Bank strategic asset allocation under a unified risk measure

Pedro Júdice *,[a], Luís Pinto[b], and José Luís Santos[b]

[a]Montepio Bank and ISCTE Business Research Unit, Portugal
[b]University of Coimbra, CMUC, Department of Mathematics, Coimbra, Portugal
pedro_judice@yahoo.com, luisp@mat.uc.pt, zeluis@mat.uc.pt

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Abstract

Most available bank asset allocation models use several risk measures as constraints; as a consequence, the comparison of the risk between different asset allocation strategies is often difficult, since each strategy is subject to several risks.

With this research, we create a simulation-optimization methodology that measures interest rate, credit and liquidity risks in a unified manner. The associated risk events, such as interest rate increases, liquidity outflows or spikes in defaults are generated using the same simulation engine, giving as output a single risk measure (the probability of failure, used by ratings agencies) that aggregates those risks under the same simulation engine.

Finally, we use our methodology to determine Pareto fronts for the optimal balance sheet allocations and minimum-risk strategies. As a result, several findings emerge, such as: 1) Risk is dependent on the income stream; 2) Allocation to book value assets is preferable; 3) Under low rate environments, a full allocation to cash is very risky and is not the minimum risk strategy; 4) Banks can make investments in stocks in environments of high prospective returns and low leverage.

1 Introduction

Risk aggregation is one of the most important topics in risk management and bank strategic asset allocation, as it is the main methodology used in internal capital adequacy exercises (ICAAP) for banks. For years, academics and practitioners have tried several solutions to risk aggregation.

In typical bank risk management, interest rate risk is usually measured using Economic Value of Equity (EVE), while credit risk is evaluated with CreditVaR and liquidity risk is assessed with Liquidity at Risk (LaR). All these risks are measured separately, under measures that are not comparable; as a consequence, the aggregation of risks used in exercises such as ICAAP is not rigorous, and fails to account for liquidity risk.

When addressing the different risks in the balance sheet it is important to ensure that there is comparability in order to assess the decisions. For example, if one wants to compare three strategies, one that has a CreditVaR of 100 and no other risk, one that has an interest rate risk (EVE) of 100, and one that has an LaR of 100, which one is the riskier? The methodologies are different, the measures are not comparable. CreditVaR typically uses a book-value approach to measuring losses, whereas EVE uses fair-value impacts, and liquidity at risk usually measures the outflows against the lack of liquid resources to compensate for those outflows. Having a simulation approach that models under the same framework credit, market, interest rate and liquidity risk is important to make such assessments.

The usual way to circumvent this difficulty is to mix these risks in an *ad hoc* fashion. Step 1 is measuring each risk (credit, market, interest rate); Step 2 is to mix them under some rule, which either sums the risks, assumes independence of the risks or assumes correlation between the different types of risk. The more sophisticated approaches use copulas, but the lack of comparability of the different risks persists. The literature is quite vast, but possible approaches are described in Brockmann and Kalkbrener [9], Chong, Feng, and Jin [17], Di Lascio, Giammusso, and Puccetti [27], Rosenberg and Schuermann [57], or Uryasev, Theiler, and Serraino [61]. However, as we mentioned before, these methodologies aggregate risks that are not comparable. Alessandri and Drehmann [1] and Jobst, Mitra, and Zenios [42] model different risks under the same simulation engine, but these methodologies do not account for liquidity risk. Gubareva and Borges [34] propose integrating credit and interest rate risk with the information from traded derivatives, but again do not incorporate funding liquidity risk stemming from customer potential withdrawals. The reader can also refer to the survey by Li et al. [45] on risk integration.

As we will see below, our research will go beyond risk integration. Risk integration is just the starting point for the paper, whose primary focus is to provide strategic asset allocation for banks based on unified risk measures. To the best of our knowledge, the risk integration papers focus solely on risk and do not provide insights on the optimal strategies to be followed by banks.

This problem of risk integration feeds into bank strategic asset allocation. When maximizing return against risk, it is important to compare different strategies, particularly those that are credit risk-intensive to those that are interest rate risk-intensive or liquidity risk-intensive. If there is no unified way to compare interest rate, credit and liquidity risks, there is no possibility to evaluate the best strategy in a comparable manner. Going back to the example above, assume a bank has three possible strategies: one that has a CreditVaR of 100 and a return of 2 (assuming no other risks), a second strategy that has interest rate risk of 100 and a return of 3, and a third strategy that has liquidity at risk of 100 and a return of 2; which strategy is the best? Without a single framework to evaluate the different risk factors, it will be difficult to evaluate the strategies. From the point of view of the manager, he is interested in assessing the risk of failure of the bank, which can be liquidity-driven or solvency-driven.

We solve the lack of comparability in the bank asset allocation problem by simulating liquidity, credit and interest rate risks under the same engine: liquidity risk is modeled with a liquidity volume econometric model, devised by [21]; our engine simulates the interest rates and defaults using a methodology close to [2]. The simulation of these risk factors feeds into the balance sheet equations, and with that we are able to calculate risk and return measures. To assess risk, we use the probability of failure over a certain horizon although other measures may be used. The probability of failure is the measure ratings agencies use to assess financial strength, and encapsulates both solvency-led failures (depletion of capital) and liquidity-driven failures (running out of liquid resources to compensate outflows).

We have used the approach for certain asset classes which are more relevant in the case of commercial banks, but the approach can be extended to several other aggregates in the balance sheet and even different for banks operating in different jurisdictions. This seems like a promising avenue for future research and for use at commercial banks.

Having the simulation for the balance sheet ready, we can then pursue optimization. As we show in Section 5.1, the optimization problem is non-continuous, non-differentiable and non-convex. In addition, we aim to obtain a global optimum. These would be significant drawbacks, but the structure of the problem leads to two great advantages: it has few optimization variables, and also does not need very sharp tolerances, as the simulation outputs also have errors. Therefore, a grid search on the possible combinations (within a certain tolerance), generates very satisfactory and intuitive solutions, as we will see below. In the grid search we set a tolerance of 1% on the allocations, which does not make a significant difference for a bank in a practical context. In effect, from the point of view of establishing asset allocations, the difference between a 15% or a 16% allocation is usually not important in practice.

The solutions obtained by this method will be good approximations for global minimizers, avoiding the convergence of the algorithms to local minimizers that may be far away from global solutions. If more accurate solutions need to be obtained, multi-objective optimization algorithms [35, 54] can be used starting from the referred solutions. In a practical context, the solutions with more accurate precision would not add value, given the tolerances that are needed in practice. Also, the objective function stems from a simulation, so its accuracy is $O(1/\sqrt{numPaths})$, where $numPaths$ is the number of simulated paths. Specifying sharp tolerances in the optimization would render an exercise that would still have a significant margin of error that comes from the objective function.

In this fashion, we are able to steer away from the problems associated with local and global optimization problems (for example, converging to local minima in the case of local optimization, or failing to generate enough initial solutions properly in the case of global optimization problems so that the problem does not converge to the global maximum).

We generate Pareto fronts, yielding very intuitive results which highlight critical issues in bank strategic asset allocation. First, the optimizer gives a clear prevalence to book value asset classes (namely mortgages), which, unlike fair value classes, do not generate volatility in the balance sheet from changes in market prices. This finding is consistent with [8], who conduct an asset optimization methodology for securities at fair value and amortized cost. We also observe that leverage is correlated to risk (not surprisingly). The level and risk of the Pareto fronts reflect the economic environment and the prospective risk premia on the different asset classes.

Our methodology also shows a critical interaction between return and risk. Usually risk and return are measured separately, but risk is highly dependent on return. Let us give another example. Suppose that, in an environment of low interest rates, a bank manager decides to invest all his assets in cash, as this would be the textbook-type riskless portfolio taught in mean-variance analysis. Since the bank has an operating cost structure to pay for, the bank would be certainly destined to fail, since it would have consecutive losses with certainty. The textbook riskless allocation strategy is clearly not the riskless strategy in our setting, and does not have the lowest risk, as we discuss in Section 5.4. In other words, if a bank does not generate return, it will be risky in the medium term.

Our approach also shows that stock investments make sense particularly in environments of low leverage and high prospective returns.

In summary, our integrated strategic asset allocation methodology has three main steps:

- Step 1: Scenario generation framework for the relevant risk factors; this is described in Section 2.

- Step 2: Simulation of the bank's asset allocation, based on the simulation of the risk factors, and the computation of the risk-return measures; this is described in Section 3.

- Step 3: Optimization of the bank strategies and determination of the Pareto fronts, based on the simulation of the bank's balance sheet; this is described in Section 4.

Our introduction would not be complete without a brief survey on the research output on balance sheet management and bank asset allocation.

Portfolio allocation models had great developments in the second half of the last century. The seminal work of Markowitz [47] influenced generations of asset allocation methods in several contexts. Merton [48] and Samuelson [58] developed the theory for investments under a lifelong consumption stream. The book by Campbell and Viceira [11] describes many models on asset allocation. Pandolfo, Iorio, Siciliano, and D'Ambrosio [55] use a non-parametric estimation method based on statistical data depth functions to obtain a model that is less sensitive to changes in the asset return distribution.

Allocation models for insurance and pension funds also had numerous advances. We highlight a few of the pioneers in this field. The book by Ziemba and Mulvey [69] provides excellent references to the subject. Other references include Boender [3], Cariño et al. [12], Consigli and Dempster [20], Gondzio and Kouwenberg [31], Kouwenberg [43], Lucas and Zeldes [46], Mulvey and Thorlacius [50], Mulvey and Vladimirou [51], and Zenios [68]. These papers often build stochastic scenarios and optimize portfolio allocations based on those scenarios.

In the context of banking, Chambers and Charnes [13] and Eatman and Sealey [29] developed deterministic models. Stochastic models can be traced back to Brodt [10], Charnes and Thore [15], Kuzy and Ziemba [44], and Pyle [56]. Bradley and Crane [5] [6] and Wolf [67] developed sequential decision models. All these models, naturally, do not incorporate the financial theory and regulation that were subsequently developed.

Looking into more recent advances, Birge and Júdice [2] created a methodology for simulating scenarios on the balance sheet over many periods. Halaj [37] devised an optimal balance sheet framework for a single-period model. Halaj [38] created an optimal balance sheet model with liquidity risk, but does the computational work over a two-period model (although a multi-period model is mentioned). The model does not take into account interest rate risks, and also does not have a unified measure of the different risks. Coelho, Santos and Júdice [18] have recently created an optimal balance sheet model for one period, ensuring robustness with turnover constraints. The approach, however, does not have a comparable framework to model different risks. Dewasurendra, Júdice and Zhu [26], have recently created an optimal balance sheet model based on a modified Kelly criterion, but this model does not account for credit risk or liquidity risk. Schmaltz, Pokutta, Heidorn and Andrae [59] devised an optimal framework for balance sheets for a non-compliant bank under Basel III, but their model does not account for interest rate risk, nor does the paper have the comparable framework we pursue in this research.

