ISCTE Business School
INSTITUTO UNIVERSITÁRIO DE LISBOA

REVERSE STRESS TESTING: IDENTIFYING WEAKNESSES TO PREVENT FAILURES

Miguel Oliveira dos Reis Barroso

Dissertation submitted as partial requirement for the conferral of

Master in Finance

Supervisor:

Professor Paulo Viegas de Carvalho, Invited Assistant Professor,

ISCTE-IUL Business School, Department of Finance

July 2020

Abstract

This dissertation uses a methodology for attributing a stock portfolio most likely negative scenarios given a pre-defined loss.

Using an extensive dataset spanning from 2007 through 2019, we calculated stock returns and their sample covariance matrix is estimated to obtain the portfolio Value at Risk (VaR). Due to idiosyncratic risk, we aggregate the returns into their corresponding indices to obtain the systematic component (the one explained by the market) and, afterwards, the Systematic Value at Risk was determined.

Backward induction is then applied. Considering that returns follow a multivariate normal distribution, we derive the main scenario which could lead to the calculated VaR or even to a worst loss – the decision is up to the user. Reverse Stress Testing should be used as a framework, otherwise the risk manager could simply recalculate the VaR for different confidence intervals and investigate the evolution of the corresponding risk factors. Thus, the objective is to find multiple plausible scenarios –not only the most probable one.

Principal component analysis (PCA) is applied to identify additional, less likely scenarios. These scenarios are linked to the basis scenario, which ensures plausibility. The relative likelihood is then defined manually as 0.1, meaning the central scenario is ten times more likely than the less likely one. Consequently, four scenarios were generated along with the calculation of their corresponding likelihoods.

Overall, we identify the most probable loss scenarios for our portfolio given an input loss. Additionally, we explore the methodology further to determine scenarios under market extreme volatility events.

JEL classification:

G28, G32, C13

Keywords: Value at Risk, Stress Testing, Principal Component Analysis

Resumo

Esta dissertação aplica uma metodologia que identifica as perdas mais prováveis de uma carteira de ações, considerando como *input* uma perda definida.

Através da utilização de um extenso conjunto de dados correspondentes ao período de 2007 até 2019, são calculados os retornos das ações e a matriz de variâncias-covariâncias é estimada de forma a obter o *Value at Risk (VaR)*. Devido ao risco idiossincrático, os retornos foram agregados em função dos índices correspondentes, a fim de obter uma componente sistemática, i.e., explicada pelo mercado, procedendo-se ao cálculo do *Systematic VaR*.

Invertendo o processo, e considerando que os retornos seguem uma distribuição normal multivariada, obtém-se um cenário central que dá origem ao *Systematic VaR* calculado, ou caso o utilizador entenda, uma perda superior. Posteriormente, o objetivo passará por encontrar diversos cenários plausíveis – e não apenas o mais provável.

O método *Principal Component Analysis (PCA)* permitirá a obtenção de cenários menos prováveis. Estes encontram-se relacionados ao cenário mais provável através de verosimilhança, o que garante a plausibilidade dos cenários gerados. A verosimilhança relativa é definida manualmente como 0.1, refletindo um cenário central dez vezes mais provável que o menos provável. Assim, foram gerados quatro cenários, juntamente com o cálculo das respetivas verosimilhanças.

Em suma, identificamos os cenários de perda mais prováveis para a carteira em questão, considerando uma perda como *input.* Adicionalmente, exploramos a metodologia de forma a determinar outros cenários em contexto de extrema volatilidade no mercado.

Acknowledgements

Aos meus pais, pelo apoio incondicional em todos os momentos da minha vida.

À Mariana, pelo incentivo e paciência constante.

Aos meus amigos.

Aos meus avós. Onde quer que se encontrem, espero que se sintam orgulhosos pela conclusão desta etapa na minha vida e na pessoa em que me tornei. Um carinho especial à minha avó, que iniciou este percurso comigo mas não assistiu ao seu fim.

Ao meu orientador, Paulo Viegas de Carvalho, pelo voto de confiança e ajuda.

Ao Sérgio, Pedro, Piero e Antoine. Sem vocês, nada disto seria possível.

Glossary and Acronyms

BCBS - Basel Committee on Banking Supervision

EBA - European Banking Authority

EWMA - Exponentially Weighted Moving Average

P&L – Profit and Loss

PCA – Principal Component Analysis

VaR – Value at Risk

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1 Introduction

"There are 10^{10} stars in the galaxy. That used to be a huge number. But it's only a hundred billion. It's less than the national deficit! We used to call them astronomical numbers. Now we should call them economical numbers."

Richard Feynman

Market Risk management plays a paramount role in modern financial institutions. As banks have a huge tone of positions in their trading books, it is crucial to have permanent control over possible deviations of the market and its repercussions to the bank portfolios.

Since the 2008 financial crisis, banking regulators have stricken their rules in terms of risk exposure. The ever-present need of risk analysis inherent to the market has made model creation and development a broad researched theme in both business and academic worlds. The increased demanding requests of regulators, associated with growing complexity of financial products, have resulted in a need for banks to develop fully capacitated frameworks to face such demand. Also, with the widening of capital requirements for Market Risk, banks need to fully understand the outputs of their models so that the risk (and the pricing itself) is also entirely explained.

At the time this dissertation is being written, the so-called *Fundamental Review of the Trading Book* is merely two years away, and with it a set of challenges arise to banks. With tighter commercial margins, risk plays an extensive role holistically. VaR should be used not only as a simple figure to be reported, but as a support to set up the best possible strategy for minimising the risk profile of a certain institution, taking into account the prevailing market situation, being the foundation of a solid limit framework.

Besides the VaR, but based on it, banking regulators have also required banks to perform stress-tests on their portfolios. The intent is to evaluate the impact in a bank's P&L and VaR of a shock in a determined asset class caused by some trigger effect. The scenarios are defined *a priori* and reported to the banking authorities. A comparison is then made, comprising all banks.

Besides the regulators-based guidelines for stress testing exercises, institutions also have their own Stress-Testing programs, in which losses are measured taking into account many likely scenarios: yield curve drops or rises (parallel or not), spread widening or equity price drops – or every scenario previously described plugged together. Even though these conditions may seem possible, they may not be as probable as a scenario the risk manager has never thought of or that may not even be plausible to happen, taking into account the specifications of his portfolios or the current market situation.

With the requirement of the European Banking Authority for banks to perform Reverse Stress Tests, banks will have a way of knowing possible harmful scenarios designed specifically for the portfolio in question, which may then be scaled into an organisational challenge, from a portfolio considerable loss to a possible bank default. The contribution to the field is bifold: it introduces a methodology which allows compliance with regulation requirements and presents itself as a tool to be employed on a daily basis by risk management practitioners.

In the pages that follow, there will be a comprehensive literature review (section 2) of the major writings on Value at Risk and (Reverse) Stress-Testing, pointing out the most used models, how to use them and what procedures should be applied for the creation of a reliable and fully explained model.

Afterwards, in section 3, the database used in this work is presented. This is followed by the hypothesis in which the model is based and the methodology is accomplished. After this, the results of the project are shown, along with an interpretation and discussion. The conclusion of this dissertation (section 6) contains the last remarks, where the major results of our study are reviewed and summarised.

2 Literature Review

In practice, banks use Value at Risk (Risk Metrics, 1996) to report their possible loss over a predetermined horizon of time, with a certain degree of confidence. The model, first developed by J.P. Morgan Chase in 1996 and originally sold to many entities, has undergone many changes throughout the years, as pricing models have evolved and so has computational power. The crisis has also played an immense role in the development of such models. Blind faith in the Gaussian Distribution and correlations are pointed out as possible drivers of model failure, along with a lack of regulation in some aspects of finance and banking in general. Since the Basel II accord (2004), banks are compelled to calculate (Pillar I) and fully disclose their VaR figures (Pillar III), causing literature over Value at Risk topics to rise exponentially.

2.1 Value at Risk

Introduced by J.P. Morgan (RiskMetrics, 1996), Value at Risk (VaR) is a statistical measure of *"the maximum potential change in a value of a portfolio"*, over an indicated time horizon within a specified confidence interval due to market shifts, expressing the expected loss limit according to a chosen confidence interval. It provides a single, summarised, easily understood measure of a possible portfolio loss and is, therefore, a useful tool in Risk Management.

The VaR is dependent on the defined liquidity horizon (holding period). According to Jorion (2006), this liquidity horizon refers to the period of time over which the VaR measures potential losses within the confidence level. A longer horizon provides the potential for larger market moves and hence larger losses.

As Alexander (2008) explains, one necessary assumption generally made in VaR calculations is that portfolio holdings remain constant over the liquidity horizon. This imposes restrictions upon a realistic time horizon and potentially decreases the usefulness of extending the horizon too much given the frequency of inventory turnover and hedging strategies in the trading business, either for banks or hedge funds. In addition to its use for market risk assessment and management, the VaR is also employed in the calculation and reporting of the regulatory capital for Market Risk of the trading book. In instances where the risk management view of the bank may differ from the regulatory requirements (inclusion or exclusion of certain risk factors in the internal model or in the 'regulatory' model), different

VaR measures may be reported - one being used in the regulatory capital calculation and reporting another one used for internal risk management.

One should also consider that Value at Risk is often described as a non-coherent risk measure, a consequence of not always complying with the sub-additive axiom (Artzner et al., 1999[1]). As a consequence, the sum of risks generated by individual assets may be lower than the total risk in the portfolio to which such assets belong. Summing up, the total market risk of the bank may be greater than the sum of the VaR's of the individual business lines, which contradicts the existence of diversification effects.

There are several commonly used alternative calculation approaches for VaR, each with its own advantages and disadvantages. The three most accepted are the Parametric VaR, the Historical VaR and the Monte Carlo VaR.

According to Wilmott (2006), there is not a perfect approach and the choice should account for the following three parameters:

- **Market Evolution:** notion of how the market may move over the liquidity horizon
- **Valuation:** method to determine the value of the asset under the market movements
- **Extraction:** method to extract the worst asset value at the chosen confidence level.

