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Markovian Model for Forecasting Financial Time Series

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Master (MSc) in Business Administration

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RESUMO

O estudo tem como objectivo criar um modelo markoviano para a previsão de séries temporais e medir a eficácia deste nas previsões de preços das ações. No estudo, o novo previsor foi inspirado em várias técnicas de aprendizagem de máquinas e incluiu abordagens estatísticas e probabilidades condicionais. Ou seja, as cadeias de Markov são a principal inspiração das técnicas para a aprendizagem das máquinas.

Para ser capaz de processar séries temporais com algorítmo do tipo Cadeias de Markov, a nova técnica é desenvolvida com base em preços diários e ações. Foram considerados treze anos de preços diários de ações para teste dos modelos.

Para medir a eficácia do novo previsor, foram obtidos resultados comparados com métodos convencionais, como os modelos ARIMA, a regressão linear, a regressão a partir da árvore de decisão. Esta comparação foi efetuada com base no Erro Absoluto Médio Percentual (MAPE) e na Raiz do Erro Quadrático Médio (RMSE). De acordo com os resultados obtidos, o novo previsor tem melhor desempenho do que a regressão da árvore de decisão, e o ARIMA tem o melhor desempenho entre eles.

Palavras-chave: Séries Temporais, Aprendizagem das Máquinas, Cadeias de Markov, Previsão

Códigos de Classificação: 2240, 4100

ABSTRACT

The study aims to create a Markovian model for forecasting financial time series and measure its effectiveness on stock prices. In the study, the new forecaster was inspired by several machine learning techniques and included statistical approaches and conditional probabilities. Namely, Markov Chains and Hidden Markov Chains are the main inspiration for machine learning techniques.

To be able to process time series with Markov Chains like algorithm, new transformation developed with the usage of daily stock prices. Thirteen years of daily stock prices have been used for the data feed.

For measuring the effectiveness of a new predictor, the obtaıned results are compared with conventional methods such as ARIMA, linear regression, decision tree regression and support vector regression predictions. The comparisons presented are based on Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE). According to the achieved results, the new predictor performs better than decision tree regression, and ARIMA performs best among them.

Keywords: Time Series, Machine Learning, Markov Chains, Forecasting

Classification Codes: 2240, 4100

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CHAPTER 1

INTRODUCTION

The thesis aims to create a new predictor by chopping and conducting data and applying statistical models based on Markov chains to forecast time series. Time-series prediction roots go back to centuries ago. Humans tried to estimate many events by observing and saving them in a timely manner, then finding related patterns to dominate nature or their species. For example, Egyptian governors recorded Nile river floods to prevent possible damage and founded out floods follow specific patterns. They did even leverage this information to increase agriculture around the river beds (Hawkins, 2005).

Technology has improved tremendously since then. It gives us tools to record events in a healthier way and for more extended periods of time or by shorter time intervals. Moreover, it provided more powerful tools for computational methods. Today's forecasts go beyond the seasonality effect. Of course, it would be wrong to attribute this only to the development of technology. Mathematicians, scientists, business people and many more have been striving for centuries to get healthier predictions. With the same purpose, this thesis aims to provide a new Markovian model for forecasting time series.

Machine learning is adjusting programs to optimize a performance criterion using sample data or past experience. Machine learning uses statistics for building mathematical models. Its core task is making interference from a sample (Alpaydın, 2010).

Models can be defined with some parameters. The learning is the execution of the program to optimize the parameters of the model using the experience. The model might make predictions for the future or describe observations to gain knowledge from data or both (Alpaydın, 2010). Artificial intelligence gives decisions or takes actions according to machine learning predictions or descriptions.

The new forecaster is inspired by Hidden Markov Chains, Decision Tree Expected Benefit and Gaussian type Non-parametric density function. Hidden Markov chains are re-examined and modified for the desired output and shape of the transformed data.

As expected, to be able to predict future events, the new predictor requires already observed events description, data. Data has been preprocessed for this study until it gets a multidimensional form. The historical stock price of The Navigator Company is selected as data, and the new predictor aims to forecast future stock prices as output.

The Navigator Company is interested in forestry products, which are mainly pulp & paper, tissue, and energy. It operates on modern, large-scale industrial units.

The Navigator Company is Portugal's third-biggest exporter and the largest national added value generator. It approximately generates 1% of GDP, around 0,3% of all Portuguese exports of goods, and near 6% Portuguese containerized cargo. Their products are shipped to approximately 130 countries, with emphasis on Europe and the USA, thus achieving the most expansive international presence among Portuguese companies (http://en.thenavigatorcompany.com, 2020).

Stock prices are converted to the multidimensional form as sequence and altitude. By classifying altitudes, another attribute of transformed data, labels are created. Even though the final estimations are given as the price itself (input shape), Hidden Markov Chains are constructed by classified data to be able to function. Therefore, the new method might be called a supervised machine learning technique.

For the new predictor, by Hidden Markov Chains, sequence and label group occurrence probabilities are calculated, then Gaussian non-parametric density functions are fitted to each group. The growth with the highest likelihood according to density function is selected as expected growth (a derivative of logarithmic price) of the related group. The final output is calculated with the decision tree expected benefit method.

The predictions are compared to conventional methods such as ARIMA, Decision tree regression, linear regression, support vector regression estimations to evaluate the performance of the new model. Comparison results are given by mean absolute percentage error (MAPE), root mean square error (RMSE).

The next chapter presents literature reviews of inspired methodologies and similar studies. The third chapter introduces existing theories and evaluation techniques. Also, the new methodology for this study is presented in the third chapter. The fourth chapter contains empirical studies application of the developed methodology, the application of conventional methods, and the comparison of results. The last chapter is dedicated to sharing conclusions.

CHAPTER 2

LITERATURE REVIEW

Over the years, many research efforts have been carried out for proper characterization, modeling, and forecasting financial time series (Tyree & Long, 1995; Hussain, Knowles, Lisoba, & El-Deredy, 2008; Sewell, 2009). The linear statistical models, such as exponential smoothing (Lemke & Gabrys, 2010) and autoregressive integrated moving average (Box, Jenkins, Reinsel, & Ljung, 2016), have been used for forecasting financial time series. Within the last decades, researchers have extensively used the random walk model for forecasting financial time series (Meese & Rogoff, 1983). At present, ARIMA is the most dominant linear model in the financial time series (especially, exchange rate) literature (Zhang, 2003). Various modifications, such as RW with drift and error correction terms, have also been developed (Sun, 2005; Ghazali, Hussain, Nawi, & Mohamad, 2009).

Meanwhile, economists are concerned with modeling volatility in asset returns. This is important as volatility is a measure of risk, and a premium for investing in risky assets is desirable for investors. For this purpose, returns are modeled as independent and identically distributed over time. In a classic work, Mandelbrot applied stable Paretian distributions to characterize the distribution of returns (Mandelbrot, 1963). Rachev and Mittnik's work (2000) contains an informative discussion of stable Paretian distributions and their use in finance and econometrics.

