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Planning the delivery of Home Social Services: A Mathematical Programming-Based Approach to support Routing and Scheduling assignments

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BUSINESS
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Department of Marketing, Strategy and Operations

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ABSTRACT

The increased average lifespan, together with low birth rates, are transforming the European Union's age pyramid. Currently, we are experiencing a transition towards a much older population structure. Given that the institutions that provide care to these population groups are limited by budgetary constraints, it is imperative to optimize several processes, among which route planning and staff scheduling stand out.

This dissertation aims to develop a mathematical programming model to support the planning of routes and human resources for providers of Home Social Services. Beyond general Vehicle Routing Problems assumptions, the proposed model also considers the following features: i) working time regulations, ii) mandatory breaks, iii) users' autonomy, and iii) meals' distribution. The present model, implemented using GAMS software, focuses simultaneously on two objective functions: minimization of operating costs, and maximization of equity through the minimization of differences in teams' working times. Chebyshev's method was chosen to solve the developed multiobjective model.

The model was built based on a Portuguese Private Institution of Social Solidarity. Through the application of the model, significant improvements are obtained when compared to the current planning of the partner institution, such as it is the case of an improved workload distribution between caregivers and routes that will result in lower costs for the institution. This model is fully enforceable to other institutions that provide services similar or equal to the institution used as a reference.

Keywords: Route Planning; Scheduling; Optimization; Mathematical Programming Models; Domiciliary Support Service; MILP, GAMS

JEL Classification System:

C61 – Optimization Techniques; Programming Models; Dynamic Analysis

L91 – Transportation: General

Resumo

O aumento da esperança média de vida, juntamente com baixas taxas de natalidade, estão a transformar a pirâmide etária da União Europeia. Atualmente, estamos a vivenciar uma transição direcionada para uma estrutura populacional muito mais envelhecida. Dado que as instituições que prestam cuidados a esta fração se encontram limitadas por restrições orçamentais, torna-se imperativo otimizar vários processos, dos quais se destacam planeamento de rotas e escalonamento de funcionárias.

Esta dissertação visa introduzir um modelo de programação matemática com a finalidade de apoiar o planeamento de rotas e recursos humanos para prestadores de Serviços de Apoio Domiciliário. O modelo assenta, além dos pressupostos de um *Vehicle Routing Problem*, nos seguintes: i) regulações de tempo de trabalho, ii) pausas obrigatórias, iii) autonomia dos utentes, e iv) distribuição de refeições. O modelo, desenvolvido através de software GAMS, foca-se em duas funções objetivo, simultaneamente: minimização dos custos operacionais, e maximização da equidade, através da minimização das diferenças nos tempos de trabalho das equipas. O método de Chebyshev foi o escolhido para desenvolver o modelo multiobjetivo.

O modelo foi construído tendo por base uma Instituição Particular de Solidariedade Social em Portugal. Através da aplicação do modelo, obtêm-se melhorias significativas, quando comparado com o atual planeamento da instituição parceira, como é o caso de uma melhor distribuição da carga de trabalho entre as funcionárias e das rotas que resultam da redução dos custos operacionais da instituição. Este modelo é totalmente extensível a outras instituições que prestem serviços semelhantes ou iguais à instituição utilizada como referência.

Keywords: Planeamento de Rotas; Escalonamento; Otimização; Modelos de Programação Matemática; Serviços de Apoio Domiciliário; MILP; GAMS

JEL Classification System:

C61 – Optimization Techniques; Programming Models; Dynamic Analysis

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Main Abbreviations

CSPPSA	Centro Social e Paroquial da Póvoa de Santo Adrião
CVRP	Classical Vehicle Routing Problem
DSS	Domiciliary Support Service
DVRP	Dynamic Vehicle Routing Problem
GAMS	Generic Algebraic Modelling System
GVRP	Green Vehicle Routing Problem
HC	Home Care
HCRSP	Home Care Routing and Scheduling Problem
HCVRP	Home Care Vehicle Routing Problem
HHC	Home Health Care
HSC	Home Social Care
MDVRP	Multi-Depot Vehicle Routing Problem
MILP	Mixed-Integer Linear Programming
OVRP	Open Vehicle Routing Problem
PISS	Private Institution of Social Solidarity
PVRP	Periodic Vehicle Routing Problem
SVRP	Stochastic Vehicle Routing Problem
VRPPD	Vehicle Routing Problem with Pickup and Delivery
VRPTW	Vehicle Routing Problem with Time Windows

1. INTRODUCTION

In this chapter, the main goal is to give context to the research problem, through the identification of the background that originated the need for developing the present investigation, highlight the main objectives as well as its contribution to the existing literature, and finally define the structure that this thesis will follow.

1.1 PROBLEM STATEMENT

The pace of population ageing is much more accelerated than before. By 2050, the global population of the elderly will escalate to 2.1 billion (WHO, 2019). Along with the ageing of the population, studies suggest that the elderly no longer desire institutional care, privileging the care provided in their homes. This way, Home Care has become one of the most fundamental ways of providing care to old people (WHO, 2012).

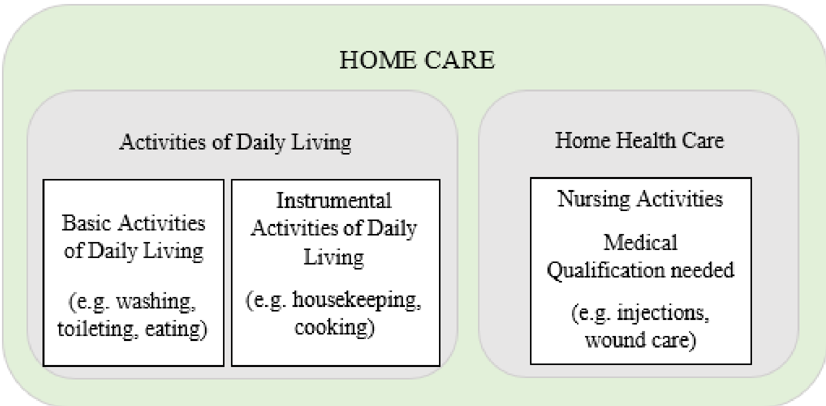
Home Care is defined as the satisfaction of people's health and social needs, in their daily living, through the provision of high-quality home-based services, within the ultimate goal of continuum care (WHO, 2008). Nowadays, it is considered a significant and flourishing sector in the western countries. The main advantages are the pressure decline in hospitals, by releasing beds for people who rely solely on hospitalization, the reduction in health system costs and the improvement in the quality of life for patients, since they remain in their home (Carello & Lanzarone, 2014; Sahin & Matta, 2015).

According to WHO (2008), triggering the need to promote care for the elderly is a recent path for European countries and there is no single history that describes the appearance of the Home Care approach. The development and growth of this sector were essentially due to the urgent necessity to provide care to the neediest, resulting in different policies, funding systems, and care provisions. Initially, health care was only provided to the poorest by hospitals and religious charities. However, this rapidly expanded to social care provision, to other population segments.

In Portugal, it can be considered that the beginning occurred with the publication of the Program of Integrated Support to the Elderly (PAII, *Programa de Apoio Integrado a Idosos*), which established several objectives, namely the promotion of the autonomy of the elderly, development of preventive measures against isolation and the promotion of the training of informal and formal care providers (Santana *et al*., 2007). Given the concerted effort of the Ministries of Labour, Solidarity and Social Security and Health, whom recognize the importance of these services for the community, Home Care has grown over the years through the establishment of social care partnerships and infrastructures (Santana *et al*., 2007).

As demand is becoming farther convoluted, different types of Home Care are becoming more prevailing. Even so, Home Care is located at the intersection between Home Social Care and Home Health Care, in which the provision of services ranges from basic activities, such as housekeeping or personal hygiene, to medical or paramedical activities. The separation between these two services relies on the nature of the service provided.

As shown in the *Figure 1.1*, three types of activities can arise under the Home Care approach. Basic Activities of Daily Living encompass essentially activities considered imperatives to the community life, such as bathing, dressing, eating or getting in and out of bed. Instrumental Activities of Daily Living include services that, although not being necessary daily, still important to the human being, as housekeeping, shopping or taking medication. Nursing Activities relate to medical activities that demand qualified employees, such as giving intravenous injections. As perceived, while Home Health Care is focused on providing medical and paramedical care, the Activities of Daily Living are analogous to Home Social Care, i.e., an area focused on providing social services (Murray, 2008; WHO, 2012). Given that Home Care combines these two distinct facets, its management is convoluted, since it is mandatory to manage physical and material resources that are very different from each other (Carello & Lanzarone, 2014).



Source: Adapted from Murray (2008) and WHO (2012)

Figure 1.1: Home Care, including types of services

The purpose of this thesis is to bring Home Social Care into the domain of research. The main reason why this theme should be integrated within the debate is that, although being a major component in our contemporary society, there are still a set of conditions that limit the provision and/or the quality of services.

In Portugal, Home Social Care services are commonly referred to as Domiciliary Support Service (SAD, *Serviço de Apoio Domiciliário*). Domiciliary Support Service can be developed from a structure created for this purpose or from an existing one, such as nursing home or day care center (Bonfim & Veiga, 1996). In general, Private Institutions of Social Solidarity (IPSS, *Instituições de Solidariedade Social*) and Holy Houses of Mercy (*Misericórdias*) are the main providers of these services (Santana *et* *al*., 2007). Designated as formal care, it is provided by teams of professional caregivers, who provide individualized and personalized care within patients' own home. These services are contracted when, for reasons of illness, disability or others, individuals are not able to handle, temporarily or permanently, their own basic needs. Therefore, the individuals concerned are not only *"frail elderly people, but also patients in need of home care after hospitalization and adults with disabilities"* (Gil, 2009; WHO, 2012).

Within such circumstances, the big goal for Home Social Care across European countries is the improvement of an efficient structure, to promote cost reduction and effective delivery of care (WHO, 2008). With the ever-increasing Home Social Care exigency, staff deficit, rising pressure for improved care, and the increasing demand and costs, developing useful models for adequately planning operations is imperative. Particularly, it is essential to have planning tools to support the outlining of routes that should be established to ensure the delivery of care to all the people in need, as well as, to schedule the activity of the professionals involved in the delivery of such services.

Nonetheless, this adequate planning is not an easy task: there are several actors involved in the process whose activity needs to be carefully planned; there is a variety of services that need to be delivered while respecting the different needs and requirements/preferences of the individuals in need; and there is a need to coordinate both human and material resources while ensuring the provision of quality care under a complex environment.

Hence, two main shreds of evidence are widely recognized: first, most Home Social Care providers perform a hand-planning of the routes, which can result in sub-optimal solutions; secondly, in most cases, the employees dedicated to this responsibility have limited skills in mathematics, since they typically come from a medical background (Eveborn, *et al.*, 2006; Trautsamwieser & Hirsch, 2014; Cissé *et al*., 2017; Fikar & Hirsch, 2017). In fact, this is the reality faced in many institutions in Portugal: the individuals assigned to the planning process have limited skills, perform manual planning, which often consumes too much time, resulting in solutions that are not guaranteed to be optimal.

Within this context, there is a clear need to develop planning tools to support an adequate provision of Home Social Care services, being this particularly relevant for the partner institution, Centro Social e Paroquial da Póvoa de Santo Adrião. This particular organization is used as case study to illustrate the usefulness of the proposed approach.

1.2 Research Question

According to the problem presented above, the research question is defined as follows: ***"How can one improve routing and scheduling decisions for Home Social Care providers currently facing a context of limited resources?"***

1.3 OBJECTIVES

Taking into consideration what was previously mentioned concerning the motivation and the research question of this thesis, its main purpose is to propose a user-friendly model, that can be used by Home Care providers in real-life circumstances, that support the daily planning decisions in the provision of these services.

Within this major objective, there are several specific goals to accomplish, such as:

a) Characterize the Home Social Care provision planning context considered as reference for the dissertation;
b) Building and implementing a planning tool to support the routing and scheduling decisions for home social care providers;
c) Applying the developed tool to the case study of Centro Social e Paroquial da Póvoa de Santo Adrião;
d) Propose managerial recommendations.

1.4 THESIS OUTLINE

With regard to the structure, this thesis is formally split into seven chapters. In this *Chapter 1*, it is presented a brief introduction to the theme and the objectives to be accomplished. *Chapter 2* provides a general framework around the concepts of Vehicle Routing Problem and Home (Health / Social) Care, as well as the importance of all features that impact the planning of routing and scheduling tasks. Also in this chapter are presented some of the studies carried out so far with regard to the routing and scheduling planning, individually and jointly. In *Chapter 3*, it is presented the methodology to be accomplished. In *Chapter 4*, an overview of the partner institution is presented, highlighting essentially their operational procedures, which allows to understand how to build a mathematical model designed specifically for organizations in Home Social Care sector. *Chapter 5* materializes the empirical approach of this thesis and describes the steps followed to build the routing and scheduling model in the mathematical program GAMS, in the context of the partner institution. *Chapter 6* presents the developed model for two mono-objective models, and later for a multiobjective model. The results obtained by running these models are also displayed, according to the materialized in the methodology. Lastly, *Chapter 7* is dedicated to conclusions and the proposal of future research.

2. LITERATURE REVIEW

This chapter introduces the field of Home Care, and a review of several approaches to assist Home Care providers in their daily routine. Section 2.1 outline the general context of Home Care planning, followed by a review of route planning in Section 2.2, and a revision of schedule planning in Section 2.3. Section 2.4 presents a summary of considerations regarding the routing and scheduling simultaneously, which defines the scope of this thesis. By the end of this chapter, the model chosen to be developed will be presented and supported according to the considerations exposed in the literature review and compared with other relevant models.

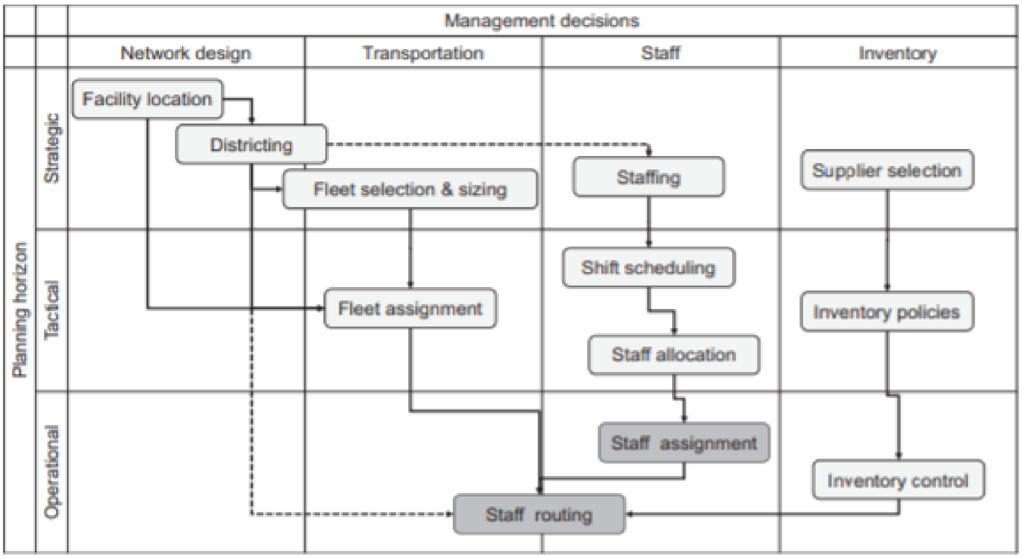
The literature review consisted of a search process based on online databases such as Abi/inform, B-on, Google Scholar and Science Direct. The keywords used, individually or in combination, within the research were: Home Care; Home Social Care; Routing Planning; Scheduling; Multiobjective.

2.1 HOME CARE PLANNING

Home Care (HC) *"can be conceived of as any care provided behind someone's front door or, more generally, referring to services enabling people to stay living in their home environment"* (WHO, 2012). HC services encompass medical, paramedical and social services that are provided to individuals in their homes, relieving the pressure put into hospitals, particularly as regards hospitalization costs (Carello & Lanzarone, 2014). Given the importance of HC in developed countries, the expected growth in demand for the next years, the consequent costs and reasonable restrictions on budget and resource availabilities, it becomes critical to understand the sector specificities and operations (Sahin & Matta, 2015).

As already mentioned, the two valences of HC can be distinguished through the services provided, but not only: while Home Health Care (HHC) services are provided by health care professionals, Home Social Care (HSC) services are delivered by aides, i.e., non-skilled professionals. Likewise, in HHC patients may require different types of care, which means that different professional' categories are needed; in HSC any caregiver can visit the patient since no skills are mandatory. Beyond, for the provision of health care, caregivers need to bring specific medical and paramedical' instruments, while in the case of social care, the equipment required is found at the patients' house (Madigan, 2008; Gomes & Ramos, 2019).

As presented by Gutiérrez & Vidal (2013), HC planning can be designed according to a three-dimensional framework (*Figure 2.1*): i) the planning horizon is defined according to the extent and influence of the planning decision; ii) management decisions are specified according to the logistics functions; and iii) service processes range from support services to medical services. Regarding the planning horizon, and depending on the level of planning, namely strategic, tactical or operational, the time horizon may be long (years), medium (months or weeks) or short (days or real-time), respectively.



Source: Adapted from Gutiérrez & Vidal (2013)

Figure 2.1: Home Care Logistics Framework (Only dimensions one and two are shown here)

The three levels of HC planning depend on how often decisions are made, and their impact period: i) strategic, considering a time horizon of over one year; ii) tactical, considering a time horizon of up to one year; and iii) operational, considering a short period (Cissé *et al*., 2017; Emiliano *et al.*, 2017). These levels are aligned in such a way that the decisions with impacts at a farther horizon are placed at a higher level in the framework. Strategic planning is associated with project decisions that typically last for long periods and involve high levels of investment, such as facility location, fleet selection, resource dimensioning and districting. Consequently, they do not suffer frequent change but involve high levels of uncertainty. Thus, the strategic level tasks are related to how the territory is portioned, creating customers clusters. When compared to the strategic level decisions, tactical planning extends over shorter periods and the level of uncertainty is more controlled. The tactical level comprises fleet assignment, staff dimensioning and shift scheduling. At this level, care providers are concerned about the efficient provision of the services. Lastly, at the operational level, planning involves periods ranging from an hour to a week or month, including staff routing and assignment, and inventory control. Recently, several authors have recognized the fourth stage of planning, real-time level, as in certain situations there is a need to make or change decisions in a very short period (Gutiérrez & Vidal, 2013). These include, e.g., unplanned demands, which urge the redefinition of routes (Sahin & Matta, 2015).

Lanzarone & Matta (2014) point out that HC human resource planning concerns three main topics: i) districting and resource dimensioning, ii) operator assignment, iii) scheduling, routing and control. These three stages relate to the three levels of planning horizon mentioned above. Therefore, the strategic level corresponds to the districting and resource dimensioning, the tactical level to the operator assignment, and the operational level corresponds to the scheduling, routing, and control (Gutiérrez & Vidal, 2013). It is important to notice that while some authors place a scheduling task as relating the operational level, others consider it to belonging to the tactical level (Emiliano *et al*., 2017).

Within the context of these services, several entities have their distinct influence. The terms defined as follows are those used throughout this thesis: i) the client/user/patient is the individual who live in their own home, and request one or more services per week; ii) the employees/caregivers/care providers are the individuals who provide the services at the clients' homes; and iii) the HC provider is the institution that employs the caregivers and has the responsibility to plan the routes and schedule of visits (Guericke, 2016). Hence, the routing and scheduling can be characterized as follows (L. Xiao *et al.*, 2018): each client has several requested services per week; a set of caregivers provides services at the clients' homes; each visit has a set of requirements, namely the possible days, caregivers' availability, among others; considering these requirements, the HC provider plan the routing and the scheduling of each caregiver.

Therefore, two main problems appear, and end up being transversal to all HC approaches: decide which caregiver will be assigned to each client, and establish a schedule for each visit, and consequently the order of appointments (Lanzarone *et al.*, 2012; Cissé *et al.*, 2017; Emiliano *et al.*, 2017).

With what being said, it becomes more perceptible that this thesis aims to support operational planning of HSC services, in order to improve routing and scheduling tasks. This way, the next two chapters provides a review of routing and scheduling planning.

2.2 ROUTE PLANNING

In the routing task, the HC provider arrange the jobs to tours, and establish the sequence of visits, which is known as routes (Guericke, 2016). In the field of Operations Research, several models have been developed to solve HC routing problem. One of them is the well-known Vehicle Routing Problem (VRP). Given that this method is the most applied in route planning (Cissé *et al*., 2017), a detailed review is presented in the following section.

