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ASSET ALLOCATION USING TREND FOLLOWING AND RISK PARITY APPROACHES

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Master in Finance

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Acknowledgments

Two years after intense studying and major dedication, I take some time to look back, recall my academic journey and dedicate my accomplishments to those that contributed most to the end of this challenging ride. The consummation of this dissertation denotes one of the most significant stages of my life, concretizing a personal objective to which I proposed to work hard. The unconditional support of all people involved was crucial to the completion of this path full of achievements. I would like to express my gratitude to all of them.

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Resumo

O período pós-crise revelou a ineficácia de muitas estratégias de alocação e a necessidade dos investidores evitarem perdas significativas durante períodos de recessão e recuperação. Após um desempenho notável, em 2008, a *Trend Following* – uma estratégia que adota posições longas em ativos com retorno positivo e posições curtas em ativos com retorno negativo – tornou-se infrutífera na geração de retornos positivos constantes. Geralmente, estas estratégias são construídas usando contratos de futuros das várias classes de ativos. São, ainda, geralmente formadas usando pesos que são inversamente proporcionais à volatilidade de cada ativo e têm historicamente experienciado boas características de diversificação. No entanto, no período 2009-2013, a estratégia tornou-se ineficiente devido ao aumento das correlações entre ativos e classes de ativos.

O aumento de co-movimentos levou ao aumento da importância da estratégia *Risk Parity*, que proporciona uma contribuição igualitária de risco. Alargando a posições longas e curtas, constrói-se a estratégia *Trend Following Risk Parity* estudada nesta dissertação. Com a combinação, espera-se conseguir um melhor desempenho em comparação com as estratégias individuais, dado que permite retornos mais elevados associados à *Trend Following* com menor volatilidade ligada à *Risk Parity*.

Utilizando 38 contratos de futuros de 6 classes de ativos num período de 31 anos, portfolios baseados no risco mostraram uma melhoria eficaz face às estratégias tradicionais, com algumas estratégias a terem um desempenho superior. A *Trend Following Risk Parity* obteve resultados superiores nos rácios de desempenho, especialmente em períodos de recessão e maiores correlações, proporcionando um retorno maior do que as outras carteiras.

Palavras-chave: Alocação de Ativos, Diversificação, Escolha de Portfólio, Paridade de Risco, Tendência

Classificações JEL: G10, G11

Abstract

The post-crisis period revealed the ineffectiveness of many allocation strategies and the need of investors to avoid significant losses during recession and recovery periods. After a remarkable performance, in 2008, Trend Following – a strategy that takes long positions in assets with positive returns and short positions in assets with negative returns – became unsuccessful in generating constant positive returns. Generally, these strategies are constructed using futures contracts across all asset classes. Additionally, they are usually formed using weights that are inversely proportional to assets' volatilities and have historically experienced good diversification features. However, in the period 2009-2013, the strategy became suboptimal due to the increase of pairwise correlations between and across asset classes.

The increase in co-movements led to the rise of Risk Parity, a long-only allocation approach that provides equal risk contribution. Extending this approach to a long-short approach, it constructs the Trend Following Risk Parity strategy that is being studied in this dissertation. With the combination of both, it is expected that it achieves better performance compared with the strategies alone since it allows higher returns associated with Trend Following and lower volatility linked with Risk Parity.

Using 38 futures contracts from 6 different asset classes over a 31-year period, risk-based portfolios show an effective improvement to traditional strategies, with some approaches performing better than others. The Trend Following Risk Parity approach achieved superior results in performance ratios, especially in high correlation periods and tougher market downturns, delivering an overall better risk-adjusted return than all other portfolios.

Keywords: Asset Allocation, Diversification, Portfolio Choice, Risk Parity, Trend Following

JEL Classifications: G10, G11

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Glossary

CAPM Capital Asset Pricing Model

ETF Exchange Traded Funds

EW Equal Weighting

FX Foreign Exchange

GMV Global Minimum Variance

MD Most Diversified

MDD Maximum Drawdown

MPT Modern Portfolio Theory

MRC Marginal Risk Contribution

QE Quantitative Easing

REIT Real Estate Investment Trusts

RP Risk Parity

STIR Short Term Interest Rates

T Tangent Portfolio

TF:RP Trend Following Risk Parity

TF:VP Trend Following Variance Parity

TRC Total Risk Contribution

1. Introduction

Portfolio management is the process of coherently focusing and managing a set of one or two different investment vehicles, and, in some cases, a set of investments. The subject attracts a lot of attention either by managers or investors with the objective of managing the collection of investments within the goals of each individual – their tolerance to risk given the different states of the global financial environment and their time horizon for needing money – thus meeting multiple needs tied to their investments. The key elements in portfolio management include, for example, diversification, asset allocation, and rebalancing. It is noteworthy that these elements are dependent on the roots of investment strategies, which include the reward for investing money on a given product/portfolio and the premia for taking an individual's money at stake.

Risk and return have always been linked since the ancestries of portfolio management. Over decades, traditional strategies like the 60/40 or the minimum-variance optimization have taken the lead, with a significant number of investors enchanted with this approach. Over time, other heuristic strategies based on equity and bonds have ascended, delivering better performance during most of the periods. Although such strategies have generally performed well over the years, during turbulent times they reveal significant drawbacks as investors are probably going to lose a substantial part of their returns. In the late 2000s, especially during and after the 2008 financial crisis, this matter became more relevant because of the increasing co-movement between and within asset classes, giving rise to many questions on the effectiveness of traditional strategies. Besides, many authors argue the optimality in portfolio construction, but many times practitioners find flaws when applying them in the real world.

How should investors allocate assets? How can they protect themselves from turning points? These are probably two of the questions that remain unanswered nowadays. It is arguable that some strategies suit better such conditions, while others seem to be outdated. The question that probably everyone has at this point is: Is there a perfect portfolio management strategy?

For many, the straight answer is no. Every investor is unique in its feelings, tastes, sensitivities, and motivations, resulting in different investors having different strategies, thus different asset allocation strategies. These are the so-called behavioral biases such as herding, regret and conservatism. Apart from these preconceptions, during downturns, most investors are impatient and end up bailing out before the market effectively recovers – despite personal characteristics and perceptions, there is always one thing that all investors have in common: when raging times come, they are more likely to panic. The reason is obvious: their money is at stake. What can investors do to avoid such discrepancies?

The goal of this dissertation is to tackle the importance of a good asset allocation strategy to increase portfolio returns and reduce its volatility. It aims to evaluate the performance of Trend Following (considering a suboptimal Volatility-Parity weighting scheme) and Risk Parity (which focuses on equal risk contribution) separately, but also the strategies combined through the introduction of a framework that takes advantage of both strategies – attributes positive or negative positions based on historical performance and weights them considering the risk (volatility) contribution to the portfolio. As the amount of portfolio attributed to each asset has a significant impact on performance, this framework outlines an optimization that changes weights accordingly, to achieve better performance. It is predictable that we find an approach that does not give up much of the upside potential but significantly reduces the downside risk, particularly when compared with traditional approaches. Considering a set of strategies (from simpler to more sophisticated ones) and a range of performance ratios that contemplate different metrics, this dissertation contributes to the literature on the subject and equates the impact of economic events in the performance of each strategy, in an attempt to find one that can warn those events. Through the presentation of pertinent literature, relevant data used for analysis and comparison along with a framework on the topic, this dissertation aims to study a relatively new strategy and compare it with other traditional strategies to prove its superior performance, especially during raging times.

2. Literature Review

In 1952, Harry Markowitz (1952:77) wrote "The process of selecting a portfolio may be divided into two stages. The first stage starts with observation and experience, and ends with beliefs about the future performances of available securities. The second stage begins with the relevant beliefs about future performances and ends with the choice of portfolio". Focusing on the second stage, it is important to anticipate returns, avoid downturns and unlock upside potential to achieve the very last goal: beat the market and make a profit. In such a process, correct asset allocation is vital and strongly dependent on the relationship between risk and return.

The decision of investing is challenging for many individuals and investors. They range from conventional investors – who prefer to have their money secured and saved for their retirement by investing in a risk-free asset – to bold investors – who are willing to put their money at stake to possibly get a premium for their investment. When the latter occurs, it raises several questions, mainly about their long-term financial objective, their risk tolerance and the ability to equate the two main concepts to meet investor needs.

Asset allocation is an old concept that aims to balance risk and return, by adjusting the percentage of the portfolio's wealth attributed to each asset based on risk tolerance, its contribution to return and the time horizon of the investment. It usually combines two or more assets from the different classes – equities, fixed-income (bonds) and cash or cash equivalents – based on their contribution to risk and return. Even though equities have higher upside potential, they also have a higher risk. In contrast, treasury bonds are safer but offer lower returns. Portfolios should be built in a way that allows them to prosper during market booms and to insulate during market downs – hence the importance of diversification. As expected, numerous strategies suit this idea.

2.1 Traditional Approaches

Traditional approaches to strategic asset allocation are strongly entrenched with the framework proposed by Markowitz in the '50s, which deals with the mean-variance portfolio for constructing equity portfolios. This methodology is difficult to implement due to the struggle in accurately estimating expected returns and covariances. Furthermore, Gross (2009) refers to a relatively new paradigm for the global economy that complicates the future evolution of asset returns (Chaves, Hsu, Li, & Shakerna, 2011).

Clare et al. (2016) show that the ability to invest from a wide variety of assets has never been easy and the introduction of electronic trading and expansion of ETFs (Exchange Traded Funds) demonstrated that traditional methods are obsolete. For example, the traditional method of allocating 60% in domestic equities and 40% in domestic bonds, appears to be outdated as it does not consider the benefits from diversification nor other alternative asset classes. Besides, they reveal the inefficiency of the 60/40 portfolio strategy given that volatility of equity dominates risk, concluding that investors should allocate an equal amount of risk to stocks and bonds to achieve Risk Parity – a concept that will be detailed later (Clifford, Andrea, & Lasse, 2012).

Most investment funds are composed of the two main asset classes: equities and bonds. Though, there are other classes commonly used such as foreign currencies, real estate and commodities. These asset classes have different risk-return profiles (for example, equity futures compared with bond futures), therefore impacting portfolio allocation in diverse ways.

Apart from the 60/40 portfolio strategy, there are others traditionally used. Firstly, the equal-weighting allocation scheme, which is an approach that attributes the same weight to each asset, regardless of their risk-return profile. This technique is very simple as investors do not need any information on the return of the assets to attribute weights. Mathematically, the weights are given by:

$$\bar{\boldsymbol{w}} = \frac{1}{N} \tag{1}$$

where, $\bar{\boldsymbol{w}}$ represents the $N \times 1$ vector of portfolio weights. However, with this strategy, assets with higher volatility (such as equity-like assets) dominate the portfolio, resulting in a portfolio that holds much more risk than the one possibly desired (Chaves, Hsu, Li, & Shakerna, 2011).

In addition, the mean-variance portfolio strategy is an investment decision that analyses how much risk investors are willing to take, given the different levels of return. By acting according to this strategy, investors should select portfolios that are in the mean-variance efficient frontier. This allows investors to find the best reward for a given level of risk or, in contrast, the least risk at a given level of return, to find the optimal balance.

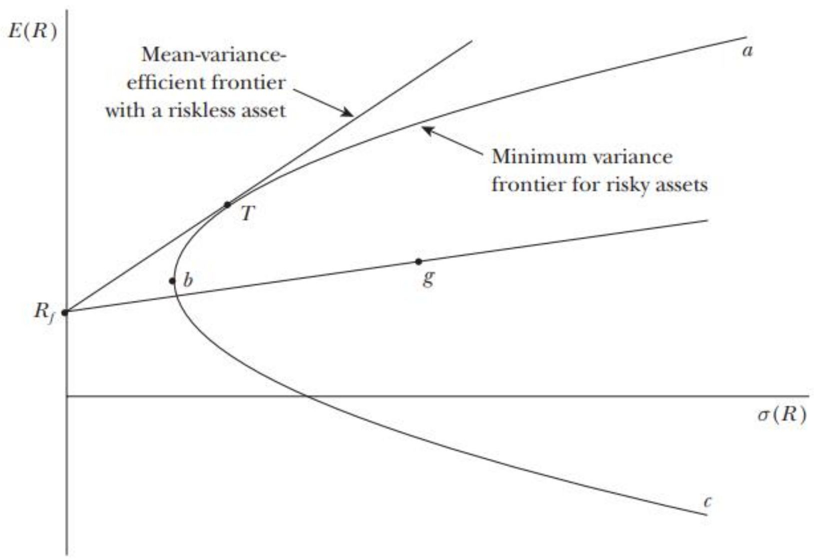


Figure 2.1 – Efficient Frontier
Source: Fama and French (2004)

Figure 2.1 shows the hyperbola-shaped frontier that comprises all efficient combinations of risky assets. In the case of the absence of the risk-free asset, efficient portfolios must only rely on the upper part of the hyperbola, that is, above the minimum-variance portfolio (point b, in the graph). In the case of laying inside the parabola or in the lower bound, the portfolio is not efficient since it displays a lower return for a given level of volatility. Considering a riskless asset, the efficient portfolio becomes the straight line connecting the tangent portfolio (T) and the risk-free rate (R_f). Furthermore, under the capital asset pricing model (CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) a logical linear relationship can be drawn and held in equilibrium. When that is the case, every investor holds portfolio T and R_f in varied portions.

Yet, this model is based on strong assumptions such as market efficiency and access to all available information. It also assumes no taxes nor transaction costs, investor's risk aversion (i.e. investors who choose the prevention of capital over the potential for a higher-than-average return) and investor's non-satiation (i.e. given two assets with the same volatility, investors prefer the one with the higher expected return). Moreover, the mean-variance strategy involves engaging in the assessment of the unknown future distribution of returns and poor estimation of parameters that may lead to suboptimal results. According to a study conducted by Jobson and Korkie (1981), to estimate risk and return accurately and unbiasedly, at least 200 monthly observations of returns are required, making it very inefficient for most investors.

2.2 A new paradigm in portfolio strategies

The key concepts of portfolio decision, choice of asset classes and allocation methods, have been challenged by academics and financial professionals in recent decades. The inclusion of commodities and other assets that have historically played less important roles (such as Real Estate, Hedge funds, FX and emerging markets) pressured investors towards defining asset classes and risk types rather than asset types. This gave rise to a strengthening of portfolio construction based on clear diversification of risk types and a deviation from the Modern Portfolio Theory (MPT). Additionally, investors became more aware of global methods aiming to provide additional diversification through the geographical dispersion of investments.

Maximization approaches for portfolio construction under the MPT have been identified with notable shortcomings. They are usually associated with significant downside risk, particularly the risk of negative returns; the use of estimates producing bad forecasts and consequently affecting the overall portfolio; the negligence of transaction costs and capacity constraints in most circumstances; and finally, the inability to incorporate constraints in the risk/return optimization, giving rise to a solution that usually lacks diversification and leads to avoidable allocation outcomes. Besides, these strategies are very subjective and dependent on investors' views and perceptions, justifying the appearance of new allocation methods.

2.2.1 The evolution of quality inputs and decreasing dependency on expected returns

The simplest and earliest models for estimating input factors were based on the single-index model, which is the case of the market model developed by Sharpe (1964). The CAPM assumes that the expected return of any asset is linked to its sensitivity to the overall market return, thus reducing the need for many estimates and improving the quality of portfolio optimization. Yet, this strategy fails to constantly consider many factors affecting the return of that specific asset. Later, the three-factor pricing model proposed by Fama and French (1992) attracted a lot of attention by suggesting an important change through the inclusion of multi-index models. Besides market return, they identified size and value as central drivers of individual security's return.