In our opinion, the approach we propose can be also used in practice to advise boards at banks on strategic asset allocation. To the best of our knowledge, in practice many existing bank asset-liability management (ALM) techniques take the asset and maturity structures of a bank as a given and try to optimize the trade-off between funding costs, interest rate risk and liquidity risk, by changing the maturity profile of the liabilities, or by using derivatives such as interest rate swaps. Our approach is different. Given a bank capital structure formed by equity and deposits, our methodology devises optimal asset allocations for a bank, taking into account the risk factors which have been discussed, including credit risk. This output can be used for instance in strategic plans.

We can view our framework as an Expert System (ES) for banking management: a computerized application that advises and helps decision-makers based on quantitative and/or qualitative information. The application and development of ES have already a long tradition in the financial domain, covering several fields such as financial analysis, banking management, investment advisory, and financial marketing [52]. Earlier ES were mainly rule-based algorithms, like Port-Man, which helped banks to advise their customers on their investments [14]. Port-Man consists of a search algorithm looking for feasible products based on personal information about the investor, e.g., risk appetite and tax and pension implications.

Nowadays, researchers investigate much more complex ES, some based on the recent trend of artificial intelligence and machine learning algorithms. One such case is Ferreira et al. [30], where the authors propose a fuzzy multiple-attribute framework for portfolio optimization in private banking. The approach consists of two steps: first, a fuzzy sorting method to match the investors' profile to the banks' investment options; next, a multiobjective optimization model to find the optimal allocation. The optimal allocation considers risk, return, and the investors' profile. Other research includes combining the mean-variance model with machine learning algorithms to select assets and predict future returns. For such task, Wang, Li, Zhang, and Liu [64] use deep learning long-short term memory networks, while Chen, Zhong, and Chen [16] use a combination of clustering and a radial basis function neural network.

While the contributions above focus on optimal asset allocation for portfolio management, our research aims to find the optimal asset allocation for a bank, given a capital structure consisting of deposits and equity. Portfolio management problems deal with traded assets, which have market prices. Bank asset allocation is a different problem. First, most assets do not trade in liquid markets, so they are accounted at book value rather than at fair value. Second, most assets are illiquid, thus the need to manage funding risk much more carefully than in the context of portfolio management. Third, the problem depends on the bank's capital structure, i.e., the proportion of shareholders' equity to creditors' funds. In contrast, in portfolio management, typically investors in the fund share the same characteristics.

The paper is organized as follows: Section 2 develops the scenario simulation engine, largely based on results in [2] and [21]; in Section 3, we conduct the simulations for the balance sheet that enable us to conduct the risk and return assessments, which will be optimized in Section 4 obtaining the Pareto fronts; Section 5 discusses the results; we conclude in Section 6.

2 The scenario generation framework for the risk factors

For asset allocation models, scenario generation often resorts to vector autoregressive processes [60] or stochastic differential equations. Among the first market interest rate models are the short-rate models of Brennan and Schwartz [7], Cox-Ingersoll-Ross [22], and Vasicek [62]. These models, however, do not incorporate the interactions between market and retail banking rates, which have been subsequently studied by Diebold and Sharpe [28], Hutchison and Pennacchi [39], Jarrow and Van Deventer [41], and Janosi, Jarrow, and Zullo [40]. Birge and Júdice [2] have built an interest rate risk model that explores the interactions between market and retail banking rates using a vector autoregression that accounts for auto-correlation.

The literature on credit risk is quite extensive (see for instance Gordy [32] and Crouhy et al. [24]). Seminal references include CreditMetrics [36], CreditRisk+ [23], CreditPorfolioView [65, 66], Gordy [33], the KMV model [4], and Vasicek [63]. Many of the models are based on the framework of Merton [49]. The model by Birge and Júdice [2] builds upon the Vasicek credit model by introducing autocorrelation and a momentum term.

Stochastic models for liquidity volumes appeared in the literature previously. Jarrow and van Deventer [41] and O'Brien [53] developed stochastic deposit volume models. Recently, Costa et al. [21] have introduced a panel data model for simulating deposits, with the advantage that this model can simultaneously account for both episodes of boom and failed banks in the sample.

We start then by developing the scenario generation framework for the risk factors. We simulate the risk factors which are relevant to our research, namely the interest rates on different classes, equity returns, credit losses, and deposit volumes. Using a single simulation engine, we are able to generate the relevant risks. Interest rate risk is included by using a vector auto-regressive model on the different relevant interest rates; equity returns (stocks) are modeled by an autocorrelated process fitted to the Standard & Poor's index; credit losses are also modeled using an autocorrelated process; and liquidity flows are modeled according to an auto-regressive model with momentum, calibrated to a cross-section of different banks. The estimation results given in this section are taken from the research conducted by Costa, Faias, Júdice and Mota, whose most interesting findings were published in [21]. The interest rates and charge-off model is inspired by the research of Birge and Júdice [2].

The motivation, effectiveness, and estimation of this type of models was investigated by those authors, and so some details are omitted. For the sake of clarity, we review the main points. First, we will address the data sources and notation and then we discuss the models and methods, where we present the estimation of the interest rate model, followed by the credit loss model and the stock price model, and finally, the deposit volume model.

2.1 Data and Notation

As argued in [2] and [21], financial crises have highlighted the need for better long-term bank asset allocation policies that allow the banks to remain profitable and solvent through economic cycles. Optimal long-term asset allocation policies are crucial to provide adequate returns to the bank stakeholders in the long run. Moreover, the long-term nature of most assets in bank balance sheets only reinforces the need for long-term asset allocation strategies.

With this in mind we take scenario generation framework from [2] and [21] that allows the long-term simulation of interest rates, equity returns, charge-offs, and deposit volumes. To adjust the underlying stochastic models, the authors of these papers have used long historical data spanning several decades and covering periods of economic growth and economic downturn. In particular, the interest rates data is from FRED (Federal Reserve Economic Data) and spans from 1971 to 2016, covering periods of very high rates (early eighties) and the recent periods of very low rates. The charge-off data is also from FRED and spans from 1985 to 2016, covering a period of increasingly high rates corresponding to the 2007-2008 subprime crisis. The S&P price index is from the Robert Shiller Irrational Exuberance Database and spans from 1946 to 2016.

Regarding the liquidity flows the authors of [21] used the annual deposit data from 9 Portuguese banks spanning from 1992 to 2016. The banks were selected based on the criterion of having an average of at least 10 billion euros in client deposits during this time span, considering a total of 128 observations. The panel data of Portuguese banks constitutes a very rich dataset, as it encapsulates periods of crises and failed banks, so that the model proposed by the authors can simultaneously account for episodes of boom and periods of financial crises and bank failures, thus yielding realistic scenarios for liquidity management and LaR estimates. In fact, the nineties were characterized by a boom in the Portuguese banking system, fueled by the privatization of formerly nationalized banks (in the aftermath of the Carnation Revolution of 1974) and the 1993 European single market for financial services. On the other hand, in 2011 Portugal needed a bailout, and from 2011 to 2014 was under severe austerity measures imposed by the Troika (European Union, European Central Bank, and the International Monetary Fund). The data analyzed in [2] and [21] are depicted in Figure 1. The model's parameters were estimated using this data set and ordinary least squares.

In Table 1 we show the model asset classes, risk factors and the data sources for each risk factor.

Asset class	Risk factors	Data
Cash	Wholesale rates	FRED
Mortgage rates	Interest rates and charge-off rates	FRED
Public debt (bonds)	Yields	FRED
Equities (stocks)	S&P index and dividend data	Robert Shiller Irrational Exuberance Data
Deposits	Deposit rates and Liquidity flows	Rates – FRED Cross section of bank deposit volume series from Portuguese banking association

Table 1: Asset classes considered in this work. For each asset class, we highlight the risk factors and the data sources.

2.2 Interest rate simulation

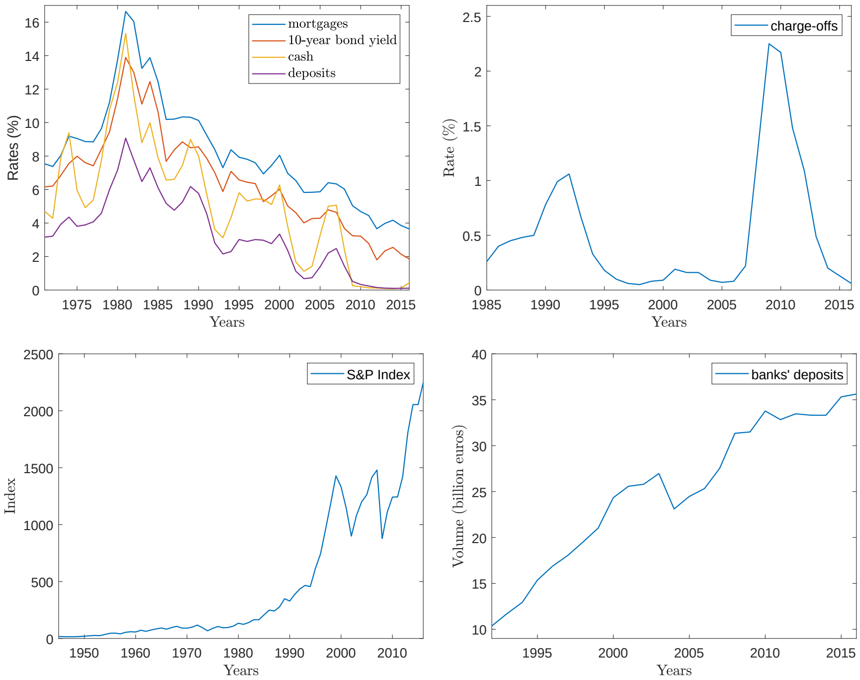

Figure 1: Historical data used to fit the risk factor models in [2] and [21]. On the upper left-hand corner, we show the US historical data for mortgage rates, 10-year Treasury bond rates, wholesale funding rates, and deposit rates. On the upper right-hand side, we show the historical credit losses (charge-off rate) for mortgages. The lower left-hand side shows the historical S&P price index, used to estimate the equity process. On the lower right-hand side we show the historical data for the bank deposits used to estimate the liquidity volume process; since the model was estimated to panel data of 9 banks, we show the historical average.

The interest rate scenario generation is very similar to Birge and Júdice [2]. As argued by the authors, their interest rate model blends elements from previous research. First, it is a discrete-time vector autoregressive process [60], with different dynamics for the short-term rates and long-term rates, as in Diebold and Sharpe [28] and Brennan and Schwartz [7]. The inclusion of momentum terms stems from their significance when conducting the time-series estimates.

However, the model has some differences when compared to Birge and Júdice [2]. First, it incorporates bond yields, which were not present in the previous model. Also, while Birge and Júdice [2] use square root residuals in the estimation, this model uses two regimes, one lognormal for low rates and one normal for high rates. Finally, this new model accommodates more recent data better. As mentioned before, this new model was estimated during research project with Costa et al. [21], whose most relevant findings were published.