2.2.1 VEHICLE ROUTING PROBLEM

Dantzig & Ramser (1959) were the first to introduce the Truck Dispatching Problem, considered as a generalization of the Traveling Salesman Problem, presented by Flood (1956). They were the pioneers by introducing the first mathematical programming formulation and algorithm approach of VRP, with a real-world application. In its elementary way, this problem deals with the design of the shortest route through the linkage of n given points. Consequently, publishing how a fleet of homogeneous trucks could meet the demand of a various number of gas stations, from a central hub, traveling the minimum distance as possible, gave rise to increasing interest and a rich body of research. A few years later, Clarke & Wright (1964) improved the approach stated above by proposing a heuristic, and generalized to an optimum routing of a fleet of trucks with different capacities from a central depot, comprising an extensive number of routes, as much as the number of delivery points increased. This became known as the VRP. Currently, it is a widely studied combinatorial optimization problem in the area of Operational Research, with applications in many fields, e.g. food delivery (Russell, 1995) and newspaper distribution (Toth & Vigo, 2002).

The VRP is usually defined as the design of optimal routes from a central depot to a set of geographically dispersed customers, under certain constraints, e.g. the capacity of the vehicles, distance, time windows and precedence relations between customers (Laporte, 2007). This way, $G = (V, A)$ is a graph where V is the set of vertices and A the set of arcs. The set of vertices represents all the customers (or requests) that need to be served and the single depot from which any route must begin and end.

In the route setting, two different levels of the decision can be considered (Carvalho, 2012):

i) The tactical problem, in which the decision level is linked to routing planning in a medium-high term since regular demand and deliveries are assumed;
ii) The operational problem, in which routes are defined daily, as demand and customers are not regular (routes turn out to be variable).

Díaz-Madroñero *et al*. (2015) proposed classification criteria relating to production and routing problems, in which they defined five subgroups of features that characterize this problem. One of these is routing aspects, where they pointed out the forward elements:

i) A fleet of vehicles: characteristics of vehicles concerning their number and capacity (if the vehicle is capacitated and if capacity is equal for all vehicles);
ii) Number of trips and visits per vehicle: number of trips prosecuted by each vehicle during a given period with a beginning and ending at the central depot;
iii) Transport data: related to transport parameters, namely transport time and distance between a pair of nodes, loading or unloading times, waiting times and time windows.

The goal of the VRP is the determination of a set of routes, each of them performed by a single vehicle while fulfilling customers' requirements and constraints and minimizing transportation costs. The most broadly studied objectives are as follow (Toth & Vigo, 2002; Y. Xiao *et al.*, 2020):

i) Minimization of the global transportation cost, dependent on global distance traveled or global travel time, and on fixed costs (see Bala *et al.*, 2017);
ii) Minimization of the number of vehicles required to face the demand (see Vonolfen & Affenzeller, 2016);
iii) Balancing of the routes, relating travel time and vehicle load (see Jozefowiez *et al.*, 2007);
iv) Minimization of the penalties, relating partial service of the customers (see Soman & Patil, 2020);
v) Minimization of fuel or CO2 emissions by optimizing vehicle payload (see Gaur *et al.*, 2013);
vi) Minimization of CO2 emissions through vehicle speed optimization (see Jabali *et al.*, 2012);
vii) Minimization of CO2 emissions via avoiding traffic jam (see Y. Xiao & Konak, 2017);
viii) Minimization of driver and fuel costs, through the optimization of speed and load simultaneously (see Bektaş & Laporte, 2011).

According to Daneshzand (2011), two main reasons explain why researchers are so interested in studying the VRP: its relevance to real-life applications and its difficulty. Lenstra & Kan (1981) have analyzed the complexity of the VRP and have concluded that nearly all VRPs are NP-hard (Non-deterministic Polynomial-time hard), which means that an efficient algorithm for solving this problem to optimality is unavailable within a reasonable time window. As stated by Ho *et al.* (2008), VRP is easier said than done.

Eksioglu, Vural & Reisman (2009) state that, although some Operational Research disciplines present exceptional rates of growth, the VRP literature has been growing exponentially at a rate of 6% per year. Despite not being as rapid as others, we can say that VRP literature is growing at a pace hard to keep on track. However, current VRP models are far as different from the one introduced by Dantzig & Ramser (1959), since they increasingly incorporate real-life characteristics and constraints, resulting in an enormous amount of publications in this field.

Since the researchers study the problem in their environment, i.e. with the reality of the country, under a certain manner of organization and with models applied within a specific institution, the conclusions that are accomplished are considerably different from each other. In fact, according to Laporte (2007), the reason why does not exist a consensus on the definition of VRP is due to the diversity of constraints encountered. Over the years, due to the considerable variability of real-life scenarios, the overall models contributed to the development of several Vehicle Routing Problem variants.

2.2.1.1 VEHICLE ROUTING PROBLEM VARIANTS

The Classical VRP (CVRP), also known as Capacitated VRP, constructs optimal delivery routes where each vehicle only performs one route, all the vehicles are equal in terms of characteristics and there is only one central hub. Each vehicle has a pre-set capacity, and so extra-loading is not allowed. This way, a fleet of m capacitated vehicles have to supply n customers with demand d. The main objective is to minimize the total cost, which can be obtained through the sum of total distance traveled, the number of vehicles required or a combination of these two factors (Taş *et al.*, 2014).

Yet, the CVRP is not suitable in various cases. Since organizations aim to accomplish the constantly increasing demand for HC services, they readjust themselves to the reality they intend to serve, generating distinct variants of the same main problem (Braekers *et al.*, 2016). From the existing literature, it is possible to recognize several VRP classes, as described below.

Multi-Depot Vehicle Routing Problem (MDVRP)

In Multi-Depot Vehicle Routing Problem, unlike CVRP and several variants, there is more than one depot, since, in some practical situations, a single depot is not suitable. Then, it is considered a homogeneous fleet, and customers' location and demand are known *a priori*. MDVRP can be solved in a two-stage approach, where first customers are assigned to depots, and then are designed routes for each depot (Ho *et al*., 2008). However, it is more efficient to handle the two steps as one (Daneshzand, 2011). Similar to the CVRP, the major objective is to minimize the total distance of routes and consequently the number of vehicles required (Espadinha & Grilo, 2018).

Open Vehicle Routing Problem (OVRP)

Open Vehicle Routing Problem appears for the companies that do not own a fleet of vehicles or vehicle fleet is insufficient for the demand and need to contract this kind of service. Thus, in this variant of VRP, hired vehicles are assigned to routes and are not required to return to the company's distribution center (Daneshzand, 2011). In general, minimization of the total distance traveled and minimization of the number of vehicles needed to serve all the customers are the objectives considered (Li *et al.*, 2007).

Periodic Vehicle Routing Problem (PVRP)

In Periodic Vehicle Routing Problem, a set of customers is visited one or more times within an exact time horizon. Thus, visits take place on a multi-day time horizon: several customers are assigned to specific days, according to their needs, and routing is performed daily (Daneshzand, 2011). As stated by Irnich *et al.* (2014), PVRP encompasses three different types of decisions: when and how frequently to visit each customer, how to cluster customers into tours on each day and how to design the routing of each vehicle for each cluster. This way, the main objective is to minimize the total distance traveled, while fitting customers to appropriate visiting days, which constraint the design of the minimum routes for each day (Cacchiani *et al.*, 2014).

Vehicle Routing Problem with Pickup and Delivery (VRPPD)

In this variant, goods need to be transported from several points to different destinations (Battarra *et al.*, 2014). Essentially two tasks are performed, namely gathering and delivering goods at customers' locations. Thus, as stated by Daneshzand (2011), each request is entitled by a pick-up point, a delivery or drop-off point and demand that needs to be transported between these two spots. The objective is the minimization of total distance traveled regarding capacity constraints (Espadinha & Grilo, 2018).

Vehicle Routing Problem with Time Windows (VRPTW)

Given that most practical situations include restrictions on the visiting time, it is opportune to present a more suitable solution. In this generalization of the CVRP, it is defined upper and lower bounds in time to delimit the occurrence of a certain event, designed as a_i and b_i, such as service at each customer must start within a specific time window $[a_i, b_i]$ (Cordeau *et al.*, 2007). Along with this obligation, each vehicle must remain at the customer's location during the assistance (Daneshzand, 2011). According to Tan *et al.* (2001), the objective of the Vehicle Routing Problem with Time Windows is to serve several customers at the minimum total distance possible, while accounting for capacity and time constraints. This variant continues to be one of the most difficult problems in combinatorial optimization (Xu *et al.*, 2015).

Time Windows can be categorized as hard or soft. In doing so, the VRPTW can be subdivided into the VRP with Soft Time Windows constraints and VRP with Hard Time Windows constraints. Concerning a hard time window, it is allowed for a vehicle to arrive before a_i and wait for the customer to be disposable, but not after b_i. This waiting time does not incur any costs. In turn, with a soft time window, constraints can be violated by augmenting the cost function with a penalty (Taş *et al*., 2014).

Green Vehicle Routing Problem (GVRP)

Over the past few years, Green Logistics has emerged and received great attention in organizations. Green Vehicle Routing Problem is a variant of the VRP that, in addition to considering monetary costs (such as conventional VRPs), also incorporates environmental impacts of the routes, and try to optimize both the functions together. Therefore, the reduction of pollution and emissions, and the sustainability of environment are the major topics studied in this variant (Y. Xiao *et al*., 2020). The main objective is to establish sustainable routes that allow the reduction of the negative impact on the environment (Wang *et al*., 2019).

Stochastic Vehicle Routing Problem (SVRP)

Since Dantzig & Ramser's (1959) work, the majority of the studies have proceeded under the assumption that all the information required to develop the problem is known and fully available. Yet, in a real world-life application, the presence of uncertainty makes this assumption not verifiable. This way, unlike the deterministic CVRP, where the associated parameters are deterministic, the Stochastic Vehicle Routing Problem considers some elements as random (Daneshzand, 2011). According to Gendreau *et al.* (2014), essentially three elements are considered to be stochastic within the VRP, namely demands (since the product' volumes are random), customers (given the probability of availability or unavailability) and times (usually service and traveling). As this problem merges characteristics of stochastic and integer programming, it is not easily solved (Gendreau *et al.*, 1996).

Dynamic Vehicle Routing Problem (DVRP)

VRP is defined as Dynamic when information evolves during the process, such as happens when some parameters are expressed through a function of time. This means that the inputs can change both during model execution and planned route execution (Psaraftis, 1988). Thus, this variant allows changes through additional decisions and attempts to deal with uncertainty in real-time. Bektas, Repoussis, & Tarantilis (2014) exemplify activities that can occur under DVRP, namely vehicle vacancy or cancellations, which are defined as interacting activities. According to Pillac *et al.* (2013), online routes are defined, since the dispatcher is informed of any changes instantly.

It is important to note that, although DVRP type allows to deal with routing problem in a real-time fashion, an investment in technological structures and machinery is needed to enable an effective and efficient communication (Bektas *et al.*, 2014).

Along with the development of VRP variants, the solution methods also ended up becoming more complex and applicable in practice, as one searches for more realistic approaches.

2.2.1.2 Solution Methods

VRP is an NP-hard problem which, according to Laporte (2007), becomes exponentially more complex to solve along with the size of the problem. However, technological advances, namely processing speed and memory capacity, allow researchers to pursue and develop models more convoluted and with even more constraints, making it more a real-life approach.

Therefore, two types of solution methods commonly appear in the literature, which are presented subsequently.

Exact Algorithms

Exact algorithms can be defined as algorithms that always solve an optimization problem to an optimal solution. Exact algorithms are essentially based on integer programming, dynamic programming, and branch-and-bound. Despite its wide application, these methods require large mathematical programming equipment and has a variable success rate. According to Fikar & Hirsch (2017), branch-and-price and branch-and-price-and-cut algorithms procedures prevail in the literature.

Heuristics

Heuristics are processes for determining good quality solutions in a short time for difficult-to-solve problems. In Laporte (2007), two predominant groups of heuristics are studied: (i) classical heuristics, and (ii) metaheuristics. The classical classification stands for the fact that the process starts from a point towards an optimal one in its neighborhood, until no improvement can be obtained. Within classical heuristics, it is admitted constructive and improvement heuristics, as follow:

i) A constructive heuristic allows obtaining an acceptable solution based on a set of stipulated rules, adapted to the problem in question. The solution method begins with an empty solution, and iteratively extends until a feasible solution. The most widely known constructive heuristic is the already referred work proposed by Clarke & Wright (1964), which is called Savings Algorithm.
ii) Improvement heuristics are based, as the name implies, on improving an existing solution, called the initial solution, which must be admissible. In VRPs structures, two improvement algorithms are applied: while (i) intra-route methods optimize each solution individually by means of a Traveling Salesman Problem improvement heuristic, (ii) inter-route heuristics are based on shifting vertices from one to another route (Laporte, 2007).

For instance, metaheuristics allow the consideration of non-improving or infeasible intermediate solutions. They can be applied to several optimization problems with relatively few adaptations, and the aim is to examine the search space to find near-optimal solutions. Generally, these procedures embody some classical form of heuristics and can be classified into three types: (i) local search, (ii) population search and, (iii) learning mechanisms (Laporte, 2007).

Now that the VRP, its variants and solution methods have been revised, it is important to begin to contextualize HC services under a VRP. This way, in this section, various dissimilarities, objectives, constraints and solution methods that managers face when planning the HC delivery are reviewed.

2.2.2 HOME CARE AS A VEHICLE ROUTING PROBLEM

To the authors' best knowledge, routing planning is not usually applied to HSC, which makes few studies in this area. Besides, some authors disregard HHC definition, studying this topic under the assumption of medical and social services provision (yet, defining it as HHC). Given all this, the objectives, constraints and solution methods' review is presented as a combination of HHC and HSC features, thus providing a holistic view of HC.

Based on Gutiérrez & Vidal (2013), Cissé *et al.* (2017), and Fikar & Hirsch (2017) frameworks, and regarding HC literature review, Home Care Vehicle Routing Problems (HCVRPs) differ in several dimensions, particularly:

i) Single- or Multi-Period Models;
ii) Deterministic or Stochastic Models;
iii) Hard or Soft Constraints;
iv) Single- or Multiobjective Models.

After presenting the before mentioned categories of problems, key constraints of HCVRPs are also presented.

2.2.2.1 SINGLE- OR MULTI-PERIOD MODELS

The planning period can go from a single day to multiple months. Single-period models are the most widely used when addressing HHC instances, which reflects the fact that few authors are concerned about the continuity of care (Wirnitzer *et al.*, 2016; Fikar & Hirsch, 2017). Yet, as denoted by Trautsamwieser & Hirsch (2014), the daily planning period is a relaxation of weekly planning, since working time is only planned for one specific day and some features are disregarded, such as maximum working time per week and rest breaks between working days. Therefore, when clients request multiple care services spread over a given planning horizon (which can be a week or a month), multi-period models are outlined. This latter approach appears in HSC cases, where the continuity of care is imperative (Carello & Lanzarone, 2014; Gomes & Ramos, 2019).

Fikar & Hirsch (2017) state that metaheuristics solution procedures are the most applied for single-period models. On the other hand, to solve multi-period problems, the authors focus essentially on both metaheuristics and exact methods. Given the uncertainty inherent in the multi-period models, some works present simulations approach to deal with dynamic events, e.g. new patients' requests. Thus, the most generally developed are Monte Carlo, Markov decision processes and discrete-event simulations.

2.2.2.2 DETERMINISTIC OR STOCHASTIC MODELS

As mentioned earlier, models can be considered deterministic or stochastic, depending on how they deal with uncertainty. While deterministic models assume that information is known in advance, in stochastic models data is considered not known *a priori*, and uncertainty is incorporated in the models (Daneshzand, 2011).

Although stochastic models play a key role in designing real-life features, most literature in HC has been proceeding under the assumption of deterministic models, ignoring the impact that uncertainty has on planning decisions (Fikar & Hirsch, 2017). Under this presumption, service quality may become substandard if changeableness is neglected at the planning level. To overcome this problem, stochastic variants have been introduced (Taş *et al*., 2014), namely the duration of the appointment or the patient/caregiver' absenteeism (Gomes & Ramos, 2019).

2.2.2.3 HARD OR SOFT CONSTRAINTS

Similar to the idea behind soft and hard time windows, other variables in HC models can be split into soft and hard constraints. This way, where hard constraints need to be obligatorily satisfied, soft constraints may be violated, although the infringement should be as minimum as possible. The brands "hard" or "soft" are defined following the HC analyst(s), i.e., the person/people who provide information about a problem and represent their(s) requirements (Hiermann *et al.*, 2015).

As an example, Bertels & Fahle (2006) nominated some hard and soft constraints. Thus, hard ones include working time regulations and qualification requirements. By contrast, soft constraints encompass the good relationship between patients and care providers, maintaining the same nurses per patient, maintaining the visit' daytime and staff satisfaction. For instance, Nickel *et al.* (2012) defined the continuity of care as a soft constraint.

2.2.2.4 SINGLE- OR MULTIOBJECTIVE MODELS

According to Cissé *et al*. (2017), to evaluate the quality of solutions, authors have been proposing single and multiobjective functions. In the case of multiobjective models, the number of criteria differs from 2 (see Braekers *et al.*, 2016; Fathollahi-Fard *et al.*, 2018; Haddadene *et al.*, 2019) to 13 (see Hiermann *et al*., 2015).

Many authors opt to consider only one single objective (see Akjiratikarl *et al.*, 2007; Bachouch *et al.*, 2011), disregarding multiobjective functions. However, multiple and conflicting objectives need to be pursued, since real-world problems are multiobjective (Cohon, 1978). Hence, in recent literature, models begin to address multiple objectives simultaneously, such as travel time or cost, service quality and patients or care providers' preferences (Decerle *et al.*, 2019). When considering the application of different objectives, the majority of studies formulate the problem with weighted objective functions (Fikar & Hirsch, 2017).

Concerning the main objectives, essentially two groups can be specified: objectives regarding the VRP (transversal to almost all models, including in HC), and objectives regarding to HC particularities.

In general, most literature contemplates the following objectives (Cissé *et al*., 2017): i) minimization of total routing cost; ii) minimization of the number of care workers; iii) minimization of the number of unassigned services; and iv) maximization of satisfaction/service level. The minimization of total routing costs is a standard objective in the formulation of the Home Care Routing Problem (HCRP). This objective is, in many studies, expressed as minimization of total traveling cost (see Shi *et al.*, 2019), total traveling time or duration (see Trautsamwieser & Hirsch, 2011), total travel distance (see Mankowska *et al.*, 2014) or even total working time (see Decerle *et al.*, 2019). The minimization of the number of unassigned services arises when HC provider may not have sufficient caregivers to serve the patients (see Liu *et al.*, 2017). The minimization of the number of care workers has the advantage of reducing staff costs, but the HC provider need to ensure that the provision of services is not compromised (see Hewitt *et al.*, 2016). Lastly, concerning the maximization of satisfaction, two types of actors can be considered, namely the patient and the caregiver. The majority of papers regards the maximization of service level accounting for patients' preferences, such as visit times (see Braekers *et al*., 2016).

According to Fikar & Hirsch (2017) other objectives include i) the minimization of the overtime (see Bard *et al.*, 2014), ii) the maximization of fairness or equity, such as, workload balance (see Cappanera & Scutellà, 2015; Gomes & Ramos, 2019), and iii) the maximization of the number of tasks performed (see Nickel *et al.*, 2012).

As a result, many papers have considered multiobjective within HCRP, many of them to reflect the goals/preferences of decision-makers (Decerle *et al*., 2019).

2.2.2.5 MAIN CONSTRAINTS IN HOME CARE AS A VEHICLE ROUTING PROBLEM

In a recent work, Cissé *et al.* (2017) schematized the constraints within a cross table. Thus, constraints can be characterized as temporal, assignment or geographic, depending on the related actors, namely the provider, the patient or the caregiver. The first group concerns essentially aspects associated with time, such as the planning horizon, the frequency of visits, or the caregivers' working hours. The assignment constraints encompass key features that help understand which caregiver can be assigned at each patient, namely the continuity of care and the preferences. The third group relate geographical aspects, as the location of the caregivers.

Additionally, Fikar & Hirsch (2017) highlight the most prevalent constraints in the literature, namely the time windows defined to initiate the visit (see Allaoua *et al.*, 2013), patients' skills requirements, i.e., matching the skills of nurses and patients' prerequisites (see Carello & Lanzarone, 2014), and working time regulations (see Bräysy *et al.*, 2009). On the other hand, caregiver's breaks (see Trautsamwieser & Hirsch, 2014) and continuity of care (see Maya Duque *et al.*, 2015) are the less considered. One critical constraint (especially in big city centers) that has a huge impact on travel times and consequently in traveling costs is traffic conditions, which has been also disregarded in almost all works.

2.3 SCHEDULE PLANNING

One of the first works in the area of scheduling was developed by Fernandez *et al.* (1974). In this work, the authors proposed several assigning methods to different rural areas, estimating travel times and assessing the maximum number of services that each nurse could perform, without any optimization technique. Currently, several authors define the scheduling problem based on a generalization of the VRP, allowing the addition of some particular features, typically temporal or spatial constraints associated with tasks, caregivers, and patients. This way, it is generally defined as a VRPTW, which involves the seeking of efficient routes, where the service starts within a given time interval (Kallehauge *et al.*, 2005), along with the minimization or maximization of pre-defined objective(s) function(s) (Akjiratikarl *et al.*, 2007).

