On the Design of Spreading Sequences for CDMA Systems with Nonlinear OQPSK-type Modulations

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Abstract—**The main focus of this paper is to present an analytical method for calculate the correlation between OQPSK (Offset Quadrature Phase-Shift Keying) signals modulated by several spreading sequence families. Our analytical model includes nonlinearities, namely by treating the transmitter amplifier as highly nonlinear. Grossly nonlinear power amplification is highly desirable as it allows power-efficient low-cost receivers, with applications in satellite, underwater or deep space communications. OQPSK-type modulations include as special cases CPM (Continuous Phase Modulation) schemes and can be designed to have low envelope fluctuations or even a constant envelope, essential in many communications systems (e.g., satellite). This model and the derived analytical results for the correlation of sequences in such a nonlinear environment are then applied to a DS-CDMA (Direct Sequence – Code Division Multiple Access) system. We present simulation results for three types of spreading sequences: ML (Maximum-Length), Kasami and TCH (Tomlinson, Cercas, Hughes) sequences. The simulation results are presented to compare the performance of those types of sequences in the linear and nonlinear DS-CDMA systems.**

Keywords – Nonlinear systems, spreading sequences, power-efficient communications, correlation

I. Introduction

There are many systems that are inherently limited by the power source, so their efficiency must be maximized, although there is a current trend towards generalized green communications. The most critical ones, such as satellite, underwater, deep space communications or simply mobile terminals, should not only use efficient modulations, but also grossly nonlinear amplifiers, as these can help save a huge amount of power.

Since most results known until recently tend to consider systems as linear as possible, namely the transmitter amplifier, the correlation of spreading sequences after modulation in a nonlinear system is not known, although nonlinear distortions caused by the amplifier are sometimes considered, although isolated from the other mentioned factors. Furthermore, it is possible to improve the spectral efficiency and to reduce the impact of nonlinearities by considering others types of modulations, instead of the traditional and conventional PSK (Phase-Shift Keying).

As referred, an obvious nonlinear component of the transmitting chain is the amplifier. In this study, our model assumes it as definitely nonlinear. In fact, the use of nonlinear amplifiers reduces the complexity and cost of systems and it also increases their efficiency and autonomy, as for example on mobile terminals which depend on batteries. These amplifiers are less complex, simpler to implement and have higher amplification efficiency and output power than linear amplifiers, which is important in most communications systems, such as wireless communications. However, non-constant envelope signals should not be amplified by grossly nonlinear amplifiers in order to avoid nonlinear distortion effects. CPM (Continuous Phase Modulation) schemes [1] are constant or almost-constant envelope modulations that include as special cases MSK (Minimum Shift Keying) [2] and GMSK (Gaussian MSK) [3] modulations, among others. These modulations provide high power and spectral efficiency. They are OQPSK-type modulations as they can be decomposed as the sum of OQPSK components [4, 5] keeping their OQPSK-type structure when submitted to band pass memory-less nonlinear devices.

Pseudorandom sequences are widely used in many communications systems, e.g., satellite systems, whose performance is dictated by their correlation behavior. For example, to keep cross-correlation as small as possible in most system applications, many families of pseudorandom sequences have been investigated, so as to improve the performance of the communication system under study. However, the majority of these studies usually assume ideal conditions that can be easily modeled and studied, such as linear modulations and ideal linear transmitters (i.e., with linear amplifiers), which do not exist in the real world. For this purpose, we have analytically derived the real correlation of several types of sequences after being processed by a nonlinear system. We then extend and apply these expressions to a DS-CDMA (Direct Sequence – Code Division Multiple

Access) model and finally we use Monte Carlo method to simulate the system for both linear and nonlinear conditions, comparing the performance of ML (Maximum-length) [6], Kasami [6] and TCH (Tomlinson, Cercas, Hughes) [7] sequences and evaluating their BER (Bit Error Rate) performance in a CDMA nonlinear system, as previously described.