The interest rate data (in percentage) are first transformed by using the following function

$$g(x) = \begin{cases} \ln(x), & 0 < x < 1 \\ x - 1, & x \geq 1 \end{cases} .$$

Type	Parameter	Description
Asset allocation vector (decision variables)	$\overline{\alpha_0}$	proportion allocated to cash
	$\overline{\alpha_1}$	proportion allocated to loans
	$\overline{\alpha_2}$	proportion allocated to (ten-year) bonds
	$\overline{\alpha_3}$	proportion allocated to stocks
Liabilities	E_t	amount of shareholder capital/equity
	D_t	volume of deposits (stochastic)
Costs	c_t	operating costs
	c	operating cost factor
Interest rates	r_t	Interest rate on new loans (mortgages)
	f_t	interest rate on cash
	y_t	ten-year bond yield
	d_t	deposit rate
Loans	λ_t	charge-offs/credit losses
	p	amortization factor for legacy loans
	L_t	volume of total loans
	I_t	income obtained from legacy loans
Bonds	$Dur(y)$	duration of the par ten year bond
Stocks	S_t	stock prices
	div_t	stock dividends

Table 2: Notation for asset allocations, liabilities, costs, interest rates, loan variables, bonds and stocks.

The function thus specified allows two regimes: a lognormal regime for low rates, which ensures that rates do not fall below zero, as observed in the US interest rate data; and a normal regime, for higher rates, that prevents the model from having explosions as observed in lognormal interest rate models. Next, the g-transformed data is centered around the long-term mean as suggested by [25]. Define

$$
\begin{aligned}
r_t^* &= g(r_t) - \widehat{g(r_t)} \\
y_t^* &= g(y_t) - \widehat{g(y_t)} \\
f_t^* &= g(f_t) - \widehat{g(f_t)} \\
d_t^* &= g(d_t) - \widehat{g(d_t)},
\end{aligned}
$$

where the hat over a quantity denotes the long-term mean.

The evolution of the transformed interest rates is given by the following vector autoregression:

$$
\begin{aligned}
r_{t+1}^* &= \phi_r^r r_t^* + \phi_r^f f_t^* + \phi_r^y y_t^* + \phi_r^d d_t^* + \phi_r^m m_t^r + \epsilon_{t+1}^r \\
f_{t+1}^* &= \phi_f^r r_t^* + \phi_f^f f_t^* + \phi_f^y y_t^* + \phi_f^d d_t^* + \phi_f^m m_t^f + \epsilon_{t+1}^f \\
y_{t+1}^* &= \phi_y^r r_t^* + \phi_y^f f_t^* + \phi_y^y y_t^* + \phi_y^d d_t^* + \phi_y^m m_t^y + \epsilon_{t+1}^y \\
d_{t+1}^* &= \phi_d^r r_t^* + \phi_d^f f_t^* + \phi_d^y y_t^* + \phi_d^d d_t^* + \phi_d^m m_t^d + \epsilon_{t+1}^d,
\end{aligned}
$$

where $\epsilon = (\epsilon_{t+1}^r, \epsilon_{t+1}^f, \epsilon_{t+1}^y, \epsilon_{t+1}^d)$ is normally distributed with mean zero and covariance Σ, and the momentum terms m_t^r, m_t^f, m_t^y and m_t^d are defined by

$$m_t^r = r_t^* - r_{t-1}^*; \quad m_t^f = f_t^* - f_{t-1}^*; \quad m_t^y = y_t^* - y_{t-1}^*; \quad m_t^d = d_t^* - d_{t-1}^*.$$

ϕ	r_t^*	f_t^*	y_t^*	d_t^*	m_t	sd	R^2	t_0	t_{-1}
r_{t+1}^*	0.42	0.21	0.28	0.02	0.21	0.94	0.92	3.65	3.85
f_{t+1}^*	-0.93	0.41	1.29	0.35	0.51	1.57	0.86	0.43	0.12
y_{t+1}^*	-0.12	0.20	0.79	0.05	0.09	0.97	0.91	1.85	2.14
d_{t+1}^*	-0.44	0.04	0.64	0.69	0.37	0.75	0.93	0.112	0.102

Table 3: Least-square estimates coefficients ϕ for rates. r_t^*, f_t^*, y_t^* and d_t^* represent the interest rates after subtracting the long-term mean. sd represents the standard error, whereas t_0 and t_{-1} represent the initial values for the simulation at $t = 0$ and $t = -1$.

The vector autoregression parameters ϕ are estimated by ordinary least squares and given in Table 3. The obtained R-squared values are also shown in Table 3. The high values, ranging from 0.86 to 0.93, suggest that the model fits the data quite well. Still in Table 3, we denote by sd the standard deviation and by t_0 and t_{-1} the initial values for the simulation, at $t = 0$ and $t = -1$, respectively.

	ϵ_t^r	ϵ_t^f	ϵ_t^y	ϵ_t^d
ϵ_t^r	1	0.76	0.96	0.80
ϵ_t^f		1	0.74	0.94
ϵ_t^y			1	0.79
ϵ_t^d				1

Table 4: Interest rate residuals correlation matrix, for the mortgage rate, the wholesale rate, the bond yields and the deposit rates. The different rate residuals are positively correlated as expected.

The estimated residuals correlation matrix is given in Table 4. Following [25], the long-term means are defined as the average of the sampling data, thus obtaining

$$\widehat{g(r_t)} = 7.2465; \quad \widehat{g(f_t)} = 4.0347; \quad \widehat{g(y_t)} = 5.5550; \quad \widehat{g(d_t)} = 2.1343.$$

To generate the interest rate trajectories, one uses the inverse transformation, i.e, adds the long-term mean and applies the inverse function g^{-1} to the simulated rates from the vector auto-regressive model.

2.3 Credit losses

In order to simulate charge-offs or credit losses, one transforms the data by using the inverse of the standard normal cumulative distribution function, N^{-1} (see [2] for details), before deriving the regression model coefficients:

$$\lambda_t^* = N^{-1}(\lambda_t).$$

Define also the momentum term:

$$m_t^\lambda = \lambda_t^* - \lambda_{t-1}^*,$$

so that the dynamics is given by the autoregression process:

$$\lambda_{t+1}^* = c_\lambda + \phi_\lambda^\lambda \lambda_t^* + \phi_\lambda^m m_t^\lambda + \epsilon_{t+1}^\lambda.$$

	c_λ	ϕ_λ^λ	ϕ_λ^m	R^2	σ_λ	t_0	t_{-1}
λ_{t+1}^*	-0.59	0.79	0.78	0.90	0.12	0.06	0.13

Table 5: Least-square estimates for the charge-off process. The regression is conducted on the changed variables $\lambda_t^* = N^{-1}(\lambda_t)$. Here, c_λ is the intercept, ϕ_λ^λ and ϕ_λ^m are the coefficients for the lag-one charge-off rate and the momentum term, σ_λ is the standard error. As before, t_0 and t_{-1} are the initial values.

Here, ϵ_{t+1}^λ is normally distributed with mean zero and standard deviation σ_λ. The least square estimated parameters ϕ, as well as the initial simulation values and the R-squared for the charge-off rates, are given in Table 5.

The R-squared of 0.90 indicates that the model fits the data well. After generating the simulation, the charge-off trajectories are obtained by applying the cumulative distribution function of the standard normal distribution, $\lambda_t = N(\lambda_t^*)$.

2.4 Stocks

For stock prices S_t and dividends div_t, consider the logarithmic transformation of the total return and the dividend yield for the Standard & Poor's index. Namely, take

$$S_t^* = \ln(\frac{S_t + div_t}{S_{t-1}})$$

and

$$\delta_t^* = \ln(\frac{div_t}{S_t}).$$

The dynamic model for S_t^* and δ_t^* is defined by:

$$S_{t+1}^* = c_S + \alpha_S \delta_t^* + \epsilon_{t+1}^S$$
$$\delta_{t+1}^* = c_\delta + \phi_\delta \delta_t^* + \epsilon_{t+1}^\delta.$$

Total returns on stocks are thus dependent on dividend yields, in line with the literature on stock returns (see for instance the book by Campbell and Viceira [11]).

The least squares estimated coefficients are given in Table 6. Note that the initial values at t_0 are associated with the total return and the dividend yield, $(S_t + div_t)/S_{t-1}$ and div_t/S_t, respectively.

To get the actual total return values, we apply the transformation $\exp(S_t^*)-1$ to the simulated data.

	c_S	c_δ	α_S	ϕ_δ	sd	t_0
S^*_{t+1}	0.49		0.11		0.15	1.1159
δ^*_{t+1}		-0.27		0.93	0.17	0.0203

Table 6: Least-square estimates coefficients for the stock prices and dividend yields, using the transformations $S_t^* = \ln(\frac{S_t+div_t}{S_{t-1}})$ and $\delta_t^* = \ln(\frac{div_t}{S_t})$. Here, c_s and c_δ represent the intercepts, α_S and ϕ_δ are the regression coefficients, sd are the standard errors and t_0 are the initial values.

2.5 Deposit volumes

Deposits volumes are estimated using the following panel data model [21]:

$$D_{t+1} = c_D + \beta_1 D_t + \beta_2(D_t - D_{t-1}) + \epsilon_t^D.$$

The parameters of the model are presented in Table 7; we can write the model as

$$D_t = 1229300 + 0.98804 D_{t-1} + 0.22016(D_{t-1} - D_{t-2}) + \epsilon_t^D.$$

This equation gives an intuitive understanding of the model, as deposit volumes are influenced by the previous volumes D_{t-1}, a momentum term $D_{t-1} - D_{t-2}$, and a residual ϵ_t^D. The dependence on previous volumes is the auto-regressive part. The momentum term generates the auto-correlation present in the model: increases in deposits are likely to be followed by increases, and decreases in deposits are likely to be followed by decreases. The residual term, as shown by the authors in [21], has negative skewness.

The skewness of the residuals, coupled with the momentum term, significantly increases the risk associated with liquidity outflows, thus enabling the model to be realistically used for liquidity risk purposes. Also, the model is calibrated to a panel data set of banks, that includes failed banks, thus allowing the possibility of significant decreases in deposits.

	c_D	β_1	β_2	$\widehat{D}$	t_0	t_{-1}
D_t	1 229 300	0.98804	0.22016	27 251 747	27251747	27251747

Table 7: Least-square estimates coefficients for the stochastic deposits. Here, c_D is the intercept, β_1 and β_2 are are the model parameters, $\widehat{D}$ is the sample mean, and t_0 and t_{-1} are the initial values.

Since the distribution of the residuals is not normal, the simulation needs to use the bootstrap method. First the authors of [21] estimate the probability density function of the residuals by a kernel distribution. Then they calculate the cumulative density function $F(x)$, and the residuals are sampled generating random numbers θ between 0 and 1 and calculating $F^{-1}(\theta)$. Since we are in a discrete setting the inverse transformation is performed by linear interpolation.

Algorithm 1 gives a sketch of the framework presented here for the risk factors simulation, for each of the $numPaths$ trajectories with time horizon T. Therefore, all the parameters of the model described in Tables 3 to 7 are loaded (step 2) as well as vector ω_0 containing the initial values (step 3). The simulation of trajectory k, $k \in \{1, \dots, numPaths\}$ is done in steps 5-8. Since the simulation process only uses the transformed values accordingly to Section 2, the initial value ω_0 is first transformed into ω_0^* (step 4), and at the end of the simulation process of each trajectory, the risk factors generated ω_t^* must be reversed with the inverse transformation to obtain the "real" values ω_t (step 8). At the end of generating all ω_t values, $t \in \{1, \ldots, T\}$, they are stored in a matrix ω^k corresponding to the k-th trajectory that will be used later in the other algorithms. This algorithm has a complexity order $O(numPaths\ T)$.