2.1.1 Parametric Approach

This approach assumes that market risk factors follow an independent and identically distributed (i.i.d.) Gaussian process, which is specified by the covariance matrix. Thus, the calculation of VaR depends on the estimates of volatility of the asset or portfolio returns. As J.P. Morgan (1996) describes, volatility is multiplied by the inverse standard Normal cumulative distribution, i.e. z-score value (ϕ^{-1}), according to a defined significance level (α):

$$VaR_{\alpha\%} = \phi^{-1}(1-\alpha)\sqrt{\sigma_p^2}$$

This method is simple to compute and the result can be relatively easy to explain. Jorion (2006) recalls the fact that this method may be appropriate for simple portfolios, as it is quite simple to compute, and the method is fairly easy to explain. However, the method may lead

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[1] Consider ρ as risk of assets X and Y. If subadditivity holds, $\rho\,(X+Y) \leq \rho(X) + \rho(Y)$, meaning that the risk of holding asset X and Y simultaneously must be less or equal to the sum of their individual risks.

to biased estimates whenever products with non-linear payoffs are inserted. Also, non-normality will not be captured. This question is addressed by Holton (2003), which states that the method is valid as long as the portfolio is diversified, a primary condition to evoke the central limit theorem.

As expressed by Alexander (2008), risk managers usually map their portfolios into a small number of risk factors, in order to reduce the dimensionality of the problem. The mapping of the VaR allows a decomposition between Systematic Risk (risk factors) and Residual Risk (idiosyncratic risk).

2.1.2 Historical Approach

Under this approach, the future market price is assumed to move as observed historically in the chosen look-back period (Linsmeier and Pierson, 1996). Market price shifts seen on each historical day create a sample size equal to the look-back period and the VaR is calculated as a percentile, representing the n-th worst result, depending on the confidence interval chosen by the risk analyst (Hendricks, 1996).

As the distribution considered is the distribution of the portfolio historical returns itself, this methodology allows us to capture non-normality effects taking into account that the moments of the distribution are preserved. Auer (2018) also adds the fact that the method is totally based in the past to generate a future estimate is a drawback; the sample may not contain an important past event, and thus will not appear in the future.

2.1.3 Monte-Carlo Approach

According to Vlaar (2000), market evolution is simulated, and thus not limited to historical movements. After gathering a model to describe the evolution of the market, one is able to apply a shock by drawing a random variable from a selected distribution and calculate its return (Glasserman, 2004). After repeating this process iteratively, the quantile for the selected confidence interval is extracted. Even though this final step represents a simple observation of the distribution, the complete process requires thousands of simulations representing a substantial amount of computational power.

As the modelling premises are defined by the risk analyst/modeller, the problem can easily become truly complex, as described by Miller (2018). The user may want to simulate hundreds of risk factors, each one requiring a different type of model assumptions (interest-rates, bond prices, stock prices, commodities, forex, volatility, repos, dividends). Even though the VaR model will be more robust, complexity comes with a cost as model risk arises which can have an enormous impact in the final result (Holton, 2003).

2.1.4 Stressed Value at Risk

In July 2009, the Basel Committee on Banking Supervision (BCBS) issued its final version of the 'Revisions to Basel II market risk framework'. Through the revision, which built upon the BCBS's Basel II framework published in June 2006, further capital requirements were introduced.

These additional measures sought to address several gaps in key market risks. One of these newly introduced measures was the 'Stressed Value-at-Risk'(SVaR). This regulation has been adopted by the European Union with the Directive Capital Requirements Directives III and further specified by the Guidelines issued by the European Banking Authority (EBA).

SVaR is a backward measure of risk that attempts to quantify tail risk in that this measure should 'replicate' the VaR calculation that would be generated on the bank's current portfolio if the relevant risk factors were experiencing a period of stress. Specific guidelines were further published in 2012, by EBA, to define a proper time window to specify the SVaR figure - the model inputs should be calibrated to historical data from a continuous 12-month of significant financial stress relevant to the bank's portfolio. This procedure may be based on judgement (risk manager expertise) or mathematical proof (usage of statistical methods), and the need for documentation describing the decision is mandatory.

2.2 Stress Testing and Reverse Stress Testing

Even though banks were already stress-testing their portfolios, BCBS decided after the 2008 crisis to publish Stress Testing Principles (2009) in order to "*address key weaknesses in stress testing practices*".

More than creating a set of rules to address Stress Testing programmes, BCBS strongly encouraged banks to embrace Stress Testing as a recent piece of its already complex risk governance model to complement it with a new perspective.

Aside from promoting discussion around the board and managers, the key message over Stress Tests lies within the fact that scenarios created by the risk stakeholders should be a balance between severity and plausibility, assuring also its non-identically feature. Breuer et al. (2009) also add an extra *'suggestiveness of risk-reducing action'* to the previous ones, recalling the importance of creating risk reduction procedures assuming the scenario happens.

As the BCBS publication states, the increment in market risk is taken by the difference between the “Normal Daily” Value at Risk and the calculated one for the created scenario. It should be then up to the board and the Chief Risk Officer to decide if they are comfortable with the calculated result. Otherwise, some changes should be done to the portfolio by position increment/unwinding in some asset class (rebalancing).

According to the 2020 European Union Wide Stress-Tests[2], requirements to banks were related to stress scenarios comprising equity, commodities, FX, Interest-Rate yields, Sovereign Credit Spreads, Corporate Credit Spreads, and Volatility shocks.

Within its final review over Stress Testing, the European Banking Authority (EBA) (2018) published a set of guidelines to stress testing, which require institutions to “*perform adequate reverse stress tests as part of the stress testing programme (...) which should be carried out by all types of institutions*”. Reverse stress testing are defined as “*an institution stress test that starts from the identification of the pre-defined outcome (...) and then explores scenarios and circumstances that might cause this to occur*”., The literature over Reverse Stress Testing is expected to grow substantially following this requirement by the regulators.

The first writings were developed by Glasserman et al. (2014), in which a method for detecting most likely stress scenarios was developed under empirical likelihood of the conditional mean. Peter Grundke et al. (2016) performed reverse stress testing methodologies over a liquidity failure on a broader, macroeconomic perspective.

Kopeliovich et al. (2018) developed a reverse stress testing approach at the portfolio level using Principal Component analysis over the portfolio distribution moments. The foundation of this dissertation is based on this previous referred paper.

[2] https://eba.europa.eu/eba-launches-2020-eu-wide-stress-test-exercise

3 Data Description

Since the subject of this research concerns banks and their applications, we analyse a *dummy*[3] portfolio of 1,000,000 € (1 million euros), distributed among HSBC Holdings, JP Morgan Chase and Deutsche Bank. The idea behind this selection is to capture market risk over different equity indices (FTSE 100, NYSE and DAX), and add over some currency risk to enhance our Systematic VaR. Summed up, there will be five Risk Factors: DAX, NYSE, FTSE 100, USD/EUR, GBP/EUR. These risk factors were also chosen taking into account their simplicity, since they have a standard interpretation: they have observed daily price quotes in the market, and the portfolio P&L is driven by the evolution of these parameters.

For the sake of ease, no further risk factors have been included, although this possibility is absolutely viable.

Our study is based on dividend-adjusted closing prices have been obtained, which results in a dataset comprised between 2007 and 2019. This dissertation is fully based in the RiskMetrics (1996) VaR model – parametric normal, with an EWMA covariance matrix estimate with parameter $\lambda = 0.94$, in line with the referred document.

The methodology requires all historical returns to be populated so that the covariance matrix is fully estimated. Thus, linear interpolation was performed over missing prices for days where the market was closed or when data quality issues arise.

The VaR liquidity horizon is 1-day. We opt for arithmetic returns so we keep the property of aggregation across assets (logarithmic returns would be useful if the main objective was to scale the VaR to longer holding periods - aggregation across time). Our currency will be Euro and the calculation date will be the 31st of December 2019. Every single calculation is performed resorting to Python, a general-purpose programming language. The code used to apply the methodology may be made available upon request.

In the appendix we display more detailed data information.

[3] Fictitious with the appearance of being real

4 Methodology

4.1 Volatility Modelling

The simplest variance estimation model over T days is specified by:

$$\hat{\sigma}_t^2 = \frac{1}{T}\sum_{i=1}^{T} r_{t-1}^2 \tag{1}$$

Although mathematically correct, the same weight is given to all sample observations, regardless of whether it is (or not) a recent event. As such, the objective should be to capture the current market conditions. In order to increase the sensitivity of the covariance matrix to more recent events, it is mandatory to give greater weight to the data obtained last. In this way, volatility is modelled using an Exponentially Weighted Moving Average (EWMA) approach:

$$\hat{\sigma}_t^2 = (1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1} r_{t-1}^2 \tag{2}$$

As we move far back in time, the weight given to a specific observation decreases exponentially. Through the use of this methodology, all observation will be taken into account (there is no elimination). However, at some point they will have an effect close to 0 on the estimated variance.

4.2 Risk Aggregation

Regarding equity risk, stock returns are mapped into their corresponding index through a one-factor CAPM Model, and so an asset i return X is represented by a β position in the correspondent risk factor (index) m:

$$X^i = \beta^i X^m + \epsilon^i \tag{3}$$

where

$$\beta^i = \frac{covar(X^i, X^m)}{var(X^i)} \tag{4}$$

Handling currency risk is quite simple: it will represent an investment of a certain amount of euros in a foreign currency.

Assuming a covariance matrix Σ and amounts Θ, the portfolio variance may be obtained as:

$$\sigma_p^2 = \Theta^T \Sigma\, \Theta \tag{5}$$

which enables the calculation of VaR, given by:

$$VaR_{99\%} = \phi^{-1}(0.99) * \sqrt{\sigma_p^2} \tag{6}$$

4.3 Systematic Risk Factor Loadings

Assuming covariance matrix C and a vector Q of invested amounts, the systematic portfolio variance may be obtained as in equation (5):

$$\sigma_{sp}^2 = Q^T C\, Q \tag{7}$$

and Systematic VaR may be computed as:

$$Systematic\ VaR_{99\%} = \phi^{-1}(0.99) * \sqrt{\sigma_{sp}^2} \tag{8}$$

4.4 Gradient Calculation

Additional information may be obtained through the Systematic VaR. The gradient vector will allow further analysis as the risk manager is able to measure the added impact of a unit of some asset to the VaR.

