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journal homepage: www.elsevier.com/locate/jeboOptimal constrained interest-rate rules under heterogeneous expectations[☆]Emanuel Gasteiger^{a,b,*}^a TU Wien, Institute of Statistics and Mathematical Methods in Economics, Wiedner Hauptstr. 8–10, 1040 Wien, Austria^b Instituto Universitário de Lisboa (ISCTE-IUL), Business Research Unit (BRU-IUL), Av. das Forças Armadas, 1649-026 Lisboa, Portugal

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ABSTRACT

This paper examines optimal monetary policy under heterogeneous expectations. To this end, we develop a stochastic New Keynesian model with a cost-push shock and coexistence of one-step-ahead rational and adaptive expectations in decentralized markets. On the one side, heterogeneous expectations imply an amplification mechanism that has many adverse consequences missing under the rational expectations paradigm. On the other side, even discretionary optimal monetary policy can manipulate expectations via a novel channel. We argue that the incorporation of heterogeneous expectations in both the design and implementation of discretionary optimal monetary policy to exploit this channel lowers macroeconomic volatility. We find that: (1.) a more hawkish policy can reduce losses due to volatility, but an overly hawkish policy does not; (2.) overestimating the share of rational expectations in the design and implementation of policy creates additional losses, while the underestimation does not; (3.) credible commitment eliminates or mitigates many of the ramifications of heterogeneous expectations.

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1. Introduction

Leading central bankers conclude that the New Keynesian inflation targeting framework developed in the last decades has been an effective tool for macroeconomic stabilization policy before and throughout the Great Recession. One reason often mentioned is that the framework provides guidance on how to manage expectations, which appears to be crucial for stabilization policy.

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However policymakers also conclude that the framework is missing important features. One of the frequently demanded extensions is to abandon the paradigm of the rational expectations hypothesis (REH) and to incorporate heterogeneous expectations. The reason is that policymakers regard heterogeneous expectations as a potential source of business cycle amplification (King, 2012; Carney, 2013; Yellen, 2016).

The perceived need to incorporate heterogeneous expectations into the framework is not borne out of theoretical curiosity, but is grounded in the ample empirical and experimental evidence on the presence of heterogeneous expectations in private sector (see, Branch and McGough, 2018; Hommes, 2021, and the references therein). One conclusion that can be drawn from this evidence is that the diversity of available forecasting models in private sector is manifold. Thus, a precise description of the expectation formation mechanism of private sector is difficult. In light of this difficulty, we presume that policymakers are not only interested in a New Keynesian inflation targeting framework that incorporates heterogeneous expectations consistent with one specific piece of evidence. Instead, given the uncertainty surrounding the precise nature of heterogeneous expectations, policymakers may be interested in insights on how the trade-offs faced by monetary policy in an economy with aggregate shocks vary with the degree of expectations heterogeneity.

Based on this idea, our paper seeks to provide an understanding of the implications of heterogeneous expectations for the inflation output variability trade-off faced by central bankers.¹ For this purpose we extend the approach developed in Branch and McGough (2009) and derive a micro-founded and stochastic New Keynesian model with extrinsic heterogeneous expectations. Households are *ex-ante* identical and only differ in the way they form expectations. Households with one-step ahead model-consistent, or, rational expectations (RE) coexist with households that have adaptive expectations (AE).² In consequence, households with RE use a correctly specified forecasting model, while households with AE use a misspecified forecasting model.³

We use our model to show how a central bank can design and implement optimal monetary policy under heterogeneous expectations. In this context, we argue that there are two important and novel insights. First, a time-consistent design should incorporate heterogeneous expectations even under discretion. This is a key difference compared to optimal monetary policy design in models with (asymptotically) model-consistent homogeneous expectations. Incorporating heterogeneous expectations in the policy design opens a new channel for manipulating private sector expectations that is similar under discretion and commitment, but absent under the REH. In particular, the central bank can exert influence on AE. As a result, in response to a cost-push shock, the larger the policy induced contraction of aggregate demand on impact, the more pessimistic are AE in the subsequent period. In consequence, the optimal contraction of aggregate demand consistent with the central bank's inflation target is larger than the contraction without incorporating heterogeneous expectations and helps to counteract the model's heterogeneous expectations amplification mechanism. Therefore, the inflation output variability trade-off under both discretion and commitment improves. Second, recall that gains from commitment under the REH result from manipulating model-consistent RE. We argue that this channel of manipulating RE has to lose traction under heterogeneous expectations. The reason is that there is a smaller portion of agents with RE, whose expectations can be manipulated. Building on these insights, we derive several novel results and implications for optimal monetary policy that we discuss in detail below.

The key novelty of our model is that households and firms interact in decentralized markets. Hence, the heterogeneity in expectations in our model stems ultimately from households that own the firms. Firms expectations are the average of their owners expectations. Given their expectations, households and firms make inter-temporal decisions. The aggregation allows to establish a mapping from these individual decisions under aggregate uncertainty to the behavior of macroeconomic variables such as inflation and output. Admittedly the aggregate demand and supply relationships turn out to be broadly equivalent to the ones in Branch and McGough (2009) and Massaro (2013). However, our environment is micro-founded and stochastic with shocks to aggregate demand and supply. Thus, our model offers two advantages first, it allows for a novel and intuitive interpretation of the transmission of aggregate shocks and monetary policy in decentralized markets. Second, we can rank policies based on losses due to macroeconomic volatility.

In more detail, households make utility maximizing decisions on consumption, savings and labor supply given their type of expectations. Firms set prices to maximize profits subject to a nominal rigidity à la Calvo (1983). Moreover, price setting depends on the average of individual household inflation forecasts. As the perceived law of motion (PLM) of households with RE accounts for the economy's inflation persistence induced by households with AE, there is an expectations-based amplification mechanism. For instance, consider a transitory cost-push shock and assume that policymakers fully accommodate the shock. Under the REH, firms raise prices on impact, but their homogeneous expectations about future inflation are zero. Thus, inflation increases on impact and returns immediately to zero. In contrast, under heterogeneous expectations, firms raise prices by more than would be the case under the REH even on impact, because on average they expect

¹ This trade-off is also often referred to as Taylor (1979) curve, as the concept was introduced therein.

² An alternative would be to incorporate intrinsic heterogeneity. Then one would assume that agents face a cost for acquiring model-consistent expectations. In addition, following the seminal work of Brock and Hommes (1997), agents may evaluate the forecasting performance of both model-consistent expectations and AE. Then, in each period agents would compare the cost and benefits of each model of expectation formation. However, then the cost and the intensity of choice would emerge as free parameters that pin down the distribution of model-consistent and AE.

³ We limit the analysis to RE and AE for two reasons. First, the presence of a significant share of AE in private sector is a robust finding in both empirical and experimental work (again, see Branch and McGough, 2018, and the references therein). Second, our model nests the representative agent model under the REH as a natural benchmark.

future price changes due to the influence of AE. Moreover, average expectations are persistently high, which implies a persistent response of aggregate inflation. As a result, the heterogeneous expectations model exhibits more price dispersion. Impact effects and the transition toward the steady state are amplified and the inflation output variability trade-off is less favorable.

Next, the central bank is assumed to implement the optimal policy via an expectations-based reaction function.⁴ A key objective of this paper is then to compare the resulting inflation output variability trade-off and the implications of expectations-based reaction functions for the central bank's loss function depending on the degree of expectations heterogeneity. However, as we are explicit about the reaction function that the central bank uses to implement the optimal monetary policy, the determinacy properties of the reaction function have to be examined. Determinacy throughout the parameter space cannot be taken for granted. The reason is that, in principle, a reaction function can be associated with possibly infinitely many different equilibria, including the one consistent with the optimal monetary policy (see, Woodford, 1999). We address this 'stability problem for optimal monetary policies' (Evans and Honkapohja, 2003) by following Evans and McGough (2007). We constrain the optimal interest rate rules to yield a determinate equilibrium under three commonly used calibrations. This ensures that our results remain valid among several calibrations.

Our main results can be summarized as follows: (i) without commitment, determinacy can only be obtained in part of the structural parameter space; (ii) relative to the REH benchmark, the amplification mechanism under heterogeneous expectations increases central bank losses due to aggregate shocks irrespective of the particular policy; (iii) the inflation variability trade-off moves outward in a non-monotonic manner with the share of AE; (iv) the higher the central bank's preference for output stabilization the larger the losses for the central bank; (v) commitment is less effective under heterogeneous expectations than under the RE benchmark, but still, many of the potential hazards of heterogeneous expectations can be either eliminated or mitigated, and, most surprisingly, determinacy is guaranteed throughout the structural parameter space among all calibrations. Overall our analysis confirms the conjecture by Eusepi and Preston (2018, p.52) that '[t]he efficacy of monetary policy might be compromised by heterogeneous expectations... '.

We reach several policy implications from our findings. First, a policy design should incorporate expectations heterogeneity, because ignoring this heterogeneity is not time-consistent and inefficient. Second, the use of *ad hoc* loss functions under heterogeneous expectations has many practical advantages relative to a model-consistent loss function, but the resulting optimal policy does not guarantee determinacy. Third, obtaining credibility to be able to commit is highly important for a benevolent central bank as discretion is outperformed by commitment along several new dimensions. Commitment has a stabilizing effect, as it allows to manipulate RE and to counteract the heterogeneous expectations amplification mechanism to some extent. Fourth, under discretion, it is always optimal for the central bank to hire a central banker that is more hawkish than the central bank itself, but not overly hawkish. The latter can result in worse outcomes of the loss function relative to not hiring such central banker. Fifth, if the central bank overestimates the share of households with RE in the design and implementation of policy, losses increase. The opposite is true if the central bank underestimates this share. Finally, our results suggest that welfare analyses under the REH understate the true welfare losses in a world with heterogeneous expectations.

The remainder of this paper is organized as follows. Section 2 outlines the model and discusses the amplification mechanism implied by heterogeneous expectations. All considerations regarding the design and implementation of optimal monetary policy are elaborated in Section 3. The main results and their policy implications are discussed in Section 4. We connect our paper to the related literature in Section 5, while Section 6 concludes.

2. The model

2.1. Households

The economy is populated with a continuum of infinitely lived households. All households are *ex ante* identical except the way they form expectations. For practical purposes, we assume that each household $i \in [0, 1]$ can be of one of the two types $\gamma \in \{1, 2\}$. Similar to the Euler Equation approach (see Honkapohja et al., 2013), both types form one-step ahead subjective expectations. Average expectations for the output gap, x_t , and inflation, π_t , are

$$\widehat{E}_t x_{t+1} \equiv \chi E_t^1 x_{t+1} + (1 - \chi) E_t^2 x_{t+1} = \chi E_t x_{t+1} + (1 - \chi) \theta^2 x_{t-1}, \quad \text{and} \quad (1)$$

$$\widehat{E}_t \pi_{t+1} \equiv \chi E_t^1 \pi_{t+1} + (1 - \chi) E_t^2 \pi_{t+1} = \chi E_t \pi_{t+1} + (1 - \chi) \theta^2 \pi_{t-1}, \quad (2)$$

with coefficient $\theta > 0$ and $\chi \in [0, 1]$ is the share of agents with expectations of type $\gamma = 1$. Following Branch and McGough (2004), one can think of type $\gamma = 1$ as *anticipatory* or *forward-looking* behavior. In particular, these agents have one-step ahead RE, i.e., their PLM is model-consistent. Expectations $\gamma = 2$ show *reflective* or *backward-looking* behavior. The latter may be *purely adaptive/mean-reverting* ($0 < \theta < 1$), *naïve* ($\theta = 1$), or *extrapolative/trend-setting* ($\theta > 1$).

⁴ Evans and Honkapohja (2003, 2006) show that this reaction function leads to a determinate REE that is stable under adaptive learning in homogeneous expectations models throughout the entire range of structural parameters. Moreover, Gasteiger (2014) has shown that the determinacy result carries over to the deterministic Branch and McGough (2009) model for the commitment case.

A household i consists of two decision makers, a worker and a consumer. The worker supplies one differentiated type of labor, $N_t(i)$, on a perfectly competitive labor market in order to earn labor income in each period t . The consumer is responsible for the inter-temporal decisions of the household. This involves the optimal choice of real consumption, $C_t(i)$, and bond holdings, $B_t(i)$. Households are also the shareholders of the continuum of firms in the economy, i.e., a household i earns nominal lump-sum profits denoted $\Upsilon_t(i)$ from the non-tradable share in firms.

Households have a life-time utility function of the form

$$E_0^\gamma \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t(i), N_t(i)) \right\},$$

where $\beta \in (0, 1)$ is the period discount factor. As is common in the learning literature, both types of agents do not observe contemporaneous aggregate endogenous variables. They hold subjective expectations about unobserved variables and given these expectations they choose $(C_t(i), B_t(i), N_t(i))_{t=0}^{\infty}$ to satisfy their perceived Euler equations (see, e.g., Branch and McGough, 2009, p.1039).⁵ Each household's instantaneous utility is given by

$$U(C_t(i), N_t(i)) = \frac{C_t(i)^{1-\sigma}}{(1-\sigma)} - \frac{N_t(i)^{1+\varphi}}{(1+\varphi)},$$

where $\sigma \geq 0$ is the inverse of the inter-temporal elasticity of substitution of consumption and $\varphi \geq 0$ is the inverse of the Frisch elasticity of labor supply.

$C_t(i)$ denotes the usual Dixit and Stiglitz (1977) continuum of differentiated goods, with individual good $j \in [0, 1]$, defined as

$$C_t(i) \equiv \left(\int_0^1 C_t(i, j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $C_t(i, j)$ is real consumption of good j by household i and $\epsilon > 1$ is the price elasticity of demand. The corresponding aggregate price index can be defined as

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}},$$

where $P_t(j)$ is the price of good j . Likewise the demand for good j by a household i , that maximizes his consumption basket for any given level of consumption expenditures, is (e.g., Walsh, 2017, p.321ff.)

$$C_t(i, j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} C_t, \quad \forall j \in [0, 1]. \tag{3}$$

In what follows, we will consider the *cashless limit* as in Woodford (2003a, p.31ff.), abstract from government spending, and assume that dividends are lump-sum, i.e., $\Upsilon_t(i) = \Upsilon_t$. Thus, household i is constrained by the nominal flow budget constraint

$$P_t C_t(i) + E_t^\gamma \{Q_{t|t+1}\} B_t(i) + I_{C,t}(i) \leq W_t N_t(i) + B_{t-1}(i) + I_{P,t}(i) + \Upsilon_t. \tag{4}$$

In real terms this is

$$C_t(i) + E_t^\gamma \{Q_{t|t+1}\} \frac{B_t(i)}{P_t} + \frac{I_{C,t}(i)}{P_t} \leq \frac{W_t}{P_t} N_t(i) + \frac{B_{t-1}(i)}{P_{t-1}} \Pi_{t-1,t}^{-1} + \frac{I_{P,t}(i)}{P_t} + \frac{\Upsilon_t}{P_t}, \tag{5}$$

where $\Pi_{t-1,t} \equiv P_t/P_{t-1}$ is the gross inflation rate from period $t-1$ to t .

Constraint (4) suggest that household i earns income from labor, one-period bond holdings, insurance payments, $I_{P,t}(i)$, and profits respectively. This income is spent on consumption, one-period bond holdings at price $Q_{t|t+1}$ (the nominal stochastic discount factor), and insurance contributions, $I_{C,t}(i)$.

In Appendix B we demonstrate that assuming complete financial markets is not sufficient for perfect risk sharing among agents. In fact, agents face two kinds of uncertainty, aggregate shocks and the idiosyncratic labor income risk due to the Calvo (1983) pricing assumed below. This implies that agents may have differing expectations about their expected real income. As a consequence, the model would lose tractability. In particular, the analysis could no longer focus on two agents, each one representative for their type $\gamma \neq \gamma'$. This can be avoided by introducing an insurance mechanism.⁶

⁵ An alternative modelling approach is the Infinite Horizon approach as applied in Massaro (2013). The approach assumes that agents have an infinite horizon for forecasts and decision rules. This enables aggregation under less restrictive assumptions on higher-order beliefs of agents. Thus, the approach in Massaro (2013) can be viewed as a generalization of the approach in Branch and McGough (2009) and herein.

⁶ Such insurance mechanisms are often used in sticky information models that base the derivation of the New Keynesian Phillips Curve on the assumption of inattentive agents instead of sticky prices. Some remarks regarding the insurance mechanism we are going to use are provided in Mankiw and Reis (2007). An alternative way of providing income insurance would be to assume that each household consists of infinitely many members with a similar type of expectations and these members pool the incomes.

We will assume that at the outset of period $t = 0$ the household's consumer signs a contract with an *actuarially fair insurance agency*. The contract obliges the household to contribute its dividends, i.e., $I_{C,t}(i) = \Upsilon_t$. In return, the agency guarantees each household the average nominal income of their type γ via an individual payment. This payment depends on the individual labor income, i.e., $I_{P,t}(i) = P_t \Phi_t^\gamma - W_t N_t(i)$, where Φ_t^γ is the average real income of type γ .⁷ The latter is given by

$$\Phi_t^\gamma = \begin{cases} \frac{\int_0^\chi \Upsilon_t(i) di + \int_0^\chi W_t N_t(i) di}{\chi P_t} = \frac{(\chi \Upsilon_t + \int_0^\chi W_t N_t(i) di)}{\chi P_t} & \text{for } \gamma = 1, \text{ and} \\ \frac{\int_\chi^1 \Upsilon_t(i) di + \int_\chi^1 W_t N_t(i) di}{(1-\chi)P_t} = \frac{((1-\chi)\Upsilon_t + \int_0^\chi W_t N_t(i) di)}{(1-\chi)P_t} & \text{for } \gamma = 2. \end{cases}$$

It follows that aggregate nominal income can be expressed as the sum of dividends and the aggregate wage bill

$$P_t \Phi_t = P_t (\chi \Phi_t^1 + (1 - \chi) \Phi_t^2) = \Upsilon_t + W_t \left(\int_0^\chi N_t(i) di + \int_\chi^1 N_t(i) di \right) = P_t Y_t,$$

which makes use of $\Upsilon_t = P_t Y_t - W_t N_t$, i.e., nominal income stems from dividends or labor. Notice that the insurance contract has further implications. The payment that a household's consumer receives, given his type, is

$$\Phi_t^\gamma(i) = \begin{cases} \Upsilon_t + \frac{W_t \int_0^\chi N_t(i) di}{\chi} - W_t N_t(i) = \Upsilon_t + W_t \left(\frac{\int_0^\chi N_t(i) di}{\chi} - N_t(i) \right) & \text{for } \gamma = 1, \text{ and} \\ \Upsilon_t + \frac{W_t \int_\chi^1 N_t(i) di}{(1-\chi)} - W_t N_t(i) = \Upsilon_t + W_t \left(\frac{\int_\chi^1 N_t(i) di}{(1-\chi)} - N_t(i) \right) & \text{for } \gamma = 2. \end{cases} \tag{6}$$

For each type in (6), the payment consists of the sum of the share in aggregate profits and the difference between type-dependent average and individual labor income. The second term captures the key of the risk-sharing mechanism implemented via the insurance contract. Thus, while (5) is the relevant constraint for the household's worker, the relevant flow budget constraint for the household's consumer can be rewritten as

$$C_t(i) + E_t^\gamma \{ Q_{t|t+1} \} \frac{B_t(i)}{P_t} \leq \frac{B_{t-1}(i)}{P_{t-1}} \Pi_{t-1,t}^{-1} + \Phi_t^\gamma. \tag{7}$$

Households are assumed to maximize their utility subject to a sequence of flow budget constraints. As we detail in [Appendix A.1](#), the optimal choices of a household's consumer, $C_t(i)$ and $B_t(i)/P_t$, and worker, $N_t(i)$, imply a household Euler condition

$$E_t^\gamma \{ Q_{t|t+1} \} = E_t^\gamma \left\{ \beta \left(\frac{C_{t+1}(i)}{C_t(i)} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}, \quad \forall t \tag{8}$$

and a consumption-leisure trade-off

$$\frac{W_t}{P_t} = N_t(i)^\varphi C_t(i)^\sigma, \quad \forall t. \tag{9}$$

These are the perceived optimality conditions of households from solving the utility maximization problem under subjective expectations. These are behavioral rules and household choices also have to satisfy the subjective transversality condition

$$\lim_{k \rightarrow \infty} E_t^\gamma \left\{ \beta^{t+k} C_t(i)^{-\sigma} Q_{t+k|t+k+1} \frac{B_{t+k}(i)}{P_{t+k}} \right\} = 0, \tag{10}$$

and the *ex-post* inter-temporal household budget constraint.⁸ Initially households have zero bond holdings, i.e., $B_0(i) = 0$.

Next, around the steady state $B(i) = 0, C(i) = Y = \Phi^\gamma, \Pi = 1$, the flow budget constraint (7) can be log-linearized as

$$c_t(i) = b_{t-1}(i) - \beta b_t(i) + \phi_t^\gamma \equiv c_t^\gamma,$$

where $c_t(i) \equiv (C_t(i) - C(i))/Y, \phi_t^\gamma \equiv (\Phi_t^\gamma - \Phi^\gamma)/Y, b_t(i) \equiv ((B_t(i)/P_t) - (B(i)/P))/Y$ and c_t^γ is consumption of type γ in period t . Thus,

$$c_t^\gamma = E_t^\gamma \{ c_{t+1}^\gamma \} - \sigma^{-1} (i_t - E_t^\gamma \{ \pi_{t+1} \} - \rho) \quad \text{for } \gamma \in \{1, 2\}. \tag{11}$$

Note that, in equilibrium, (11) must be satisfied for both types of households and therefore there is no need to explicitly consider individual bond holding (see, [Branch and McGough, 2009](#), p.1041). Next, we can iterate (11) forward to obtain⁹

$$c_t^\gamma = E_t^\gamma \{ c_\infty^\gamma \} - \sigma^{-1} E_t^\gamma \left\{ \sum_{k=0}^{\infty} (i_{t+k} - \pi_{t+k+1} - \rho) \right\}. \tag{12}$$

⁷ Notice that one could alternatively assume that $I_{C,t}(i) = \Upsilon_t + W_t N_t(i)$ and then $I_{P,t}(i) = P_t \Phi_t^\gamma$.

⁸ In contrast, in the Infinite Horizon approach in [Massaro \(2013\)](#), agents have infinite horizon forecasts and their decisions satisfy the inter-temporal budget constraint *ex-ante*. As shown by [Massaro \(2013\)](#), both the Infinite Horizon and Euler Equation approach yield qualitatively similar results.

⁹ See [Appendix A.2](#) for the details.

2.2. Firms

There exists a unit continuum of firms where a typical firm $i \in [0, 1]$ consists of two decision makers, one for hiring and one for sales. The typical firm produces a differentiated good $j \in [0, 1]$, and operates under monopolistic competition. Price setting follows Calvo (1983). Thus, in each period t a fraction of firms $1 - \theta_p$ receives a signal to reset its optimal price, P_t^* . The remaining fraction θ_p cannot reset its price. This implies that the aggregate price level dynamics are given by

$$P_t = [\theta_p P_{t-1}^{1-\epsilon} + (1 - \theta_p) P_t^{*1-\epsilon}]^{1/1-\epsilon}.$$

Each firm i operates with the identical technology and produces its output of variety j , $Y_t(i, j) = Y_t(i)$, according to

$$Y_t(i, j) = Y_t(i) = A_t N_t(i)^{1-\alpha}. \tag{13}$$

Multi-factor productivity, A_t , is assumed to follow the exogenous process

$$a_t = \rho_a a_{t-1} + \epsilon_t^a,$$

where $a_t \equiv (A_t - A)/A$, $0 < |\rho_a| < 1$, and $\epsilon_t^a \sim \text{iid}(0, \sigma_a^2)$.

Firms take wages as given and their cost-minimizing hiring decision is given by

$$MC_t(i) = \frac{W_t}{P_t MPN_t(i)} = \frac{W_t N_t(i)}{P_t(1 - \alpha)Y_t(i)}, \quad \text{where}$$

$MPN_t(i) = (1 - \alpha)A_t N_t(i)^{-\alpha}$ is the marginal product and $MC_t(i)$ are real marginal cost.