The first conditional heteroskedasticity model was autoregressive conditional heteroskedasticity (ARCH). According to Engle (2004), finding a model that could assess the validity of Friedman's (1977) conjecture that the unpredictability of inflation was a primary cause of business cycles was the original idea. Unpredictability caused by this uncertainty would affect investment behaviors. Following this idea required a model in which this uncertainty could change over time. For parameterizing conditional heteroskedasticity in a wage-price equation, Engle (1982) applied his resulting ARCH model. Bollerslev (1986) and Taylor (1986) simultaneously proposed the conditional variance, which is also a linear function of its own. The model called generalized ARCH (GARCH)

Linear models can be relatively poor for capturing economic behavior for a western economy subject to the business cycle; one example would be a linear (Box, Jenkins, Reinsel, & Ljung, 2016) model of output growth, where the properties of output growth in expansions are quite different from recessions (Hamilton, 1989; Sichel, 1994). Highly volatile regimes caused by shock accumulation can be different from relatively less volatile financial regimes as well.

With a different perspective, Ralph Nelson Elliott proposed in the 1930s that market prices unfold in specific patterns, which practitioners today call Elliott waves, or simply waves. Elliott wave analysts hold that each individual wave has its own signature or characteristic, which typically reflects the moment's psychology (Poser, 2003).

R.N. Elliot defines patterns as dominant or corrective trends and introduces them as sequenced waves. Dominant trends consist of five waves in order, and corrective trends consist of three waves particularly (Volna, Kotyrba, Janosek, Habiballa, & Brazina, 2013). For this reason, R.N. Elliott claimed that there are certain patterns among price movements.

On the other hand, A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event (Gagniuc, 2017). The theory has been created by Russian mathematician Andrey Markov. Following years theory finds many applications in fields such as meteorology, biology, chemistry, bioinformatics, information technology, and economy (Gagniuc, 2017).

W.K. Hastings composed Markov chains and Gibs Sampling methods and introduced Monte Carlo Markov Models (MCMM). MCMM describes price movements as drift and impulse. Their sequential relations are calculated by Markovian models (HASTINGS, 1970).

Leonard E. Baum used Markov Chains, not only for one independent variable but two. He used conditional probability to identify their relations to predict one by another with Markov probabilities (Baum & Petrie, 1966). One of the theory's first application was speech recognition (Baker, 1975). Although today it has wide usage over many fields such as computational finance (Sipos, et al., 2016), sequence classification (Blasiak & Rangwala, 2011) and DNA motif discovery (Wong, Chan, Peng, Li, & Zhang, 2013).

As an extension of the HMM, a hidden semi-Markov model (HSMM) is traditionally defined by allowing the underlying process to be a semi-Markov chain. Each state has a variable duration, which is associated with the number of observations produced while in the state. The HSMM is also called “explicit duration HMM” (Ferguson, 1980), “hidden semi-Markov model” (Murphy, 2002) and segment model (Ostendorf, Digalakis, & Kimball, 1996) in the literature, depending on their assumptions and their application areas.

The first approach to the hidden semi-Markov model was proposed by Ferguson (1980) which is partially included in the survey paper by Rabiner (1989). This approach suggests the explicit duration HMM different than the implicit duration of the HMM. It suggests that the state length or duration is generally distributed depending on the current state of the underlying semi-Markov process. It also depends on the “conditional independence” of outputs.

Levinson replaced the probability mass functions of duration with continuous probability density functions to form a continuously variable duration HMM (Levinson, 1986). As Ferguson (1980) pointed out, an HSMM can be realized in the HMM framework in which both the state and its state occupancy time. This idea was exploited in 1991 by a 2-vector HMM (Krishnamurthy, Moore, & Chung, 1991) and a duration-dependent state transition model (Vaseghi, 1991). Similar approaches were proposed in many applications.

For the probability estimation Parzen (1962) used kernels and defines the probability function as

$$\hat{f}_h(x) = n^{-1} \sum_{i=1}^{n} K_h(x - X_i) \tag{2.1}$$

Where h is called the bandwidth and K is a kernel. While Gaussian kernels are used, Turlach (1993) suggests using the loss function to minimize the estimation distance between the density estimator and the real density function to decide optimal bandwidth.

CHAPTER 3

METHODOLOGY

The new method is developed based on Markov chains. It is necessary to compare its results with traditional methods to evaluate its overall performance. Hence, ARIMA, linear regression, decision tree regression, support vector regression has been chosen to use for comparison.

The methodology chapter is divided into four subsections. In the first subsection, Markov chains is introduced. The second subsection contains methodologies for traditional predictors that are used for comparison. The third chapter is dedicated to discuss and introduce a new methodology. The fourth subsection introduces evaluation techniques that are used in the study.

3.1. Markov Chains

3.1.1. Probabilistic Distribution and State Space

Let I be a countable set. Each $i \in I$ is called a state, and I is called the state-space. $\lambda = (\lambda_i : i \in I)$ is a measure on I if $0 \leq \lambda_i < \infty$ for all $i \in I$. Additionally, if the total mass $\sum_{i \in I} \lambda_i$ equals 1, then λ is a probability distribution. We work throughout with probability space (Ω, F, P). Recall that random variable X with values in I is a function $X: \Omega \rightarrow I$. Suppose:

$$\lambda_i = P(X = i) = P(\{\omega: X(\omega) = i\}) \quad (3.1)$$

Is setted. Then λ defines a probability distribution of the distribution of X. X can be thought of as modeling a random state which takes the value i with probability λ_i. Note that a matrix $P = (p_{ij} : i, j \ \in I)$ is stochastic if every row $(p_{ij} : i, j \ \in I)$ is a probabilistic distribution (Norris, 1997).

A discrete parameter stochastic process $\{X(f), t \ = \ 0, 1, 2, \dots \}$ is said to be a Markov process if, for any set of n time points $t_1 < t_2 < \cdots < t_n$ in the index set of the process, the conditional distribution of $X(t_n)$, for given values of $X(t_1), \dots, X(t_{n-1})$ depends only on $X(t_{n-1})$, the most recent known value; more precisely, for any real numbers $x_1, \dots, x_n$

$$P[X(t_n) < x_n | X(t_1) = x_1, \dots, X(t_{n-1}) = x_{n-1}] = P[X(t_n) < x_n | X(t_{n-1}) = x_{n-1}] \quad (3.2)$$

Intuitively, one interprets the equation that, given the "present" of the process, the "future" is independent of its "past" (Norris, 1997).

Markov processes are classified according to the nature of the index set of the process and the nature of the state space. A real number x is said to be a possible value, or a state, of a stochastic process $\{X(t), t \in T\}$ if there exists a time t in T such that the probability$P[x - h < X(t) < x + h]$ is positive for every $h > 0$. The set of possible values of a stochastic process is called its state space (Parzen, 1965).