According to Akjiratikarl *et al.* (2007), optimization techniques are developed to provide a schedule solution under the minimization of the total distance traveled. This way, the potential advantages of efficient scheduling are as follow:

i) The reduction of the traveling distance, and consequently, the traveling costs;
ii) The improvement in worker utilization by reducing the time spent traveling, and hence reducing the number of workers needed;
iii) The growth of customer service by meeting their requirements within a specific time window;
iv) Better time management for executives, freeing them for other regulatory and strategic tasks.

Scheduling is a decision-making process and a fundamental component for the daily life of any company, in any industry. Similarly to the routing problem, giving the rising demand for HC services, the caregivers' deficit and constant pressure to reduce or contain costs, it becomes imperative to optimize scheduling operations (Mutingi & Mbohwa, 2013). As stated by Jemai *et al.* (2013), only if the routing solution already considers the assignment problem and the minimization of transportation costs, it is possible to obtain a good scheduling guide.

Given that this area of research is relatively new, whose interest has been rising along these years, no single classification has been defined. Thus, this family of problems is known as Home Care Staff Scheduling (Mutingi & Mbohwa, 2013), Home Care Scheduling Problem (Martin *et al.*, 2020), Home Care Worker Scheduling (Akjiratikarl *et al.*, 2007) or even Home Care Crew Scheduling (Rasmussen *et al.*, 2012). In all cases, it consists on assigning caregivers to optimal routes, to provide care to patients, at specific times, in their homes, located in a definite geographic area, and the scheduling is repeated periodically (Akjiratikarl *et al.*, 2007; Martin *et al.*, 2020).

As mentioned earlier, the terms of HHC and HSC are often used interchangeably. This exchange of terms, although done in the wrong way, does not cause different results when defining a routing problem. However, when developing a scheduling problem, it is important to recognize what kind of services a particular institution provides, given that the nature of the activities has an impact on the objective(s) and constraint(s) to be defined (Rendl *et al.*, 2012).

Firstly, HSC workers generally spend more time providing their services. As an example, while a HSC worker may take a few hours in the habitational hygiene maintenance, a HHC worker when providing medical or paramedical services, may take just a few minutes to administer an injection or collect blood. Secondly, given the non-medical nature of the tasks performed under the HSC services, there is greater flexibility, when compared to HHC services. According to the same example presented above, habitational hygiene maintenance has greater flexibility, both at the time scheduled for its realization, i.e., it can be carried out in the morning or the afternoon, taking into account the users' preferences, as well as in its duration. Likewise, the frequency with which this task is performed can also be reduced; however, when it comes to the administration of medicines, this flexibility does not apply, given the medical nature of the tasks (Mosquera *et al.*, 2019).

One of the common and current practices when designing a scheduling plan is the construction of what is known as scheduling with aggregated tasks. In this kind of planning, tasks are scheduled in blocks, not concerning what kind of individual services are programmed. Yet, as introduced by Mosquera *et al.* (2019), patients' contracts are becoming more detailed, which makes scheduling with aggregated tasks out-fashioned, since they did not consider the desired frequency, duration and priority. Thus, scheduling with disaggregated tasks allows better management of time by considering individuals tasks.

As presented by Pinedo (2008), the scheduling configuration can also be divided into two main classes: (i) deterministic models, and (ii) stochastic models. As mentioned earlier, the main difference between these two is that, in stochastic models, are considered sources of uncertainty or randomness, which are modeled in several ways.

When computing HC scheduling, the overall goal is to find valid solutions that satisfy both HC provider, caregivers and patients. Therefore, next two sections provide a review of the objectives and constraints most frequently applied in this formulation.

2.3.1 MAIN OBJECTIVES

As mentioned above, the scheduling planning can be formulated as an extension of the VRP, with the introduction of the time windows, conceiving therefore one of the VRP variants specified already. This way, Rendl *et al.* (2012) and Jemai *et al.* (2013) outline the following objectives, that can be a complement to the previous routing problem:

i) The minimization of traveling distance and/or traveling time, and consequently the traveling costs;
ii) The maximization of patients' satisfaction, e.g., through the minimization of patients' preferences deviations or by meeting their requirements within a specified time frame;
iii) The maximization of caregivers' satisfaction, e.g., through the minimization of caregivers' preferences deviations;
iv) The improvement of caregiver utilization and, hence the reduction of the number of care providers needed;
v) The balance of work distribution among caregivers.

2.3.2 MAIN CONSTRAINTS

The literature review disclosed that common constraints in HC scheduling can be widely classified into two main groups, namely hard and soft constraints. The aim is to schedule caregivers through the complete satisfaction of hard constraints while trying to accomplish soft constraints. As already noticed, hard constraints must be satisfied, while soft constraints can be violated, even if it is not desirable, incurring into a penalty (Mutingi & Mbohwa, 2013).

Time windows is, by far, the most considered constraint, since in every model it is defined a time interval at which the service should start, which can be labeled as hard (see Akjiratikarl *et al*., 2007) or soft (see Rendl *et al*., 2012).

Mutingi & Mbohwa (2013) classify several constraints based on three main categories, namely i) time, ii) demand, and iii) preferences. Time-based constraints include essential aspects related to employees' working hours, the sequence of assignments that must be followed and the time window in which each task must be performed (see Akjiratikarl *et al*., 2007). In turn, demand-based constraints are related to the minimum number of staff required to carry out the tasks (see Eveborn *et al.*, 2006). Finally, preference-based constraints encompass the desires of the staff, patients and the organization: patient-workers loyalty, predefined time windows, management goals and staff preferences on schedule times (see Trautsamwieser & Hirsch, 2011).

One of the most important constraints, especially for HSC structures, was initially introduced by Begur *et al.* (1997), which is known as the spreading of tasks. This specific feature ensures that, whenever an activity is required by a patient more than one time in a week, it is distributed over different days, to prevent the provision of the service on consecutive days (Maya Duque *et al.*, 2015; Mosquera *et al*., 2019).

Another recent feature that has received attention is fairness, where it can be found essentially two approaches: i) balance workload of staff members and ii) equity regarding task assignments (Mosquera *et al*., 2019). While the former approach regards the works' schedule and overtimes (see Mutingi & Mbohwa, 2013), the last one notices essentially the demand balancing among employees (see Mankowska *et al*., 2014).

Continuity of care is also considered in these models and assure that the same worker or group of workers are assigned to the same patient during the whole treatment process, which preferably provides all the services required (Maya Duque *et al*., 2015). The main advantages are the prevention of loss information among employees and the maintenance of a reliability relationship between patient and caregiver (Lanzarone & Matta, 2014). Related to this constraint is loyalty, which conveys the same idea. To the authors' best knowledge, Gomes & Ramos (2019) designed a unique loyalty scheme, where loyalty was pursued within a week, but weekly, the caregivers rotated among patients (non-loyalty), to prevent musculoskeletal injuries.

Patient and caregivers' preferences can also play a part, whether individually or simultaneously (see Rendl *et al*., 2012; Hiermann *et al*., 2015).

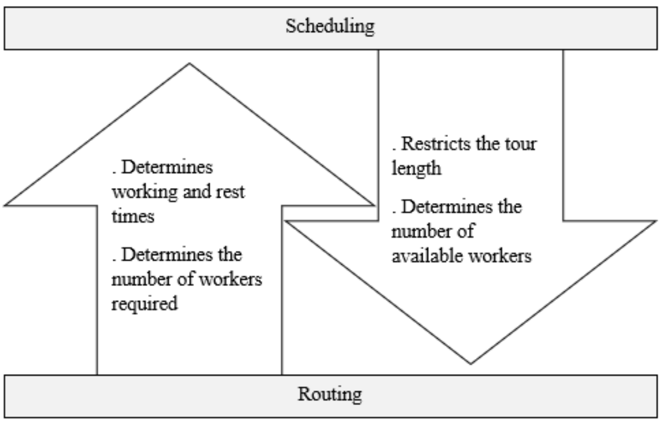
As highlighted, all along this review, the central aim is to develop and test a mathematical model that includes the routing and scheduling of a HC Provider. Therefore, the next chapter presents a summary of the dependency relationship between these two tasks, the main objectives and constraints applied in models that contemplate simultaneously both routing and scheduling, and the final conclusions.

2.4 HOME SOCIAL CARE ROUTING AND SCHEDULING PROBLEMS

The operational outlining of a HSC provider is established in two planning tasks, namely routing and scheduling. After being determined individually, the two plans need to be combined in a joint schedule. In this joint planning, certain requirements have to be taken into account (Guericke, 2016):

i) Time windows: the time interval that restricts the starting time of a specific service (see Kallehauge *et al.*, 2005);
ii) Shift: the time interval that restricts the start and ending time of employees' availability, i.e., the daily working time (see Trautsamwieser & Hirsch, 2011);
iii) Caregivers' availability: the availability of employees can be restricted due to vacations, days off or specific days for training (see Shao *et al.*, 2012);
iv) Breaks: legal interruptions of activity during working hours (see Trautsamwieser & Hirsch, 2011);
v) Rest times: legal pauses between shifts (see Trautsamwieser & Hirsch, 2014).

Considering a holistic view, it is important to respect all the aspects from both approaches, since the dependency between those is evident (see *Figure 2.2*). The number of visits is limited by the number of care providers assigned. If there are few caregivers, not all jobs can be considered on the routes. Otherwise, if the caregivers are too many, the planning becomes inefficient. In addition, it is important to know the exact working times and breaks, since the daily working time of the employees limits the length of the routes (Guericke, 2016).



Source: Adapted from Guericke (2016)

Figure 2.2: Dependency between Routing and Scheduling

During the planning process, two different settings can arise, namely the static and dynamic ones, which relates to the evolution of information, i.e. the fact that information provided to the planner may change during the execution of the routes (Pillac *et al.*, 2013). In the previous feature, there are no modifications of patients and caregivers during an established planning horizon. The great advantage is that employees know, in advance, the working hours, and can consequently plan their life. Thus, as patients' demands and caregivers' working hours are stable, the aim is to minimize the tour lengths. Yet, given the HC circumstances, many organizations account for uncertainty, i.e., several changes of demands, availabilities, and capacities. Therefore, under this context, along with the minimization of the tour lengths, it is necessary the consideration of the continuity between periods (Guericke, 2016). As stated by Di Mascolo *et al.* (2017), almost all HC problems deal with static features, with the minority dealing with dynamic aspects (see Nickel *et al*., 2012; Carello & Lanzarone, 2014).

The Home Care Routing and Scheduling Problem (HCRSP) was originally solved on a daily planning horizon, and integrated practical constraints, such as the maximization of preferences (see Eveborn *et al.*, 2006; Braekers *et al.*, 2016). Thenceforth, models continued to evolve to a weekly horizon, integrating even more practical and real-life constraints, such as continuity of care or patients care plan (see Maya Duque *et al*., 2015).

Given the evolution of the models, which over the years have increasingly incorporated aspects closer to real-life instances, it becomes important to know and monitor these features. Therefore, the next two sections provide a review of the most commonly applied objectives and restrictions.

2.4.1 MAIN OBJECTIVES

Without a doubt, the most widespread objective is the minimization of transport costs, total distance or total travel time, which is often combined with patients' or workers' preferences (Braekers *et al*., 2016).

The most common objectives in these models can range from economic to satisfaction goals. As stated above, the most prevalent relates the routing costs, as defined in Trautsamwieser & Hirsch (2011), or Mankowska *et al.* (2014). The second commonplace goal is the introduction of nurse or clients preferences, which are considered in Bredström & Rönnqvist (2008). Another common feature is the consideration of working time regulations, as in Begur *et al.* (1997) or Trautsamwieser & Hirsch (2014).

Additional characteristics also contemplated in these studies are known as continuity metrics, namely continuity of care and continuity of time, which can be considered in the objective function or as a constraint (Milburn, 2012). The previous one minimizes the number of caregivers visiting a patient during the planning horizon. Within this feature, Wirnitzer *et al*. (2016) specified six objective functions that are related to the satisfaction level of patients, particularly the minimization of the: i) number of different nurses per cluster, ii) number of different nurses per patient; iii) relative number of different nurses per patient; iv) number of nurse-switches per patient, and v) number of relative nurse-switches per patient. On the other hand, continuity of time minimizes the deviations in visit times.

2.4.2 MAIN CONSTRAINTS

As already shown in the previous chapters, some constraints must be considered when defining routing and scheduling problems individually. However, when considering the two plans simultaneously, the ultimate goal is to find optimal routes that must be followed by the employees, to provide the care required by the patients, within specified time windows (Jemai *et al*., 2013). According to Eveborn *et al.* (2006), the optimal solution entails the following constraints:

i) Each worker must attend the proposed assignment and must visit the patients in a precise scheme;
ii) Each patient must be visited the exact number of required visits;
iii) Visits must be performed within the predefined time windows;
iv) Each staff member has programmed breaks, legally accounted;
v) Travel time between patients' visits need to be appraised;
vi) Each route must start and end at the specified central point (generally the depot).

According to Di Mascolo *et al*. (2017), HC constraints can be divided into two main categories: one relating patients, and the other relating the caregivers. The constraints concerning the patients can be detached into two subgroups:

i) Temporal constraints, related essentially to the starting hour of the visit, preference days, and precedence of services (see Bredström & Rönnqvist, 2008);
ii) Assignment constraints, concerning basically the patients' preferences and dislikes, and the continuity of care (see Bertels & Fahle, 2006).

For instance, the concerning the caregivers can be divided into three branches:

i) Temporal constraints, concerning the maximum working time, the availability days, breaks, days off and travel time depending on public transport (see Bard *et al.*, 2014);
ii) Assignment constraints, related to the employees' preferences and vehicle capacity (see Trautsamwieser & Hirsch, 2014);
iii) Geographical constraints, concerning the location of patients' home and organization headquarters (see Nickel *et al*., 2012).

2.5 CONCLUSIONS

The literature review provided an overview, from the general to the particular, of distinct topic areas. First of all, it allowed distinguishing the two valences of Home Care, their main differences, and how these characteristics can impact the planning of several tasks. Secondly, it enabled an understanding of the most studied problem in the area of Operations Research – the Vehicle Routing Problem, especially its typical aspects. Thirdly, the analysis of the Vehicle Routing Problem within the Home Care context allowed the awareness of its specificities, which are generally common, and also the understanding of additional features that can be originated within specific contexts, which conceive different constraints, and consequently distinct problem formulations.

Table 2.1 summarizes the key features that were introduced in certain models, considered relevant by the authors, that configure the routing and scheduling problems within the Home Care context. The first consideration is that a great part of the models focuses on Home Health Care to the detriment of Home Social Care, making a few models developed within this field. Secondly, most of the models only contemplate the costs inherent to the caregivers' travels, ignoring costs related to their wages. At last, it can be concluded that no study considers, simultaneously, the minimization of operating costs, and the caregivers' equity in terms of working time. Additionally, the inclusion of particular features, such as the shifts, users' autonomy, and meals' distribution is scarce or even null.

Table 2.1: Key features analyzed in Home Care Routing and Scheduling problems

	Valences		Objectives		Constraints				
	HHC	HSC	Operating Costs	Equity	Working Time Regulations	Break Requirements	Shifts	Patients' Autonomy	Meals' Distribution
Braekers *et al.* (2016)	x		x		x				
Guericke & Suhl (2017)	x				x	x	x		
Xiao *et al.* (2018)	x		x		x	x			
Gomes & Ramos (2019)		x		x		x			x
Our Study		x	x	x	x	x	x	x	x

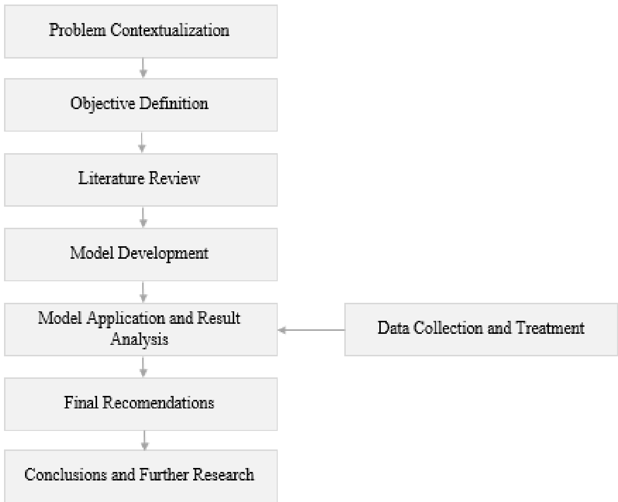
Source: Own Elaboration

Taking into consideration all the aspects mentioned, it is worth noting the opportunity to develop a model that contemplates aspects little or never introduced in the Home Social Care routing and scheduling context, to fill a gap in the literature.

Up to the authors' knowledge, the model most similar to the one presented is proposed by Xiao *et al*. (2018), but as it is applied to Home Health Care, and it does not include certain features relevant to the Home Social Care sector. Our study thus departs from the model proposed by Xiao *et al*. (2018) and adapts it to consider the specificities of the Home Social Care sector, as well as additional features that are relevant when making planning decisions in this sector. These additional features include accounting for the impact of patients' autonomy, the planning of different shifts, work time regulations, and accounting for additional planning objectives other than cost-related ones.

3. METHODOLOGY

Concerning the methodology that will be used during this thesis, the following *Figure 3.1* schematizes the sequence of phases.



Source: Own Elaboration

Figure 3.1: Scheme of the Methodology implemented

According to Yin (2017), the methodology followed in this thesis is based on a case study since it respects the following features: i) the key research question is a "how" question; ii) the research worker has barely or no control over behavioral events; ii) the target of the study is a contemporary phenomenon (as opposed to entirely historical).

As already mentioned, HC embodies countless advantages, with the main one being the reduction of hospitalization costs (Sahin & Matta, 2015). Yet, like many other organizations facing competitive pressures, HC providers need to continuously establish new ways to contain costs, improve the services' provision, their quality, and consequently the productivity (Begur *et al.*, 1997). Since the better management of human resources is presented as the initial approach to accomplish these objectives (Shao *et al.*, 2012), this dissertation aims to present an improved resource allocation under a routing and scheduling model.

The scope of this thesis is the Social and Solidarity Sector, namely the Home Social Care Sector. The case study research will be conducted with a Portuguese HSC provider, Centro Social e Paroquial da Póvoa de Santo Adrião (CSPPSA). The information collected enabled to understand the organization's operations. Thus, the main ambitions of this dissertation are:

a) Analyze the HC provider's operations;
b) Collect basic information and map the users, to understand their geographic distribution and the corresponding services provided to each one;

c) Support the HC provider by proposing a solution approach for the operational planning that has to be performed regularly;
d) Explore the impact of accounting simultaneously several objectives relevant in this sector, which can be conflicting;
e) Integrate relevant working regulations, such as maximum daily work, breaks, and days-off;
f) Integrate relevant users' aspects, such as their home location and autonomy;
g) Solve the integrated routing and scheduling for real-world sized problem instances;
h) Develop a generic mathematical model that can be applied by other HSC institutions in Portugal.

First, the methodological strategy of this study involves a review of the literature, collecting the main theoretical concepts to frame the theme and identify the contributions from previous studies. Considering the main objective of developing a routing and scheduling model, it is imperative to understand all the features that impact the way the caregivers are allocated to the routes and the users.

Second, regarding the empirical component, the decision was to collect information from the CSPPSA, to develop a model closer to reality. The information collected focused essentially on the users, with emphasis on the required services and the respective frequency. Given the current context of the COVID-19 pandemic and considering the aggravation of visiting elderly people, often with weakened medical conditions, we were only allowed to follow the routes for one week. Even so, it was possible to understand the logistics among the employees, the decisions regarding the sequence of visits, how much time is spent at each user's home, among others. This information enabled to construct of the total daily visits, discriminating the exact number of personal hygiene, habitational hygiene, and meals' distribution. Generally, weekly planning is repeated, thus being our basic planning for the construction of the model.

Additionally, a meeting was held with the direction of the CSPPSA to understand not only the planning of these services but also financial management issues. In Chapter 9, Annex A, it is displayed a guide with the questions made to the direction of the CSPPSA. Alongside this, semi-structured interviews were conducted with the caregivers and management team for more logistical matters.

Given the above, and considering the literature review, the decision was to develop a model-based in Mixed-Integer Linear Programming (MILP). To obtain the optimal solution for both planning tasks, the model was implemented in the software GAMS (General Algebraic Modelling System). This algebraic modeling language was chosen to detriment of others as one can solve a deterministic optimization problem, it is easy to write and read since is an algebra-based notation, it can help save time and reduce input errors since it inserts a coded error message immediately. In this regard, this methodological approach assists decision-makers in acknowledging complex decision making.