Each household owns an equal share in each firm and a firm has to take into account each shareholders expectations according to his share. This assumption allows us to elaborate the optimal price setting behavior of a firm i 's sales department under expectations operator \widehat{E}_t .

A firm i receiving the random signal to reset its price, takes into account that the price will be in place for k periods. Thus, it faces the problem of choosing the profit-maximizing price for its good $P_t^*(i)$, level of employment $N_{t+k|t}(i)$ and quantity of output $Y_{t+k|t}(i)$ given its constraints, i.e.,

$$\begin{aligned} \max_{N_{t+k|t}(i), Y_{t+k|t}(i), P_t^*(i)} \quad & \sum_{k=0}^{\infty} \theta_p^k \widehat{E}_t \{ Q_{t,t+k} (P_t^*(i) Y_{t+k|t}(i) - W_{t+k} N_{t+k|t}(i)) \} \quad \text{s.t.} \\ & Y_{t+k|t}(i) = \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \\ & Y_{t+k|t}(i) = A_{t+k} N_{t+k|t}(i)^{(1-\alpha)}. \end{aligned}$$

It follows that profit maximization in period t , given its constraints, requires that

$$\widehat{E}_t \left\{ \sum_{k=0}^{\infty} (\beta \theta_p)^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \Pi_{t,t+k}^{-1} Y_{t+k|t}(i) \left(\frac{P_t^*(i)}{P_t} - \mathcal{M} \Pi_{t,t+k} MC_{t+k|t}(i) \right) \right\} = 0, \tag{14}$$

where $\mathcal{M} \equiv \epsilon/(\epsilon - 1)$ is the desired constant mark-up and $MC_{t+k|t}(i)$ are period $t + k$ real marginal cost of a firm that last reset its price in period t . Linearization of (14) yields

$$p_t^*(i) - p_t = (1 - \beta \theta_p) \widehat{E}_t \left\{ \sum_{k=0}^{\infty} (\beta \theta_p)^k (\pi_{t,t+k} + \widehat{mc}_{t+k|t}(i)) \right\}. \tag{15}$$

Note that deriving (14) and (15) involves exactly the same steps as discussed in Galí (2015, pp.56–57). Moreover, Assumption A1 of Branch and McGough (2009) was invoked for arriving at (15), i.e., subjective expectations fix observables.

2.3. Equilibrium

In equilibrium, all decisions of households and firms need to be consistent with each other and all markets need to clear. The goods market clearing condition is

$$Y_t = C_t, \quad \forall t. \tag{16}$$

Bond market clearing requires that all bonds issued by agents of type γ must be held by agents of type $\gamma' \neq \gamma$ and vice versa. In linearized terms, this condition is given by

$$\chi b_t^1 = -(1 - \chi) b_t^2 \quad \forall t.$$

Furthermore, in steady state, expectations of all types coincide and $B^1(i) = B^2(i) = 0$ holds. Therefore, the average real wealth in period t can be expressed as

$$\begin{aligned} \chi c_t^1 + (1 - \chi)c_t^2 &= \chi(b_{t-1}^1 - \beta b_t^1 + \phi_t^1) + (1 - \chi)(b_{t-1}^2 - \beta b_t^2 + \phi_t^2) \\ &= \chi\phi_t^1 + (1 - \chi)\phi_t^2 = y_t. \end{aligned} \tag{17}$$

The latter allows one to show that the Dynamic IS equation is given by

$$y_t = \widehat{E}_t\{y_{t+1}\} - \sigma^{-1}(i_t - \widehat{E}_t\{\pi_{t+1}\} - \rho), \tag{18}$$

as outlined in [Appendix A.3](#).

Labor market clearing in this model is known to imply

$$y_t = a_t + (1 - \alpha)n_t \tag{19}$$

up to a first-order linear approximation (see, [Galí, 2015](#), p.59).

Next, we derive the aggregate supply relationship. As we show in [Appendices A.4](#) and [A.5](#) the *inflation equation* is given by

$$\pi_t = \kappa \widehat{m}c_t + \beta \widehat{E}_t\{\pi_{t+1}\}, \tag{20}$$

where $\kappa \equiv \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p} \Theta$.

The model, as outlined so far, has the property of *divine coincidence* (see, [Blanchard and Galí, 2007](#)), as it is free of any real rigidities. Thus, in order to introduce a trade-off between stabilizing inflation and the welfare relevant output gap, we will assume a time-varying exogenous aggregate wage markup along the lines of [Galí et al. \(2007, p.46\)](#), i.e.,

$$\mu_t^w \equiv (w_t - p_t) - mrs_t.$$

[Appendix A.5](#) shows that average real marginal cost in log deviations can be written as¹⁰

$$\widehat{m}c_t = mc_t - mc = \left[\sigma + \frac{(\varphi + \alpha)}{(1 - \alpha)} \right] (y_t - y_t^n) + (\mu_t^w - \mu^w), \tag{21}$$

where y_t^n denotes the output under flexible prices. The *New Keynesian Phillips Curve* is therefore obtained by applying (21) to (20), i.e.,

$$\pi_t = \beta \widehat{E}_t\{\pi_{t+1}\} + \lambda(y_t - y_t^n) + \kappa(\mu_t^w - \mu^w),$$

where $\lambda \equiv \kappa \left[\sigma + \frac{(\varphi + \alpha)}{(1 - \alpha)} \right]$. Let us define $x_t \equiv y_t - y_t^n$ as the relevant output gap for policymakers and $u_t \equiv \kappa(\mu_t^w - \mu^w)$ as the cost-push shock, which is assumed to follow

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \tag{22}$$

where $0 < |\rho_u| < 1$ and $\varepsilon_t^u \sim \text{iid}(0, \sigma_u^2)$. Then it follows that

$$\pi_t = \beta \widehat{E}_t\{\pi_{t+1}\} + \lambda x_t + u_t. \tag{23}$$

Finally, using [Assumption A3](#) and the definition from above yields

$$x_t = \widehat{E}_t\{x_{t+1}\} - \sigma^{-1}(i_t - \widehat{E}_t\{\pi_{t+1}\}) + g_t, \tag{24}$$

where $g_t \equiv \sigma^{-1}\rho + \Delta \widehat{E}_t\{y_{t+1}^n\}$ can be thought of as the *natural rate of interest*. Given that a_t is a AR(1) process, we can assume that

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g, \tag{25}$$

where $0 < |\rho_g| < 1$ and $\varepsilon_t^g \sim \text{iid}(0, \sigma_g^2)$.¹¹

Under an assumption for the monetary policy instrument i_t , the aggregate economy summarized by [Eqs. \(22\)–\(25\)](#), (1) to (2) states a stochastic second-order difference system

$$\mathbf{y}_t = \mathbf{A}E_t\mathbf{y}_{t+1} + \mathbf{C}\mathbf{y}_{t-1} + \mathbf{D}\mathbf{z}_t \tag{26}$$

$$\mathbf{z}_t = \mathbf{R}\mathbf{z}_{t-1} + \mathbf{e}_t, \tag{27}$$

where $\mathbf{y}_t \equiv [x_t, \pi_t]'$, $\mathbf{z}_t \equiv [g_t, u_t]'$, $\mathbf{e}_t \equiv [\varepsilon_t^g, \varepsilon_t^u]'$, and suitable matrices \mathbf{A} , \mathbf{C} , \mathbf{D} , and \mathbf{R} . System (26) can be interpreted as an associated RE model and then the results of [Branch and McGough \(2004\)](#) imply that standard solution methods for models under the REH can be applied. A *minimum state variable* solution to this system can be obtained by applying [Klein's \(2000\)](#) method and is given by

$$\mathbf{y}_t = \mathbf{\Omega}\mathbf{y}_{t-1} + \mathbf{\Lambda}\mathbf{z}_t, \tag{28}$$

where $\mathbf{\Omega}$ and $\mathbf{\Lambda}$ are the solution matrices.

¹⁰ We assume that there exists a subsidy to neutralize the distortions caused by market power. Thus, y_t^n is efficient and [Appendix A.6](#) contains its computation.

¹¹ Notice that this economy is similar to the aggregate stochastic economy considered in [Branch and Evans \(2011\)](#), however their specification of expectations is different.

2.4. The private sector amplification mechanism

We have not yet specified policy, nevertheless, in order to understand the transmission of monetary policy later on, it is instructive to describe the effect of a cost-push shock on private sector behavior in isolation, i.e., without an endogenous response of monetary policy. In our framework, one can think of it as $i_t = 0$, which can also be interpreted as a central bank that fully accommodates a cost-push shock.

Consider a transitory cost-push shock and its effect under the REH, $\chi = 1$. Moreover, assume that the model is locally determinate. Because there is no persistence in the economy, and households with RE have a model-consistent PLM, it holds that average expectations of aggregate variables are zero, i.e., $\widehat{E}_t \pi_{t+1} = E_t^1 \pi_{t+1} = 0$ and $\widehat{E}_t x_{t+1} = E_t^1 x_{t+1} = 0$. The PLM is of the form (28), but due to $\chi = 1$ with $\Omega = 0$ and $C = 0$.

In our model the shock means a positive deviation of the exogenous wage markup, which raises marginal costs for firms. This creates inflationary pressures, because firms that can reset their price, will raise their price, $P_t^*(i)$. In consequence, inflation exhibits the same transitory increase as the cost-push shock. The output gap is unaffected.

If the central bank were to respond to the shock by raising the nominal interest rate, i_t , then it would increase the real interest rate for given expectations. This would in turn lower aggregate demand, which tends to lower marginal cost. The result would be less inflationary pressure compared to $i_t = 0$. As can be seen from (24), the output gap would decline on impact. Accordingly, via (23), inflation would increase by less than the cost-push shock.

Next, consider the very same transitory cost-push shock and its effect under heterogeneous expectations, i.e., $\chi \in (0, 1)$. Now, there is persistence in the economy due to the presence of households with AE. As a direct consequence, households with RE account for this persistence due to the presence of AE in their model-consistent PLM. Now we have $\Omega \neq 0$ due to $C \neq 0$ as $\chi < 1$.

When monetary policy again fully accommodates the shock, i.e., $i_t = 0$, given the above described inflationary pressures, inflation will definitely rise. However, on impact households with RE now expect inflation to rise also in the subsequent period due to their PLM, i.e., $E_t^1 \pi_{t+1} > 0$. As AE are zero on impact by assumption, we know that average inflation expectations are positive, i.e., $E_t^1 \pi_{t+1} = \widehat{E}_t \pi_{t+1} > 0$. Moreover, aggregate demand will be higher as the real interest rate for households with RE will be lower, compare (24). This also implies higher expected demand, i.e., $E_t^1 x_{t+1} = \widehat{E}_t x_{t+1} > 0$ and higher aggregate demand.

In sum, higher aggregate demand and higher average inflation expectations results in even more inflationary pressure, which results in larger price increases and inflation on impact via (23). This states an amplification mechanism. Due to the presence of AE, the model-consistent expectations of households imply larger impact effects relative to the REH benchmark with $\chi = 1$. Only in the periods after impact AE are different from zero, affect the average expectations, and imply persistent deviations from steady state.

Again, if the central bank were to respond to the shock by raising the nominal interest rate, i_t , then it would increase the real interest rate. The output gap would fall on impact and inflation would still rise on impact. Still average expectations would differ from zero, i.e., $E_t^1 \pi_{t+1} = \widehat{E}_t \pi_{t+1} > 0$ and $E_t^1 x_{t+1} = \widehat{E}_t x_{t+1} < 0$, and the transition would be affected by AE. The potential effect of this policy is to mute the amplification mechanism, but due the existence of heterogeneous expectations, policy cannot entirely shut down this mechanism.

The presence of heterogeneous expectations in our model results in an amplification mechanism absent in a homogeneous REH benchmark model. This mechanism creates higher price dispersion. Therefore the inflation output variability trade-off should be less favorable. The policy issue is then to manipulate current and eventually expected future aggregate demand such that the economy is stabilized around the steady state is such a heterogeneous expectation environment. Moreover, we highlight that the inflation persistence due to expectations heterogeneity also amplifies the impact effects of the cost-push shock. Such impact effects stand in contrast to what is known for other forms of inflation persistence such as price indexation modeled under the REH (see, e.g., Steinsson, 2003; Woodford, 2003a, p.499ff.).

3. Robust optimal constrained monetary policy

3.1. The design of optimal monetary policy under heterogeneous expectations

For the design of optimal monetary policy, the central bank's objective is to minimize the *ad hoc* loss function

$$E_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{1}{2} (\pi_{t+s}^2 + \omega_x x_{t+s}^2) \right\}, \tag{29}$$

where $\omega_x \geq 0$ is the weight that a central bank assigns to output gap stabilization relative to inflation stabilization. We interpret ω_x as the central bank's exogenous preference parameter. Such loss functions are common in the literature, see, for instance, Evans and Honkapohja (2003, 2006), Gasteiger (2014).¹² Our first objective in this section is to show how the

¹² An alternative would be to utilize the model-consistent loss function for our model as derived by Di Bartolomeo et al. (2016), which involves eight additional terms. We discuss the practical advantages of our approach in Section 3.2 below.

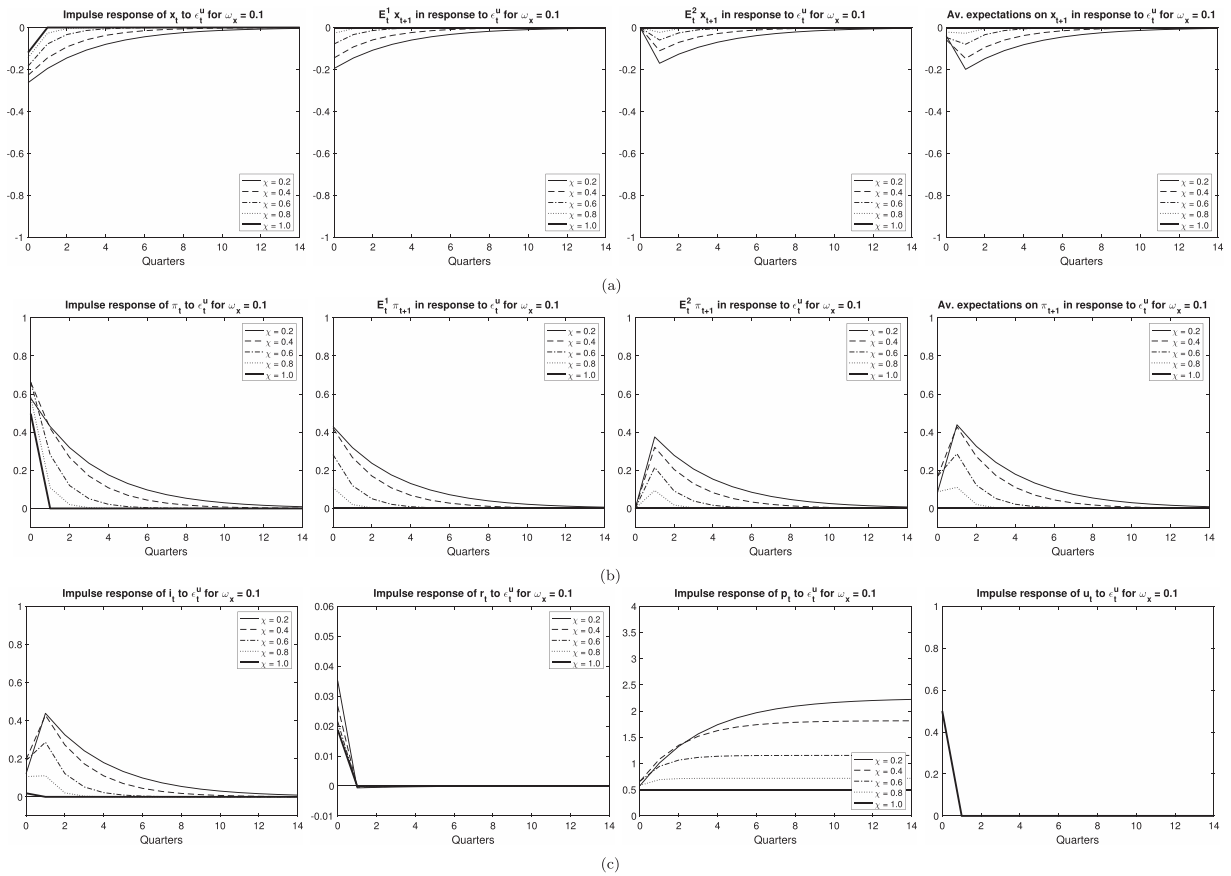


Fig. 1. Effect of a transitory cost-push shock under discretion with W calibration.

assumed heterogeneity in expectation formation affects the design of optimal monetary policy under both discretion and commitment.

3.1.1. Discretion

Under discretion, as shown in Appendix C, the first-order necessary conditions are

$$E_t \kappa_{2|t+s} = -E_t \pi_{t+s} + \beta^2 (1 - \chi) \theta^2 E_t \kappa_{2|t+s+1} \tag{30}$$

$$0 = -\omega_x E_t x_{t+s} + \lambda E_t \kappa_{2|t+s}, \tag{31}$$

where $\kappa_{2|t+s}$ is a Lagrange multiplier. From (30) to (31) one can eliminate the Lagrange multipliers and arrives at a specific targeting rule

$$\pi_t = -\frac{\omega_x}{\lambda} (x_t - \beta^2 (1 - \chi) \theta^2 E_t x_{t+1}). \tag{32}$$

In order to satisfy (32) in response to a cost-push shock, the central bank needs to contract output sufficiently such that both output and inflation satisfy this rule. Thus, it is not optimal for the central bank to fully accommodate cost-push shocks.

However notice that there is an important difference compared to the case of homogeneous expectations. Optimal stabilization policy under discretion means that the central bank takes expectations heterogeneity in private sector into account.

In order to clarify this point, recall the above thought experiment of a transitory cost-push shock. First, consider the REH benchmark with $\chi = 1$. Then (32) collapses to $\pi_t = -(\omega_x/\lambda)x_t$. Inflation is allowed to rise only by $(\omega_x/(\omega_x + \lambda^2))\varepsilon_t^u$ instead of ε_t^u on impact. The consequences of the contraction in aggregate demand are less inflationary pressures and price dispersion on impact. Average expectations of inflation and output gap coincide with RE and remain unchanged at zero. The thick solid line in Fig. 1 illustrates this policy.¹³

¹³ The simulated IRFs are for illustrative purposes. We use the W calibration. The IRFs are qualitatively similar in the MN and CGG calibration. All calibrations are detailed in Table 1 below.

However, in the case of heterogeneous expectations, $\chi \in (0, 1)$, the presence of households with AE plays an important role as can be seen directly from (32). The central bank is still required to contract aggregate demand in response to a cost-push shock. However, the central bank now takes into account the effect of a fall in output on impact on AE one period ahead. The contraction will induce pessimism reflected in a decrease in average output gap expectations one period ahead. In addition, this pessimism has consequences for the impact effects. Households with RE will account for this pessimism one period ahead via their PLM and will be more pessimistic on impact. The expectations of households with RE imply a reduction in inflationary pressures and expected future average inflation on impact.

We can show this formally by considering a solution for inflation of the form¹⁴

$$\pi_t = \Omega_{2,2}\pi_{t-1} + \Lambda_{2,2}u_t. \tag{33}$$

As we show in Appendix C, for u_t serially uncorrelated, i.e., $\rho_u = 0$, from (33) follows that (32) implies

$$x_t = -\left(\frac{\lambda}{\omega_x[1 - \beta^2(1 - \chi)\theta^2\Omega_{2,2}]}\right)\pi_t. \tag{34}$$

Two remarks regarding (34) are in order. First, our discussion that lead up to (34) does not imply that optimal stabilization policy under heterogeneous expectations yields a more favorable inflation output variability trade-off compared to the REH benchmark. In contrast, under heterogeneous expectations the above described amplification mechanism is in place and the trade-off is worse than under the REH. Fig. 1 illustrates the amplification mechanism yielding to larger impact effects and more persistent deviations from steady state.

Second, (34) suggests that optimal policy under heterogeneous expectations needs to explicitly account for expectations heterogeneity. In face of the amplification mechanism, as long as $[1 - \beta^2(1 - \chi)\theta^2\Omega_{2,2}] < 1$, stabilizing inflation requires a larger contraction in output compared to the case where expectations heterogeneity is ignored in the policy design, i.e., $x_t = -(\lambda/\omega_x)\pi_t$. A benevolent central bank has an incentive to incorporate expectations heterogeneity in order to face an improved trade-off under heterogeneous expectations. As monetary policy affects current output and inflation, the central bank has also an effect on AE in the subsequent period. The central bank can manipulate AE and it should do so. This seems natural as otherwise an issue of time-inconsistency would emerge. The central bank would have to credibly ignore structural information that it could exploit in the current period to reduce losses by affecting future behavior.¹⁵

In the main analysis below, we consider implementation of the central bank's specific targeting rule by an expectations-based reaction function as the existing literature on implementation strategies in homogeneous and heterogeneous expectations models has shown that this reaction function has many desirable properties.¹⁶

The reaction function under discretion is given by

$$i_t = \delta_x E_t x_{t+1} + \delta_\pi E_t \pi_{t+1} + \delta_{Lx}^d x_{t-1} + \delta_{L\pi} \pi_{t-1} + \delta_u u_t + \delta_g g_t, \tag{35}$$

where $\delta_x \equiv \sigma[\chi - (\omega_x/(\omega_x + \lambda^2))(1 - \chi)(\beta\theta)^2]$, $\delta_\pi \equiv [1 + (\lambda\sigma/(\omega_x + \lambda^2))\beta]\chi$, $\delta_{Lx}^d \equiv \sigma(1 - \chi)\theta^2$, $\delta_{L\pi} \equiv [1 + (\lambda\sigma/(\omega_x + \lambda^2))\beta](1 - \chi)\theta^2$, $\delta_u \equiv [\sigma\lambda/(\omega_x + \lambda^2)]$, and $\delta_g \equiv \sigma$.¹⁷

3.1.2. Commitment

Next, before we characterize the model's inflation output variability trade-off in greater detail, we examine the potential implications of heterogeneous expectations for the commitment case. Given that the central bank can credibly commit to its optimality conditions from a *timeless perspective* to overcome the problem of *time-inconsistency*, the first-order necessary conditions are

$$E_t \kappa_{2|t+s} = -E_t \pi_{t+s} + \chi E_t \kappa_{2|t+s-1} + \beta^2(1 - \chi)\theta^2 E_t \kappa_{2|t+s+1} \tag{36}$$

$$0 = -\omega_x E_t x_{t+s} + \lambda E_t \kappa_{2|t+s}, \tag{37}$$

for each date $s \geq 0$ and initial condition $\kappa_{2|-1} = 0$.¹⁸ Gasteiger (2014) derives the *specific targeting rule* under commitment from a timeless perspective in this model via (36) and (37) as

$$\pi_t = -\frac{\omega_x}{\lambda}(x_t - \chi x_{t-1} - \beta^2(1 - \chi)\theta^2 E_t x_{t+1}). \tag{38}$$

¹⁴ We detail the solution of the model in Appendix D below. The subscripts of coefficients $\Omega_{2,2}$ and $\Lambda_{2,2}$ denote rows and columns of their position in the respective matrices Ω and Λ .

¹⁵ Ideally, one would now turn to a comparison of outcomes under (32) relative to $i_t = 0$ in order to assess effectiveness of optimal policy. However, this is not feasible as $i_t = 0$ does not generate determinacy for $\chi \in (0, 1]$ for standard calibrations (see, e.g., Branch and McGough, 2009; Gasteiger, 2011). Nevertheless, we compare optimal policy and alternatives below in Section 4.3.3.

¹⁶ An alternative would be to consider implementation via fundamentals-based reaction functions. For instance, Gasteiger (2014) has shown that the latter do not generate determinacy in this model for a large share of the parameters space under commitment. The same can be shown for the case of discretion.

¹⁷ Branch and Evans (2011, p.388ff.) derive this reaction function in the context of a model with heterogeneous expectations, where both types of expectations are misspecified.

¹⁸ See Appendix C.