3.1.2. Transition Probability Matrices

In order to specify the probability law of a discrete parameter Markov chain {Xn} it suffices to the state for all times $\infty > m > 0$ and states j and k, the probability mass function

$$p_j(n) = P[X_n = j] \tag{3.3}$$

and the conditional probability mass function

$$p_{j,k}(m, n) = P[X_n = k | X_m = j] \tag{3.4}$$

The function $p_{j,k}(m, n)$ is called the transition probability function of the Markov chain. Since for all integers q, and any q time points $n_1 < n_2 < \cdots < n_q$ and states $k_1, \dots, k_q$

$$\begin{aligned} P\left[X_{n_1} = k_1, \dots, X_{n_q} = k_q\right] \\ = p_{k_1}(n_1) p_{k_1,k_2}(n_1, n_2) p_{k_2,k3}(n_2 n_3) \dots p_{k_{q-1},k_q}\left(n_{q-1}, n_q\right) \end{aligned} \tag{3.5}$$

A Markov chain is said to be *homogeneous* (or to be homogeneous in time or to have stationary transition probabilities) if $p_{j,k}(m, n)$ depends only on the difference $n — m$. We then call

$$p_{j,k}(n) = P[X_{n+t} = k | X_t = j]\ for\ any\ integer\ t > 0 \tag{3.6}$$

the n-step transition probability function of the homogeneous Markov chain $\{X_n\}$. In words, $p_{j,k}(n)$ is the conditional probability that a homogeneous Markov chain now in state j will move after n steps to state k. The one-step transition probabilities $p_{j,k}(1)$ are usually written simply $p_{j,k}$ in symbols,

$$p_{j,k} = P[X_{t+1} = k | X_t = j]\ for\ any\ integer\ t \geq 0 \tag{3.7}$$

The transition probabilities of a Markov chain $\{X_n\}$ with state-space $\{0,1,2, \dots\}$ are best exhibited in the form of a matrix:

$$P(m,n) = \begin{bmatrix} p_{0,0}(m,n) & p_{0,1}(m,n) & p_{0,2}(m,n) & \dots & p_{0,k}(m,n) & \dots \\ p_{1,0}(m,n) & p_{1,1}(m,n) & p_{1,2}(m,n) & \dots & p_{1,k}(m,n) & \dots \\ \vdots & \vdots & \vdots & \dots & \vdots & \dots \\ p_{j,0}(m,n) & p_{j,1}(m,n) & p_{j,2}(m,n) & \dots & p_{j,k}(m,n) & \dots \\ \vdots & \vdots & \vdots & \dots & \vdots & \dots \end{bmatrix}$$

Note that the elements of a transition probability matrix $P(m,n)$ satisfy the conditions:

$$p_{j,k}(m,n) \geq 0 \qquad for\ all\ j,k \tag{3.8}$$

,

$$\sum_k p_{j,k}(m,n) = 1 \quad for\ all\ j,k \tag{3.9}$$

Given a $p \times q$ matrix A and a $q \times r$ matrix B,

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1q} \\ a_{21} & a_{22} & \dots & a_{2q} \\ \vdots & \vdots & \dots & \vdots \\ a_{p1} & a_{p2} & \dots & a_{pq} \end{bmatrix}, \qquad B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1r} \\ b_{21} & b_{22} & \dots & b_{2r} \\ \vdots & \vdots & \dots & \vdots \\ b_{q1} & b_{q2} & \dots & b_{qr} \end{bmatrix}$$

the product C = AB of the two matrices is defined as the $p \times r$ matrix whose element c_{jk}, lying at the intersection of the jth row and the kth column, is given by

$$c_{jk} = a_{j1}b_{1k} + a_{j2}b_{2k} + \cdots + a_{jq}b_{qk} = \sum_{i=1}^{q} a_{ji}b_{ik} \tag{3.10}$$

Similarly, given two infinite matrices A and B, one can define the product AB as the matrix C whose element c_{jk}, lying at the intersection of the jth row and the kth column, is given by

$$c_{jk} = \sum_i a_{ji}b_{ik} \tag{3.11}$$

(Norris, 1997).

3.1.3. Hidden Markov Chains

In previous chapters, how Markov states depend on each other is discussed, especially by only one step. Additionally, it is good to keep in mind that our transformed data is discrete and has two-dimensional observations as sequence and altitude.

The case of discrete data includes several possibilities: univariate unbounded counts, univariate bounded counts, including binary observations, observations of categories, and multivariate versions of these. Hidden Markov models consist of two parts: first, an unobserved parameter process $\{X_t : t \in N\}$ that is a Markov chain on (1, 2, .. , m), and second, an observed process $\{C_t : t \in N\}$ such that the distribution of C_t is determined only by the current state X_t, irrespective of all previous states and observations. (The symbol N denotes the natural numbers.) This structure is represented summarized by the following equations, in which $C^{(t)}$ and $X^{(t)}$ denote the histories of the processes C_t and X_t from time 1 to time t:

$$P\left[X_t \middle| X^{(t-1)}\right] = P[X_t|X_{t-1}], t = 2,3, \dots \tag{3.12}$$

,

$$P(C_t \,|C^{(t-1)}, X^{(t)}) = P(C_t \,|X_t), t \in N \tag{3.13}$$

The Markov chain is assumed here to be irreducible, aperiodic, and (unless it is stated otherwise) homogeneous (Rabiner & Juang, 1986).

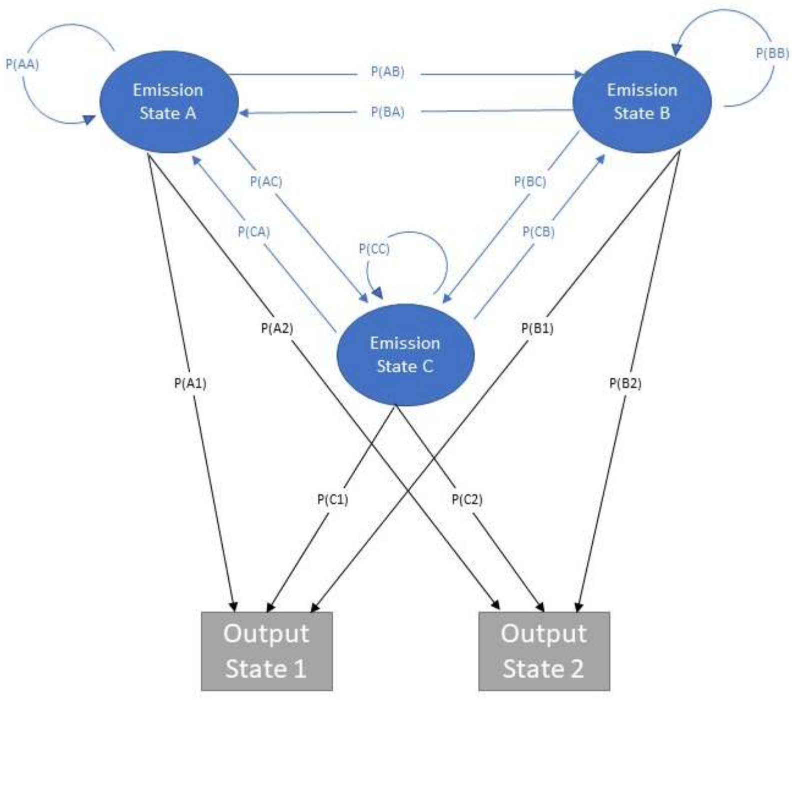

Figure 3.1 Hidden Markov Chains Illustration

The example scheme above presents the basis of HMC. While blue arrows present Markov Chain transmissions and regarding probabilities, black arrows present HMC emission transmissions and regarding probabilities.

State A, State B, and State C present example hidden states which follow Markov properties. $p_{A,A}$, p_{AB}, and p_{AC} denote transition probabilities from State A to State A, State A to State B, State A to state C. p_{AB}, p_{BB}, and p_{BC} denotes transition probabilities from state B to State A, State B to State B, State B to State C. p_{CA}, p_{CB}, and p_{CC} denote transition probabilities from State C to State A, State C to State B, State C to state C respectfully.