4. CASE STUDY

This chapter will focus essentially on the specifities of the partner institution. For this, a general review of HSC in Portugal will be presented, followed by a detailed characterization of the CSPPSA, namely its operational procedures, the services provided, the caregivers and the users.

4.1 HOME SOCIAL CARE IN PORTUGAL

As well as in other European countries like Spain or Italy, the two components of HC in Portugal are divided into different systems. Therefore, while HHC belongs to the health system, provided by the central or regional government, HSC, under the responsibility of the local government or municipality, is part of the social system (WHO, 2008).

Within the scope of the Ministries of Health and Labour, Solidarity and Social Security, DSS allows to respond not only to the needs of the elderly, but also to situations of dependency, in which problems such as disability, chronic illnesses and dementia stand out (Gil, 2009). Given the great relevance of infrastructures to ensure support for these users, partnerships with municipalities and non-profit institutions are crucial. As mentioned earlier, the definitions of DSS and HSC services are analogue (Bonfim & Veiga, 1996; Santana *et al.*, 2007).

Therefore, the DSS is a social response contracted between the State and each institution, through Social Security. In general, this service should provide the following services (Bonfim & Veiga, 1996):

i) Provision of hygiene and comfort care;
ii) Housekeeping and small cleaning at home;
iii) Preparation, transportation and/or distribution of meals;
iv) Treatment of clothes.

The DSS can also provide the following ones (Bonfim & Veiga, 1996):

i) Monitoring outside;
ii) Acquisition of food and other items;
iii) Social interaction;
iv) Minor home repairs.

Thus, at first glance, HSC services requires the definition of care plans, under the users' needs, namely the services and respective frequency. Then, the execution of these plans asks for deep coordination between human and material resources, taking into consideration the services' costs and the resource constraints (Borsani *et al.*, 2006).

4.2 THE PARTNER INSTITUTION

The development of this work and its recognition is based on a real-world case study. In an attempt to establish a partnership, we contacted the CSPPSA, that was receptive to the suggestion of participating in this project.

The institution's headquarters are located in Póvoa de Santo Adrião, in Odivelas. Although Portugal presents a high ageing index (161,3%), Odivelas is an area that does not have one of the highest percentages of the elderly population (126,5%) (PORDATA, 2019).

CSPPSA is a Private Institution of Social Solidarity (PISS), which means that it was constituted by private initiative, with a non-profitable nature, to provide solidarity services to those in need. CSPPSA's mission is to promote social intervention actions in partnership with civil society in the promotion and dignification of the human person, and aims to be a reference in the community, responding to the needs of families and people, continuously improving its intervention. The Catholic inspiration does not imply that it practices any discrimination based on religious beliefs; it only guide the execution of religious activities that may be programmed in the activity plan (CSPPSA, 2020).

In this institution's case, the services provided fall under three types of classification: Childhood Daycare Support (childcare services for children between four months and fifteen years), Community Support (which includes support services for unemployed youth and adults, the provision of meals to economically disadvantaged families, among others), and Senior Support (which includes Daycare Center and DSSs for the elderly or people with dementia) (CSPPSA, 2020). Our spotlight is on HSC, i.e., DSSs.

4.2.1 LEGAL FRAMEWORK

The so-called Social and Solidarity Sector is a fundamental instrument for the practice of solidarity and social representation in Portugal. Thus, nonprofit organizations of social action are commonly known as PISS. These organizations are legally recognized in the Portuguese Constitution[1], namely by the

[1] The Constitution of the Portuguese Republic, dated 2.4.1976 and last revised by Constitutional Law no. 1/2005, 7th revision, 12th August, art. 63, No. 5

Decree-Law nº 172-A/2014, within the scope of the Ministry of Labour, Solidarity and Social Security (2014).

According to the specific statute, PISS develop social solidarity activities in areas such as social security, health, and education. Through a close relationship with the population and cooperation with the State, PISS seeks to respond to social emergencies and to support the most vulnerable citizens.

PISS's mission is achieved through the provision of goods, services and other initiatives to promote the well-being and quality of life of families and communities. Among the various actions developed by the PISS, it can be pointed out the support for children, elderly, people with disabilities, support for social and community integration and social protection in the event of illness, old age, disability and death.

These organizations comprise several types of institutions such as social solidarity associations, social action voluntary associations, mutual benefit institutions, social solidarity foundations or Holy Houses of Mercy.

4.2.2 HOME SOCIAL CARE IN CSPPSA

One of the valences of social support is the DSS, which consists of providing individualized and personalized home care to people and families when, due to illness, disability and/or impediment, they cannot guarantee the satisfaction of basic needs and/or the fulfillment of daily living activities. Therefore, the main objectives are the contribution to the improvement of the quality of life, the contribution in delaying or trying to avoid the institutionalization, the provision of services that satisfy the basic needs, and the promotion of self-esteem and autonomy of the patients (CSPPSA, 2020).

Nevertheless, several difficulties emerge when it is necessary to allocate scarce resources, such as caregivers working at the institution, to users spread over a wide area, with different necessities and available times. It is thus essential to have an adequate planning of the routes that should be performed to ensure those visits and to provide the services required. To better understand how to make this allocation, the following subsections are dedicated to the functioning of the institution's HSC network.

4.2.3 THE SERVICES

DSSs allows the provision of a large range of assistance, essentially sorted into four major groups: i) personal hygiene, ii) habitational hygiene, iii) laundry care and iv) food. Each of these services includes a series of several tasks, as shown in *Table 4.1*.

Table 4.1: Examples of services available to the users in the CSPPSA

Personal Hygiene	Habitational Hygiene	Laundry Care	Food
Diaper maintenance	Dusting	Collection	Distribution
Head washing	Floor mopping	Delivery	
Body washing	Vacuuming		
Body hydration	Taking out the trash		
Moisturizing cream application	Kitchen Cleaning		
Nail's cutting	Washing the dishes		
Dressing	Exchange of beds sheets		
Making the bed			

Source: Own Elaboration

It is important to recognize that not all users contract the four different services simultaneously, and even when they do, they do not always require all the available sub-activities. Therefore, the services provided by the institution have some flexibility, with different users requiring different packages of care. In particular circumstances, additional services that are not previously available may be contracted, given the users' physical and mental limitations. Some of these examples are the administration of medicines and the preparation and serving of breakfast, lunch or dinner.

Another important feature is the frequency with which the services are provided. Along with the fact that each user requires different types of services, they are provided according to their specific needs. In general, services such as personal hygiene or food distribution are the most frequent, since there are users requiring these services seven days a week. In turn, habitational hygiene and clothes washing are less frequent.

Laundry washing is the only service that can be provided internally (in the user's home) or externally (in the CSPPSA). If it is provided internally, the caregivers only need to insert the clothes inside the washing machine; if it is provided externally, a specific day of the week is contracted with each user to collect the clothes (generally an established day when personal hygiene or habitational hygiene is provided) and deliver them one week later. This service implies some free time beyond the provision of services so that the caregivers develop the laundry task at the CSPPSA.

4.2.4 THE USERS

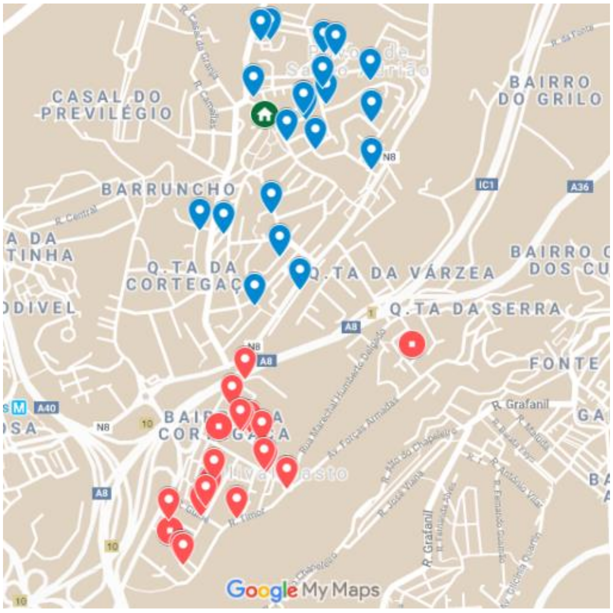
In the CSPPSA, the users are elderly with varying degrees of dependence. Therefore, the services requested by them will be distinctive. On the one hand, there are autonomous users who, when requesting various services, do so with the main objective of supervision, and not so much due to the lack of mobility. On the other hand, there are dependent users, who need total aid to perform their basic daily activities. It is important to emphasize that although dependent users request more visits, the frequency of visits desired by autonomous users is variable.

The Social Charter (MTSS & Nogueira, 2009) considers the following degrees of dependence:

i) Autonomous – Able to perform basic care needs without support from third parties;
ii) Partially Dependent – Needs third party support for personal hygiene and/or movement;
iii) Dependent – Cannot independently perform the tasks essential to the satisfaction of everyday life basic needs, namely activities related to food, movement or personal hygiene care;
iv) High level of dependency – Accumulates situations of dependency that characterize dependents, and the user is bedridden or presents severe dementia.

The classification presented above is the one used by the CSPPSA. Hence, when scheduling employees, the dependence degree is always considered, with one caregiver being assigned in the case of autonomous users ((i)), and two caregivers being allocated in the remaining cases ((ii)-(iv)). In addition to this feature, the users' social context is also considered. Typically, users live complex realities, in which they can reject to receive care, and even refuse to receive the caregivers in their homes. In these convoluted cases, users are flagged, and two caregivers are allocated to try to streamline the provision of care.

In *Figure 4.1*, the locations of the patients' homes are identified, discriminating between the two covered zones by the CSPPSA caregivers, Póvoa de Santo Adrião and Olival Basto. The *Figure 4.1* also exhibits the location of the CSPPSA headquarters. In the moment of data collection, there were forty-five patients requiring the HSC services. It is important to note that there are three cases in which two CSPPSA' users live in the same house.



Source: Own Elaboration

Figure 4.1: Geographical distribution of users' homes and location of CSPPSA headquarters

Caption:

- CSPPSA's Headquarters
- Users' Home in Póvoa de Santo Adrião
- Users' Home in Olival Basto
- Home with two users

Whenever possible, a fix routine per user is ensured when planning the routes that should be established to ensure those visits. This means that the CSPPSA establishes with each user a visiting time, considering the users' preferences and the caregivers' availability. The visit will be larger or smaller depending on the extent and duration of tasks to be provided. Most users require several visits per day, but currently, they can only receive at most four visits per day, i.e. morning care, lunch delivery, afternoon care, and dinner delivery.

4.2.5 The Caregivers

For each one of the social support valences that CSPPSA offers, there are restrictive and unique employees. Currently, there are nine women caregivers assigned exclusively to the DSS. Considering the current route planning, it is necessary to assure that care is provided by, at least, seven employees daily. In an anomalous case of only having six employees available, a tenth caregiver belonging to other

CSPPSA' team may be allocated. So far, this caregiver has already performed functions in the DSS in similar situations (i.e., a caregiver who belongs to another team, but who can help the DSS team in extreme cases).

Regarding the schedules, working hours may vary according to the caregivers but are always continuous intervals. In general, the schedules have a time horizon of one month. Since in each day there are several users requesting different services, every day the routes are different. In order to avoid overloading of the CSPPSA caregivers, a measure implemented indicates that all the employees should visit all users. This method makes the most demanding work divided by all, avoiding a more pronounced physical effect in certain caregivers. Therefore, in the current system, there is no continuity of care (i.e. there is not a limited number of caregivers who can serve each user) and so all caregivers visit each one. Also, another practice implemented is the rotation of pairs, i.e., each of the caregivers works with the remaining employees. This measure aims to avoid the deterioration of interpersonal relationships between two caregivers.

These two practices mentioned above do not interfere with the possible qualifications required when providing care, since all employees have the same competences. Thus, as the qualifications are homogeneous, it is not necessary to take this feature into account when assigning caregivers to the users.

4.2.6 OPERATIONAL PROCEDURES

According to the current legislation, employees are required to attend forty hours of training annually. Therefore, it is expected for employees to attend training during the work period.

During the business days (Monday to Friday), CSPPSA opens at 8:30 a.m. and ceases its activities at 6:30 p.m.. During these days, the schedule planning encompasses two different shifts: the first one between 8:30 a.m. and 5:30 p.m., with an hour of lunch, and a second one from 8:30 a.m. to 6:30 p.m., with two hours of lunch, resulting in a total operating period of seven hours and a half (considering the two daily breaks). On Saturdays and National Holidays, the working period begins at 8:30 a.m. and ends at 5:30 p.m., with one hour of lunch and no reduction in the total operating period. On Sundays, the working period initiates at 8:30 a.m. and culminates at 12:30 p.m., a reduction in the operating period to four hours. The CSPPSA does not provide care on Christmas, New Year and Easter Sunday. Each caregiver is required to work thirty-seven hours a week and is entitled to two days off per week.

Generally, in case there is an even number of employees working, they are allocated in pairs; in the case of an odd number, there is a caregiver alone, who essentially ensures the care of autonomous users. Route planning requires two separate schedules, depending on the morning or afternoon's agenda. This way, during the morning, only personal hygiene is performed and, during the afternoon, habitational hygiene, purchase of goods for a user and laundry care are executed. Currently, in the morning, there are five routes (green, deep blue, light blue, yellow and orange), each of which has between seven to eleven users. In the afternoon there are four routes (1, 2, 3 and 4) when there are between seven to nine caregivers, or three routes (1, 2 and 3) when six caregivers are working. The higher number of routes in the morning's agenda is due to the fact that users who are currently served require more personal hygiene services, when compared to the other services available. It is important to notice that in this planning users that only require meals are not accounted for, which makes this planning unrealistic concerning the people to visit and the time spent, both in the visits themselves and in the distances to be covered.

The caregivers have a total availability of four vehicles to perform their services and share the fifth vehicle with another CSPPSA team. Assuming a business day when, e.g., seven caregivers are working, the current workday is as follows:

i) All caregivers gather at the CSPPSA, preferably a few minutes before 8.30 a.m., to understand which route and with whom they will take it. The seventh caregiver is, by default, alone, assigned to autonomous users and use the fifth shared vehicle;
ii) At 8:30 a.m., the teams leave CSPPSA and perform users' hygiene;
iii) In the middle of the morning, without any predefined schedule, the caregivers are entitled to a break of approximately thirty minutes, and there is no obligation to return to the CSPPSA;
iv) Around 11:30 a.m., the seventh caregiver with the fifth vehicle has to travel to the institution that cooks the meals, pick them for patients belonging to other CSPPSA team, and travel to the CSPPSA to deliver them. When this task is finished, the caregiver can continue to visit the scheduled users;
v) Of all teams that are working, the first one that ends the assigned users' hygiene will need to go to the same institution to pick up the food for the DSS' users and begins the meals' distribution. As the remaining teams finish their morning visits, they join the other ones (notice that all teams perform this task);
vi) When the meals' distribution ends, the teams return to the CSPPSA for a lunch break. Each caregiver is entitled to one or two hours of lunch, depending on the shift of each one;
vii) In the afternoon, the employees perform habitational hygiene and dinner deliveries. In the case of finishing the tasks before the departure time, the caregivers return to the CSPPSA to carry out laundry duties. Otherwise, the caregivers will still have to return to the CSPPSA at the end of the day.

Given that within the services offered, the most requested is the personal hygiene, that there is some uncertainty relating the time spent in providing these services, that they are performed in the morning and that there is no maximum time to finish the lunch' deliveries, the caregivers may see their time for lunch reduced.

Regarding the pickup of the users' meals, the teams need to return to the institution that prepares them as much as necessary. The first team that dispatches the morning visits does not pick up all the lunches since generally, they are not all ready to be gathered up. This way, the teams maintain permanent contact through a specific smartphone application, to understand who is missing meals. The same happens when delivering dinners.

4.3 THE CHALLENGE

Given the previous description, it is straightforward to apprehend the complexity inherent in building the routing and scheduling of CSPPSA's caregivers. Some evidence can be drawn:

i) The current planning is developed manually by a social educator, which can make the routes inefficient, that do not respect all the conditioning factors that involve this process, and for which is spent a great deal of time;
ii) The current planning does not account for users to whom only meals are distributed, causing routes to be incomplete and unreliable in terms of the time that is actually spent by the caregivers in the performance of their tasks. As these deliveries are not included in the aforementioned planning, the sequence of lunch and dinner distributions are done randomly and without a specified time window;
iii) The current planning does not consider laundry tasks, which means that there is no regularity in the completion of these functions, i.e., these tasks are generally performed during the free time which usually exists at the end of the day, but which is not periodic and a routine;
iv) Given that not all tasks are included in the current planning, and that there is no detailed schedule, which includes how much time is needed to perform each task, it is usual for caregivers to finish their tasks beyond the time stipulated by the institution;
v) Considering the aforementioned, it is usual for the caregivers to have little or no time to perform other functions in the CSPPSA, such as laundry tasks, among others that would be desirable.

Along with what was mentioned, one of the issues noted was that planning is not structured in terms of time, which means that there is no planning in which it is visible, for example, that the duration of service *x* in patient *M* plus the time that is lost in the journey between patient *M* and patient *N* equals to the time the service begins on patient N. The CSPPSA has only relatively equitable planning during the week so that it is possible to serve all the patients scheduled, thus there is no strict planning in terms of hours. For this reason, employees often see the start of lunch being delayed.

These observations allow us to realize that the operational processes of the CSPPSA can be improved, mainly by optimizing the allocation of scarce resources. Thus, the main objective is to develop a mathematical programming planning model that allows obtaining not only the optimal allocation of caregivers to the users but also the optimal routes that should be established when performing the Home Social Care visits. Accordingly, the following planning decisions should be explored:

i) Which should be the routes established by each team of caregivers, i.e., which is the sequence of visits to be followed per day?
ii) How should the workload be distributed across caregivers working in different shifts?

These decisions should be made while considering the patients' autonomy, the work time regulations, and the need to ensure visits with different purposes (personal or habitational hygiene, or even meals distributions). And this while simultaneously considering the following planning objectives:

i) Minimization of operational costs, including travel costs and costs with the wages of the caregivers;
ii) Maximization of equity, through the minimization of the differences in daily working time between the teams.

5. BUILDING THE PLANNING MODEL

5.1 MILP MODEL

A model based in mixed-integer linear programming, as the one under study, involves variables that are constrained as integers and variables constrained as non-integers. The proposed MILP, presented in the next sub-chapters, addresses real-world features, being considered a faithful construction of the logistics behind the provision of HSC services in the CSPPSA.

5.1.1 MATHEMATICAL FORMULATION OF THE MODEL

We model the Home Social Care routing and scheduling problem as a directed graph $G = (V, A)$, where V is the set of nodes and A the set of arcs.

Each visit is represented by a separated node in the graph, whereas two or more visits may be associated with the same physical location (e.g., if the same user requires a visit in the morning for personal hygiene and a second visit in the afternoon for housing hygiene). This means that, given a user who needs at least two daily visits, a node will be created for each visit, and they will both have the same geographical location (Decerle *et al*., 2019). Given this, the set of arcs is defined as $A = \{(g, v) \mid g, v \in V, g \neq v\}$, where g and v represent the beginning and the end node of each arc, respectively – possible arcs include: HSCC - user visit $\in J$, user visit $\in J$ - user visit $\in J$, user visit $\in J$ - break node $\in B$, break node $\in B$ - user visit $\in J$, user visit $\in J$ – institution that provide the meals $\in V$, institution that provide the meals $\in V$ - user visit $\in J$, user visit $\in J$ - HSCC. Each arc $\{g, v\} \in V$ is characterized by d_{gv} and c_{gv} that denote travel distance and travel time between locations of node g and node v.

In the objective function, two variables are introduced to account for costs, such as wages of caregivers, €α per hour, and the oil cost, €β per kilometer. ε and δ are defined as a small and big positive value, respectively, and introduced into the proposed model.

5.1.2 NOTATION

The following sections present the notation used for the model, along with the mathematical formulation of the objective functions and key constraints.