Assuming a confidence interval of 99%, the gradient is calculated as:

$$\nabla = \phi^{-1}(0.99) \frac{1}{\sqrt{Q^T C Q}} C Q \tag{9}$$

4.5 Reverse Stress Test

4.5.1 Central Scenario

Assuming vectors of returns $R = [R_1, \quad \dots \quad , R_n]$ following a multivariate normal distribution with mean 0 and Covariance Matrix $C = E(XX')$, and $Q = [Q_1, \quad \dots . \quad , Q_n]$ is a vector of quantities in a portfolio, the total portfolio return is equal to:

$$R = Q^T R \tag{10}$$

Assuming the portfolio mean is 0, the variance is additionally given by:

$$Q^T R Q \tag{11}$$

Denoting L as the required loss amount, we want to find a scenario that results in the pre-defined loss, resulting in a hyperplane H, such as:

$$H = \{R : L = Q^T R\} \tag{12}$$

As the previously presented equation has infinite solutions, with the initial assumptions about the distribution we may find the most probable one, which yields:

$$R^* = L \frac{C\ Q}{Q^T C\ Q} \tag{13}$$

Likewise, we would like to single out a few more plausible scenarios satisfying equation (10). In order to control for plausibility, a likelihood factor q is added to constrain the new scenarios to have a likelihood between q and 1.

4.5.2 Principal Component Analysis and Gram-Schmidt Process

PCA is a linear transformation technique for dimensionality reduction, as defined by Pearson (1901). It eliminates structural redundancies without sacrificing information if there are highly correlated variables, as it finds features that are uncorrelated directions of maximum variance in the data, i.e. the principal components. These features are linear combinations of the original data and are used to construct the new feature space to project the data into.
The first step relies on applying a Gram-Schmidt process over the vector Q. Our transformed orthogonal matrix P will be calculated, with Q being the first row.

The portfolio returns R will be also transformed using the orthogonal matrix P:

$$Y = P R^* \tag{14}$$

PQ yields in a unit vector $\tilde{Q}$, with the first element being the norm of the original Q vector. The hyperplane $H_y = \{Y: \tilde{Q}\, Y = L\}$ is now generated from H. By carrying out the Gram-Schmidt process, one can check that the first element of γ is the quotient of L by the norm of the Q vector. Such a statement allows for a simpler calculation over the conditional distribution, as H_y is parallel to H and perpendicular to the (1, 0,…, 0) vector.

Recall our covariance matrix C, which will be then computed into the transformed space:

$$D = P\, C \tag{15}$$

We can then perform a D decomposition as a block matrix. We already know Y_1 (the first element of Y), so we are merely considering Y_2 to Y_n. Assuming I = {2,…, n}:

$$C_Y = \begin{bmatrix} d_{11} & D_{1I} \\ D_{I1} & d_{II} \end{bmatrix} \tag{16}$$

Assuming the already known quantity of Y_1, the distribution of $\{Y_2, \dots, Y_n\}$ is characterized by:

$$\mu_Y^H = \frac{L}{d_{11}\|Q\|} D_{I1} \tag{17}$$

$$C_Y^H = d_{1I} \frac{1}{d_{11}} D_{I1} D_{1I} \tag{18}$$

The next step lies within calculating the eigenvectors V of C_Y^H by decreasing order of the eigenvalues E:

The subsequent phase is purely mathematical. The main concern is to generate $M + 1$ (we will consider $M = 3$) equidistant points, with one in the origin and the others lying in circumference's limit with radius R. The t vertex points shall be (the points are given by the methodology author):

$$t_1 = (1{,}0), t_2 = , (-0.5, -0.866)\ t_3 = (-0.5{,}0.866)$$

A rotation to the principal components' subspace is performed:

$$z^j = Vt^j, j = 1{,}2, .., M \tag{19}$$

After the rotation procedure, we shall determine the sphere radius in order to preserve the likelihood (q = 0.1) selected by the risk analyst:

$$Radius = \sqrt{-\frac{2\ln q}{z^{M^T} C_Y^{H^{-1}} z^M}} \tag{20}$$

The scenario 0 has already been calculated, and the alternative three vectors result in the product of:

$$y^j = Radius * z^j * \mu_Y^H, j = 1{,}2, .., M \tag{21}$$

The first component will be appended into the first row of each vector:

Rotating back to the original space using the orthogonal matrix P:

$$R^j = P\, y^j, j = 1{,}2, .., M \tag{22}$$

The Euclidean distance between central and non-central scenarios is, as required, the radius.

5 Empirical Study

5.1 Relevance and purpose of the study

The primary purpose of the study relies on creating specific loss scenarios designed exclusively for the portfolio in question, taking into account its distribution properties, and to build a structure in which a risk user can rely on a regular basis to perform its tasks.

5.2 Portfolio Details

The reference date market quotes are the following:

-	USD/EUR	GBP/EUR	DBK	JPM	HSBC
27/12/2019 quote	1.11744	0.85337	6.906€	139.41$	597.8£

Table 1 - Spot Market Quotes

Our stock portfolio is composed by 48,268 stocks of DBK, 2,143 stocks of JPM, 654 of HSBC and 48,268 of DBK. The betas were calculated on a year basis from equation (4):

Stock	Amount (Original currency)	Amount (€)	Beta
DBK	333,339€	333,339€	1.22
JPM	298,177$	333,195€	1.53
HSBC	390,961£	333,635€	1.03

Table 2 - Portfolio Information

Adding currency risk:

Stock/Currency	Amount (θ)	Beta
DBK	333,339€	1.22
JPM	333,195€	1.53
HSBC	333,635€	1.03
USD/EUR	333,195€	-
GBP/EUR	333,635€	-

Table 3 - Portfolio Information with Currency Risk

5.3 Value at Risk

5.3.1 VaR calculation

The usage of equation (2) to determine the portfolio returns covariance matrix generates:

$$\Sigma = \begin{bmatrix} 0.00026 & \dots & \dots & \dots & \dots \\ 0.000073 & 0,000065 & \dots & \dots & \dots \\ 0.00011 & 0,000049 & 0,00010 & \dots & \dots \\ -0.000011 & 0,00000016 & 0,00000016 & 0,0000070 & \dots \\ -0.0000013 & 0,0000056 & 0,000016 & 0,0000032 & 0,000038 \end{bmatrix}$$

The application of equation (4) results in a portfolio standard deviation equal to 10,325.237€, which results in a Total VaR of 24,020€ through the utilization of equation (6).

5.3.2 Systematic VaR calculation

The application of equation (3) and (4) leads to:

Stock	Index	Amount	Beta	Amount * Beta
DBK	DAX	333,339€	1.22	407,746€
HSBC	FTSE 100	333,635€	1.03	343,918€
JPM	NYSE	333,195€	1.53	510,556€

Table 4 – Equity Risk Factor mappings

With the inclusion of currency risk, we get:

<table>
<tr><th>Stock/Currency</th><th>Index</th><th>Amount</th><th>Beta</th><th>Amount * Beta /Currency</th></tr>
<tr><td>DBK</td><td>DAX</td><td>333,339€</td><td>1.22</td><td>407,746€</td></tr>
<tr><td>HSBC</td><td>FTSE 100</td><td>333,635€</td><td>1.03</td><td>343,918€</td></tr>
<tr><td>JPM</td><td>NYSE</td><td>333,195€</td><td>1.53</td><td>510,556€</td></tr>
<tr><td>GBP/EUR</td><td colspan="3" rowspan="2"></td><td>333,635€</td></tr>
<tr><td>USD/EUR</td><td>333,195€</td></tr>
</table>

Table 5 - Portfolio Mapping with Currency Risk

As pointed out in the previous sub-chapter, equation (2) allows for the systematic portfolio returns covariance matrix to be obtained:

$$C = \begin{bmatrix} 0{,}0000366 & \dots & \dots & \dots & \dots \\ 0.00000238 & 0{,}000049 & \dots & \dots & \dots \\ 0.0000163 & 0{,}0000179 & 0{,}000014 & \dots & \dots \\ -0{,}0000170 & 0{,}00000074 & -0{,}0000018 & 0{,}000038 & \dots \\ -0{,}0000049 & -0{,}0000034 & -0{,}0000022 & 0{,}00000326 & 0{,}00000705 \end{bmatrix}$$

Which results in a portfolio standard deviation of 5,760.86€ according to equation (7) and consequently a Systematic VaR equal to 13,401€ as equation (8) represents.

5.3.3 Gradient VaR

For the sake of completeness, Gradient Value at Risk can likewise be achieved. Equation (9) generates:

Identifier	**Gradient**
DAX	0.009758
FTSE	0.014719
NYSE	0.007520
GBP/EUR	-0.000364
USD/EUR	0.002483

Table 6 - Portfolio Risk Gradient Vector

5.4 Central Scenario

For the purpose of this dissertation, a loss of 13,401€will be considered, as it represents the 99% percentile Systematic VaR. According to equation (13), the optimal R vector return is the following:

$$R^* = \begin{bmatrix} -0.98\%\ (DAX) \\ -1.42\%(FTSE) \\ -0.75\%\ (NYSE) \\ -0.25\%\ (GBP/EUR \\ 0.04\%(USD/EUR) \end{bmatrix}$$

5.5 Reverse Stress Test

Usage of Gram Schmidt Process over $\boldsymbol{Q}$ yields an orthogonal matrix corresponding to:

$$P\ = \begin{bmatrix} 0.465 & 0.393 & 0.583 & 0.381 & 0.38 \\ -0.199 & 0.92 & -0.249 & -0.163 & -0.162 \\ -0.414 & 0 & 0.774 & -0.339 & -0.339 \\ -0.414 & 0 & 0 & 0.845 & -0.339 \\ 0,633 & 0 & 0 & 0 & -0.774 \end{bmatrix}$$

Which will be applied to transform the vector of returns. Equation (14) results in:

$$Y = \begin{bmatrix} -0.0153 \\ -0.0889 \\ -0.00105 \\ 0.002 \\ -0.006 \end{bmatrix}$$

With $\tilde{Q} = (876091{,}0{,}0{,}0{,}0)$.