State 1 and State 2 presents an example of output or emission states. P(A1) and P(A2) denote emission probabilities while Markov State is A and emission probabilities are 1 and 2. P(B1) and P(B2) denote emission probabilities while the output state is B and emission probabilities are 1 and 2. P(C1) and P(C2) denote emission probabilities while the output state is C and emission probabilities are 1 and 2, respectfully.While the hidden states depend on the time, the emission states depend on the initial hidden states.

For example, let us consider hidden states A, B, and C occur on a daily basis and assume that today's state is A. Let the P' (B2) denotes the probability of having hidden state B and outcome state 2 tomorrow. P' (B2) can be calculated as follow:

$$P'(B2) = p_{AB} \times P(B2) \tag{3.14}$$

Here, State A to State B Markov transmission is the initial condition; when the initial state is B having outcome state 2 is the desired outcome for having B2 state in one day.

If hidden states are irrelevant and the expected hidden state not necessarily to be B, having desired output state 2 in one day P' (2) can be calculated as follow:

$$\begin{aligned} P'(2) &= P'(A2) + P'(B2) + P'(C2) \\ &= p_{AA} \times P(A2) + p_{AB} \times P(B2) + p_{AC} \times P(C2) \end{aligned} \tag{3.15}$$

For this study, consider the sequence Status of waves as hidden states and the label status of waves as desired output states. Therefore, each time sequence probability calculations have been done according to the Markov process, and probabilities are considered as initial conditions for emission probabilities. (Rabiner & Juang, 1986).

3.2. Methods for Comparison

3.2.1. ARIMA

Exponential smoothing and ARIMA models are the two most widely used approaches to time series forecasting. ARIMA models aim to describe the autocorrelations in the data (Hyndman & Athanasopoulos, 2018). For further investigation, let us check autoregressive and moving average processes.

1.1.1.1. Autoregressive Process

The autoregressive process can be thought as, $\widetilde{z_t}$ from the linear filter of stationary time series and consider B as the backward shift operator:

$$\widetilde{z_t} = \sum_{j=1}^{\infty} \pi_j \tilde{z}_{t-j} + a_t \tag{3.16}$$

,

$$Bz_t = z_{t-1}\,,\ \ B^j_{z_t} = z_{t-j} \tag{3.17}$$

where the shock input is white noise a_t. Thus, $\widetilde{z_t}$ can be introduced with transfer function $\phi^{-1}(B)$ as follow strictly:

$$\widetilde{z_t} = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \cdots + \phi_p \tilde{z}_{t-p} + a_t \tag{3.18}$$

Where $\phi_1, \phi_2, \ldots, \phi_p$ symbols are a finite set of weight parameters and different than zero. Also, Box and Jenkins define presented autoregressive process with an order of p or only AR(p) (Box, Jenkins, Reinsel, & Ljung, Time Series Analysis Forecasting and Control, 2016).

3.2.1.1. Moving Average Process

When time series is presented as linear form;

$$\widetilde{z_t} = a_t + \sum_{j=1}^{\infty} \Psi_j a_{t-j} \tag{3.19}$$

Consider only the first q of Ψ weights are non-zero. Thus, the process may be written as :

$$\tilde{z}_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \dots - \theta_q a_{t-q} \tag{3.20}$$

Where $-\theta_1, -\theta_2, .., -\theta_q$ for the finite set of weight parameters. We can call the process as moving average process of order q or only MA(q) (Box, Jenkins, Reinsel, & Ljung, 2016).

3.2.1.2. Autoregressive Moving Average

In previous parts, The autoregressive process and the moving average process have been introduced adequately. The autoregressive moving average can be thought of as a mixture of these two processes. Thus, mathematically it can be presented as follow:

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \cdots + \phi_p \tilde{z}_{t-p} - \theta_1 a_{t-1} - \theta_2 a_{t-2} \dots - \theta_q a_{t-q} + a_t \tag{3.21}$$

Or

$$\phi(B)\tilde{z}_t = \theta(B) a_t \tag{3.22}$$

Where p is the order of autoregression, and q is the order of moving average. We can name the all process shortly ARMA(p, q). Once again, principally, the time series should be stationary (Box, Jenkins, Reinsel, & Ljung, 2016).

After presenting essential relative concepts and terms, we can proceed to the method that will be followed. The method includes four steps to obtain the optimum ARMA model:

3.2.1.3. Data preparation for ARIMA

It involves transformations and differencing. Realtime data is mostly non-stationary. Transformations of the data can help stabilize the variance in a time series where the variation changes with the level. Differencing natural logarithmic closing prices applied as data transformation (Nelson & GRANGER, 1979). $G_t = \ln(P_t) - \ln(P_{t-k})$ for integration level k.

Thus, with the transformation, we can obtain a stationary time series and avoid heteroscedasticity.

3.2.1.4. Model selection

In the Box-Jenkins framework, we can use ACF and PACF based on the transformed and differenced data to try to identify potential ARMA processes that provide a good fit to the data. Later developments have led to other model selection tools such as Akaike's Information Criterion which we are going to include as well (Tong, 1975).

3.2.1.5. Parameter Estimation

It means finding the values of the model coefficients, which provide the best fit for the data.

3.2.1.6. Model Checking

Model-checking involves testing the assumptions of the model to identify any areas where the model is inadequate. If the model is insufficient, it is necessary to go back to Step 2 and try to identify a better model.

3.2.2. Linear Regression

The simple linear regression model is a model with a single regressor x that has a relationship with a response y that is a straight line or linear. This simple linear regression model is

$$y(x) = \beta_0 + \beta_1 x + \varepsilon \tag{3.23}$$

where the intercept β_0 and the slope β_1 are unknown constants and ε is a random error component.

The parameters β_0 and β_1 are called regression coefficients. These coefficients have a useful interpretation. The slope β_1 is the change in the mean of the distribution of y produced by a unit change in x. If the range of data on x includes x = 0, then the intercept β_0 is the mean of the distribution of the response y when x = 0 (MONTGOMERY, PECK, & VINNING, 2012).

For modeling time series with linear regression, logarithmic difference transformation applied. For observed values P_t, growth for estimation length as estimated sequence selected. For depended variable $P_{t+\bar{e}_s}$, according to the length adjustment (will be mentioned in the following chapters), data shifted one unit so it is the target variable as future growth for the training dataset.

3.2.3. Decision Tree Regression

The decision tree is the visualization form that has a root node and the leaf node. The leaf node contains the results. There are two types of nodes present in the decision tree: the inner node and the terminal node.

Two types of decision trees can be drowned in the forecasting: the classification tree and the other is the regression tree. Classification tree analysis is preferable when the prediction result needs to be classified into which class the data belongs, and when the predicted result can be considered a real number, Regression tree analysis is more efficient (Navin, 2013).

A decision tree is a method to find the target value and check the possibility of the trends with the different branches. In the decision tree, all instances are represented as the attribute values. It automatically performs the reduction of the complexity, selection of the features and regarding predictive analysis.

It starts from the root node and step by step, and it goes down until the terminal node to interprets the result (Navin, 2013).