In 1992, Black and Litterman proposed the Black-Litterman Model to reduce the uncertainty related to sample estimates. Accordingly, the equilibrium expected returns could be resultant from observable information, in particular, asset pricing, and a consecutive modification to meet optimization's specific opinions about their future performance. Similarly, Michaud (1998) contributed to the sensitivity reduction of the final allocation by presenting the resampled efficient frontier approach, with a portfolio that is not optimal but considerably more stable in terms of input parameters.

Despite the significant improvement in accuracy over the years of the mean-variance portfolio theory, they continue to be highly dependent on expected returns. In the most recent years, many risk-based portfolios became progressively more pertinent. An example of these risk-based portfolios is the global minimum variance portfolio (GMV), which arises as a natural solution of the Markowitz efficient frontier. The GMV is a strategy that has performed very well when applied to an equity portfolio construction. It combines assets in a way that minimizes the total volatility of the portfolio, by using covariance data but neglecting expected returns information (Clarke, De Silva, & Thorley, 2011). Algebraically, it can be derived by solving:

$$\min \frac{1}{2}\bar{\boldsymbol{w}}' \sum \bar{\boldsymbol{w}} \qquad s.t. \bar{\boldsymbol{w}}' \, \mathbf{1} = 1 \tag{2}$$

where $\mathbf{1}$ represents the $N \times 1$ vector of ones and $\sum$ represents the variance-covariance matrix. By only considering variance inputs, this strategy does not rely on any expected return estimates. However, as it is generally associated with lack of diversification, it may significantly suffer from extreme events (for example, in the 2008 financial crisis it was possible to observe a high concentration of risk in few assets). The strategy has also been criticized by using the variance as a good measure for risk because it penalizes unwelcome high losses and desired low losses and for its high sensitivity to parameter estimation errors.

A second example of the risk-based approach, more recently used, is the Most Diversified (MD) portfolio that was first introduced by Choueifiaty and Coignard (2008). It intends to maximize the ratio between the weighted average individual volatilities and the portfolio's total volatility. The maximization is a derivation of the Diversification Ratio that is given by:

$$D^p = \frac{w^{P'}\Sigma}{\sqrt{w^{P'}\Omega w^p}} \tag{3}$$

where the vector of asset volatilities is given by $\sum = \{\sigma_1, \ldots, \sigma_n\}$ for a portfolio which weights are represented by $w^p = \{w_1^p, \ldots, w_n^p\}$ and Ω denotes the $N \times N$ variance-covariance matrix. One theoretical property of this diversification measure is that the ratio is only equal to one when the portfolio is exclusively composed of one asset. When this is not the case, the ratio will always be higher than one. Therefore, the higher the value, the more diversified the portfolio is. Algebraically, the optimization is derived as:

$$w^{MD} = \max f(w)$$

$$f(w) = \frac{\sum_{i=1}^{n} w_i \sigma_i}{\sigma_p} = \frac{w^{p'}\Sigma}{\sqrt{w^{p'}\Omega w^p}} \tag{4}$$

$$s.t. W'\mathbf{1} = 1$$

where $\mathbf{1}$ represents the $N \times 1$ vector of ones, $\sum = \{\sigma_1, \ldots, \sigma_n\}$ the vector of asset's volatilities for any portfolio $w^p = \{w_1^p, \ldots, w_n^p\}$ and Ω represents the $N \times N$ variance-covariance matrix. The difference between the numerator and the denominator relies on correlations. Given the maximization problem and the fact that correlations are only considered in the denominator, this portfolio attributes higher weights to assets with lower correlations and lower weights to assets with higher correlations. The authors demonstrate that in a situation where all assets have the same volatility, the MD portfolio becomes the GMV portfolio. Fernholz (2002) extended these findings and found that the difference between the parameters provides a positive contribution to the overall portfolio expected return and can sometimes be understood as a free lunch coming from diversification.

The last case is the so-called Risk Parity portfolio – a strategy that will be the subject of analysis in this dissertation. For this purpose, investors can choose assets that match their risk tolerance but may need to lever up or down each asset constituent to contribute the same amount of risk and the desired target level. Empirical studies were carried by different authors in an attempt to analyze whether these strategies provide a superior risk/return. Anderson et al. (2012) found that it is not possible to conclude that the Risk Parity approach is superior to the traditional 60/40 allocation strategy given that results are significantly influenced by start and end dates, transaction costs and leverage. Nevertheless, it is noteworthy seeing that there has been a shift from static investment strategy to dynamic risk-based approaches.

2.2.2 Low-yield environment

Asset allocation can be significantly affected by the economic outline. It is not by chance that investors are becoming more uncertain about the future, mainly since the beginning of the pandemic. This situation is recurrent during downturn situations, as occurred in the post-2008 financial crisis. Among the risks of a negative outlook, there is the concern of low-yield environments.

The persistence of low short-term interest rates alongside the quantitative easing (QE) policy at the beginning of the last decade resulted in uncertainty for investors, especially regarding bond yields. In January 2013, the chairman of the Federal Market Committee, Ben Bernanke, reiterated that an increase in interest rates promoted by a premature QE policy would result in a longer recovery period and persisting interest rates. However, this situation may be favorable to equities given that rises in bond yields (which behave inversely to interest rates) reflect rising growth expectations and lower systematic risks. It is also common that in these situations, investors perceive equities as a good alternative to bonds since the opportunity cost (when compared with the opportunity cost of holding bonds) is lower. Nevertheless, it is noteworthy that, in some circumstances, rising bond yields may be harmful to equities due to rising inflation expectations and inflation risks. As a result, the relationship between bond yields and equity prices is not constant. While in the post-war period this correlation was inverted, following the tech-bubble eruption in 2000, the correlation reversed and became more closely related since the 2008 global financial crisis.

In the current low-yield environment, investors are challenged to achieve their return targets within the constraints of their investment plans. Lower interest rates have led to lower bond yields, which in turn resulted in decreasing yields for other assets. In this sense, returns become more interesting when shifted towards capital gains – the more the increase in prices, the more investors are willing to sell them. However, there are limitations to this effect. For investors to sell their assets, there must be someone willing to buy them. As the yields across all assets decrease, the incentives for buying these assets will decrease at some point, resulting in an effect that will be offset in the long run.

While bond returns are likely to remain low, prompted by near-zero interest rates in the current COVID-19 pandemic and low inflation is a common tool to achieve higher yields, urges the risk of earlier inflation causing capital losses. As a result, the key focus of investors should be shifted towards assets that deliver sustainable income and remain competitive under different economic outlooks, while adopting strategies that can comprise and consider the uncertainty caused by the current macro environment. These strategies include targeting opportunities across different asset classes (other than equities and bonds) and investing in a stronger portfolio diversification using a dynamic portfolio approach that can generate regular income for a given investor's risk profile.

2.2.3 The increasing relevance of Trend Following

In the last decade, the world economy has entered a stage where old concepts and approaches are unlikely to continue to work. This is mainly driven by the environment of stagnating global growth, low market returns, high volatility and increasing correlations across traditional asset classes.

The aftermath of the 2008 global financial crisis, revealed the importance of correctly choosing and accurately allocating assets in portfolio management. One of the most effective ways of doing so is to actively manage the portfolio through a frequent adjustment of asset mixes to better suit market trends. Generally, such adjustments include reducing positions in poor-performing assets and increasing them in well-performing ones. Although such a strategy allows minimizing losses, it is not enough for constant well-performing portfolios since we must also consider risk and its manageable portion (the idiosyncratic risk) and ponder the impact of diversification.

Based on the above characteristics, two important tools take the lead in dynamic asset allocation: Trend Following and Risk Parity. The first one is a simple market-timing model that focuses on which assets to hold based on the signals while the latter reveals how much of these assets to hold. Whereas Trend Following has always been relevant for portfolio strategists, Risk Parity is a relatively new portfolio strategy and it has recently attracted a lot of attention after the global financial crisis due to its good performance.

The US research group Dalbar found that the overwhelming driver of discrepancy is bad timing by investors, especially during extreme events. In concrete, in October 2008, the S&P500 fell 16.8% while many investors lost on average about 24%. The same happened at the time of the Black Monday Crash in 1987 and the Russian Crisis in 1998, showing that investors are more likely to panic at big market turning points. However, everyone must bear in mind that for some active portfolios to outperform, others must underperform. The following paragraphs attempt to provide an overview of strategies that alone or combined will decrease the probability of underperformance under downturn circumstances (Authers, 2015).

Momentum strategies usually refer to approaches involving past returns, rewarding the good performers and penalizing worse performers. Ostgaard (2008) claims that the concept of Trend Following (used for decades in futures markets and particularly in commodities) has been widely used, employed by various methods such as moving averages crossovers. Moving averages are commonly referred to in Technical Analysis – a method of evaluating and identifying investment opportunities using trends from the trading activities such as price changes – and involve weighing the closing prices of stocks or broad market indices. When the trend is positive, long positions are adopted while short positions are taken when the trend is negative. The effectiveness of this strategy alone has attracted a lot of attention among the academic community. While Faber (2010) demonstrated that Trend Following strategies can achieve equity-level returns with bond-level volatility, Clare et al. (2016) give the perspective of other authors who questioned the effectiveness of the strategy in the US equity market, providing a variety of explanations that include investor's underreaction to market news and their propensity to display herding behavior (Friesen, Weller & Dunham, 2009; Ilmanen, 2011).

Despite being advocated by many strategists, the Trend Following approach must be carefully employed since it extrapolates price trends. The reason is simple: the strategy is only profitable when assets with negative past returns go down and when positive past returns continue to rise. When the opposite occurs, due to significant changes in the macroeconomic paradigm, investors will be worse-off. For example, let us assume a 50%-50% portfolio with an asset that has positive past returns and another asset with negative past returns, the investor will go long (invest) in the positive one and short (divest) in the negative one. If everything remains as expected, the maximum loss is zero. In contrast, if the opposite occurs, the downside potential is huge. This situation is very likely to occur since portfolios are usually dominated by equity-like assets, which in turn carry a great amount of risk, thus evidencing the challenges related to risk allocation.

Combining assets from different asset classes in a portfolio requires superior attention and Volatility-Parity urges as the natural solution to fairly distribute the risk across assets and asset classes. This weighting scheme explored by Moskowitz et al. (2012) states that the weight of each asset should be inversely proportional to its historical volatility, implying that all assets come in the portfolio with the same level of volatility. Yet, this strategy ignores the importance of correlations and can become suboptimal as pairwise correlations increase over time.

2.3 Correlations everywhere

After an impressive performance, since 2009, Trend Following has continuously delivered poor performance justified by an increase in co-movements between assets. In addition, assets from different asset classes became more correlated with each other, thus decreasing the benefits of diversification between and across asset classes. By ignoring covariations and co-movements, the Volatility-Parity weighting scheme became obsolete and failed to achieve the objective of allocating the same amount of risk to each asset.

Usually, this degree of co-movement is measured by correlations that relate to the dependency between two or more assets. On one hand, when the correlation is positive, it indicates that the assets move in the same direction (up and down) and are similarly affected by different events. On the other hand, when assets have a negative correlation, they move in opposite directions. When there is a case of perfect negative correlation (that is, correlation is equal to minus one), the combined assets will eliminate risk. There is also the case when the correlation is zero which represents the situation where the assets have no relationship between them. This situation is great for diversification effects because the volatility/risk of the portfolio is theoretically minimized. However, in the real world, this situation is extremely unlikely and non-correlation assets are difficult to find. Mathematically, correlations can be defined as:

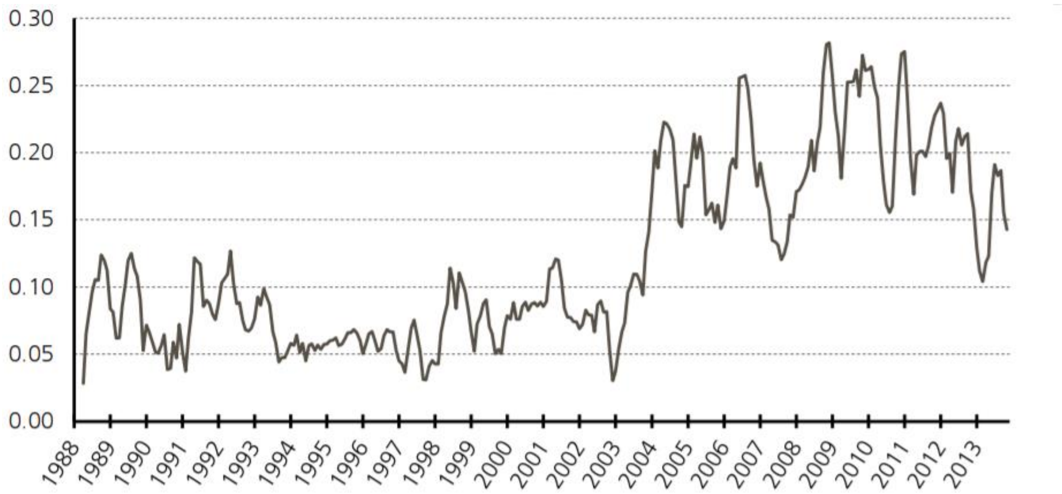
$$\rho_{xy} = \frac{Cov\ (r_x, r_y)}{\sigma_x \sigma_y} \tag{5}$$

where σ_x and σ_y represent the standard deviation of asset x and y, respectively and the numerator represents the covariance between assets x and y. The covariance is also an important tool in modern portfolio theory and is statistically defined as the measure of joint volatility of two random variables. It can be defined as:

$$Cov\left(r_x, r_y\right) = \frac{\sum_i^n (r_x^i - \overline{r_x}) \times (r_y^i - \overline{r_y})}{n-2} \tag{6}$$

where $\overline{r_x}$ and $\overline{r_y}$ represent the mean value of asset x and y, respectively.

It is imperative to understand the financial notion of correlation because the goal of asset allocation is related to the combination of assets with low correlation to lower portfolio volatility. When putting negative or low correlation investments in the portfolio, investors are decreasing the volatility of the portfolio. The combination of asset classes with low correlation reduces the overall volatility of the portfolio and allows the investor to achieve a higher return in the long run through a more aggressive investment strategy. Nevertheless, the opposite can happen if assets are highly correlated.



Notes: Monthly average pairwise correlation across all contracts using a 90-day rolling estimation window. The sample period is from April 1988 to December 2013.

Figure 2.2 – Rolling Average Pairwise correlations across all assets
Source: Baltas (2015)

According to Baltas (2015:10) "One of the most prevalent claims for this recent lackluster performance of Trend Following has been the post-crisis increased level of pairwise correlations". Figure 2.2 shows the three-month average pairwise correlations across all futures contracts implied in his study. The level of pairwise correlations has significantly increased after the global financial crisis. He found evidence that the dramatic shift in pairwise correlations is largely driven by the significant increase of inter-asset class correlations except for the correlation between fixed-income and the rest of the asset classes (energy, commodities, FX and Equities).

The profitability of the Trend Following strategy is highly dependent on two main factors: persistence of price trends and efficient combination of assets. On the one hand, it is simple to understand that the strategy will not work if the asset movements significantly change. On the other hand, considering a weighting scheme that accounts for correlations, any inefficiency will be due to poor risk allocation among portfolio constituents.

Considering the hypothesis that the poor performance of the Trend Following approach is related to the suboptimal allocation of the Variance-Parity weighting scheme, Baltas (2015) found that when evaluating the performance of the strategy with Sharpe Ratios in four different correlation regimes (low: less than 5%; medium: from 5% to 10%, high: from 10% to 20% and extreme: more than 20%) using daily returns, he found that the performance of the Trend Following strategy using Variance Parity weighting scheme drops significantly when the correlation diverges from zero into positive territory and it is even more affected when moving away towards a higher correlation regime (the Sharpe Ratio fall from 1.28 in high correlation environment to 0.27 in extreme correlations one).