II. PARALLEL AND SERIAL OQPSK SCHEMES

OQPSK-type modulations can be represented in both serial and parallel formats. These types of modulations are derived from QPSK (Quaternary Phase-Shift Keying) where the band pass signal is $x_{BP}(t) = \mathrm{Re}\left\{x_p(t)e^{2j\pi f_c t}\right\}$ [8], f_c is the carrier frequency and $x_p(t)$ the complex envelope given by

$$x_p(t) = \sum_n a_n r_p(t - 2nT) \tag{1}$$

where $a_n = a_n^I + ja_n^Q$ with $a_n^I = \pm 1$ and $a_n^Q = \pm 1$ represent the 'in-phase' and 'quadrature' bits and $r_p(t)$ is the adopted pulse shape, where T is the bit duration. QPSK symbols are separated by $2T$ because we are transmitting 2 bits per symbol.

Let us now consider the transmission of the same band pass signal but now assuming OQPSK-type schemes. This modulation is also based on the transmission of two pulses of $2T$ time duration but with a delay T between them. The complex envelope of the signal represented in parallel format is given by [5]

$$x_p(t) = \sum_n a_n^I r_p(t - 2nT) + \sum_n a_n^Q r_p(t - 2nT - T) \tag{2}$$

A compact way to represent the same signal, is given by

$$x_p(t) = \sum_n a_n^p r_p(t - nT) \tag{3}$$

where a_n^p corresponds alternately to the "in-phase" and "quadrature" bits, that is,

$$a_n^p = \begin{cases} a_{n/2}^I = \pm 1 & ,n \ even \\ ja_{(n+1)/2}^Q = \pm j & ,n \ odd \end{cases} \tag{4}$$

To obtain the serial format, we consider a carrier frequency $f_c' = f_c + 1/4T$. In this case the band pass signal is $x_{BP}(t) = \mathrm{Re}\left\{x_s(t)e^{2j\pi f_c' t}\right\}$ where

$$x_s(t) = x_p(t)e^{-j\pi\frac{t}{2T}} \tag{5}$$

substituting $x_p(t)$ according to (3), gives

$$x_s(t) = \left[\left(\sum_n a_n^p \delta(t - nT)\right) * r_P(t)\right] e^{-j\pi\frac{t}{2T}} \tag{6}$$

The order of the translation and convolution operations can be changed enabling the following mathematical deduction

$$\begin{aligned} x_s(t) &= \left[\left(\sum_n a_n^p \delta(t-nT)\right) e^{-j\pi\frac{t}{2T}}\right] * \left[r_p(t) e^{-j\pi\frac{t}{2T}}\right] = \\ &= \sum_n a_n^p e^{-j\pi\frac{t}{2T}} r_p(t-nT) e^{-j\pi\frac{t}{2T}} \end{aligned} \tag{7}$$

The complex envelope of OQPSK signal in the serial format can then be written as

$$x_s(t) = \sum_n a_n^s r_s(t - 2nT) \tag{8}$$

and, using equation (7), we can conclude that

$$r_s(t) = r_p(t)e^{-j\pi\frac{t}{2T}}, \tag{9}$$

$$a_n^s = a_n^p e^{-j\pi\frac{n}{2}} \tag{10}$$

which means that, a_n^s always presents a real value $a_n^s = \pm 1$, enabling the use of sequences with only real values because only a real sequence of symbol coefficients is needed, rather than sequences with *real* and *imaginary* values, which are used in the parallel format. This serial format also allows both the serial modulator and demodulator to have a single-branch structure [5].

III. NONLINEAR OQPSK MODULATIONS

Let us now consider the amplification of OQPSK signals over nonlinear band pass systems, using a nonlinear band pass memoryless amplifier. The signal at the amplifier input is expressed by

$$x_{in}(t) = \mathrm{R}e^{j\arg(x_{in}(t))} \tag{11}$$

where $R = |x_{in}(t)|$ is the envelope. At the output, the complex envelope is given by [9]

$$x_{out}(t) = A(R)e^{j(\Theta(R)+\arg(x_{in}(t)))} \tag{12}$$

where $A(R)$ and $\Theta(R)$ denote the AM-to-AM and AM-to-PM conversion functions, respectively, of the nonlinear amplifier. We modeled the amplifier as an ideal BPHL (Band-Pass Hard Limiter) where $A(R) = 1$ and $\Theta(R) = 0$, which is the worst case scenario for nonlinear distortion. Thus, the complex envelope at the output of the nonlinear amplifier can be simplified to

$$x_{out}(t) = e^{j(\arg(x_{in}(t)))} \tag{13}$$

Considering now the following signal at the input of the nonlinear device

$$x_{in}(t) = \sum_n a_n r(t - nT) \tag{14}$$

where $r(t)$ is the pulse of the used modulation. Since we are assuming a serial representation of the OQPSK signal, the a_n values only take real values, that is, $a_n = \pm 1$.

