six subperiods to verify the stationarity properties of each one of them (Figure 5.5). The ADF test, especially with a drift, showed that the first four subperiods were marked by evidence of stationarity in the differentials. According to the PP test results, the unit root hypothesis could not be rejected in all subperiods. For a complete assessment of the stationarity in the subperiods, we also present the DF-GLS test results (Table 5.22). According to the DF-GLS test, the unit root hypothesis could be rejected in two subperiods (2001M11-2006M03 and 2009M04-2012M03). Based on the results from the three tests (ADF, PP, and DF-GLS), we observe that the two subperiods with the weakest indication of stationarity (2016M02-2019M01 and 2019M02-2022M02) were characterized by two critical shocks: one domestic and the other international. The first was the domestic economic and political turmoil of 2015-2016 that resulted in President Dilma Rousseff´s impeachment. The second was the Covid-19 worldwide pandemic. It seems that the two shocks can explain the lack of stability of coefficients during both subperiods. It is noteworthy that the global economic shock of the Great Recession from 2007 to 2009 occurred during subperiods with evidence of stationarity. Table 5.22 summarizes the unit root tests' results.

Table 5.22

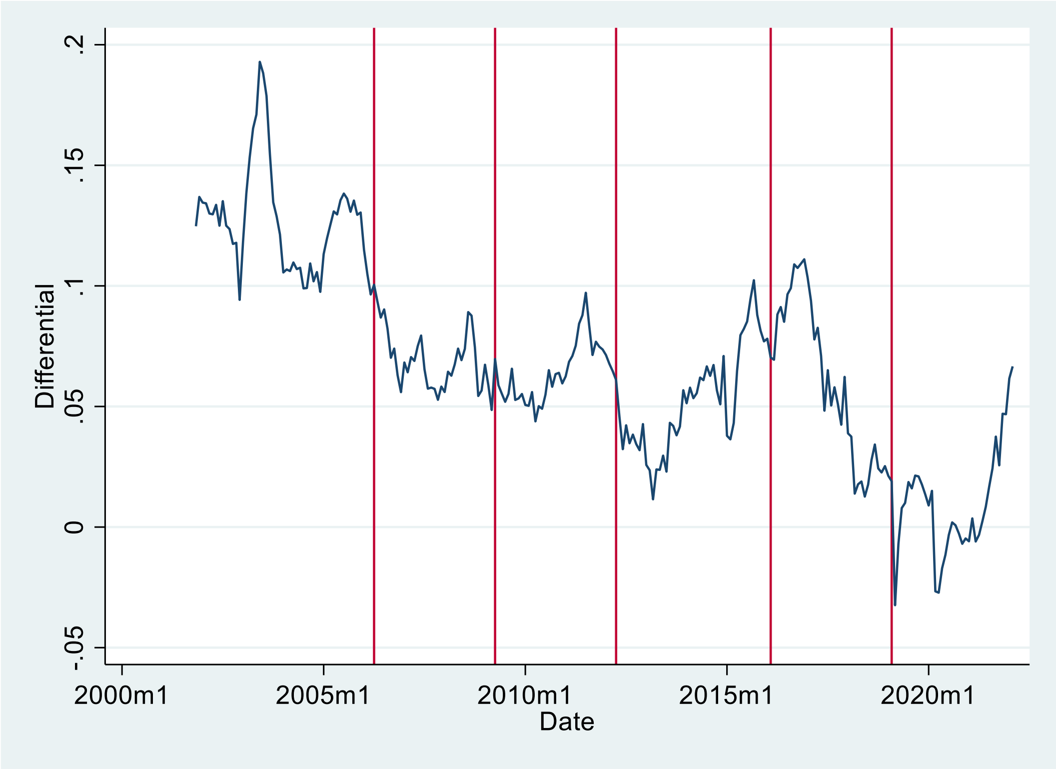
ADF, PP, and DF-GLS Test Results, Real Interest Rate Differentials (6 subperiods)

Subperiod	ADF Test	PP Test	DF-GLS Test
2001M11-2006M03	2 lags with drift at 1%	Non-stationary	2 lags at 1%
2006M04-2009M03	0 lag with drift at 5%	Non-stationary	Non-stationary
2009M04-2012M03	0 lag with drift at 5%	Non-stationary	0 lag at 10%
2012M04-2016M01	0 lag with trend at 10%	Non-stationary	Non-stationary
2016M02-2019M01	Non-stationary	Non-stationary	Non-stationary
2019M02-2022M02	Non-stationary	Non-stationary	Non-stationary

Source: Performed by author in STATA

Figure 5.5

Real Interest Rates Differentials Between Brazil and the United States and 5 Structural Breaks



5.1.4. Difference stationary processes

Bearing in mind that both Brazilian real interest rates and the differentials displayed no tendency to return to a linear trend if we consider the entire sample, but present evidence of stationarity after multiple structural breaks were considered (especially according to the ADF test with drift), we advance our analysis to verify if they follow a difference stationary process.