In regimes of high correlation, it is usually observable that the Trend Following strategy reduces its performance because of the absence of strong price trends. However, the suboptimal distribution of the risk constituents should not be neglected, giving rise to the need for a relatively new allocation strategy.

2.4 The rise of Risk Parity

The alternative solution to the commonly used Variance-Parity weighting scheme is the previously mentioned Risk Parity, an approach that computes portfolio weights so that each asset contributes the same amount of risk to the global portfolio. In other words, Chaves et al. (2011:109) refer that "Proponents of risk parity approach argue that a more efficient approach to asset allocation is to equally weight the class by its risk (volatility) contribution to the portfolio".

Unlike the equal-weighting scheme, Risk Parity focuses on risk allocation rather than capital allocation. For example, in the case of a two-asset portfolio where asset A has 10% volatility and asset B has 20% volatility, an equal-weighting scheme recommends a 50-50% capital allocation, which would result in an overweighting of asset B in the portfolio's overall risk. On the other hand, based on the Risk Parity weighting scheme, the amount of capital allocated to assets A and B should be such that the relative risk contribution to the portfolio is equal to each asset, thus resulting in a portfolio that would be dominated by asset A risk. This situation results in a portfolio that is generally heavy in fixed income, therefore with low volatility and returns.

As understood, the Risk Parity strategy aims to equate the risk contributions of portfolio assets to the portfolio's overall volatility. On the way to algebraically understand this concept, it is important to highlight the concept of Marginal Risk Contribution (MRC). The concept can be defined as the increase/decrease in portfolio volatility for a marginal change in the asset weight or, in other words, the partial derivative of the portfolio's total volatility with respect to the weight of each asset:

$$MRC_i = \frac{\partial \sigma}{\partial w^i}, \forall_i \tag{7}$$

From MRC, it is possible to reach the Total Risk Contribution (TRC):

$$TRC_i = w_t^i \times \frac{\partial \sigma_t}{\partial w^i}, \forall_i$$

$$\sigma_t^p = \sum_{i=1}^{n} TRC_i \tag{8}$$

Qian (2006) mentioned that one can derive the individual percentage contribution to risk from the previous equation through the division of TRC by σ_t^p and the sum of percentage contributions equal to one:

$$p_i^p = \frac{TRC_i}{\sigma^p}$$

$$\sum_{i=1}^{n} p_i^p = 1 \tag{9}$$

The percentage contribution to risk is the ratio of the covariance between the contribution of asset i and overall portfolio contribution, in terms of portfolio volatility. Qian (2006) concludes that in a universe of noteworthy large portfolio losses, percentage contributions can be interpreted as the actual percentage contribution to portfolio loss of a certain asset. For example, if the percentage contribution of asset i is 20%, investors may expect that 20% of the loss is attributed to asset i, in the case of experiencing a severe portfolio loss.

According to a study by Qian et. al (2005), risk contribution can effectively be seen as a precise method of loss contribution, that is, when a negative scenario occurs, risk contribution can be an accurate indicator of what contributed more to that loss. They found that in the 60/40 portfolio approach when a loss of more than two percent occurs, over 95% is attributed to stocks while when the value is above three or four percent, it represents above 100% of the loss contribution. These results show that using variances and covariances to calculate this value, the economic interpretation of the risk contribution is the approximation of the expected loss (Qian, 2005).

The objective of the Risk Parity approach is to equate all the components of the summation and effectively set them to $\frac{1}{N}th$ of the portfolio's volatility in terms of risk contribution, rather than portfolio volatility. Asness et al (2012) propose a simple approach that attributes weights to each asset i according to the inverse of its volatility and disregards any optimization problem:

$$W_i = \frac{1/\sigma_i}{\sum_{i=1}^{N} 1/\sigma_i} \qquad (10)$$

$$s.t. W'\mathbf{1} = 1$$

where **1** represents a vector of ones. Despite being seen as the roots of the Risk Parity strategy and partially considering risk diversification by penalizing high volatile assets and rewarding low volatility ones, it does not consider asset correlations, thus resulting in an overall volatility potentially higher than the actual value. In the Methodology section, we present this strategy associated with the Trend Following approach and an alternative approach to the Risk Parity strategy using non-linear optimization methods.

When compared with the above-mentioned strategies (60/40, equal-weighting, minimum-variance and mean-variance strategy), Risk Parity reveals the ability to consider all asset classes and different risk profiles. Beyond this, one of the major benefits of Risk Parity over mean-variance portfolios is that "(…) investors do not need to formulate expected return assumptions to form portfolios" (Chaves, Hsu, Li, & Shakerna, 2011:109). The only contribution that is needed is variances (and covariances) which are typically easier to estimate using previous data. In addition, Risk Parity allows for better beta diversification since it decreases the exposure of the portfolio to equity components while providing a framework that is seen as a capital protection strategy, by giving up some of the return of equity-like assets to defend the strategy from market downturns.

Critics of the strategy question whether, in the long run, commodities and government bonds provide enough premium over cash and leveraging to be worth investing. Also, as the number of assets in the portfolio increases, the number of possible combinations in portfolio weights grows exponentially. This gives rise to the need of considering the subjective views of the investor to pre-select and group assets into a manageable set. Moreover, the Risk Parity strategy is long-only, meaning that the weight attributed to each asset must be positive. The drawbacks of this model can be surpassed through the integration of both Trend Following and Risk Parity strategies by integrating a subjective parameter, the target volatility that is dependent on investor's perceptions and the signal of the past returns to extend the strategy to a long-short framework. Further details on this will be discussed in section 4 (Inker, 2010; Lee, 2011).

2.4.1 The opportunity cost of Risk Parity

The Risk Parity strategy has, obviously, shortcomings and assumptions that are crucial for the correct functioning of the model. This is the case of the estimation of correlations and states of the world. On the one hand, correlations are assumed to be stable over time. On the other hand, and perhaps more importantly, it assumes that all states of the world are of equal probability, which has not occurred historically. Besides, it is sensitive to borrowing costs as the value of leverage increases with the decrease of interest rates. Based on a study conducted by Chaves et al. (2011) that compares Risk Parity with other portfolio strategies from January 1980 to June 2010, Risk Parity strategy outperforms most of the traditional approaches, with the second-best Sharpe Ratio and second-lowest volatility, as one could expect by the robust predominance of fixed-income assets. However, it fails to consistently beat the 60/40 and equal-weighted portfolio, showing one of the drawbacks of the model: its sensitiveness to the inclusion of assets.

Chaves et al. (2011) found that the performance of the Risk Parity approach is highly sensitive on the type of assets chosen and to how much to allocate to each of them (similar to what happens in the case of the equal-weighting scheme). In the scenario of a reduction from nine to a five-asset class sample (keeping only the U.S. long Treasury, U.S. investment-grade corporate, S&P 500, commodities, and REITs – Real Estate Investment Trusts), they experienced a reduction of about six basis points. Then, they included one of the most invested intermediate-term US Treasury, the BarCap Aggregate Bond Index which has historically one of the best Sharpe Ratios (about 0.82) with an average return of 7.3% and a volatility of 4% over the last 30 years. The result is a significant improvement in the Sharpe Ratios of both the nine and five-asset class universes (three basis points in the first case and five basis points in the second one). The evaluation of that study suggests that including more assets results in improved Risk Parity portfolios. However, the authors moderate expectations by considering a two-asset class Risk Parity portfolio (S&P500 and BarCap Aggregate) that has a significantly better Sharpe Ratio than the best performer (0.62 vs 0.54). As a result, the inclusion of more asset classes appears to increase the performance of the Risk Parity strategy, however, investors should bear in mind that the choice of portfolio constituents still plays an important role.

Another important note is that the Risk Parity strategy seems to reject the CAPM model because it considers a different allocation scheme, thus revealing to be an inefficient strategy according to this model. The Risk Rarity portfolio appears to be somewhere between the efficient frontier and the line passing through the minimum-variance portfolio. As a result, we can look at the strategy as an allocation scheme that provides a better return than the minimum-variance portfolio but lower than the tangency portfolio. This is particularly relevant for investors that may want to increase their level of exposure for an increase in the expected return or to investors that prefer to decrease a little of the expected return to decrease their risk exposure. However, in the case of assets that have the same correlations and Sharpe Ratios, the Risk Parity becomes the tangency portfolio and, therefore, efficient.

This situation is extremely unlikely leading investors to question whether they prefer to invest in a strategy that has been widely used while exposing them to more risk or account for that risk and invest in a relatively new but highly productive strategy (Maillard, Roncalli, & Teiletche, 2010).

2.4.2 When Risk Parity meets Trend Following

Clare et al. (2016) extended the findings of the Risk Parity weighting scheme by applying Trend Following strategies. By benefiting from the choice of assets produced by Trend Following and equal risk contribution of Risk Parity, it tries to evaluate and compare the performance with a pure Risk Parity strategy. The study uses historical data based on five broad asset classes from 1994 to 2015, from diverse geographies and indices. The portfolios that consider Risk Parity strategies are usually rebalanced on a monthly basis.

Following the method of Faber (2007), the signs are determined at the end of the month, short selling is not allowed, and transaction costs are ignored. “More precisely, if the price of the asset class index is above its x-moving average then one can say that the asset class is in an uptrend and it is purchased” (Clare, Seaton, Smith, & Thomas, 2016:67). If the opposite occurs, the asset is in a downtrend and should be sold. Besides this, it applies a Volatility-Parity weighting scheme, so that all asset classes enter with the same ex-ante volatility. The strategy can be described as Trend Following Volatility-Parity (TF:VP) and will be mathematically detailed in subsequent sections.

Table 2.1 – Performance Statistics based on five broad asset classes (1994-2015)

	Equal weighting	**Trend Following (8 months)**	**Risk Parity**
Annualized return (%)	6.61	8.09	6.59
Annualized volatility (%)	12.09	6.78	5.91
Sharpe Ratio	0.33	0.80	0.67
Max. monthly return (%)	10.21	6.75	3.96
Min. monthly return (%)	-18.99	-6.55	-8.40
Maximum downturn (%)	46.60	6.86	20.46
Skewness	-1.06	-0.16	-0.99

Source: Clare et al. (2016)

The results of this study show that, from a Sharpe Ratio perspective, the combined strategy produces about 0.80 against the 0.67 of Risk Parity alone. Moreover, when compared with the equally weighted, which attributes 20% to each asset class regardless of historical performance, the strategy outperforms once again in terms of Sharpe Ratio (0.80 against 0.33) at a much lower annualized volatility (6.78% against 12.09%). Therefore, the TF:VP strategy can perform much better, at a lower risk.

The TF:VP strategy assumes that all pairwise correlations are equal to zero, that is, it does not account for relationships amongst the available assets. Therefore, the variance-covariance matrix (to be defined at a later stage) is diagonal and when that is the case, the strategy is an accurate Risk Parity approach. Nevertheless, in practice, this is not the regular case and the TF:VP turns out to be a suboptimal allocation strategy.

Given that Risk Parity corrects the problem related to pairwise correlations, it is expected that the Trend Following Risk Parity (TF:RP) strategy performs even better and arises with an optimal solution. The construction of this strategy using a more sophisticated construction methodology leads to superior performance, especially in higher correlation regimes. Let us recall the example from 2.3 where the Sharpe Ratio of the Trend Following strategy changed from 1.28 to 0.27 when moving from a high correlation state to an extreme one. Baltas (2015) extended those findings applying a Risk Parity approach to the Trend Following results:

Table 2.2 – Sharpe Ratio in different correlation environments (adapted)

	Low	**Medium**	**High**	**Extreme**
TF:VP	2.06	1.50	1.28	0.27
TF:RP	2.01	1.69	1.19	0.86

Source: Baltas (2015)

It is evident from the table above that the improvement of the performance of the strategy is more pronounced in the extreme correlation environment, giving rise to the idea that in such a scenario the volatility parity weighting scheme may be suboptimal. However, the diversification effect of the strategy is not so pronounced in a low correlation environment, a scenario where the economy tends to be in a boom, as the strategy is penalized by the inclusion of lower return assets.

Taking advantage of the signal of the past returns as used in the Trend Following framework and equal risk contribution from the Risk Parity framework, the objective in the Methodology section is to detail the strategy and analyze the portfolio return over the sample period.

2.5 Investor's constraints in portfolio optimization

Typically, many investors (including RP investors) use numerous tools like leverage to increase the expected return and attain a certain level of desired risk. In addition, the inclusion of portfolio constraints must also be considered as they may have a significant effect on portfolio construction.

Chow et al. (2016) show that on minimum-variance portfolios, investors can experience increased volatility caused by any additional increase in portfolio constraints. While these constraints may increase the investment ability and meet most of the investor's needs, it will shift the portfolio features towards the market capitalization-weighted portfolio – a portfolio where each component is weighted according to the size of its total market capitalization. This portfolio is seen as stable and reflective of the broader markets, in which bigger companies have greater influence over small ones. Evidence shows that these simulated minimum-variance portfolios outperformed traditional passive investment strategies while providing higher Sharpe Ratios in comparison to the market capitalization-weighted benchmarks.

Lee (2014) studied the optimality of the pure Risk Parity portfolios and reveals that in the case of where correlations and Sharpe Ratios among all assets (and asset classes) are the same, the Risk Parity portfolio becomes a mean-variance portfolio. Therefore, some practitioners believe that Risk Parity is a special case of the mean-variance portfolio strategy. The author argues that deviations in the Sharpe Ratio in the short-term are likely to occur, though, they have been more comparable in the long run. Considering that this assumption holds, the Risk Parity strategy is more efficient (in terms of Sharpe Ratio) compared to the traditional strategies, for example, the 60/40 stock-bond allocation strategy. However, it is possible to observe that regardless of diversification effects, the Risk Parity strategy may give low expected returns given its high weighting of low-risk assets.

To sum up, the superior advantage of Risk Parity is largely related to the fact that an unconstrained efficient frontier is more efficient than a constraint efficient frontier, *ceteris paribus*. Although Risk Parity is not perfect and may seem more constrained than other strategies, it can provide a significant improvement by using standard constraints (that is, constraints that are applicable and generally used in other strategies), allowing the introduction of short selling through the long-short framework explained in the Methodology section.

3. Data Description

Concerning data, this dissertation considers the monthly closing prices of futures contracts (from all asset classes – Energy contracts, Commodity contracts, Fixed Income contracts, Foreign Exchange contracts, Equity Index contracts and Short-Term Interest Rate contracts – STIR henceforth) as they are assumed to be fairly dispersed and representative of all asset classes around the globe. For this purpose, we looked at the S&P Systematic Global Macro Index (S&P SGMI) which is intended to represent the global futures universe. It uses a quantitative methodology to update prices from a diversified portfolio that includes over 30 assets from commodities to foreign exchange and financial future contracts. As such, we analyze a total of 38 futures contracts (37 from the S&P SGMI) plus the FTSE100 futures contract. In Figure 3.1 we present the total number of contracts per asset class.