If the OQPSK signal does not have constant envelope, extra-components will appear after nonlinear amplification, but interestingly, the resulting nonlinear signal can be decomposed as the sum of OQPSK components. Thus, the signal at the output of the nonlinear device is given by [5]

$$x_{out}(t) = \sum_{m=0}^{M-1} \sum_n a_n^{(m)} r^{(m)}(t - nT) \tag{15}$$

where the number of components after amplification, is quantified by variable M [5]. In this study we only consider the four most important components, i.e., $M = 4$. Figure 1, shows variables $r^{(0)}(t)$, $r^{(1)}(t)$, $r^{(2)}(t)$ and $r^{(3)}(t)$ associated with the resulting components from the nonlinear amplification of the pulse $r(t)$.

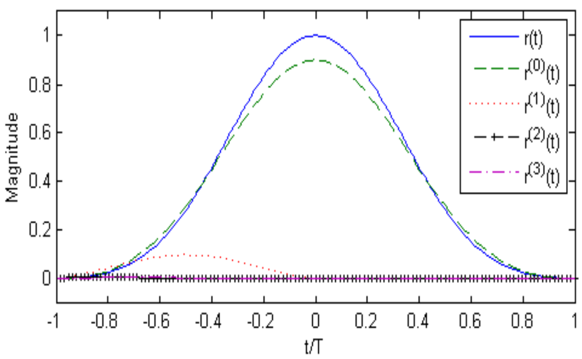

Figure 1 - Pulse shapes $r(t)$, $r^{(0)}(t)$, $r^{(1)}(t)$, $r^{(2)}(t)$ and $r^{(3)}(t)$.

IV. SIMULATION MODEL

In this paper we consider the downlink transmission in a DS-CDMA system in which a base station (BS) simultaneously transmits data blocks for P users. Each bit of the data blocks, will be spread considering three types of spreading sequences. In the receiver we dispread the incoming signal, employing the same spreading sequences, to compare their performance.

A. Transmitter and Receiver models

To simulate the system, we assume a simple model for the transmitter of a BS, where data is spread considering ML, Kasami and TCH sequences, modulated by MSK and GMSK, followed by a nonlinear amplifier as shown in Figure 2.

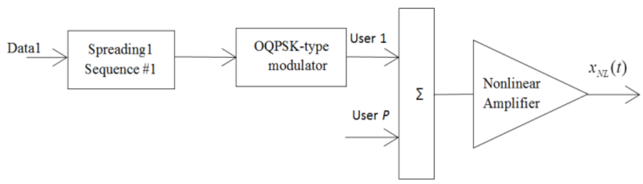

Figure 2 - Block diagram for the DS-CDMA transmitter.

For a GMSK pulse, the transmitted signal at the output of the BS is given by

$$x_{NL}(t) = \sum_{p=0}^{P-1} \sum_{m=0}^{M-1} \sum_n b^p a_n^{(m,p)} r^{(m,p)}(t - nT) \tag{16}$$

where b is the information bit of user p.

To recover the transmitted bits of user p we must correlate the received signal, containing data spread by sequences from all other users, nonlinear effects and noise, with the same sequence used at the transmitter. We assume an AWGN (Additive White Gaussian Noise) channel represented by $n(t)$. We consider the receiver structure shown in Figure 3.

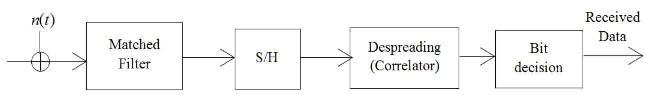

Figure 3 – Block diagram for the receiver.

Since the main operation of the receiver is the cross-correlation of sequences in this nonlinear environment, we now present an analytical method to evaluate the correlation of sequences using nonlinear OQPSK-type modulations, applied to DS-CDMA systems. This allows us to compare and to determine the behavior and characteristics of pseudo-random sequences in this non-linear multi-user system.