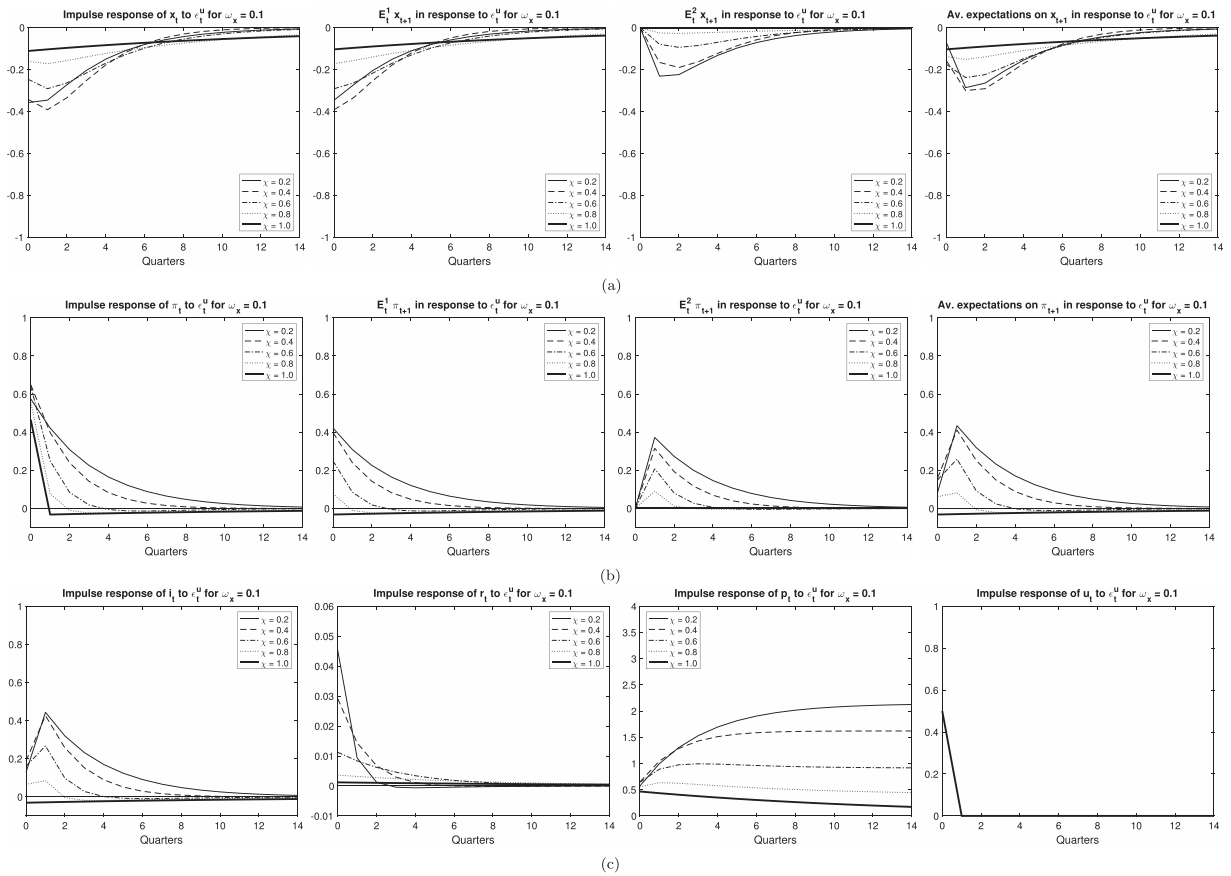


Fig. 2. Effect of a transitory cost-push shock under commitment with W calibration.

A comparison of (32) to (38) shows the additional implication of commitment via the additional term involving x_{t-1} . For the sake of clarity, let us first consider the REH benchmark with $\chi = 1$. Then (38) collapses to $\pi_t = -(\omega_x/\lambda)(x_t - x_{t-1})$. The fact that the targeting rule has inertia in the output gap introduces persistence in the economy.

Consider the thought experiment of a transitory cost-push shock. Under discretion, the economy does not have any kind of persistence and inflation and output gap just deviate on impact in order to satisfy (32). RE of households remain unaffected by the cost-push shock. However, under (38) the central bank induces persistence due to the inertia in the output gap. As a consequence, under the REH, model-consistent expectations of households incorporate this persistence and deviate from zero. In particular, in response to a cost-push shock, the central bank will contract aggregate demand in order to mitigate inflationary pressures. Inflation will rise on impact. However, the persistence induced by the central bank will render expectations about future inflation and output gap pessimistic. Thus, one can think of this as the manipulation of RE. The commitment can be thought of as a credible threat by the central bank to contract output in the subsequent periods until inflation reaches its target. The pessimism in expectations feeds back into household responses on impact. Firms perceive less inflationary pressure compared to the case of discretion and therefore inflation will rise by less. Thus, the deviations of inflation and the output gap on impact are smaller compared to discretionary policy and both variables remain off target for an extended period of time. The thick solid line in Fig. 2 illustrates optimal monetary policy under the REH.¹⁹ In sum, this policy is known to improve the inflation output variability trade-off under the REH.

How do heterogeneous expectations, i.e., $\chi \in (0, 1)$, affect the inflation output variability trade-off under commitment? First, heterogeneous expectations imply the amplification mechanism described above. Therefore one can expect a deterioration of the inflation output variability trade-off compared to the REH benchmark. Second, notice the term in (38) that captures the central bank's commitment is multiplied by χ . Therefore, the lower the share of households with RE, the less households RE can be manipulated and in consequence the effect of commitment on average expectations shrinks. Thus, heterogeneous expectations reduce the central bank's ability to manipulate average expectations via this channel. Third, (38) calls for the incorporation of expectations heterogeneity in the same way as under discretion, as the central bank can and should manipulate AE as discussed above. Ignoring expectations heterogeneity does again yield larger losses.

¹⁹ Depending on ω_x and λ , commitment can have such a strong negative effect on $E_t^1 \pi_{t+1}$ that the nominal interest rate is in fact lowered.

In sum, we conclude that optimal stabilization policy under heterogeneous expectations in the commitment case is more effective in stabilizing the economy compared to the case of discretion. However, again, this does not imply that the resulting inflation output variability trade-off is more favorable compared to the REH benchmark due to the heterogeneous expectations amplification mechanism. The simulations in Fig. 2 support this conclusion. Under heterogeneous expectations, the impact effects are amplified relative to the REH benchmark. Moreover, the impulse response functions converge faster than in the REH benchmark. Both observations are consistent with the view that expectations heterogeneity curbs the central banks ability to manipulate expectations in the sense that it can manipulate less households with RE and therefore commitment has less effect on average expectations. The effects of the cost-push shock under commitment are more similar to the ones under discretion. In particular, notice that the price level may change permanently and that there is no under-shooting of the steady state. This observation is typical for models with inflation persistence (see, Woodford, 2003a, p.499ff.).

Finally, similar to the case of discretion, in the subsequent analysis we will assume that optimal monetary policy is implemented via an expectations-based reaction function, i.e.,

$$i_t = \delta_x E_t x_{t+1} + \delta_\pi E_t \pi_{t+1} + \delta_{Lx} x_{t-1} + \delta_{L\pi} \pi_{t-1} + \delta_u u_t + \delta_g g_t, \quad (39)$$

where $\delta_{Lx} \equiv \sigma[(1 - \chi)\theta^2 - (\omega_x/(\omega_x + \lambda^2))\chi]$ and coefficients δ_x , δ_π , δ_{Lx} , δ_u , and δ_g are the same as under discretion.

3.2. A practical perspective on the loss function

This section argues that the optimal monetary analysis based on the *ad hoc* loss function (29) is of high practical relevance. Such an *ad hoc* loss function involving only output gap and inflation deviations can be justified as a reasonable approximation of a second-order approximation of a standard utility function in the homogeneous expectations benchmark.

Compared to this benchmark, the model-consistent loss function for our model has *two* additional components (see Di Bartolomeo et al., 2016). One component is due to the higher price dispersion caused by adaptive agents, similar to a model with price indexation (see Steinsson, 2003). The second component is due to the dispersion in consumption. The two components imply *eight* additional terms with two types of agents as in our model. In case of three or more types of expectation formation, the number of terms in such a model-consistent loss function grows further. Therefore such a loss function raises several practical issues.

First, as with any welfare-based loss function, the conduct of fully optimal policy requires that the relative weights cannot be freely chosen, but rather should be the result of the estimation of the structural parameters. This stands in contrast with policymakers that frequently interpret the trade-off as a policy menu in practice.

Second, effectively communicating a loss function with a total of *ten* different terms involving aggregate and average consumption levels for different types of agents is certainly challenging. In case of ineffective communication, there is a risk of in-transparency with regard to the loss function. Moreover, in the Branch and McGough (2009) model, one of the underlying axioms structures higher-order beliefs. It is assumed that agents are not explicitly aware of the presence of heterogeneous expectations. Thus, a central bank that communicates a loss function that explicitly depends on heterogeneous expectations undermines the consistency of the axiom within the context of this model.

Third, the extent to which this loss function is operational is unknown. Assume that a specific targeting rule can be derived. This will most likely depend on average consumption levels for different types of agents. Paralleling the debate on reliable measures of aggregate variables, the issue is whether reliable measures of these dis-aggregate consumption levels are available, or, can be obtained in the near future. The same concern applies to potential implementation strategies. Suppose that such a strategy next to private sector expectations on aggregate variables also conditions on private sector expectations on dis-aggregate consumption. The issue is then, to what extent these private-sector expectations on dis-aggregate consumption can be tracked by the central bank.

Fourth, while the loss function obtained by Di Bartolomeo et al. (2016) is consistent with the Branch and McGough (2009) model, it is not consistent with any other model of the economy. This point applies to any model-specific loss function. In the presence of model uncertainty, this point raises the issue of whether such a loss function remains to have favorable properties in extended versions of the same model, or, in distinct models.

Thus, we conjecture that most of the applied policy considerations may be centered around an *ad hoc* loss function. This view is consistent with a central bank thinking of the inflation output variability trade-off as a policy menu, where the relative weights reflect the preferences of the central bank. However, in light of the findings of Di Bartolomeo et al. (2016), an *ad hoc* loss function will understate the model-consistent welfare losses. To what extent this understatement matters is ultimately an empirical question: are the coefficients on the eight additional terms in the model-consistent loss function of Di Bartolomeo et al. (2016) significantly different from zero?

3.3. Definition of robust monetary policy and measures of loss

As our analysis also takes a stand on the implementation of optimal policy, the determinacy properties of the considered reaction function have to be examined. The reason is that, in principle, a reaction function can be associated with possibly infinitely many different equilibria, including the one consistent with the optimal monetary policy (see, Woodford, 1999). For instance, there may exist policy preferences ω_x that lead to indeterminacy or explosiveness. Furthering on this point,

general conditions for determinacy in this set-up are not available. Thus, we rely on numerical methods, which require a calibration. Moreover, the use of numerical methods is also required for a more detailed characterization of the inflation output variability trade-off.

However, a numerical analysis is generically subject to robustness concerns. One peculiar concern is structural parameter uncertainty and the implications for determinacy raised in Evans and McGough (2007). A policy preference ω_x may yield determinacy and rather low losses under one particular combination of λ and σ , whereas the very same policy preference may yield indeterminacy or explosiveness and therefore rather high losses for different values. Thus, as suggested by Evans and McGough (2007) we constrain the implementation strategies in the sense that we analyze them only for the policy preferences ω_x that generate determinacy across the widely used calibrations of Woodford (2003b), Clarida et al. (2000), McCallum and Nelson (1999), henceforth W, MN, CGG, for (λ, σ) . In our context, uncertainty regarding λ is particularly important. λ measures the degree of price stickiness and therefore the degree to which the central bank can influence inflation and inflation expectations via affecting aggregate demand. In short, λ is crucial for characterizing the inflation output variability trade-off faced by the central bank.

Another particular concern may be the uncertainty regarding expectations heterogeneity. It seems fairly unlikely that a central bank is able to exactly measure χ and θ , even if our model would perfectly approximate the true structure of expectations heterogeneity. We address this concern, by computing the different measures of loss in our analysis for different degrees of expectations heterogeneity. Later on, when discussing the policy implications of our main results, we will elaborate the consequences of expectations mismeasurement.

Consistent with (29), the long-run losses are computed as

$$\mathcal{L}^i(\chi, \theta, \omega_x) = (1 - \beta)^{-1} [\text{Var}(\pi) + \omega_x \text{Var}(x)] \tag{40}$$

for each reference model (or calibration) $i \in \{W, MN, CGG\}$, where $\text{Var}(\cdot)$ denotes the unconditional long-run variance of a variable. A robust optimal constrained monetary policy under heterogeneous expectations is then defined as a policy that yields determinacy across reference models for given χ and θ .

As an alternative for accounting for parameter uncertainty, we also compute the Bayesian loss as done in Evans and McGough (2007) and Levin and Williams (2003). This implies that the central bank has a prior, i.e., a subjective probability distribution over reference models $i \in \{W, MN, CGG\}$ and compute a weighted average of the losses in the three reference models, i.e.,

$$\mathcal{L}^B(\chi, \theta, \omega_x) = \omega_W \mathcal{L}^W(\chi, \theta, \omega_x) + \omega_{MN} \mathcal{L}^{MN}(\chi, \theta, \omega_x) + \omega_{CGG} \mathcal{L}^{CGG}(\chi, \theta, \omega_x). \tag{41}$$

For simplicity, we assume that $\omega_i = 1/3$ for $i \in \{W, MN, CGG\}$.

4. Main results

4.1. The case of discretion

For now, the central bank designs optimal monetary policy under discretion and implements it via reaction function (35). The reduced form matrices in (26) are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\omega_x(1-\chi)(\beta\theta)^2}{\omega_x+\lambda^2} & -\frac{\lambda\beta\chi}{\omega_x+\lambda^2} \\ \frac{\lambda\omega_x(1-\chi)(\beta\theta)^2}{\omega_x+\lambda^2} & \frac{\omega_x\beta\chi}{\omega_x+\lambda^2} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0 & -\frac{\lambda\beta(1-\chi)\theta^2}{\omega_x+\lambda^2} \\ 0 & \frac{\omega_x\beta(1-\chi)\theta^2}{\omega_x+\lambda^2} \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0 & -\frac{\lambda}{\omega_x+\lambda^2} \\ 0 & \frac{\omega_x}{\omega_x+\lambda^2} \end{bmatrix}, \text{ and } \mathbf{R} = \begin{bmatrix} \rho_g & 0 \\ 0 & \rho_u \end{bmatrix}. \tag{42}$$

Notice that we have relegated all further details regarding the reduced form and the solution of the model to Appendix D.

Our calibration is detailed in Table 1. Inspection of the matrices in (42) makes clear that the matrices are independent of σ . Thus, all else equal, differences in results among the three reference models W, MN, and CGG are related to the choice of λ and therefore related to the degree of price stickiness. Comparing the values for λ in the W, MN, and CGG calibration, we observe that the degree of price rigidity in the W and CGG calibration is much higher than in the MN calibration, as λ is much larger in the latter case. The larger λ , the more flexible are prices, the lower is the potential for relative price dispersion for a given cost-push shock. Thus, we expect a more favorable inflation output variability trade-off in the MN calibration.

Next, the properties of the exogenous shocks are taken from Evans and Honkapohja (2003, p.1059). The range for the expectations set-up, χ and θ , includes the values used in Branch and McGough (2009, p.1046ff.), Di Bartolomeo et al. (2016), or, Gasteiger (2014), and is arguably large. Finally, while we vary $\omega_x \in (0, 2]$, the tables below report solely results within an empirically relevant range, $\omega_x \in (0, 0.05]$ (see, e.g., Dennis, 2006; Givens, 2012) due to space constraints. The empirical literature, overall, appears to support small values for ω_x . Moreover, the range of micro-founded weights as discussed in Woodford (2003a, p.401) is usually also close to zero.

Our main results hold among the three reference models W, MN, and CGG unless stated otherwise. Comparisons between the REH benchmark and heterogeneous expectations are constrained by the requirement that both economies are determinate. The results can be summarized as follows.

RESULT 1. For discretionary optimal monetary policy under heterogeneous expectations, implemented via expectations-based reaction function (35), we find that:

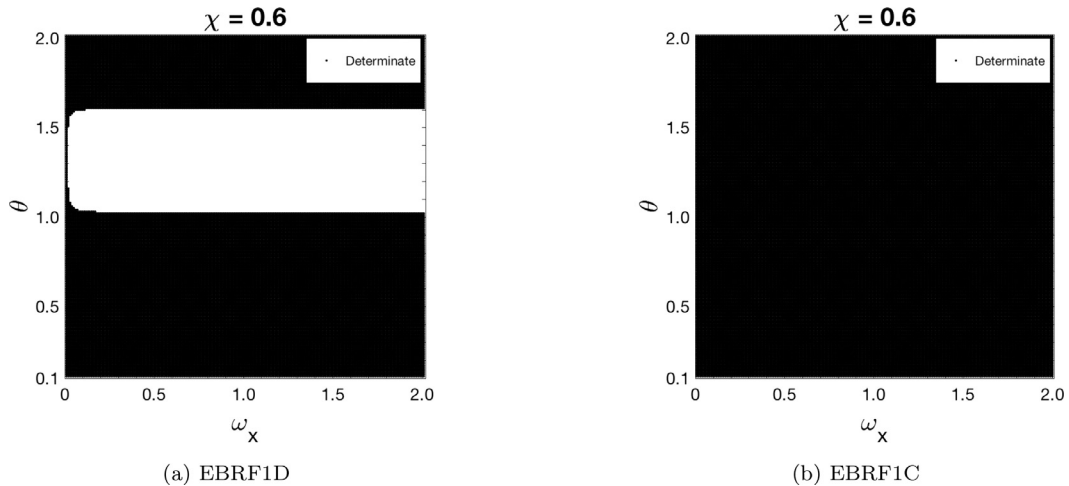


Fig. 3. Region of determinacy across the W, MN, and CGG calibrations for the Expectations-Based Reaction Function in the $\omega_x - \theta$ -space for $\chi = 0.6$.

Table 1
Calibrations used in the numerical analyses, quarterly frequency.

Parameter	Description	Value		
χ	Share of agents with RE	$\in \{1.00, \dots, 0.10\}$		
θ	Forecasting coefficient of type $\gamma = 2$ agents ^a	$\in \{0.10, \dots, 2.00\}$		
β	Period discount factor	0.99		
(ρ_g, ρ_u)	Coefficients of exogenous shocks ^b	(0.40, 0.40)		
(σ_g, σ_u)	Standard deviations of exogenous shocks ^b	(1.00, 0.50)		
		W	MN	CGG
λ	Parameter relating to the degree of price stickiness ^c	0.024	0.300	0.075
σ	Inverse of the inter-temporal elasticity of substitution ^c	0.157	1/0.164	1/4

^aThe range includes the values used in Branch and McGough (2009, p.1046ff.) and is arguably large. ^bTaken from Evans and Honkapohja (2003, p.1059). ^cTaken from W = Woodford (2003b), MN = McCallum and Nelson (1999), CGG = Clarida et al. (2000).

- 1.1 when AE are *purely adaptive* and *naïve*, $\theta \leq 1$, the model is determinate. When AE are *extrapolative*, $\theta > 1$, the model is indeterminate or explosive for a large share of the parameter space;
- 2.2 Losses are strictly larger compared to the REH benchmark for the W and CGG calibration as well as the Bayesian case;
- 3.3 The inflation output variability trade-off shifts out in non-monotonic way, depending on the share of AE;
- 4.4 A higher preference for output stabilization, ω_x , implies higher losses.

Result 1.1 is based on Fig. 3a. One can see that determinacy prevails for the entire parameters space considered for ω_x close to zero. When AE are *purely adaptive* and *naïve* expectations, i.e., $\theta \leq 1$ this result generalizes to $\omega_x \in (0, 2]$. In contrast, when AE are *extrapolative*, i.e., $\theta > 1$, but not too large, then indeterminacy and explosiveness is possible for a wide range of $\omega_x \in (0, 2]$; this finding is robust for $\chi \in (0, 1)$. An intuitive explanation for this result is that the central bank faces a dilemma due to its preference ω_x . In response to a cost-push shock, given ω_x , the contraction in aggregate demand on impact consistent with the targeting rule (32) is not large enough to mitigate the acceleration mechanism described in Section 2.4 sufficiently to ensure determinacy. Put differently, a contraction in output sufficiently large to ensure determinacy would violate the targeting rule for given ω_x .²⁰

Result 1.2 follows from Table 3. All else equal, the losses $\mathcal{L}^i(\chi, \theta, \omega_x)$ defined in (40) are the lowest under the REH in the reference models $i \in \{W, CGG\}$ and for Bayesian model averaging $\mathcal{L}^B(\chi, \theta, \omega_x)$ defined in (41). This result is due to the amplification mechanism implied by heterogeneous expectations and confirms that heterogeneous expectations imply a deterioration of the inflation output variability trade-off resulting in higher losses.

Result 1.3 stems from Figs. 4 and 5 as well as Table 3.²¹ Conditional on determinacy, the relationship between the loss $\mathcal{L}^i(\chi, \theta, \omega_x)$ and the share of AE $(1 - \chi)$ is non-monotonic $\forall i \in \{W, MN, CGG, B\}$. $\mathcal{L}^i(\chi, \theta, \omega_x)$ peaks when $\chi \approx [0.4, 0.6]$. In order to gain some intuition for the result, focus on Panel 4a. With an increase in the share of AE, i.e., declining χ , the amplification mechanism is gaining strength. Compared to the REH benchmark, this implies higher inflationary pressure and higher price dispersion. The trade-off shifts outward. Consistent with the nature of the amplification mechanism, the

²⁰ Table 2 compares our determinacy findings to the literature. We also verified numerically that our results hold if we assume that agents with RE are instead behaving like econometricians. For this verification we use the E-stability conditions from Evans and Honkapohja (2006).

²¹ Notice that the horizontal axes in Fig. 5a depicts $1 - \chi$ from the left to the right.

Table 2
Overview on results.

Policy Implementation and Calibration	Expectations Set-Up			
	$E_t^1 = E_t$		$E_t^1 = E_t^*$	
	$\chi = 1$	$\chi < 1$	$\chi = 1$	$\chi < 1$
(35)	EH2003 ^a		This Paper	
W ^b , MN, CGG	D ^c	$\theta \leq 1$ D	$\theta > 1$ D / I / E	E-stable
(39)	EH2006		This Paper	
W, MN, CGG	D	$\theta \leq 1$ D	$\theta > 1$ D	E-stable

^a EH2003 = Evans and Honkapohja (2003), EH2006 = Evans and Honkapohja (2006). ^b W = Woodford (2003b), MN = McCallum and Nelson (1999), CGG = Clarida et al. (2000). ^c D = determinate, I = indeterminate, or, E = explosive.