The graphic representation of Brazilian real interest rates in first differences can be observed in Figure 5.6. The ADF and PP tests show strong evidence of stationarity in the Brazilian real interest rates after the first differences (Tables 5.24 and 5.25). Therefore, we evaluate the dynamics of *ex-ante* real interest rate differentials based on the differentiated real interest rate time series (both American and Brazilian). We carry out the same set of unit root tests to characterize the dynamic behavior of those differentials.

Figure 5.6

Brazil Real Interest Rates in First Differences

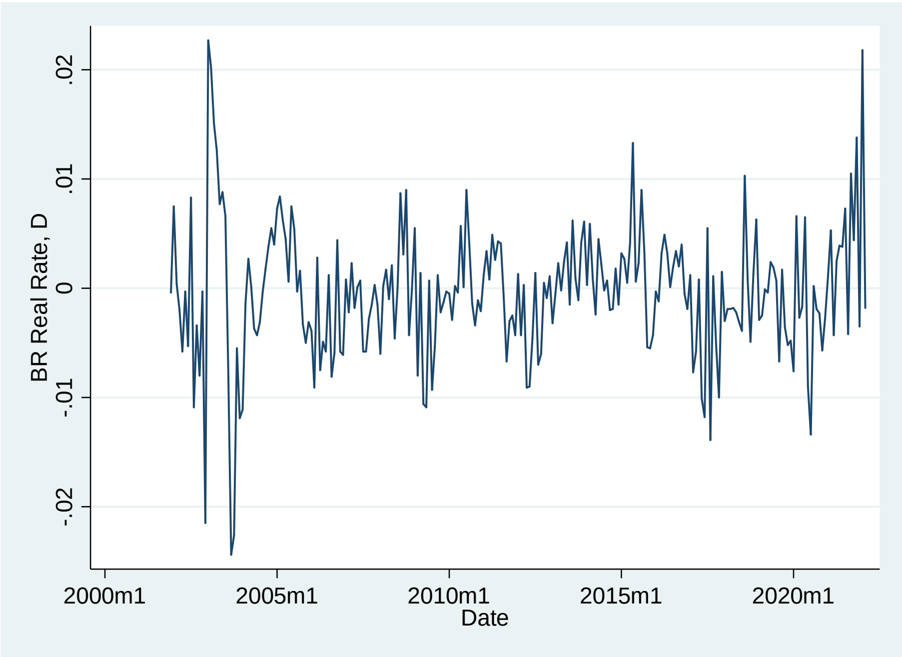


Table 5.23

Lag-order selection statistics, Brazil Real Rates in First Differences

	LR	FPE	AIC	HQIC	SBIC
Brazil Real Rates	2 lags	3 lags	3 lags	2 lags	2 lags

Source: Performed by author in STATA

Table 5.24

ADF Tests Results, Brazil Real Rates in First Differences

	Test Statistic	1%	5%	10%
Brazil Real Rates (1 lag)	-7.293	-3.463	-2.881	-2.571
Brazil Real Rates (1 lag, trend and drift)	-7.309	-3.993	-3.431	-3.131
Brazil Real Rates (1 lag, drift)	-7.293	-2.342	-1.651	-1.285

Source: Performed by author in STATA

Table 5.25

PP Test Results, Brazil Real Rates in First Differences

	Test Statistic	1%	5%	10%
Brazil Real Rates	-11.999	-3.463	-2.881	-2.571

Source: Performed by author in STATA

As in the differentiated Brazilian rates stationarity analysis, the differentials between both the American and Brazilian real interest rates in first differences showed strong evidence of stationarity according to the ADF and PP tests (Tables 5.27 and 5.28).