The next step lies within rotating the covariance matrix C into the transformed space.

Equation (15) produces the following:

$$D = \begin{bmatrix} 0{,}0000432 & \dots & \dots & \dots & \dots \\ 0{,}0000251 & 0{,}0000295 & \dots & \dots & \dots \\ 0{,}000002981 & 0{,}0000040 & 0{,}00000609 & \dots & \dots \\ -0{,}00000515 & -0{,}00000661 & -0{,}00000804 & 0{,}0000428 & \dots \\ 0{,}0000182 & 0{,}00001151 & 0{,}00000557 & -0{,}0000195 & 0{,}0000237 \end{bmatrix}$$

Equation (16) enables a decomposition of D as a block matrix, which yields:

$$d_{11} = 0{,}000043 \text{ , } D_{I1}, D_{1I} = \begin{bmatrix} 0{,}0000251 \\ 0{,}000002981 \\ -0{,}00000515 \\ 0{,}0000182 \end{bmatrix},$$

$$D_{II} \begin{bmatrix} 0{,}0000295 & \dots & \dots & \dots \\ 0{,}0000040 & 0{,}00000609 & \dots & \dots \\ -0{,}00000661 & -0{,}00000804 & 0{,}0000428 & \dots \\ 0{,}00001151 & 0{,}00000557 & -0{,}0000195 & 0{,}0000237 \end{bmatrix}$$

Which allows to equation (17) and (18) to be computed:

$$\mu_Y^H = (-0.00889, -0.00105, 0.00182, -0.00646)$$

$$C_Y^H = \begin{bmatrix} -0.00001495 & \dots & \dots & \dots \\ 0.0000023 & 0.000005885 & \dots & \dots \\ -0.00000361 & -0.00000768 & 0.0000422 & \dots \\ 0.00000091 & 0.00000431 & -0.0000173 & 0.0000160 \end{bmatrix}$$

The eigenvalues and eigenvectors of C_Y^H are given by:

$$V = \begin{bmatrix} -0.10 & -0.979 & 0.073 & 0.155 \\ -0.18 & -0.12 & 0.16 & -0.96 \\ 0.87 & -0.07 & 0.47 & -0.08 \\ -0.43 & 0.14 & 0.86 & 0.21 \end{bmatrix}$$

$$E = \begin{bmatrix} -0.00005 & 0.0000148 & -0.0000390 & 0.00000743 \end{bmatrix}$$

V may be used to rotate the t-vertex points into the principle components space, through the usage of equation (19):

$$z^1 = \begin{bmatrix} -0.105 \\ -0.187 \\ 0.874 \\ -0.434 \end{bmatrix}, z^2 = \begin{bmatrix} 0.90 \\ 0.198 \\ -0.375 \\ 0.0919 \end{bmatrix}, z^3 = \begin{bmatrix} -0.795 \\ -0.0105 \\ 0.499 \\ 0.342 \end{bmatrix}$$

The radius of 0.09 calculated resorting to equation (20) allows equation (21) to be computed:

$$y^0 = \begin{bmatrix} -0.0153 \\ -0.00889 \\ -0.00105 \\ 0.00182 \\ -0.00646 \end{bmatrix}, y^1 = \begin{bmatrix} -0.0153 \\ -0.00984 \\ -0.0027 \\ 0.00981 \\ -0.01043 \end{bmatrix}, y^2 = \begin{bmatrix} -0.0153 \\ -0.00066 \\ 0.00076 \\ -0.0016 \\ -0.00562 \end{bmatrix}, y^3 = \begin{bmatrix} -0.0153 \\ -0.01615 \\ -0.00115 \\ -0.00273 \\ -0.00332 \end{bmatrix}$$

Rotation to the original space may be achieved through equation (22). The final results are:

$$R^0 = \ R^*, R^1 = \begin{bmatrix} -0.1468 \\ -0.01523 \\ -0.00861 \\ 0.0053 \\ 0.00147 \end{bmatrix}, R^2 = \begin{bmatrix} -0.01019 \\ -0.00066 \\ 0.00816 \\ -0.0073 \\ -0.0010 \end{bmatrix}, R^3 = \begin{bmatrix} -0.0044 \\ -0.0208 \\ -0.00579 \\ -0.00512 \\ 0.00069 \end{bmatrix}$$

5.6 Reverse Stress Testing Scenarios

The methodology is fully applied following Section 4. The results are illustrated in the table below:

Asset	Quantity	Scenario 0 Return	Scenario 1 Return	Scenario 2 Return	Scenario 3 Return
DAX	407,746	-0.98%	-1.47%	-1.02%	-0.44%
FTSE	343,918	-1.42%	-1.51%	-0.66%	-2.09%
NYSE	510,556	-0.75%	-0.86%	-0.82%	-0.58%
GBP/EUR	333,635	-0.25%	0.50%	-0.73%	-0.51%
USD/EUR	333,195	0.04%	0.15%	-0.11%	0.07%
Total	1,929,050	13,401	13,401	13,401	13,401

Table 7 - Reverse Stress Test Scenarios

From the interpretation of the results of Table 7 we can deduce that the pre-defined 99% loss is reached, reflecting the most probable outcomes to generate the 99% Systematic VaR.

As seen in the table, we present four scenarios with Scenario 0 being the central (most probable) one. The subsequent three scenarios are based on it, in line with the previously described procedure, and possess a relative likelihood of 0.4554, 0.1 and 0.1, respectively.

Also, for further comparisons, the methodology was applied once more, comprising only 2008 returns in the VaR calculation to capture the extreme market volatility at the time. This process identifies the portfolio dynamics and its corresponding stress scenarios taking into account a major impact due to volatility.

In practical terms, we can observe what would need to happen to have a loss that may have happened in 2008 environment, capturing the effect of a high volatile period in our up-to-date portfolio.

In summary, the VaR at the end of the 2008 year:

	99%	99.9%
Total VaR	95,696€	-
Systematic VaR	81,388€	130,110€

Table 8 - Portfolio VaR under extreme volatility

Stating L = 130,110€, the scenarios are given by:

$$R^0 = \begin{bmatrix} -0.095 \\ -0.137 \\ -0.07 \\ -0.024 \\ 0.003 \end{bmatrix}, R^1 = \begin{bmatrix} -0.099 \\ -0.1385 \\ -0.074 \\ -0.0166 \\ 0.004 \end{bmatrix}, R^2 = \begin{bmatrix} -0.095 \\ -0.13 \\ -0.0736 \\ -0.028 \\ 0.002 \end{bmatrix}, R^3 = \begin{bmatrix} -0.089 \\ -0.144 \\ -0.07 \\ -0.0026 \\ 0.003 \end{bmatrix}$$

Which in table form:

Asset	Quantity	Scenario 0 Return	Scenario 1 Return	Scenario 2 Return	Scenario 3 Return
DAX	407,746	-0.095	-0.099	-0.095	-0.089
FTSE	343,918	-0.137	-0.1385	-0.13	-0.144
NYSE	510,556	-0.07	-0.074	-0.0736	-0.07
GBP/EUR	333,635	-0.0024	-0.0166	-0.028	-0.0026
USD/EUR	333,195	0.003	0.004	-0.02	0.003
Total	1,929,050	-130,110	-130,110	-130,110	-130,110

Table 9 - Reverse Stress Tests Scenarios under extreme volatility

The scenarios generated in Table 9 are again in line with the 99.9% 2008 VaR and fully represent the reverse stress potential, achieving scenarios the financial institution may have never acknowledged.

6 Conclusions and Limitations

Over the past 10 years, regulation has revolutionized risk management. As regulators demand complex frameworks, banks have been investing a huge amount of resources to comply with such requests.

With this dissertation, a methodology was suggested to perform Reverse Stress Tests. In order to conduct such method, some stages were implemented.

As the methodology section progressed, Covariance Matrices were calculated to achieve the portfolio variance which was then used for Value at Risk. The figures were calculated to define a starting point, as it reflects a statistically plausible loss for our dummy portfolio. Assuming multivariate normality, we could find the most probable scenario that would cause such loss. Through the use of Principal Component Analysis and a method created by Kopeliovich et al. (2018), we were able to define some other scenarios linked to the central one, which assures the plausibility of the scenario – one of the BCBS requirements for developing stress tests. To complement the empirical study, we derived the most probable scenarios over a possible loss in the 2008 extreme volatility scenario, creating portfolio-specific loss scenarios under market stress periods.

As the purpose of the study was narrowed to that simple (but complex) objective, the problem was kept simple, composed of products with linear payoffs. As the problem expands into more complex products (Exotics) the difficulty of applying such methodology arises, as the complexity of such pricing models (or even the inexistence of one) increases and so does the demand of computational power. Also, the primary assumption of returns following multivariate normal distribution tends to be inaccurate, as financial returns usually exhibit fat tails in their distributions. Even though the Central Limit theorem may be evoked through additional risk factors, in the worst-case scenario, a way was found to present the risk manager a greater number of stress scenarios Taylor-made for his portfolio.

The results are presented in a totally transparent way, and the method presented may help investors with simple portfolios to have a better, systematic view over the risk of their investments and understand its vulnerabilities.

