Quantitative Measures of Crowd Patterns in Agent-Based Models of Street Protests

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***Abstract*— In this work we describe the introduction of quantitative measures of emergent crowd patterns in an existing Agent-Based model (ABM) of street protests with multiple actors (police, protester and 'media' agents). The model was applied to a scenario of a police force defending a government building which protesters seek to invade. The improved model provided a coherent 'narrative' of the simulations and highlighted the realistic and unrealistic aspects of the agents' interactions. Two new types of police agents – 'defensive' and 'offensive' – were introduced, leading to a realistic model representation of police cordons defending a site or charging to disperse clusters of violent protesters. The new quantitative measures provided information on cluster size and orientation of clusters of violent protesters, as well as police coverage and protester breaching of the defensive perimeter, together with the time history of the bursts of localized fights and arrests. It was shown how the quantitative measures of the emergent properties can be used for both parameterization and validation of the model.**

Keywords – Agent-Based model; protests; violence; clustering; emergent properties

I. INTRODUCTION

Protest demonstrations are a means by which citizens press established authorities for change. Sometimes, protesters demand for regime change (e.g. the "Arab Spring" uprising) whereas in other cases the motif is a specific issue that triggers action because of existing social tension and conflict (e.g. protests in Brazil in 2014 against the FIFA World Cup or the unrests in Ferguson and other U. S. cities after the shooting of Michael Brown).

The study of protest demonstrations is an important problem in sociology, social psychology and political science, as well as in social simulation and complex systems studies. However, this study is very difficult because of the multiplicity of actors, interactions and scales involved. Thus, existing ABM of conflict and violence are usually centered on specific scales, mechanisms and emergent phenomena, and can be broadly classified as 'abstract', 'middle-range' or 'facsimile' models [1]. 'Abstract' models provide macroscopic level descriptions of emergent phenomena such as bursts of rebellion against a central authority [2], the influence of Information and Communications Technology (ICT) on revolution [3], or large scale ethnic violence [2]. At this level, the key aspects are the choice of rules and social context variables, and the feedback of emergent properties and information diffusion (e.g. media coverage) on those variables [4], [5]. 'Middle-range' models attempt to describe the dynamics of conflict phenomena with specific space and time scales at a 'mesoscopic' level, such as the London riots of 2011 [6]. Finally, 'facsimile' models attempt to describe the dynamics of avoid-approach, clustering, confrontation and reaction to events (e.g. fires or explosions) at small scales [7], [8]. In 'middle-range' and 'facsimile' models, the key questions are related to keeping the model as simple as possible, otherwise parameterization and validation (and consequently realism and usefulness) of the model may be lost.

Recently, we proposed an ABM of street protests and violent confrontation for simulating the interaction between protesters and police forces, together with 'Media' agents to represent the effects of news coverage. The scenario includes features such as attraction points, obstacles, entrances and exits [9]. Following the approach in [7] we considered three types of protesters – 'hardcore', 'hanger-on' and 'passer-by' – with different behaviors. Also, protesters can be in four states, 'quiet', 'active', 'violent', and 'fighting'. Police agents try to defend specific areas in the protest space, engage violent protesters, and arrest them if they have sufficient backup (local superiority).

In this model, all agents are reactive and have one 'move' and one 'behave' rule. The agents' movement is multi-goal driven, as they may approach or avoid other agents and spatial features, depending on their percept and on the weights of a 'personality vector' which encodes their tendencies or 'motivations'. The weights of the personality vector may change depending on the percept, so that the agents' architecture, although simple, can represent a rich variety of interaction behaviors. The protesters' change of state is modeled using two variants of Epstein's threshold rule [2], which embodies the effects of individual grievance (which acts as motivation for action) and risk assessment depending on the balance between local support (number of violent or active agents within the vision cone) and deterrence (number of police agents within the vision cone).

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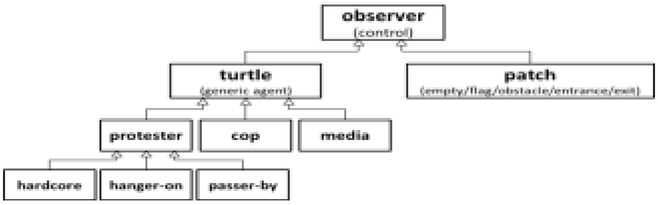

Figure 1. Simplified class diagram for model entities of the ABM of street protests (see [9] for a full description).