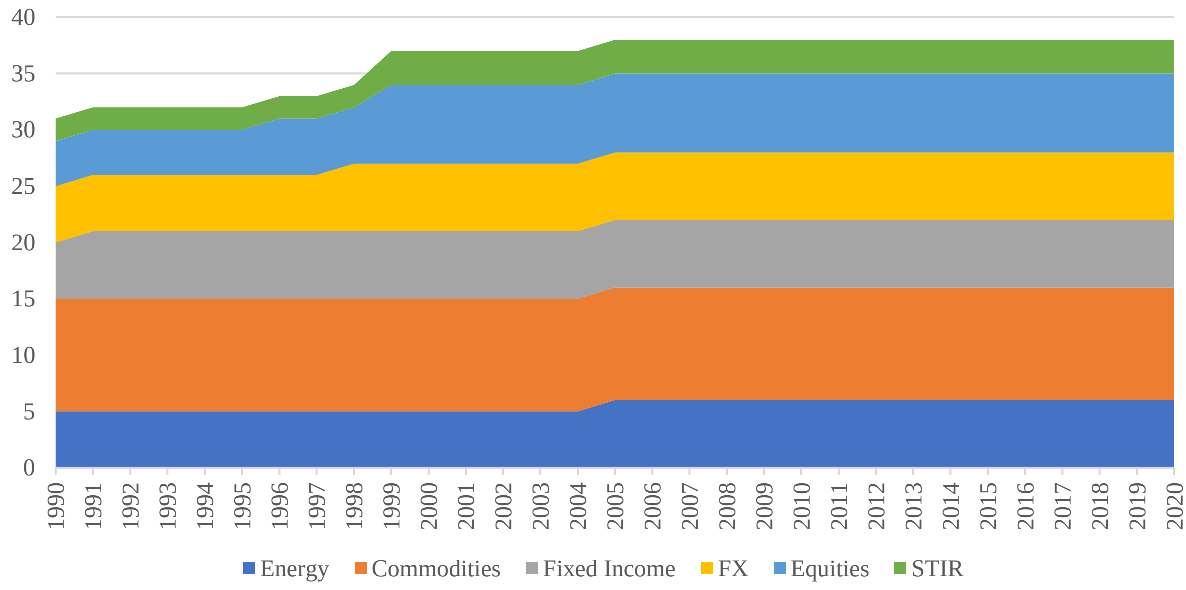


Figure 3.1 – Number of contracts per asset class
Source: Own calculations

The inclusion of future contracts from all asset classes in this dissertation is justified by its increasing employment in hedge funds, to increase diversification.[1] These contracts are usually associated with positive skewness, low transaction costs, high liquidity and a high degree of transparency. It is important to highlight that futures contracts have specific features that differ from the most common spot cash instruments. Firstly, futures contracts have finite maturity, meaning that they are traded for a short period, mainly before maturity. Secondly, futures contracts do not require a significant initial margin payment, and, in theory, no capital is required to start a long or short position.

The starting date for the data considered is 1990 and will be restricted to 2020 to contemplate a significant number of contracts. As shown in Figure 3.1, the number of contracts included increases as

[1] As showed by Clare et al. (2016)

future contracts became more and more traded over time. Furthermore, there will be a focus on the last decade to evaluate the impact of the post-2008 financial crisis on portfolio's returns, as well as over the last two years to infer the impact of the current pandemic in the performance of the portfolios. It is expected that the data from 2020 suffers a significant impact in terms of volatility of some assets due to the COVID-19 outbreak and its impact on financial markets. The main sources of data were Yahoo Finance and Bloomberg. Moreover, to avoid currency fluctuations, we analyze prices quoted in US Dollars, thus currency effects are inexistent in our research.

Table 3.1 – Descriptive statistics per asset class

	Energy	Commodities	Fixed Income	FX	Equities	STIR
Mean Return	11.15%	0.73%	3.56%	1.18%	7.31%	0.37%
Excess return	2.69%	-7.73%	-4.90%	-7.28%	-1.15%	-8.09%
Volatility	31.58%	22.46%	5.31%	8.40%	16.80%	0.41%
Sharpe Ratio	0.09	-0.34	-0.92	-0.87	-0.07	-19.65
Skewness	0.62	0.46	0.19	-0.02	0.42	1.77
Kurtosis	6.19	2.75	5.87	2.36	8.55	11.33

Note: Excess returns are calculated based on the US 30-year yield rate of 8.46% as of January 1990. Sources: Bloomberg, Federal Reserve Economic Data and own calculations.

Table 3.1 describes the main descriptive statistics of the six asset classes. For the full sample period, all asset classes present a positive mean return, with Energy contracts on the lead while STIR are the contracts with the lowest value. In terms of excess return, Energy contracts are the only class to display a positive excess return and a positive Sharpe Ratio. As expected, STIR contracts are the less volatile class while the highest excess return of the Energy class is at the cost of the highest volatility among all classes. Table 3.2 displays the entire list of contracts per asset class employed in the study along with summarized statistics. It stands out that STIR contracts exhibit small volatilities (3 Month Euribor exhibits the smallest average annual volatility with a value of 0.26%) while Energy contracts are the most volatile (Natural Gas exhibits the largest average annual volatility with a value of 49.05%). Similarly, the 3 Month Euribor contract presents the lowest annual average return and Brent Crude the highest value. Figure 3.2 displays the monthly return densities. Annexes A – F display the correlations between assets within each asset class.

Table 3.2 – Descriptive statistics per contract

	Initial date	Obs.	Avg return	Avg volatility	Skewness	Kurtosis
Energy						
Natural Gas	May-90	369	6.32%	49.05%	0.88	2.64
Heating Oil	Feb-90	372	10.53%	30.21%	0.43	2.27
Gas Oil	Feb-90	372	13.68%	29.57%	0.27	1.50
Crude Oil	Feb-90	372	10.32%	33.21%	1.19	12.13
Brent Crude	Feb-90	372	14.37%	30.42%	0.09	4.65
RBOB Gasoline	Nov-05	183	11.65%	17.01%	-0.99	5.58
Commodities						
Sugar #11	Feb-90	372	3.30%	27.04%	0.19	0.74
Live Cattle	Feb-90	372	-0.24%	12.74%	-0.48	1.79
Coffee "C"	Feb-90	372	-2.69%	31.59%	1.14	3.26
Cotton #2	Feb-90	372	-1.86%	23.67%	0.20	0.64
Soybeans	Feb-90	372	5.41%	20.80%	-0.04	0.76
Corn	Feb-90	372	-4.85%	22.77%	0.34	1.19
Wheat	Feb-90	372	-4.43%	25.23%	0.43	1.56
NA Copper	Feb-90	372	8.20%	21.83%	-0.01	2.74
Gold (100 oz.)	Feb-90	372	2.12%	13.77%	0.17	1.21
Silver	Feb-90	372	2.32%	25.16%	0.26	1.06
Fixed Income						
T-Notes (10 Yr)	Feb-90	372	3.68%	5.17%	0.18	1.90
T-Notes (5 Yr)	Feb-90	372	2.54%	3.45%	0.16	1.11
T-Bonds (30 Yr)	Feb-90	372	4.97%	8.36%	0.15	2.00
Euro-Bund	Dec-90	362	4.80%	9.10%	0.03	1.96
Euro-Bobl	Nov-91	351	2.37%	2.53%	0.01	0.09
JGB	Feb-90	372	3.01%	3.22%	-0.45	4.66
FX						
Euro FX	Jun-98	272	1.17%	6.32%	0.00	1.14
Japanese Yen	Feb-90	372	-0.25%	9.25%	0.56	3.25
British Pound	Feb-90	372	1.12%	8.29%	-0.54	2.14
Australian Dollar	Feb-90	372	2.79%	10.04%	-0.26	1.55
Canadian Dollar	Feb-90	372	0.81%	6.84%	-0.29	2.77
Swiss Franc	Feb-90	372	1.43%	9.64%	0.11	1.21
Equities						
Nasdaq Complex	Jul-99	259	10.61%	14.03%	-0.41	1.78
S&P 500 Index Complex	Feb-90	372	7.95%	12.74%	-0.57	1.35
Euro Stoxx 50	Feb-99	264	6.51%	13.73%	-0.30	2.75
Dax	Dec-90	362	8.06%	19.18%	-0.45	1.63
Kospi 200	Jun-96	296	9.01%	23.74%	0.99	6.19
Nikkei 225 Futures	Feb-90	372	3.57%	19.38%	0.03	1.29
FTSE 100	Feb-90	372	5.44%	14.82%	-0.24	1.32
STIR						
Eurodollars (3-month)	Feb-90	372	0.62%	0.60%	1.42	6.02
3 Month Euribor	Jan-99	265	0.16%	0.26%	3.03	15.32
3 Month Euroyen	Feb-90	372	0.34%	0.37%	0.67	5.69

Source: Bloomberg and own calculations.

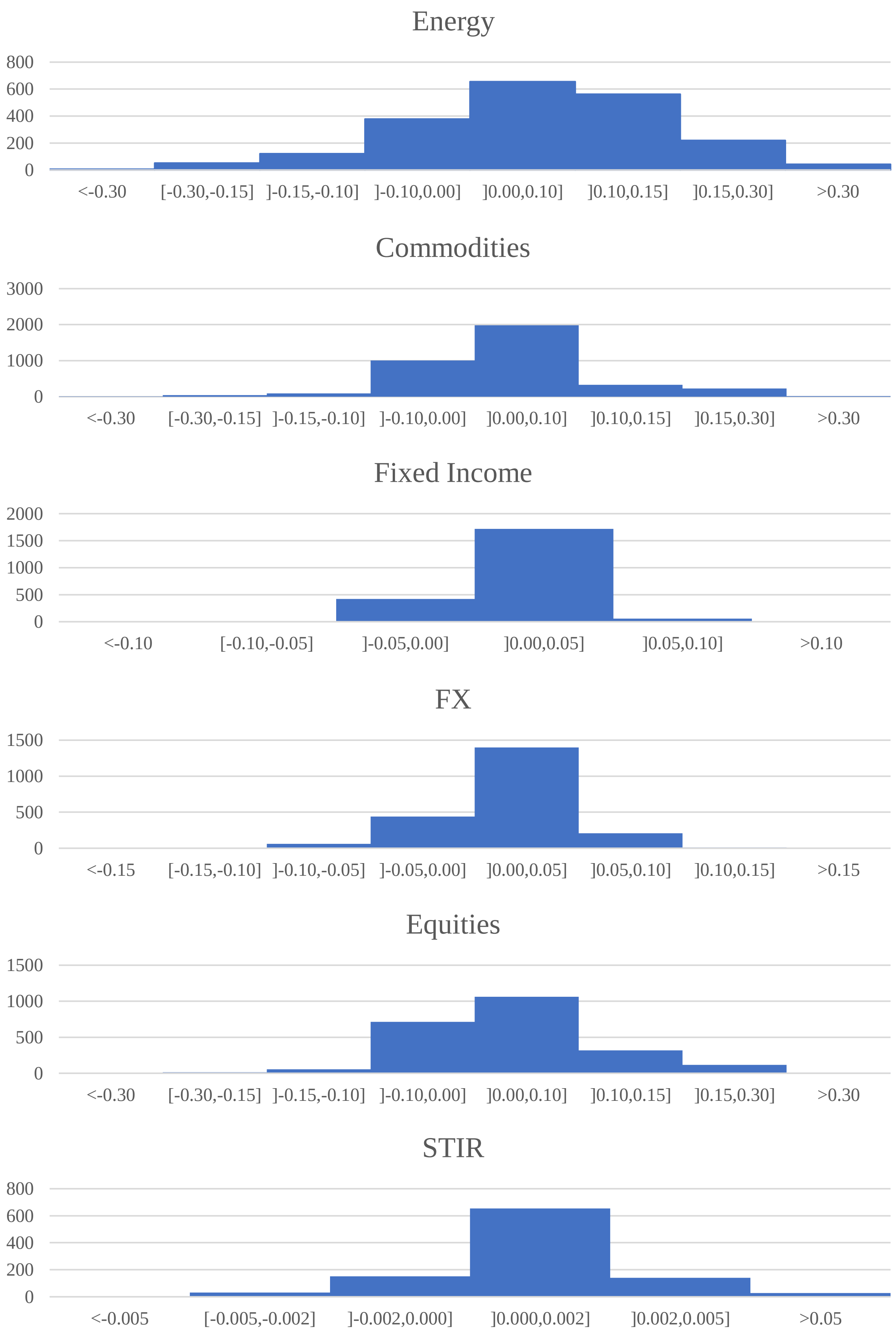


Figure 3.2 – Monthly return densities
Source: Own calculations

3.1 Continuous price series in future contracts

Working with future contracts is not straightforward because they are short-lived instruments that expire from time to time. For example, let us consider a future contract starting in 1990, the beginning of the sample period. After the maturity date, that specific contract is not active anymore, causing the series of that contract to end up at that point. The standard approach to avoid this difficulty is to find neighboring contracts and splice together the time series to effectively replicate the investor's position and the, before maturity, roll-over the position onto a different contract of the same underlying asset with a later date of maturity.

The application of this scheme presupposes the use of the most heavily traded contract, usually measured by open interest or volume and is almost always the contract that is closer to maturity (commonly referred to as front contract). Hence, the roll-over should take place when the liquidity jumps to the contract that is closer to maturity, usually referred to as a first-back contract. By doing this, we can assure the use of the most liquid contract at each point in time, i.e. at each month (Mokowitz, Ooi, & Pedersen, 2012).

Although this approach seems rational, we must consider an important issue from the roll-over strategy. In theory, future contracts are unfunded investments, that is, there is no initial margin to be paid to start a position. However, in practice, this may not be true because there is no particular reason why the price of the two similar contracts (where the only difference is the maturity date) that are part of the roll-over should be the same. Therefore, splicing the time series of these futures contracts together would result in a price, which would then result in an untrue return that is not capitalized by the investor. In addition, in the case of a contract that is usually in a contango (a situation when futures contracts are trading at a premium to the spot price), this would bias upwards the average return. Conversely, when a contract is almost always in a backwardation (a situation when the current price of an underlying asset is higher than prices trading in the futures market), the average return of the asset is shifted downwards.

Undoubtedly, we need to carefully look at and ease this issue. To get rid of the price jump from $\left|F_{t,T_2} - F_{t,T_1}\right|$ of a continuous price series when a roll-over takes place, let us consider F_{t,T_1} and F_{t,T_2} denote the time t futures contracts of two contracts of the same underlying with maturities T_1 and T_2, we can:

- Multiply backwards the entire price path up to time t with the price ratio $\frac{F_{t,T_2}}{F_{t,T_1}}$,
- Add backwards to the entire price path up to time t the price difference $F_{t,T_2} - F_{t,T_1}$

Independently of the chosen scheme, the entire price path of the roll-over will be appropriately shifted to guarantee that the prices of the neighboring contracts are the same and equal to the price of the more recent contract. Nevertheless, we can immediately consider a complication with the backwards-difference adjustment: the historical distortion of the percentage change. For instance, the return between 10 and 20 is not the same as $10 + k$ and $20 + k$, a feature that complicates the back-testing setting. This would systematically bias the average return downwards or upwards in the case of the previously mentioned contango or backwardation. Besides, in the case of backwardation, the difference would be negative, resulting in negative historical prices.

These issues can be solved using the backwards ratio adjustment. Considering the previous example, the return between 10 and 20 is the same as the return from $10k$ and $20k$. Hence, we can use this methodology for the roll-over of contracts, resulting in a series that can be used for back-testing purposes.

In order to get the backwards ratio adjusted continuous price series with the roll-over taking place when liquidity shifts between contracts, we make use of Bloomberg's generic contracts using the adjustment through the <GFUT> screen.

4. Methodology

In this chapter, we present the implementation of the investment strategies presented in section 2, with a special focus on the Trend Following Risk Parity framework. Firstly, to evaluate the performance of the strategy during the periods considered in the previous section, we are going to use quantitative data while other qualitative matters (such as perceptions, time horizon expectations and adverse selection) will not be directly considered. Before analysis, the data will be gathered and checked for missing information, as for the futures contracts available at the beginning of the sample period. The data were analyzed and compared using Excel.