5.1.2.1 INDICES AND SETS

Table 5.1: Indices and Sets' Definition

$G = (V, A)$	V is the set of nodes, A the set of arcs
$g, v \in V, V = J \cup B \cup IN \cup \{0, n+1\}$	Set of nodes
$A = \{(g, v) \mid g, v \in V, g \neq v\}$	Set of arcs
$j, i \in J$, $J = J' \cup J'' \cup J''' \cup J'''' = \{1, \ldots, j, \ldots, J\}$	Set of daily visits, with J', J'', J''' and J'''' corresponding to visits for personal hygiene, visits for habitational hygiene, visits for lunch distributions and visits for dinner distributions, respectively
$b \in B, B = \{n+2, n+3\}$	Set of break nodes
$in \in IN, IN = \{n+4, n+5\}$	Set of visits for the institution providing the meals
$k \in K, K = \{1, \ldots, k, \ldots, K\}$	Set of teams
$t \in T, T = T' \cup T'' = \{1, \ldots, t, \ldots, T\}$	Planning horizon, in days, with T' and T'' corresponding to the five business days (Monday to Friday) and to the weekend, respectively
$0, n+1$	Origin and destination of all the caregivers' routes
$n+2$	Morning break node
$n+3$	Lunch break node
$n+4$	Visit for the institution that provides the meals for lunch
$n+5$	Visit for the institution that provides the meals for dinner
$A_1 = \{(j, b): j \in J, b \in B\}$	Visits $j \in J$ preceding breaks $b \in B$
$A_2 = \{(b, j): b \in B, j \in J\}$	Breaks b $\in B$ preceding visits $j \in J$
$A_3 = \{(j, in): j \in J, in \in IN\}$	Visits $j \in J$ preceding institution nodes $in \in IN$
$A_4 = \{(in, j): in \in IN, j \in J\}$	Institution nodes in $\in IN$ preceding visits $j \in J$

$A_5 = \{(g, t): g \in V, t \in T\}$	Association between nodes g $\in V$ and days $t \in T$
$A_6 = \{(0, j): 0 \in V, j \in J\}$	Origin node 0 $\in V$ preceding visits $j \in J$
$A_7 = \{(j, n+1): j \in J, n+1 \in V\}$	Visits $j \in J$ preceding destination node $n+1$ $\in V$
$A_8 = \{(k, t): k \in K, t \in T\}$	Association between teams k $\in K$ and days $t \in T$

5.1.2.2 PARAMETERS

Table 5.2: Parameters' Definition

$a_{kt} = \sum_{b \in B} q'_{bkt}$	Break length for team $k \in K$ on day $t \in T$
c_{gv}	Travel time between node $g \in V$ and node $v \in V$
d_{gv}	Travel distance between node $g \in V$ and node $v \in V$
g_t	Maximum number of teams working per day $t \in T$
l'_{kt}	Maximal daily working time of team $k \in K$ per day $t \in T$
q'_{bkt}	Duration of each break $b \in B$ of team $k \in K$ on day $t \in T$
q_{jkt}	Duration of each visit $j \in J$ by team $k \in K$ on day $t \in T$
$[m_{kt}, n_{kt}]$	Time window of working time for team $k \in K$ on day $t \in T$, that corresponds to their available time
α_k	Hourly wage by team $k \in K$ (€/hour)
β	Oil cost per kilometer (€/km)
ε	Small positive value
δ	Big positive value
θ	Time up to which all lunch distributions must be delivered

5.1.2.3 VARIABLES

Table 5.3: Auxiliary Variables' Definition

o'_{bkt}	Actual starting time of break $b \in B$ for team $k \in K$ on day $t \in T$
h_g	Variable to be used for eliminating subtours involving each node $g \in V$, as suggested by Miller & Zemlin (1960)
dt_{kt}	Total travel distance for team $k \in K$ on day $t \in T$
l_{kt}	Daily working time of team $k \in K$ on day $t \in T$
m'_{kjt}	Time at which the team $k \in K$ starts serving the first visit $j \in J$ on day $t \in T$
n'_{kjt}	Time at which the team $k \in K$ ends serving the last visit $j \in J$ on day $t \in T$
u_{kt}	Time at which the team $k \in K$ ends serving the last lunch distribution on day $t \in T$
l''_t	Greater working time for teams working on day $t \in T$

Table 5.4: Decision Variables' Definition

x_{gvkt}	Binary variable that equals 1 if a node $v \in V$ is visited by a team $k \in K$ directly after node $g \in V$ on day $t \in T$; 0 otherwise
f_1	Operational costs minimization variable
f_2	Equity maximization variable

5.1.3 OBJECTIVE FUNCTIONS

The objective functions proposed as follows regard two main intervenients in the HSCRSP. These two objectives are accounted simultaneously in our model:

i) Objective function 1 minimizes the operating costs of the CSPPSA, including the wages of caregivers and fuel costs. This objective admits not only travel costs but also fixed costs with caregivers' wages. This last aspect allows the consideration of full and part-time caregivers.

$$f_1 = min \sum_{k \in K} \sum_{t \in T} (\alpha l_{kt} + \beta dt_{kt}) \tag{1}$$

ii) Objective function 2 aims to maximize equity between teams, by minimizing differences in working times. Thus, the goal is to minimize these differences, making sure that there are no discrepant differences in the total working time of each team.

$$f_2 = min \sum_{k \in K} \sum_{t \in T} l_t'' - l_{kt} \tag{2}$$

5.1.4 CONSTRAINTS

A set of constraints is considered in the model, as described below.

(a) Routes-Related Constraints

(a.1) Equation 3 ensure that each visit $j \in J \cup B \cup IN \cup 0$ is done exactly once, for $t \in T'$.

$$\sum_{\substack{k \in K \\ k:(k,t) \in A_8}} \sum_{\substack{v \in J \cup B \cup IN \cup 0 \\ v:(v,t) \in A_5}} x_{vjkt} = 1 \quad \forall j \in J, t \in T', (j,t) \in A_5 \tag{3}$$

(a.2) Equation 4 ensure that each visit $j \in J \cup B \cup IN \cup n+1$ is done exactly once, for $t \in T'$.

$$\sum_{\substack{k \in K \\ k:(k,t) \in A_8}} \sum_{\substack{v \in J \cup B \cup IN \cup n+1 \\ v:(v,t) \in A_5}} x_{jvkt} = 1 \quad \forall j \in J, t \in T', (j,t) \in A_5 \tag{4}$$

(a.3) Equation 5 ensure that each visit $v \in J \cup B \cup 0$ is done exactly once, for $t \in T''$. This condition differs from Equation 3 because, at the weekend, there are no meals distribution, so caregivers do not need to go to the institution.

$$\sum_{\substack{k\in K\\ k:(k,t)\in A_8}} \sum_{\substack{v\in J\cup B\cup 0\\ v:(v,t)\in A_5}} x_{vjkt} = 1 \quad \forall j \in J, t \in T'', (j,t) \in A_5 \tag{5}$$

(a.4) Equation 6 ensure that each visit $v \in J \cup B \cup n+1$ is done exactly once, for $t \in T''$. This condition differs from Equation 4 because, at the weekend, there are no meals distribution, so caregivers do not need to go to the institution.

$$\sum_{\substack{k\in K\\ k:(k,t)\in A_8}} \sum_{\substack{v\in J\cup B\cup n+1\\ v:(v,t)\in A_5}} x_{jvkt} = 1 \quad \forall j \in J, t \in T'', (j,t) \in A_5 \tag{6}$$

(a.5) Equation 7 ensure that each team $k \in K$ enters only once at each node $g \in V$, for $t \in T$.

$$\sum_{\substack{g\in V\\ g:(g,t)\in A_5}} x_{gvkt} \leq 1 \quad \forall\, v \in V, k \in K, t \in T, (v,t) \in A_5, (k,t) \in A_8 \tag{7}$$

(a.6) Equation 8 ensure that each team $k \in K$ leaves only once at each node $g \in V$, for $t \in T$.

$$\sum_{\substack{v\in V\\ v:(v,t)\in A_5}} x_{gvkt} \leq 1 \quad \forall\, g \in V, k \in K, t \in T, (g,t) \in A_5, (k,t) \in A_8 \tag{8}$$

(a.7) Equation 9 determine the total traveling distance of team $k \in K$, for $t \in T'$.

$$dt_{kt} = \sum_{\substack{g\in J\cup B\cup IN\cup 0\\ g:(g,t)\in A_5}} \sum_{\substack{v\in J\cup B\cup IN\cup n+1\\ v:(v,t)\in A_5\\ v\neq g}} x_{gvkt}\, d_{gv} \quad \forall\, k \in K, t \in T', (k,t) \in A_8 \tag{9}$$

(a.8) Equation 10 determine the total traveling distance of team $k \in K$, for $t \in T''$. This condition differs from Equation 9 because at weekends the caregivers do not have to go to the institution that provides the meals.

$$dt_{kt} = \sum_{\substack{g\in J\cup B\cup 0\\ g:(g,t)\in A_5}} \sum_{\substack{v\in J\cup B\cup n+1\\ v:(v,t)\in A_5\\ v\neq g}} x_{gvkt}\, d_{gv} \quad \forall\, k \in K, t \in T'', (k,t) \in A_8 \tag{10}$$

(a.9) Equations 11 to 19 formulate a flow conservation constraint, which must be held for each team $k \in K$ separately and ensure connected and cyclic routes. These restrictions algebraically state that the sum of the flow through the arcs directed to a node is equal to the sum of the flow through the arcs directed away from that node.

(a.9.1) Equation 11 ensures that the sum of the flow from the node $g \in V$ to the node $v \in V$ is equal to the sum of the flow from node $v \in V$ to the node $g \in V$, for $t \in T'$.

$$\sum_{\substack{g \in J \cup B \cup IN \cup 0 \\ g:(g,t) \in A_5 \\ g \neq v}} x_{gvkt} = \sum_{\substack{g \in J \cup B \cup IN \cup n+1 \\ g:(g,t) \in A_5 \\ g \neq v}} x_{vgkt} \quad \forall\, k \in K, t \in T', (v,t) \in A_5, (k,t) \in A_8 \tag{11}$$

(a.9.2) Equation 12 ensures that the sum of the flow from the node $g \in V$ to the node $v \in V$ is equal to the sum of the flow from node $v \in V$ to the node $g \in V$, for $t \in T''$.

$$\sum_{\substack{g \in J \cup B \cup 0 \\ g:(g,t) \in A_5 \\ g \neq v}} x_{gvkt} = \sum_{\substack{g \in J \cup B \cup n+1 \\ g:(g,t) \in A_5 \\ g \neq v}} x_{vgkt} \;\; \forall\, k \in K, t \in T'', (v,t) \in A_5, (k,t) \in A_8 \tag{12}$$

(a.9.3) Equation 13 ensures that the sum of the flow from the node $in \in IN$ to the node $j \in J$ is equal to the sum of the flow from node $j \in J$ to the node $in \in IN$, for $t \in T'$.

$$\sum_{\substack{j \in J''' \cup J'''' \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=in)jkt} = \sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{j(v=in)kt} \quad \forall\, t \in T', (in, j) \in A_4, \forall\, k \in K, (k,t) \in A_8 \tag{13}$$

(a.9.4) Equation 14 ensures that the sum of the flow from the node $b \in B$ to the node $j \in J$ is equal to the sum of the flow from node $j \in J$ to the node $b \in B$.

$$\sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=b)jkt} = \sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{j(v=b)kt} \quad \forall\, t \in T, (j,b) \in A_1 \;\; \forall\, k \in K, t \in T', (k,t) \in A_8 \tag{14}$$

(a.9.5) Equation 15 ensures that the sum of the flow from the node $0 \in V$ to the node $j \in J$ is equal to the sum of the flow from node $j \in J$ to the node $n+1 \in V$.

$$\sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=0)jkt} = \sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{j(v=n+1)kt} \quad \forall\, t \in T, (0,j) \in A_6, (n+1, j) \in A_7 \tag{15}$$

(a.9.6) Equation 16 ensures that the sum of the flow from the node $0 \in V$ to the node $j \in J$ is equal to the sum of the flow from node $j \in J$ to the node $in \in IN$, for $t \in T'$.

$$\sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=0)jkt} = \sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{j(v=in)kt} \quad \forall\, t \in T', (0,j) \in A_6, (j, in) \in A_3 \tag{16}$$

(a.9.7) Equation 17 ensures that the sum of the flow from the node $0 \in V$ to the node $j \in J$ is equal to the sum of the flow from node $in \in IN$ to the node $j \in J$, for $t \in T'$.

$$\sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=0)jkt} = \sum_{\substack{j \in J''' \cup J'''' \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=in)jkt} \quad \forall\, t \in T', (0,j) \in A_6, (in, j) \in A_4 \tag{17}$$

(a.9.8) Equation 18 ensures that the sum of the flow from the node $0 \in V$ to the node $j \in J$ is equal to the sum of the flow from node $j \in J$ to the node $b \in B$.

$$\sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=0)jkt} = \sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{j(v=b)kt} \quad \forall\, t \in T, (0,j) \in A_6, (j, b) \in A_1 \tag{18}$$

(a.9.9) Equation 19 ensures that the sum of the flow from the node $0 \in V$ to the node $j \in J$ is equal to the sum of the flow from node $b \in B$ to the node $j \in J$.

$$\sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=0)jkt} = \sum_{\substack{j \in J \\ j:(j,t) \in A_5}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=b)jkt} \quad \forall\, t \in T, (0,j) \in A_6, (b, j) \in A_2 \tag{19}$$

(a.10) Equation 20 determine that flows between nodes can only take place for teams who have left the origin.

$$x_{gvkt} \leq \sum_{\substack{j \in J \\ j:(0,j) \in A_6}} x_{(v=0)jkt} \quad \forall\, g, v \in V, k \in K, t \in T, (g,t) \in A_5, (v,t) \in A_5, (k,t) \in A_8 \tag{20}$$

(a.11) Equation 21 prevents subtours between nodes, as suggested by Miller & Zemlin (1960).

$$h_g - h_v + |V| \times x_{gvkt} \leq |V| - 1 \;\forall\, g, v \in V, k \in K, t \in T, (g,t) \in A_5, (v,t) \in A_5, (k,t) \in A_8, v \neq g \tag{21}$$

(b) Caregivers-Related Constraints

(b.1) Equation 22 establish that the number of teams leaving the origin should not exceed the maximum number of teams available to work per day $t \in T$.

$$\sum_{\substack{j \in J \\ j:(j,t) \in A_5 \\ j:(0,j) \in A_6}} \sum_{\substack{k \in K \\ k:(k,t) \in A_8}} x_{(v=0)jkt} \leq g_t \;\; \forall\, t \in T \tag{22}$$

(b.2) Equation 23 determine the hour of the first visit of each team $k \in K$.

$$m'_{kt} = m_{kt} + \sum_{\substack{j \in J \\ j:(j,t) \in A_5 \\ j:(0,j) \in A_6}} x_{(v=0)jkt}\, c_{(v=0)j} \;\forall\, k \in K, t \in T, (k,t) \in A_8 \tag{23}$$

(b.3) Equation 24 determine the hour of the last visit of each team $k \in K$. It is important to note that, for the calculation of this function, we have the following parcels: the time at which the first visit of each team begins, the sum of displacement between the visiting nodes multiplied by the time spent on the trips, the sum of displacement between the visiting nodes and the break nodes multiplied by the time spent on the trips, the sum of displacement between the visiting nodes and the institution that provides the meals multiplied by the time spent on the trips, the duration of each visit served, and the duration of each break taken by the caregivers.

$$
\begin{aligned}
n'_{kt} = m'_{kt} &+ \sum_{\substack{j\in J\\ j:(j,t)\in A_5}} \sum_{\substack{i\in J\\ i:(i,t)\in A_5}} x_{jikt}\, c_{ji} + \sum_{\substack{j\in J\\ j:(j,t)\in A_5}} \sum_{\substack{i\in J\\ i:(i,t)\in A_5}} x_{ijkt}\, c_{ij} \\
&+ \sum_{\substack{j\in J\\ j:(j,t)\in A_5}} \sum_{\substack{b\in B\\ b:(j,b)\in A_1}} x_{jbkt}\, c_{jb} + \sum_{\substack{j\in J\\ j:(j,t)\in A_5}} \sum_{\substack{b\in B\\ b:(b,j)\in A_2}} x_{bjkt}\, c_{bj} \\
&+ \sum_{\substack{j\in J\\ j:(j,t)\in A_5\\ j:(j,in)\in A_3}} x_{j(v=in)kt} c_{j(v=in)} + \sum_{\substack{j\in J'''\cup J''''\\ j:(j,t)\in A_5}} x_{(v=in)jkt}\, c_{(v=in)j} \\
&+ \sum_{\substack{v\in V\\ v:(v,t)\in A_5}} \sum_{\substack{i\in J\\ i:(i,t)\in A_5}} x_{vikt}\, q_{ikt} + \sum_{\substack{j\in J\\ j:(j,t)\in A_5}} \sum_{\substack{b\in B\\ b:(j,b)\in A_1}} x_{jbkt}\, q'_{bkt} \;\; \forall\, k \\
&\in K, t \in T, (k,t) \in A_8
\end{aligned} \tag{24}
$$

(b.4) Equation 25 determine the working time of each team $k \in K$.

$$
l_{kt} = n'_{kt} + \sum_{\substack{j\in J\\ j:(j,t)\in A_5\\ j:(j,n+1)\in A_7}} x_{j(n+1)kt} c_{j(n+1)} \;\; \forall\, k \in K, t \in T, (k,t) \in A_8 \tag{25}
$$

(b.5) Equation 26 restrict the working time to avoid overtime for each team $k \in K$.

$$
l_{kt} \le l'_{kt} \;\; \forall\, k \in K, t \in T, (k,t) \in A_8 \tag{26}
$$

(b.6) Equation 27 establish that the greater working time of each team $k \in K$ must be equal or greater than the daily working time for each team $k \in K$ per day $t \in T$.

$$
l''_t \ge l_{kt} \;\; \forall\, k \in K, t \in T, (k,t) \in A_8 \tag{27}
$$

(c) Meals-Related Constraints

(c.1) Equation 28 indicate the time for the last lunch distribution of each team $k \in K$. It is important to note that, for the calculation of this function, we have the following parcels: the time at which the first visit of each team begins, the sum of displacement between the visiting nodes for personal hygiene multiplied by the time spent on the trips, the sum of displacement between the visiting nodes and the morning break multiplied by the time spent on the trips, the sum of displacement between the visiting nodes and the institution that provides the meals multiplied by the time spent on the trips, the sum of displacement between the visiting nodes for lunch distribution multiplied by the time spent on the trips, the duration of each personal hygiene performed, the duration of each lunch distribution served, and the duration of morning break taken by the caregivers.

$$u_{kt} = m'_{kt} + \sum_{\substack{j\in J' \\ j:(j,t)\in A_5}} \sum_{\substack{i\in J' \\ i:(i,t)\in A_5}} x_{jikt} c_{ji} + \sum_{\substack{j\in J' \\ j:(j,t)\in A_5 \\ j:(j,n+2)\in A_1}} x_{j(v=n+2)kt}\, c_{j(v=n+2)}$$
$$+ \sum_{\substack{j\in J' \\ j:(j,t)\in A_5 \\ j:(n+2,j)\in A_2}} x_{(v=n+2)jkt}\, c_{(v=n+2)j}$$
$$+ \sum_{\substack{j\in J' \\ j:(j,t)\in A_5 \\ j:(j,n+4)\in A_3}} x_{j(v=n+4)kt}\, c_{j(v=n+4)}$$
$$+ \sum_{\substack{j\in J''' \\ j:(j,t)\in A_5}} x_{(v=n+4)jkt}\, c_{(v=n+4)j} \tag{28}$$
$$+ \sum_{\substack{j\in J''' \\ j:(j,t)\in A_5}} \sum_{\substack{i\in J''' \\ i:(i,t)\in A_5}} x_{jikt} c_{ji} + \sum_{\substack{j\in J' \\ j:(j,t)\in A_5}} \sum_{\substack{i\in J' \\ i:(i,t)\in A_5}} x_{jikt} q_{ikt}$$
$$+ \sum_{\substack{j\in J''' \\ j:(j,t)\in A_5}} \sum_{\substack{i\in J''' \\ i:(i,t)\in A_5}} x_{jikt} q_{ikt}$$
$$+ \sum_{\substack{j\in J' \\ j:(j,t)\in A_5 \\ j:(j,n+2)\in A_1}} x_{j(v=n+2)kt}\, q'_{(v=n+2)kt} \quad \forall\, k \in K, t$$
$$\in T, (k,t) \in A_8$$

(c.2) Equation 29 ensure the maximum time for u_{kt} of each team $k \in K$.

$$u_{kt} \leq \theta, \forall\, k \in K, t \in T, (k,t) \in A_8 \tag{29}$$

(c.3) Equation 30 ensure that u_{kt} can only be calculated for teams working on each day $t \in T$.

$$u_{kt} \leq \sum_{\substack{g\in V \\ g:(g,t)\in A_5}} \sum_{\substack{v\in V \\ v:(v,t)\in A_5}} x_{gvkt}\, \delta \quad \forall\, k \in K, t \in T, (k,t) \in A_8 \tag{30}$$

(d) Non-Negativity and Binary Variables

(d.1) Equations 31 to 37 define the non-negativity.

$$o'_{bkt} \geq 0 \tag{31}$$

$$m'_{kt} \geq 0 \tag{32}$$

$$n'_{kt} \geq 0 \tag{33}$$

$$dt_{kt} \geq 0 \tag{34}$$

$$l_{kt} \geq 0 \tag{35}$$

$$u_{kt} \geq 0 \tag{36}$$

$$h_g \geq 0 \tag{37}$$

(d.2) Equations 38 define feasible values for decision variables.

$$x_{gvkt} \in \{0,1\} \tag{38}$$

5.2 MULTIOBJECTIVE APPROACH

Multiobjective planning allows one to contemplate several objectives simultaneously, which are usually conflicting with each other (Cohon, 1978).