The ABM of street protests was implemented in NetLogo [10] and is fully described in [9], using the ODD protocol [11]. The model's simplified class diagram is shown in Figure 1 (see the Appendix of [9] for a full description).

In [9] we describe the application of the model to a scenario of a police force protecting the entrance of a government building which protesters seek to occupy. The model reproduced several crowd patterns often found in such events, such as clustering of 'violent' and 'active' protesters, formation of a confrontation line with localized fights and occasional arrests, 'media' agents moving near the 'hot spots' and trying to register episodes of violence, and 'passer-by' protesters wandering in the protest space trying to avoid the confrontation area. This simulated scenario can also be interpreted as a 'game' between protesters and police, in which the policeman must protect the perimeter and simultaneously engage 'violent' protesters with local superiority, and the latter try to breach the perimeter, engage police agents and avoid being arrested. This leads to the following questions: *i*) Which emergent properties are important in this 'game' (i.e. that clarify who is the 'winner')? and *ii*) How can these properties be quantified, integrated in a model interface, and used to better program the agents' behavior?

In this work we describe the implementation of new quantitative measures of crowd patterns and relevant variables for expressing the intensity of the protests and the relative 'success' of the police force and protesters in achieving their respective goals. These measures were integrated in a 'dashboard' on the NetLogo interface tab that provides a precise and rich 'narrative' of the system's behavior during a simulation. The improved interface was used to evaluate the performance of two new types of 'cop' agents, 'defensive' and 'offensive'.

The rest of this paper is organized as follows. In section II, we describe the new quantitative measures and describe their implementation in the ABM proposed in [9]. In section III we describe the two new types of police agents introduced in the model. In section IV, we present the results of simulations of the scenario considered in [9] with 'multi-role' (or 'default'), 'defensive' and 'offensive' cops. Section V contains a discussion of the results, and section VI a summary of conclusions.

II. Quantitative Measures of Emergent Properties

We considered four classes of emergent properties: *i*) crowd patterns; *ii*) protest intensity and collective behavior (mimetic effects); *iii*) potential 'news' impact; and *iv*) police vs. protesters (outcome of the 'game'). For each of these for classes we selected specific quantities to be computed at each time step.

TABLE I. Quantitative Measures of the Emergent Properties

Emergent properties	Quantitative measures	Observations
Crowd patterns	Cluster size histogram, 'violent' and 'active' protesters	New
	Degree of clustering of 'violent' protesters (% large cluster)	New
	Cluster orientation of largest cluster of 'violent' protesters	New
Police vs. Protesters	# 'violent' protesters on defensive perimeter # 'active' protesters on defensive perimeter	New
	% defensive perimeter covered by policemen	New
Protest intensity & collective behavior (mimetic effects)	# 'quiet' protesters # 'active' protesters # 'violent' protesters # 'fighting' protesters	Implemented in [9]
Potential 'news' impact	# records of fights taken by 'media' agents	Implemented in [9]

Table I summarizes the quantitative measures of emergent properties implemented in the current version of the model. In [9] we described the implementation of the quantitative measures of protest intensity and potential 'news' impact, which is relatively simple. The quantitative measures of the 'success' of the police agents can also be programmed in a straightforward manner using NetLogo primitives, but the implementation of the measures of crowd patterns is more involved. In what follows, we briefly describe our implementation of these latter in NetLogo procedures.

A. Cluster detection

We define a cluster of a given agent set as a subset of agents such that the union of their Moore neighborhoods of radius equal to the move range is a connected patch set. To identify all clusters we use a recursive depth-first search algorithm on the Moore neighborhood of a randomly chosen starting agent in the agent set, which is a variant of the depth-first search algorithm for finding the connected components in a network (with the links replaced by Moore neighborhood proximity). This is implemented using two NetLogo procedures, `find-clusters` which initializes the process, and `explore` which implements the recursive search. The output is a list of agent clusters, each cluster being a list of agents. From the list of clusters, it is possible to produce a histogram of the cluster sizes and orientations.