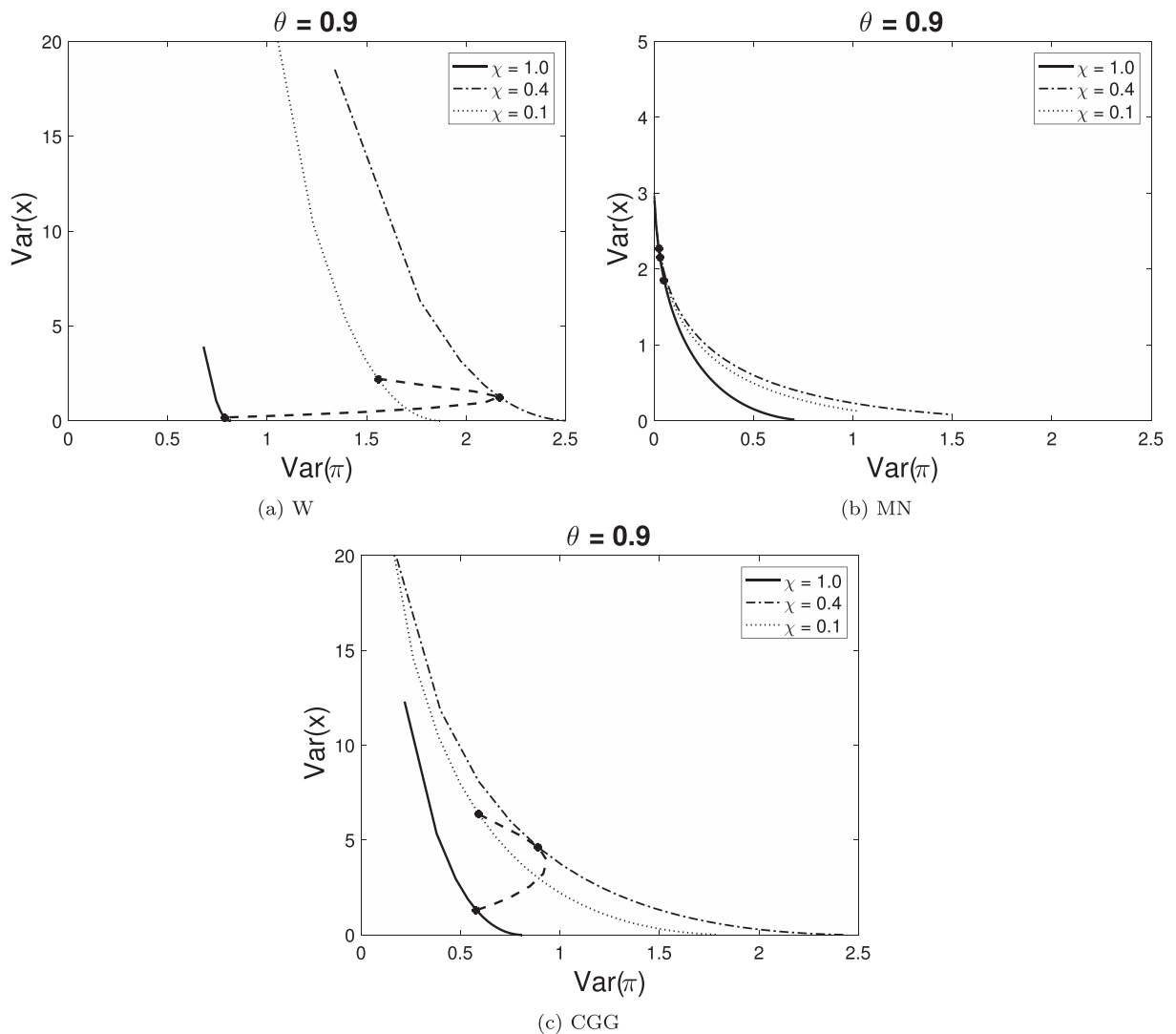


Fig. 4. Taylor (1979) curves for the W, MN, and CGG calibrations for the Expectations-Based Reaction Function under discretion. We depict $\omega_x \in (0, 2]$ as smaller values imply a very large $\text{Var}(x)$. The * on the curve indicates $\omega_x = 0.05$, smaller (larger) values of ω_x are to the left (right) along the curve.

Table 3
Results for the Expectations-Based Reaction Function under discretion.

Central Bank preferences	Expectations parameters		$\mathcal{L}^i(\chi, \theta, \omega_x)$ evaluated in				ω_x^* and $\mathcal{L}^i(\chi, \theta, \omega_x \omega_x^*)$ evaluated in				% $\Delta\mathcal{L}^i(\chi, \theta, \omega_x \omega_x^*)$ evaluated in							
	θ	χ	W	MN	CGG	B	W	MN	CGG	B	W	MN	CGG	B				
0.01	0.9	1	71.91	3.23	34.18	36.44	0.01	71.91	0.01	3.23	0.01	34.18	36.44	0	0	0	0	
		0.8	101.81	3.21	37.28	47.43	0.01	101.81	0.01	3.21	0.01	37.28	47.43	0	0	0	0	
		0.6	139.17	3.19	38.65	60.34	0.01	139.17	0.01	3.19	0.01	38.65	60.34	0	0	0	0	
		0.4	152.52	3.18	37.86	64.52	0.01	152.52	0.01	3.18	0.01	37.86	64.52	0	0	0	0	
	1	0.2	131.85	3.16	35.8	56.94	0.01	131.85	0.01	3.16	0.01	35.8	56.94	0	0	0	0	
		0.8	123.49	3.22	39.42	55.38	0.01	123.49	0.01	3.22	0.01	39.42	55.38	0	0	0	0	
		0.6	235.84	3.21	42.55	93.87	0.01	235.84	0.01	3.21	0.01	42.55	93.87	0	0	0	0	
		0.4	306.34	3.2	42.31	117.28	0.01	306.34	0.01	3.2	0.01	42.31	117.28	0	0	0	0	
	1.1	0.2	245.94	3.19	40.28	96.47	0.01	245.94	0.01	3.19	0.01	40.28	96.47	0	0	0	0	
		0.8	160.39	3.23	42	68.54	0.01	160.39	0.01	3.23	0.01	42	68.54	0	0	0	0	
		0.6	728.63	3.22	47.19	259.68	0.01	728.63	0.01	3.22	0.01	47.19	259.68	0	0	0	0	
		0.4	926.91	3.22	47.06	325.73	0.01	926.91	0.01	3.22	0.01	47.06	325.73	0	0	0	0	
	0.03	0.9	0.2	428.35	3.22	44.75	158.77	0.01	428.35	0.01	3.22	0.01	44.75	158.77	0	0	0	0
			1	78.1	9.17	56.41	47.89	0.02	77.53	0.02	8.85	0.02	54	46.8	-0.73	-3.4	-4.28	-2.29
			0.8	116.75	9.14	70.72	65.54	0.02	114.5	0.02	8.87	0.02	65.04	62.8	-1.92	-2.96	-8.04	-4.17
			0.6	175.46	9.06	82.55	89.02	0.01	162.55	0.02	8.86	0.01	73.61	81.67	-7.36	-2.18	-10.83	-8.26
1		0.4	206.65	8.96	83.37	99.66	0.01	189.56	0.02	8.85	0.02	75.43	91.28	-8.27	-1.16	-9.52	-8.41	
		0.2	176.44	8.84	75.01	86.76	0.02	171.08	0.02	8.84	0.02	71.98	83.97	-3.04	-0.02	-4.04	-3.22	
		0.8	146.67	9.22	80.05	78.65	0.01	140.23	0.02	8.93	0.01	71.65	73.6	-4.39	-3.13	-10.5	-6.41	
		0.6	381.71	9.21	106.35	165.76	0.01	283.59	0.02	8.98	0.01	83.79	125.45	-25.7	-2.41	-21.22	-24.31	
1.1		0.4	729.93	9.15	111.7	283.59	0.01	417.29	0.02	9.02	0.01	91.82	172.71	-42.83	-1.37	-17.8	-39.1	
		0.2	549.89	9.07	99.15	219.37	0.01	415.61	0.02	9.05	0.02	92.83	172.49	-24.42	-0.24	-6.38	-21.37	
		0.8	-	-	-	-	0.01	183.68	0.02	9	0.01	77.49	90.06	-	-	-	-	
		0.6	-	-	-	-	0.01	929.92	0.01	9.25	0.01	96.8	345.32	-	-	-	-	
0.05		0.9	0.4	-	-	-	0.01	1538.42	0.01	9.34	0.01	109.17	552.31	-	-	-	-	
			0.2	2015.81	9.3	125.99	717.03	0.01	998.86	0.02	9.26	0.01	115.87	374.66	-50.45	-0.41	-8.03	-47.75
			1	79.46	14.42	64.5	52.79	0.03	79.08	0.03	13.75	0.03	62.35	51.73	-0.47	-4.66	-3.32	-2.01
			0.8	120.26	14.52	85.93	73.57	0.02	118.25	0.03	13.86	0.03	79.29	70.47	-1.67	-4.55	-7.72	-4.22
	1	0.6	185.2	14.44	107.72	102.45	0.02	175.61	0.03	13.92	0.02	92.79	94.11	-5.18	-3.58	-13.86	-8.14	
		0.4	222.97	14.21	112.26	116.48	0.02	208.47	0.03	13.93	0.02	99.09	107.17	-6.5	-1.9	-11.73	-7.99	
		0.2	190.11	13.91	98.92	100.98	0.03	185.52	0.04	13.82	0.03	94.39	97.91	-2.42	-0.68	-4.58	-3.04	
		0.8	152.35	14.76	100.55	89.22	0.02	147.51	0.03	14.03	0.02	88.45	83.33	-3.18	-4.99	-12.03	-6.6	
	1.1	0.6	438.65	14.85	154.87	202.79	0.01	331.35	0.03	14.23	0.02	115.13	153.57	-24.46	-4.17	-25.66	-24.27	
		0.4	1045.79	14.72	173.89	411.47	0.01	528.24	0.03	14.36	0.02	130.56	224.39	-49.49	-2.45	-24.92	-45.47	
		0.2	787.26	14.51	147.88	316.55	0.02	563.03	0.04	14.4	0.03	134.1	237.18	-28.48	-0.77	-9.32	-25.07	
		0.8	-	-	-	-	0.02	201.22	0.02	14.36	0.02	100.58	105.39	-	-	-	-	
	1.1	0.6	-	-	-	-	0.01	1131.2	0.01	15.27	0.01	146.4	430.96	-	-	-	-	
		0.4	-	-	-	-	0.01	2149.93	0.01	15.47	0.01	171.29	778.9	-	-	-	-	
		0.2	7100.56	15.1	208.75	2441.47	0.01	1569.37	0.04	14.98	0.02	180.6	588.32	-77.9	-0.8	-13.48	-75.9	

Table 4
Results for the Expectations-Based Reaction Function under commitment.

Central Bank preferences	Expectations parameters		$\mathcal{L}^i(\chi, \theta, \omega_x)$ evaluated in				ω_x^* and $\mathcal{L}^i(\chi, \theta, \omega_x \omega_x^*)$ evaluated in				% $\Delta\mathcal{L}^i(\chi, \theta, \omega_x \omega_x^*)$ evaluated in								
	ω_x	θ	χ	W	MN	CGG	B	W	MN	CGG	B	W	MN	CGG	B				
0.01	0.9	1	50.28	2.94	22	25.08	0.01	50.28	0.01	2.94	0.01	22	25.08	0	0	0	0		
		0.8	71.65	3.02	26.17	33.61	0.01	71.65	0.01	3.02	0.01	26.17	33.61	0	0	0	0		
		0.6	102.09	3.09	30.43	45.2	0.01	102.09	0.01	3.09	0.01	30.43	45.2	0	0	0	0		
	1	0.4	126.66	3.13	33.61	54.47	0.01	126.66	0.01	3.13	0.01	33.61	54.47	0	0	0	0		
		0.2	124.86	3.15	34.69	54.23	0.01	124.86	0.01	3.15	0.01	34.69	54.23	0	0	0	0		
		0.8	77.29	3.03	26.84	35.72	0.01	77.29	0.01	3.03	0.01	26.84	35.72	0	0	0	0		
	1.1	0.6	128.59	3.1	32.23	54.64	0.01	128.59	0.01	3.1	0.01	32.23	54.64	0	0	0	0		
		0.4	189.8	3.15	36.68	76.55	0.01	189.8	0.01	3.15	0.01	36.68	76.55	0	0	0	0		
		0.2	208.96	3.18	38.78	83.64	0.01	208.96	0.01	3.18	0.01	38.78	83.64	0	0	0	0		
	0.03	0.9	0.8	83.52	3.04	27.58	38.04	0.01	83.52	0.01	3.04	0.01	27.58	38.04	0	0	0	0	
			0.6	162.86	3.11	34.2	66.72	0.01	162.86	0.01	3.11	0.01	34.2	66.72	0	0	0	0	
			0.4	276.12	3.17	39.92	106.4	0.01	276.12	0.01	3.17	0.01	39.92	106.4	0	0	0	0	
		1	0.2	325.85	3.21	42.93	123.99	0.01	325.85	0.01	3.21	0.01	42.93	123.99	0	0	0	0	
			1	61.53	7.41	35.62	34.85	0.03	61.53	0.03	7.41	0.03	35.62	34.85	0	0	0	0	
			0.8	93.14	7.9	46.41	49.15	0.03	93.14	0.03	7.9	0.03	46.41	49.15	0	0	0	0	
		1.1	0.6	143.17	8.32	59.41	70.3	0.03	143.17	0.03	8.32	0.03	59.41	70.3	0	0	0	0	
			0.4	182.49	8.61	69.61	86.9	0.03	182.49	0.03	8.61	0.03	69.61	86.9	0	0	0	0	
			0.2	170.1	8.76	71.32	83.39	0.03	170.1	0.03	8.76	0.03	71.32	83.39	0	0	0	0	
		0.05	0.9	0.8	103.53	7.95	48.63	53.37	0.03	103.53	0.03	7.95	0.03	48.63	53.37	0	0	0	0
				0.6	208.15	8.43	67.05	94.54	0.03	208.15	0.03	8.43	0.03	67.05	94.54	0	0	0	0
				0.4	372.55	8.79	84.57	155.3	0.03	372.55	0.03	8.79	0.03	84.57	155.3	0	0	0	0
			1	0.2	413.41	8.98	91.5	171.3	0.03	413.41	0.03	8.98	0.03	91.5	171.3	0	0	0	0
				0.8	114.35	8.01	51.12	57.83	0.03	114.35	0.03	8.01	0.03	51.12	57.83	0	0	0	0
				0.6	303.27	8.55	75.94	129.25	0.03	303.27	0.03	8.55	0.03	75.94	129.25	0	0	0	0
1.1			0.4	723.64	8.96	101.62	278.07	0.03	723.64	0.03	8.96	0.03	101.62	278.07	0	0	0	0	
			0.2	895.81	9.21	113.93	339.65	0.03	895.81	0.03	9.21	0.03	113.93	339.65	0	0	0	0	
			1	65.69	10.81	42.16	39.55	0.05	65.69	0.05	10.81	0.05	42.16	39.55	0	0	0	0	
0.05			0.9	0.8	101.35	11.84	57.22	56.81	0.05	101.35	0.05	11.84	0.05	57.22	56.81	0	0	0	0
				0.6	159.3	12.76	76.84	82.96	0.05	159.3	0.05	12.76	0.05	76.84	82.96	0	0	0	0
				0.4	203.55	13.41	92.49	103.15	0.05	203.55	0.05	13.41	0.05	92.49	103.15	0	0	0	0
			1	0.2	185.12	13.71	93.58	97.47	0.05	185.12	0.05	13.71	0.05	93.58	97.47	0	0	0	0
				0.8	114.31	11.97	60.7	62.32	0.05	114.31	0.05	11.97	0.05	60.7	62.32	0	0	0	0
				0.6	250.32	13.04	90.43	117.93	0.05	250.32	0.05	13.04	0.05	90.43	117.93	0	0	0	0
	1.1		0.4	497.33	13.85	121.33	210.84	0.05	497.33	0.05	13.85	0.05	121.33	210.84	0	0	0	0	
			0.2	552.77	14.29	132.41	233.16	0.05	552.77	0.05	14.29	0.05	132.41	233.16	0	0	0	0	
			0.8	127.18	12.1	64.57	67.95	0.05	127.18	0.05	12.1	0.05	64.57	67.95	0	0	0	0	
	0.05		0.6	0.6	391.38	13.34	106.89	170.54	0.05	391.38	0.05	13.34	0.05	106.89	170.54	0	0	0	0
				0.4		14.3	156.33	441.2	0.05		0.05	14.3	0.05	156.33	441.2	0	0	0	0
				0.2	1152.96				0.04	1152.96			0.05				0	0	0
				0.2	1454.4	14.86	179.08	549.45	0.04	1454.4	0.05	14.86	0.05	179.08	549.45	0	0	0	0

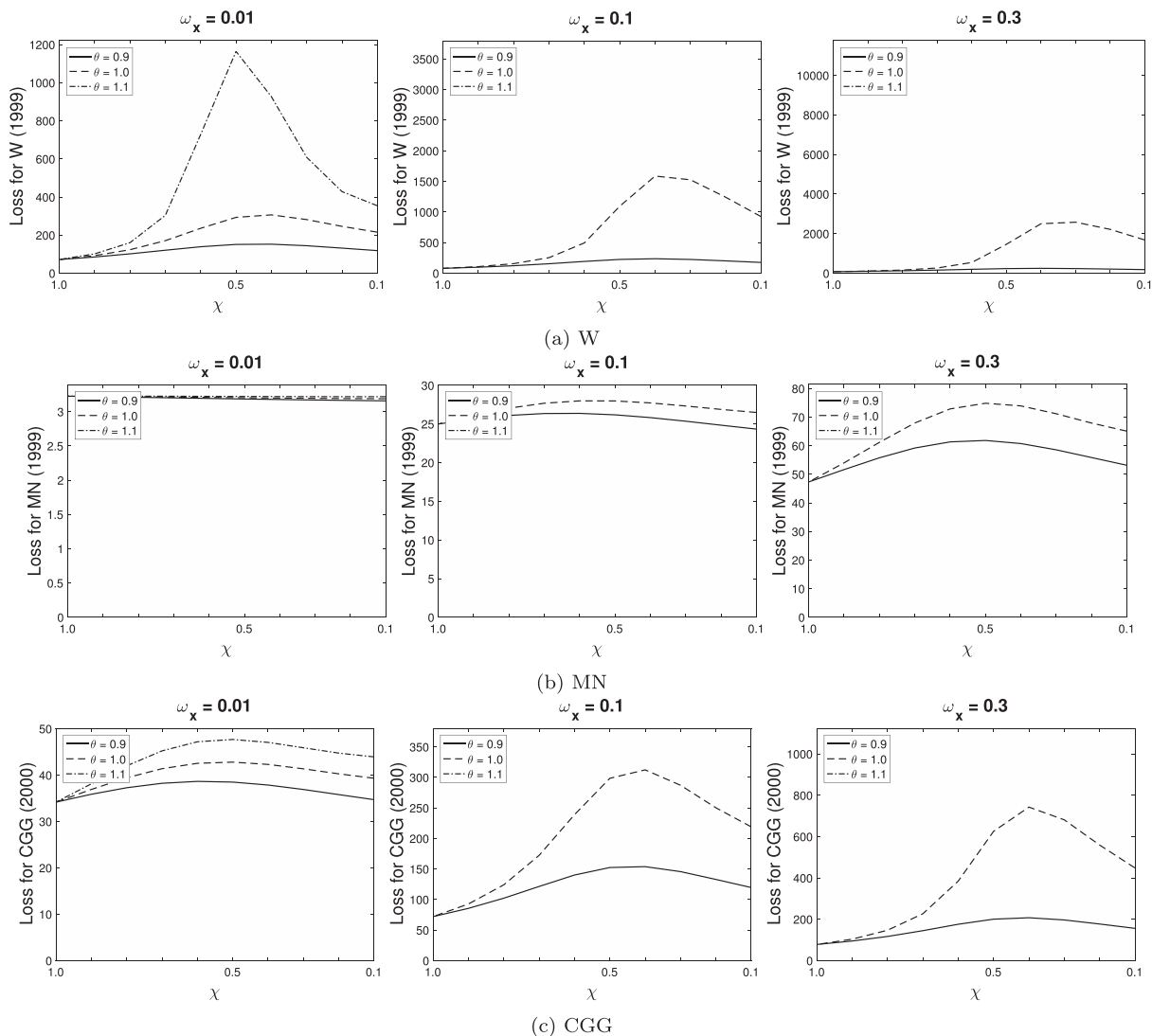


Fig. 5. Losses for the W, MN, and CGG calibrations for the *Expectations-Based Reaction Function* under discretion.

increase in $\text{Var}(\pi)$ is larger than the increase in $\text{Var}(x)$. Recall that at the heart of the mechanism are households with RE amplifying the impact effects of a cost-push shock because of the persistence induced by households with AE. For some χ , the share of households with RE is so low that the amplification mechanism must lose strength. Therefore the trade-off shifts back inward to some extent.

Result 1.4 can be seen from Fig. 6 and Table 3. $\mathcal{L}^l(\chi, \theta, \omega_x)$ and $\omega_x, \forall l \in \{W, MN, CGG, B\}$, are positively related to each other, i.e., the higher ω_x , the larger $\mathcal{L}^l(\chi, \theta, \omega_x)$. This is intuitive as a higher ω_x gives more potential to the amplification mechanism due to heterogeneous expectations resulting in a worse volatility outcome. While the reduction in output volatility due to a higher ω_x is relatively small, there is a relatively large increase in inflation volatility. However, this interpretation can be questioned on the grounds of the normalisation of weights in loss function (40). In particular, for a given inflation and output volatility, a higher ω_x has to result in a larger loss. Therefore, in order to corroborate our interpretation of results, we have carried out an additional exercise. We fix the relative weight in the loss function at a particular value $\omega_x \in (0, 2]$. Then we take the variances generated by any $\tilde{\omega}_x \in [\omega_x, 2]$ used in the policy implementation and compute the losses $\mathcal{L}^l(\chi, \theta, \omega_x | \tilde{\omega}_x) \forall l \in \{W, MN, CGG\}$. The results of this exercise in Fig. 10 show that, for a given central bank preference ω_x used to compare losses, implementing policy with a higher $\tilde{\omega}_x$ leads to a larger loss. This confirms Result 1.4.²²

²² We are grateful to an anonymous referee for highlighting this issue and suggesting the additional exercise.

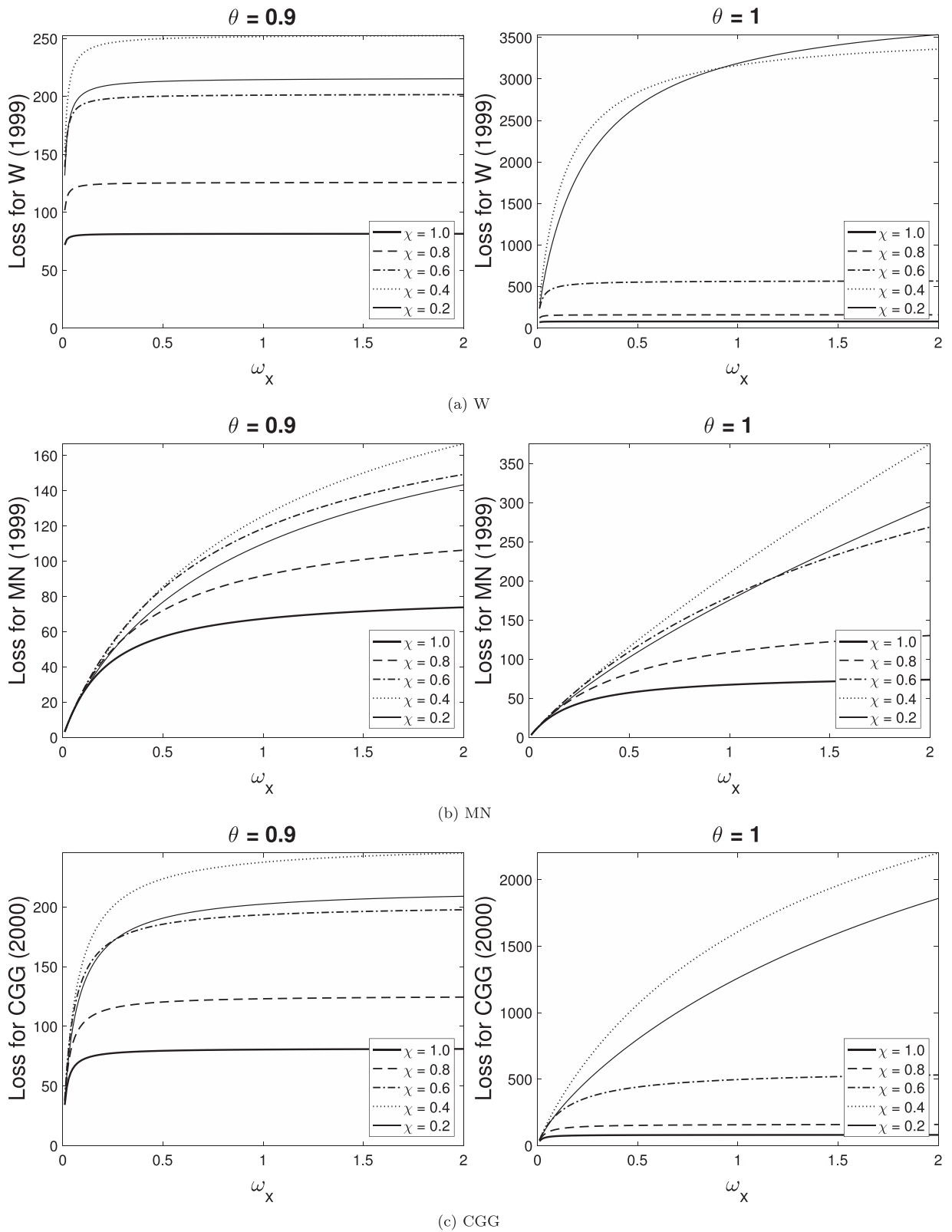


Fig. 6. Losses for the W, MN, and CGG calibrations for the *Expectations-Based Reaction Function* under discretion.