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Appendices

Appendix A. Time Series

Deutsche Bank

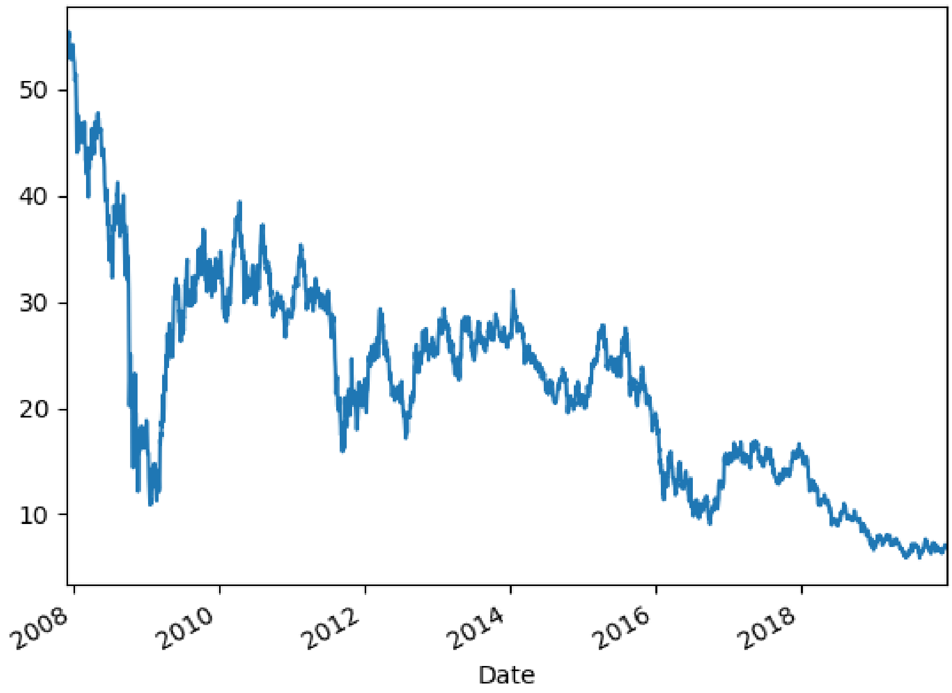

Figure 1- Deutsche Bank Adjusted Close Price

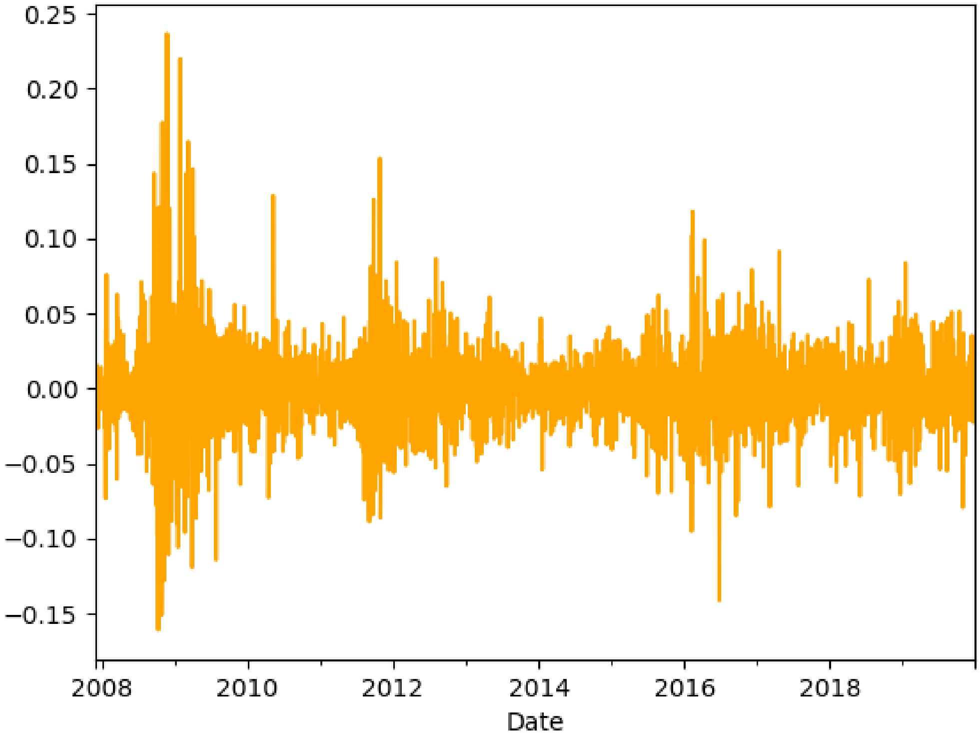

Figure 2 - Deutsche Bank Adjusted Close Return

DAX

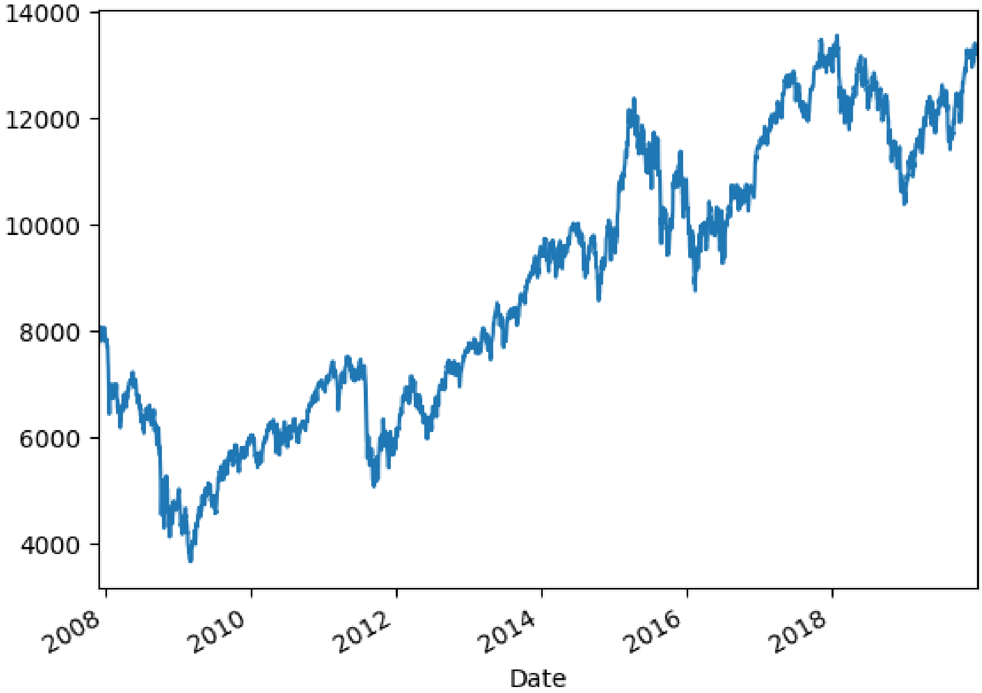

Figure 3 - DAX Adjusted Close Price

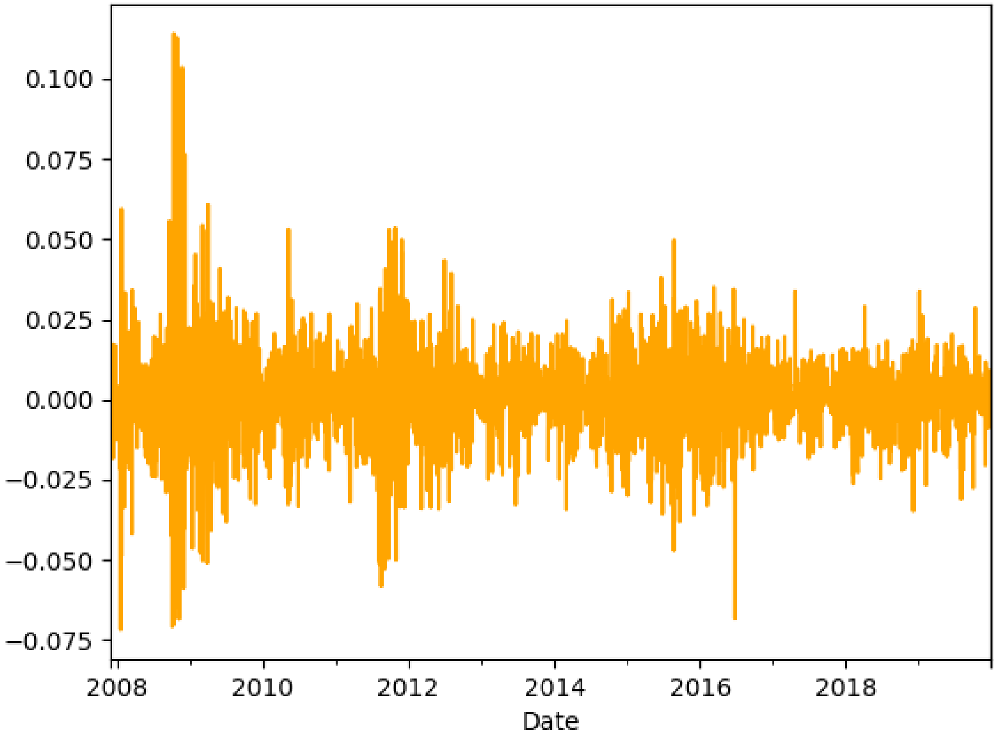