B. Determining clusters' orientation

In our model, we determine the spatial orientation of a cluster by computing the inertia tensor and the principal axes of inertia from the agents' coordinates (we consider that all agents have the same mass). The three independent components of the inertia tensor are obtained by first computing the coordinates of the barycenter of the cluster and then the moments of inertia I_x and I_y with respect to the x and y axes and the product of inertia P_{xy}. From the moments and product of inertia, the cluster orientation is obtained by finding the rotation angle that makes the product of inertia zero and minimizes the moment of inertia with respect to the rotated axes. The whole process is implemented using the three Netlogo reporters, `cg` for finding the barycenter of an agentset, `inertia` for finding the components of the 2D inertia tensor and `orientation`, which determines the orientation of the principal axes of inertia. If the absolute value of the difference between the two moments of inertia or the product of inertia are smaller than a specified tiny value (which we arbitrarily set to 10^{-5}), the orientation angle is set to the average heading of the cluster. In the 'dashboard' we only included a histogram of the orientation for clusters of 'violent' protesters with more than ten agents. This is because the spatial orientation of small clusters is strongly variable and erratic. For large clusters, the histogram gives useful indications on the influence of the configuration of the protest space on the evolution of 'violent' groups.

III. USING MEASURES TO IMPROVE THE MODEL: IMPLEMENTING 'DEFENSIVE' AND 'OFFENSIVE' COPS

A. Agents' movement and behavior

In our ABM agents are reactive and have one 'behave' and one 'move' rule. 'Hardcore' protesters have the highest propensity for turning 'violent', trying to cluster, occupying attraction points and engaging cops. 'Hanger-on' protesters correspond to 'susceptible' protesters in a crowd with moderate incentive to turn 'active' and 'violent' and approach other 'active'/ 'violent' protesters, whereas 'passer-by' protesters mostly remain 'quiet' and try to avoid 'hot spots'. 'Cops' try to defend attraction points, keep close to comrades to avoid numerical inferiority, engage 'violent' protesters and arrest them if they have enough local superiority. 'Media' agents are attracted to sites where police agents and 'violent' protesters are confronting, and record ("take pictures") of episodes of violence.

Purposeful goal-driven movement is obtained by defining a 'personality vector' whose components encode the tendencies or 'motivations' to approach/avoid spatial features and other agents within the agent's vision cone [12], [9]. The personality vector is updated by setting its components equal to a pre-defined 'default personality' (different for each agent type/subtype) and then applying 'context-rules' that change the components depending on the agent's percept.

B. 'Default', 'defensive' and 'offensive' cops

Table II shows the description of the components of the personality vector and their values for the 'multi-role' (or 'default') cop agents introduced in [9] and two new subtypes of 'cop' agents, 'defensive' and 'offensive'. The settings for the other agent types and subtypes can be found in [9].

TABLE II. COMPONENTS OF THE AGENTS' PERSONALITY VECTOR

Feature	'violent' protesters	'active' protesters	'quiet' protesters	cop	'Media'	flag (attraction point)	obstacle	Exit
Component	ω_0	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
'multi-role' cop	2	½	½	5	0	5	½	0
'defensive' cop	0	0	0	5	0	5	5	0
'offensive' cop	4	1	1	5	0	0	0	0

An agent in position (x_0,y_0) with personality vector $\boldsymbol{\omega}_A$ moves with a probability p = 0.9 to the empty patch within its move range that minimizes the penalty function $\boldsymbol{\omega}_A/\|\boldsymbol{\omega}_A\|_1 \cdot (\mathbf{S}_A(x,y)-\mathbf{S}_A(x_0,y_0))$, where $\mathbf{S}_A$ is a vector whose components are the sum of distances from point (x,y) to each visible element in the feature set; and to a random patch within its move range with probability $1 - p = 0.1$. The weights range from +5 (strong attraction) to –5 (strong repulsion), a weight equal to zero meaning that the agent is insensitive to the feature. Table II shows that by default the interactions are all attractive (positive sign) among all subtypes of 'cops', and that 'defensive' and 'multi-role' cop agents are strongly attracted to patches they must protect (flags). 'Defensive' cops keep 'on station' and close to other cops but by default do not engage protesters. 'Offensive' cops are strongly attracted towards 'violent' protesters and slightly attracted to 'active' and 'quiet' protesters whereas 'default' cops have mixed behavior.