4.2. Gains from commitment

Commitment from a timeless perspective is known to generate determinacy throughout the parameter space and smaller losses in homogeneous expectations models. Thus, it is straight-forward to ask, to what extent this finding carries over to the heterogeneous expectations model?

Recall from our discussion in Section 3.1.2 that commitment introduces persistence in the output gap. This can be seen in the targeting rule (38) and the related implementation (39). As a consequence, the dynamics of the economy can be described by reduced form (26), where all system matrices are as in the case of discretion except for

$$C = \begin{bmatrix} \frac{\omega_x \chi}{\omega_x + \lambda^2} & -\frac{\lambda \beta (1 - \chi) \theta^2}{\omega_x + \lambda^2} \\ \frac{\omega_x \lambda \chi}{\omega_x + \lambda^2} & \frac{\omega_x \beta (1 - \chi) \theta^2}{\omega_x + \lambda^2} \end{bmatrix}. \tag{43}$$

Matrix (43) allows two observations. First, the reduced form is again independent of σ and potential differences among reference models must again stem from λ . Second, consistent with the specific targeting rule under commitment (38), there is persistence in output that depends on χ . The first column in matrix C is now different from zero. Moreover, as it is multiplied by χ , one can see that heterogeneous expectations curb the potential gains from commitment.

Consistent with these observations, our findings are as follows.

RESULT 2. For optimal monetary policy with commitment from a timeless perspective under heterogeneous expectations, implemented via expectations-based reaction function (39), we find that:

- 2.1 the model is determinate throughout the parameter space;
- 2.2 Losses are strictly higher compared to the REH benchmark, but strictly lower compared to the case of discretion;
- 2.3 The inflation output variability trade-off shifts out in non-monotonic way, depending on the share of AE;
- 2.4 A higher preference for output stabilization, ω_x , implies higher losses.

Result 2.1 is based on Fig. 3b and generalizes this finding of Gasteiger (2014) in the sense that it holds among several reference models. Consistent with our arguments in Section 3.1, commitment enables the central bank to manipulate RE. In response to a cost-push shock, this ability to manipulate RE provides scope for the central bank to satisfy the targeting rule and to mitigate the acceleration mechanism in price setting due to heterogeneous expectations at any preference for output stabilization.²³

Result 2.2 can be seen from Table 4 and extends our result 1.2 for discretion. All else equal, the losses $\mathcal{L}^l(\chi, \theta, \omega_x)$ defined in (40) are the lowest under the REH in all reference models $\forall \iota \in \{W, MN, CGG\}$ and for Bayesian model averaging $\mathcal{L}^B(\chi, \theta, \omega_x)$ defined in (41). The explanation is again the amplification mechanism introduced by heterogeneous expectations. However, the effect of commitment is worthwhile. Commitment lowers losses as the amplification mechanism can always be mitigated to some extent via manipulating RE. This explains, why losses are lower compared to the case of discretion. However, the gains from commitment in terms of losses are lower, when the share of agents with RE, χ , is lower. We illustrate this in Table 5. This shows that expectations heterogeneity can curb the central bank’s ability to manipulate average expectations.

Result 2.3 can be seen from Figs. 7 and 8 as well as Table 4. Conditionally on determinacy, the relationship between the loss $\mathcal{L}^l(\chi, \theta, \omega_x)$ and the share of AE $(1 - \chi)$ is non-monotonous $\forall \iota \in \{W, CGG, B\}$ if $\theta \leq 1$. $\mathcal{L}^l(\chi, \theta, \omega_x)$ peaks when $\chi \approx 0.2$. Compared to discretion, the peak is at a lower level, consistent with the beneficial effect of commitment.

For an intuition, focus on Panel 7a. With a declining share of households with RE, χ , the amplification mechanism is strengthened as in the case of discretion, which raises both $\text{Var}(\pi)$ and $\text{Var}(x)$, therefore the trade-off shifts outward. However, there is now an additional effect. A decline in χ also implies that the central bank’s ability to manipulate RE is reduced, which implies that $\text{Var}(x)$ increases at a faster rate compared to the case of discretion. At some share of AE, $(1 - \chi)$, the amplification mechanism then becomes weaker, as the share of households with RE is getting lower. In consequence, the trade-off shifts back toward to the origin.

Next, when AE are extrapolative, then $\mathcal{L}^l(\chi, \theta, \omega_x)$ increases monotonically when χ declines. The latter is consistent with destabilizing properties of extrapolative expectations in backward-looking models.

Result 2.4 stems from Fig. 9 and Table 4 and extends result 1.4 to the case of commitment. $\mathcal{L}^l(\chi, \theta, \omega_x)$ and $\omega_x, \forall \iota \in \{W, MN, CGG, B\}$, are positively related to each other, i.e., the higher the central bank’s preference ω_x , the larger losses $\mathcal{L}^l(\chi, \theta, \omega_x)$. Again, this is due to the fact that more weight on output stabilization strengthens the amplification effect is initiated by inflationary pressures.²⁴

4.3. Policy implications

We now turn to the policy implications of heterogeneous expectations. We will first elaborate the implications of a more hawkish monetary policy. Next, we examine the implications of mismeasuring expectations in the design and implementation of monetary policy. Finally, we discuss straightforward implications emerging from our results above. The former two

²³ Again compare Table 2: our results are plausible from an adaptive learning viewpoint.

²⁴ We have done a similar exercise as related to Result 1.4 to corroborate our interpretation of results, see Fig. 11.

Table 5
Gains from commitment for the Expectations-Based Reaction Function.

Central Bank preferences	Expectations parameters		$\mathcal{L}^W(\chi, \theta, \omega_x)$			$\mathcal{L}^{MN}(\chi, \theta, \omega_x)$			$\mathcal{L}^{CGG}(\chi, \theta, \omega_x)$			$\mathcal{L}^B(\chi, \theta, \omega_x)$			
	ω_x	θ	χ	Discr.	Comm.	% Δ	Discr.	Comm.	% Δ	Discr.	Comm.	% Δ	Discr.	Comm.	% Δ
0.01	0.9	1	71.91	50.28	-30.08	3.23	2.94	-8.79	34.18	22	-35.61	36.44	25.08	-31.18	
		0.8	101.81	71.65	-29.63	3.21	3.02	-5.85	37.28	26.17	-29.79	47.43	33.61	-29.13	
		0.6	139.17	102.09	-26.65	3.19	3.09	-3.39	38.65	30.43	-21.27	60.34	45.2	-25.09	
	1	0.4	152.52	126.66	-16.96	3.18	3.13	-1.53	37.86	33.61	-11.22	64.52	54.47	-15.58	
		0.2	131.85	124.86	-5.31	3.16	3.15	-0.39	35.8	34.69	-3.1	56.94	54.23	-4.75	
		0.8	123.49	77.29	-37.41	3.22	3.03	-5.88	39.42	26.84	-31.92	55.38	35.72	-35.49	
	1.1	0.6	235.84	128.59	-45.47	3.21	3.1	-3.41	42.55	32.23	-24.25	93.87	54.64	-41.79	
		0.4	306.34	189.8	-38.04	3.2	3.15	-1.55	42.31	36.68	-13.29	117.28	76.55	-34.73	
		0.2	245.94	208.96	-15.04	3.19	3.18	-0.39	40.28	38.78	-3.71	96.47	83.64	-13.3	
	0.03	0.9	0.8	160.39	83.52	-47.92	3.23	3.04	-5.91	42	27.58	-34.35	68.54	38.04	-44.49
			0.6	728.63	162.86	-77.65	3.22	3.11	-3.44	47.19	34.2	-27.53	259.68	66.72	-74.31
			0.4	926.91	276.12	-70.21	3.22	3.17	-1.56	47.06	39.92	-15.17	325.73	106.4	-67.33
		1	0.2	428.35	325.85	-23.93	3.22	3.21	-0.39	44.75	42.93	-4.07	158.77	123.99	-21.9
			0.8	78.1	61.53	-21.22	9.17	7.41	-19.14	56.41	35.62	-36.86	47.89	34.85	-27.23
			0.6	116.75	93.14	-20.22	9.14	7.9	-13.5	70.72	46.41	-34.38	65.54	49.15	-25
1.1		0.4	175.46	143.17	-18.4	9.06	8.32	-8.19	82.55	59.41	-28.03	89.02	70.3	-21.03	
		0.2	206.65	182.49	-11.69	8.96	8.61	-3.83	83.37	69.61	-16.51	99.66	86.9	-12.8	
		0.8	176.44	170.1	-3.6	8.84	8.76	-0.98	75.01	71.32	-4.91	86.76	83.39	-3.89	
0.05		0.9	0.8	146.67	103.53	-29.41	9.22	7.95	-13.72	80.05	48.63	-39.25	78.65	53.37	-32.14
			0.6	381.71	208.15	-45.47	9.21	8.43	-8.42	106.35	67.05	-36.96	165.76	94.54	-42.96
			0.4	729.63	372.55	-48.96	9.15	8.79	-3.95	111.7	84.57	-24.29	283.59	155.3	-45.24
		1	0.2	549.89	413.41	-24.82	9.07	8.98	-1.01	99.15	91.5	-7.72	219.37	171.3	-21.92
			0.8	-	114.35	-	-	8.01	-	-	51.12	-	-	57.83	-
			0.6	-	303.27	-	-	8.55	-	-	75.94	-	-	129.25	-
	1.1	0.4	-	723.64	-	-	8.96	-	-	101.62	-	-	278.07	-	
		0.2	2015.81	895.81	-55.56	9.3	9.21	-1.03	125.99	113.93	-9.57	717.03	339.65	-52.63	
		1	79.46	65.69	-17.32	14.42	10.81	-25.02	64.5	42.16	-34.64	52.79	39.55	-25.08	
	0.05	0.9	0.8	120.26	101.35	-15.72	14.52	11.84	-18.44	85.93	57.22	-33.41	73.57	56.81	-22.79
			0.6	185.2	159.3	-13.99	14.44	12.76	-11.62	107.72	76.84	-28.67	102.45	82.96	-19.02
			0.4	222.97	203.55	-8.71	14.21	13.41	-5.58	112.26	92.49	-17.61	116.48	103.15	-11.44
		1	0.2	190.11	185.12	-2.62	13.91	13.71	-1.45	98.92	93.58	-5.4	100.98	97.47	-3.48
			0.8	152.35	114.31	-24.97	14.76	11.97	-18.95	100.55	60.7	-39.63	89.22	62.32	-30.15
			0.6	438.65	250.32	-42.93	14.85	13.04	-12.18	154.87	90.43	-41.61	202.79	117.93	-41.85
1.1		0.4	1045.79	497.33	-52.44	14.72	13.85	-5.9	173.89	121.33	-30.23	411.47	210.84	-48.76	
		0.2	787.26	552.77	-29.79	14.51	14.29	-1.54	147.88	132.41	-10.46	316.55	233.16	-26.34	
		0.8	-	127.18	-	-	12.1	-	-	64.57	-	-	67.95	-	
1.1		0.6	-	391.38	-	-	13.34	-	-	106.89	-	-	170.54	-	
		0.4	-	1152.96	-	-	14.3	-	-	156.33	-	-	441.2	-	
		0.2	7100.56	1454.4	-79.52	15.1	14.86	-1.58	208.75	179.08	-14.21	2441.47	549.45	-77.5	

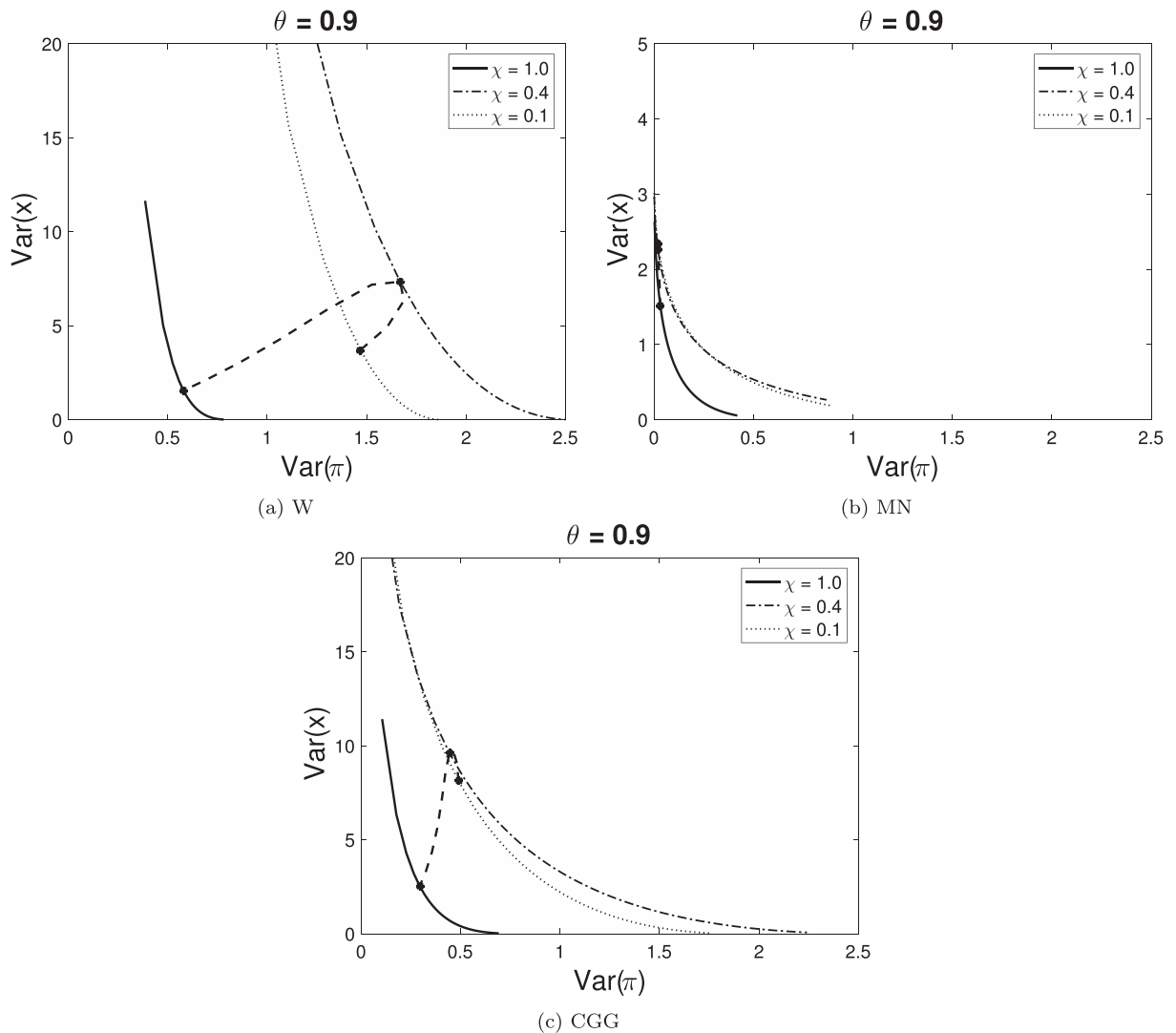


Fig. 7. Taylor (1979) curves for the W, MN, and CGG calibrations for the *Expectations-Based Reaction Function* under commitment. The * on the curve indicates $\omega_x = 0.05$, smaller (larger) values of ω_x are to the left (right) along the curve.

exercises can be motivated by inspecting (32). All else equal, either a lower ω_x or a lower χ imply that the contraction in output required to satisfy (32) is relatively larger. This can imply a better exploitation of the novel channel described in Section 3.1 to counteract the heterogeneous expectations amplification mechanism and therefore can have important consequences for central bank losses.

4.3.1. A more hawkish monetary policy

Point of departure is the finding that losses increase with the preference of the central bank for output gap stabilization, ω_x . As the central bank’s preference is given, one may wonder whether there exists an incentive for the central bank with preference ω_x to hire a more ‘conservative’ central banker that implements $\omega_x^* \leq \omega_x$. This question is also addressed in Orphanides and Williams (2008). We compute the losses for this scenario as

$$\mathcal{L}^l(\chi, \theta, \omega_x | \omega_x^*) = (1 - \beta)^{-1} [\text{Var}(\pi | \omega_x^*) + \omega_x \text{Var}(x | \omega_x^*)]. \tag{44}$$

An incentive for the central bank to hire a more ‘conservative’ central banker then exists if

$$\% \Delta \mathcal{L}^l(\chi, \theta, \omega_x | \omega_x^*) = \left[\left(\frac{\mathcal{L}^l(\chi, \theta, \omega_x | \omega_x^*)}{\mathcal{L}^l(\chi, \theta, \omega_x)} \right) - 1 \right] \times 100 < 0.$$

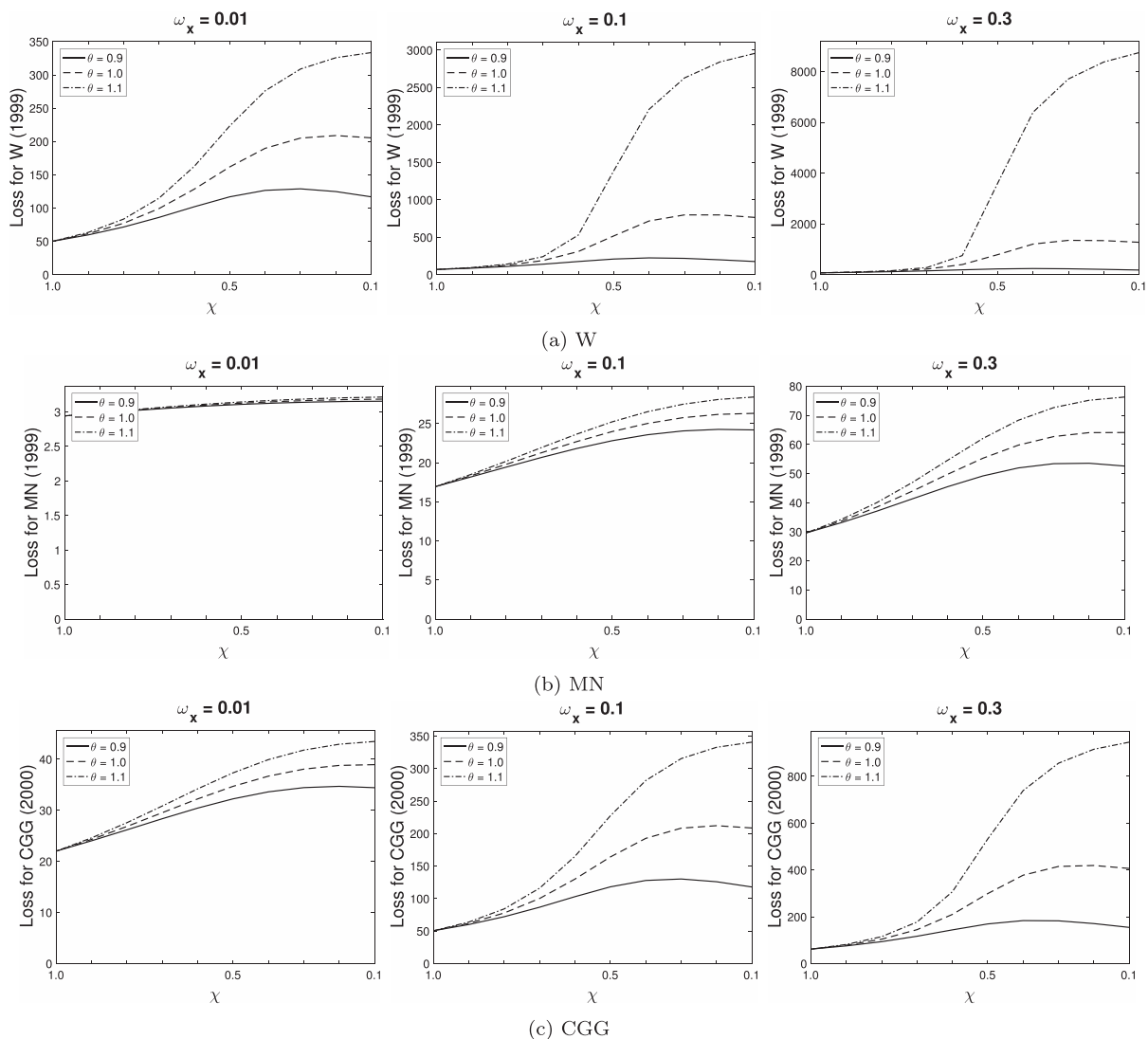


Fig. 8. Losses for the W, MN, and CGG calibrations for the *Expectations-Based Reaction Function* under commitment.

One can find (44) by (1.) taking the variances from Tables 8 (or 9 in case of commitment) and interpreting them as the outcome of a candidate central banker; (2.) computing losses for $\omega_x \in (0, 2]$ for each pair of variances resulting from a candidate central banker; (3.) running a grid search over all losses to find $\mathcal{L}^i(\chi, \theta, \omega_x | \omega_x^*)$ and ω_x^* .²⁵

The results for discretion and commitment in the empirically plausible range of ω_x are displayed in the last four columns of Tables 3 and 4 respectively. A negative sign suggests that in this particular case the central bank has an incentive to hire a more ‘conservative’ central banker. Our computations show that under discretion this incentive is present throughout, but is not present in the case of commitment. Inspecting the values of ω_x^* in Table 3 also shows that it is not optimal to have a central banker that is too conservative. As $\omega_x \rightarrow 0$, the right-hand side in (32) (or (C.4) for serially correlated shocks) approaches $-\infty$. Thus, the variance of the output gap and losses become potentially infinitely large. This mechanism counteracts the reduction of the inflation variance and losses due to a smaller ω_x that also exploits the novel channel to manipulate heterogeneous expectations.

We conclude that under heterogeneous expectations and discretionary policy, it is always optimal for the central bank to hire a central banker that is more hawkish than the central bank itself. However, hiring an overly hawkish central banker can result in worse outcomes of the loss function relative to not hiring such central banker.²⁶ Finally, when the central

²⁵ In a previous version of this paper, we just considered one particular ‘conservative’ central banker with preference $\omega_x^* = 0.01$. We are grateful to an anonymous referee for suggesting the improved exercise.

²⁶ We have found this in the previous version of this paper, when we only examined the case of $\omega_x^* = 0.01$.

