

# Information Theory based Feature Selection for Customer Classification

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**Abstract.** The application of Information Theory techniques in customer feature selection is analyzed. This method, usually called information gain has been demonstrated to be simple and fast for feature selection. The important concept of mutual information, originally introduced to analyze and model a noisy channel is used in order to measure relations between characteristics of given customers. An application to a bank customers data set of telemarketing calls for selling bank long-term deposits is shown. We show that with our method, 80% of the subscribers can be reached by contacting just the better half of the classified clients.

**Keywords:** Customer segmentation, feature selection, mutual information

## 1 Introduction

Customer segmentation in marketing campaigns is an important and classical problem in Business Intelligence. Since the marketing campaign is cost extensive, an optimal selection of the customers to whom direct the campaign results in saving a lot of money. Since the larger the number of customer attributes, the slower and complex the algorithm, there is a search for optimal performance with good results. Many algorithms based on several mathematical methods have been developed, being support vector machines (SVM) and neural networks the most popular, see for example a review in [6] and [11]. Since customers are described by a lot of attributes, the first stage in order to classify them is to select not all the attributes but the most relevant in order to avoid redundant or irrelevant information, this stage is also called feature selection, a common data mining (DM) problem. Selecting just the most relevant variables helps to simplify the prediction algorithm, and makes the segmentation result easy to be understood by decision makers and managers. Despite that the knowledge of the problem domain is usually used to manually discard some irrelevant features, getting a

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good mathematical method to automatically perform this task is quite important, see for example [4] for a review. In this work, a variable selection method based on entropy and mutual information is presented. Since they are estimated from probabilities just making sums and multiplications, the estimation of these functions is performed in a simple way. Then, the estimation process is easy and fast, allowing to deal even with a big number of data, albeit requiring efficient sampling methods. Applications of Information Theory in feature selection has been largely analyzed, see for example [2], [7] and [10], though practical applications involve mainly pattern and text classification, with few contributions to CRM. In this work, we show that the application of Information Theory in feature selection can be also applied with good results to customer classification. Once an optimal number of input variables are selected, a simple algorithm can be used for prediction. We apply this method to a bank telemarketing campaign. The Lift table reveals that almost all the affirmative responses can be achieved by contacting just half of the whole dataset, as it will be shown. Entropy and mutual information can be used to quantify the information content of the selected attributes from any method on one hand, and how much of this information is transmitted to the output variable on the other.

The aim of this work is to show that concepts of information theory are an important tool in DS related to customer segmentation, either to extract the optimal number of attributes carrying most of the information of the output variable, or to measure the efficiency of any segmentation method.

This paper is organized as follows: Some concepts from Information Theory and their application to classification are presented in section 2, results and evaluation of applying this method to a real dataset are presented in section 3, final conclusions and future work are given in section 4.

## 2 Information Theory

### 2.1 Entropy and Mutual Information

Entropy and Mutual Information are well known concepts in Communications and Information Theory. They were originally introduced by Claude Shannon in a seminal paper, [14], in order to find the optimal coding of a source on one hand and a noisy channel on the other. Entropy is related to uncertainty or information content of a random variable and is defined as:

$$H(X) = - \sum_{x \in \mathcal{X}} p(x) \log p(x) \quad (1)$$

When more probable is the event, less information gives its occurrence. Entropy is bounded by the cardinality of the set of outcomes:  $H(X) \leq \log |\mathcal{X}|$  and attains its maximum when the events follow a uniform distribution  $p_i = \frac{1}{|\mathcal{X}|}$ . The bigger the entropy, the more random are the events, then, the occurrence of an event gives more information, though they are less predictable.

Mutual information between sets of random variables  $X^n = \{X_1, X_2, \dots, X_n\}$  and  $Y^n = \{Y_1, Y_2, \dots, Y_n\}$  is defined as follows:

$$\begin{aligned}
 I(X^n; Y^n) &= \sum_{x^n \in \mathcal{X}^n, y^n \in \mathcal{Y}^n} p(x^n, y^n) \log \frac{p(x^n, y^n)}{p(x^n) p(y^n)} \\
 &= H(X^n) - H(X^n | Y^n) \\
 &= H(Y^n) - H(Y^n | X^n) \\
 &= H(X^n) + H(Y^n) - H(X^n, Y^n)
 \end{aligned} \tag{2}$$

where  $\mathcal{X}^m$  and  $\mathcal{Y}^m$  are the set of outcomes of  $x^m$  and  $y^m$ , see [3] for details.

## 2.2 The noisy communication channel

A communication channel is a device or medium capable of transmitting information. Because of noise, the input and output information are not the same but related. We can use the mutual information as a measure of that relation. We will consider thereafter that  $X$  is the random variable at the input and  $Y$  is the output random variable. According to the source coding theorem, we know that the entropy is a measure of the average bits of information necessary to code the outcomes of a given random variable. In this way,  $H(X)$  is a measure of the input information to the channel,  $H(Y)$  the information content at the output,  $I(X; Y)$  the transmitted information, and taking into account the relations (2):  $H(Y|X)$  is a measure of the noise introduced by the channel. The conditional entropy  $H(X|Y)$  is called equivocation or ambiguity and it must be subtracted to the input information in order to obtain the transmitted information. According to the channel coding theorem, the Mutual Information gives the channel capacity and determines the maximum rate of information transmitted by the channel, see details in [14] and [3].