4.1 Risk and Return Measures

As previously mentioned, one of the major problems of portfolio construction is the ability to accurately access expectations on future returns, volatility and covariances. Regarding volatility, we can infer the general volatility of a portfolio through the individual asset risk. Once the variance-covariance matrix, Ω, is defined and the vector of weights determined, the overall portfolio's volatility is given by:

$$\sigma_p = \sqrt{\boldsymbol{w}_p' \Omega \boldsymbol{w}_p} \tag{11}$$

where $\boldsymbol{w}_p$ represents the $N \times 1$ vector of weights and Ω the $N \times N$ variance-covariance matrix.

Given that the true values are unknown, the most common way to estimate them is by recurring to historical data. To calculate the holding period return, we make use of the rationale suggested by Baltas and Kosowski (2013) about future price series. As previously mentioned, in theory, futures are instruments that do not require any capital to open a new position. However, in practice, this is not exactly true because opening a new position involves posting cash as collateral (usually referred to as initial margin). Let us consider F_t and F_{t+1} the continuous futures price at the end of t and $t+1$, respectively. The initial margin is denoted by M_t that earns the risk-free rate r_t^f, so that the margin is expected to grow $M_t(1 + r_t^f)$ at the end of the month. The cash amount at the end of the month in the margin account, assuming no variation in margin payments, is then the expected growth of the margin plus the capital gain/loss of the contract:

$$M_t(1 + r_t^f) + (F_{t+1} - F_t) \tag{12}$$

As a result, the holding period return in excess of the risk-free rate is given by:

$$r_t^i = \frac{[M_t(1 + r_t^f) + (F_{t+1} - F_t)] - M_t}{M_t} - r_t^f = \frac{F_{t+1} - F_t}{M_t} \tag{13}$$

Assuming the extreme scenario where the initial margin requirement is equal to the prevailing futures prices, that is $M_t = F_t$, then the return in excess of the risk-free rate becomes:

$$r_t^i = \frac{F_{t+1} - F_t}{F_t} \tag{14}$$

We can derive the asset's i annual compounded mean return, $\bar{R}_i$, from with T monthly observations, $R_i = \{r_1^i; \dots, r_T^i\}$ as the geometric mean:

$$\bar{R}_i = \left(\prod_{i=1}^{T} r_i\right)^{\frac{1}{T}} \tag{15}$$

Consequently, the portfolio's expected return can be defined as the linear product of the individual asset's weights and mean returns:

$$\bar{R}_p = E[R_p] = \sum_{i=1}^{N} w_i \bar{R}_i \tag{16}$$

4.2 Assessing Portfolio Performance

The evaluation of a portfolio performance involves the determination and comparison of a portfolio relative to a comparable benchmark. Risk-adjusted performance methods adjust the return to account for the different levels of risk and capture the return per unit of risk. To evaluate and compare the performance several approaches can be used and resulting in different interpretations. The most common approach is to use the Sharpe Ratio introduced by Sharpe (1966), a risk-adjusted measure that measures the average excess expected return (the difference between the portfolio's return and risk-free) per unit of total risk (volatility) of an investment. This metric requires two inputs: Excess returns and volatilities. While the numerator captures the reward of investing in the portfolio over the risk-free rate, the denominator captures the variability of the returns of the portfolio. Commonly, in the case of a portfolio whose Sharpe Ratio is higher than the benchmark, we can say that the portfolio has outperformed in terms of excess return per unit of risk. A common practice is the use of the annualized Sharpe Ratio obtained by multiplying with a factor of $\sqrt{T} = \sqrt{12}$ given that data will be gathered monthly. Considering individual and independent variables, the annualized Sharpe Ratio is represented by the following formula:

$$SR_p = \frac{E[R_p] - R_f}{\sigma_p} \times \sqrt{T} \tag{17}$$

Apart from the Sharpe Ratio, the other two approaches used are the Sortino Ratio and the Calmar Ratio. The Sortino Ratio is a derivation from the Sharpe Ratio that considers harmful volatility instead of total volatility. In practice, the inputs used are almost the same with a small difference on the denominator because of the consideration of pure downside risk. Since the Sortino Ratio focuses on the negative deviation from the mean, it provides a better view of the risk-adjusted performance of a portfolio. Algebraically, it is derived as:

$$Sortino\ Ratio_p = \frac{E[R_p] - R_f}{\sigma_d} \tag{18}$$

where σ_d represents the standard deviation of negative portfolio returns. Regarding the Calmar Ratio, it is a measure of risk-adjusted returns that was developed and introduced by Terry Young in 1991. The ratio uses the annual rate of return (usually considering a time frame of 36 months) divided by the Maximum Drawdown (MDD):

$$Calmar\ Ratio_p = \frac{E[R_p] - R_f}{MDD} \tag{19}$$

where the Maximum Drawdown is an indicator of downside risk and measures the maximum fall in the value of the investment, given by:

$$MDD = \frac{Trough\ Value - Peak\ Value}{Peak\ Value} \tag{20}$$

Despite being criticized for focusing on drawdown and ignoring volatility, the Calmar Ratio is more understandable than other strategies and it is said to be better the higher it is. The use of the three performance evaluators relies on the hope of a more stable and reliable analysis. Finally, to check the statistical significance of the results, we plan to use a one-sided and two-sided t-test.

4.3 Trend Following and Risk Parity frameworks

Baltas (2015) presents other algebraical frameworks to the approaches presented in section 1. Based on the same assumptions, Baltas proposes constructing generic strategies resorting to optimizations that are constrained by non-linear equations. After a few derivations and adjustments, we can arrive with Methodologies for Trend Following and Risk Parity alone and Trend Following and Risk Parity combined.

In regards to Trend Following, it is important to recall that the strategy resorts in long and short positions in each asset i, that is asset i can have a positive or negative solution. Then, let x_t^i denote the amount of US dollars invested in asset i at time t, such that $x_t^i > 0$ stands for long positions and $x_t^i < 0$ stands for short positions. In addition, let N_t denote the number of available futures contracts at time t. The return of this strategy is calculated as:

$$r_{t,t+1}^{TF} = \sum_{i=1}^{N_t} \frac{x_t^i}{\sum_{j=1}^{N_t} |x_t^j|} \times r_{t,t+1}^i = \sum_{i=1}^{N_t} w_t^{Net,i} \times r_{t,t+1}^i \tag{21}$$

where, $w_t^{Net,i} = \frac{x_t^i}{\sum_{j=1}^{N_t} |x_t^j|}$ denotes the net weight invested (or divested) in asset i at time t. These weights do not have to add up to 100% since they can take positive or negative values. Trend Following allows for long and short positions. However, many supporters of the strategy argue that it should be run with a target constant level of volatility (σ_{TGT}). This is a theoretical and subjective level that refers to investors' perceptions, desires and feelings. As explained above, since the Risk Parity strategy seems to fall between the Capital Market Line and the minimum-variance portfolio line, it is reasonable to assume that the target level of the investors must fall somewhere between the minimum-variance portfolio and the tangency portfolio. Let us consider that the minimum-variance portfolio has a standard deviation of 5% while the tangency portfolio has a standard deviation of 15%. This means that, assuming no rebalancing costs, every investor willing to take a target standard deviation between 5% and 15% may consider Risk Parity while investors willing to take either 5% or 15% may be indifferent between the Risk Parity or one of the other two strategies.

Introducing the Volatility-Parity described in the Literature Review, the amount of US dollars invested asset i at time t is given by:

$$x_t^{TF:VP,i} = \frac{sign\ (r_{t-J,t}^i)}{\sigma_t^i} \tag{22}$$

where J represents the lookback period, typically equal to 12 months.

By joining the two conditions, we end up with the strategy given by TF:VP strategy:

$$r_{t,t+1}^{TF:VP} = \frac{\sigma_{TGT}}{\sigma_t^{TF}} \sum_{i=1}^{N_t} sign\left(r_{t-J,t}^i\right) \times \frac{(\sigma t_t^i)^{-1}}{\sum_{i=1}^{N_t} (\sigma t_t^i)^{-1}} \times r_{t,t+1}^i \tag{23}$$

where, σ_t^{TF} denotes the unlevered realized running volatility of the Trend Following strategy.

Regarding Risk Parity alone, it represents an extension of the Volatility-Parity scheme that is only defined in a long-only framework. The objective function of this strategy can be given by the maximization of logarithmic weights as such:

$$\begin{gathered}\max \sum_{i=1}^{N_t} \log\left(w_t^i\right) \\ subject\ to: \\ 1)\ \sqrt{\boldsymbol{w_t'} \times \textstyle\sum_{\boldsymbol{t}} \times \boldsymbol{w_t}} \leq \sigma_{TGT} \\ 2)\ w_t^i > 0, \forall i \\ 3)\ \sum_{j=1}^{N_t} w_t^j = 1\end{gathered} \tag{24}$$

However, the formulation presented is only applicable to long-only portfolios as the $\log\left(w_t^i\right)$ can only be defined for positive weights. Allowing the framework to long and short positions requires the introduction of more asset-specific information to score the optimization and allowing diminishing returns. In regards to the first one, the most common approach is to use the information in the form of expected returns. On the other hand, the inclusion of diminishing returns requires that the marginal increase in the return decreases as the position on that asset increases. To achieve these purposes, considering μ_t^i the score of asset i at time t, the optimization of equation 24 becomes:

$$\max \sum_{i=1}^{N_t} |\mu_t^i| \times \log(w_t^i)$$
$$subject\ to:$$
$$1)\ \sqrt{\boldsymbol{w_t'} \times \boldsymbol{\Sigma_t} \times \boldsymbol{w_t}} \le \sigma_{TGT}$$
$$2)\ w_t^i > 0, if\ \ \mu_t^i > 0 \tag{25}$$
$$3)\ w_t^i \le 0, if\ \mu_t^i < 0$$
$$4)\ \sum_{j=1}^{N_t} |w_t^j| = 1$$

The key difference between equations 24 and 25 relies on the introduction of a score in the form of an absolute value in the objective function and, consequently, in the optimization constraints. More significantly, this new formulation provides the introduction of an important feature that regards the types of positions – long or short. The positions are fully determined by the sign of the scores as such:

- Assets with positive returns have a long position in the portfolio;
- Assets with negative returns have a short position in the portfolio.

For simplification reasons, we assume that in the case of a score equal to zero, investors should undertake a positive position.

4.3.1 When Trend Following meets Risk Parity

Considering the previous framework, it is now very simple to combine the Risk Parity approach with the sign of the Trend Following one. On the one hand, the Trend Following strategy provides a proper trading rule based on the sign of the past returns, it lacks an efficient weighting scheme that accounts for pairwise correlations. On the other hand, Risk Parity provides an efficient weighting scheme and distributes the overall risk based on specific rules but absences a proper scoring methodology. So, considering that the two strategies can complement each other, let us consider that the score is given by the Trend Following signal (in this case the signal of the past 12-months returns), we can state that:

$$\mu_t^i = sign\left(r_{t-12,t}^i\right) \tag{26}$$

By applying the previous equality in equation 25, we create the Trend Following Risk Parity (TF:RP) approach:

$$\max \sum_{i=1}^{N_t} \log\left(|w_t^i|\right)$$
$$subject\ to:$$
$$1)\ \sqrt{\boldsymbol{w_t'} \ \times \ \sum_{\boldsymbol{t}} \times \boldsymbol{w_t}} \leq \sigma_{TGT}$$
$$2)\ w_t^i > 0, if\ sign\left(r_{t-12,t}^i\right) = 1 \tag{27}$$
$$3)\ w_t^i \leq 0, if\ sign\left(r_{t-12,t}^i\right) = -1$$
$$4)\ \sum_{j=1}^{N_t} |w_t^j| = 1$$

Finally, in regards to the return of this framework, let $w_t^{RP,i}$ denote the weights from the above optimization such that the return of the strategy becomes:

$$r_{t,t+1}^{TF:RP} = \frac{\sigma_{TGT}}{\sigma_t^{TF}} \sum_{i=1}^{N_t} w_t^{RP,i} \ \times r_{t,t+1}^i \tag{28}$$

4.3.2 Variance-Parity versus Risk Parity

To conclude the analysis on the different optimizations used, it is important to highlight the main difference between the use of the Risk Parity weighting scheme and the volatility parity one. Considering that the Risk Parity weighting scheme considers that each portfolio constituent contributes to the same amount of the portfolio risk, we can derive the following expression:

$$w_t^{RP,i} \times MRC = constant, \forall i \tag{29}$$

It is also known that the weight of a portfolio constituent is proportionally attributed to the respective marginal contribution to risk of each portfolio's constituent. Additionally, the factor of proportionality means that the absolute weights must sum up to one. Overall, we can conclude that the weights of the Trend Following Risk Parity strategy (given the choice of μ_t^i) can be rewritten as the inverse marginal risk contribution weighted portfolio:

$$w_t^{RP,i} = \frac{(MRC_t^i)^{-1}}{\sum_{i=1}^{N_t}(MRC_t^i)^{-1}} \tag{30}$$

In contrast, the volatility-parity portfolio assumes that the weight of each asset is given by:

$$w_t^{VP,i} = \frac{(\sigma_t^i)^{-1}}{\sum_{i=1}^{N_t}(\sigma_t^i)^{-1}} \tag{31}$$

By looking at the two expressions, we can see the difference between them. As discussed in section 1, while the Risk Parity weighting scheme focuses on the risk contribution of each asset constituent by considering the MRC, the volatility-parity scheme focuses on the individual risk of each asset to allocate the weighting scheme. However, it is important to realize that in the case of all correlations being equal, the two weighting schemes are identical.

4.4 Hypotheses

The general purpose of this dissertation is to evaluate the performance of the Risk Parity frameworks and compare it with other traditional strategies, such as EW and the GMV. It also aims to understand whether the Trend Following Risk Parity approach can constantly outperform Trend Following alone (with a Volatility-Parity weighting scheme) and the Risk Parity strategy alone. The hypotheses listed below are based on the main findings of the literature review. The existence of evidence in the superior performance of the Risk Parity alone is mixed with some authors being in favor of the strategy, especially during market friction times, while others seem reluctant in attributing such a good performance to the strategy. Since the existing literature shows a little skewness towards the superiority of Risk Parity weighting schemes towards traditional strategies, the first hypothesis is as follows:

- **Hypothesis 1:** Risk Parity strategies can consistently outperform traditional strategies from a Sharpe Ratio perspective.

Since momentum strategies provide a good signal of which assets are expected to over and underperform in a given period, the introduction of such strategies in the Risk Parity weighting scheme is expected to provide a significant improvement to the strategy, when compared with the pure Risk Parity Strategy. As result, the second hypothesis can be driven as follows:

- **Hypothesis 2:** The TF:RP strategy provides better risk-adjusted returns than the RP strategy alone.

After evaluating whether the TF:RP provides better returns than the RP strategy alone, it is important to question what the best weighting scheme is to apply for a Trend Following strategy, namely, whether TF:RP provides a better risk-adjusted return than the TF:VP strategy. The third hypothesis is given by:

- **Hypothesis 3:** The TF:TR strategy can provide better risk-adjusted returns than the TF strategy using a Volatility-Parity weighting scheme.

Finally, considering the effects of global events in asset returns and risk, we observe the performance of all strategies over a recent period and expect a superior performance of the TF:RP strategy. Therefore, the last hypothesis is expressed as follows:

- **Hypothesis 4:** The TF:RP strategy can outperform other strategies over a 10-year investment period.

The next section presents the main findings of our study to conclude about these hypotheses.