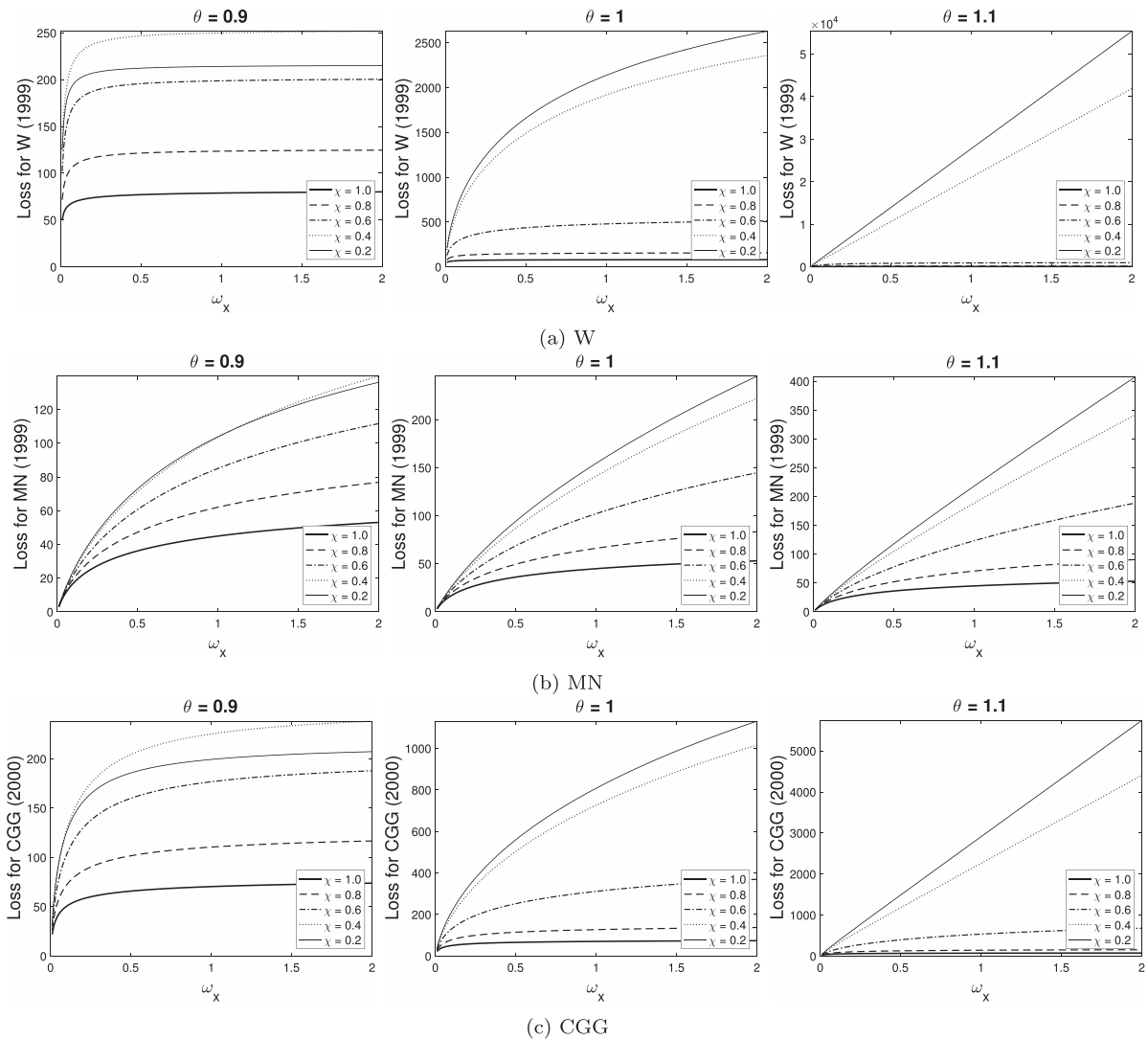


Fig. 9. Losses for the W, MN, and CGG calibrations for the *Expectations-Based Reaction Function* under commitment.

bank has a very large preference for output stabilization, ω_x and AE are extrapolative, but not too large, there is always an incentive to hire a very ‘conservative’ central banker, as otherwise the economy is indeterminate or explosive (recall Panel 3a).

4.3.2. Mismeasuring expectations

So far the analysis has assumed that policymakers can perfectly measure heterogeneous expectations. However, in practice there may be mismeasurement of expectations. Such mismeasurement affects both the design and implementation of optimal monetary policy. As can be seen from (32), the central bank’s measure of χ affects size of the output gap contraction in response to a cost-push shock and therefore also the extent to which the central bank exploits the novel channel to manipulate heterogeneous expectations. At least under discretion it is therefore natural to ask what the losses and policy implications of a mismeasurement of χ are?²⁷ We address the mismeasurement issue by computing

$$\mathcal{L}^i(\chi, \theta, \omega_x | \bar{\chi}) = (1 - \beta)^{-1} [\text{Var}(\pi | \bar{\chi}) + \omega_x \text{Var}(x | \bar{\chi})]$$

for $i \in \{W, MN, CGG\}$. $\bar{\chi}$ denotes the share of households with RE as measured by the central bank. Table 6 has results for the case of $\theta = 0.9$ and $\omega_x = 0.05$ and considers situations where both $\bar{\chi}$ and $\chi \in \{0.2, 0.4, 0.6, 0.8\}$. For instance, when the central bank measures $\bar{\chi} = 0.6$ and the true share is $\chi = 0.8$, it underestimates χ . For prices sufficiently sticky, i.e., in case

²⁷ In contrast, under commitment mismeasurement would imply that policy is time-inconsistent (see Gasteiger, 2014, p.1548) and cannot lead to an equilibrium.

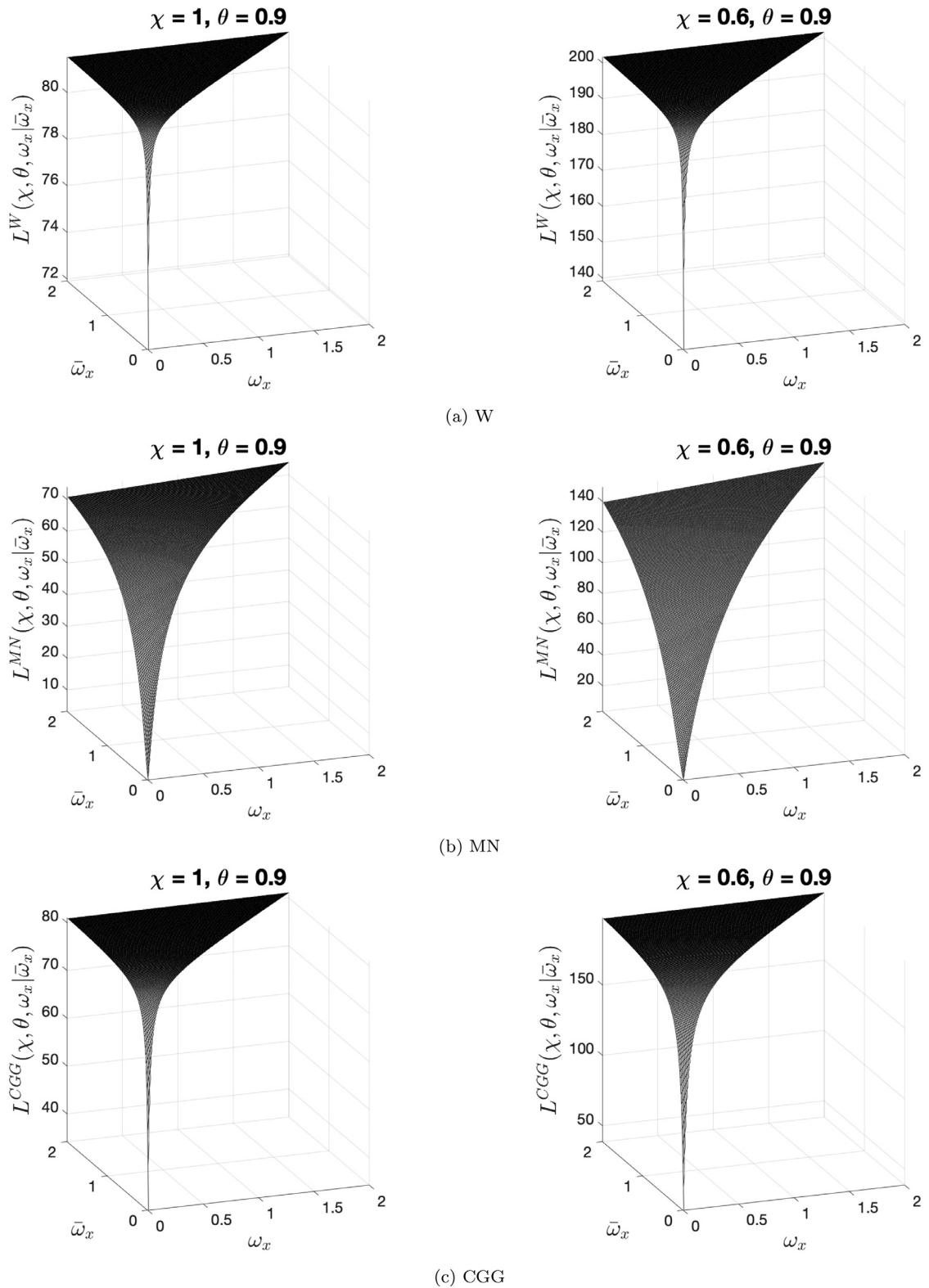


Fig. 10. Losses for the W, MN, and CGG calibrations for the *Expectations-Based Reaction Function* under discretion in the $\omega_x - \bar{\omega}_x$ -space for $\chi = \{0.6, 1\}$ and $\theta = 0.9$.

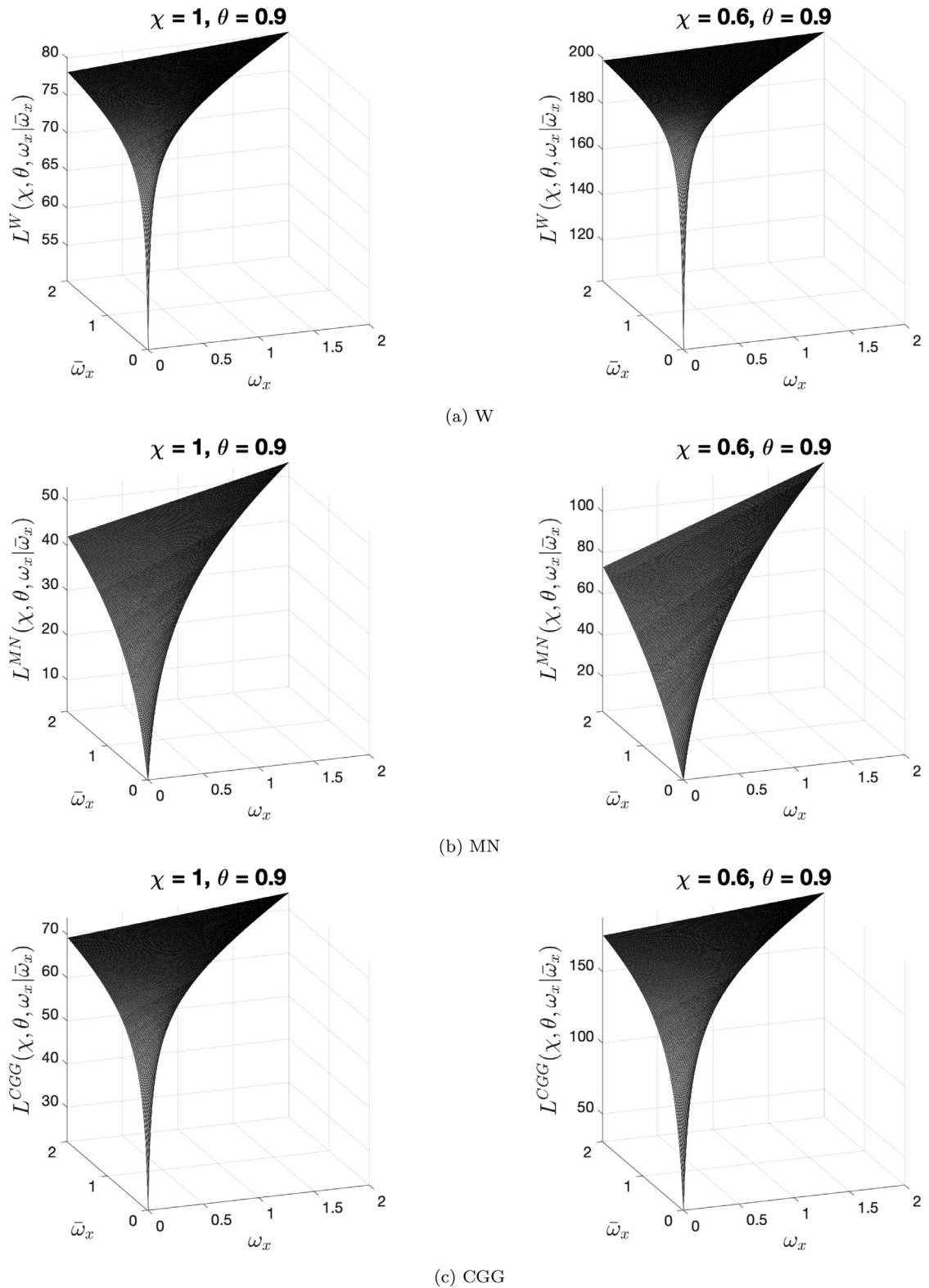


Fig. 11. Losses for the W, MN, and CGG calibrations for the *Expectations-Based Reaction Function* under commitment in the $\omega_x - \bar{\omega}_x$ -space for $\chi = \{0.6, 1\}$ and $\theta = 0.9$.

Table 6
Mismeasuring expectations.

$\bar{\chi}$	$\mathcal{L}^W(\chi, \theta = 0.9, \omega_x = 0.05 \bar{\chi})$				$\mathcal{L}^B(\chi, \theta = 0.9, \omega_x = 0.05 \bar{\chi})$
	χ				
	0.8	0.6	0.4	0.2	
0.8	120.26	206.98	243.92	203.23	193.60
0.6	115.11	185.20	236.12	200.06	184.12
0.4	118.21	171.93	222.97	195.41	177.13
0.2	126.08	170.88	209.60	190.11	174.17

$\bar{\chi}$	$\mathcal{L}^{MN}(\chi, \theta = 0.9, \omega_x = 0.05 \bar{\chi})$				$\mathcal{L}^B(\chi, \theta = 0.9, \omega_x = 0.05 \bar{\chi})$
	χ				
	0.8	0.6	0.4	0.2	
0.8	14.52	14.16	15.36	17.02	15.27
0.6	16.17	14.44	14.22	15.50	15.08
0.4	17.64	15.79	14.21	14.19	15.46
0.2	18.64	17.12	15.20	13.91	16.22

$\bar{\chi}$	$\mathcal{L}^{CGG}(\chi, \theta = 0.9, \omega_x = 0.05 \bar{\chi})$				$\mathcal{L}^B(\chi, \theta = 0.9, \omega_x = 0.05 \bar{\chi})$
	χ				
	0.8	0.6	0.4	0.2	
0.8	85.93	120.34	123.13	105.59	108.75
0.6	77.56	107.72	120.40	104.86	102.63
0.4	74.41	96.12	112.26	103.02	96.45
0.2	74.13	90.68	102.77	98.92	91.63

of the W and CGG calibration, the consequence is that losses are mostly lower than in the case of correct measurement. Exceptions can be explained with the strength of the amplification mechanism that peaks around $\chi = 0.4$. In contrast, in case of overestimating χ , the opposite is true. Thus, the policy implication is that overestimating the share of households with RE is costly in terms of losses. Notice that the last column contains the loss from a Bayesian perspective, where the policymaker assigns equal probability to $\chi \in \{0.2, 0.4, 0.6, 0.8\}$, but designs and implements policy based on a particular $\bar{\chi}$. One can observe that when prices are sufficiently sticky, the Bayesian loss tends to be lower than the loss if $\bar{\chi} = \chi$ were true unless χ is rather high.

Intuitively, as $\bar{\chi} \rightarrow 1$, the right-hand side in (32) (or (C.4) for serially correlated shocks) becomes more similar to the optimal policy under homogeneous RE. This implies that a central bank that overestimates the share of RE makes less use of the novel channel to manipulate heterogeneous expectations as it actually should. Relative to $\bar{\chi} = \chi$, a smaller output gap and larger inflation variance translate into a larger central bank loss. Contrary, a central bank that underestimates the share of RE makes more use of the novel channel to manipulate heterogeneous expectations. Similar to a more hawkish policy in Section 4.3.1, this can generate the opposite effect: a larger output gap and smaller inflation variance translate into a lower central bank loss.²⁸

4.3.3. Implications emerging from our main results

There are a couple of further novel implications for our results in Sections 4.1 and 4.2. One is that an alternative way of rendering the economy determinate is to obtain the ability to commit, as we have found that commitment always yields determinacy throughout the parameter space for all considered calibrations. Therefore commitment is even more desirable, when concerns of robustness with regard to the calibration of structural parameters are taken seriously and the nature of heterogeneity is unknown, because commitment allows the central bank to obtain determinacy irrespective of the reference model. Moreover, commitment also outperforms discretion on the grounds of generating lower losses.

Next, following the arguments in Sections 3.1 regarding the design and implementation of optimal monetary policy under heterogeneous expectations, ignoring expectations heterogeneity in the design of discretionary optimal monetary policy is not time consistent and raises losses. Thus, policymakers should take heterogeneous expectations into account in the design of discretionary optimal monetary policy. The same is true under commitment. In order to support our claims, we consider the ignorance case $x_t = -(\lambda/\omega_x)\pi_t$ as an example of inappropriate policy for a comparison with (32) regarding outcomes under discretion. Table 7 presents the results, which show that policy design that accounts for expectations heterogeneity generates outcomes that are closer to the RE benchmark relative to the ignorance case. Depending on χ , the difference in losses can become substantial.

Finally, there are more policy implications that follow directly from the results in the above Sections 4.1 and 4.2. First, our results support the view that welfare analyses under the REH, as carried out by many modern central banks, understate the true losses. Second, optimal policy design and implementation based on an *ad hoc* loss function has several practical

²⁸ Equivalently, consider the coefficients of reaction function (35). If the central bank overestimates χ , then the feedback to RE (AE) in (35), δ_π ($\delta_{L\pi}$), is larger (lower) than it should actually be. So the central bank is leaning too much against the less relevant type of inflation expectations.

Table 7
Gains from incorporating expectations heterogeneity in policy design under discretion.

ω_x	Central Bank preferences		$\mathcal{L}^W(\chi, \theta, \omega_x)$			$\mathcal{L}^{MN}(\chi, \theta, \omega_x)$			$\mathcal{L}^{CGG}(\chi, \theta, \omega_x)$			$\mathcal{L}^B(\chi, \theta, \omega_x)$			
	Expectations parameters		Design with			Design with			Design with			Design with			
	θ	χ	$\chi = 1$	$\chi \leq 1$	% Δ	$\chi = 1$	$\chi \leq 1$	% Δ	$\chi = 1$	$\chi \leq 1$	% Δ	$\chi = 1$	$\chi \leq 1$	% Δ	
0.01	0.9	1	71.91	71.91	0	3.23	3.23	0	34.18	34.18	0	36.44	36.44	0	
		0.8	102.79	101.81	-0.96	3.22	3.21	-0.45	38.17	37.28	-2.34	48.06	47.43	-1.31	
		0.6	145.64	139.17	-4.44	3.22	3.19	-0.82	41.29	38.65	-6.39	63.38	60.34	-4.81	
	1	0.4	170.77	152.52	-10.69	3.21	3.18	-1.11	42.59	37.86	-11.09	72.19	64.52	-10.62	
		0.2	154.14	131.85	-14.46	3.2	3.16	-1.26	41.72	35.8	-14.18	66.35	56.94	-14.19	
		0.8	125.57	123.49	-1.66	3.24	3.22	-0.57	40.81	39.42	-3.4	56.54	55.38	-2.05	
		0.6	270.66	235.84	-12.86	3.24	3.21	-1.1	47.55	42.55	-10.53	107.15	93.87	-12.4	
		0.4	511.35	306.34	-40.09	3.25	3.2	-1.55	52.54	42.31	-19.48	189.05	117.28	-37.96	
		0.2	554.43	245.94	-55.64	3.25	3.19	-1.87	54.39	40.28	-25.94	204.02	96.47	-52.72	
	1.1	0.8	165.42	160.39	-3.05	3.25	3.23	-0.72	44.17	42	-4.91	70.95	68.54	-3.4	
		0.6	-	728.63	-	-	3.22	-	-	47.19	-	-	259.68	-	
		0.4	-	926.91	-	-	3.22	-	-	47.06	-	-	325.73	-	
		0.2	-	428.35	-	-	3.22	-	-	44.75	-	-	158.77	-	
		1	78.1	78.1	0	9.17	9.17	0	56.41	56.41	0	47.89	47.89	0	
		0.8	117.2	116.75	-0.38	9.24	9.14	-1.11	72.13	70.72	-1.96	66.19	65.54	-0.99	
0.03	0.9	0.6	179.06	175.46	-2.01	9.27	9.06	-2.25	88.76	82.55	-7	92.36	89.02	-3.62	
		0.4	218.51	206.65	-5.43	9.26	8.96	-3.26	97.1	83.37	-14.15	108.29	99.66	-7.97	
		0.2	190.83	176.44	-7.54	9.2	8.84	-3.9	92.11	75.01	-18.57	97.38	86.76	-10.9	
	1	0.8	147.67	146.67	-0.68	9.36	9.22	-1.46	82.62	80.05	-3.11	79.88	78.65	-1.55	
		0.6	415.17	381.71	-8.06	9.51	9.21	-3.16	124.09	106.35	-14.29	182.92	165.76	-9.39	
		0.4	1213.61	729.93	-39.85	9.61	9.15	-4.82	164.25	111.7	-31.99	462.49	283.59	-38.68	
		0.2	1331.07	549.89	-58.69	9.66	9.07	-6.1	174.59	99.15	-43.21	505.11	219.37	-56.57	
		0.8	-	-	-	-	-	-	-	-	-	-	-	-	
		0.6	-	-	-	-	-	-	-	-	-	-	-	-	
	1.1	0.4	-	-	-	-	-	-	-	-	-	-	-	-	
		0.2	-	2015.81	-	-	9.3	-	-	125.99	-	-	717.03	-	
		1	79.46	79.46	0	14.42	14.42	0	64.5	64.5	0	52.79	52.79	0	
		0.9	0.8	120.55	120.26	-0.24	14.75	14.52	-1.55	87.25	85.93	-1.52	74.18	73.57	-0.82
			0.6	187.63	185.2	-1.29	14.94	14.44	-3.39	114.74	107.72	-6.12	105.77	102.45	-3.14
			0.4	231.32	222.97	-3.61	14.98	14.21	-5.14	129.6	112.26	-13.38	125.3	116.48	-7.04
1	0.2	200.23	190.11	-5.06	14.85	13.91	-6.31	120.32	98.92	-17.78	111.8	100.98	-9.68		
	0.8	153	152.35	-0.43	15.08	14.76	-2.09	103.13	100.55	-2.51	90.41	89.22	-1.31		
	0.6	465.73	438.65	-5.82	15.61	14.85	-4.92	180.91	154.87	-14.39	220.75	202.79	-8.14		
	0.4	1654.4	1045.79	-36.79	15.98	14.72	-7.89	273.53	173.89	-36.43	647.97	411.47	-36.5		
	0.2	1821.04	787.26	-56.77	16.16	14.51	-10.21	293.62	147.88	-49.64	710.27	316.55	-55.43		
	0.8	-	-	-	-	-	-	-	-	-	-	-	-		
1.1	0.6	-	-	-	-	-	-	-	-	-	-	-	-		
	0.4	-	-	-	-	-	-	-	-	-	-	-	-		
	0.2	-	7100.56	-	-	15.1	-	-	208.75	-	-	2441.47	-		
	0.2	-	-	-	-	-	-	-	-	-	-	-	-		

Table 8
Details for Table 3.