The personality vector changes depending on the agent's percept via context rules. 'Default' and 'offensive' cops have four context rules: *i*) 'on station' which sets flag attraction to zero when the agent is already on a flagged patch; *ii*) 'pursuit' for pursuing 'violent' protesters when in advantage; *iii*) 'avoid/retreat' to avoid 'violent' protesters when outnumbered and alone; and *iv*) 'support' to help outnumbered comrades in 'hot spots'. 'Defensive' cops have the following context rules: *i*) 'on station' with the same meaning as before; *ii*) 'engage trespassing protesters' which sets attractive weights for protesters when they are trying to breach the defensive perimeter (strongest for 'violent' protesters); and *iii*) 'support' with the same meaning as before.

IV. RESULTS

To illustrate the use of the quantitative measures of the emergent properties to improve the model, we performed simulations for the scenario of a police force defending the entrance of a government building from protesters reported in [9] using 'multi-role' ('default'), 'defensive' and 'offensive' cops, and analyzed their relative performance.

TABLE III. SIMULATION PARAMETERS (ALL SIMULATIONS)

Parameter	Value	Description
min-pxcor	0	minimum x-coordinate
max-pxcor	149	maximum x-coordinate
min-pycor	0	minimum y-coordinate
max-pycor	37	maximum y-coordinate
initial-num-cops	100	initial number of cops
initial-num-hardcores	50	initial number of 'hardcore' protesters
initial-num-hangers-on	750	initial number of 'hanger-on' protesters
initial-num-passers-by	200	initial number of 'passer-by' protesters
num-media	4	number of "Media" agents
max-steps	360	duration of the simulation (ticks)
vision-radius	15	vision radius (in patches)
vision-angle	185	vision angle (in degrees)
move-radius	1	agents' move radius (in patches)
fight-duration	10	fight duration (ticks)
population-threshold*	0.1	threshold for state transition (protesters)
government-legitimacy*	0.82	government legitimacy

*used to determine state transitions in Epstein's threshold rule [2], [9].

Table III shows a summary of the input parameters common to all simulations (see [9] for a more detailed description).

Figure 2 shows the quantitative measures of protest intensity and potential 'news' impact, police coverage and protesters' breaching of the defensive perimeter, and percentage of the largest clusters of 'violent' and 'active' protesters, for a simulation with 'multi-role' cops.

The graph on top of figure 2 shows that fights occurred in bursts. All fights were 'covered' by 'media' agents (sometimes by all four 'media' agents), and 'violent' protesters were progressively arrested (34 arrests and 180 'pictures taken' at the end of the simulation).

The graph in the middle shows that there was occasional breaching of the defensive perimeter by 'violent' protesters and significant breaching by 'active' protesters. This was due to poor coverage of the defensive area (79% average), to the insufficient interaction between cops and 'active' protesters.

The graph on the bottom shows that 'violent' protesters clustered initially (owing to their initial placement and to their 'default personality' and context rules [9]) but the largest cluster shrank as they were engaged by the 'cops', because of dispersion and arrests. In contrast, 'active' protesters clustered progressively and because they were not engaged by the 'cops' they formed a large cluster (88% of all 'active' protesters in the simulation space) at the end of the simulation. It is interesting to note the irregular oscillations of the percentage of the largest cluster, especially for the case of 'active' protesters.

Figure 3 shows a snapshot of the simulation space. It can be observed that cops tended to form a wedge and engaged 'violent' protesters, but they left the flanks of the defensive perimeter relatively unprotected, which created opportunities for breaching. At this time step (tick) 'active' protesters were concentrated in two separate clusters and the percentage of their largest cluster was only 37%. 'Violent' protesters formed a confrontation line with the 'cops' on the left flank of the defensive perimeter, with localized fights and 'media' agents well placed to 'take pictures'.