Algorithm 1 Scenario generator

```
procedure SCENARIOGENERATOR(numPaths, T)
{ω_t = (r_t, f_t, d_t, y_t, λ_t, S_t, div_t, D_t)};
    Load model parameters from Table 3 to Table 7;
    Read ω_0, the initial values for the scenario;
    Compute ω_0^* using the transformation functions defined in Section 2;
    for k = 1 to numPaths do
        for t = 1 to T do
            Compute ω_t^* using formulas in Section 2;
            Compute ω_t using the inverse of the transformation functions;
        Save the k-th path scenario as ω^k;
```

3 Integrated balance sheet simulation and optimization

In this section we perform the simulation of the balance sheet, using the simulation of the risk factors described above. After simulating the balance sheet we devise risk and return indicators which are the main measures used to calculate the Pareto Fronts in Section 4.

Let us consider the evolution of the bank and of its risk factors, exogenous to the bank, for an horizon of T and periods $t = t_0, \ldots, T$, where $t = t_0$ represents the initial state. In order to proceed, we first need to establish some notation. Let us denote by ω the stochastic variable that allows us to represent each trajectory for the risk factors and Ω the space of all possible trajectories. We denote by ω_t the realization of trajectory ω at time t, that encapsulates all the information at time t, namely interest rates, charge-off rates, stock prices, and the volumes of core deposits. Specifically,

$$\omega_t = (r_t, f_t, d_t, y_t, \lambda_t, S_t, div_t, D_t).$$

We assume that the bank's initial capital structure is given, i.e., at time t_0, the bank has E_{t_0} from shareholder capital an D_{t_0} from deposits. Total funding comes from these two sources. As time evolves, shareholder capital increases if the bank makes a profit; otherwise it will decrease. Deposits evolve according to the stochastic volume method explained in Section 2.5. As a result, we assume that management does not fully control deposits and bank runs are possible.

As presented in Table 2, let us denote by $\overline{\alpha_0}$, $\overline{\alpha_1}$, $\overline{\alpha_2}$ and $\overline{\alpha_3}$ the constant proportions of the funding allocated to cash, loans, bonds and stocks. By $\overline{\alpha} = (\overline{\alpha_0}, \overline{\alpha_1}, \overline{\alpha_2}, \overline{\alpha_3})$ we represent the vector of constant proportions allocated to each asset class.

We denote by $Fail(\omega)$ the Bernoulli random variable that describes if the bank fails under trajectory ω; in this case it will be equal to 1, otherwise it will be zero (the bank survives). This random variable will be dependent on the percentage allocations referred to above, but for the sake of clarity of notation, we will not specify this dependence.

We describe the model for the balance sheet. As we go along the equations, we omit the dependency on ω on each random variable for the sake of notation, except when this dependency is needed. For example, the evolution of shareholders' capital $E_t(\omega)$, which depends on the trajectories for the risk factors, is denoted by E_t.

- The *balance sheet equation*:

  Since total funding (equity plus deposits) can be allocated to cash, loans, bonds and stocks, we denote by α_0, α_2 and α_3 the dollar or euro amounts in cash, bonds and stocks at time t. α_1 denotes the dollar or euro amount in new loans at time t. The fundamental balance sheet equation

  $$\alpha_0 + (L_t + \alpha_1) + \alpha_2 + \alpha_3 = E_t + D_t \tag{1}$$

  has to hold (i.e., assets are equal to liabilities plus shareholders' equity). Therefore we set the dollar or euro amounts as

  $$\alpha_0 = \overline{\alpha_0}(E_t + D_t), \quad \alpha_2 = \overline{\alpha_2}(E_t + D_t), \quad \alpha_3 = \overline{\alpha_3}(E_t + D_t),$$

  and

  $$\alpha_1 = \max(\overline{\alpha_1}(E_t + D_t) - L_t, 0).$$

  The amount of new loans α_1 will be positive only when the desired loans determined by $\overline{\alpha_1}$ exceeds the legacy loans in the books. In the case that α_1 is zero, i.e., legacy loans exceed the desired loans, α_0, α_2 and α_3 are proportionally adjusted accordingly to satisfy the fundamental balance sheet equation (1). We are assuming that loans are not callable or transferable.

- A bank needs to be compliant with the common equity tier 1 ratio (CET1) limit T_l^1. This is specified by the following restriction:

  $$\frac{E_t}{\max(w_L L_t + w_S \alpha_3, 0.01)} > T_l^1, \tag{2}$$

  where w_L and w_S are the risk weights assigned to mortgages and stocks by regulators, respectively. Cash and bonds have zero risk weights, so they do not show in the denominator. We also specify a tolerance of 0.01 in the minimum risk weight to avoid explosions in the CET1 ratio, which is infinite in case the bank invests all the funds in cash or Treasuries. We assume that the bank is compliant with the common equity tier 1 limit, and that it fails if it reaches this limit, i.e., we set $E_t = 0$ and

  $$Fail(\omega) = 1.$$

  This can be seen as an *solvency-driven failure*. We also need to specify *liquidity-driven failures*, which occur when the bank does not have enough liquid securities to compensate for outflows. Since in our setting all the assets but loans are liquid, this occurs when there is a shortage of liquid assets, i.e., when the loan balance is lower than total liabilities:

$$D_t + E_t \geq L_t. \tag{3}$$

If the previous restriction is not fulfilled the bank fails and we assume that $E_t = 0$ and

$$Fail(\omega) = 1.$$

As we can observe, by flagging both liquidity-driven and solvency-driven failures we are able to integrate solvency and liquidity risks into a single framework. As we will see later, we will use as a risk measure the probability of failure or default of the bank, that evaluates the strength of the balance sheet in a single measure. The probability of default is possibly the most important indicator to make such an assessment and is widely used by ratings agencies.

- I_t is the income from loans in the books at time t, which will be positively influenced by new loans:

$$I_{t+1} = (I_t + r_t\alpha_1)(1 - p - \lambda_{t+1}) \tag{4}$$

where p is the amortization rate.

- The total loans L_t are given by legacy loans (loans in the books) at time t plus new loans, and their evolution in time is influenced by amortizations and defaults:

$$L_{t+1} = (L_t + \alpha_1)(1 - p - \lambda_{t+1}). \tag{5}$$

- $Dur(y_t)$ is the modified duration of the par ten year bond (sensitivity to interest rates), which can be approximated by:

$$Dur(y_t) = \frac{1}{y_t(y_t+1)^{10}} - \frac{1}{y_t}. \tag{6}$$

- I_t^T is the total income on the assets, which depends on the income on legacy loans I_t, the income on new loans $r_t\alpha_1$, the return on cash $\alpha_0 f_t$, the total return on stocks $\alpha_3 S^*_{t+1}$, and the total return on bonds $\alpha_2(y_t + Dur(y_t)m^y_{t+1})$ (given by the coupon plus the change in the bond prices, using the modified duration). The total income is also negatively affected by credit losses $\lambda_{t+1}(L_t + L_t^{new})$ and the interest rate charged on deposits $-d_tD_t$:

$$\begin{aligned} I^T_{t+1} = I_t + r_t\alpha_1 + \alpha_0 f_t - d_tD_t - \lambda_{t+1}(L_t + L_t^{new}) \\ + \alpha_3 S^*_{t+1} + \alpha_2(y_t + Dur(y_t)m^y_{t+1}). \end{aligned} \tag{7}$$

- The variable c_t accounts for operating costs that depend on the size of the balance sheet, which is fully funded by equity and deposits:

$$c_{t+1} = cA_t = c(E_t + D_t), \tag{8}$$

where c is the cost factor to the balance sheet size.

- Earnings are given by the total income minus the operating costs:

$$e_{t+1} = I^T_{t+1} - c_{t+1}. \quad (9)$$

- Div^R_t represents the accumulated dividends that shareholders receive, that we assume that are given as constant payout ratio R_p in terms of earnings. Since we need to keep track of accumulated dividends, we assume that shareholders reinvest them at the cash rate:

$$Div^R_{t+1} = (1+f_t)Div^R_t + Div_{t+1}, \quad \text{with} \quad Div_{t+1} = \max(R_p e_{t+1}, 0). \quad (10)$$

- E_t is the bank's equity or shareholders' capital, which increases with the amount of earnings that is not distributed through dividends:

$$E_{t+1} = \max(E_t + e_{t+1} - Div_{t+1}, 0). \quad (11)$$

- At the final year of simulation, and for all paths, we calculate the return Ret, it is based on the expectation $\boldsymbol{E}$ of equity plus reinvested dividends:

$$Ret = \left(\frac{\boldsymbol{E}(E_T + Div^R_T)}{E_0} \right)^{\frac{1}{T}} - 1,$$

  with T the number of years in the simulation.

- As we anticipated, we use the probability of failure as our central risk measure.

Our unified risk measure allows us to differentiate the interest rate risk between fixed-rate assets at book value (in our case fixed-rate mortgages) and at fair value (in this setting bond securities), unlike change in EVE, which is used in the interest rate risk in the banking book (IRRBB) regulation for medium to long-term interest rate risk. First, let us elaborate on the sources of interest rate risk for assets at book value and assets at fair value. The interest rate risk for assets at fair value is essentially the risk of devaluations in these assets due to a rate shock (for instance an increase in interest rates). The interest rate risk for assets at book value is different: it comes from a long-term potential loss in net interest margin on these assets in case there is an increase in funding costs. As shown by [8] using a simulation model, the interest rate risk for assets at fair-value is typically much higher than the interest rate risk for assets at book-value.

In order to assess medium and long-term interest rate risk, the IRRBB regulation uses essentially changes in economic value of equity (EVE) and duration-based measures. These measures are price-based and are suitable for assets and liabilities at fair value; in our view, these measures are not suitable for assets and liabilities at book value, since these are not exposed to price fluctuations. To the best of our knowledge, the IRRBB measures do not differentiate between assets at book value and at fair value. In other words, the IRRBB risk measures (such as change in EVE after a rate shock) for an asset are the same irrespectively of being classified at fair value or book value.

Many assets and liabilities are not measured at fair value. Our approach uses the accounting treatment instead of measuring interest rate risk under a price-based measure. This enables us to calculate the interest rate risk for assets at fair value (such as bond securities) and at book value (such as loans).

Algorithm 2 gives a sketch of the procedure to evaluate the bank balance sheet given a vector of allocation portions $\bar{\alpha}$ and a specific scenario ω. Thus, for each instant $t \in \{0, \ldots, T-1\}$ of trajectory ω (step 2), the auxiliary values described in the formulas (2) to (11) in Section 3 are calculated to obtain the final values E_{t+1}, Div^R_{t+1} and $Fail$ (step 3). The simulated values in the last period correspond to the output of Algorithm 2 (E_T, Div^R_T and $Fail$) and they will serve as input for the Algorithm 3. This procedure determines the average risk-return measures under all the generated scenarios for the bank balance sheet for a given $\bar{\alpha}$, i.e., the average return capitalized over the period under analysis and the probability of default. The complexity order of Algorithm 2 is $O(T)$ and for Algorithm 3 is $O(numPaths\ T)$.