3.2.4. Support Vector Regression

Vapnik's SVM technique is based on the Structural Risk Minimization (SRM) principle. The objective of SVM is to find a decision rule with good generalization ability through selecting some particular subset of training data, called support vectors. This method nonlinearly maps the input space into high dimensional feature space, then constructs an optimal separating hyperplane (Cortes & Vapnik, 1995).

An important characteristic of SVR is that here the training process is equivalent to solving a linearly constrained quadratic programming problem. So, the SVM solution is always unique and globally optimal (Adhikari & Agrawal, 2013).

3.3. The New Method

While shaping the new model, the following steps were followed briefly.

- Firstly, the data obtained was transformed into multidimensional data by passing through a number of transformations, and since the Markov chains are state-based (Norris, 1997), the transformed data is classified into adequate labels in order for the Markov chains to work.
- Following the principles of Markov Chains, new sets of waves that are going to be used for estimations are formed.
- The principles of hidden Markov chains were used to compute dependent probabilities based on these sets.
- Using the non-parametric density function, most likely expected growth rates from these sets were calculated.
- And finally, the growth rates and probabilities obtained were processed using the decision tree expected benefit method, the expected growth and the corresponding time frame of this growth were determined.

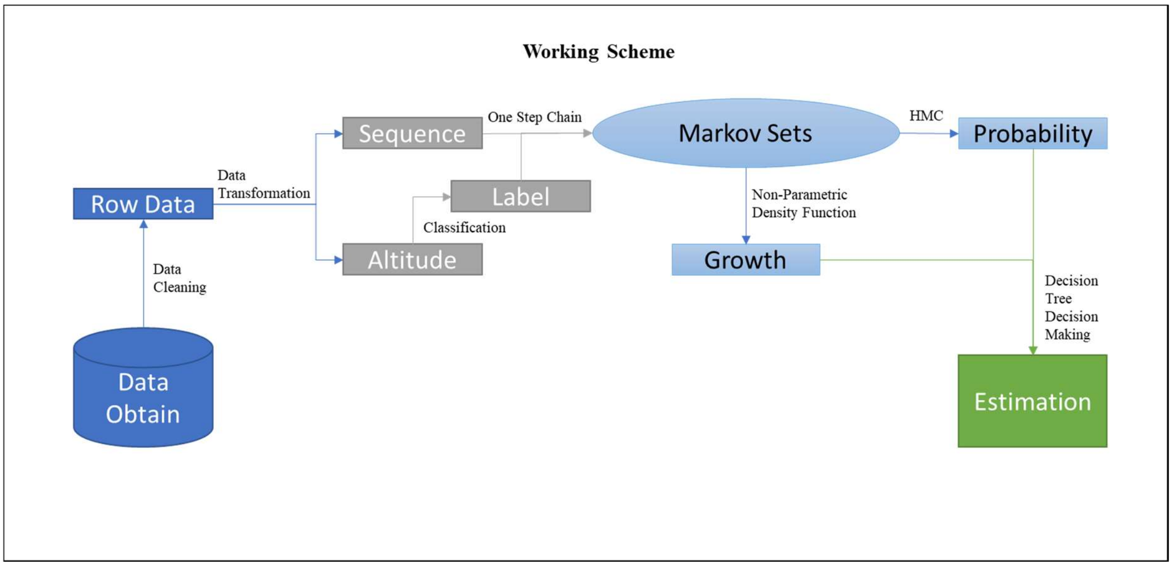

Figure 3.2 Working Scheme

The figure above presents the steps followed. The following chapters explain related methodologies respectfully the presentation.

3.3.1. Transformation

Most of the time serie models are based on specific rules and design parameters for their own requirements. The stationarity condition might be given as an example for the ARIMA process, which is a highly common methodology for modeling time series.

There are many academic and non-academic studies about the topic. The power transformation (Hwang & Kim, 2004), logarithmic transformation (Nelson & GRANGER, 1979), arcsin transformation, square-root transformation (Bromiley & Thacker, 2002) methods can be given as an example to obtain data that is following related model requirements such as heterogeneity, stationarity, reduced skewness.

A different approach has been developed to represent the data with different attributions in this study. The new transformation function is developed for transforming daily closing prices to waves.

Wave is a term for variables in the converted data. The method is about detecting consecutive daily growths that have the same sign in the sense of positivity and negativity.

Accordingly, it creates waves by grouping consecutive daily growths. As an example, if today's return sign is equal to yesterday's, the method adds today's growth to the current wave. If the return sign is different from yesterday's return sign, the current wave brakes, and it creates a new wave starting from the last closing time. Waves are two-dimensional observations that include sequence and altitudes. These two properties of waves will be introduced in the next sections.

3.3.1.1. Attributions of Waves

Waves have three subdomains that are sequence, altitude, and the label. The label is not directly observable. Instead, it is a product of altitude. Therefore, Wave transformation output contains two-dimensional observation.

$$Seq\left(W_{n_t}\right): Sequence\ of\ the\ wave\ n\ in\ day\ t$$

$$Alt\left(W_{n_t}\right): Altitude\ of\ the\ wave\ n\ in\ day\ t$$

$$Lab\left(W_{n_t}\right): Label\ of\ the\ wave\ n\ in\ day\ t$$

3.3.1.1.1. *Sequence*

The sequence (wavelenghth) of the wave presents consecutive days that have the same return sign. Thus, it is presented with the time unit, which is the day format in this study, respectfully to the obtained dataset. It can be defined as the length of the wave as well. It shows the duration of the wave with days unit.

$$Seq(W_{n_{x+c}}): \Delta t \tag{3.24}$$

$$Seq(W_{n_{x+c}}) = (x + c) - x \tag{3.25}$$

$$Seq(W_{n_{x+c}}) = c \tag{3.26}$$

Where $W_{n_{x+c}}$ presents the wave n in day x+c, t_x is the starting date of the wave, t_{x+c} is the last date of the wave.

$$P_t: Closing\ price\ of\ the\ stock\ for\ day\ t$$

$$R_t: Natural\ logarithmich\ return\ for\ day\ t \therefore R_t = \ln(P_t - P_{t-1})$$

$$Seq(W_n) = \begin{cases} Seq(W_{n_t}) = Seq\left(W_{n_{(t-1)}}\right) + 1 & if \ \dfrac{|R_t|}{R_t} = \dfrac{|R_{t-1}|}{R_{t-1}} \\ Seq(W_{n_t}) = Seq\left(W_{n_{(t-1)}}\right) + 1 & if \ R_t = 0 \\ \exists W_{n+1} \ and \ \ seq(W_{n+1_t}) = 1 & if \ \dfrac{|R_n|}{R_n} \neq \dfrac{|R_{n-1}|}{R_{n-1}} \end{cases} \quad (3.27)$$

The sequence of waves depends on the return sign. If the day's return has the same sign as the last day's return or the day has zero return, the wave continues, and the sequence of it grows one unit respectfully. Otherwise, the wave breaks, and a new wave starts.