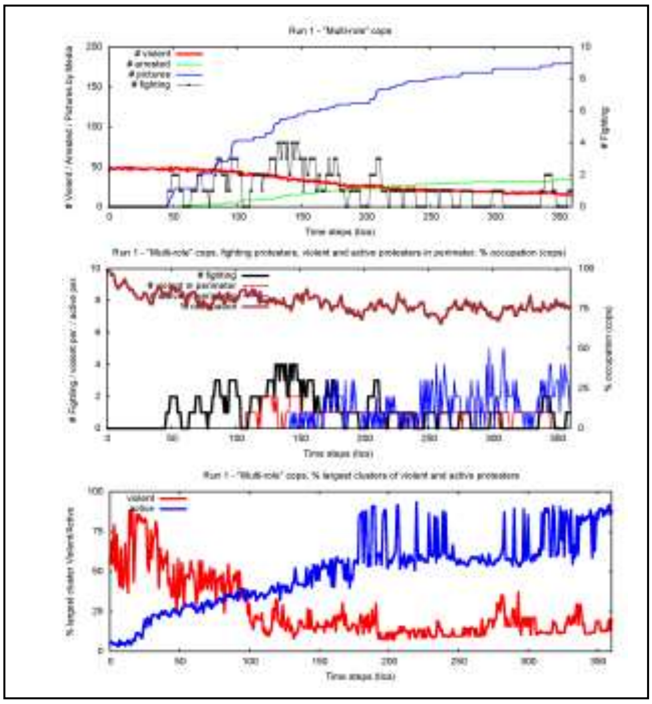

Figure 2. Time history of the quantitative measures of protest intensity and mediatic impact (top), police coverage and protesters' breaching of the defensive perimeter (middle), and percentage of the largest cluser of 'violent' and 'active' protesters (bottom), for 'multi-role' cops.

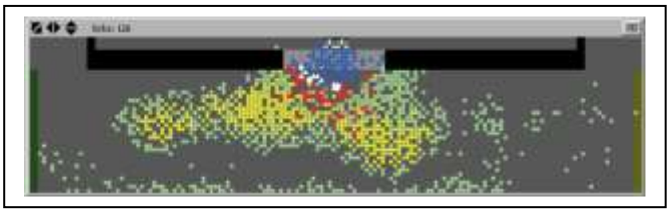

Figure 3. Snapshot of the simulation space 'multi-role' cops protecting a government building (scenario as in [9]), for $t = 126$. Blue triangles are 'cops'; red circles are 'violent' protesters; yellow circles are 'active' protesters; green circles are 'quiet' protesters; little human figures are 'media' agents and fighting agents (protesters and cops) are represented in white. The defensive perimeter in front of the government building is represented in light gray, and black patches represent obstacles.

Figure 4 shows the information corresponding to figure 2 for the case of 'defensive' cops. Fighting occurred in bursts that started later than in the previous case owing to the 'defensive' cops sticking to the defensive perimeter. The number of arrests at the end of the simulation was smaller than in the previous case, and so was the breaching of the perimeter because the perimeter coverage was much better (94% on average). This was due to increasing the attraction weight for obstacle cells in their 'default personality' (avoiding discontinuity of their spatial attraction 'motivation' at the flanks), refraining from engaging protesters outside the defensive perimeter (table II) and context rules to 'keep station' and 'engage trespassing protesters'. The largest cluster of 'active' protesters was significantly smaller in this case, because the crowd of 'active' protesters was split into two large clusters.

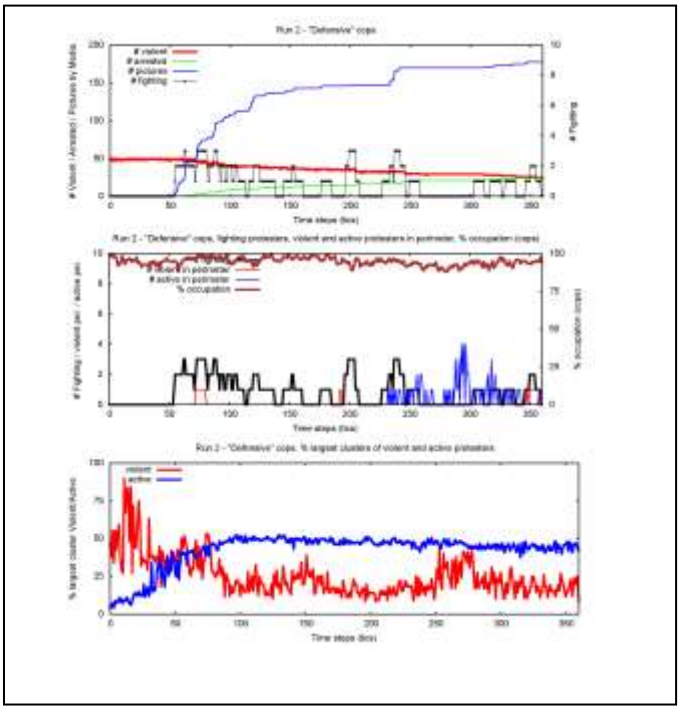

Figure 4. Time history of the quantitative measures of protest intensity and mediatic impact (top), police coverage and protesters' breaching of the defensive perimeter (middle), and percentage of the largest cluser of 'violent' and 'active' protesters (bottom), for 'defensive' cops.