Algorithm 2 Bank balance sheet simulator

1: **procedure** $(e,\ d,\ f)$ = BANKBALANCESHEET($\bar{\alpha}$, ω)
2: **for** $t = 0$ to $T-1$ **do**
3: Compute E_{t+1}, Div^R_{t+1} and $Fail$ using the formulas in Section 3;
4: $e = E_T$; $d = Div^R_T$; $f = Fail$;

Algorithm 3 Computation of the risk-return measures for the bank balance sheet

1: **procedure** (FAIL, RET) = BANKPERFOMANCE($\bar{\alpha}$)
2: **for** $k = 1$ to $numPaths$ **do**
3: Load the k-th path scenario as ω^k;
4: $(e^k,\ d^k,\ f^k)$ = bankBalanceSheet($\bar{\alpha}$, ω^k);
5: $Fail = (\sum_{k=1}^{numPaths} f^k)/numPaths$; {probability of default}
6: $Ret = \left(\frac{\sum_{k=1}^{numPaths}(e^k+d^k)/numPaths}{E_0}\right)^{\frac{1}{T}} - 1$; {average return}

4 Optimization of the bank strategies

Since $Fail$ is a random variable with Bernoulli distribution, then the expected value for this variable corresponds to the probability of failure, that is, $Risk = E(Fail) = P(Fail(\omega) = 1)$. This key indicator summarizes in a single number the financial strength of the bank, so that it accounts for all the risks simultaneously. As we mentioned before, this probability computes the likelihood of the bank defaulting by both solvency-driven or liquidity-driven shocks. The importance of this measure also stems from its wide use by ratings agencies.

We would also like to note that other measures could be possible. For instance, one could use an average maximum drawdown measure (by computing the maximum drawdown on each of the trajectories and averaging these numbers), or an expected shortfall on losses. On these computations, one would assume that a liquidity-driven failure would amount to a total loss; this would also allow the inclusion of liquidity risk into the single-measure framework. However,

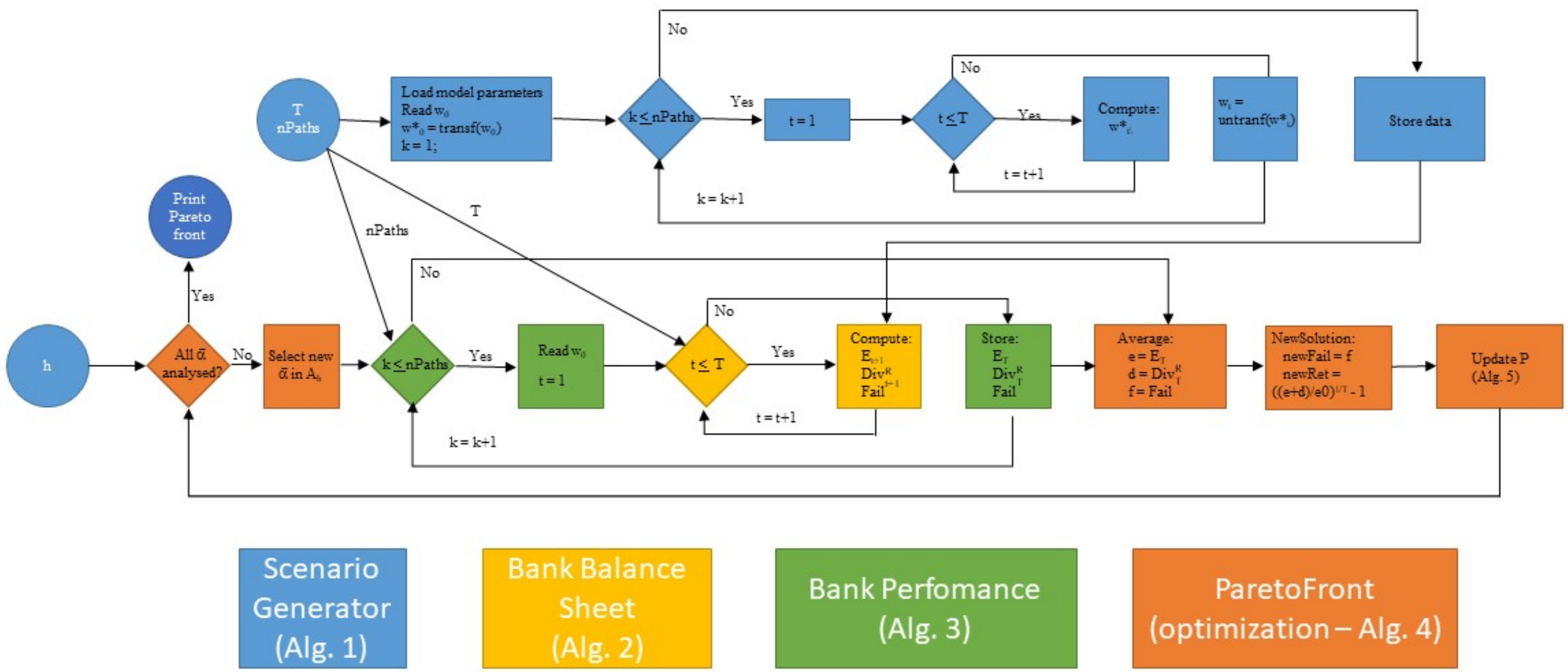

Figure 2: Fluxogram

we chose the probability of default, due to its great importance and widespread use in Banking and Finance.

In this way, the problem can be formulated as follows:

$$\begin{aligned} \max_{\bar{\alpha}} \quad & E(Ret) \\ \min_{\bar{\alpha}} \quad & E(Fail) \\ s.t. \quad & (1)-(11) \end{aligned}$$

We denote by A_h the discretization with step length h of the set of admissible solutions. The model was tested using the step $h = 0.02$ for all the decision variables, making $\bar{\alpha}_i \in \{0, 0.02, 0.04, \ldots, 1\}$, $i \in \{0, 1, 2, 3\}$, satisfying the additional constraint $\sum_{i=0}^{3} \bar{\alpha}_i = 1$. More than 23000 admissible solutions were analyzed and the points at the Pareto frontier were selected. A solution $\bar{\alpha}$ belongs to the Pareto frontier if there is no admissible solution β such that

$$E(Ret(\beta)) \geq E(Ret(\bar{\alpha})) \text{ and } E(Fail(\beta)) \leq E(Fail(\bar{\alpha})),$$

where one of the inequalities is strict.

Algorithm 4 gives us the general sketch of the optimization routine to compute the Pareto Front. It starts by calling the *scenarioGenerator* routine to generate all the scenarios which will be used in the simulation-optimization procedure (step 2). Next, the set of admissible solutions, A (step 3), is discretized considering a step h to build $\{\bar{\alpha} \in A : \bar{\alpha}_i/h \in \mathbb{N}_0\}$ (step 4). Therefore, each solution in A_h (step 6) is evaluated (step 7) and the Pareto front is updated (step 8).

Algorithm 5 is used to update the Pareto front, where P is the current Pareto front set and $(newFail, newRet)$ is the risk and return of a new solution in A_h which is under analysis. If in the current Pareto front P there exists some $(Fail_\ell, Ret_\ell)$ that dominates $(newFail, newRet)$ - step 2, i.e., a point with better risk and return, the new solution is discarded (step 3). Otherwise, the new solution is included in P and this set is updated keeping the solutions with better risk ($\bar{P}_1$) or better return ($\bar{P}_2$) – step 5-7. As $\#A_h = O(h^{-3})$ and in the worst-case all the elements of A_h produce new elements in P, the worst-case complexity of Algorithm 5 is $O(h^{-3})$ and consequently the worst-case complexity of Algorithm 4 is $O(h^{-6}\ numPaths\ T)$. We would like to emphasize that the worst-case scenario is very unrealistic and that the average-case complexity of Algorithm 5 should be much smaller than $O(h^{-3})$.

Algorithm 4 Computation of the Pareto front for the risk-return measures

1: **procedure** P = ParetoFront($numPaths$, T, h)
2: scenarioGenerator($numPaths$, T); {generates the scenarios}
3: $A = \{\bar{\alpha} : \bar{\alpha}_i \geq 0 \wedge \sum_{i=0}^{3} \bar{\alpha}_i \leq 1\}$; {set of admissible solutions}
4: $A_h = \{\bar{\alpha} \in A : \bar{\alpha}_i/h \in \mathbb{N}_0\}$; {discretization of A with step length h}
5: $P = \emptyset$; {actual Pareto front in lexicographic order}
6: **for each** $\bar{\alpha} \in A_h$ **do**
7: $(newFail,\ newRet)$ = bankPerfomance($\bar{\alpha}$);
8: P = updateParetoFront(P, $(newFail,\ newRet)$);

Algorithm 5 Update the Pareto front with the new solution

```
procedure Pnew = UPDATEPARETOFRONT(P, (newFail, newRet))
    if ∃(Fail_ℓ, Ret_ℓ) ∈ P: Fail_ℓ ≤ newFail ∧ Ret_ℓ ≥ newRet then
        Pnew = P; {the new solution is not efficient}
    else
        P̃_1 = {(Fail_ℓ, Ret_ℓ) ∈ P : Fail_ℓ < newFail};
        P̃_2 = {(Fail_ℓ, Ret_ℓ) ∈ P : Ret_ℓ > newRet};
        Pnew = P̃_1 ∪ {(newFail, newRet)} ∪ P̃_2;
```

If additional precision is required or additional variables are included in the problem, the points generated with this strategy can be used as initial solutions to compute local optimizers using non-continuous and non-differentiable methods such as direct search (for a reference, see [19]). In our case, we did not use these methods because the number of variables is low, the solutions have the needed precision from a practical standpoint, and the objective function stems from a simulation process. Specifying sharp tolerances in the optimization would render an exercise that would still have the margin of error that comes from the objective function.

5 Computational results

In this section, we conduct several computational results on the methodology that we propose. We will start by conducting univariate tests, where we examine changes in only one asset class, so that we can better understand the risk and return profiles associated with each of these classes in our unified and multiperiod framework.

We will then proceed to the analysis of efficient frontiers, assuming different economic environments and prospective returns. As we will see, given the low prospective returns associated with the last few years, in an environment of low rates, the corresponding Pareto fronts will result lower prospective returns but also higher risk profiles. The model will also prefer book-value assets, in this case mortgages, given the lower volatility when compared to fair-value assets, whose price changes severely create balance sheet volatility.

A third subsection will evaluate how the results change when in the presence of more conservative leverage ratios. We will see that equity investments may make sense for banks with lower leverage which choose to gain more risk.

Finally, we will address minimum risk portfolios. We will observe a revealing but intuitive finding: unlike textbook treatments of asset allocation, the lowest risk portfolio is not full investment in cash. In fact, full allocation to cash can be very risky, as the bank will not generate enough return to compensate for operating costs, thus facing a likely failure.

In the whole section, we will see that risk is highly dependent on the return profile. If a bank generates a steady return, it will be better capitalized and thus the likelihood of failure will decrease.

5.1 Univariate tests

We start by analyzing the univariate effect of changing the asset allocation on only one asset class in Figures 3 - 5 . For example, an allocation of 40% to loans assumes that the remainder is allocated to cash. Therefore, in the graphs we test the effect of substituting cash by other asset classes. The simulation parameters are described in Table 8.