## 2.3 Feature selection using Mutual information

Feature selection is a very important issue not just in DM but in other scientific areas, [4]. This is a quite important task in customer segmentation since an optimal selection of attributes or variables simplifies the prediction algorithm.

Recalling explanations about the entropy remarked before, since the entropy is lower for more predictable random variables, we look for a given set of input variables having the lowest joint entropy on one hand, and also carrying most of the information of the output variable on the other. This method is usually called Information Gain or Max-Relevance Min-redundancy in the literature, see [5] and [10] for details. Mutual Information can be used to choose the set of attributes carrying out most of the information. Then, from the whole set of attributes  $X^n = \{X_1, X_2, \dots, X_n\}$ , we choose a subset  $X^m \subset X^n$  in such a way that being  $Y$  the output variable:  $I(X^m; Y)$  carries enough information according to a given criterion, see for example [5], [12] and [15]. Mutual Information allows eliminating attributes not related to the output variable on

one hand, and attributes with redundant information on the other. A situation where redundant information can be removed arises when variables are connected in a Markov chain:  $X_1 \rightarrow X_2 \rightarrow Y$ . A well known relation for this case is given by the data processing inequality  $I(X_1; X_2) \geq I(X_1; Y)$ , the relations:  $I(X_1, X_2; Y) = I(X_2; Y)$ ,  $I(X_2; Y) \geq I(X_1; Y)$  can be also demonstrated in a similar way. Then, we can point out that these functions from the information theory are quite suitable to eliminate redundant information, not generally taken into account with other methods.

### 3 Application to a real case

We will apply next the information theory concepts to real data first analyzed in [8] and available at [13] corresponding to a directed bank marketing campaign performed by its own contact-center. This dataset has also been used in [11] and has been recently completed and extensively analyzed in [9].

#### 3.1 The dataset

The dataset collected is related to a bank telemarketing campaign to sell long-term deposits that occurred between May 2008 and November 2013, corresponding to a total of 52,944 contacts. For each contact, several number of attributes were stored despite the result (success or not success). For the whole database considered, there were 6557 successes (12.38% success rate). We use the available dataset: a testing set with 41188 records and a training set with 4120 records randomly chosen, having the 21 attributes listed below:

- Bank client data:
  1. age (numeric)
  2. job : type of job (categorical: "admin.", "unknown", "unemployed", "management", "housemaid", "entrepreneur", "student", "blue-collar", "self-employed", "retired", "technician", "services")
  3. marital : marital status (categorical: "married", "divorced", "single"; note: "divorced" means divorced or widowed)
  4. education (categorical: "unknown", "secondary", "primary", "tertiary")
  5. default: has credit in default? (binary: "yes", "no")
  6. balance: average yearly balance, in euros (numeric)
  7. housing: has housing loan? (binary: "yes", "no")
  8. loan: has personal loan? (binary: "yes", "no")
- Related with the last contact of the current campaign:
  9. contact: contact communication type (categorical: "unknown", "telephone", "cellular")
  10. day: last contact day of the month (numeric)
  11. month: last contact month of year (categorical: "jan", "feb", "mar", ..., "nov", "dec")
  12. duration: last contact duration, in seconds (numeric)
- Other attributes:

13. campaign: number of contacts performed during this campaign and for this client (numeric, includes last contact)
  14. pdays: number of days that passed by after the client was last contacted from a previous campaign (numeric, -1 means client was not previously contacted)
  15. previous: number of contacts performed before this campaign and for this client (numeric)
  16. poutcome: outcome of the previous marketing campaign (categorical: "unknown", "other", "failure", "success")
- social and economic context attributes
    17. emp.var.rate: employment variation rate - quarterly indicator (numeric)
    18. cons.price.idx: consumer price index - monthly indicator (numeric)
    19. cons.conf.idx: consumer confidence index - monthly indicator (numeric)
    20. euribor3m: euribor 3 month rate - daily indicator (numeric)
    21. nr.employed: number of employees - quarterly indicator (numeric)
  - Output variable (desired target):
    22. y - has the client subscribed a term deposit? (binary: "yes", "no")

### 3.2 Feature selection

In the variable selection stage, a set of relevant variables are selected from the training set as those who carry sufficient information. Since it is important to evaluate models having no other knowledge than previous campaigns we remove from the dataset variables corresponding to the current campaign "contact", "month", "day", "duration" and "campaign". Since the software powerhouse<sup>TM</sup>[1] performs a segmentation process based on information theory, we use that product in order to get the metrics. The software calculates the mutual information between a subset of attributes  $X^m \subset X^n$  and the output variable separately by estimating the joint probability getting a percentage indicator (information gain) as:  $IG(X^m) = \frac{I(Y;X^m)}{H(Y)} \times 100$ , clearly  $0 \leq IG(X^m) \leq 100$ . A goodness of estimation of the joint probability  $p(X^m, Y)$  is performed by using a  $\chi^2$  statistical test that gives a reliability percentage  $Rel(X^m) = \text{Percentage confidence of the } \chi^2 \text{ estimation test of } p(X^m, Y)$ . The chosen subset  $X^m$  is that with maximum product  $IG(X^m) \times Rel(X^m)$ .