Figure 5.7

Real Interest Rates Differentials in First Differences Between Brazil and the United States 2001-2022

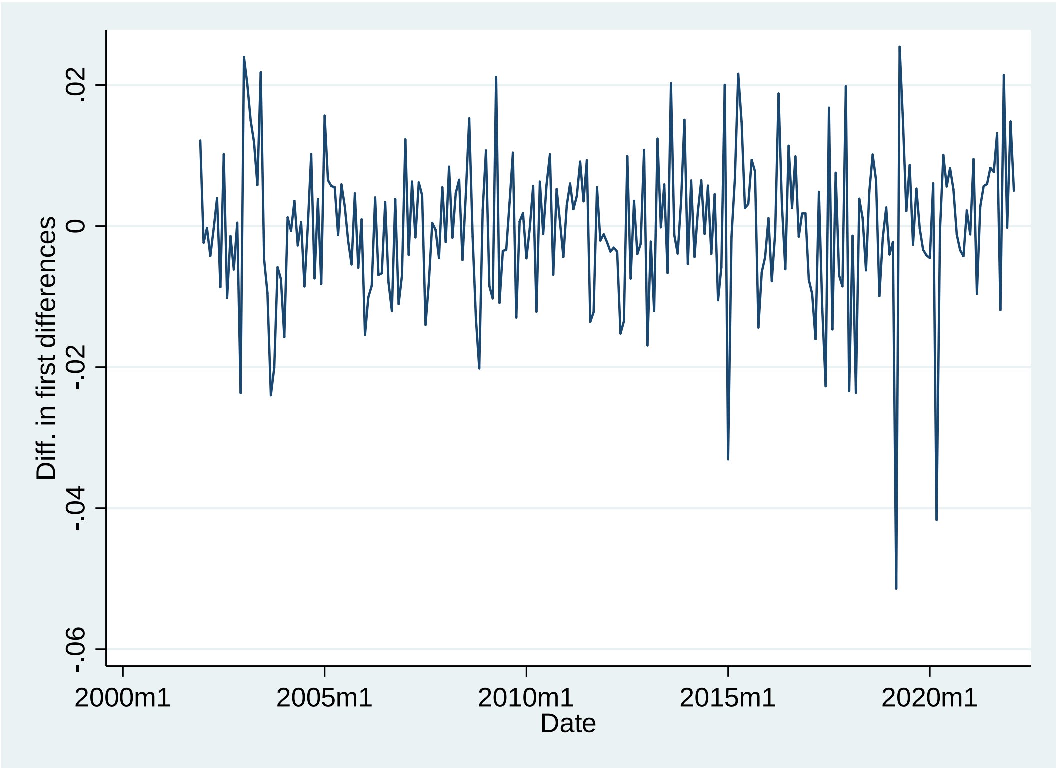


Table 5.26

Lag-order selection statistics, Real Interest Rates Differentials in First Differences

	LR	FPE	AIC	HQIC	SBIC
Differentials	-	0 lag	0 lag	0 lag	0 lag

Source: Performed by author in STATA

Table 5.27

ADF Tests Results, Real Interest Rates Differentials in First Differences

	Test Statistic	1%	5%	10%
Differentials (0 lag)	-16.770	-3.463	-2.881	-2.571
Differentials (0 lag, trend and drift)	-16.774	-3.993	-3.431	-3.131
Differentials (0 lag, drift)	-16.770	-2.342	-1.651	-1.285

Source: Performed by author in STATA

Table 5.28

PP Test Results, Real Interest Rates Differentials in First Differences

	Test Statistic	1%	5%	10%
Differentials	-16.722	-3.463	-2.881	-2.571

Source: Performed by author in STATA

5.1.5. Nonlinear adjustment and unit roots

Nonlinear adjustment to the RIRP is theoretically justified by market imperfections (Dumas, 1992). Considering that nonlinear dynamics in the real interest rate – generated, for example, by transaction costs – could reduce the power of the conventional unit root tests performed, we apply the procedure to allow for nonlinear adjustment proposed by the KSSUR test proposed by Kapetanios et al. (2003). The results present evidence of nonlinear dynamics in the Brazilian real interest rates. At the 5% level, we can reject the unit root hypothesis in favor of a stationary nonlinear ESTAR model (Table 5.29).