Mortgage risk weight w_L	0.35	Costs to balance sheet c	0.015
Stocks risk weight w_S	1	Payout ratio R_p	0.5
Tier 1 ratio limit T_l^1	0.1	Initial equity E_0	$0.05D_0$
Amortization ratio p	0.1		

Table 8: Baseline simulation parameters. We specify the risk weights given in the Tier 1 capital ratio, the Tier 1 limit ratio and the amortization proportion of loans. Annual costs represent 1.5% of the balance sheet. We assume that the dividend payout ratio is 50%, whereas the initial equity base is 5% of the initial deposit volume.

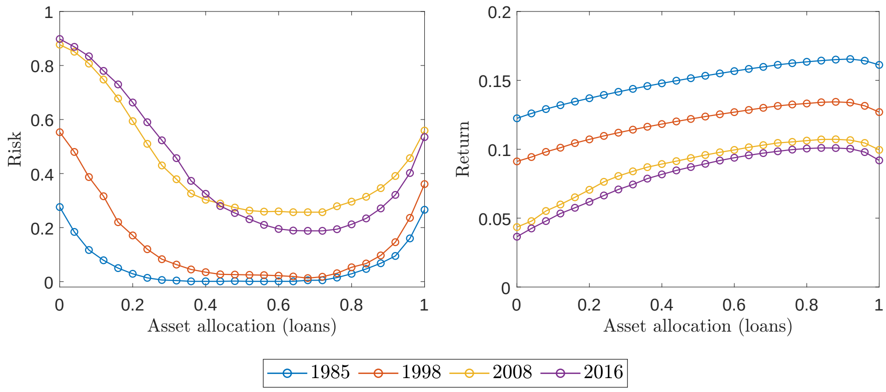

Figure 3: Evolution of risk and return with loans, considering as initial points the interest rates, charge-offs and stock prices in 1985, 1998, 2008 and 2016. We test the univariate effect on risk and return of changing the allocation to loans and replacing them with cash.

We conduct the tests assuming as initial environments those of the end-of-year for 1985, 1998, 2008 and 2016, which correspond to different and varied periods in the sample. For each of these periods, we take the interest rates, the charge-off rates and the stock price variables as the initial points in the simulation.

When looking at the graphs, one can immediately observe that the functions are not differentiable, not convex and not continuous (in the case of the returns).

The eighties were associated with high interest rates, that subsequently fell, along with sharp rises in the equity markets. By the end of the 1990s, equity markets were severely overvalued. 2008 is the year of the collapse of Lehman Brothers, so it's also an important point in the sample.

When examining the risk results, one can immediately see that loans tend to be much less risky than equities and bonds. This finding is revealing of how the accounting treatment impacts very considerably the risk profile. Whereas loans are classified at book value, and therefore market prices do not influence their Profit and Loss (P&L), in our setting we are assuming that bonds and equities

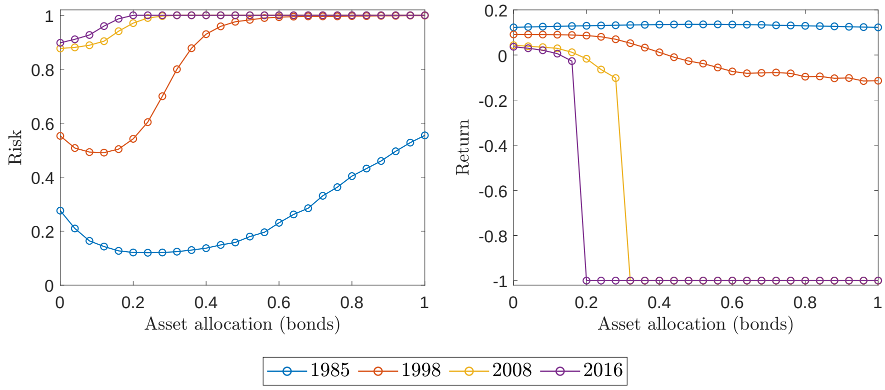

Figure 4: Evolution of risk and return with bonds, considering as initial points the interest rates, charge-offs and stock prices in 1985, 1998, 2008 and 2016. We test the univariate effect on risk and return of changing the allocation to bonds and replacing them with cash.

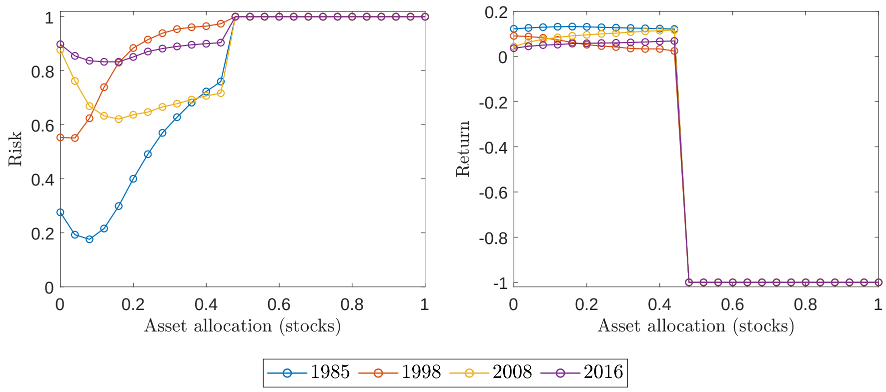

Figure 5: Evolution of risk and return with stocks, considering as initial points the interest rates, charge-offs and stock prices in 1985, 1998, 2008 and 2016. We test the univariate effect on risk and return of changing the allocation to stocks and replacing them with cash.

are classified at fair value, so that the variation in market prices impacts the earnings and the capital on the bank. This variation in prices accounts for the much higher risk profiles of bonds and equities, i.e., securities in general. It is also a very clear indication of the volatility that fair-value accounting induces in general.

Another feature that we observe is the risk profile of loans which has a parabola-like shape. When the allocation to loans is zero, the bank is essentially putting all the resources into cash. This may not be a problem in periods of high rates such as 1985, but in a context of ultra-low interest rates such as recent years, the bank is possibly earning a very low interest margin when considering the rates on cash against the rates that the bank pays on deposits. Particularly in times of low rates, these ultra-low net interest margins far from compensate the operating costs associated with the bank. Therefore, it is no surprise that

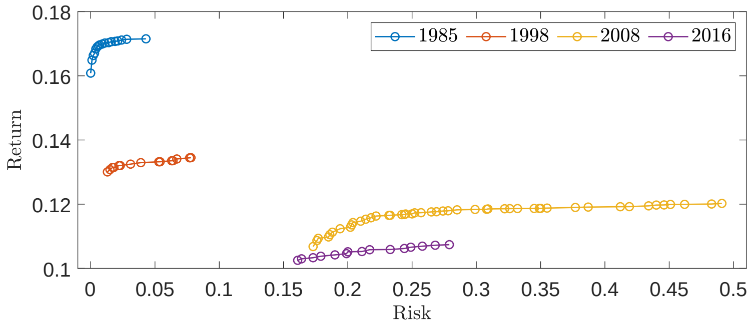

Figure 6: Pareto front obtained with the proposed model, for the different economic environments of 1985, 1998, 2008 and 2016. As the prospective returns have been decreasing over the years, the returns on the balance sheet tend to be lower as the years advance.

for the 2016 environment, putting all the resources into cash can be extremely risky and dictates the almost certain failure of the bank. Also, putting all the resources in loans can be a risky strategy from the point of view of the bank: first, loans have credit risk which can be higher in times of crises; second, mortgages are not liquid and the bank may face failure because of deposit runs. Both these features are captured in our model.

The return profiles are heavily influenced by the likelihood of default by the bank. If the bank faces default, then it will not be able to generate more returns. As we can see, both bonds and equity show a cut-off point after which failure is certain and therefore returns are very low from then on.

5.2 Efficient frontiers

In this section we analyze efficient frontiers and the corresponding allocations, so that we can better understand their shape and also the properties of the allocations. In Figure 6 we plot the efficient frontiers for the four different economic environments that we have mentioned above. In Figures 7 - 10 we document the allocations associated with the different points in the efficient frontier, along with the risk and return measures. The last two bars in each graph show the comparison of the risk and return measures with common heuristic asset allocation strategies (to be analyzed later).

As a first observation, we notice that efficient frontiers tend to be upward sloping, which is not surprising. If risk is relaxed, then the bank can achieve a better return.

We also observe in general that prospective returns have been decreasing over the years. The eighties were characterized by higher interest rates and higher dividend yields, which in turn influenced the returns on the banks.

In general, we observe that the model suggests high allocations to mortgages. As we mentioned, the accounting classification here plays an important part. Mortgages are classified at book value rather than fair value, making them ideal instruments for mitigating balance sheet volatility. Stocks and bonds induce much more volatility in the balance sheet. As a consequence, one can observe that for most years the model selects an almost zero amount to Treasury bonds

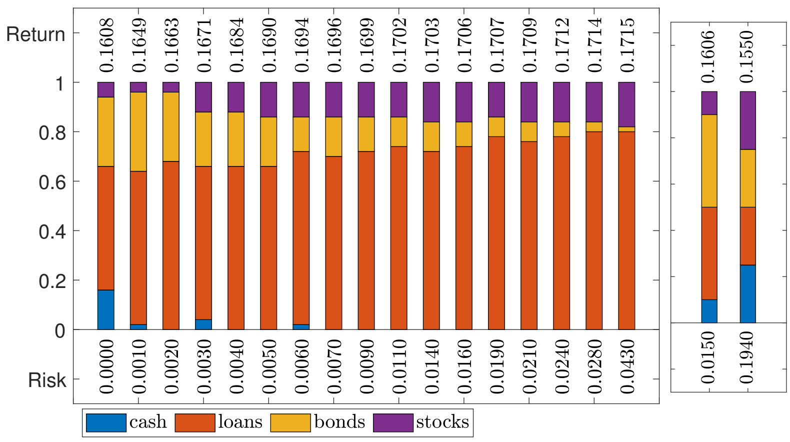

Figure 7: Pareto optimal solutions obtained with the proposed model (1985). The two bars on the right-hand side describe the risk and the return for two heuristic allocations: 40% loans/40% bonds and equal weight.

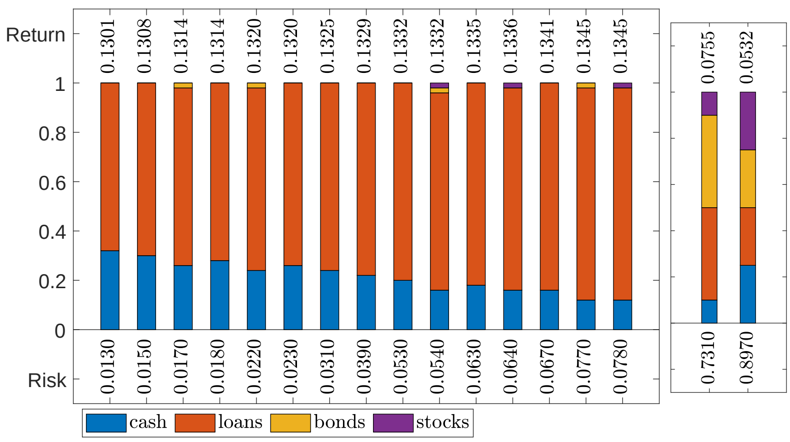

Figure 8: Pareto optimal solutions obtained with the proposed model (1998). The two bars on the right-hand side describe the risk and the return for two heuristic allocations: 40% loans/40% bonds and equal weight.