Figure 4 - DAX Adjusted Close Return

HSBC

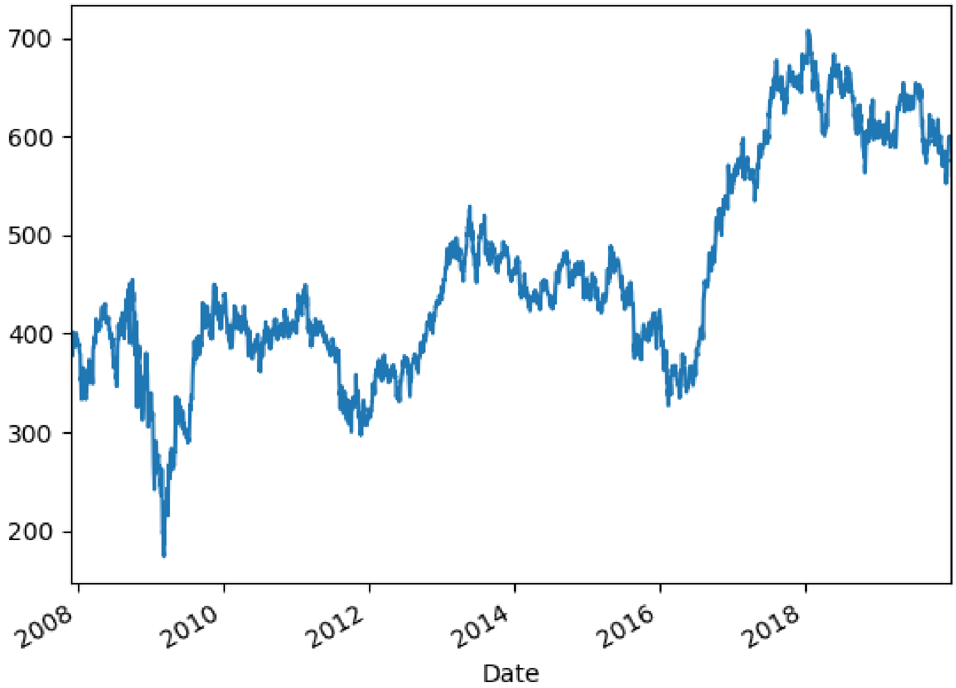

Figure 5 - HSBC Adjusted Close Price

Figure 6 - HSBC Adjusted Close Return

FTSE 100

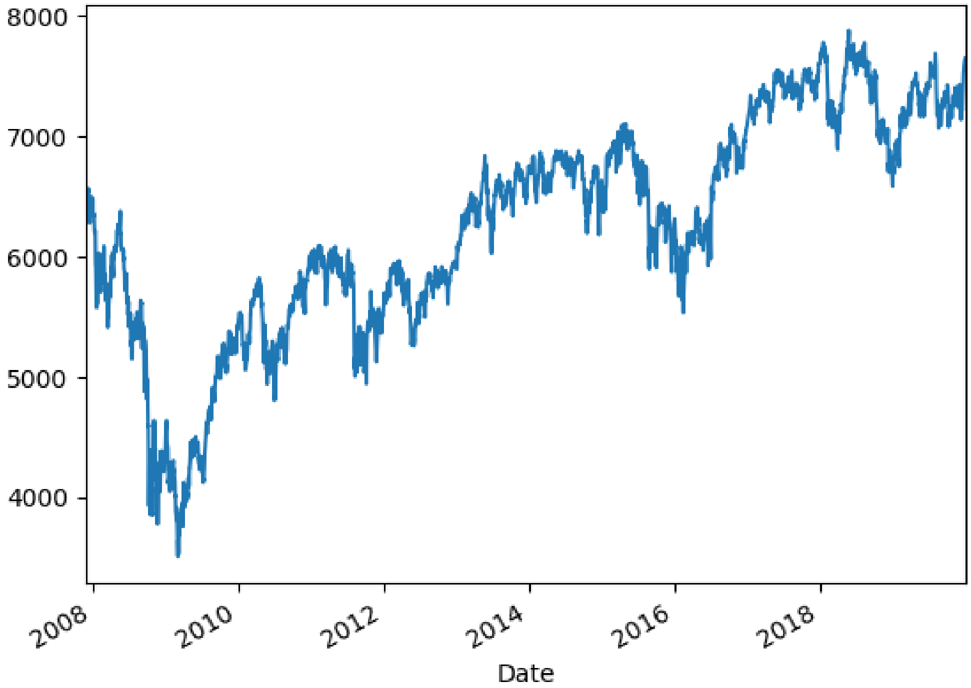

Figure 7 - FTSE 100 Adjusted Close Price

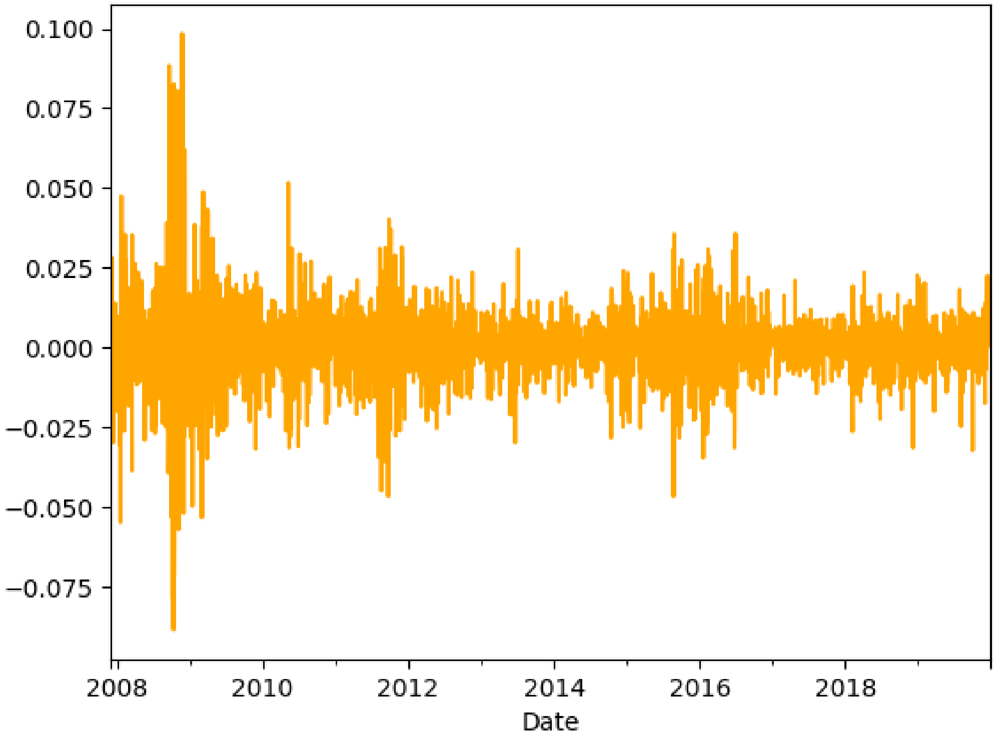

Figure 8 - FTSE 100 Adjusted Close Return

J.P. Morgan Chase

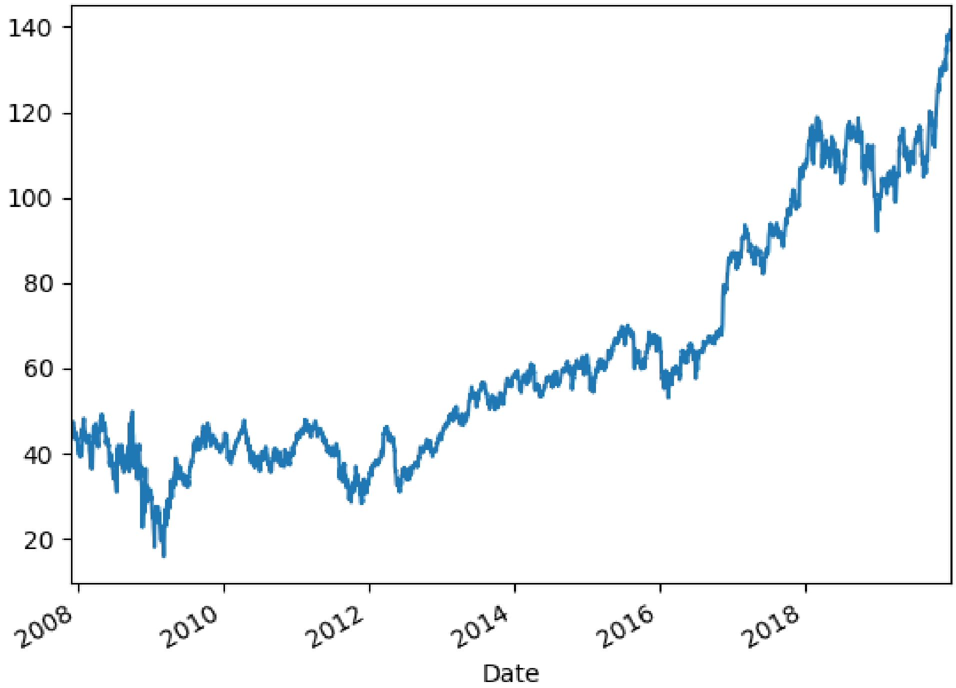