B. Correlation of binary sequences

Let us now consider the DS-CDMA signal at the output of the transmitter, shown in Figure 2

$$x_{NL}(t) = \sum_{p'=0}^{P-1} \sum_{m'=0}^{M-1} \sum_{n'} b^{p'} a_{n'}^{(m',p')} r^{(m',p')}(t - nT) \qquad (17)$$

and the nonlinear signal from user p

$$x(t) = \sum_{m=0}^{M-1} \sum_{n} a_n^{(m,p)} r^{(m,p)}(t - nT) \qquad (18)$$

the correlation between these signals is given by

$$R_x(\tau) = \int_0^{NT} x_{NL}(t) x^*(t-\tau) dt \qquad (19)$$

clearly,

$$R_x(\tau) = \sum_{p'}^{P-1} b_{p'} \left(\sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \sum_{n} R_{mm'p'}(n) p_{mm'p'}(t - (n'-n)T - (\tau_{p'} - \tau_p)) \right) \qquad (20)$$

where

$$R_{mm'p'}(n) = \sum_{n'=0}^{N-1} a_{n'-n}^{(m',p')} a_{n'}^{(m,p)*} \qquad (21)$$

and

$$p_{mm'p'}(t) = r^{(m',p')}(-t) r^{(m,p)*}(t) \qquad (22)$$

In our model we assume that the spreading sequences for the interfering users are randomly delayed, so $a_{n,p'} = a_{n-\Delta N,p}$ where ΔN represents a random variable uniformly distributed between zero and the length of the employed spreading sequences. We then conclude that,

$$R_x(\tau) = \sum_{p'=0}^{P-1} b_{p'} \left(\sum_{m=0}^{M-1} \sum_{m'=0}^{M-1} \sum_{n} R_{mm'p'}(n) p_{mm'p'}(t - (n'-n)T) \right) \qquad (23)$$

If the pulse type used, e.g., a MSK pulse, complies with Nyquist conditions, i.e., $p(kT) = 0, k \neq 0$, then this correlation is given by

$$R_x(\tau) = \sum_{p'=0}^{P-1} b_{p'} \sum_{n=0}^{SF-1} a_{n',p'} a_{n,p}^* \qquad (24)$$

V. Performance Results

In this section we present the performance results for each pseudorandom sequence type together with the effects of nonlinearity, by comparing the obtained results for linear and non-linear systems. Figure 4 presents the autocorrelation of each sequence after nonlinear modulation and amplification, evaluated according to our analytical expressions.

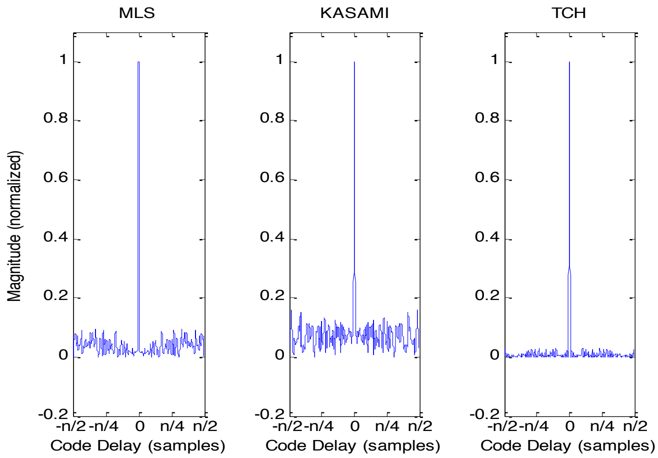

Figure 4 - Autocorrelation function of all OQPSK components for each pseudorandom sequence.

As we can see, TCH sequences present the best performance on its autocorrelation in this nonlinear scenario. In order to evaluate the performance of ML, Kasami and TCH sequences in a DS-CDMA system, for both linear and nonlinear conditions, we used Monte Carlo simulation.