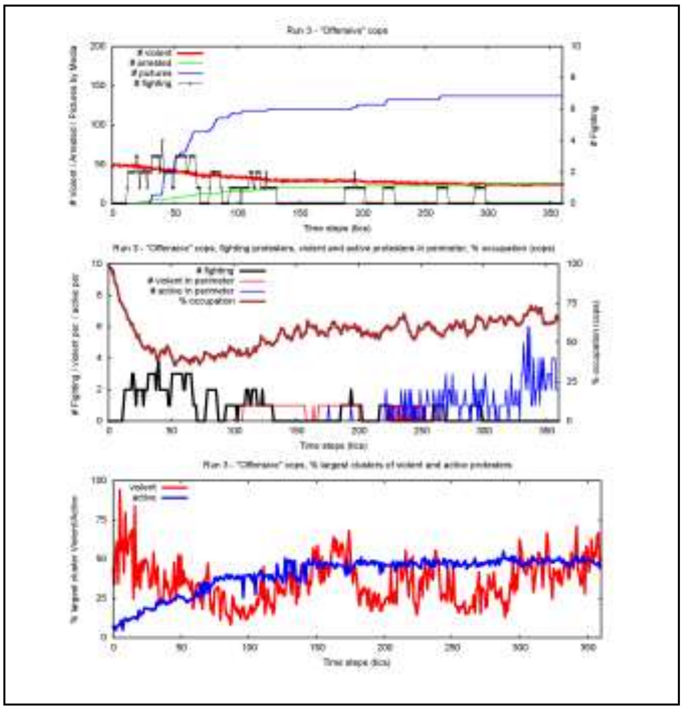

Figure 5. Time history of the quantitative measures of protest intensity and mediatic impact (top), police coverage and protesters' breaching of the defensive perimeter (middle), and percentage of the largest cluser of 'violent' and 'active' protesters (bottom), for 'offensive' cops.

Figure 5 shows the information corresponding to figures 2 and 4 for the case of 'offensive' cops. In this case, fighting started much earlier (top and middle graphs) since the cops left the perimeter to engage the 'violent' protesters. The number of arrests was smaller than for 'multi-role' cops and higher than for 'defensive' cops, but the number of violence episodes recorded by 'media' agents was smaller than in the two previous cases. This happened because after the initial fighting 'violent' protesters dispersed and later reformed a cluster, but not in direct contact with the police force. In this way, there were no further opportunities ('hot spots') for 'media' agents to seek. There was significant breaching of the defensive perimeter (middle graph), particularly by 'active' protesters, because of the low perimeter coverage by the cops (57% average). The bottom graph in figure 5 shows that the clustering behavior of 'active' protesters was almost identical to that in the previous case, but 'violent' protesters clustered, dispersed and regrouped towards the end of the simulation.

Figure 6 shows two snapshots of the simulation space, for simulations with 'defensive' cops (top) and 'offensive' cops (bottom). The top image shows 'cop' agents covering of the defensive perimeter well. A confrontation line appeared with two 'cops' fighting with a 'violent' protester and the 'media' agents well placed to record the events. 'Active' protesters clustered behind the 'violent' protesters. 'Quiet' protesters with 'hanger-on' personality were attracted towards the confrontation zone whereas 'passer-by' agents flowed from right to the exit on the left by avoiding 'hot spots'.

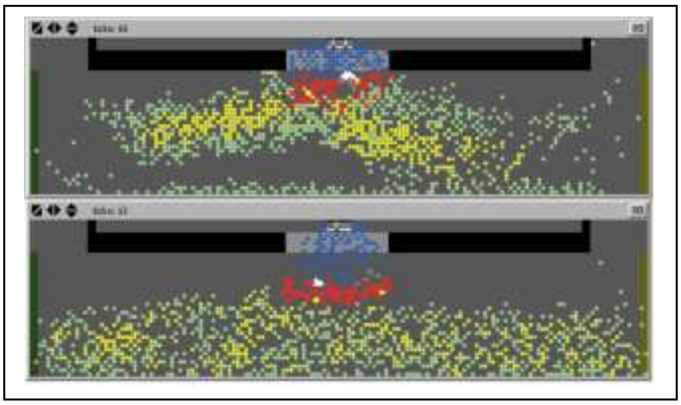

Figure 6. Snapshots of the simulation space for the simulations with 'defensive' cops (top) and 'offensive' cops (bottom). Agents and scenario features are represented as described in figure 5.