5. Results

This chapter details the results of the portfolio constructions emphasized in the Methodology section and in the Literature Review. Through the examination of features such as return, risk, performance ratios and risk weighting, we aim to analyze the benefits of the various allocation strategies over the sample period, with a special focus on the portfolios using a Risk Parity weighting scheme. There is also a simulation of a hypothetical investment on some of the portfolio strategies. The main conclusions on the performance of the strategies during the 31-year investment period are presented in Table 5.1, which displays the average statistics for each strategy.

Table 5.1 – Portfolio Statistics (average values)

	EW	GMV	RP	TF:VP	TF:RP	MD
Return	3.98%	1.30%	4.89%	1.96%	10.02%	2.99%
Excess return	1.17%	-1.51%	2.08%	-0.84%	7.22%	0.18%
Volatility	7.90%	0.62%	2.08%	2.18%	6.76%	4.18%
Max. Drawdown	-24.65%	-17.50%	-21.99%	-14.04%	-17.94%	-28.26%
Sharpe Ratio	0.21	-1.73	0.34	-0.29	1.09	0.01
Sortino Ratio	0.12	0.05	0.19	0.34	0.85	0.00
Calmar Ratio	0.02	0.05	0.18	0.15	0.39	0.01

Source: Own calculations

All portfolios realized positive returns, ranging from 1.30% to 10.02% for the GMV and TF:RP, respectively. Similarly, in what concerns volatility, we observe the GMV presenting the lowest average value over the sample period (0.62%), while the EW presents the highest average value close to 8%. Looking at the differences in the weighting schemes, it is clear that portfolios with a Risk Parity weighting scheme surpass the gains from others, namely the volatility parity weighting scheme. When analyzing Sharpe ratios, we observe a shift from a negative position in the TF:VP strategy to a positive one in the RP portfolio. The shift is even more noticeable for the TF:RP, which achieves better performance ratios than the latter two. However, the drawdown of the simple RP strategy is, on average, close to 8 pp. higher. The hypothesis of equality in the mean return between the Volatility-Parity and Risk-Parity strategies is strongly rejected with a two-sided p-value of 3.59%. In the following pages, we analyze the strategies in more detail considering the above-presented results and their detail over the sample period.

5.1 Performance Analysis

In this section, we present the main results of our analysis of the different strategies for the full sample. Firstly, we analyze the 6 strategies in terms of Return and Excess Return. In Table 5.2 we display these returns from the best rewarder strategy to the lowest one. From a single return viewpoint, we observe the TF:RP strategy being a clear winner among all other strategies. Besides, we observe a superior performance of the Risk Parity strategies, being the ones that present higher returns and consequently higher excess returns. In this dimension, the TF:RP stands out with an average Excess Return that is three times higher than the pure Risk Parity strategy. Regarding the worst performers, we observe the TF:VP and GMV being the ones that would reward fewer investors over the sample period, generating a negative premium over risk-free. Apart from these two, all other strategies reward investors with a significant premium over risk-free, despite the discrepancies in this value.

Table 5.2 – Portfolio Returns (average values)

	TF:RP	RP	EW	MD	TF:VP	GMV
Return	10.02%	4.89%	3.98%	2.99%	1.96%	1.30%
Excess return	7.22%	2.08%	1.17%	0.18%	-0.84%	-1.51%

Source: Own calculations

Additionally, we present Figure 5.1 which exhibits the yearly returns for the different strategies. As we can observe, annual returns follow a similar pattern in terms of peak and trough values, that is, the strategies seem to change alongside, with the TF:RP strategy being the one which presents the highest returns, especially in the period after the 2008 financial crisis where results show a stronger dispersion when compared with the other strategies. It is also noteworthy that the EW strategy, which is often used as a benchmark strategy in the financial literature regardless of its lack of sophistication, presents a similar shape to optimized strategies, namely the Risk Parity approach. However, we should not regret saying that the EW strategy is the one that presents the lowest annual return among all strategies over the 31-year period (-23.50%) in the year 2008. The highest value was achieved by the MD strategy in 2009 reaching a value of (24.25%) following its second-poorest performance in the previous year (-10.27%).

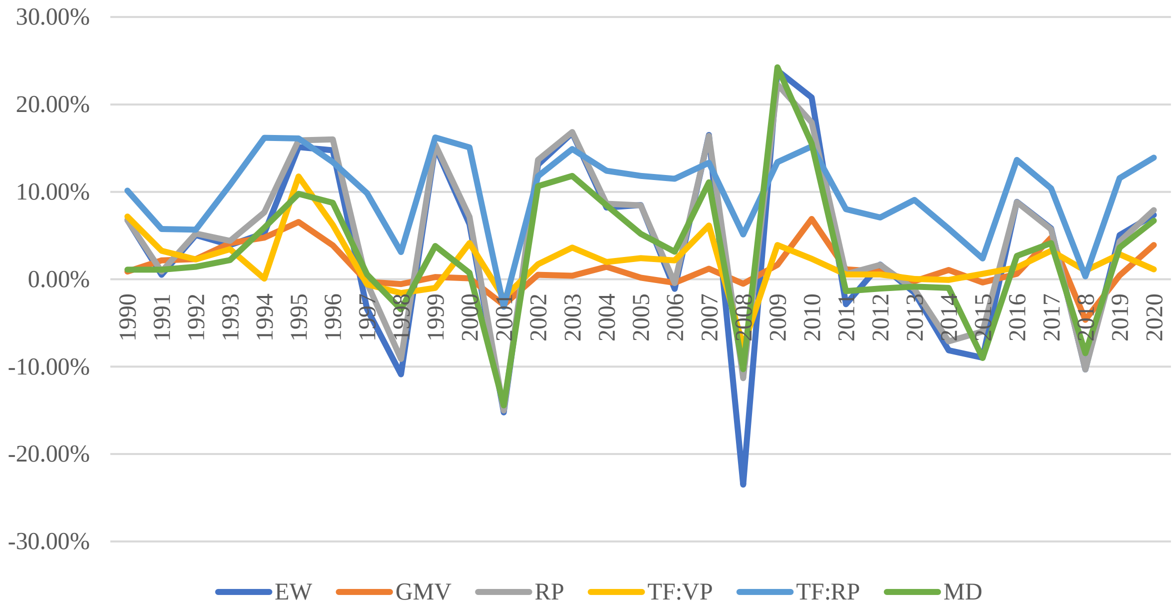


Figure 5.1 – Annual returns
Source: Own calculations

Interestingly, we can observe that for the vast majority of the strategies, the year of 2008 is highly penalizing while for the best performing strategy the most penalizing year in terms of annual returns is the year of 2001. Comparing the traditional strategies, we can observe that the EW strategy achieves better returns when these strategies achieve positive values while the opposite occurs when both GMV and EW achieve negative returns. In 2011, GMV achieved a positive return of 1.14% while the EW scheme obtained a return of -2.85%.

Portfolio returns are connected to the risk inherent in each strategy. Concerning the average performance of these strategies, as shown in Table 5.3, we witness the GMV strategy with the lowest volatility among all strategies, despite presenting the second-lowest drawdown. As expected, the EW approach is the one that presents higher volatility and a Maximum Drawdown of around 25%. From this perspective, that is from a downside risk perspective, the Most Diversified portfolio is the one that has suffered more from severe losses with average drawdowns of around 28%.

Table 5.3 – Portfolio Volatilities (average values)

	GMV	**RP**	**TF:VP**	**MD**	**TF:RP**	**EW**
Volatility	0.62%	2.08%	2.18%	4.18%	6.76%	7.90%
Max. Drawdown	-17.50%	-21.99%	-14.04%	-28.26%	-17.94%	-24.65%

Source: Own calculations

Figure 5.2 presents the yearly volatilities for each strategy from 1990 to 2020. We can observe that the most noteworthy volatilities are registered in periods of higher uncertainty, namely the financial crisis and the COVID-19 outbreak. During the 2008 financial crisis, most portfolio volatilities spiked with the biggest values being registered in this period. The exemption to this behavior is GMV, which was able to maintain its objective. It is not by chance that this strategy was the best performer in terms of volatility, since the focus of this approach is in minimizing the volatility through the change in individual asset weights. On average terms, we observe that strategy outperformed reaching a value of 0.62% despite the good performance of the TF:VP approach. The latter, through the inverse volatility weighting scheme, can closely approach the GMV strategy. However, the strategy diverged during the 2008 financial crisis, reaching one of its highest values in the entire period (6.82%), thus revealing a huge sensitivity to asset volatilities and covariances.

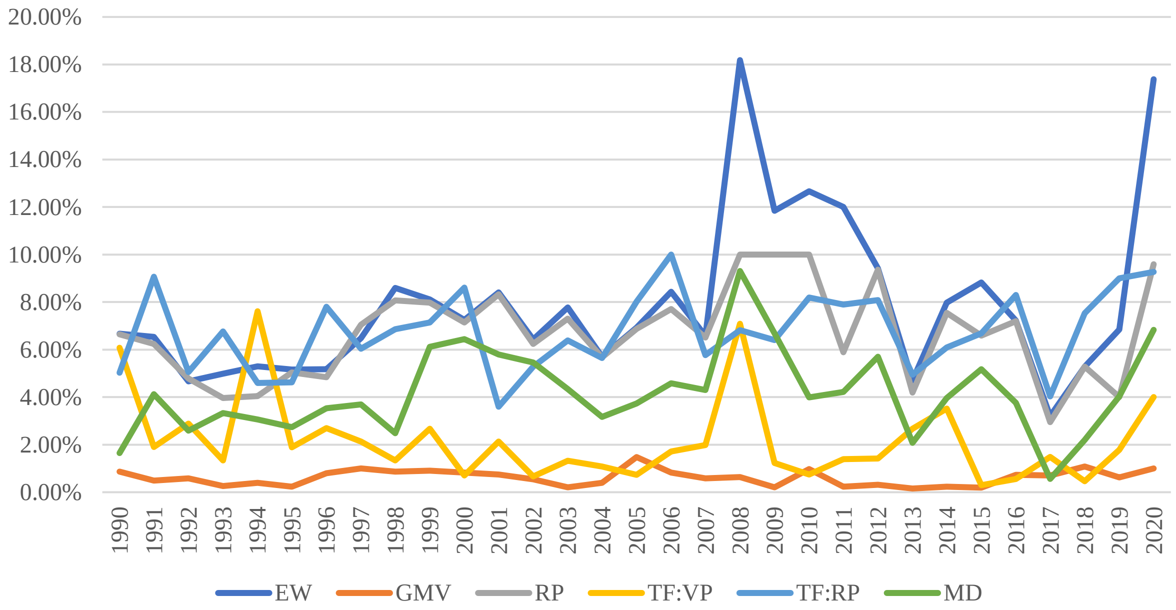


Figure 5.2 – Yearly volatilities
Source: Own calculations

In terms of the worst-performing strategies, we can observe significant changes in the portfolios following different behaviors. The EW scheme is the one that presents higher volatilities across the period, reaching the two highest values of 18.18% and 17.37% in 2008 and 2020, respectively. The results of this strategy should not surprise anyone since this portfolio is inherent to disregard any information in the attribution of its weights. Regarding the Risk Parity strategy alone and the TF:RP approach, both strategies achieve similar returns during the most uncertain periods, with some fluctuations during the rest of the sample period.

Recalling Figure 2.2 of the Literature Review, we observe the increase in pairwise correlations that became more pronounced after the global financial crisis. When we observe the MD portfolio strategy, we can see the decrease after that period, thus showing the effect of diversification in a portfolio strategy. However, the portfolio seems to adopt a similar shape in terms of volatility for 2020.

Regarding the performance ratios, in Table 5.4, we present the performance ratios for all strategies in the sample period. The TF:RP stands out with a noteworthy performance and the highest values for the Sharpe, Sortino and Calmar ratios. The Sharpe ratio over the entire period is three times higher than the second-best performer for this indicator, the single RP approach. Interestingly, the EW approach presents the third-best Sharpe Ratio, despite the simple method of allocation. In what consists of the worst-performers, we observe the GMV as the poorest performer, especially due to constant negative performances in terms of Sharpe ratios in the period before the 2000s. Looking at the Calmar Ratio, we can observe the optimized strategies being the ones that comprise the highest values.

Table 5.4 – Portfolio Performance ratios (average values)

	TF:RP	**RP**	**EW**	**MD**	**TF:VP**	**GMV**
Sharpe Ratio	1.09	0.34	0.21	0.01	-0.29	-1.73
Sortino Ratio	0.85	0.19	0.12	0.00	0.34	0.05
Calmar Ratio	0.39	0.18	0.02	0.01	0.15	0.05

Source: Own calculations

Regarding the main findings on Yearly Sharpe Ratio performance that are exhibited in Figure 5.3, we can find a significant fluctuation, especially in the TF:VP and GMV portfolios. This change is mainly explained by the small values in volatility accompanied by smaller (sometimes negative) returns when compared with other strategies, resulting in significant changes in the numerator and denominator, leading to higher fluctuations. This condition results in a situation where for the full sample the GMV portfolio achieves the highest (6.81) and lowest (-6.91) numbers in terms of Sharpe Ratios. In terms of the most stable portfolio, we can witness the great outcomes of the TF:RP strategy with constant positive returns and above most of the other strategies, apart from the years of 1998, 2001 and 2018 where it achieved negative returns, yet far better than all other strategies. In the last few years, the portfolios seem to converge towards a value around 1.00 with the TF:RP being the one that can beat other strategies nearly in every period.

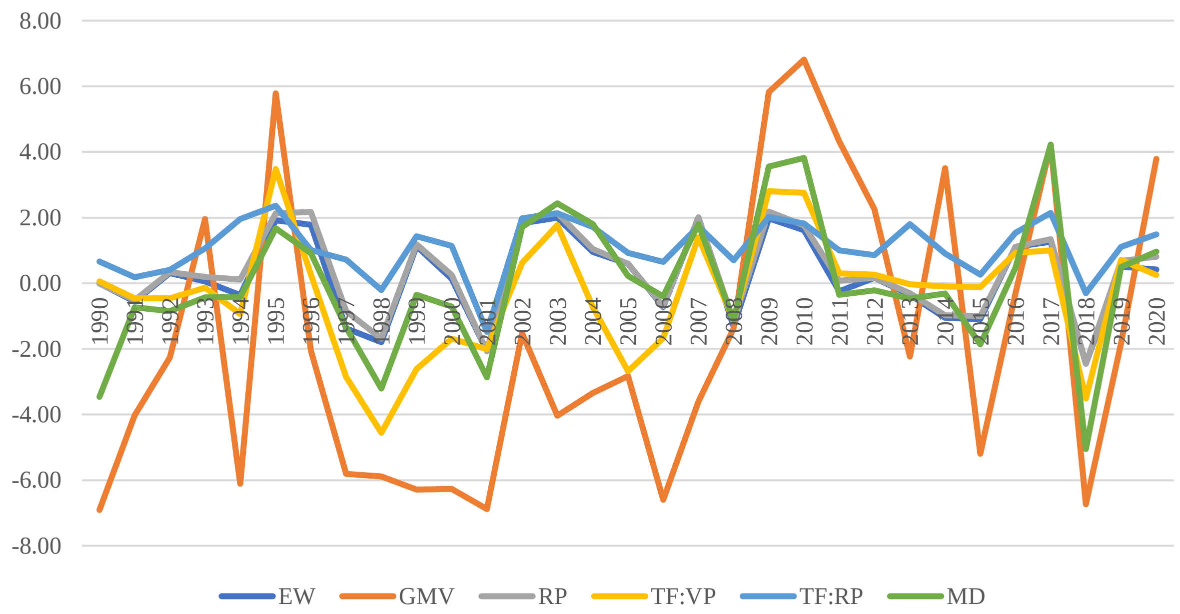


Figure 5.3 – Yearly Sharpe ratios
Source: Federal Reserve Economic Data and own calculations. Yearly Excess returns are calculated based on the 1-Year Treasury Constant Maturity Rate.