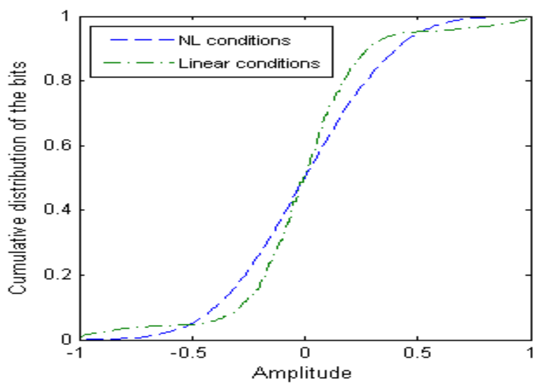

Figure 5- Cumulative distribution of the received bits in linear and NL conditions.

Figure 5 presents the cumulative distribution of the received bits for both systems, using the same parameters. As expected, in the nonlinear system the dispersion is higher and the majority of the received bits in the linear system are closer to the right decision value. Figure 6 shows and compares the results obtained for ML, Kasami and TCH sequences in both systems.

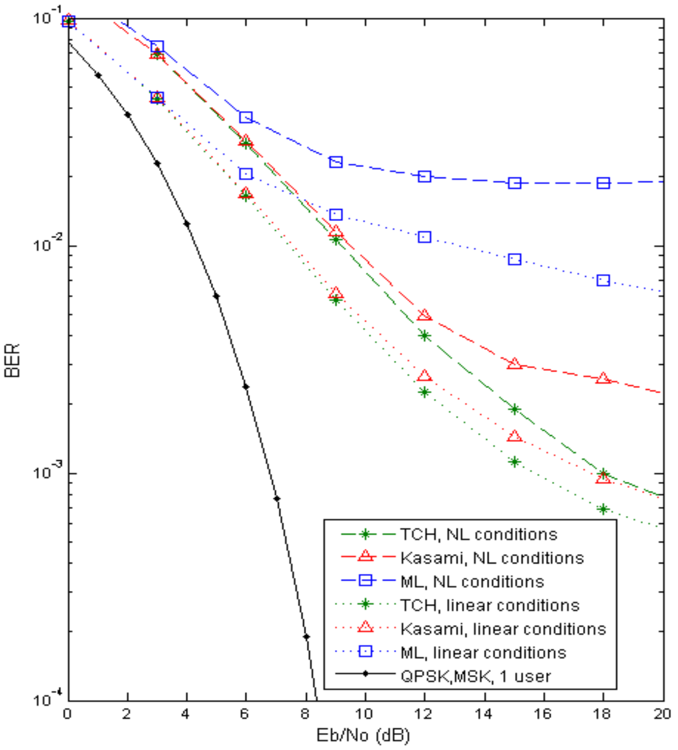

Figure 6 - Performance of sequences in a DS-CDMA system.

In this study we considered sequences length 256, that is, a spreading factor of $SF = 256$, for which we tried to maximize the system's capacity. We also assumed a value for the BER of $BER = 10^{-3}$, which is usually taken as the minimum value for the QoS (Quality Of Service) in many applications. As previously mentioned, and to get a more realistic scenario, we considered random delays for interfering users. We found that TCH sequences allow the maximum capacity in both linear and nonlinear systems, which is about 25 simultaneous users. It is clear that MAI (Multiple Access Interference) and nonlinearities degrade the system's performance as we should expect for all types of sequences. ML sequences have the worst behavior, which is due to their bad cross-correlation properties. On the other hand, we can see that TCH sequences outperform the other sequence types considered. This behavior is consistent for both linear and nonlinear situations.

VI. CONCLUSIONS

In this paper we presented the performance of ML, TCH and Kasami sequences for a system modeled as the downlink of a DS-CDMA system using OQPSK-type modulations and highly nonlinear amplification, in a AWGN channel. The model used is based on the analytical evaluation of correlation for these nonlinear systems, which was derived, followed by Monte Carlo simulation, to evaluate the system's performance and capacity at BER=10^{-3}. Both linear and nonlinear systems were simulated. The results have shown that TCH sequences present better performance on both systems, allowing a capacity of 25 simultaneous users in those highly nonlinear conditions.

ACKNOWLEDGEMENTS

This work was partially supported by FCT/ Instituto de Telecomunicações projects OE/EEI/LA0008/2011 and PTDC/EEA-TEL/120666/2010.

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