Figure 9 – J.P. Morgan Chase Adjusted Close Price

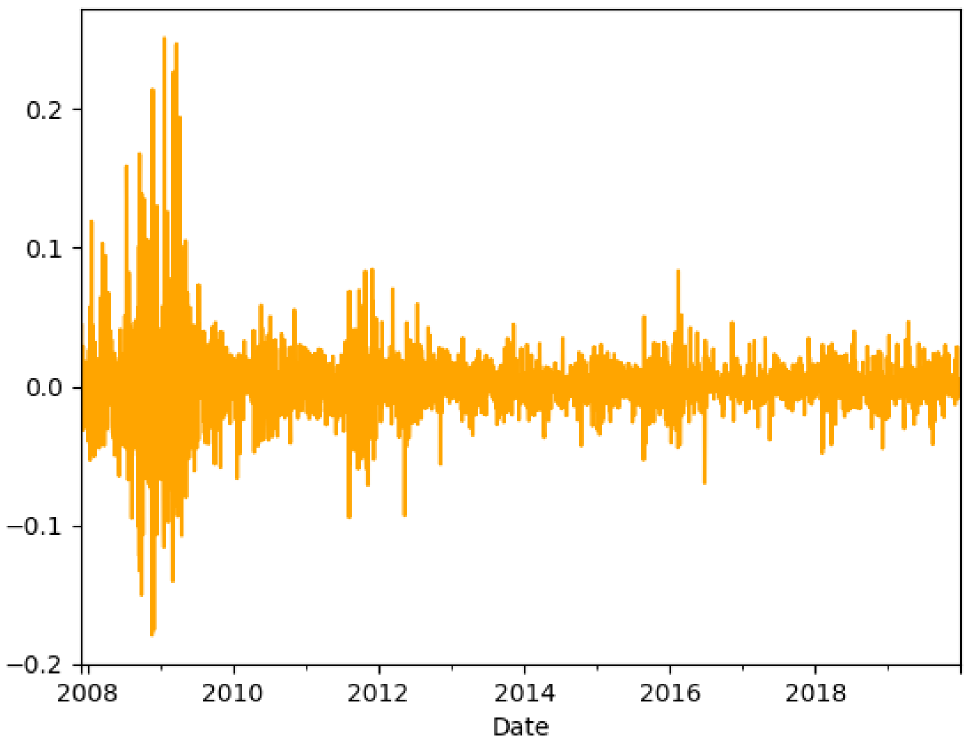

Figure 10 – J.P. Morgan Chase Adjusted Close Return

NYSE

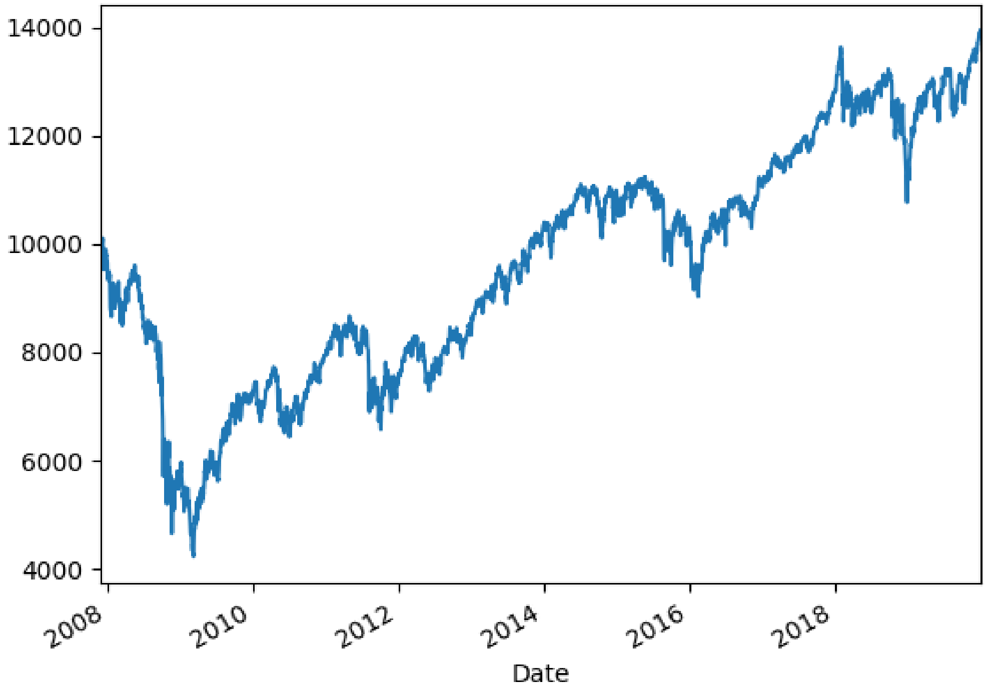

Figure 11 - NYSE Adjusted Close Price

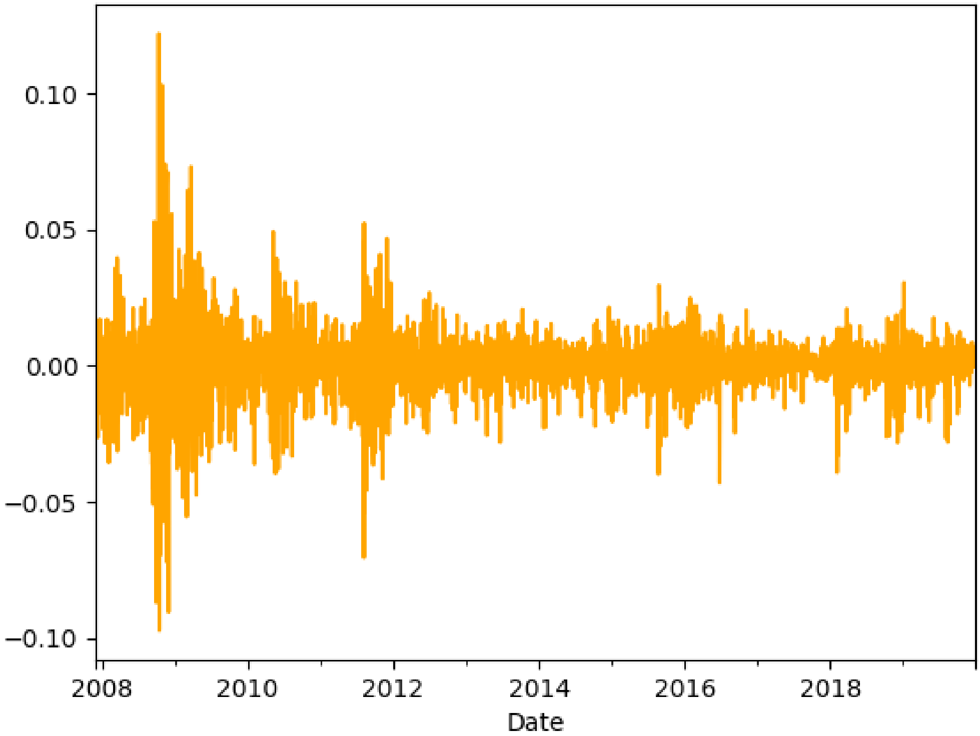

Figure 12 - NYSE Adjusted Close Return

GBP/EUR

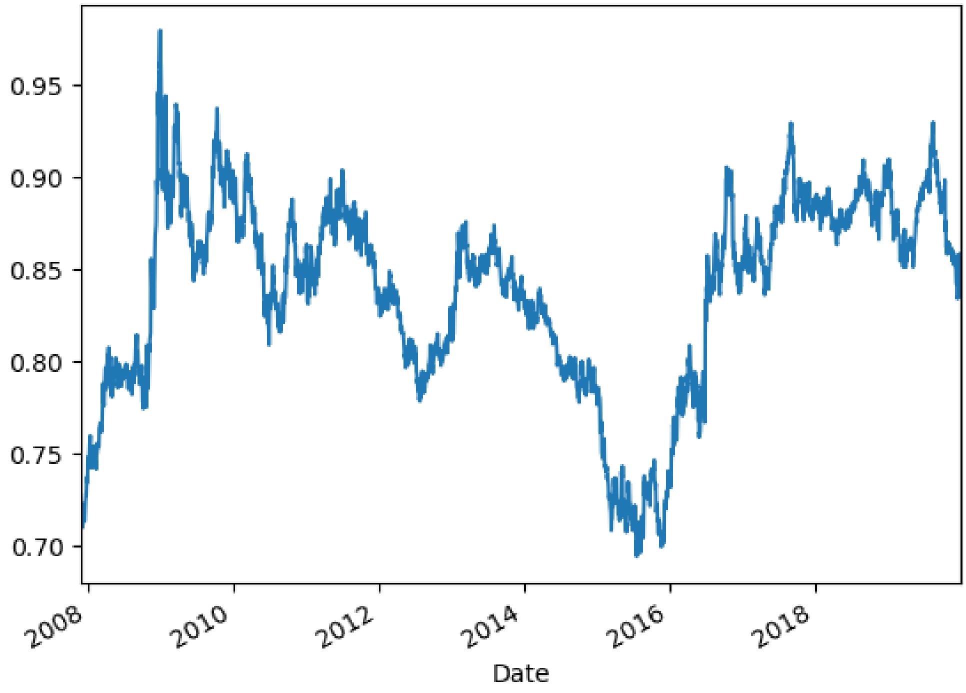

Figure 13 - GBP/EUR Prices

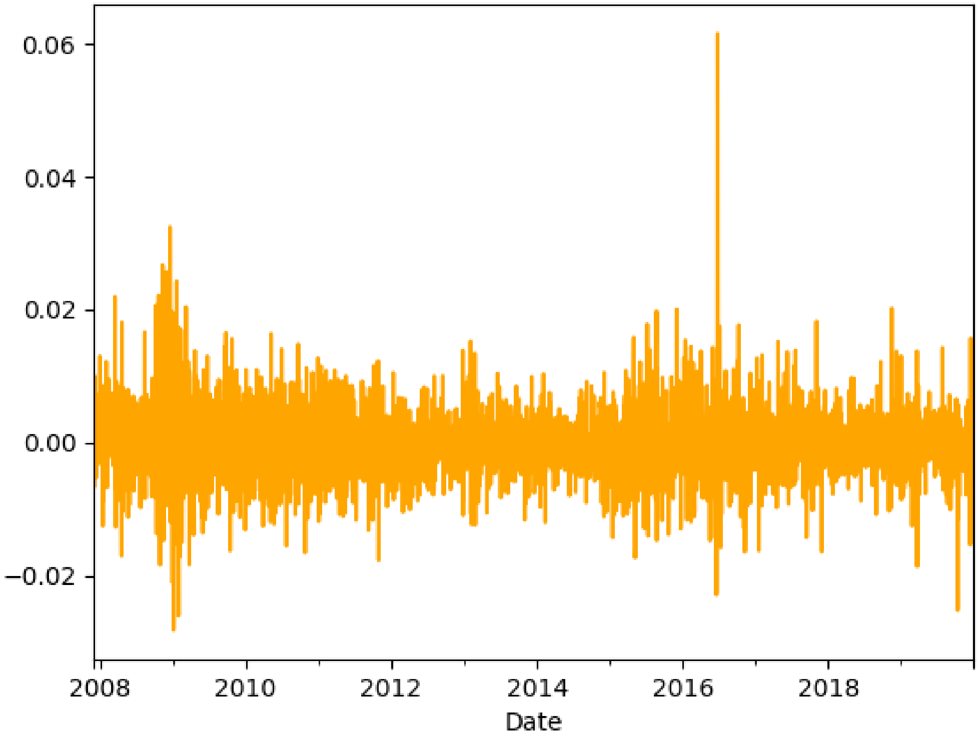