Central Bank preferences	Expectations parameters		Unconditional Variances under Calibration						
			W		MN		CGG		
			x_t	π_t	x_t	π_t	x_t	π_t	
0.01	0.9	1	3.916	0.68	2.904	0.003	12.303	0.219	
		0.8	6.54	0.953	2.926	0.003	14.815	0.225	
		0.6	11.691	1.275	2.948	0.002	17.479	0.212	
	1	0.4	18.522	1.34	2.97	0.002	19.836	0.18	
		0.2	24.659	1.072	2.991	0.002	21.769	0.14	
		0.8	8.372	1.151	2.942	0.003	16.112	0.233	
		0.6	23.876	2.12	2.979	0.002	20.62	0.219	
		0.4	55.475	2.509	3.014	0.002	24.757	0.175	
		0.2	84.834	1.611	3.048	0.001	28.113	0.122	
	1.1	0.8	11.646	1.487	2.96	0.003	17.744	0.243	
		0.6	100.644	6.28	3.013	0.002	24.803	0.224	
		0.4	305.754	6.212	3.061	0.002	31.059	0.16	
		0.2	285.256	1.431	3.104	0.001	35.559	0.092	
		1	0.49	0.766	2.291	0.023	2.969	0.475	
		0.8	0.873	1.141	2.362	0.02	4.285	0.579	
	0.03	0.9	0.6	1.76	1.702	2.429	0.018	6.164	0.641
			0.4	3.166	1.972	2.49	0.015	8.096	0.591
			0.2	4.536	1.628	2.545	0.012	9.694	0.459
1		0.8	1.162	1.432	2.404	0.02	5.065	0.649	
		0.6	4.796	3.673	2.51	0.017	9.012	0.793	
		0.4	19.409	6.717	2.607	0.013	13.653	0.707	
		0.2	36.268	4.411	2.693	0.01	17.474	0.467	
		1.1	0.8	-	-	-	-	-	-
		0.6	-	-	-	-	-	-	-
0.05		0.9	0.4	3.166	1.972	2.49	0.015	8.096	0.591
			0.2	462.342	6.288	2.843	0.008	29.926	0.362
			1	0.181	0.786	1.854	0.051	1.304	0.58
		1	0.8	0.327	1.186	1.962	0.047	2.027	0.758
			0.6	0.679	1.818	2.064	0.041	3.227	0.916
			0.4	1.266	2.166	2.154	0.034	4.604	0.892
			0.2	1.853	1.808	2.233	0.027	5.776	0.7
			0.8	0.439	1.502	2.021	0.047	2.49	0.881
			0.6	2.038	4.285	2.182	0.039	5.413	1.278
	1.1	0.4	10.9	9.913	2.325	0.031	9.63	1.257	
		0.2	22.557	6.745	2.448	0.023	13.237	0.817	
		0.8	-	-	-	-	-	-	-
		0.6	-	-	-	-	-	-	-
		0.4	-	-	-	-	-	-	-
		0.2	1025.123	19.749	2.671	0.017	28.598	0.658	

advantages, but bears a risk of destabilizing the economy as it does not necessarily guarantee determinacy under discretion. This finding stands in contrast to the findings for discretionary optimal policy either based on an *ad hoc* loss function in related homogeneous expectations models (see [Evans and Honkapohja, 2003](#), p.814) or based on a model-consistent loss function in this model (see [Di Bartolomeo et al., 2016](#)), where determinacy prevails throughout the parameter space.

5. Related literature

Our findings regarding the inflation output variability trade-off are broadly consistent with the ones for the imperfect knowledge case in [Orphanides and Williams \(2005, 2008\)](#) or [Gaspar et al. \(2006\)](#), which has become a standard result in the literature on bounded rationality ([Eusepi and Preston, 2018](#)): the optimal response of the central bank to a cost-push shock is to put more weight on inflation stabilization than under the REH. Moreover, [Orphanides and Williams \(2005\)](#) show that the trade-off can shift out in a non-monotonic fashion under imperfect knowledge. However, the mechanism generating their results is that a temporary rise in inflation makes homogeneous agents conclude from their regression model that inflation will be higher in the long-run. In contrast, in our model the mechanism is based on agents with heterogeneous expectations, which is favored by the data.

[Ilabaca and Milani \(2021\)](#) estimate the model developed herein with a rolling-window approach. They show that the data favours the heterogeneous expectations version over homogeneous RE. This may be explained by the heterogeneous expectations amplification mechanism discussed herein. They also find that the expectations coefficients χ and θ are time varying which reinforces the arguments in favor of a robustness approach pursued herein. Another important aspect of their work is that they carry out a test of a novel and testable asymmetry hypothesis emerging from our model as discussed

Table 9
Details for Table 4.

Central Bank preferences	Expectations parameters		Unconditional Variances under Calibration						
			W		MN		CGG		
			x_t	π_t	x_t	π_t	x_t	π_t	
0.01	0.9	1	11.651	0.386	2.635	0.003	11.42	0.106	
		0.8	21.395	0.503	2.777	0.002	15.682	0.105	
		0.6	38.936	0.632	2.892	0.002	20.642	0.098	
	1	0.4	52.911	0.737	2.972	0.002	24.472	0.091	
		0.2	46.019	0.788	3.013	0.001	25.33	0.094	
		0.8	25.002	0.523	2.79	0.002	16.515	0.103	
		0.6	63.27	0.653	2.919	0.002	23.231	0.09	
		0.4	122.474	0.673	3.013	0.001	29.221	0.075	
		0.2	136.08	0.729	3.066	0.001	31.724	0.071	
	1.1	0.8	28.759	0.548	2.803	0.002	17.442	0.101	
		0.6	98.018	0.648	2.947	0.002	26.145	0.081	
		0.4	235.984	0.401	3.054	0.001	34.365	0.056	
0.2		292.773	0.331	3.12	0.001	38.415	0.045		
1		3.016	0.525	1.912	0.017	4.303	0.227		
0.8		5.918	0.754	2.163	0.014	6.882	0.258		
0.03	0.9	0.6	11.53	1.086	2.386	0.012	10.652	0.275	
		0.4	15.136	1.371	2.546	0.01	13.725	0.284	
		0.2	11.036	1.37	2.617	0.009	13.573	0.306	
	1	0.8	7.491	0.811	2.193	0.014	7.565	0.259	
		0.6	26.756	1.279	2.455	0.011	13.644	0.261	
		0.4	71.741	1.573	2.653	0.008	20.344	0.235	
		0.2	78.993	1.764	2.758	0.007	22.513	0.24	
		1.1	0.8	8.785	0.88	2.226	0.013	8.327	0.261
		0.6	52.084	1.47	2.528	0.01	17.325	0.24	
	0.05	0.9	0.4	216.497	0.741	2.763	0.007	28.489	0.161
			0.2	279.717	0.567	2.897	0.005	33.46	0.135
			1	1.552	0.579	1.514	0.032	2.535	0.295
1		0.8	3.042	0.861	1.8	0.028	4.329	0.356	
		0.6	5.851	1.3	2.07	0.024	7.228	0.407	
		0.4	7.346	1.668	2.268	0.021	9.609	0.444	
		0.2	4.948	1.604	2.347	0.02	9.086	0.482	
		1.1	0.8	4.059	0.94	1.839	0.028	4.876	0.363
		0.6	16.955	1.655	2.167	0.022	10.109	0.399	
1.1		0.4	54.365	2.255	2.424	0.017	16.695	0.379	
		0.2	59.429	2.556	2.553	0.015	18.546	0.397	
		0.8	4.73	1.035	1.883	0.027	5.476	0.372	
	0.6	36.395	2.094	2.271	0.02	13.869	0.375		
	0.4	212.443	0.907	2.585	0.014	26.33	0.247		
	0.2	277.611	0.663	2.758	0.011	31.673	0.207		

right below. Next, [Elias \(2020\)](#) estimates a heterogeneous expectations model under the assumption of adaptive learning and coexistence of correctly and misspecified PLMs. He also finds that the data favors heterogeneous expectations over (asymptotically) model-consistent homogeneous expectations.

Our modelling approach builds on [Branch and McGough \(2009\)](#) and is also related to [Massaro \(2013\)](#). It is an alternative way of obtaining similar aggregate demand and supply relations.²⁹ However, our model is a novelty as it is a stochastic and decentralized markets version of the [Branch and McGough \(2009\)](#) model with distinct households and firms. Together with the introduction of aggregate uncertainty, the approach states a stark contrast to the widely used yeoman farmer model. An advantage of the approach is that household labor income is dependent on household effort and there is no free-rider problem as in the yeoman farmer model that requires fully enforceable contracts. Furthermore, the separation between households and firms implies that the expectations heterogeneity is related to the households. The latter own the firms. Therefore firms have average expectations that depend on the distribution of households among RE and AE. Thus, the model also implies a novel and testable hypothesis: there is a general asymmetry between expectation formation between households and firms. [Ilabaca and Milani's \(2021\)](#) results support this hypothesis. Moreover, this asymmetry is a potential narrative to explain the differing forecasting behavior of households and firms documented in [Fuhrer \(2015\)](#).³⁰

²⁹ Notice that we use an insurance mechanism to keep the model tractable. This route was suggested by [Branch and McGough \(2009, p.1041\)](#), but not pursued therein.

³⁰ Notice that the coexistence of RE and AE in our model is also consistent with [Fuhrer's \(2015\)](#) conclusion that '[...] individuals who do not possess full information about the economy link their own expectations to previous aggregate expectations[...]'.

This paper is also connected to the literature that examines the design and implementation of optimal monetary policy in a representative agent framework under the REH (e.g., Clarida et al., 1999) or under adaptive learning (Evans and Honkapohja, 2003; 2006; Duffy and Xiao, 2007). A key conclusion from this literature is that expectations-based reaction functions generate determinacy under both discretion and commitment throughout the parameter space. Gasteiger (2014) has shown that this finding extends to the deterministic Branch and McGough (2009) model of heterogeneous expectations when the central bank considers an *ad hoc* loss function and is able to commit. Moreover, Gasteiger (2014) shows that the central bank should take heterogeneous expectations into account when designing and implementing optimal monetary policy under commitment. This paper extends Gasteiger (2014) in several directions. First, the model herein is a micro-founded stochastic decentralized economy that addresses several modelling shortcomings of a yeoman farmer model and allows for a more intuitive interpretation of results based on individual behavior. Second, considering a stochastic set-up gives scope to richer policy implications. Without stochastic shocks the losses converge to zero asymptotically, and the central bank's problem is just to bring about determinacy and it is impossible to rank the performance of any two equilibria due to different preferences of the policymaker with regard to losses. Third, this paper elaborates the design and implementation of optimal monetary policy under discretion and contrasts this with the findings under commitment. The case of discretion is important as not all central banks may have the ability to commit, but undoubtedly central banks conduct policy under heterogeneous expectations.

This paper is also related to Di Bartolomeo et al. (2016) who focus on how to obtain the fully optimal policy from a second-order approximation to household utility in the Branch and McGough (2009) yeoman farmer model, which precludes an intuitive explanation of results based on the interaction of households and firms on decentralized markets. Di Bartolomeo et al. (2016) append a shock to the aggregate Phillips curve and numerically analyse the case of discretion and commitment under this model-consistent loss function and also report that, in both the case of discretion and commitment, determinacy prevails throughout the parameter space. However, these findings are obtained without elaborating implementation strategies. In consequence, the determinacy properties of implementation strategies under discretion in a model with heterogeneous expectations are unknown. Therefore, in light of the reasoning in Woodford (1999), our examination of the determinacy properties of the expectations-based reaction function as a possible implementation strategy of optimal policy under discretion addresses a concern of high interest for policymakers. It turns out that our determinacy results for discretionary policy contrast (Di Bartolomeo et al., 2016) results. Determinacy *does not* prevail throughout the parameter space.

A further important difference to Di Bartolomeo et al. (2016) is that we use an *ad hoc* loss function for practical reasons that we have discussed above. On the one hand this choice most likely implies that our analysis understates the true losses due to heterogeneous expectations. This is illustrated in Di Bartolomeo et al. (2016) or Hagenhoff (2018). Hagenhoff (2018) also shows that, depending on the share of AE, $(1 - \chi)$, these losses can become substantial. On the other hand, we are able to show how to implement optimal monetary policy under heterogeneous expectations. More generally, we think that the case of *ad hoc* loss functions is important as in practice policymakers may use large models with many different features, which makes the use of model-consistent loss functions infeasible. Thus, policymakers may actually use an *ad hoc* loss function.

As regards our findings more generally, we are confident that most of them are robust to the use of the model-consistent loss function. When calibrating the structural parameters, Hagenhoff (2018) finds that the term with the relative weight on inflation deviations is much larger in the model-consistent loss function than the corresponding term in the *ad hoc* loss function. Also the weights on the additional inflation terms are larger. In turn, the relative weight on the output gap deviations are relatively smaller. The model-consistent loss function then yields lower losses relative to the *ad hoc* loss function. Overall, this is consistent with our finding that a more hawkish stance toward inflation, e.g., because of a conservative central banker ($\omega_\chi^* < \omega_\chi$) or an underestimated share of RE ($\bar{\chi} < \chi$), reduces central bank losses, because in such cases the central bank exploits the novel channel to manipulate heterogeneous expectations to a further extent.

Finally, it is important to emphasize that our model of heterogeneous expectations introduces macroeconomic persistence. However, it should not be confused with models that incorporate widely used 'bells and whistles' in order to improve the fit to certain features of the data. Examples are rule-of-thumb households (see, e.g., Galí et al., 2004), rule-of-thumb firms with price indexation (see, e.g., Steinsson, 2003), or households with external additive habit formation in consumption (see, e.g., Smets and Wouters, 2003). For instance, the design of optimal monetary policy based on an *ad hoc* loss function is not affected by the existence of rule-of-thumb households or habit formation. Next, the implications for the design of optimal monetary policy emerging from firms with price indexation are different to the ones emerging from heterogeneous expectations. Moreover, there is another crucial difference compared to rule-of-thumb consumers and firms. The nominal interest rate has no direct effect on the inter-temporal decisions of such rule-of-thumb households or firms as they do not make inter-temporal decisions. In contrast, in our model all agents take monetary policy into account and make inter-temporal decisions. In fact, each type of household faces a distinct *ex ante* real interest rate, which is a crucial part of the unique amplification mechanism mentioned above. Thus, the presumption that the heterogeneous expectations model developed herein is equivalent to a homogeneous model with 'bells and whistles' and therefore exhibits similar properties and yields similar policy implications is misleading.

6. Conclusions

This paper develops a stochastic version of the New Keynesian model with heterogeneous expectations. The novelty is that it can be used to analyze the transmission of aggregate shocks and monetary policy on households and firms that interact in decentralized markets.

Our analysis calls for the incorporation of heterogeneous expectations in both the design and implementation of optimal monetary policy to manipulate expectations via a novel channel. Naive policy design that does not incorporate heterogeneous expectations leads to inefficient outcomes. The reason is that heterogeneous expectations also imply an amplification mechanism absent in homogeneous expectations models. This amplification mechanism poses several challenges for policymakers. Under discretion, determinacy only prevails in part of the structural parameter space. Relative to the benchmark of the rational expectations hypothesis, the mechanism also raises losses and implies that the inflation output variability trade-off shifts out in a non-monotonic way with the share of adaptive expectations. The potential pitfalls of a high preference for output stabilization are exacerbated under heterogeneous expectations. A more hawkish monetary policy can lower central bank losses, but an overly hawkish policy does not. Moreover, overestimating the share of rational expectations in the design and implementation of monetary policy increases losses. In sum, our findings render a central banks' ability to commit highly desirable as optimal monetary policy under commitment can eliminate or alleviate many of the ramifications of heterogeneous expectations.

Our results also raise questions on the implications of interest rate stabilization in the loss function or simple monetary policy rules with interest rate smoothing. For instance, our findings suggest that in order to render the economy determinate under expectations heterogeneity, a larger contraction of aggregate demand is necessary in response to a cost-push shock. However, a preference for interest rate stabilization or interest rate smoothing then clearly seems counterproductive and leads to the conjecture that a preference for interest rate stabilization or interest rate smoothing implies unnecessary losses under heterogeneous expectations. We take up such considerations in future research.

Declaration of Competing Interest

The author has no relevant interests to declare other than the financial support mentioned in the acknowledgements of the title page.

Appendix A. Model derivations

Assumptions A1 to A7 are taken from Branch and McGough (2009).

A1. Household Lagrangian

The household's problem can be solved by the Lagrangian

$$\mathcal{L}(\cdot, i) = E_t^\gamma \left\{ \sum_{k=0}^{\infty} \beta^{t+k} \left[\frac{C_{t+k}(i)^{1-\sigma}}{(1-\sigma)} - \frac{N_{t+k}(i)^{1+\varphi}}{(1+\varphi)} - \lambda_{t+k}^H \left(C_{t+k}(i) + Q_{t+k|t+k+1} \frac{B_{t+k}(i)}{P_{t+k}} \right. \right. \right. \\ \left. \left. \left. + \frac{I_{C,t+k}(i)}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}} N_{t+k}(i) - \frac{B_{t+k-1}(i)}{P_{t+k-1}} \Pi_{t+k-1,t+k}^{-1} - \frac{I_{P,t+k}(i)}{P_{t+k}} - \frac{\Upsilon_{t+k}}{P_{t+k}} \right) \right] \right\},$$

where λ_{t+k}^H is the Lagrange multiplier. This yields the first-order necessary conditions

$$\frac{\partial \mathcal{L}(\cdot, i)}{\partial C_{t+k}(i)} \stackrel{!}{=} 0 \Leftrightarrow E_t^\gamma \left\{ \beta^{t+k} [C_{t+k}(i)^{-\sigma} - \lambda_{t+k}^H] \right\} \stackrel{!}{=} 0, \tag{A.1.1}$$

$$\frac{\partial \mathcal{L}(\cdot, i)}{\partial \left(\frac{B_{t+k}(i)}{P_{t+k}} \right)} \stackrel{!}{=} 0 \Leftrightarrow E_t^\gamma \left\{ \beta^{t+k} [-\lambda_{t+k}^H Q_{t+k|t+k+1}] + \beta^{t+k+1} [\lambda_{t+k+1}^H \Pi_{t+k,t+k+1}^{-1}] \right\} \stackrel{!}{=} 0, \tag{A.1.2}$$

$$\frac{\partial \mathcal{L}(\cdot, i)}{\partial N_{t+k}(i)} \stackrel{!}{=} 0 \Leftrightarrow E_t^\gamma \left\{ \beta^{t+k} \left[-N_{t+k}(i)^\varphi - \lambda_{t+k}^H \frac{W_{t+k}}{P_{t+k}} \right] \right\} \stackrel{!}{=} 0. \tag{A.1.3}$$

Combining (A.1.1) and (A.1.2) yields (8), likewise (A.1.1) and (A.1.3) yield (9). Moreover, (A.1.1) and (A.1.2) also imply the subjective transversality condition (10).

Log-linearization of (8) and (9) around a steady state, together with definitions $\rho \equiv -\ln(\beta)$, $i_t \equiv -\ln(E_t^\gamma \{Q_{t|t+1}\})$, $c_t(i) \equiv (C_t(i) - C(i))/Y$, where $C(i) = Y$, and $\pi_t \equiv (\Pi_t - \Pi)/\Pi$ yields

$$c_t(i) = E_t^\gamma \{c_{t+1}(i)\} - \sigma^{-1} (i_t - E_t^\gamma \{\pi_{t+1}\} - \rho), \quad \text{and}$$

$$w_t - p_t = \varphi n_t(i) + \sigma c_t(i) \equiv mrs_t(i),$$

i.e., the real wage equals the marginal rate of substitution, $mrs_t(i)$. Furthermore, $w_t \equiv (W_t - W)/W$, $p_t \equiv (P_t - P)/P$, and $n_t(i) \equiv (N_t(i) - N(i))/N$.

A2. Forward iteration of household wealth

Eq. (11) one period ahead is

$$E_{t+1}^Y \{c_{t+1}^Y\} = E_{t+1}^Y \{c_{t+2}^Y - \sigma^{-1}(i_{t+1} - \pi_{t+2} - \rho)\},$$

which by Assumptions A1 and A3 is

$$c_{t+1}^Y = E_{t+1}^Y \{c_{t+2}^Y\} - \sigma^{-1}(i_{t+1} - E_{t+1}^Y \{\pi_{t+2}\} - \rho). \tag{A.2.1}$$

Combining (11) with (A.2.1) and applying Assumptions A1 and A3 again yields

$$c_t^Y = E_t^Y \{E_{t+1}^Y \{c_{t+2}^Y\}\} - \sigma^{-1}(E_t^Y \{i_{t+1}\} - E_t^Y \{E_{t+1}^Y \{\pi_{t+2}\}\} - \rho) - \sigma^{-1}(i_t - E_t^Y \{\pi_{t+1}\} - \rho),$$

which, by using Assumptions A5 can be rewritten as

$$c_t^Y = E_t^Y \{c_{t+2}^Y\} - \sigma^{-1} \sum_{k=0}^1 E_t^Y \{(i_{t+k} - \pi_{t+k+1} - \rho)\}.$$

Using Assumptions A4 and A5 as well as repeating the steps outlined above yields

$$c_t^Y = \lim_{k \rightarrow \infty} E_t^Y \{c_{t+k+1}^Y\} - \sigma^{-1} E_t^Y \left\{ \sum_{k=0}^{\infty} (i_{t+k} - \pi_{t+k+1} - \rho) \right\}.$$

Finally, defining $\lim_{k \rightarrow \infty} E_t^Y \{c_{t+k+1}^Y\} \equiv E_t^Y \{c_{\infty}^Y\}$ yields (12).

A3. Equilibrium derivation of dynamic IS curve

On the goods market, each firm i will supply sufficient goods to meet demand for its variety j , i.e.,

$$Y_t(j) = C_t(j), \quad \forall t. \tag{A.3.1}$$

Moreover, demand (3) and the definition of aggregate output,

$$Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

imply the goods market clearing condition (16).

Given (17), we can use (12) to get

$$y_t = \chi E_t^1 \{c_{\infty}^1\} + (1 - \chi) E_t^2 \{c_{\infty}^2\} - \sigma^{-1} \widehat{E}_t \left\{ \sum_{k=0}^{\infty} (i_{t+k} - \pi_{t+k+1} - \rho) \right\}. \tag{A.3.2}$$

One period ahead this is

$$\widehat{E}_t \{y_{t+1}\} = \chi E_t^1 \{c_{\infty}^1\} + (1 - \chi) E_t^2 \{c_{\infty}^2\} - \sigma^{-1} \widehat{E}_t \left\{ \sum_{k=1}^{\infty} (i_{t+k} - \pi_{t+k+1} - \rho) \right\}, \tag{A.3.3}$$

where we rule out terms of higher order beliefs by Assumption A6 of Branch and McGough (2009). Finally, subtracting (A.3.3) from (A.3.2) and applying Assumptions A6 and A7 of Branch and McGough (2009) yields (18).