3.3.1.1.2. *Altitude*

Another essential attribute of the wave is altitude. It presents the natural logarithmic growth of the wave from the beginning of the wave until it breaks. Therefore it is a scale for wave power or natural logarithmic growth of closing prices along with the wave. It does not have a unit because it is a growth indicator that means the logarithmic proportion of quantities with the same units.

$$P_t: Closing\ price\ of\ the\ stock\ for\ day\ t$$

$$R_t: Natural\ logarithmich\ return\ for\ day\ t$$

$$R_t = \ln(P_t) - \ln\,(P_{t-1}) \quad (3.28)$$

$$Alt(W_{n_{x+c}}) = \sum_{t=x}^{x+c} R_t \quad (3.29)$$

Where $Alt(W_{n_t})$ presents the altitude of the wave n in day t, x is the first date of the wave, $x + c$ is the last date of the wave.

3.3.1.1.3. *Label*

The label of the wave is additional attribution of the wave. It is an assistance attribution to function hidden Markov Chains. It is calculated by classifying the quantiles of negative and positive attitudes.

Let the $D=d_1,d_2,\ldots,d_{10}$ to present decile intervals of altitude of positive waves. Waves that are in related decile interval will be a label with L=1,2,…,10 respectfully. A similar approach is applied for negative waves if $D^- = d^-_{-1}, d^-_{-2}, \ldots, d^-_{-10}$ denotes decile intervals of altitude of negative waves. Therefore, waves in related decile intervals will be labeled with L=-1,-2,…,-10.

3.3.2. Markov Process

In this part, the Markov process will be applied to transformed data. These sections will be aimed at applying the Markov process by rearranging it according to the wave structure. To be able to do that, several wave sets suitable for the purpose were formed and The Markov probabilities are calculated over these sets.

3.3.2.1. Set of All Observed Waves

Let the S denotes observed waves from the first wave until the current wave:

$W_{n=m}: Current\ Wave$

$W_{n=1}: First\ Wave\ Observed$

$$S = \{W_n: \sum_{n=1}^{m} W_n\} \qquad (3.30)$$

In the presentation above, n denotes the consecutive index of waves, which start from the first wave "1" until the current wave that is presented by W_m and denotes the last observed wave at the observation moment where $n \in N$. Therefore S includes all waves that have been observed, and it is the population of observed waves.

3.3.2.2. Set of Continue or Break Waves

Core principles and general formulation for Markov Process are discussed and presented in chapter 3.1. Only one step forward event estimation will be applied to transformed data to increase estimation accuracy, prevent a class of states or irreducible classes and calculation simplicity.

Let the sequence of waves $Seq(W_n)$ presents states for the Markov Chains. If we organize the equation (3.6) accordingly, we will obtain the following presentation.

$$P[Seq(W_n) = z|Seq(W_n^{(n-1)}) = z_1, \dots, z_{n-1})] = P[Seq(W_n) = z|Seq(W_{n-1}) = z_{n-1})] \tag{3.31}$$

One step estimation refers that the current event only depends on the last observed event, not more. Thus, to determine the current wave's sequence, which might not be finished yet, we can use Markov probabilities by forming the probability vector based on the previous wave sequence.

The presentation above would be efficient if sequences were observable at once but consider that waves do not occur at once. They are the product of continuous observation of the closing prices. The closing prices are periodic. When the stock exchange is closed, a new closing price occurs. Nevertheless, the wave transformed data is not periodic, and the new wave does not necessarily occur at the end of every day; a new wave occurs respectfully to descriptions in chapter 3.3.1.1.1.

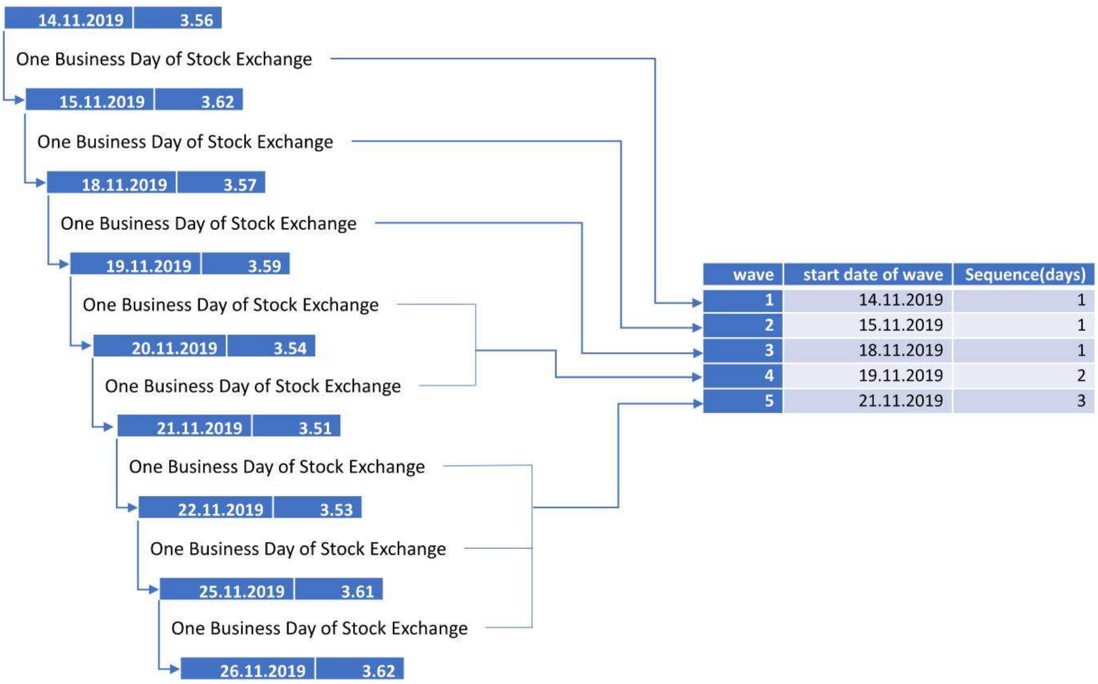

Figure 3.3 Wawe Occurance

Thus the current wave always has the potential to grow more and the possibility of having the current sequence as more than one day. Those possibilities create the necessity to scrutinize the Markov process for waves.

Because when the discrete event occurs at once, probabilities have been calculated only by observations made. However, in the case of waves, the current sequence of the current wave might provide additional information. The current wave's sequence logically can not be less than the current sequence. Thus, state-space for the Markov process should not include waves with less sequence than the current wave's current sequence.

Shrinking state space, according to the current sequence, makes using pre-defined transition matrixes impractical because transition matrixes are efficient when probability vectors are constant and pre-defined. In contrast, probability vectors vary on the current sequence for the current wave that should be estimated. Therefore, defining probability vectors according to the current sequence of the current wave will be wise.

Let S_{CB} denotes Set of Continue or Break Waves, which is formed to determine if the current wave is going to continue or break. Every wave which exists in the continue or break set has the same altitude sign as the current wave (positive or negative). Every wave that exists in the continue or break set's previous wave has a sequence equal to the previous wave’s sequence. Every wave that exists in the continue or break set has an equal or bigger sequence than the current wave's current sequence. Every wave in the break or continue set is an element of the set of waves. Simply S_{CB} can be presented as below:

$$S_{CB} = \begin{cases} W_n : W_n \in S \\ W_n : Seq(W_{n-1}) = Seq(W_{m-1}) \\ W_n : Seq(W_n) \geq Seq(W_m) \\ W_n : \dfrac{|Alt(W_n)|}{Alt(W_n)} = \dfrac{|Alt(W_m)|}{Alt(W_m)} \end{cases} \tag{3.32}$$

Respectfully to the description of S_{CB} and S that is the set of all waves. S is the superset of continue or break set of waves $S_{CB} \subset S$. In other words, the continue or break set of waves is the subset of the set of waves.