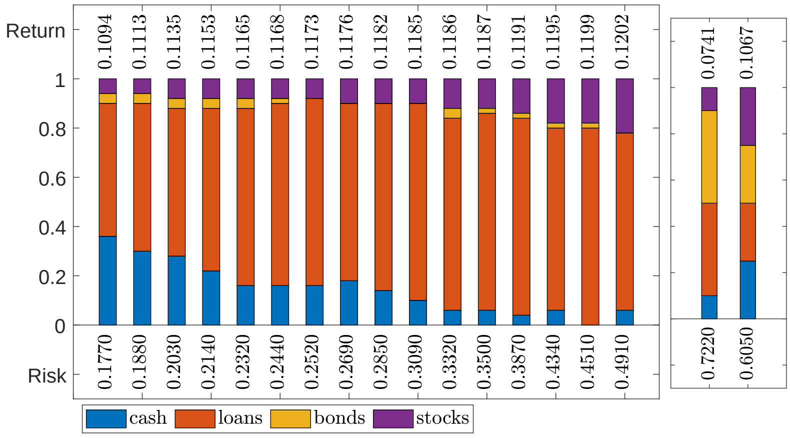

Figure 9: Pareto optimal solutions obtained with the proposed model (2008).The two bars on the right-hand side describe the risk and the return for two heuristic allocations: 40% loans/40% bonds and equal weight.

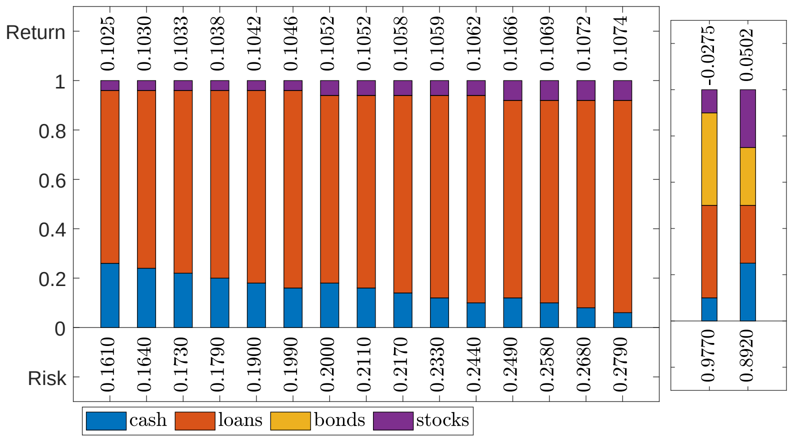

Figure 10: Pareto optimal solutions obtained with the proposed model (2016).The two bars on the right-hand side describe the risk and the return for two heuristic allocations: 40% loans/40% bonds and equal weight.

because of these fluctuations.

The allocation to stocks seems to be highly dependent on the economic environment. It is no coincidence that, in a period of market bubble such as 1998, when dividend yields were at historically low levels of 1.4%, the model selects almost no stocks in the portfolio. In contrast, using the data of the year of 2008, i.e., the year of Lehman Brothers' collapse, the model would have chosen a relatively high allocation to stocks, since the stock devaluations in 2008 caused an increase in prospective returns on stocks by the end of that year.

In general, the model always leaves a considerable stock of liquid assets, so that it can avoid failure due to deposit run-offs.

Finally, we clearly observe the domination of efficient frontier strategies versus two commonly adopted heuristic strategies. To compare the efficient frontier with heuristic strategies, we use an equal-weight strategy that allocates one quarter to each asset class, and a strategy that allocates 40% to loans, 40% to Treasury bonds, 10% to cash and 10% to stocks. We can clearly observe the suboptimal performance of these strategies.

5.3 Tests with more conservative leverage levels

In our previous tests, we assumed that equity corresponds to 5% of the deposit base, which corresponds to a bank that is 20 times leveraged. This is a very high leverage level, although common in practice. In this section, we evaluate how the results change as a function of the leverage of the bank.

In Figure 11, we analyze how the univariate tests behave when changing the leverage levels, whereas in Figures 12 - 14 we show the Pareto frontier and the associated asset allocations.

In the univariate tests, we can still observe the parabola-shaped effect of risk, particularly on loans. We can also observe that, for each asset class, risk is also increasing when leverage is higher, which is also not surprising. The returns also show revealing patterns. As we have seen before, there is a cut-off point for stocks and bonds from which the return is -100%. What we can clearly observe is that this cut-off point increases when leverage decreases, revealing again that lower leverage is associated with the lower certainty of having bankruptcy.

The Pareto frontier has also some very interesting features. When we compare the stock allocation in our baseline leveraged bank in Figure 10, with $E_0 = 0.05D_0$, to the lower leverage levels in Figures 13 - 14, we observe that lower leverage produces a higher allocation to stocks for the same level of risk. This is quite intuitive. When the leverage is lower, risk decreases, if all else is constant. Therefore, if a bank chooses to decrease its leverage, it can still maintain the same level of risk if it increases the exposure to stocks.

Looking at Figure 12, we can observe that, when leverage decreases, expected returns decrease, but also the point with the least risk decreases. The minimum risk allocation will be addressed below.

5.4 Portfolios with minimum risk

In this section, we analyze the portfolios with minimum risk. In typical mean variance portfolio problems, when assuming a riskless asset, the minimum risk portfolio is the full investment in riskless cash.

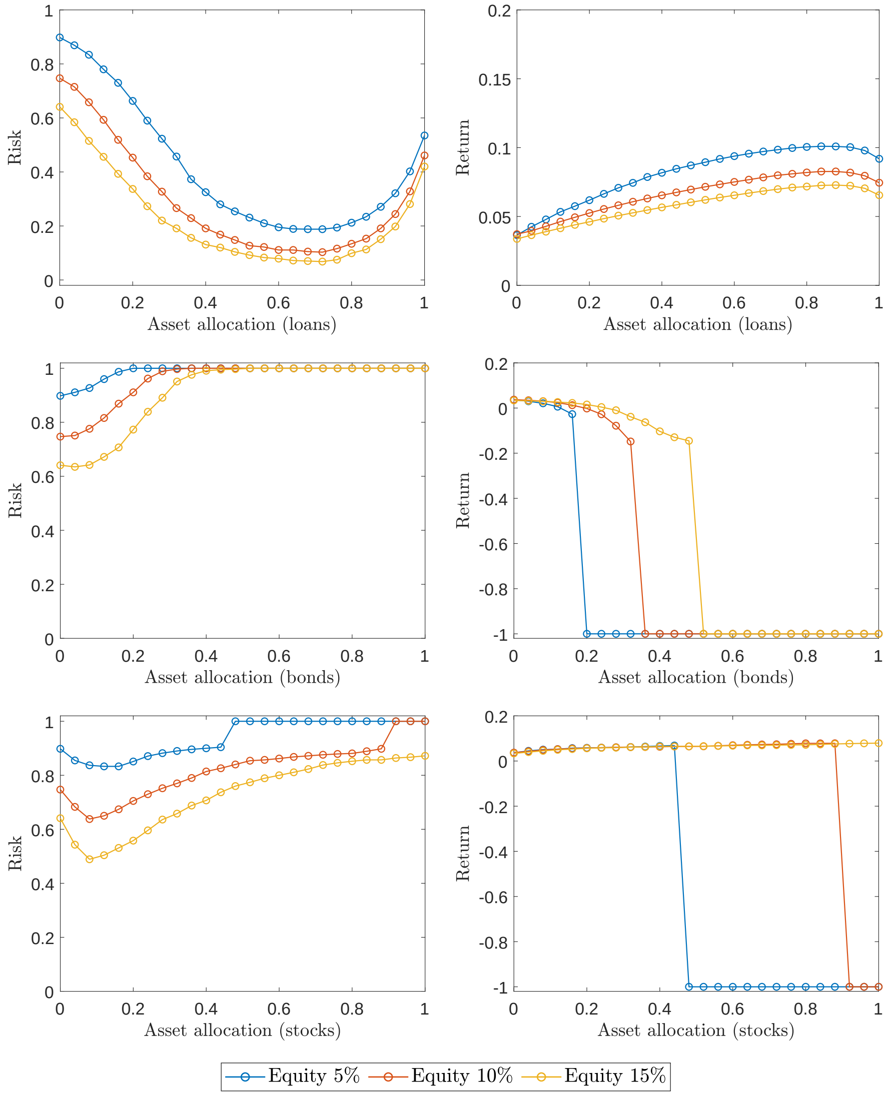

Figure 11: Evolution of risk and return with loans, bonds and stocks for the 2016 economic environment with different leverage levels, at $E_0 = 0.05D_0$, $E_0 = 0.1D_0$ and $E_0 = 0.15D_0$. We test the univariate effect on risk and return of changing the allocation in each asset class and replacing it with cash.

As we have seen in the sections before, full investment in cash is not riskless in our setting, because operating costs will increase the probability of a loss, therefore increasing risk.

We calculate the minimum risk portfolios in Figure 15, assuming that the ratio of equity to deposits is equal to 5%. First we observe that the allocation to cash is different from 100%. Also, mortgage loans represent a significant amount of the allocation, given their low risk profile when compared to bonds and stocks. As we have mentioned before, the low risk profile associated with mortgages is also linked to its accounting classification: since mortgages are accounted at

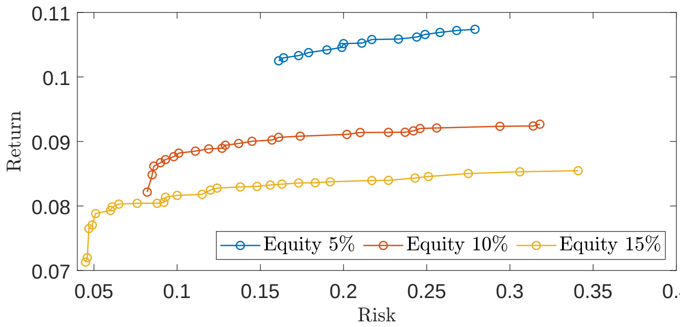

Figure 12: Pareto fronts, for the 2016 economic environment, with different leverage levels, at $E_0 = 0.05D_0$, $E_0 = 0.1D_0$ and $E_0 = 0.15D_0$. One observes that higher leverage produce higher returns but also higher risk.

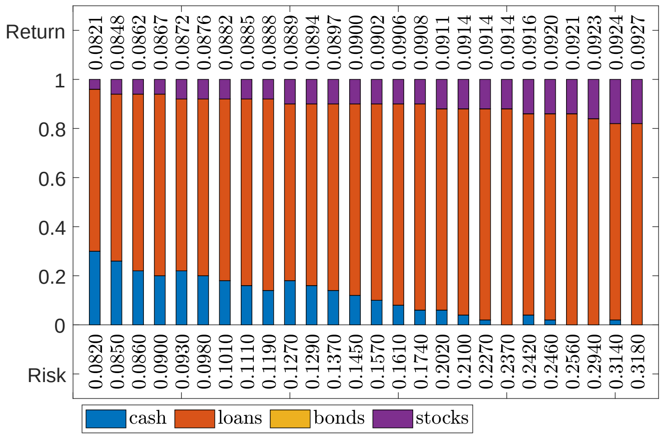

Figure 13: Pareto optimal solutions obtained with the proposed model, considering 2016 as the initial economic environment and $E_0 = 0.1D_0$.