Considering the HSC features for an optimal allocation of caregivers as presented in section 5.1, a multiobjective model was built for the CSPPSA HSC services. This model has two objectives: the minimization of the operating costs (measured in euros; Eq. (1)), and the minimization of the total working time of the caregivers (measured in minutes; Eq. (2)).

Over the past few years, several methods have been proposed to solve multiobjective problems. According to Jozefowiez *et al.* (2008), three main categories are distinguished:

i) Scalar Approaches;
ii) Pareto Approaches;
iii) Non-Scalar, Non-Pareto Approaches.

The key idea of scalar approaches, that is, to scalarize the objective, is to combine *m* objectives into a single criterion. Since the parameter can alter, optimal outcomes for the single-objective problem correspond to the Pareto outcomes for the multiobjective model. Without any kind of loss, the weights can be scaled so their sum equals one. The selection of different weights produces a different single-objective problem (Ralphs *et al.*, 2004; Jozefowiez *et al*., 2008).

Commonly, three key scalarizing techniques are used to compute nondominated solutions (Antunes *et al*., 2016):

i) The e-constraint technique (optimization of one of the *m* objectives considering the others *m*-1 as constraints by stipulating the lower levels that the decision-maker is eager to accept);

ii) The weighted-sum technique (optimization of a weighted-sum of the *m* objective functions through the assigning of weighted coefficients);
iii) The reference-point based techniques (minimization of a distance function to a reference point, generally the ideal solution, such as the Manhattan metric or the Chebyshev metric).

Chebyshev's method was chosen to solve the developed multiobjective model. This approach is noteworthy since it minimizes the maximum difference in all objectives and enables to obtain the entire set of nondominated solutions to a multiobjective linear programming model.

Given the space dimension *n*, the distance function that assigns a scalar $\|y_i - z_i\| \in \mathbb{R}$ to each pair of points y_i, $z_i \in \mathbb{R}^n$ is designed as metric. For the L_q metric, the distance among two points in $\mathbb{R}^n$ is

$$\|y_i - z_i\|_q = \left[\sum_{i=1}^{n} |y_i - z_i|\right]^{1/q} \tag{39}$$

$$\|y_i - z_i\|_\infty = \max_{i=1,\dots,n} |y_i - z_i| \quad q \in \{1, 2, \dots\} \tag{40}$$

L_1 is the sum of all components of $|y_i - z_i|$, L_2 the Euclidean distance, and L_∞ is the Chebyshev distance, in which the largest-difference component in $|y_i - z_i|$ is considered.

A weighted family of L_q^λ metrics can also be constructed, where a different scale factor can be assigned to the multiple components through the application of the vector $\lambda \geq 0$, such as

$$\|y_i - z_i\|_q^\lambda = \left[\sum_{i=1}^{n} \lambda_i |y_i - z_i|\right]^{1/q} \tag{41}$$

$$\|y_i - z_i\|_\infty^\lambda = \max_{i=1,\dots,n} \lambda_i |y_i - z_i|, q \in \{1, 2, \dots\} \tag{42}$$

Considering the augmented weighted Chebyshev's metric, $L_\infty^{\lambda,\rho}$ can be identified as a combination of L_∞^λ and L_1^λ metrics, in which the external isodistance contour $L_\infty^{\lambda,\rho}$ relates to $\|y_i - z_i\|_\infty^\lambda + \sum_{i=1}^{n} \rho_i |y_i - z_i|$, with a small positive value ρ_i.

Thus, this approach can be presented as

$$\min_{x \in X} \left\{ \max_{i=1,\dots,n} \lambda_i [f_i(x) - z_i^*] - \sum_{i=1}^{n} \rho_i f_i(x) \right\}, \lambda \geq 0 \tag{43}$$

where λ_i are the weight vectors, $f_k(x)$ are the objective functions (f_1 for the cost-related objective and f_2 for the equity-related objective), z_i^* are the ideal solutions (f_1^* for the minimum cost and f_2^* for the maximum equity) and ρ_i are small positive scalars.

It is important to notice that λ_i do not represent the relative importance of the objectives, but the direction. This way, augmenting λ_i increases the direction for improving the outcomes of the objective function i, while reducing λ_i decreases the direction for the outcomes.

Equation (43) is equivalent to

$$\min_{x \in X} \left\{ \nu - \sum_{i=1}^{n} \rho_i f_i(x) \right\} \tag{44}$$

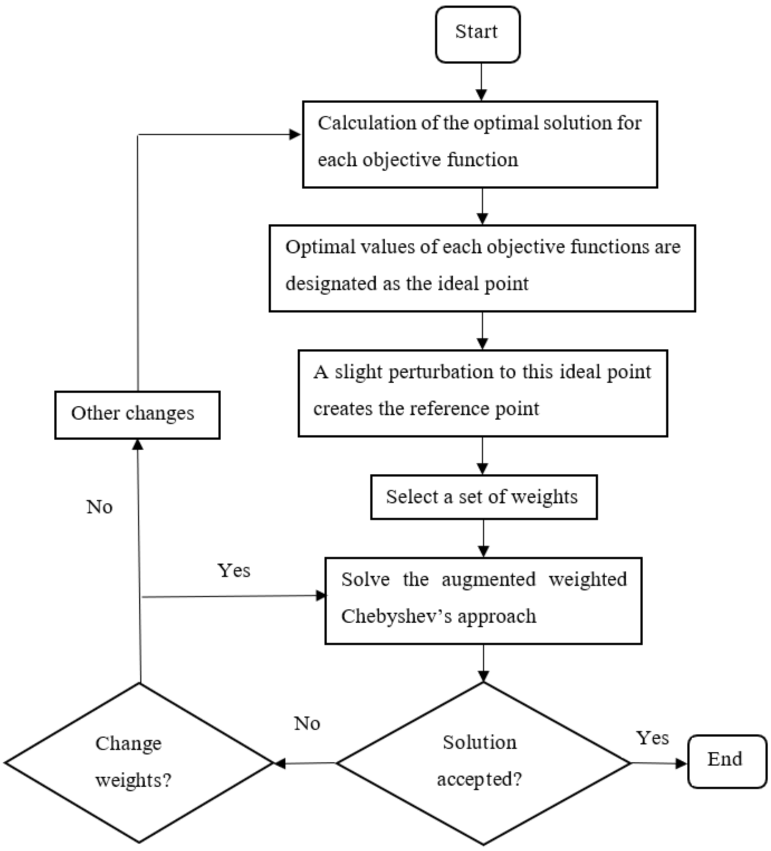
s.t.

$\lambda_i [f_i(x) - z_i^*] \leq \nu, i = 1, \ldots, n$

$x \in X$

$\nu \geq 0$

Figure 5.1 is a flowchart that shows all the steps that were taken during this process. It is interesting to observe the reference point because it is the point that the decision maker would like to reach, given that its components are the best attainable values within the feasible region.



Source: Adapted from Pereira et al. (2019)

Figure 5.1: Flowchart of the developed process

The implementation of the model will demonstrate the potential of this user-friendly tool, not only for the institution presented in the case study, but also for other HC providers.

6. RESULTS

In this chapter, the main goal is to examine the results of the model established earlier when it is applied to the particular case of the CSPPSA, in Portugal, and when the aim is to support the daily planning of home-based services provided by this institution. This section thus presents the data that was used, analyze the results obtained from the mono-objective model (each of the two functions individually) and the results of the multiobjective model and compare them with the CSPPSA's current solution. By the end of the chapter, it is performed a sensitivity analysis, and some recommendations will be presented for the given institution.

The model was validated before proceeding to the application to the real case. This validation was achieved through the application of fully controlled test cases. Corroboration was accomplished by verifying that the expected results were in fact attained, based on the application of the model.

The results herein explored are obtained by applying the proposed model using the CSPPSA as a case study. The model was implemented in the GAMS 23.7 and was solved with CPLEX 12.0 on a Two Intel Xeon X5680, 3.33 Gigahertz computer with 12 Gigabyte RAM.

6.1 CASE STUDY DATA

Accessing the input data is a crucial step in the application and testing of a model, as it allows having a real and significant influence on the production of solid results that are expected to be applied in the institution under study. This section aims to introduce and justify some decisions made regarding the treatment of the data that were used in the model.

Although it was not possible to visit all users (only seven remained to visit), it was possible to build a plan with estimated times for providing care, based on the visits that we were able to follow and the level of dependency of each user. Thus, in general terms, the planning was built according to the following premises:

i) Personal Hygiene:
   a) Users with several or high level of dependency: twenty minutes;
   b) Users with low or no level of dependency: fifteen minutes;

ii) Habitational Hygiene: thirty minutes;

iii) Meals' Distribution: five minutes.

As mentioned earlier, some users request services that are not available at the outset but can be contractualized if needed. Thus, the following situations arise in current planning:

i) For the users to whom it is necessary to prepare breakfast, lunch, dinner, and medication, five minutes were added to personal hygiene;
ii) Although generally, the Monday to Friday afternoons are reserved for habitational hygiene, some users need personal hygiene twice a day, and so visits lasting other than thirty minutes are personal hygiene.

Together with the users' data, we also collected information regarding the caregivers. Concerning the thirty minutes break in the morning, it is important to note that there is no fixed location associated, and so, employees can take the break where they choose. This way, it is assumed, by default, that the employees take the break in the CSPPSA.

Further, it was also important to define a maximum time by which the caregivers would have all lunch deliveries made, to not delay planning, and consequently to not delay the lunch break time. Thus, it was defined a scalar, worth 330 minutes, which defines this maximum time.

Another specificity implemented is related to the shifts. In our model, instead of adding an index *s*, we defined the maximum working time per day and per team as a strategy to distinguish between different shifts. Thus, from Monday to Friday there are two teams with a maximum working time of 600 minutes (in the CSPPSA this is the longest shift, with two hours of lunch) and three teams with a maximum of 540 minutes (the smallest shift, with one hour of lunch). At the weekend there is only one team (i.e., there are no shifts) - on Saturday the maximum working time is 540 minutes, and on Sunday it is 240 minutes. For these days, lunch breaks are only for one hour. Therefore, when the reference "1" appears after the team name, it means that the team belongs to the longest shift, whereas when the reference "2" appears, it means that the team belongs to the shortest shift. For example:

i) "k11" – Team k1 belonging to shift 1;
ii) "k42" – Team k4 belonging to shift 2.

Another main point in route planning is the scheme of days-off already implemented in the CSPPSA, which allows an understanding of how many teams are available each day. According to this scheme, two caregivers work on Saturday and Sunday. These two employees are entitled to two breaks each, which are usually given on Thursday and Friday to one caregiver, and Monday and Tuesday (after the weekend worked) to the second one. See the following scheme that is currently followed by the CSPPSA, as presented in *Table 6.1*: the nine available caregivers are designated with the names *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, and *I*.

In the first weekend of the scheme, *A* and *B* are the two caregivers working, so these two take the days-off on Thursday and Friday, and Monday and Tuesday. This scheme is repeated weekly, and each weekend the caregivers working/off are different. It is also possible to verify how many caregivers are available daily and consequently, how many teams can be assembled. This plan was adopted for the model developed in this thesis, in which the maximum number of teams per day is the one defined in the fifth column of the following scheme. This way, the model determines how many teams are needed each day depending on the scheduled visits.

Table 6.1: Scheme of days-off implemented in the CSPPSA

Week	Working	Day-off	Caregivers working	Teams
Monday	A, B, C, D, E, F, G, H	I	8 caregivers	4 teams
Tuesday	A, B, C, D, E, F, G, H	I	8 caregivers	4 teams
Wednesday	A, B, C, D, E, F, G, H, I	-	9 caregivers	5 teams
Thursday	B, C, D, E, F, G, H, I	A	8 caregivers	4 teams
Friday	B, C, D, E, F, G, H, I	A	8 caregivers	4 teams
Saturday	A, B	C, D, E, F, G, H, I	2 caregivers	1 team
Sunday	A, B	C, D, E, F, G, H, I	2 caregivers	1 team
Monday	A, C, D, E, F, G, H, I	B	8 caregivers	4 teams
Tuesday	A, C, D, E, F, G, H, I	B	8 caregivers	4 teams
Wednesday	A, B, C, D, E, F, G, H, I	-	9 caregivers	5 teams
Thursday	A, B, D, E, F, G, H, I	C	8 caregivers	4 teams
Friday	A, B, D, E, F, G, H, I	C	8 caregivers	4 teams
Saturday	C, D	A, B, E, F, G, H, I	2 caregivers	1 team
Sunday	C, D	A, B, E, F, G, H, I	2 caregivers	1 team
Monday	A, B, C, E, F, G, H, I	D	8 caregivers	4 teams
Tuesday	A, B, C, E, F, G, H, I	D	8 caregivers	4 teams

Source: Own Elaboration

In addition to issues related to the users and caregivers, aspects associated with the routes already implemented in the CSPPSA were also considered. Therefore, the caregivers only travel to the institution to collect the users' meals from Monday to Friday, at lunch and dinner (weekend meals are delivered on Friday afternoon with dinner), and on Sundays, caregivers have only the morning break.

Considering all the elements mentioned above, introduced in the model as inputs, another key question was to understand how much fuel is spent every hundred kilometers. As will be seen throughout the rest of the chapter, the management team does not perform a financial analysis of their costs. Thus, we consider two assumptions, verified when monitoring the routes:

i) The CSPPSA cars are old;

ii) Considering the locations of the users' homes (small geographical area), the congested traffic at peak hours, and the traffic signs, the caregivers stand in a constant stop-and-go.

Therefore, considering the price of fuel at the date of the present dissertation, we consider that the price of fuel is 1,20€ per liter and that one liter allows a distance of 12500 meters to be covered.

Considering all the features mentioned throughout the last chapters, the model was tested and validated together with 417 visiting nodes (including the origin, breaks, institution that provides the meals, and destination).

6.2 RESULTS AND DISCUSSION

Given the inherent complexity of the outlined constraints, not only by considering the typical characteristics of the VRP but also by the characteristics of the HSC services, the decision was to apply the MILP model presented in Chapter 5 for each day of the week in separate – this is possible because the decisions are independent for different days.

Therefore, the Mono-Objective version of the model was run for seven different cases (one for each day of the week) with the cost-related objective – this is hereafter referred to as Mono-Objective Model 1 (MONO 1) (section 6.2.1). Then, the mono-objective version of the model was run for one single day of the week using the maximization of equity as objective (this is hereafter referred to as Mono-Objective Model 2 – MONO 2) – the aim is to compare the planning results when different planning objectives are pursued (section 6.2.2). Afterward, to explore the impact of accounting for these two objectives simultaneously, a Multiobjective version of the model was built following the approach described in section 5.2 (this is hereafter referred to as MULTI), and this version was run for three different scenarios, with each scenario relying on different weights (section 6.2.3):

i) Scenario 1: the weight assigned to deviations associated with the minimum cost has a higher value when compared to the weight assigned to the deviation associated with the maximum equity (λ_1 =0,7 and λ_2 =0,3);
ii) Scenario 2: the weights assigned to deviations associated with the minimum cost and maximum equity have the same value (λ_1= λ_2 =0,5);
iii) Scenario 3: the weight assigned to deviations associated with the minimum cost has a lower value when compared to the weight assigned to the deviation associated with the maximum equity (λ_1 =0,3 and λ_2 =0,7)

A comparison between the planning solutions obtained when pursuing a single or multiple objectives is also presented, and a report of the computational results is also provided at the end of the chapter.

6.2.1 MONO-OBJECTIVE MODEL 1: OPERATIONAL COSTS

To present a comprehensive view of the planning of a week, the decision was made to choose one of the two objective functions to be analyzed in more detail. Therefore, the following analysis focuses on the Mono-Objective Model 1 (MILP model formulated in section 5.1 using Eq. (1) as an objective to be minimized), referring to the operational costs over a week.

Number of teams

It is important to focus on the number of teams working on each day, to effectively understand how many caregivers are needed to perform the visits (according to Mono-Objective Model 1), compared to the number of caregivers currently allocated (according to the CSPPSA). As specified in *Table 6.1*, the partner institution follows a scheme of days-off, which allows an understanding of how many teams are available each day. In fact, the management team always places the total number of caregivers available to work, which means that is never any team left to allocate (except for off-duty caregivers). For instance, for the Mono-Objective Model 1 case, as displayed in *Table 6.2*, the model always allocates three teams, from Monday to Friday, and at the weekend assigns one (since it was introduced, as model input, that the maximum number of teams working on the CSPPSA current model was one, so that number could not be different on the GAMS model). Hence, it turns out that over the week, the model informs that fewer teams than the number of teams actually working are required.

Table 6.2: Number of teams working for a week - CSPPSA VS Mono-Objective Model 1

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
CSPPSA	4	4	5	4	4	1	1
MONO 1	3	3	3	3	3	1	1
Difference	1	1	2	1	1	0	0

Source: Own Elaboration

Routes

As mentioned above, when monitoring the routes, it was realized that the route planning performed by the social worker does not include the sequence of visits, which means that the caregivers themselves decide the route to be carried out. As a result, no document was provided on which the sequence of visits could be verified. Even so, as it was allowed to monitor visits for a week, we have records of the sequence of visits performed in those days.

Only to exemplify, and not to make the analysis too cumbersome, the routes for a day of the week will be presented in *Table 6.3* (which includes, along with the visits, breaks, and the institution that provides the meals) and a day of the weekend (in which the routes are shorter), as in *Table 6.4*. The results presented below in these two tables represent the routes performed when running the Mono-Objective Model 1 for two days: Monday and Sunday, respectively.

Notice that, when collecting data from CSPPSA, a node planning (Annexes C to G) was built to comprehend how many visits are necessary to perform each day. This planning served as the basis for the construction of the model in GAMS. Therefore, the nomenclature used as "j" refers to a node, and the value that appears right next to it is only indicative. For a better understanding of the nomenclature used for the node's characterization, we recommend consulting Annex B.

The great advantage of route planning, as exemplified in *Table 6.3* and *Table 6.4*, compared to what is currently practiced at the CSPPSA, is because that one knows in advance the sequence of visits – i.e., which caregivers will visit which users - how long they will take to perform their visits, and how much time they have available at the end of the day to accomplish other tasks. As explained in Chapter 4, there is no plan previously defined for the visits, with each team of caregivers deciding on the sequence on their own, without considering any particular criteria.

Table 6.3: Monday Routes for the Mono-Objective Model 1

Team	Routes
k11	j414 – j18 – j11 – j13 – j8 – j5 – j24 – j10 – j20 – j412 – j15 – j19 – j27 – j7 – j6 – j21 – j9 – j416 – j40 – j49 – j42 – j413 – j59 – j417 – j79 – j73 – j72 – j78 – j64 – j68 – j67 – j76 – j69 – j415
k32	j414 – j2 – j412 – j12 – j16 – j14 – j22 – j28 – j1 – j17 – j3 – j26 – j25 – j416 – j50 – j30 – j46 – j41 – j51 – j33 – j47 – j38 – j413 – j56 – j57 – j60 – j61 – j54 – j55 – j417 – j66 – j415
k42	j414 – j4 – j412 – j23 – j416 – j29 – j39 – j32 – j45 – j43 – j31 – j35 – j34 – j44 – j48 – j36 – j37 – j413 – j53 – j62 – j58 – j63 – j52 – j417 – j71 – j65 – j77 – j75 – j74 – j70 – j415

Source: Own Elaboration

Table 6.4: Sunday Routes for the Mono-Objective Model 1

Team	Routes
k1	j414 – j406 – j405 – j403 – j404 – j412 – j408 – j407 – j410 – j411 – j409 – j415

Source: Own Elaboration

The routes for the remaining days can be accessed in Chapter 9.

Daily Working Time

Concerning the results obtained for the daily working time when the objective is to minimize the operational costs, it is relevant to compare it with the daily working time currently practiced in the CSPPSA. According to the CSPPSA, there is no control or analysis of how much time is spent by the caregivers providing the services to each user. In fact, caregivers have access to a mobile phone application in which they specify the hours at which each team enters and leaves the users' house. However, the caregivers fail to register, making it unreliable. Therefore, and considering the routes that were possible to follow, we assume the worst scenario, in which the caregivers finish their workday near the end of each shift.

In *Table 6.5* it is presented the total daily working time for both approaches. As one can see, the time needed to provide HSC services for a week according to the solution obtained by running the model is roughly 128 hours (7704 minutes) per week, contrasting with the worst scenario of the CSPPSA, which is 212 hours (12720 minutes) per week. It is interesting to note that, when CSPPSA allocates all teams (Wednesday), the solution from the Mono-Objective Model 1 places only three teams: on CSPPSA shift 1 two teams are allocated (in a total of 1200 minutes), while in Mono-Objective Model 1 only one team is assigned, with a daily working day of 572 minutes; for instance, in CSPPSA shift 2, three teams are allocated, for a total of 1620 minutes, while in Mono-Objective Model 1, two teams are assigned, for a total of 872 minutes. The reduction of working time in Mono-Objective Model 1 is 1376 minutes.