One can also observe the particularly low values for most portfolios during four main periods: the Dot-com bubble at the beginning of the century, the global financial crisis in 2008; the end of 2015; and in 2018 following the discussions on Brexit and rising interest rates environment. During these periods, most portfolios sharpened their volatilities and decreased their returns. Comparing the strategies, we observe a superior performance of the Risk Parity approaches that is even more pronounced in these periods. For example, in the year of 2018, the Sharpe Ratio of the TF:RP strategy is almost positive, reaching a value of -0.30, showing the effectiveness of the approach in these periods.

5.2 Risk Allocation outcomes

The investment strategy inherent to this portfolio relies on different strategies of asset allocations, either by traditionally choosing them or attributing a more sophisticated approach. In this sense, it is noteworthy to evaluate how an asset (or asset classes) has contributed to the volatility of a given portfolio. Considering the most volatile asset in the data set, which is the Natural Gas future contract, we can observe the dispersion of the asset allocation given by each strategy. For example, in 2020, the value for this asset among all portfolio strategies ranges from 0% in the TF:RP and 5.40% in the MD strategy. Similarly, the 3 Month Euribor future contract ranges from 2.63% in the EW scheme and 29.51% in the TF:VP portfolio strategy. This case is similar for the asset classes and varies according to the strategy employed and the period analyzed. Figure 5.4 displays the gross weights attributed to each asset class throughout the investment period.

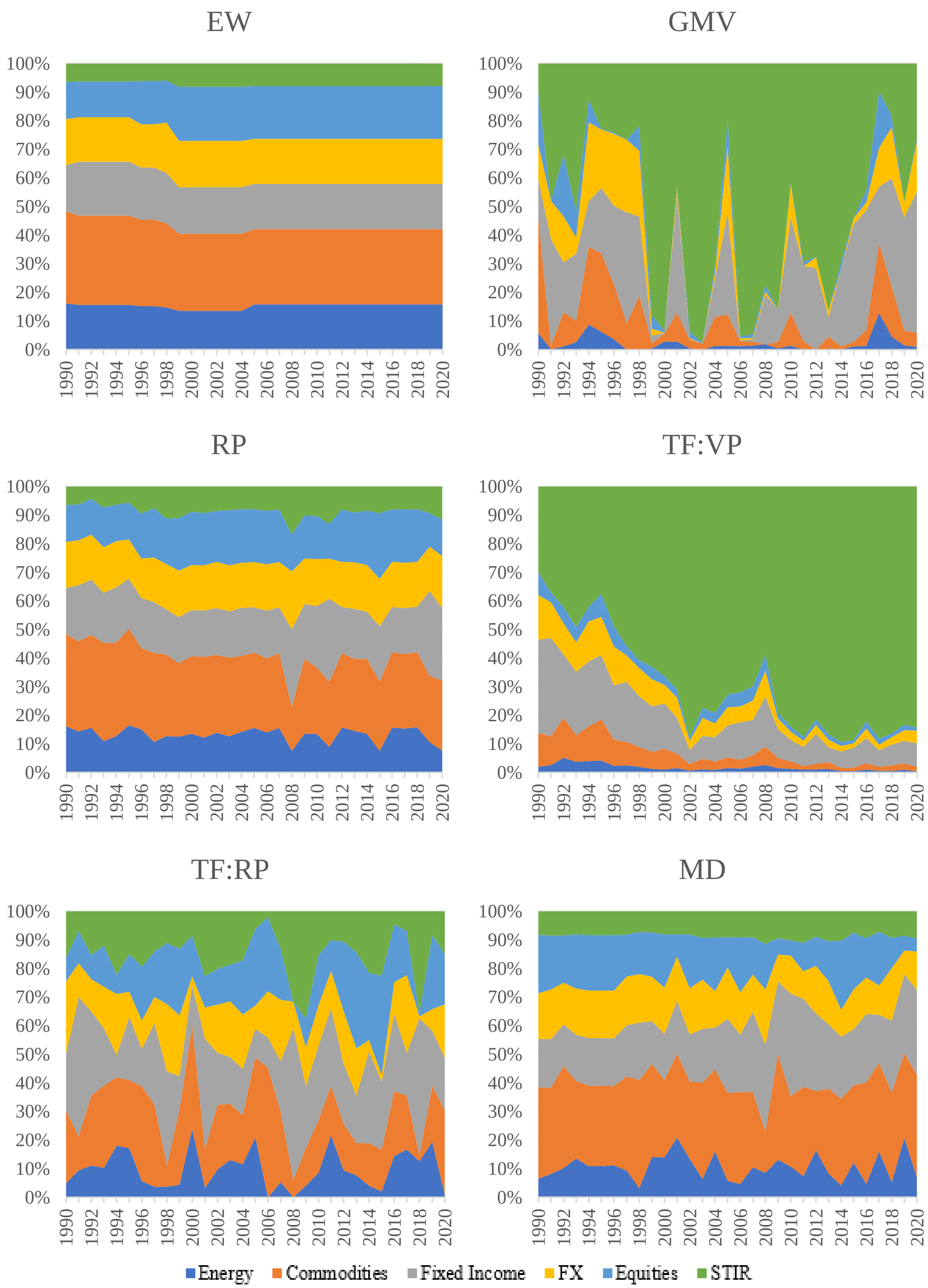


Figure 5.4 – Portfolio allocations per asset class from 1990 to 2020
Source: Own calculations

As argued by Risk Parity criticizers, portfolios that use these weighting schemes show a significant shift towards bonds. However, this is also the case of both GMV and TF:VP which are also positioned with a strong preference for short-term investments and bonds, with the STIR class representing an average value of 53% and 69% respectively. In terms of the more stable allocation strategies, the EW approach only changes due to changes in the number of assets and soothing after the introduction of the last contract in 2005. Similarly, the MD portfolio seems to follow a pattern, with an increasing relevance given to commodities contracts, which represent about 30% on average of the asset class weight attribution. Regarding the dispersion of the TF:RP strategy, it is possible to observe a significant change in the gross weight attribution over time.

To maintain the equal risk contribution target, the strategy shifts between asset classes to obtain the benefits from diversification. In the case of this strategy, the average weight for each strategy is quite similar, with a significant amount being attributed to Fixed Income contracts (23% on average) ranging from as low as 8% to as high as 53%. An interesting feature about this Figure is the fact that Risk Parity approaches tend to shift their weights towards safer assets in the periods of 2008 and 2018 to keep the equal risk contribution objective and reduce portfolio frictions.

Regarding risk contribution, Figure 5.5 denotes the comparable total risk contribution, considering the gross weights presented before. From a risk point of view, both Risk parity and Volatility parity weighting schemes are more balanced than the comparable traditional strategies. Nevertheless, the Trend Following Risk Parity approach does not balance risk contributions as well as other strategies due to high trading activity. At least one of the asset classes outweighs the other ones in terms of risk contribution in the EW, GMV and MD strategies, with Commodities and Energy contracts being the ones with the most percentage allocated. This situation is predictable for these strategies since they do not consider risk contribution in their weighting schemes, therefore using the most volatile asset classes, as described in the data section. In the case of the EW, although money allocations are equal, the risk contributions are far from being equal. In fact, total risk contributions are almost entirely dominated by Commodities and Energy contracts, with an insignificant contribution of Fixed Income and STIR. As for the case of the GMV portfolio, despite the heavy allocation in STIR contracts, the most total risk contribution is attributed to fixed income, with some spikes in favor of Energy contracts.

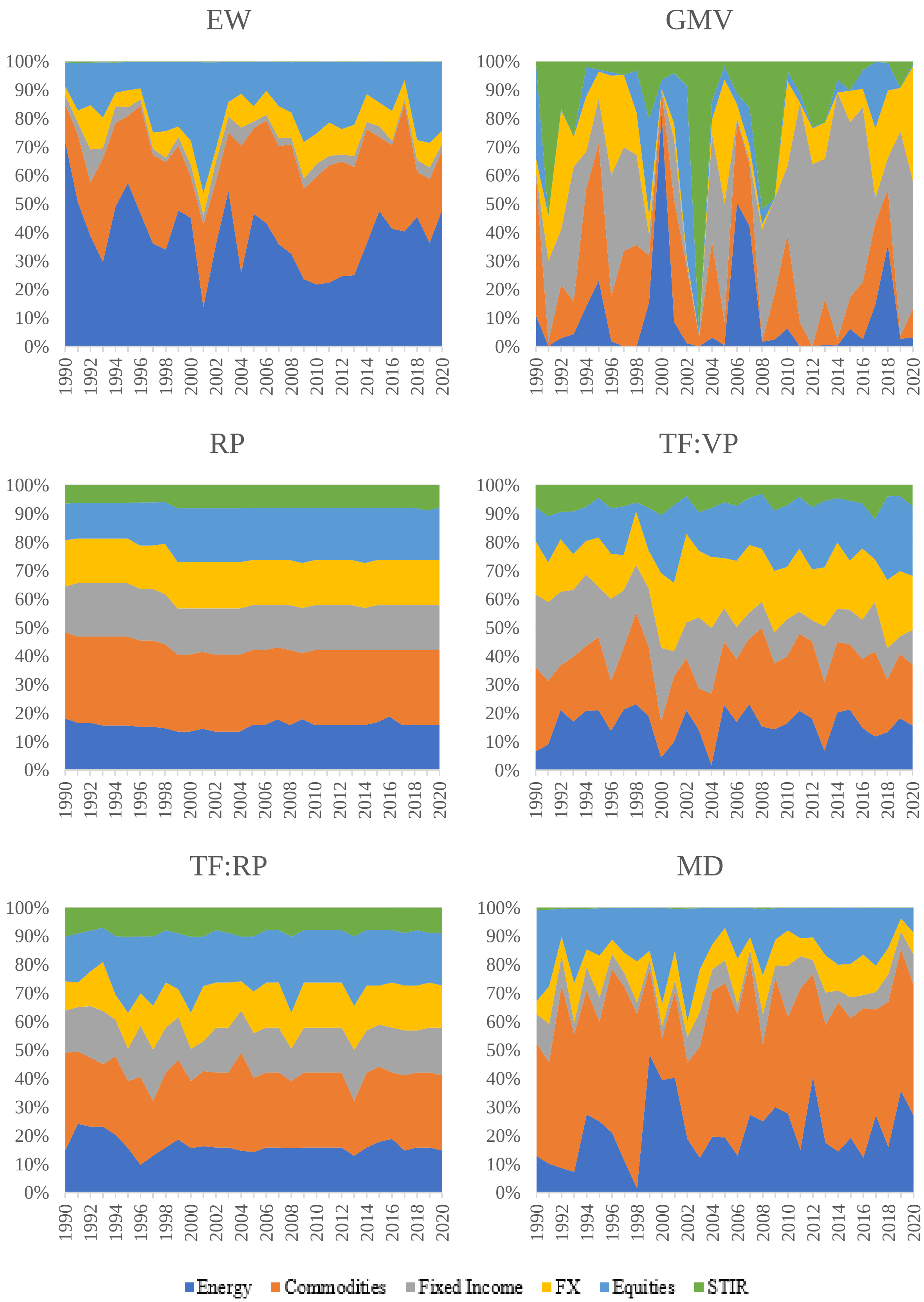


Figure 5.5 – Portfolio total risk contribution per asset class from 1990 to 2020
Source: Own calculations

5.3 Investment Analysis over a 10-year period

This section presents the performance of the different strategies in a shorter period of investment and analyzes a hypothetical investment over a 10-year period. In Figure 5.6 we start by analyzing the performance ratios during 2010-2020 to compare the best strategies over the investment period.

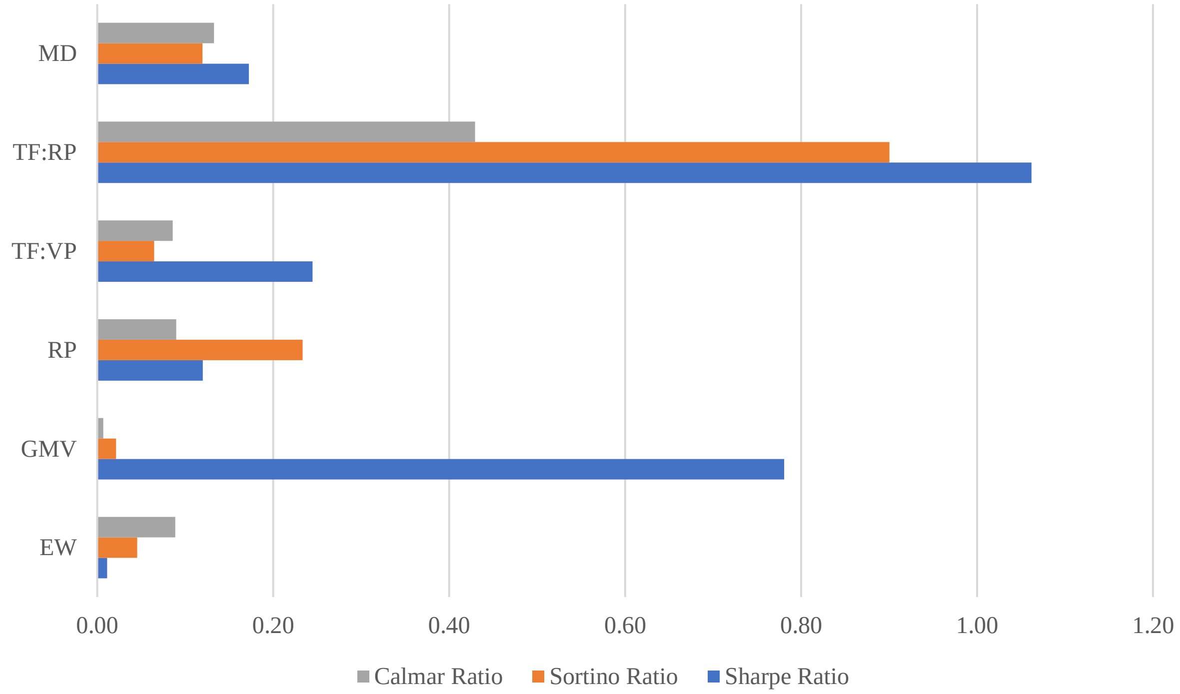


Figure 5.6 – Performance Ratios 2010-2020
Source: Own calculations

From the performance presented above, it is possible to verify a significant shift towards the TF:RP in all the statistics, especially in the Sortino Ratio, where the values significantly surpass all other strategies. Regarding this latter, we observe the Risk Parity weighting schemes as the best performers followed by MD portfolio, thus showing the significant impact of diversification in this period of increasing co-movements. The single Risk Parity approach portfolio can achieve a value that is twice the number for the MD portfolio and much better than all other traditional strategies. In the case of the worst-performing strategies, GMV stands out with a value of about 0.02. As for the case of Sharpe Ratios, we observe a strong positioning of GMV leveraged by constant small values in volatility. A value of 0.86 is still far from the one achieved by the TF:RP portfolio (1.06) which is again the best performer among the portfolios. In this case, the poorest performer is EW achieving just a small value of 0.01. Finally, in the case of the Calmar Ratio, apart from the TF:TP, the ratio is more balanced among the portfolios with values that range from 0.01 to 0.13.