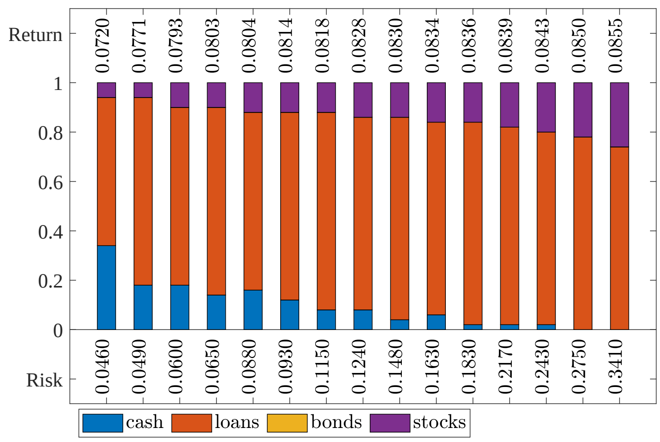

Figure 14: Pareto optimal solutions obtained with the proposed model, considering the 2016 as the initial economic environment and $E_0 = 0.15D_0$.

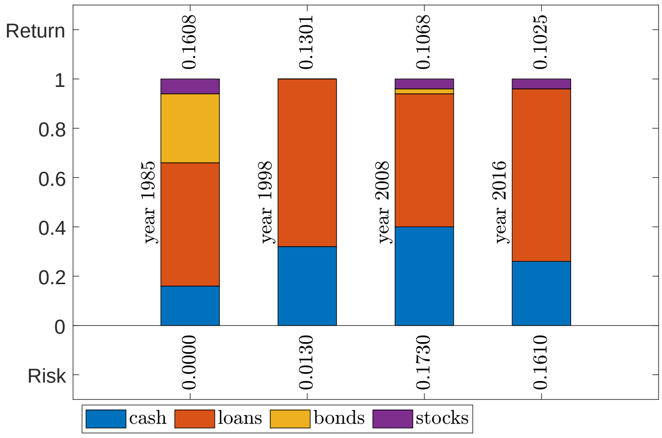

Figure 15: Minimum risk solutions, assuming different initial environments in 1985, 1998, 2008 and 2016. Due to the decrease in prospective returns over the years, as time progresses the balance sheet return is lower and risk is higher.

book value, this asset class is not exposed to severe market fluctuations as in the case of bonds and stocks.

Year	Equity to Total Assets	Earnings to Total Assets
1985	0.2598	0.0415
1998	0.1598	0.0238
2008	0.1174	0.0162
2016	0.1087	0.0148

Table 9: Average earnings to total assets and equity to total assets for the minimum risk portfolios using the simulation model for a period of 30 years, assuming different initial points corresponding to the economic environments in 1985, 1998, 2008 and 2016. We can observe that, for more recent years, the results of the simulation show lower earnings to assets and lower shareholders' equity, due to the decrease in prospective returns in the last two decades.

As we can see from Figure 15, the minimum risk varies very significantly depending on the year and the economic environment. The returns simulated by our scenario engine are much higher when the initial points are taken from 1985 than in the latter years, as observed in Table 9. This means that, for the scenarios generated based on 1985, a bank will be extremely profitable, with average earnings to total assets of 4.2% and very rapidly be capitalized, as one can see in Figure 16. When looking at the evolution of the equity to total assets, we see that the simulations that start in 1985 rapidly will generate extremely well capitalized banks, due to extremely high returns. Well capitalized banks will be less risky and default less.

On the other hand, for a bank that starts in 2016, the average earnings to total assets resulting from the simulation is 1.5%; as a consequence, the bank will not be as capitalized and will be more vulnerable to economic shocks and have more risk. Summarizing, the low risk that one observes in 1985 is essentially driven by higher prospective returns in 1985. This link between the return and

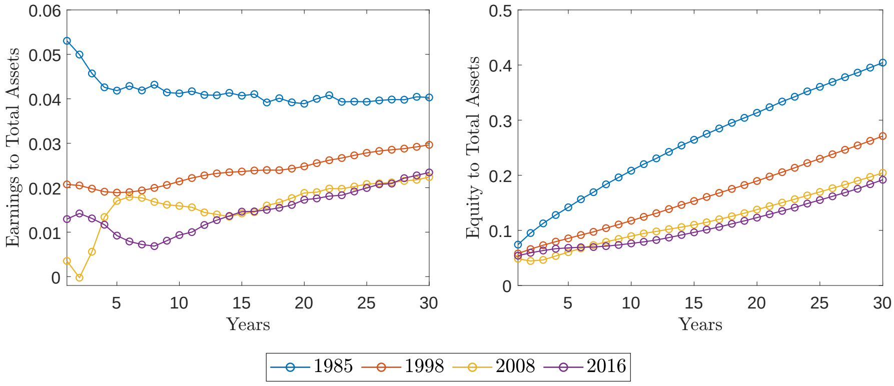

Figure 16: Evolution of earnings to total assets (on the left) and equity to total assets (on the right), considering four different initial economic environments.

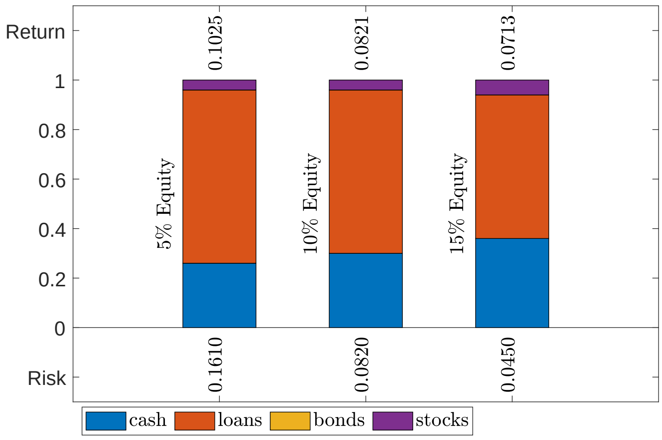

Figure 17: Solutions with minimum risk obtained with the proposed model for the year 2016 with different leverage levels.

risk in our model is one of the differentiating factors of our research: a bank that produces solid returns is less risky as well, because very quickly will be in a very well capitalized position.

In Figure 17, we change the equity level to assess the level of minimum risk. We can readily observe, not surprisingly, that the higher the capitalization the less risky the bank will be.

6 Conclusion

In this paper, we developed a unified framework for bank strategic asset allocation, encapsulating all the risks into one single measure, the probability of failure of the bank. This single measure, which is evaluated by ratings agencies, gives a single score for the financial strength of the bank, and avoids the silo-based approach for risk measurement which has been present in banks. In fact, in practice, risks are evaluated separately and then aggregated in an *ad* *hoc* fashion.

We built upon the risk factor scenario generation framework of Birge-Júdice [2] and Costa, Faias, Júdice and Mota [21] to develop a simulation methodology for the balance sheet, from which we calculated return and risk measures. As a consequence, we built a unified framework for evaluating risk and return, as it evaluates simultaneously liquidity and solvency risks under a single measure.

We subsequently formulated the optimization model. The optimization problem is non-continuous, non-differentiable, non-convex, which seemed a drawback at first. We also were interested in obtaining global, not local, optima. However, given the structure of the problem, and the required tolerances, we used a grid search to determine the Pareto fronts. The grid search on the possible combinations (within a certain tolerance), generated very intuitive solutions.

The solutions obtained by this method were good approximations for global optimizers, avoiding the convergence of the algorithms to local minimizers that may be far away from global solutions. We also argued that more accurate solutions could be obtained via multi-objective optimization algorithms [35, 54] that could be used starting from the referred solutions. In a practical context, however, the solutions with more accurate precision would not add value, given the tolerances that are needed, and the errors associated with the objective function, which is obtained by simulation.

The allocations given by the Pareto fronts generate a considerable portion in loans, given the high returns and no market fluctuations associated to the valuation at book-value. Fair-value assets, such as equity and Treasury bonds are much more volatile and thus the optimizer generates a lower allocation to these asset classes.

One critical feature of our model is that risk is dependent on return. This is also a critical feature evaluated by rating agencies. In fact, under our framework, if a bank generates returns in good years, it will become better capitalized and thus less risky. A solid income stream is a guarantee of low risk for any bank. Under this reasoning, and as a result of the lower interest rates witnessed in the past few years, the simulations indicate that under the most recent environment banks are subject to lower prospective returns and higher risk.

We also evaluated minimum risk portfolios. In standard textbooks, the minimum risk allocation would be full investment in cash. In our setting, we incorporate operating costs, so that the minimum risk allocation is not full allocation to cash. In fact, under the current environment of low rates, a bank that completely invests in cash will very likely face failure, as its income will not be sufficient to cover operating costs.

We have also documented the effect of leverage. Leverage makes the bank less riskier, so that the bank can introduce equity in its investments in case it wants to generate higher returns. For a similar level of risk, a bank with lower leverage will allocate more to stocks.

We hope that this framework will be used by academics and practitioners in the areas of risk management, asset-liability management, treasury and strategic planning. It can serve as a management flight simulator that can help boards at banks to have robot-advisory on the management of the balance sheet and the strategic choices. Our model is a first step in this direction. The approach can be used in practice to advise boards at banks on optimal asset allocation, which can be an important input for strategic plans. In this case, the methodology needs to be adapted to the segments, products and data for the bank.

Much research in this field still needs to be done. We point out a few possible directions. In particular, the methodology may be extended to other liability classes, such as repos, as these have played an important role in past financial crises. The model can also be generalized or adapted to other contexts, such as the case of investment banks, where there is typically a significant exposure to derivatives, and therefore to other risk factors, such as commodity price risk, volatility risk and correlation risk.

Finally, the multi-year scenario methodology can be extended to capture the interconnections between the different types of risks. For example, interest rates on mortgages should depend on the default intensity: one could investigate whether banks will tend to price their loans at higher spreads to the Treasury rates in times of crisis. Liquidity outflows from customers and creditors could also relate to funding costs: when there is a liquidity drought, funding costs may rise. These liquidity outflows are also possibly linked to credit risk. In many financial crises, many banks faced simultaneous defaults on their assets and withdrawals from customers and creditors (these tend to become more reluctant to lend money to banks when the balance sheet deteriorates). New research should shed light on all these possible connections.

As highlighted in the risk integration literature, addressing the nonlinear interrelations between risk factors is also of great importance. Given the examples before, the correlation between liquidity outflows, defaults, and funding costs may become higher during crises, showing its nonlinear nature. Undertaking this research will comprise understanding these interactions first and then posit a nonlinear model to explore such interactions. One possible direction is to specify the risk factors under nonlinear vector autoregressive processes or nonlinear time series processes dependent on common macroeconomic factors.

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