3.3.2.3. Set of Continue Waves

Let S_C denote the set of Continue Waves and every wave which exists in the continue set has a greater sequence than the sequence of the current wave. Every wave which exists in the continue set is a member of the continue or break set of waves.

$$S_C = \begin{cases} W_n : W_n \in S_{CB} \\ W_n : Seq(W_n) > Seq(W_m) \end{cases} \tag{3.33}$$

S_{CB} is the set of all continue or break waves. Therefore, it includes the "continue set of waves" $S_C \subset S_{CB}$ or the continue set of waves is the subset of the set of continue or break waves.

3.3.2.4. Set of Break Waves

Let S_b denotes the set of break waves. Every wave which exists in the break set's has an equal sequence with the sequence of the current wave and every wave which exists in the break set is a member of the continue or break set of waves.

$$S_B = \begin{cases} W_n : W_n \in S_{CB} \\ W_n : Seq(W_n) = Seq(W_m) \end{cases} \tag{3.34}$$

S_{CB} is the set of all continue or break waves. Therefore, it includes the "break set of waves," $S_B \subset S_{CB}$. In other words, the break set of waves is the subset of the set of continue or break waves.

$$S_B \cup S_C = S_{CB} \tag{3.35}$$

The set of all continue or break waves is the union of the set of break waves and the set of continue waves.

3.3.2.5. Continue or Break Probabilities

S, S_{CB}, S_C and S_B are described and introduced above. In this part, let us determine the probabilities to continue or break with the help of the mentioned wave sets.

$$P_C(W_m) = \frac{|S_C|}{|S_{CB}|} \tag{3.36}$$

In the equation above, $P_C(W_m)$ denotes the probability of the current wave to continue. The probability of the current wave to continue is equal to the size of the set of continue waves divided by the size of the set of Continue or Break Waves. Because, while S_{CB} includes all possible outputs in the frame of the Markov process, S_C presents the desired outputs suggesting that the current wave is going to continue; its sequence will be greater than the current sequence.

$$P_B(W_m) = \frac{|S_B|}{|S_{CB}|} \tag{3.37}$$

In the equation above, $P_B(W_m)$ denotes the probability of the current wave to break. The probability of break is equal to the size of the set of break waves divided by the size of the set of the continue or break waves. Because, while S_{CB} includes all possible outputs in the frame of the Markov process, S_B presents the desired outputs suggesting that the current wave is going to break; its sequence will be the same as the current sequence.

Chapter 3.3.2.2 explains that the set of continue or break waves is the union of the set of continue waves and the set of break waves. Therefore, the sum of probabilities of continue and break is explicitly equal to one.

$$S_B \cup S_C = S_{CB} \therefore |S_C| + |S_B| = |S_{CB}| \tag{3.38}$$

So,

$$P_C(W_m) + P_B(W_m) = 1 \tag{3.39}$$

Chapter 3.3.2.3 and 3.3.2.4 mentioned that the set of continue or break waves is a superset of the set of continue waves and the set of the break waves. Therefore, the sizes of the subsets are equal or less than the size of the continue or break waves set.

$$S_B \subset S_{CB} \therefore 0 \leq \frac{|S_B|}{|S_{CB}|} \leq 1 \tag{3.40}$$

and,

$$S_C \subset S_{CB} \therefore 0 \leq \frac{|S_C|}{|S_{CB}|} \leq 1 \tag{3.41}$$

Additionally, any output of S_C or S_B does not overlap. Those three conditions are proof of the continue and break probabilities are real and exist as a probability vector for this level.

3.3.2.6. Sequence Sets of Waves

The advantages of using adjusted probability vectors depending on the current sequence of the current wave have been discussed in the previous chapter. Additionally, another practical necessity occurs to estimate the next wave when wave transformation is applied to the closing prices. If the current wave's sequence has been estimated as the current sequence of the current wave, it means there is a high likelihood for the current wave to break at the end of the previous day. In other words, the growth sign of stock price is going to change. Thus, the approach will not forecast a future price. Instead, it will estimate the end of the wave which is already actualized.

Since the purpose of the study is forecasting future prices, providing preliminary estimates would be inefficient. Therefore, an additional condition is applied to Markov properties. Consider that if the growth sign change today, it means the current wave is going to end at the end of yesterday. To be able to estimate the next wave's sequence initial condition would be the current wave is going to be W_{m+1} and the previous wave is going to be W_m. Thus, The Markov process should be reconstructed according to W_{m+1} instead of W_m.

With the extra condition, the wave is going to continue to grow with probability $P_C(W_m)$ or the wave is not going to continue to grow with probability $P_B(W_m)$.

3.3.2.7. Sequence SubSets if Current Wave Continue and the Probability

If the current wave continues, there will be a few possible sequence outcomes of it. Markov process is used to determine the probability of occurrence of each possible sequence; Markov states are sequences.

Let $P_z(Seq(W_m))$ represent the probability that the current wave's sequence is going to be Z. $z = m, m+1, \dots, j$ where j is the observed maximum sequence upon all observed waves and m is the current sequence of the current wave. S_{CS_z} presents the set of waves which are an element of S_C and has a specific sequence of z. $S_{CS_z} = \{W_n \in S_C \mid Seq(W_n) = Z\}$ According to the description, there are sequence subsets for every sequence observed from m+1 to j. Therefore, the union of sequence subsets from m+1 to j is equal to the continue set of waves.

$$S_C = \bigcup_{z=m+1}^{j} S_{CS_z} \tag{3.42}$$

Every wave for sequence subset is chosen from the continue set of waves. Therefore, the continue set of waves is a superset for every conditional sequence subset $S_{CS_z} \subset S_C$.

The probability of each sequence occurs for the current wave can be calculated as below:

$$P_z(W_{m_s}) = \frac{|S_{CS_z}|}{|S_C|} \tag{3.43}$$

Therefore, every sequence probability is equal or more than zero and equal or less than one. The sum of the sequence probabilities is equal to one.

$$0 \le P_z(W_{m_s}) \le 1 \tag{3.44}$$

,

$$\sum_{z=m+1}^{j} P_z(W_{m_s}) = 1 \tag{3.45}$$

Additionally, any output does not overlap. Those three properties prove that the array contains every sequence probability for the continuing condition is a probability vector, and it exists.

3.3.2.8. Sequence SubSets if Current Wave Break and The Probability

A discrete-time Markov chain is a sequence of random variables X1, X2, X3, ... with the Markov property, namely that the probability of moving to the next state depends only on the present state and not on the previous states:

$$P(X_{n+1} = x | X_1 = x_1, X_2 = x_2, \dots, X_n = x_n) = P(X_{n+1} = x | X_n = x_n) \tag{3.46}$$

If the current wave breaks, it means that one more wave will be added to the S, the current wave will be W_{m+1}, and the previous wave will be W_m. Therefore, a new set of waves will be required to define sequence probabilities for the break condition.

$$S_{NB} = \begin{cases} W_n : W_n \in S \\ W_n : Seq(W_{n-1}) = Seq(W_m) \\ W_n : \dfrac{|Alt(W_n)|}{Alt(W_n)} = \dfrac{|Alt(W_{m+1})|}{Alt(W_m)} \end{cases} \tag{3.47}$$

S_{NB} denotes the new break set, which is described in the presentation above. In previous chapters, the Markov process applied to the current wave, which already exists and takes into account the previous wave W_{m-1} while matching waves. Here, waves that members of the new break set $W_n \in S_{NB}$ follow different rules because if the current wave breaks, it will be the previous wave and the Markov process should be applied accordingly with matching waves with the same sequence of if current wave breaks W_n: $Seq(W_{n-1}) = Seq(W_m)$.