Table 5.29

KSSUR Tests Results, Brazil Real Rates

	Test Statistic	1%	5%	10%
Fixed – 3 lags	-3.699	-3.460	-2.911	-2.634
AIC – 3 lags	-3.699	-3.511	-2.948	-2.664
SIC – 3 lags	-3.699	-3.480	-2.927	-2.647
GTS05 – 3 lags	-3.699	-3.499	-2.940	-2.657
GTS10 – 3 lags	-3.699	-3.508	-2.947	-2.663

Source: Performed by author in STATA

Considering the evidence of nonlinear dynamics in the interest rate time series, we apply the same test to the real interest rate differentials. We highlight that conventional stationarity tests could not reject the unit root hypothesis for those differentials. Similarly, the KSSUR test results don't allow for the rejection of the unit root hypothesis for the differentials (Table 5.30). No evidence of nonlinear adjustment is found in the differentials time series.

Table 5.30

KSSUR Tests Results, Real Interest Rates Differentials

	Test Statistic	1%	5%	10%
Fixed – 1 lag	-2.124	-3.466	-2.919	-2.641
AIC – 0 lag	-2.284	-3.497	-2.940	-2.658
SIC – 0 lag	-2.284	-3.474	-2.926	-2.647
GTS05 – 0 lag	-2.284	-3.490	-2.935	-2.655
GTS10 – 0 lag	-2.284	-3.496	-2.939	-2.657

Source: Performed by author in STATA

5.2. Cointegration Analysis

In subsection 5.1.3 (Structural break tests: differentials), to assess the stationarity of the differentials between Brazilian and American real interest rates in the whole sample, we identified a stochastic trend that was removed by differencing. Alternatively, it is possible that a linear combination of integrated variables is stationary. If that is the case, such variables are said to be cointegrated. Cointegration refers to the fact that two or more series can share a stochastic trend. We will apply cointegration tests to verify the RIRP hypothesis between Brazil and the United States.

Given that the series under comparison are integrated of different orders, as United States real rates are I (0) and Brazil real rates are I (1), it is not appropriate to use two of the main approaches to cointegration: Engle and Granger's two-step residual-based procedure or the Johansen system-based reduced rank regression test. In this case, one suitable cointegration test is the ARDL Bounds test for cointegration proposed by Pesaran et al. (2001). Considering an ARDL model in which Brazilian real rates are the dependent variable, the results indicate that the hypothesis of no level relationship cannot be rejected, i.e., no evidence of cointegration between the time series is found (Table 5.31). Reversing the order of the variables, that is, considering the American rates the dependent variable leads to results that strongly indicate cointegration (United States rates with one lag and Brazil rates with no lag). The cointegration test should be invariant to the variable selected for normalization. Because economic theory postulates that a small open economy´s interest rate is influenced by a center economy and not the opposite, this result must not be considered valid strictly from a macroeconomic perspective. Furthermore, the ARDL bounds testing approach requires that the dependent variable must be I (1) in levels, which is not the case since American real interest rates are found to be stationary.

Table 5.31

ARDL Bounds test, United States and Brazil Real Rates

(Brazil Rates with 2 lags, United States Rates with 0 lag)

	p-value I (0) / I (1)	1% I (0) / I (1)	5% I (0) / I (1)	10% I (0) / I (1)
F = 2.282	0.354 / 0.482	6.952 / 7.916	4.956 / 5.781	4.060 / 4.812
t = -1.218	0.665 / 0.755	-3.447 / -3.836	-2.869 / -3.234	-2.570 / -2.921

Source: Performed by author in STATA

Having observed no evidence of cointegration between both time series in the whole sample, we must introduce the possibility of structural breaks, to reach a more thorough diagnosis of the relationship between Brazilian and American real interest rates. In an effort to proceed with this testing, we use the five break dates estimated in subsection 5.1.3. (Structural break tests: differentials) to create six subperiods and to investigate the existence of cointegration evidence in the subperiods. The cointegration tests for all but one subperiod reach the same result: the absence of a level relationship cannot be rejected. The only exception is subperiod 2009M04-2012M03, in which the hypothesis of no cointegration can be rejected at the 5% level. Although not as clearly as indicated by the unit root tests, there is evidence of the validity of the RIRP between the United States and Brazil. However, the RIRP cannot be confirmed for the whole sample and, in relation to the cointegration test, it can only be pointed in one subperiod, 2009M04-2012M03. We must notice that both the ADF test and the DF-GLS test reveal evidence in favor of the RIRP hypothesis in that subperiod.