The image on the bottom shows the 'cops' advancing towards the cluster of 'violent' protesters and engaging them. This happened just after the beginning of the simulation, before 'active' protesters had time to cluster. However, this led to poor perimeter coverage, which created opportunities for 'active' protesters to breach the defensive perimeter afterwards.

V. Discussion

The quantitative measures of crowd patterns, protest intensity, potential 'news coverage' impact and police attack/defend performance, provided a rich and clear interpretation of the behavior of the ABM of street protests and violent confrontation. This allowed a more complete understanding of the model's representation of some protest features often observed in real events, such as clustering of 'violent' and 'active' protesters, moving confrontation lines, intermittent bursts of fighting and 'media' agents 'churning' around 'hot spots' to 'get the perfect picture'. In particular, the time history of the largest clusters of 'violent' and 'active' protesters showed how the model represented the formation, dispersal and regrouping of such clusters, as well as of the effect of their interaction with the police force.

In the simulated scenario, the police force has two goals which cannot be achieved concurrently, namely engaging protesters and protecting a perimeter. The new quantitative measures of perimeter coverage and breaching of the defensive perimeter were very useful to measure the performance of the 'multi-role' (or 'default') 'cop' agents introduced in [9]. This led to the introduction of two new types of 'cop' agents, 'defensive' and 'offensive'. 'Defensive' cops provided better perimeter coverage and ensured small breaching of the defensive perimeter (as expected), whereas 'offensive' cops provided an ABM representation of a 'police charge'. The disadvantage of both 'multi-role' and 'offensive' cops was the allowance for breaching of the defensive perimeter, because of lower coverage of the perimeter area and the temporary immobilization (two cops for each protester) due to fights and arrests. However, offensive cops have the advantage of dispersing 'violent' protesters and keeping them away from the vicinity of the defensive perimeter. This avoids the crowd compacting very near the defensive perimeter and reduces the opportunities for 'media' agents to 'take pictures'. Thus, from the viewpoint of the police force, it may be useful to combine 'defensive' and 'offensive' cops.

VI. CONCLUSIONS

In this work, we described the implementation of a set of quantitative measures of emergent properties in a previously developed ABM of street protests and violent confrontation with multiple players – 'cop', 'protester' and 'media' agents. The innovative features of the present work are: *i*) consideration of four classes of emergent properties – crowd patterns, protest intensity, potential 'news impact' and 'police vs. protester game'; *ii*) implementation of quantitative measures of those properties, particularly cluster size distribution (using a depth-first search recursive algorithm to identify clusters) and cluster orientation (by computing the clusters' principal axes of inertia); *iii*) combination of the new measures in a 'dashboard' that also includes the graphical representation of the agents and scenario features in the simulated protest space.

The improved ABM was applied to the simulation of a scenario in which a police force protects the entrance of a government building from protesters, described in [9]. The new quantitative measures provided good information on the relationship between the (defective) coverage of the defensive perimeter by the 'cop' agents and subsequent breaching of the perimeter by 'violent' and 'active' protesters. In this way, two new types of 'cop' agents with 'defensive' and 'offensive' characteristics were programmed. The new types of 'cop' agents performed as expected, with 'defensive' cops providing a better coverage of the defensive perimeter and 'offensive' cops engaging the protesters as in a police charge.

It was shown that the implementation of the quantitative measures of emergent crowd properties (patterns, measures of protest intensity, potential 'news impact' and effectiveness of police agents) provides better and more objective analysis of the behavior of ABM of street protests and violent confrontation. It was also shown that the combination of different quantitative measures provides very useful hints for developing new agent types and subtypes. They can also be used to assist parameterization of the agents' attributes (e.g. the weights of the agents' 'personality vector') and model validation (by comparing simulated time series with the corresponding measures obtained by analyzing videos of real events), leading to improved realism of the model outputs.

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