A4. Equilibrium labor market and marginal costs

Next, we consider labor market clearing. Aggregate employment is defined as

$$N_t \equiv \int_0^1 N_t(i) di.$$

Together with technology (13), condition (A.3.1) and demand (3), employment is given by

$$N_t = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} di.$$

A first-order linear approximation to the latter expression is given by (19), (see, Galí, 2015, p.59). Next, average real marginal costs in logs are

$$mc_t = (w_t - p_t) - mpn_t$$

$$= (w_t - p_t) - (a_t - \alpha n_t - \log(1 - \alpha)), \tag{A.4.1}$$

where (A.4.1) follows from (13) and

$$mpn_t = a_t - \alpha n_t + \log(1 - \alpha).$$

Labor market clearing condition (19) and rearranging terms yields

$$mc_t = (w_t - p_t) - \frac{1}{(1 - \alpha)} a_t + \frac{\alpha}{(1 - \alpha)} y_t - \log(1 - \alpha).$$

Next, we combine

$$\begin{aligned} mc_{t+k|t}(i) &= (w_{t+k} - p_{t+k}) + mpn_{t+k|t} \\ &= (w_{t+k} - p_{t+k}) - \frac{1}{(1 - \alpha)} a_{t+k} + \frac{\alpha}{(1 - \alpha)} y_{t+k} + \log(1 - \alpha) \\ &\quad + \frac{\alpha}{(1 - \alpha)} (y_{t+k|t}(i) - y_{t+k}) \\ &= mc_{t+k} - \frac{\alpha \epsilon}{(1 - \alpha)} (p_t^*(i) - p_{t+k}). \end{aligned}$$

In terms of log deviations from steady state mc we have

$$\begin{aligned} \widehat{mc}_{t+k|t}(i) &= mc_{t+k|t}(i) - mc \\ &= \widehat{mc}_{t+k} - \frac{\alpha \epsilon}{(1 - \alpha)} (\widehat{p}_t^*(i) - \widehat{p}_{t+k}). \end{aligned} \tag{A.4.2}$$

A5. Equilibrium forward iteration of the Firm's FOC, wage markup and marginal cost

Using (15) together with (A.4.2) yields

$$p_t^*(i) - p_t = (1 - \beta\theta_p) \widehat{E}_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_p)^k \left(\Theta \widehat{mc}_{t+k} + \frac{\beta\theta_p}{(1 - \beta\theta_p)} \pi_{t+k+1} \right) \right\}, \tag{A.5.1}$$

where $\Theta \equiv \frac{(1-\alpha)}{1+\alpha(\epsilon-1)} \leq 1$. By Assumption A1, (A.5.1) can be written as

$$\widehat{E}_t \{ p_t^*(i) - p_t \} = (1 - \beta\theta_p) \widehat{E}_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_p)^k \left[\Theta \widehat{mc}_{t+k} + \frac{\beta\theta_p}{(1 - \beta\theta_p)} \pi_{t+k+1} \right] \right\}. \tag{A.5.2}$$

One period ahead, (A.5.2) is given by

$$\widehat{E}_{t+1} \{ p_{t+1}^*(i) - p_{t+1} \} = (1 - \beta\theta_p) \widehat{E}_{t+1} \left\{ \sum_{k=0}^{\infty} (\beta\theta_p)^k \left[\Theta \widehat{mc}_{t+k+1} + \frac{\beta\theta_p}{(1 - \beta\theta_p)} \pi_{t+k+2} \right] \right\}, \tag{A.5.3}$$

or

$$(\beta\theta_p) \widehat{E}_{t+1} \{ p_{t+1}^*(i) - p_{t+1} \} = (1 - \beta\theta_p) \widehat{E}_{t+1} \left\{ \sum_{k=0}^{\infty} (\beta\theta_p)^{k+1} \left[\Theta \widehat{mc}_{t+k+1} + \frac{\beta\theta_p}{(1 - \beta\theta_p)} \pi_{t+k+2} \right] \right\}. \tag{A.5.4}$$

Evaluation of (A.5.4) with $\widehat{E}_t\{\cdot\}$ and invoking Assumption A5 of Branch and McGough (2009) yields

$$(\beta\theta_p) \widehat{E}_t \{ p_{t+1}^*(i) - p_{t+1} \} = (1 - \beta\theta_p) \widehat{E}_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_p)^{k+1} \left[\Theta \widehat{mc}_{t+k+1} + \frac{\beta\theta_p}{(1 - \beta\theta_p)} \pi_{t+k+2} \right] \right\}. \tag{A.5.5}$$

Subtracting (A.5.5) from (A.5.3) and applying Assumption A4 of Branch and McGough (2009) allows us to arrive at

$$p_t^*(i) - p_t = (1 - \beta\theta_p) \widehat{E}_t \left\{ \left[\Theta \widehat{mc}_t + \frac{\beta\theta_p}{(1 - \beta\theta_p)} \pi_{t+1} \right] \right\} + \beta\theta_p \widehat{E}_t \{ p_{t+1}^*(i) - p_{t+1} \}.$$

Finally, Assumptions A1 and A3 of Branch and McGough (2009) yield

$$p_t^*(i) - p_t = (1 - \beta\theta_p) \Theta \widehat{mc}_t + \beta\theta_p \widehat{E}_t \{ \pi_{t+1} \} + \beta\theta_p \widehat{E}_t \{ p_{t+1}^*(i) - p_{t+1} \}.$$

Recall that all firms make similar decisions, thus

$$p_t^* - p_t = (1 - \beta\theta_p) \Theta \widehat{mc}_t + \beta\theta_p \widehat{E}_t \{ \pi_{t+1} \} + \beta\theta_p \widehat{E}_t \{ p_{t+1}^* - p_{t+1} \}.$$

Moreover, from (Calvo, 1983) aggregate price setting assumption follows that linearized around the steady state

$$p_t^* - p_t = \frac{\theta_p}{1 - \theta_p} \pi_t.$$

Therefore

$$\frac{\theta_p}{(1 - \theta_p)} \pi_t = (1 - \beta\theta_p)\Theta\widehat{m}c_t + \beta\theta_p\widehat{E}_t\{\pi_{t+1}\} + \beta\theta_p\frac{\theta_p}{(1 - \theta_p)}\widehat{E}_t\{\pi_{t+1}\},$$

which can be simplified to the inflation Eq. (20).

Assuming an efficient consumption-leisure choice (9), we can express average marginal cost in the economy as

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (\mu_t^w + mrs_t) - (a_t - \alpha n_t + \log(1 - \alpha)) \\ &= \mu_t^w + (\varphi n_t + \sigma c_t) - (a_t - \alpha n_t + \log(1 - \alpha)) \\ &= \mu_t^w + \sigma c_t + (\varphi + \alpha)n_t - a_t - \log(1 - \alpha). \end{aligned}$$

Imposing labor market clearing (19), goods market clearing (16), and collecting terms yields

$$mc_t = \mu_t^w + \left[\sigma + \frac{(\varphi + \alpha)}{(1 - \alpha)} \right] y_t - \left[\frac{(1 + \varphi)}{(1 - \alpha)} \right] a_t - \log(1 - \alpha),$$

Likewise, in steady state it holds that

$$mc = \mu^w + \left[\sigma + \frac{(\varphi + \alpha)}{(1 - \alpha)} \right] y_t^n - \left[\frac{(1 + \varphi)}{(1 - \alpha)} \right] a_t - \log(1 - \alpha),$$

where y_t^n is the natural level of output, i.e., the level in absence of nominal rigidities under a constant price and wage markup.

A6. Flexible price equilibrium

The profit maximization problem of a monopolistically competitive firm i facing given aggregate price, wage, and constant wage markup can be solved by the Lagrangian

$$\begin{aligned} \mathcal{L}(\cdot, i) &= P_t^*(i)Y_t(i) - \mathcal{M}^w W_t N_t(i) - \lambda_{1,t}^F [Y_t(i) - A_t N_t(i)^{1-\alpha}] \\ &\quad - \lambda_{2,t}^F \left[Y_t(i) - \left(\frac{P_t^*(i)}{P_t} \right)^{-\epsilon} Y_t \right], \end{aligned}$$

where $\lambda_{1,t}^F$ and $\lambda_{2,t}^F$ are the Lagrange multipliers and \mathcal{M}^w is the constant aggregate wage markup. This yields the first-order necessary conditions

$$\frac{\partial \mathcal{L}(\cdot, i)}{\partial P_t^*(i)} \stackrel{!}{=} 0 \Leftrightarrow Y_t(i) - \lambda_{2,t}^F \left[-(\epsilon) \left(\frac{P_t^*(i)}{P_t} \right)^{-\epsilon-1} \frac{1}{P_t} Y_t \right] \stackrel{!}{=} 0, \tag{A.6.1}$$

$$\frac{\partial \mathcal{L}(\cdot, i)}{\partial P_t^*(i)} \stackrel{!}{=} 0 \Leftrightarrow Y_t(i) - \lambda_{1,t}^F - \lambda_{2,t}^F \stackrel{!}{=} 0,$$

$$\frac{\partial \mathcal{L}(\cdot, i)}{\partial N_t(i)} \stackrel{!}{=} 0 \Leftrightarrow -\mathcal{M}^w W_t - \lambda_{1,t}^F [-A_t(1 - \alpha)N_t(i)^{-\alpha}] \stackrel{!}{=} 0. \tag{A.6.2}$$

Conditions (A.6.1)–(A.6.2) can be summarized by

$$\frac{W_t}{P_t} = (\mathcal{M}\mathcal{M}^w)^{-1} (1 - \alpha) A_t N_t^\alpha, \tag{A.6.3}$$

where we also impose symmetry among the firms and we defined $\mathcal{M}^{-1} \equiv (\epsilon - 1)/\epsilon$ as the constant markup over the price of consumption goods. A linear approximation to (A.6.3) is

$$(w_t - p_t) = a_t - \alpha n_t.$$

Imposing goods market clearing (16) and labor market clearing (19) yields the natural level of output

$$y_t^n = \left[\frac{(1 + \varphi)}{[\sigma(1 - \alpha) + (\alpha + \varphi)]} \right] a_t = \psi_{ya}^n a_t.$$

Appendix B. Risk sharing

In this section, we closely follow [Ljungqvist and Sargent \(2012, ch.8.8, p.268ff.\)](#) and consider sequential-trading of [Arrow \(1964\)](#) securities. The household's problem is to choose $C_t(i, s^t)$, $N_t(i, s^t)$, $B_t(i)$, and $\{\mathcal{A}_{t+1}(i, s_{t+1}, s^t)\}$, where the latter is a vector of claims on time $t + 1$ units of the *numeraire*, i.e., currency. Hereby $s^t = [s_0, s_1, \dots, s_t]$ is the history of realized states and potential realizations in period $t + 1$ are denoted by $s_{t+1} \in S$. Furthermore, $q_t(s_{t+1}|s^t)$ is the price of one unit of the *numeraire* in $t + 1$, contingent on history s^t the realization of state s_{t+1} . $\pi_t(i, s^t)$ is the unconditional subjective probability which agent i assumes for the realization of history s^t .

The problem can be solved by the Lagrangian

$$\begin{aligned} \mathcal{L}(\cdot, i) = & \sum_{t=0}^{\infty} \sum_{s^t} \left\{ \beta^t U(C_t(i, s^t), N_t(i, s^t)) \pi_t(i, s^t) \right. \\ & + \lambda_t^H(i, s^t) \left(\frac{W_t}{P_t} N_t(i, s^t) - \frac{T_t}{P_t} + \frac{A_t(i, s^t)}{P_t} + \frac{B_{t-1}(i)}{P_t} \right. \\ & \left. \left. - C_t(i, s^t) - \sum_{s_{t+1}} q_t(s_{t+1}|s^t) \frac{A_{t+1}(i, s_{t+1}, s^t)}{P_t} - R_t \frac{B_t(i)}{P_t} \right) \right\}. \end{aligned}$$

The first-order conditions with respect to $C_t(i, s^t)$, $N_t(i, s^t)$, $B_t(i)$, and $\mathcal{A}_{t+1}(i, s_{t+1}, s^t)$ are given by

$$\begin{aligned} \beta^t U_{C,t}(i, s^t) \pi_t(i, s^t) - \lambda_t^H(i, s^t) &= 0, \text{ or} \\ \beta^{t+1} U_{C,t+1}(i, s^{t+1}) \pi_{t+1}(i, s^{t+1}) - \lambda_{t+1}^H(i, s^{t+1}) &= 0, \end{aligned} \tag{B.1}$$

$$\beta^t U_{N,t}(i, s^t) \pi_t(i, s^t) + \lambda_t^H(i, s^t) \frac{W_t}{P_t} = 0, \tag{B.2}$$

$$\lambda_t^H(i, s^t) [-R_t P_t^{-1}] + \lambda_{t+1}^H(i, s^{t+1}) [P_{t+1}^{-1}(s^{t+1})] = 0, \tag{B.3}$$

$$\lambda_t^H(i, s^t) [-q_t(s_{t+1}|s^t) P_t^{-1}] + \lambda_{t+1}^H(i, s^{t+1}) [P_{t+1}^{-1}(s^{t+1})] = 0, \tag{B.4}$$

which have to hold $\forall t, s^t$, and $s_{t+1} \in S$.

Next, define the subjective probability of agent i for the realization of state s_{t+1} conditional on the realized history s^t as $\pi_t(i, s_{t+1}|s^t) \equiv \pi_{t+1}(i, s^{t+1})/\pi_t(i, s^t)$ and combine (B.4) and (B.1) to derive

$$q_t(s_{t+1}|s^t) = \beta \left(\frac{U_{C,t+1}(i, s^{t+1})}{U_{C,t}(i, s^t)} \right) \left(\frac{P_{t+1}(s^{t+1})}{P_t} \right)^{-1} \pi_t(i, s_{t+1}|s^t). \tag{B.5}$$

Defining $Q_t(i, s_{t+1}|s^t) \equiv q_t(s_{t+1}|s^t)/\pi_t(i, s_{t+1}|s^t)$ and summing over all potential states leads to

$$\begin{aligned} \sum_{s_{t+1}} q_t(s_{t+1}|s^t) &= \sum_{s_{t+1}} \beta \left(\frac{U_{C,t+1}(i, s^{t+1})}{U_{C,t}(i, s^t)} \right) \left(\frac{P_{t+1}(s^{t+1})}{P_t} \right)^{-1} \pi_t(i, s_{t+1}|s^t) \\ \sum_{s_{t+1}} \pi_t(i, s_{t+1}|s^t) Q_t(i, s_{t+1}|s^t) &= \sum_{s_{t+1}} \beta \left(\frac{U_{C,t+1}(i, s^{t+1})}{U_{C,t}(i, s^t)} \right) \left(\frac{P_{t+1}(s^{t+1})}{P_t} \right)^{-1} \pi_t(i, s_{t+1}|s^t). \\ E^i \{ Q_t(s_{t+1}|s^t) | s^t \} &= E^i \left\{ \beta \left(\frac{U_{C,t+1}(i, s^{t+1})}{U_{C,t}(i, s^t)} \right) \left(\frac{P_{t+1}(s^{t+1})}{P_t} \right)^{-1} | s^t \right\}. \end{aligned}$$

For notational convenience, we define

$$\begin{aligned} E_t^i \{ Q_{t|t+1} \} &\equiv E^i \{ Q_t(s_{t+1}|s^t) | s^t \} \\ E_t^i \left\{ \beta \left(\frac{U_{C,t+1}(i)}{U_{C,t}(i)} \right) \left(\frac{P_{t+1}}{P_t} \right)^{-1} \right\} &\equiv E^i \left\{ \beta \left(\frac{U_{C,t+1}(i, s^{t+1})}{U_{C,t}(i, s^t)} \right) \left(\frac{P_{t+1}(s^{t+1})}{P_t} \right)^{-1} | s^t \right\}. \end{aligned}$$

Thus, we can derive a general Euler condition

$$E_t^i \{ Q_{t|t+1} \} = E_t^i \left\{ \beta \left(\frac{U_{C,t+1}(i)}{U_{C,t}(i)} \right) \left(\frac{P_{t+1}}{P_t} \right)^{-1} \right\}.$$

Furthermore, (B.1) and (B.2) imply

$$\frac{U_{C,t}(i, s^t)}{U_{N,t}(i, s^t)} = \frac{W_t}{P_t},$$

and from (B.3) to (B.4) it follows that

$$\begin{aligned} \lambda_t^H(i, s^t)P_t^{-1}R_t &= \lambda_{t+1}^H(i, s^{t+1})P_{t+1}^{-1}(s^{t+1}), \\ \lambda_t^H(i, s^t)P_t^{-1}q_t(s_{t+1}|s^t) &= \lambda_{t+1}^H(i, s^{t+1})P_{t+1}^{-1}(s^{t+1}), \\ \sum_{s_{t+1}} R_t &= \sum_{s_{t+1}} q_t(s_{t+1}|s^t)^{-1}. \\ R_t &= E_t^i\{Q_{t|t+1}\}. \end{aligned}$$

Next, for two agents $i \neq j$ Eq. (B.5) implies

$$\frac{U_{C,t}(i, s^t)}{U_{C,t}(j, s^t)} = \frac{U_{C,t+1}(i, s^{t+1})\pi_t(i, s_{t+1}|s^t)}{U_{C,t+1}(j, s^{t+1})\pi_t(j, s_{t+1}|s^t)} \tag{B.6}$$

$\forall t, s^t, s_{t+1} \in S$.

Under the REH, it holds that subjective and objective probabilities coincide, i.e., $\pi_t(i, s_{t+1}|s^t) = \pi_t(j, s_{t+1}|s^t) = \pi_t(s_{t+1}|s^t)$ and iterating backwards yields

$$\frac{U_{C,t}(i, s^t)}{U_{C,t}(j, s^t)} = \frac{U_{C,0}(i, s^0)}{U_{C,0}(j, s^0)}.$$

It follows, that, if initial wealth is the same for all agents and s^0 is observed, i.e., $\pi_0(s_0) = 1$, we have $C_0(i) = C_0(j) = C_0$, which in turn implies that $C_t(i) = C_t(j) = C_t, \forall t$. This result means that all agents can plan current and future consumption optimally across time and state by fully insuring themselves via trading state-contingent claims, i.e., *perfect risk-sharing*.

However, in the present paper subjective probabilities differ among types of agents, i.e., $\pi_t(i, s_{t+1}|s^t) \neq \pi_t(j, s_{t+1}|s^t)$, thus the general (B.6) holds $\forall t, s^t, s_{t+1} \in S$. Summing over all $s_{t+1} \in S$ results in

$$\sum_{s_{t+1}} \pi_t(i, s_{t+1}|s^t)U_{C,t+1}(i, s^{t+1}) = \frac{U_{C,t}(i, s^t)}{U_{C,t}(j, s^t)} \sum_{s_{t+1}} \pi_t(j, s_{t+1}|s^t)U_{C,t+1}(j, s^{t+1}).$$

The latter implies that

$$\frac{E_t^i\{U_{C,t+1}(i)\}}{E_t^j\{U_{C,t+1}(j)\}} = \frac{U_{C,t}(i, s^t)}{U_{C,t}(j, s^t)}.$$

Thus, given a complete market for state-contingent claims, there is *imperfect risk-sharing* due to heterogeneity in expectations.

Appendix C. The optimal monetary policy problem

The Lagrangian of this problem is given by

$$\mathcal{L}(\cdot) = E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{2}(\pi_{t+s}^2 + \omega_x x_{t+s}^2) + \kappa_{2|t+s} [\pi_{t+s} - \beta\chi\pi_{t+s+1} - \beta(1-\chi)\theta^2\pi_{t+s-1} - \lambda x_{t+s} - u_{t+s}] \right] \right\}.$$

The related first-order necessary conditions are given by

$$\frac{\partial \mathcal{L}(\cdot)}{\partial \pi_{t+s}} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad E_t \left\{ \beta^s [\pi_{t+s} + \kappa_{2|t+s}] + \beta^{s-1} [\kappa_{2|t+s-1} [-\beta\chi]] + \beta^{s+1} [\kappa_{2|t+s+1} [-\beta(1-\chi)\theta^2]] \right\} \stackrel{!}{=} 0 \tag{C.1}$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial x_{t+s}} \stackrel{!}{=} 0 \quad \Leftrightarrow \quad E_t \left\{ \beta^s [\omega_x x_{t+s} + \kappa_{2|t+s} [-\lambda]] \right\} \stackrel{!}{=} 0, \tag{C.2}$$

for each date $s \geq 0$ and initial conditions $\kappa_{2|-1} = 0$, given that the central bank employs a commitment to its optimality conditions from a *timeless perspective* to overcome the problem of *time-inconsistency*. We can equivalently express (C.1)–(C.2) as

$$\begin{aligned} E_t \kappa_{2|t+s} &= -E_t \pi_{t+s} + \chi E_t \kappa_{2|t+s-1} + \beta^2(1-\chi)\theta^2 E_t \kappa_{2|t+s+1} \\ 0 &= \omega_x E_t x_{t+s} + \lambda E_t \kappa_{2|t+s}, \end{aligned}$$

which corresponds to (30) and (31) in the paper. Notice that (30) follows from setting $E_t \kappa_{2|t+s-1}$ to zero. In our set-up this is equivalent to choosing the loss-minimizing policy at a generic time t , where the central bank accounts for the equilibrium response of endogenous variables in period $t + 1$ under the optimal policy.

In order to examine the consequences of optimal monetary policy design under discretion, one can iterate (30) forward and obtains

$$\kappa_{2|t} = - \sum_{s=0}^S [\beta^2(1-\chi)\theta^2]^s E_t \pi_{t+s} + [\beta^2(1-\chi)\theta^2]^{S+1} E_t \kappa_{2|t+S+1}.$$

Using (31) and considering the limit of $S \rightarrow \infty$ and assuming $|\beta^2(1-\chi)\theta^2| < 1$, which is true for $0 < \theta \leq 1$, but not necessarily true for $\theta \geq 1$, one obtains

$$\begin{aligned} \frac{\omega_x}{\lambda} x_t &= - \sum_{s=0}^{\infty} [\beta^2(1-\chi)\theta^2]^s E_t \pi_{t+s} \\ \frac{\omega_x}{\lambda} x_t &= - \pi_t - \sum_{s=1}^{\infty} [\beta^2(1-\chi)\theta^2]^s E_t \pi_{t+s}. \end{aligned} \tag{C.3}$$

Then from (33) follows

$$\sum_{s=1}^{\infty} E_t \pi_{t+s} = \sum_{s=1}^{\infty} \Omega_{2,2}^s \pi_t + \sum_{\tau=1}^s \Omega_{2,2}^{s-\tau} \Lambda_{2,2} \rho_u^\tau u_t = \sum_{s=1}^{\infty} \Omega_{2,2}^s \pi_t + \frac{\rho_u(\rho_u^s - \Omega_{2,2}^s)}{(\rho_u - \Omega_{2,2})} \Lambda_{2,2} u_t,$$

and in consequence, we can write (C.3) as

$$\begin{aligned} \frac{\omega_x}{\lambda} x_t &= -\pi_t - \sum_{s=1}^{\infty} [\beta^2(1-\chi)\theta^2]^s \left[\Omega_{2,2}^s \pi_t + \frac{\rho_u(\rho_u^s - \Omega_{2,2}^s)}{(\rho_u - \Omega_{2,2})} \Lambda_{2,2} u_t \right] \\ \frac{\omega_x}{\lambda} x_t &= - \left[\frac{[1 - \beta^2(1-\chi)\theta^2 \rho_u] \pi_t + \beta^2(1-\chi)\theta^2 \Lambda_{2,2} \rho_u u_t}{[1 - \beta^2(1-\chi)\theta^2 \rho_u][1 - \beta^2(1-\chi)\theta^2 \Omega_{2,2}]} \right] \\ \Leftrightarrow x_t &= - \frac{\lambda}{\omega_x} \left[\frac{[1 - \beta^2(1-\chi)\theta^2 \rho_u] \pi_t + \beta^2(1-\chi)\theta^2 \Lambda_{2,2} \rho_u u_t}{[1 - \beta^2(1-\chi)\theta^2 \rho_u][1 - \beta^2(1-\chi)\theta^2 \Omega_{2,2}]} \right]. \end{aligned} \tag{C.4}$$

Assume that u_t is serially uncorrelated, i.e., $\rho_u = 0$. Then (C.4) collapses to (34).

Appendix D. Model solution and long-run variance

Combining (27) and (28) yields

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{z}_t \end{bmatrix} = \begin{bmatrix} \mathbf{\Omega} & \mathbf{\Lambda R} \\ \mathbf{0} & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{z}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{\Lambda} \\ \mathbf{I} \end{bmatrix} \mathbf{e}_t.$$

More compact this is

$$\mathbf{Y}_t = \mathbf{G} \mathbf{Y}_{t-1} + \mathbf{K} \mathbf{e}_t$$

where \mathbf{G} is $(m+n) \times (m+n)$, and \mathbf{K} is $(m+n) \times n$. Finally, in the long-run we assume that $\text{Var}(\mathbf{Y}_t) = E[\mathbf{Y}_t \mathbf{Y}_t'] \equiv \mathbf{\Xi}$ and $\text{Var}(\mathbf{e}_t) = E[\mathbf{e}_t \mathbf{e}_t'] \equiv \mathbf{\Sigma} \forall t$. Moreover $\mathbf{\Xi}$ is $(m+n) \times (m+n)$ and $\mathbf{\Sigma}$ is $n \times n$. It follows that

$$\begin{aligned} \mathbf{\Xi} &= \mathbf{G} \mathbf{\Xi} \mathbf{G}' + \mathbf{K} \mathbf{\Sigma} \mathbf{K}' \\ \text{vec}(\mathbf{\Xi}) &= [\mathbf{I} - (\mathbf{G} \otimes \mathbf{G})]^{-1} \text{vec}(\mathbf{K} \mathbf{\Sigma} \mathbf{K}'), \end{aligned} \tag{D.1}$$

where $\text{vec}(\mathbf{\Xi})$ is $(m+n)^2 \times (m+n)^2$. The long-run variances for the variables of interest can then be found in the respective entry in this vector (D.1).

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