Figure 14 - GBP/EUR Returns

USD/EUR

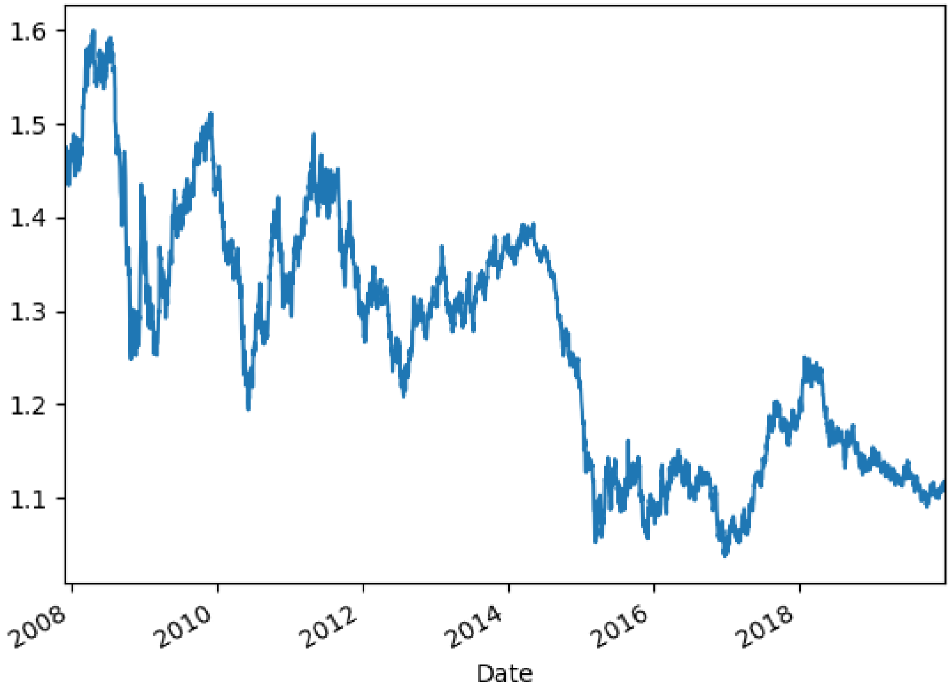

Figure 15 - USD/EUR Prices

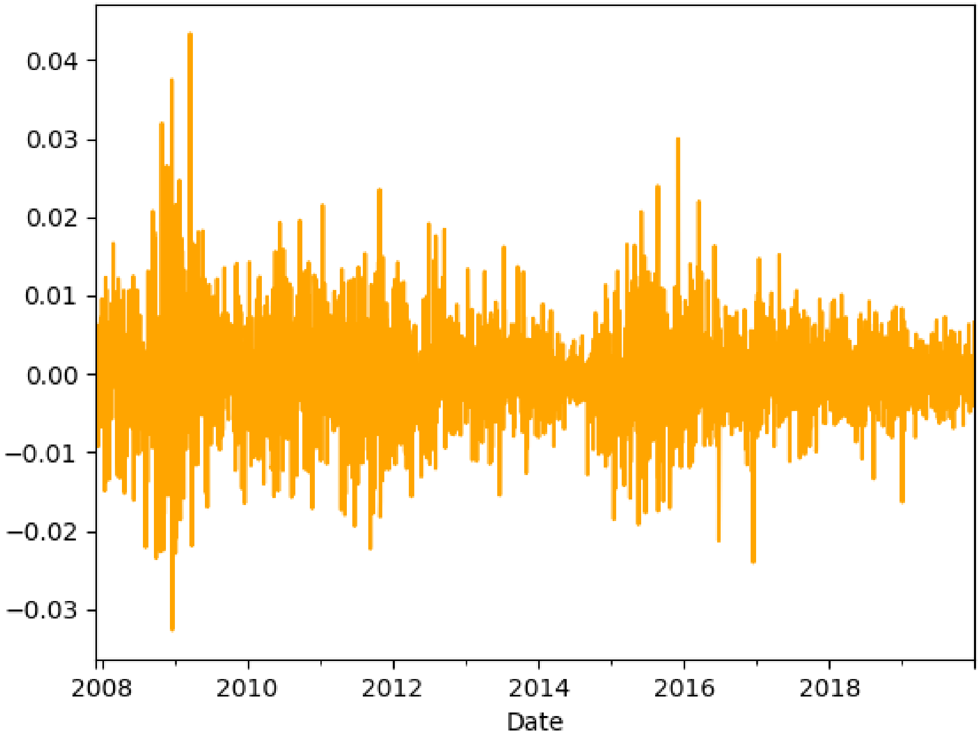

Figure 16 - USD/EUR Returns

Appendix B. Risk Factor Correlation Matrix

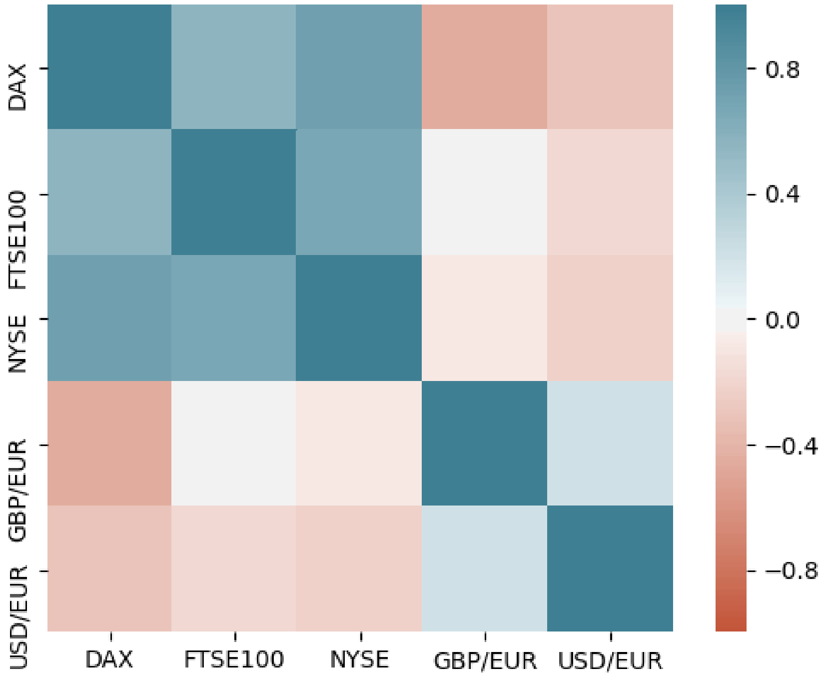

Figure 17 - Returns Correlation Matrix

Appendix C. Distribution Plots

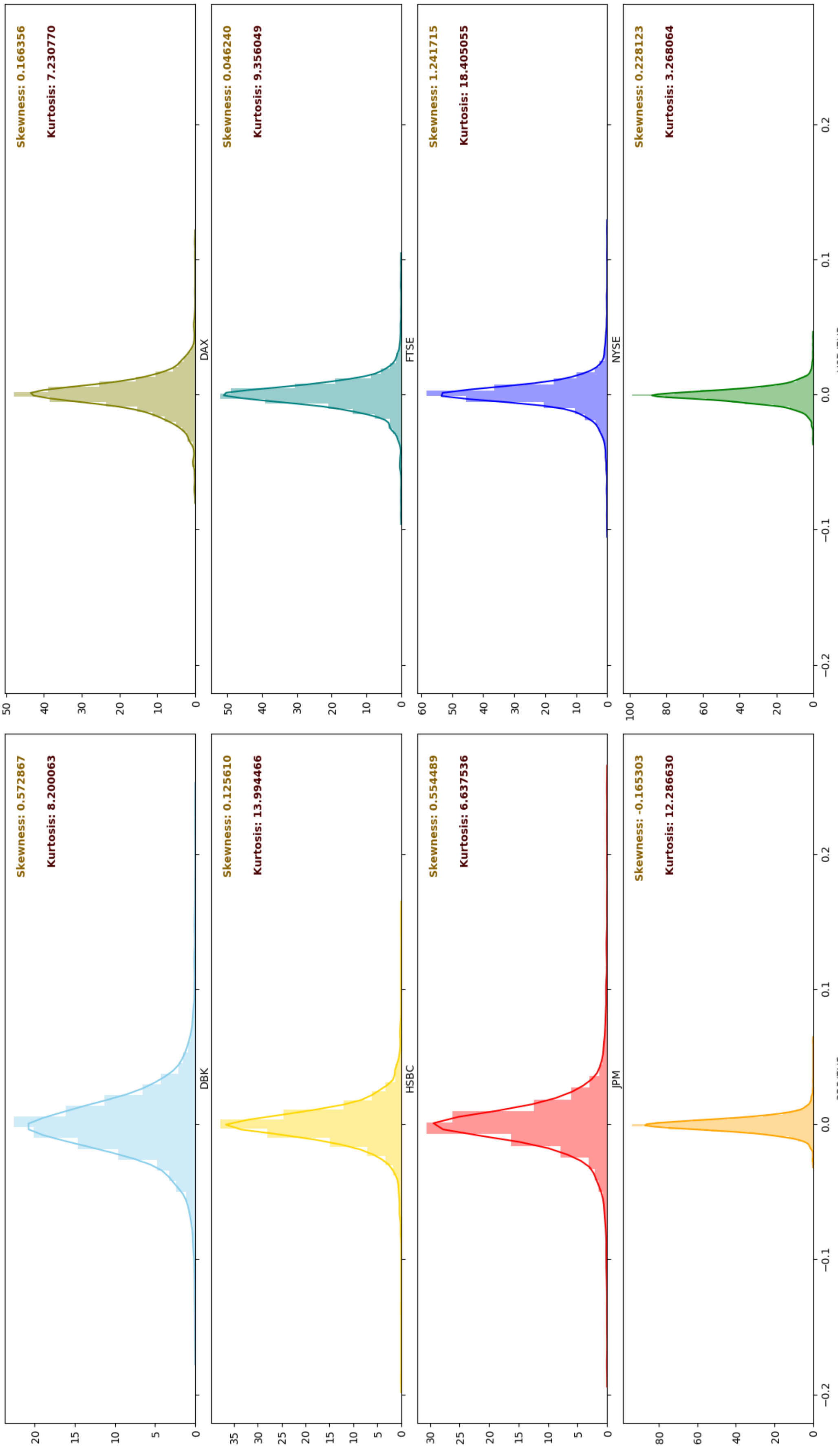

Figure 18 - Returns Distribution Plot