#	Name	$H(X)$	$H(Y X)$	Gain $I(X;Y)$	Reliability
19	cons.conf.idx	2,3449	0,8067	19,33%	96,20%
2	job	2,9295	0,7677	23,23%	95,14%
4	education	2,5463	0,6625	33,75%	89,61%
1	age	1,6597	0,5377	46,23%	80,82%
3	marital	1,3199	0,4349	56,51%	70,72%

**Table 1.** Selected variables.

Elimination of redundant information occurs in the present data with the variable "previous", despite of this variable is mentioned as quite relevant in the original paper [8], it is not taken into account in our model. This is a consequence of the relation between "previous" with "pdays" and "poutcome". This

relation is rapidly seen in the software we used, we can see that "pdays" gives approximately 80 % of information of previous. Our selected features and their relative importance differ from those obtained in [9], where an adapted forward selection method was used.

### 3.3 Prediction

Once the more relevant variables are selected based on the information content, we can develop a predictive model using different techniques from naive Bayes to the most recent support vector machines, neural networks and genetic algorithms, see for example [11], [9] for a review. Since we focus on the feature selection, we use the simple logistic regression method to predict the output variable in order to measure the efficiency of our selection.

### 3.4 Evaluation

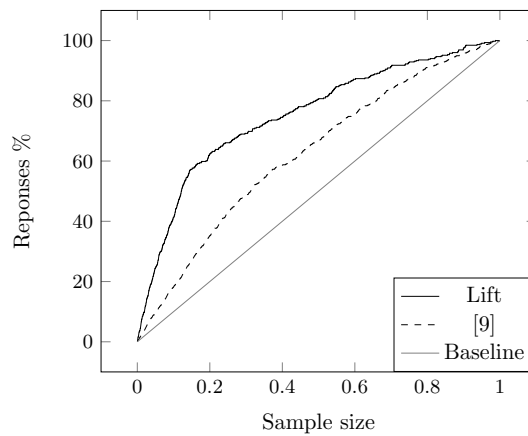
Since Lift is a usually used metrics to evaluate models, the Lift curve is shown in fig. 1. The area under the lift curve is ALIFT = 0.754. The lift reflects that with our model, 80% of the positive responses can be achieved by 50% of the sample scored with this method. The confusion matrix obtained from our model is shown in table (2). Then, the Company can get good results contacting just half of the sample, saving a lot of time and money.

Target	Predicted as	
	failure	success
failure	28583	7964
success	1817	2824

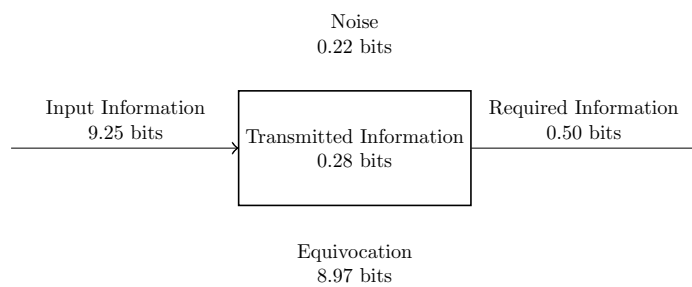
**Table 2.** Confusion Matrix

As it can be seen from the confusion matrix, by using our model, the campaign will miss just 7964 successful phone calls and avoid 28583 that would result in unsuccessful contacts, a useful information for managers. The model has low sensitivity on one hand, and a very good specificity on the other. The good specificity results in a very good Lift curve as a consequence. It is interesting to compare our results with those reported in [9] where the logistic regression algorithm of the rminer package from R using all the attributes (even more than those available online) was used. Since different test and training sets were used in [9], this must be considered as a rough comparison. Both curves are shown in fig. 1. Since no heavy CPU consume is involved, the product we use performs either the feature selection or prediction in few seconds in an Intel<sup>TM</sup> I3 processor.

Regarding the noisy communication channel described in section (2), we show in fig. 2 the estimated information content of the input variables, the output variable and the transmitted information for the present dataset. We recall that the output is a dichotomous variable with entropy bounded to 1 bit of information.



**Fig. 1.** The Lift curve obtained by the Logistic regression prediction algorithm.



**Fig. 2.** Estimated information contents

## 4 Conclusions

A customer segmentation method based on information theory was developed. We have shown the advantages of applying the information theory concepts in order to eliminate redundant or irrelevant attributes. Results of applying the method to a real data set with the analysis of the performance was shown. The Lift table shows that a successful bank marketing campaign can be achieved by contacting just half of the whole dataset, as it was illustrated. As a result, we have shown that Information Theory is an important tool for client selection decisions.

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