Table 6.5: Total Daily Working Time – CSPPSA Worst Solution VS Mono-Objective Model 1 (in minutes)

Days		Mon	Tue	Wed	Thu	Fri	Sat	Sun	**Total**
CSPPSA	k11	600	600	600	600	600	540	240	
	k21	600	600	600	600	600			
	k32	540	540	540	540	540			
	k42	540	540	540	540	540			
	k52			540					
Total DWT (min.)		2280	2280	2820	2280	2280	540	240	12720
MONO 1	k11	582	508		492	600	247	224	
	k21		518	572					
	k32	474			415	384			
	k42	401	474	337	426	515			
	k52			535					
Total DWT (min.)		1457	1500	1444	1333	1499	247	224	7704

Source: Own Elaboration

First and Last Visits

Other variables of interest, which are key points for the calculation of the daily working time, are the hours of the first and the last visit. These variables allow us a more realistic idea of the time allocated to the provision of HSC services, which includes the time spent traveling between nodes. As expected, these times do not include travel between the origin and the first visit, and between the last visit and the destination. As one can see in *Table 6.6*, for the Mono-Objective Model 1, the model assigns for each team a first visit in which the travel time from the origin is between one and three minutes. According to the CSPPSA's current solution, caregivers can move to more peripheral areas, starting the day with users furthest from the CSPPSA, which consequently leads to a longer travel time for the first trips.

Table 6.6: Time at which each team starts serving the first visit, on Monday – Mono-Objective Model 1 (in minutes)

Teams	MONO 1
k11	1
k21	2
k32	1
k42	3

Source: Own Elaboration

Operational Costs

According to the CSPPSA, no cost control is carried out, i.e., it is not possible to indicate how much is spent in terms of fuel and caregivers' wages. For this reason, one cannot juxtapose the CSPPSA and model's solutions.

Nevertheless, it is relevant to explore the results related to costs in more detail. Accordingly, it appears that a high percentage of the daily costs is related to the salaries of the caregivers, and the expressiveness of fuel costs is very low. The total cost for a week is around 1130,36€. As expected, the least costly days are at the weekend (*Table 6.7*). For the rest, the day with the lowest value is Wednesday, on which the major reduction is in wage costs. This value is the lowest for the reason mentioned above, in the daily working time tab.

Table 6.7: Fuel and Wage Costs for a week - Mono-Objective Model 1 (in euros)

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Fuel Costs	4,12	4,13	4,11	3,19	4,2	0,6	0,98
Wage Costs	218,55	218,55	176,48	199,95	224,85	37,05	33,6
Total Costs	222,67	222,68	180,59	203,14	229,05	37,65	34,58

Source: Own Elaboration

For the calculation of this function, the portion referring to the salaries includes two parcels, namely the salary of caregivers per minute, multiplied by the total number of working hours. It is important to note that the hours included in this arithmetic only refer to the hours allocated to the provision of HSC services. As expected, each caregiver has a total number of contract hours that they must comply with, i.e., the daily working hours are not minimized. However, the key point of minimizing wages advert to the number of caregivers needed each day. As mentioned above, results obtained by running the model when considering a cost-minimization perspective show that it is possible to reduce from five to three the number of teams that are required per day. Therefore, by reducing the number of caregivers required, the cost of wages is minimized.

6.2.2 MONO-OBJECTIVE MODEL 1 VS MONO-OBJECTIVE MODEL 2: OPERATIONAL COSTS VS EQUITY OBJECTIVES

While objective function 2 (Eq. (2)) aims to maximize equity, objective function 1 (Eq. (1)) has the purpose of minimizing operating costs, which include, along with travel costs, the working time associated with salary costs of the caregivers. In other words, for objective 1 it is assumed a cost approach, while for objective function 2 it is intended a time analysis.

To simplify the analysis, a day of the week was chosen to develop the comparison between the two mono-objective models and the current CSPPSA solution – the day selected was Monday.

Operational Costs VS Equity

This analysis is important because it allows decision-makers to study conflicting objectives and obtain more information to understand what the best approach to their daily planning is.

First of all, when studying operational costs and equity, one is comparing the result of the Mono-Objective Model 1 with the result of the Mono-Objective Model 2. Our great expectation when comparing these two is that the optimal result for the cost (i.e., the result with a lower value) is obtained when applying the Mono-Objective Model 1 and that the optimal result for equity (i.e., the maximum equity, which translates into the minimum difference between the workload of teams) is obtained when applying the Mono-Objective Model 2. As one can see in *Table 6.8*, our expectation is verified. When the Mono-Objective Model 1 is applied, the minimum costs implies a higher inequality in the caregivers' workload. On the other hand, when the Mono-Objective Model 2 is applied, one obtains the best possible equitable workload, which is, in this case, no difference at all (see the zero-value shown in *Table 6.8*). Yet, to obtain these results for equity, the routes end up becoming less efficient, and consequently, the operational costs increase.

Table 6.8: Operational Costs (in euros per week) VS Equity (in minutes) – Mono-Objective Model 1 VS Mono-Objective Model 2

	Operational Costs	Equity
MONO 1	222,67	155
MONO 2	394,85	0

Source: Own Elaboration

Number of teams

One of the major differences to emphasize is the number of teams allocated to the HSC services. As one can see in *Table 6.9*, Mono-Objective Model 1 allocates, on Monday, only three caregivers, while Mono-Objective Model 2 assigns four teams. This corroborates what was mentioned above: there is a trade-off between these two functions, in which to obtain a better result in equity, operating costs increase, which is largely explained by the increase in the number of caregivers.

Table 6.9: Number of teams working for a week – Mono-Objective Model 1 VS Mono-Objective Model 2

	Monday
MONO 1	3
MONO 2	4
Difference	1

Source: Own Elaboration

Routes, Daily Working Time, and Total Travel Distance

The routes from both Mono-Objective Models cover all nodes, fulfilling the additional requirements of daily working time, rest times, and time needed to provide care to the users, according to their degree of dependency, among others. The routes comply with all the constraints, and caregivers are allocated to all daily visits, none of which are neglected. This is important because in the CSPPSA's current circumstances no user remains to visit. Nevertheless, as already mentioned, caregivers can see their rest hours reduced to fulfill the planning. According to the solution obtained when considering both objectives, caregivers can visit all patients and still have a time interval until the end of the shift to perform other tasks (according to the daily working time for each Mono-Objective Model). The remaining routes can be found in Chapter 9.

As presented in *Table 6.10*, Mono-Objective Model 2 allocates one more team than Mono-Objective Model 1. It is interesting to note that, while Mono-Objective Model 1 assigns one team to shift 1 (the longest) and two teams to shift 2 (the shortest), the Mono-Objective Model 2 allocates two teams in each of the shifts. When it comes to the total travel distance, one found that the four teams in Mono-Objective Model 2 cover a much greater distance than the teams in Mono-Objective Model 1, which suggests inefficient routes. However, associated with these same routes, one has an entirely equitable distribution of daily working time, while in Mono-Objective Model 1, the biggest difference between the teams is 181 minutes of working time.

Table 6.10: Summary of the number of visits per team, daily working time, and total travel distance, on Monday - Mono-Objective Model 1 VS Mono-Objective Model 2

	MONO 1			MONO2			
	k11	k32	k42	k11	k21	k32	k42
Number of visits per team	32	30	29	20	21	27	27
DWT (in minutes)	582	474	401	470	470	470	470
TTD (in meters)	14176	13098	15595	20202	34380	38090	41100

Source: Own Elaboration

6.2.3 MULTIOBJECTIVE MODEL

When implementing Chebyshev's method, different weights are assigned to each solution. The three scenarios considered for the multiobjective version of the model - hereafter referred to as Multiobjective Model 1 (MULTI 1), Multiobjective Model 2 (MULTI 2), and Multiobjective Model 3 (MULTI 3) - were run only for one day of the week, particularly, for Monday. The choice of this day was random since one could have chosen any of the seven days of the week.

Operational Costs VS Equity

This analysis is important because it allows decision-makers to understand how the functions behave simultaneously with different weightings.

As expected, the two objectives react in the same direction to the variation of the weights assigned. That is, when the weight attributed to the operational costs function decreases/increases, its value increase/decrease. The same happens with equity: for increasingly higher (small) values of the weight, the distribution of working time among the caregivers becomes progressively equitable (more balanced).

As stated in *Table 6.11*, starting from an intermediate situation, in which the same weight is attributed to both objectives, the solution is 305,93€ for operational costs and a difference of 123 minutes in the caregivers' working time. In the case of CSPPSA privileging a situation of equity among caregivers, the improvement is 19,51%, but with an associated increase in operational costs of around 23,30%. In the opposite case, in which the CSPPSA favors a reduction in operational costs, the gain is 22,36%, but with an increase in the inequality of working time in the order of 37,39%. However, and considering the literature review, most PISS are often under budget constraints. Thus, in a comparison between the extreme cases presented here, if the CSPPSA intends to move to an extreme situation of cost reduction, the reduction would be around 37,31%, as opposed to a drastic reduction in terms of equality in working time in the order of 90,90%.

Table 6.11: Operational Costs (in euros per week) VS Equity (in minutes) – Multiobjective Model 1 VS Multiobjective Model 2 VS Multiobjective Model 3

	MULTI 1	MULTI 2	MULTI 3
Operational Costs	234,64	305,93	374,34
Equity	147	123	77

Source: Own Elaboration

Routes, Daily Working Time, and Total Travel Distance

Concerning the routes obtained, the results are in line with those observed in the Mono-Objective versions of the model. All visits are included in the planning, and all the restrictions in terms of breaks and meals' distribution are respected.

As an example, the following *Table 6.12* present the routes for the Multiobjective Model 3, given that four teams are allocated. Notice that in Multiobjective Model 3, the weight assigned to the objective function 2 (Eq. (2)) is 0,7. Thus, to achieve a higher level of equity, the model allocates one more team to work to be able to distribute visits more evenly among all employees.

Table 6.12: Monday Routes for the Multiobjective Model 3

Team	Routes
k11	j414 – j11 – j18 – j412 – j23 – j1 – j28 – j17 – j22 – j3 – j26 – j25 – j416 – j33 – j47 – j413 – j61 – j55 – j54 – j417 – j73 – j79 – j70 – j415
k21	j414 – j12 – j412 – j15 – j416 – j30 – j51 – j41 – j50 – j36 – j48 – j37 – j413 – j56 – j57 – j60 – j58 – j52 – j63 – j417 – j66 – j415
k32	j414 – j10 – j20 – j24 – j5 – j8 – j13 – j412 – j16 – j14 – j416 – j49 – j42 – j413 – j59 – j417 – j72 – j78 – j74 – j415
k42	j414 – j2 – j412 – j4 – j19 – j27 – j9 – j6 – j21 – j7 – j416 – j40 – j32 – j39 – j29 – j46 – j45 – j43 – j31 – j35 – j44 – j34 – j38 – j413 – j53 – j62 – j417 – j71 – j65 – j77 – j75 – j64 – j68 – j67 – j76 – j69 – j415

Source: Own Elaboration

Regarding the daily working time, notice that the best results in terms of equity are those of the Multiobjective Model 3, in which the biggest difference in working time is 38 minutes. For instance, while in the Multiobjective Model 2 it is 99 minutes, in Multiobjective Model 3 one recognizes 108 minutes. In terms of the number of visits per team, note that Multiobjective Models 1 and 2 are more balanced, while Multiobjective Model 3, despite being the most unbalanced in terms of number, is the most balanced in terms of time. It is also important to note that, in Multiobjective Model 1, the team that has a greater daily working time, also have a greater total traveled distance. However, when looking at Multiobjective Models 2 and 3, it appears that there are teams that have a shorter daily working time associated with greater distances covered when compared with teams from each of the models. These results indicate inefficient routes (*Table 6.13*).

Table 6.13: Summary of the number of visits per team, daily working time, and total travel distance, on Monday - Multiobjective Model 1 VS Multiobjective Model 2 VS Multiobjective Model 3

	MULTI 1			MULTI 2			MULTI 3			
	k11	k21	k32	k11	k21	k32	k11	k21	k32	k42
Number of visits per team	29	30	32	28	32	31	22	20	18	35
DWT (in min.)	559	520	451	554	530	455	503	506	470	468
TTD (in meters)	20593	14625	18298	24967	20125	26097	20052	31056	32480	36540

Source: Own Elaboration

6.2.4 MONO-OBJECTIVE MODELS VS MULTIOBJECTIVE MODELS

Operational Costs VS Equity

As expected, Mono-Objective Models 1 and 2 present the best solutions for the respective objective function, as shown in *Table 6.14*. That is, Mono-Objective Models provide the best solutions for the respective objective function that they minimize and present the worst solutions for the remaining objective function.

As noted in the previous section, suppose one starts in an intermediate situation. In the case of CSPPSA privileging a situation of equity among caregivers (moving forward to Mono-Objective 2), one obtains the optimal solution for equity, but with an associated increase in operational costs of around 29,06%. In the opposite case, in which the CSPPSA favors a reduction in operational costs (moving forward to Mono-Objective Model 1), the gain is around 27,21%, but with an increase in the inequality of working time in the order of 26,01%. Considering the situation in which the CSPPSA intends a drastic reduction in operational costs, the reduction would be around 43,60%, but with an increase in inequality in the working times of the various teams, with a difference of 155 minutes.

Table 6.14: Operational Costs (in euros) VS Equity (in minutes) – Mono-Objective Models VS Multiobjective Models

	MONO 1	MULTI 1	MULTI 2	MULTI 3	MONO 2
Operational Costs	222,67	234,64	305,93	374,34	394,85
Equity	155	147	123	77	0

Source: Own Elaboration

6.2.5 COMPUTATIONAL RESULTS

In the following tables, it is possible to evaluate some of the characteristics of the model's runs for each day, particularly the execution time, which is the time interval necessary for the model to generate an output. These computational results show the complexity of a problem such as the one under study.

Table 6.15 presents the computational results for Mono-Objective Model 1:

Table 6.15: Computational Results under Mono-Objective Model 1

Days	Execution Time (seconds)	Gap	Iterations	Single Equations	Integer Variables	Variables
Monday	28800	27,61%	3405620	58673	15008	29488
Tuesday	28800	25,34%	4471739	60051	15452	30169
Wednesday	28800	29,72%	4235597	71597	18215	35995
Thursday	28800	44,65%	5393614	55965	15940	28146
Friday	28800	23,32%	5374424	61445	15752	30862
Saturday	0,609	4,41%	30	284	54	289
Sunday	0,469	8,23%	98	336	70	313

Source: Own Elaboration

Table 6.16 presents the results for Mono-Objective Model 2, and the Multiobjective Models:

Table 6.16: Computational Results under Mono-Objective Model 2 and Multiobjective Models

Models	Execution Time (seconds)	Gap	Iterations	Single Equations	Integer Variables	Variables
MONO 2	7343	0%	2879160	58677	15008	29487
MULTI 1	28800	19,4%	18299831	58681	29380	29490
MULTI 2	28800	23,6%	9835487	58681	29380	29490
MULTI 3	28800	29,2%	15856393	58681	29380	29490

Source: Own Elaboration

7. CONCLUSIONS AND FUTURE WORK

7.1 CONCLUSIONS

One of the most significant and impactful changes of the century is the increasingly sharp ageing of the population. Widespread access to increasingly improved healthcare, coupled with low birth rates, means that the proportion of elderly people in the general population is expanding. Accompanying ageing is the increase of chronic diseases, such as dementia. Therefore, the only solution to this problem often lies in PISS, which provide care for the elderly and dependent individuals. Its non-profit nature, along with the limited budget and support that is often made available to these institutions, makes it essential to ensure adequate planning of its services' provision. Since a key service provided within the scope of these institutions involves the delivery of care in home-based settings, adequate planning at this level often implies ensuring the most efficient routing and scheduling planning decisions. These decisions usually involve an efficient allocation of scarce resources (both human and material resources, such as caregivers and vehicles used to visit people at home), the delivery of a multiplicity of services (such as personal and habitational hygiene, as well as meals' distribution), as well as accounting for cost and equity concerns in the delivery of care.

Given the complexity of this planning, the decision was to build a MILP model that would help PISS in their daily planning of services' provision. Particularly, it aims at supporting PISS making decisions on the routes that should be established by each team of caregivers, as well as on the workload distribution across caregivers, and this while considering multiple and potentially conflicting objectives, which is the case of cost and equity-related objectives. Chebyshev's method was chosen to solve the developed multiobjective model. The developed model was implemented in GAMS software, respecting a series of restrictions already followed by CSPPSA, such as working time regulations, breaks, meals' distribution, among others.

The proposed model contributes to the literature in the area as it is an adequate tool for route planning and staff scheduling, for a little-studied field such as HSC. This allows decision-makers to take more assertive measures concerning what they want to achieve for the respective institution, ultimately obtaining optimal solutions for the operations outlining.

The proposed approach was applied to a real case study, namely, to support the daily planning of home-based services provided by CSPPSA, in Portugal. Given the inherent complexity of the model itself, the decision was to run the model independently for each day of the week, to reduce the computational requirements. Despite results are obtained with substantial gaps, the solutions are quite satisfying. In case CSPPSA favors a situation of lower costs, the single-objective version of the model accounting for a single cost-related objective will be the most appropriate. In turn, if an equitable distribution of daily working time between all caregivers is privileged, the single-objective version of the model accounting for a single equity-related objective will be the most suitable. However, if the decision-maker considers the two objective functions imperative, the multiobjective approach should be implemented.

In addition to the improvement of the results, when compared to the CSPPSA, this model also allows for a reduction in planning complexity and allows for a more realistic picture of all users who are required to serve. Planning is no longer done manually, two route configurations per day are no longer mandatory, and there is no risk of obtaining sub-optimal solutions. Hence, the model indicates how many teams are needed per day, and using that information as a basis, the management team only needs to manage which caregivers should be selected for each route, respecting the rotation of employees in terms of weekly workload and shifts. By not specifying the caregivers to work each day, it also allows respecting two guidelines already imposed in the CSPPSA, namely the rotation of pairs and the distribution of the most demanding work by all. By imposing that the delivery of lunches should be finished before a pre-established hour, we ensure that the caregivers do not delay their tasks and are able to take their lunch break at the programmed time. With the introduction of the daily working time, it allows contemplating part-time caregivers (notwithstanding this valence does not exist in the CSPPSA, other PISS can apply this feature to their caregivers). Additionally, by minimizing the working time, it is possible to obtain a constant and periodic routine in which caregivers are able to perform other tasks, without neglecting the provision of HSC services.

7.2 LIMITATIONS AND FUTURE WORK

In terms of future research, some key proposals can be considered. The first concerns the institution that provides the food. As mentioned, the institution that provides the meals in an external organization, and so the CSPPSA has to adapt to the institution's schedules, particularly about the hours when meals are ready to be collected. Therefore, additional constraints could be included in the model to model the traveling to the institution after a certain hour contracted with the organization.

Secondly, it is important to define, even more accurately, the users' autonomy. When building the model, users' autonomy was considered by adding a few more minutes to the provision of services to more dependent users. However, it may be interesting to explore the teams with special requirements whenever there is a need to visit the most dependent users - for instance, it may be imposed that more than one caregiver is necessary for those visits. This may also be added as additional constraints in the model.

Thirdly, it should be noticed that, when the model was built, four daily visit subsets were created, namely personal hygiene, lunch distribution, habitational hygiene, and dinner distribution. With the allocation of caregivers proposed by the model, what happens is that a team can perform more visits focused on personal hygiene, whereas another one provides more visits focused on habitational hygiene. Therefore, it may also be interesting to add additional constraints forcing a more equitable distribution of different types of services among the teams working each day. One should note that the equity-related objective accounted for in the model as Objective Function 2 is only focused on ensuring that the daily working time is distributed equally, but it does not model the equitable distribution of types of services.

Lastly, we recommend future investigation in which decision-makers are empowered to introduce other objective functions that better model the environment in which the institution is inserted, and the services provided.