CHAPTER 6

Conclusions

The present Dissertation's main goal is to better understand the Brazilian financial integration with the rest of the world. For that, we investigate if Brazilian real interest rates converge to the international benchmark, the United States. We apply the two primary methodologies present in the recent empirical work on the RIRP. At first, we examine the existence of real interest rate parity looking into the stationarity properties of real interest rate differentials with unit root and stationarity tests. Secondly, we explore the existence of comovement between real interest rates using cointegration tests. We use monthly data from the Central Bank of Brazil (Banco Central do Brasil, BCB) and the Federal Reserve Bank of Cleveland, starting from November 2001 since the statistics for the Brazilian market expectations are available from then up to the most recent Focus-Market Readout. The most recent data available for our study was from February 2022.

We find evidence of stationarity in the data regarding the United States real interest rates using conventional unit root tests. For the Brazilian rates, the conventional tests fail to reject the hypothesis of a unit root. One possible explanation for this finding is related to the existence of structural breaks, since the power of unit root tests is affected by structural change, even if the data is stationary. We observe evidence of multiple structural breaks in the Brazilian time series. Firstly, we apply tests for a single break in the data generating process. Assuming a fixed number of breaks, usually a single *ex-ante* determined break, is a significant drawback of similar approaches. For that reason, the research is extended to test for multiple breaks.

About the structural stability issue, after a single structural break is identified for the Brazilian series and the sample divided into two subperiods, the stationarity tests could not reject the hypothesis of a unit root. More tangible signs of stationarity in the Brazilian time series emerge when multiple breaks are considered.

Five break dates are estimated using the Bai and Perron test regarding the real interest rate differentials. The first four subperiods show evidence of stationarity in the differentials, especially using the ADF test with drift. Based on the results from three tests (ADF, PP, and DF-GLS), we observe that the two subperiods with no evidence of stationarity (2016M02-2019M01 and 2019M02-2022M02) are characterized by two critical shocks: one domestic and the other international. The first was the domestic economic and political turmoil of 2015-2016 that resulted in President Dilma Rousseff´s impeachment. The second was the Covid-19 worldwide pandemic. The two shocks can explain the lack of stability of coefficients during both subperiods. It is noteworthy that the global economic shock of the Great Recession from 2007 to 2009 occurred during subperiods with evidence of stationarity.

Furthermore, we must notice that structural breaks may be smooth. When dummy variables are used in structural break tests, implicitly we assume that the break fully manifests itself at a specific date. Therefore, the study of smooth breaks presents an area for further research.

Evidence of nonlinear dynamics in the Brazilian real interest rate is found. A nonlinear adjustment process could be caused by several frictions, such as transaction costs in the Brazilian market. However, the unit root hypothesis could not be rejected when the same test for nonlinear dynamics was applied to the real interest rate differentials.

Regarding the influence of structural breaks and nonlinear dynamics on the stationarity tests, we treat both problems separately. However, they could be treated jointly, as demonstrated by Christopoulos & León-Ledesma (2010). This aspect constitutes another suggestion for future research.

In relation to the possibility of cointegration between American and Brazilian real interest rates, no evidence of cointegration between both time series is found in the sample considered as a whole, that is, from 2001 to 2022. We apply structural breaks to the analysis to reach a more thorough diagnosis of the relationship between Brazilian and American real interest rates. We use the same five break dates estimated previously for the stationarity investigation to create six subperiods and to examine the presence of cointegration in the subperiods. The absence of a level relationship could not be rejected in all but one subperiod. The only exception is subperiod 2009M04-2012M03, in which the hypothesis of no cointegration could be rejected at the 5% level.

In summary, based on the stationarity of real interest rate differentials, the findings put in evidence the RIRP hypothesis and, consequently, a high degree of market integration between Brazil and the United States. On the other hand, the cointegration tests were not able to indicate a level relationship between American and Brazilian real interest rates. Our findings concerning the stationarity of the differentials are consistent with the observed financial liberalization and the emergence of global capital markets. More specifically, we conclude that the stationarity of real interest rate differentials must be evaluated under the prism of multiple structural breaks. Furthermore, the results imply that external factors highly influence Brazilian monetary policy, specifically those originated in the American economy. Nevertheless, the rejection of the non-stationarity hypothesis does not mean the Brazilian monetary authority is unable to independently influence its domestic financial market. Depending on the degree of persistence of the real interest rate differentials, it could still be possible to implement an independent and effective domestic monetary policy in Brazil. Indeed, a deeper examination of the persistence and the mean reversion speed of those differentials should also be addressed in future research.

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