The outstanding risk-adjusted performance of the TF:RP over the sample period is also reflected in the 10-year investment period. It is also noteworthy that considering the performance ratios of Figure

5.6 MD, RP and TF:RP strategies are the portfolios that show up and should be analyzed in more detail. In particular, in Figure 5.7 our purpose is to analyze a hypothetical investment over the last 10 years to understand which strategy would deliver a higher return.

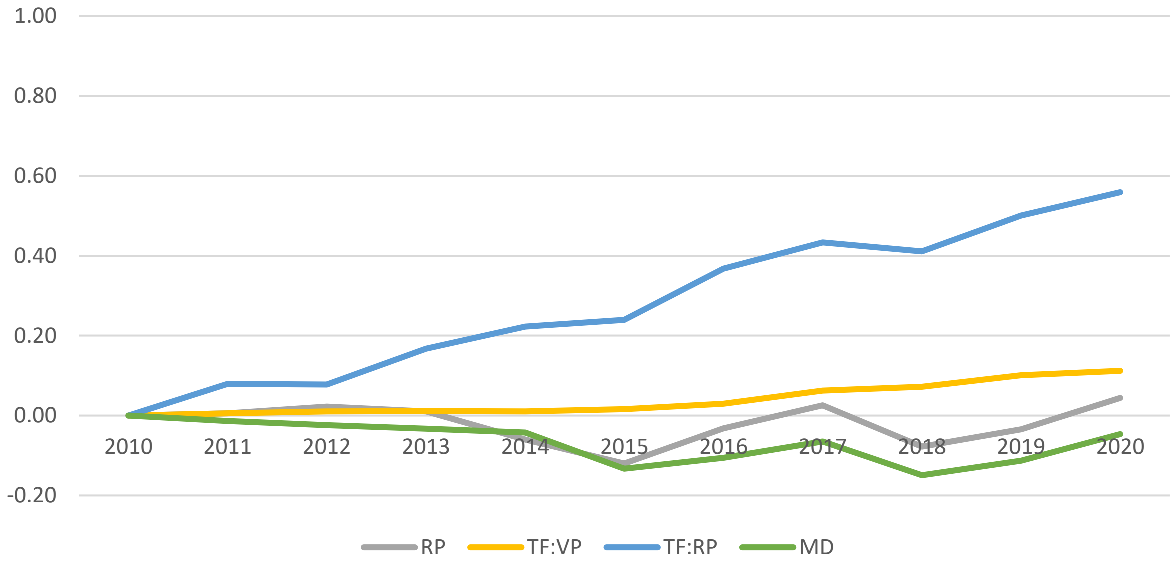


Figure 5.7 – Cumulative Returns 2010-2020
Source: Own calculations

Considering the baseline of 2010, the TF:RP strategy exponentially increases an investment that is made in this period, because of its ability to consider asset information and equate risk contribution. Due to the ups and downs during the yearly returns, all other strategies experience a small gain over the sample period. In this sense, let us consider a hypothetical investment of $100,000 in each of the strategies (disregarding any roll-over/trading costs) and a roll-over strategy adopted based on the inputs described in chapter 3. The wealth development in such circumstances would result in positive accumulated growth for all strategies. The TF:RP delivers an accumulated value of 56%, resulting in a final value of the wealth of $155,913. The second-best performer, the Trend Following Risk Parity strategy only delivers 11% and the single Risk Parity strategy only rewards the investor with an accumulated return of $4,409 over 10 years. The MD strategy would result in a total reduction of the investors' wealth of 5%.

6. Conclusion

This study compares the performance of Risk Parity approaches with other commonly used approaches, either using more traditional portfolios – such as EW and MVP – or others considered more sophisticated such as the volatility parity weighting scheme and the Most Diversified strategy. We analyzed a total of 6 strategies over the 31-year period and found a superior performance of the TF:RP strategy over its peers. The situation is more pronounced when analyzed through different performance ratios and for a shorter frame of time. Interestingly, in this shorter period, we found signs of a strong predominance of the Trend Following strategy when compared with the pure Risk Parity weighting scheme.

Regarding the strategies themselves, the Global Minimum Variance shows up as the poorest performer, showing the lowest Sharpe Ratio and average return. More precisely, the strategy presents a negative average excess return. Despite the significant ups and downs in the yearly return, Sharpe Ratios and constant lower volatility, this strategy lacks consecutive good performances. However, we found evidence of a considerably high Sharpe Ratio over the 10-year investment, showing the leverage of small values of volatility in the period. In terms of the most uncertain periods, we found evidence of constant negative Sharpe Ratios, most of the times lower than all other strategies. The strategy is heavy on Bonds contracts, namely, STIR contracts and presents a volatile risk contribution, despite the predominance of Fixed Income contracts. In the case of the Most Diversified portfolio, we found evidence of an average return that doubles the gains from the GMV. In fact, the yearly highest value is achieved by the strategy, in 2009, following a negative performance in the previous year. In the years following the 2008 financial crisis, we verified a strong improvement of the strategy, despite the strong negative downs in the years of economic drawdowns. The portfolio assumes a similar shape in terms of portfolio allocations and risk contribution, with Commodities as the primary weighted category and risk allocated contracts. When looking at the 10-year period, the strategy provides investors with a negative return, despite the good performance in most of the ratios.

Concerning the Equal Weighting scheme, evidence shows a similar pattern to the best performers, despite the poorest performance among all strategies in 2009. It presents the highest values of volatility in volatility in the periods of 2008 and 2020. This situation was expected due to the attribution of weights regardless of any return or volatility measure, thus outweighing contracts with higher volatilities and underweighting assets with lower volatilities in those periods. Nevertheless, this strategy attains the third-best Sharpe Ratio, on average terms, despite the small average values of Calmar and Sortino Ratios. The portfolio allocation per asset class is highly fair, with small variations coming from the changes in the number of contracts considered over the sample period. However, the percentage risk contribution shows the inefficiency of the strategy, especially in the periods of higher uncertainty, where it is skewed towards Energy and Commodities contracts, with a residual part attributed to STIR, the less volatile asset class.

Generally speaking, Trend Following strategies have been profitable and outperformed many other strategies showing good diversification features against most market downturns. The main source of profitability is the benefit from diversification across assets and asset classes. At first, a simple inverse volatility (volatility-parity weighting scheme) was effective to bring significant improvements in diversifying risk. In the case of the TF:VP strategy, we observe an average return that cannot surpass other strategies across the periods nor underperform them, thus showing a constant pattern. Portfolio allocations are strongly skewed towards STIR due to the nature of the strategy that relies on attributing more weight to lower volatility assets. However, the risk contribution follows a more stable pattern. In terms of volatility, the strategy can approximate GMV in most periods, showing a stronger dispersion after the 2008 financial crisis, evidencing the effects of co-movements and the need for a change in the weighting scheme.

Risk Parity strategies show a higher degree of diversification compared to the volatility parity weighting schemes. Portfolio weights show some changes across the period to attain the goal of equal risk contribution, resulting in a total risk contribution that is similar over time. When comparing the performance of the strategy to the volatility parity one, it shows a higher level of return by shortly increasing the volatility, resulting in the change of the Sharpe Ratio into positive values. Despite its good performances in the periods of the Dot-com bubble, global financial crisis and COVID-19 , this strategy cannot constantly outperform other strategies. Based on these findings, we reject the first hypothesis of this study and conclude that Risk Parity strategies cannot consistently outperform traditional strategies from a Sharpe Ratio perspective. Nevertheless, the 10-year period investment shows the effectiveness of the strategy, though still below the Trend Following strategy.

The Trend Following Risk Parity approach is positioned as the best strategy in this study. Despite being one of the strategies that presents higher levels of volatility, it is by far the one that shows better returns. The additional risk taken rewards investors with a differentiated return that is also supported with the best performance ratios over the period. Compared to the single Risk Parity strategy, we observe an excess return that is three times higher, in average terms, and a total yearly return two times higher. Hence, we accept the second hypothesis and conclude that TF:RP strategy can provide better risk-adjusted returns than the RP strategy alone. By comparing the two strategies using Trend Following, we observe a strong prevalence of the TF:RP, shown in the change towards a positive Sharpe Ratio and a higher return by a residual increase in volatility. Therefore, we accept the third hypothesis and conclude that the TF:TR strategy provides better risk-adjusted returns than the TF strategy using a Volatility-Parity weighting scheme. The same occurs when we look at the 10-year period, where several events took place, showing a clear outperformance in terms of risk-adjusted returns and performance ratios. Based on these findings, we accept the fourth hypothesis and conclude that TF:RP can outperform all other strategies over a 10-year investment period. To preserve the goal of equal risk contribution, the strategy shifts the weights of the contracts between and within asset classes, especially during periods of higher correlations.

Trend Following gives a decent perspective on the performance of each asset and a minor but very significant indication of the behavior of an individual asset over a given period. However, the changes in economic outlook show the drawbacks of this strategy alone and the importance of a relatively new approach to equate the contribution of each asset. The approach used in this dissertation shows a genuine improvement in terms of portfolio construction. Though, it assumes a constant desired level of risk that surely changes from investor to investor and from times to times. Additionally, the study consists of an exclusive inclusion of futures contracts and on the assumption of no direct nor indirect costs in the construction of the portfolios. Another important remark is the need for roll-over to convert futures prices into a continuous price series right before the maturity date. These values are crucial in the calculation of returns and all other input measures used in the optimization procedure, thus creating a strong dependency on these approximations.

Future research on this subject should consider the introduction of different types of costs and the analysis of these strategies using either stocks or other types of instruments. Besides, it should test the robustness of the results under different scenarios, such as the exclusion of some assets from portfolios and the moment of generation of the Trend Following approach – either by delaying by a few days into the following month or anticipating before the end of the current month. Moreover, it would be interesting to adjust the target level of the model according to market trends, thus ranging from higher values in bull markets – when investors may be able to take higher levels of risk – to lower values in bear markets – when investors adopt more conservative approaches.

The findings of this dissertation show the relevance of efficient asset allocation strategies to correctly assess market trends and meet investors' needs within their constraints. We found benefits and drawbacks of the strategies and periods where some strategies performed better than others, and a predominance of the Trend Following Risk Parity strategy over others. Beyond the need for an enhancement in optimization strategies, there is also the need for accurate estimates to incorporate these models. However, these strategies are not stanched as the development of asset allocation strategies is accompanied by developments in several markets. Hence, the subject of asset allocation using Trend Following and Risk Parity approaches will continue to attract much interest in the following years.

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8. Annexes

Annex A – Correlations in Energy Futures Contracts

	Natural Gas	Heating Oil	Gas Oil	Crude Oil	Brent Crude	RBOB Gasoline
Natural Gas	1.0000	0.3497	0.2778	0.1977	0.2103	0.0695
Heating Oil	0.3497	1.0000	0.8916	0.7988	0.8416	0.4888
Gas Oil	0.2778	0.8916	1.0000	0.7776	0.8532	0.4799
Crude Oil	0.1977	0.7988	0.7776	1.0000	0.9035	0.5452
Brent Crude	0.2103	0.8416	0.8532	0.9035	1.0000	0.5650
RBOB Gasoline	0.0695	0.4888	0.4799	0.5452	0.5650	1.0000

Source: Bloomberg and own calculations

Annex B – Correlations in Commodities Futures Contracts

	Sugar #11	Live Cattle	Coffee "C"	Cotton #2	Soybeans	Corn	Wheat	NA Copper	Gold (100 oz.)	Silver
Sugar #11	1.0000	0.0355	0.1191	0.1228	0.1748	0.1470	0.1220	0.2053	0.1094	0.1399
Live Cattle	0.0355	1.0000	-0.0146	0.0226	0.0267	0.0016	0.0307	0.0812	-0.0792	-0.0178
Coffee "C"	0.1191	-0.0146	1.0000	0.1132	0.2049	0.1546	0.1565	0.1647	0.1626	0.1982
Cotton #2	0.1228	0.0226	0.1132	1.0000	0.3727	0.3343	0.2253	0.2957	0.1035	0.1420
Soybeans	0.1748	0.0267	0.2049	0.3727	1.0000	0.6996	0.5079	0.2504	0.1517	0.1810
Corn	0.1470	0.0016	0.1546	0.3343	0.6996	1.0000	0.6265	0.1597	0.1528	0.1927
Wheat	0.1220	0.0307	0.1565	0.2253	0.5079	0.6265	1.0000	0.1557	0.1607	0.1264
NA Copper	0.2053	0.0812	0.1647	0.2957	0.2504	0.1597	0.1557	1.0000	0.2555	0.3277
Gold (100 oz.)	0.1094	-0.0792	0.1626	0.1035	0.1517	0.1528	0.1607	0.2555	1.0000	0.7145
Silver	0.1399	-0.0178	0.1982	0.1420	0.1810	0.1927	0.1264	0.3277	0.7145	1.0000

Source: Bloomberg and own calculations

Annex C – Correlations in Fixed Income Futures Contracts

	T-Notes (10-Yr)	T-Notes (5 Yr)	T-Bonds (30 Yr)	Euro-Bund	Euro-Bobl	JGB
T-Notes (10-Yr)	1.0000	0.9690	0.9373	0.4751	0.6144	0.2995
T-Notes (5 Yr)	0.9690	1.0000	0.8593	0.4779	0.6029	0.2775
T-Bonds (30 Yr)	0.9373	0.8593	1.0000	0.4237	0.6109	0.2978
Euro-Bund	0.4751	0.4779	0.4237	1.0000	0.2805	0.2214
Euro-Bobl	0.6144	0.6029	0.6109	0.2805	1.0000	0.3074
JGB	0.2995	0.2775	0.2978	0.2214	0.3074	1.0000

Source: Bloomberg and own calculations

Annex D – Correlations in Foreign Exchange Futures Contracts

	Euro FX	Japanese Yen	British Pound	Australian Dollar	Canadian Dollar	Swiss Franc
Euro FX	1.0000	0.2000	0.4986	0.6070	0.4861	0.6516
Japanese Yen	0.2000	1.0000	0.1671	0.1313	0.0539	0.4217
British Pound	0.4986	0.1671	1.0000	0.3952	0.3651	0.5494
Australian Dollar	0.6070	0.1313	0.3952	1.0000	0.6626	0.3878
Canadian Dollar	0.4861	0.0539	0.3651	0.6626	1.0000	0.2578
Swiss Franc	0.6516	0.4217	0.5494	0.3878	0.2578	1.0000

Source: Bloomberg and own calculations

Annex E – Correlations in Equities Futures Contracts

	Nasdaq Complex	S&P 500 Index Complex	Euro Stoxx 50	Dax	Kospi 200	Nikkei 225 Futures	FTSE 100
Nasdaq Complex	1.0000	0.7117	0.6839	0.6589	0.4615	0.4035	0.5519
S&P 500 Index Complex	0.7117	1.0000	0.7039	0.7416	0.5208	0.5136	0.7814
Euro Stoxx 50	0.6839	0.7039	1.0000	0.8732	0.5017	0.4207	0.7507
Dax	0.6589	0.7416	0.8732	1.0000	0.4926	0.4380	0.7649
Kospi 200	0.4615	0.5208	0.5017	0.4926	1.0000	0.4936	0.4566
Nikkei 225 Futures	0.4035	0.5136	0.4207	0.4380	0.4936	1.0000	0.5341
FTSE 100	0.5519	0.7814	0.7507	0.7649	0.4566	0.5341	1.0000

Source: Bloomberg and own calculations

Annex F – Correlations in STIR Futures Contracts

	Eurodollars (3-month)	3 Month Euribor	3 Month Euroyen
Eurodollars (3-month)	1.0000	0.4395	0.2382
3 Month Euribor	0.4395	1.0000	0.0644
3 Month Euroyen	0.2382	0.0644	1.0000

Source: Bloomberg and own calculation