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9. ANNEXES

Annex A: Meeting guide with the Direction

Meeting Guide with CSPPSA Direction

Day: December 19, 2019

Questions:

1) Who is the person / people responsible for planning the HSC services?
2) How is the planning carried out, and how far in advance?
3) How long does it take to finish the planning?
4) How is the team that provides these services constituted?
5) Is a technical background necessary?
6) How do the caregivers move around the visits?
7) How many cars do the caregivers have available?
8) How are teams formed?
9) Is there a rotation of pairs, or are the working pairs fixed?
10) Are the employees who visit a given user always the same, or does it vary?
11) Is there any time contracted with each user to perform the visit?
12) How are the caregivers allocated according to the users' autonomy?
13) How many breaks do the caregivers have and what is their duration?
14) Is there an obligation to take breaks at the CSPPSA?
15) Which institution provides the meals?
16) Where is the institution located?
17) Is there any control over the hours at which each visit was performed?
18) Is there any control over how long it takes to provide a specific service to each user?
19) Do caregivers visit all users, or can they leave a visit for the next day due to lack of time?
20) How many caregivers are needed, at least, to provide HSC services?
21) Are there part-time caregivers?
22) How much the caregivers earn per hour?
23) How many days off are the caregivers entitled to?
24) How it is the days-off scheme?
25) How many hours of training a year do the caregivers have?

Annex B: Nodes Nomenclature in GAMS software

Set of nodes in a week	j1 to j417
Set of daily visits in a week	j1 to j411
Set of daily visits on Monday	j1 to j79
Set of personal hygiene on Monday	j1 to j28
Set of lunch distribution on Monday	j29 to j51
Set of habitational hygiene on Monday	j52 to j63
Set of dinner distribution on Monday	j64 to j79
Set of daily visits on Tuesday	j80 to j159
Set of personal hygiene on Tuesday	j80 to j108
Set of lunch distribution on Tuesday	j109 to j131
Set of habitational hygiene on Tuesday	j132 to j143
Set of dinner distribution on Tuesday	j144 to j159
Set of daily visits on Wednesday	j160 to j237
Set of personal hygiene on Wednesday	j160 to j186
Set of lunch distribution on Wednesday	j187 to j209
Set of habitational hygiene on Wednesday	j210 to j221
Set of dinner distribution on Wednesday	j222 to j237
Set of daily visits on Thursday	j238 to j314
Set of personal hygiene on Thursday	j238 to j264
Set of lunch distribution on Thursday	j265 to j287
Set of habitational hygiene on Thursday	j288 to j298
Set of dinner distribution on Thursday	j299 to j314
Set of daily visits on Friday	j315 to 395
Set of personal hygiene on Friday	j315 to j343
Set of lunch distribution on Friday	j344 to j366
Set of habitational hygiene on Friday	j367 to j378
Set of dinner distribution on Friday	j379 to j395
Set of daily visits on Saturday	j396 to j402
Set of personal hygiene on Saturday	j396 to j402
Set of daily visits on Sunday	j403 to j411
Set of personal hygiene on Sunday	j403 to j411
Set of break nodes	j412 to j413
Origin of all routes	j414
Destination of all routes	j415
Institution that provides the meals	j416 to j417

Annex C: Summary table of the weekly provision of personal hygiene according to each user and respective duration (Creation of several nodes per user according to the number of visits)

PERSONAL HYGIENE

ID	NODES	MON	TUE	WED	THU	FRI	SAT	SUN
P2 + P6	j1,j80,j160,j238,j315,j403	30	40	30	30	40		10
P3	j2,j81,j161,j239,j316	20	15	20	15	20		
P4	j3,j82,j162,j240,j317	15	20	15	15	20		
P5	j4,j83,j163,j241,j318,j396,j404	15	15	15	15	15	15	15
P7	j5,j84,j164,j242,j319	20	20	20	20	20		
P8	j6,j85,j165,j243,j320	15	15	15	20	15		
P9	j86,j321		20			20		
P10	j7,j87,j166,j244,j322,j405	20	15	20	15	20		15
P11	j8,j88,j167,j245,j323	25	20	25	20	25		
P12	j9,j89,j168,j246,j324	15	20	15	15	20		
P14	j90,j325		15			15		
P15	j10,j91,j169,j247,j326	20	20	20	20	20		
P16	j11,j92,j170,j248,j327,j397,j406	20	15	20	15	20	15	15
P17	j12,j249	15			15			
P20	j13,j93,j171,j250,j328,j398,j407	20	20	20	20	20	20	20
P21	j14,j172,j329	20		20		20		
P22	j94		20					
P23	j15,j95,j173,j251,j330	20	15	20	15	20		
P24	j16,j96,j174,j252,j331	15	15	15	20	15		
P25	j17,j97,j175,j253,j332	15	20	15	15	20		
P26	j18,j98,j176,j254,j333,j399,j408	20	15	15	20	15	15	15
P34	j19,j99,j177,j255,j334	20	15	20	15	20		
P38	j20,j100,j178,j256,j335	15	15	15	20	15		
P31	j21,j101,j179,j257,j336	20	15	55	15	20		
P33	j22,j102,j180,j258,j337	20	15	20	15	20		
P37	j23,j103,j181,j259,j338,j400,j409	20	15	15	15	20	15	15
P39	j24,j104,j182,j260,j339,j401,j410	25	25	25	25	25	25	25
P32	j25,j105,j183,j261,j340	15	20	15	15	20		
P40	j26,j106,j184,j262,j341	15	15	15	15	15		
P43	j27,j107,j185,j263,j342	15	15	15	20	15		
P45	j28,j108,j186,j264,j343,j402,j411	20	15	20	15	20	15	15
TOTAL		j1 to j28	j80 to j108	j160 to j186	j238 to j264	j315 to j343	j396 to j402	j403 to j411

Annex D: Summary table of the weekly provision of lunch distribution according to each user and respective duration (Creation of several nodes per user according to the number of visits)

LUNCH DISTRIBUTION

ID	NODES	MON	TUE	WED	THU	FRI
P1	j29,j109,j187,j265,j344	5	5	5	5	5

P2 + P6	j30,j110,j188,j266,j345	5	5	5	5	5
P5	j31,j111,j189,j267,j346	10	10	10	10	10
P8	j32,j112,j190,j268,j347	5	5	5	5	5
P9	j33,j113,j191,j269,j348	5	5	5	5	5
P11	j34,j114,j192,j270,j349	5	5	5	5	5
P13	j35,j115,j193,j271,j350	5	5	5	5	5
P15	j36,j116,j194,j272,j351	5	5	5	5	5
P17	j37,j117,j195,j273,j352	5	5	5	5	5
P18	j38,j118,j196,j274,j353	5	5	5	5	5
P19 + P28	j39,j119,j197,j275,j354	5	5	5	5	5
P21	j40,j120,j198,j276,j355	5	5	5	5	5
P22	j41,j121,j199,j277,j356	5	5	5	5	5
P24	j42,j122,j200,j278,j357	5	5	5	5	5
P27 + P42	j43,j123,j201,j279,j358	5	5	5	5	5
P29	j44,j124,j202,j280,j359	5	5	5	5	5
P30	j45,j125,j203,j281,j360	5	5	5	5	5
P33	j46,j126,j204,j282,j361	5	5	5	5	5
P35	j47,j127,j205,j283,j362	5	5	5	5	5
P39	j48,j128,j206,j284,j363	5	5	5	5	5
P41	j49,j129,j207,j285,j364	5	5	5	5	5
P44	j50,j130,j208,j286,j365	5	5	5	5	5
P45	j51,j131,j209,j287,j366	10	10	10	10	10
TOTAL		j29 to j51	j109 to j131	j187 to j209	j265 to j287	j344 to j366

Annex E: Summary table of the weekly provision of habitational hygiene according to each user and respective duration (Creation of several nodes per user according to the number of visits)

HABITATIONAL HYGIENE

ID	**NODES**	**MON**	**TUE**	**WED**	**THU**	**FRI**
P2 + P6	j52,j132,j210,j288,j367	30	10	10	10	10
P5	j53	30				
P8	j54,j133,j211,j289,j368	10	10	10	10	10
P9	j290				30	
P10	j55,j134,j212,j291,j369	10	10	10	10	10
P11	j56,j135,j213,j292,j370	10	10	10	10	10
P13	j371					20
P18	j293				20	
P19 + P28	j136		20			
P20	j57,j137,j214,j294,j372	10	10	10	10	10
P22	j58,j138,j215,j295,j373	15	15	15	15	15
P25	j216			20		
P27 + P42	j374					20
P38	j59,j139,j217,j296,j375	10	10	10	10	10
P29	j218			20		
P34	j60	20				
P35	j140		30			

P39	j141		30			
P36	j219			30		
P40	j61,j142,j220,j297,j376	10	10	10	10	30
P41	j62	20				
P44	j377					20
P45	j63,j143,j221,j298,j378	10	10	10	10	10
TOTAL		j52 to j63	j132 to j143	j210 to j221	j288 to j298	j367 to j378

Annex F: Summary table of the weekly provision of dinner distribution according to each user and respective duration (Creation of several nodes per user according to the number of visits)

DINNER DISTRIBUTION

ID	**NODES**	**MON**	**TUE**	**WED**	**THU**	**FRI**
P5	j64,j144,j222,j299,j379	10	10	10	10	10
P8	j65,j145,j223,j300,j380	5	5	5	5	5
P9	j66,j146,j224,j301,j381	5	5	5	5	5
P11	j67,j147,j225,j302,j382	5	5	5	5	5
P13	j68,j148,j226,j303,j383	5	5	5	5	5
P15	j69,j149,j227,j304,j384	5	5	5	5	5
P18	j70,j150,j228,j305,j385	5	5	5	5	5
P19 + P28	j71,j151,j229,j306,j386	5	5	5	5	5
P21	j72,j152,j230,j307,j387	5	5	5	5	5
P22	j73,j153,j231,j308,j388	5	5	5	5	5
P24	j74,j154,j232,j309,j389	5	5	5	5	5
P27 + P42	j75,j155,j233,j310,j390	5	5	5	5	5
P29	j76,j156,j234,j311,j391	5	5	5	5	5
P30	j77,j157,j235,j312,j392	5	5	5	5	5
P39	j393					5
P41	j78,j158,j236,j313,j394	5	5	5	5	5
P44	j79,j159,j237,j314,j395	5	5	5	5	5
TOTAL		j64 to j79	j144 to j159	j222 to j237	j299 to j314	j379 to j395

Annex G: Summary table of the total number of nodes per user

ID	**NODES**
P1	j29,j109,j187,j265,j344
P2 + **P6**	j1,j80,j160,j238,j315,j403,j30,j110,j188,j266,j345,j52,j132,j210,j288,j367
P3	j2,j81,j161,j239,j316
P4	j3,j82,j162,j240,j317

P5	j4,j83,j163,j241,j318,j396,j404,j31,j111,j189,j267,j346,j53,j64,j144,j222,j299,j379
P7	j5,j84,j164,j242,j319
P8	j6,j85,j165,j243,j320,j32,j112,j190,j268,j347,j54,j133,j211,j289,j368,j65,j145,j223,j300,j380
P9	j86,j321,j33,j113,j191,j269,j348,j290,j66,j146,j224,j301,j381
P10	j7,j87,j166,j244,j322,j405,j55,j134,j212,j291,j369
P11	j8,j88,j167,j245,j323,j34,j114,j192,j270,j349,j56,j135,j213,j292,j370,j67,j147,j225,j302,j382
P12	j9,j89,j168,j246,j324
P13	j35,j115,j193,j271,j350,j371,j68,j148,j226,j303,j383
P14	j90,j325
P15	j10,j91,j169,j247,j326,j36,j116,j194,j272,j351,j69,j149,j227,j304,j384
P16	j11,j92,j170,j248,j327,j397,j406
P17	j12,j249,j37,j117,j195,j273,j352
P18	j38,j118,j196,j274,j353,j293,j70,j150,j228,j305,j385
P19 + **P28**	j39,j119,j197,j275,j354,j136,j71,j151,j229,j306,j386
P20	j13,j93,j171,j250,j328,j398,j407,j57,j137,j214,j294,j372
P21	j14,j172,j329,j40,j120,j198,j276,j355,j72,j152,j230,j307,j387
P22	j94,j41,j121,j199,j277,j356,j58,j138,j215,j295,j373,j73,j153,j231,j308,j388
P23	j15,j95,j173,j251,j330
P24	j16,j96,j174,j252,j331,j42,j122,j200,j278,j357,j74,j154,j232,j309,j389
P25	j17,j97,j175,j253,j332,j216
P26	j18,j98,j176,j254,j333,j399,j408
P27 + **P42**	j43,j123,j201,j279,j358,j374,j75,j155,j233,j310,j390
P29	j44,j124,j202,j280,j359,j218,j76,j156,j234,j311,j391
P30	j45,j125,j203,j281,j360,j77,j157,j235,j312,j392
P31	j21,j101,j179,j257,j336
P32	j25,j105,j183,j261,j340
P33	j22,j102,j180,j258,j337,j46,j126,j204,j282,j361
P34	j19,j99,j177,j255,j334
P35	j47,j127,j205,j283,j362,j140
P36	j219
P37	j23,j103,j181,j259,j338,j400,j409
P38	j59,j139,j217,j296,j375

P39	j24,j104,j182,j260,j339,j401,j410,j48,j128,j206,j284,j363,j141,j393
P40	j26,j106,j184,j262,j341,j61,j142,j220,j297,j376
P41	j49,j129,j207,j285,j364,j62,j78,j158,j236,j313,j394
P43	j27,j107,j185,j263,j342
P44	j50,j130,j208,j286,j365,j377,j79,j159,j237,j314,j395
P45	j28,j108,j186,j264,j343,j402,j411,j51,j131,j209,j287,j366,j63,j143,j221,j298,j378

Annex H: Weekly routes for Mono-Objective Model 1, with corresponding daily working time (in minutes) and total travel distance (in meters)

GAMS Solution – Tuesday

Team	Route	DWT	TTD
k11	j414 – j83 – j412 – j95 – j99 – j107 – j87 – j101 – j85 – j89 – j416 – j120 – j110 – j131 – j121 – j130 – j126 – j125 – j123 – j111 – j115 – j114 – j124 – j128 – j116 – j413 – j141 – j135 – j137 – j417 – j151 – j145 – j158 – j154 – j150 – j415	508	15684
k21	j414 – j81 – j412 – j96 – j105 – j106 – j82 – j97 – j108 – j94 – j102 – j80 – j416 – j113 – j127 – j413 – j139 – j132 – j138 – j143 – j417 – j152 – j159 – j153 – j157 – j155 – j144 – j148 – j147 – j156 – j149 – j415	518	13321
k42	j414 – j98 – j90 – j92 – j93 – j88 – j104 – j84 – j91 – j100 – j412 – j103 – j86 – j416 – j109 – j119 – j112 – j129 – j122 – j118 – j117 – j413 – j140 – j142 – j136 – j134 – j133 – j417 – j146 – j415	474	14028

GAMS Solution – Wednesday

Team	Route	DWT	TTD
k21	j414 – j176 – j171 – j170 – j167 – j182 – j164 – j412 – j163 – j177 – j185 – j160 – j180 – j175 – j416 – j187 – j197 – j190 – j203 – j201 – j200 – j196 – j413 – j211 – j212 – j220 – j219 – j417 – j229 – j223 – j227 – j415	572	15575
k42	j414 – j178 – j169 – j412 – j174 – j181 – j172 – j416 – j191 – j205 – j413 – j218 – j213 – j214 – j417 – j230 – j237 – j231 – j235 – j233 – j222 – j226 – j234 – j225 – j228 – j415	337	12740
k52	j414 – j161 – j412 – j173 – j183 – j184 – j162 – j186 – j168 – j166 – j165 – j179 – j416 – j188 – j208 – j204 – j199 – j209 – j198 – j207 – j189 – j193 – j192 – j202 – j206 – j194 – j195 – j413 – j217 – j216 – j221 – j215 – j210 – j417 – j224 – j236 – j232 – j415	535	14487

GAMS Solution – Thursday

Team	Route	DWT	TTD
k11	j414 – j249 – j412 – j250 – j310 – j255 – j263 – j312 – j262 – j261 – j257 – j244 – j243 – j300 – j289 – j291 – j417 – j308 – j288 – j238 – j264 – j253 – j416 – j268 – j413 – j292 – j245 – j302 – j415	492	8547
k32	j414 – j242 – j260 – j412 – j251 – j313 – j307 – j295 – j417 – j306 – j297 – j275 – j265 – j240 – j246 – j416 – j282 – j281 – j286 – j279 – j413 – j293 – j270 – j280 – j284 – j274 – j239 – j247 – j256 – j304 – j415	415	12127
k42	j414 – j248 – j412 – j254 – j305 – j298 – j417 – j314 – j258 – j416 – j276 – j287 – j277 – j266 – j273 – j413 – j296 – j272 – j271 – j267 – j241 – j299 – j303 – j294 – j285 – j269 – j283 – j259 – j301 – j290 – j278 – j252 – j309 – j311 – j415	426	12564

GAMS Solution – Friday

Team	Route	DWT	TTD
k11	j414 – j318 – j412 – j338 – j329 – j337 – j343 – j315 – j332 – j416 – j356 – j345 – j366 – j365 – j361 – j355 – j364 – j357 – j413 – j371 – j370 – j372 – j374 – j373 – j377 – j378 – j367 – j417 – j386 – j380 – j392 – j390 – j379 – j383 – j382 – j391 – j393 – j384 – j415	600	16119
k32	j414 – j335 – j326 – j319 – j339 – j323 – j327 – j328 – j325 – j333 – j412 – j330 – j321 – j416 – j348 – j362 – j353 – j413 – j375 – j417 – j381 – j415	384	11332
k42	j414 – j316 – j412 – j331 – j334 – j342 – j336 – j322 – j320 – j340 – j341 – j317 – j324 – j416 – j344 – j354 – j347 – j360 – j358 – j346 – j350 – j349 – j359 – j363 – j351 – j352 – j413 – j368 – j369 – j376 – j417 – j388 – j395 – j387 – j394 – j389 – j385 – j415	515	16252

GAMS Solution – Saturday

Team	Route	DWT	TTD
k11	j414 – j396 – j398 – j397 – j399 – j413 – j401 – j402 – j400 – j415	247	6230

Annex I: Time at which each team starts serving the first visit and finishes the last visit on each day, for Mono-Objective Model 1 (in minutes)

Time at which each team starts serving the first visit

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
k11	1	3		1	3	3	2
k21		1	1				
k32	1			2	1		
k42	3	1	1	2	1		
k52			1				

Time at which each team finishes serving the last visit

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
k11	580	506		490	598	245	222
k21		516	570				
k32	471			413	381		
k42	399	471	335	424	513		
k52			533				

Annex J: Monday routes for Mono-Objective Model 2, with corresponding daily working time (in minutes) and total travel distance (in meters)

GAMS Solution – Monday

Team	Route	DWT	TTD
k11	j414 – j2 – j3 – j9 – j412 – j12 – j24 – j22 – j15 – j26 – j25 – j416 – j43 – j37 – j44 – j413 – j52 – j60 – j61 – j417 – j70 – j415	470	20202
k21	j414 – j8 – j14 – j412 – j7 – j1 – j16 – j28 – j20 – j416 – j46 – j30 – j413 – j54 – j417 – j78 – j73 – j74 – j79 – j75 – j69 – j72 – j415	470	34380
k32	j414 – j18 – j412 – j5 – j4 – j13 – j19 – j416 – j51 – j39 – j48 – j45 – j47 – j29 – j38 – j32 – j34 – j42 – j35 – j40 – j413 – j57 – j53 – j58 – j62 – j417 – j76 – j77 – j415	470	38090
k42	j414 – j10 – j23 – j412 – j27 – j6 – j11 – j17 – j21 – j416 – j36 – j41 – j49 – j33 – j31 – j50 – j413 – j63 – j56 – j59 – j55 – j417 – j67 – j71 – j66 – j68 – j64 – j65 – j415	470	41100

Annex K: Time at which each team starts serving the first visit, on Monday – Mono-Objective Model 1 VS Mono-Objective Model 2 (in minutes)

Teams	MONO 1	MONO 2
k11	1	1
k21	2	2
k32	1	1
k42	3	2

Annex L: Monday routes for Multiobjective Model 1, with corresponding daily working time (in minutes) and total travel distance (in meters)

GAMS Solution – Multiobjective 1

Team	Route	DWT	TTD
k11	j414 – j18 – j11 – j13 – j8 – j10 – j20 – j412 – j15 – j27 – j9 – j416 – j41 – j46 – j50 – j30 – j51 – j40 – j49 – j42 – j38 – j413 – j53 – j62 – j417 – j71 – j65 – j77 – j75 – j64 – j415	559	20593
k21	j414 – j2 – j412 – j12 – j16 – j19 – j1 – j17 – j14 – j28 – j416 – j33 – j47 – j413 – j56 – j57 – j60 – j61 – j54 – j55 – j417 – j73 – j79 – j72 – j78 – j74 – j69 – j76 – j67 – j68 – j70 – j415	520	14625
k42	j414 – j4 – j412 – j5 – j24 – j23 – j22 – j3 – j26 – j25 – j7 – j21 – j6 – j416 – j32 – j39 – j29 – j45 – j43 – j31 – j35 – j34 – j44 – j48 – j36 – j37 – j413 – j59 – j63 – j52 – j58 – j417 – j66 – j415	451	18298