S_{NBS_z} presents the set of waves which are an element of S_{NB} and has a specific sequence of z.

$$S_{NBS_z} = \begin{cases} W_n \in S_{NB} \\ Seq(W_n) = z \end{cases} \quad (3.48)$$

According to the description above, there are sequence subsets for every sequence observed. Therefore, the union of sequence subsets from one to j is equal to the new break set of waves.

$$S_{NB} = \bigcup_{z=1}^{j} S_{NBS_z} \quad (3.49)$$

Every wave for sequence subset is chosen from the new break set of waves. Therefore, the new break set of waves is a superset for every conditional sequence subset.

$$S_{NBS_z} \subset S_{NB} \quad (3.50)$$

The probability of each sequence occurs for the current wave can be calculated as below:

$$P_z(W_{m+1_s}) = \frac{|S_{NBS_z}|}{|S_{NB}|} \quad (3.51)$$

Therefore, every sequence probability is greater than or equal to zero and less or equal to one. The sum of the sequence probabilities is equal to one.

$$0 \leq P_z(W_{m+1_s}) \leq 1 \quad (3.52)$$

,

$$\sum_{z=1}^{j} P_z(W_{m+1_s}) = 1 \quad (3.53)$$

Any output does not overlap. The last three properties prove that the array contains every sequence probability for the breaking condition is a probability vector, and it exists.

Please find below the figure depicting the sets that follow this entire Markov process. The set from which the arrow comes out contains all the members of the set at the end of the arrow. Markers on arrows represent Markov probabilities. Set names are given in accordance with the above definitions.

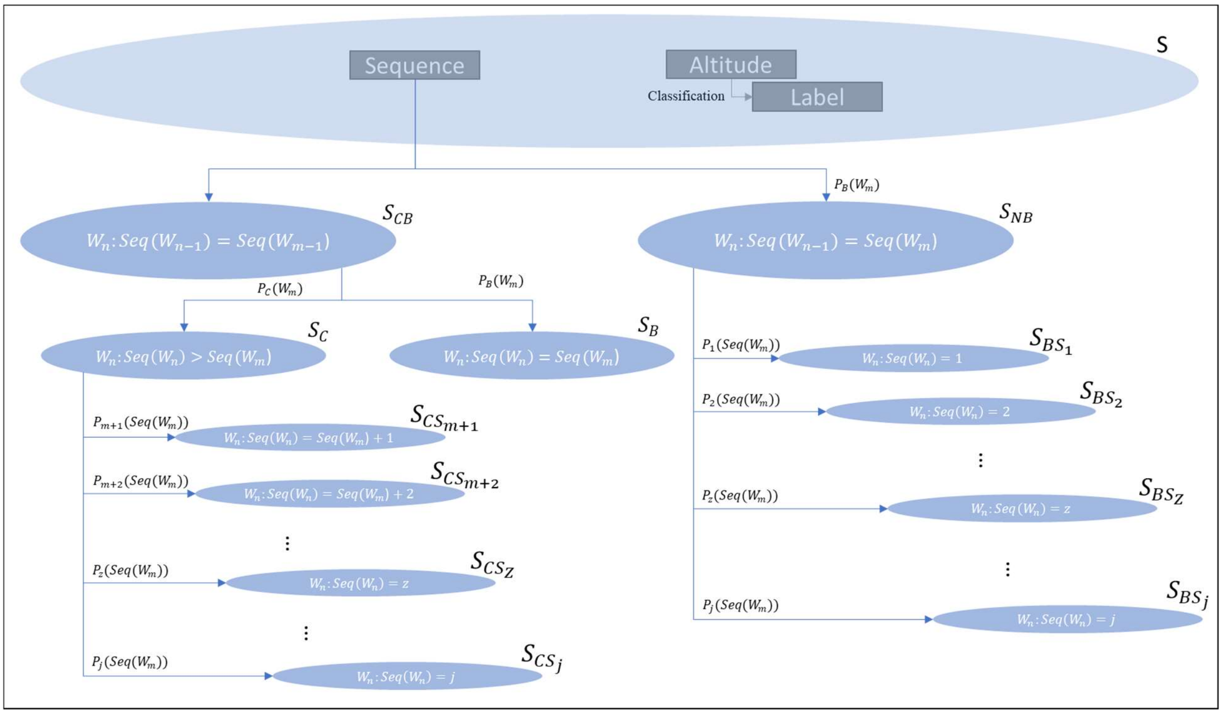

Figure 3.4 Markov Process Illustartion

3.3.3. Hidden Markov Probabilities by Sets of Waves

The second conditional probability step was added to estimate the next sequence according to the Markov Process. The third conditional step would be necessary according to Hidden Markov Process. Consider that the sequence of the wave and the altitude of the wave are separate indicators. The label of the wave has been calculated according to the altitude of the wave. Thus, the label of the wave and the sequence of the wave are different variables that can be used for the Hidden Markov chains.

$P_{l_{HMC}} = P\left(W_{m_s} = z \middle| W_{m_l} = 1,2,\dots,10\right) \vee P\left(W_{m+1_s} = z \middle| W_{m+1_l} = -10,-9,\dots,-1\right)$ where $P_{l_{HMC}}$ denotes Hidden Markov Chain label output probabilities, $P\left(W_{m_s} \middle| W_{m_l}\right)$ denotes label output possibilities with initial continue and have a sequence of z conditions for the current wave. $P\left(W_{m+1_s} \middle| W_{m+1_l}\right)$ denotes label output possibilities with initial conditions: The current wave breaks and the next wave(does not exist yet) has a sequence of z.

$P(W_{m_s})$ and $P(W_{m+1_s})$ are hidden Markov probabilities (emission probabilities). They are pre-calculated probabilities and contains known values. Thus, for the label probabilities calculations, the prior Markov chain process is relevant and can be considered as absolute. While $P_{l_{HMC}}$should be calculated based on these prerequisites, only the corresponding state-space will be used and occurrence at the same time is vitally important.

Until now, break or continue sets, sequence subsets of break or continue sets described so far. Another set of waves can be defined as label subsets. S_{zl} denotes a set of waves with label l for sequence z. for l =-10, -9,…, -1, 1, 2,…, 10. Every each label presents a decile of wave's growth in all observed wave's growth S, separately for negative and positive values.

$$S_n = \{ W_n \in S | \, Alt(W_n) < 0\} \tag{3.54}$$

and,

$$S_p = \{ W_n \in S | \, Alt(W_n) > 0\} \tag{3.55}$$

Since Waves cannot have zero altitudes by definition, negative and positive definitions above do not include zero points. S_n denotes a set of waves with negative altitude. S_p stands for a set of waves with a positive altitude.

By definition, all negative waves tagged with the label from minus ten to minus one and all of them belong to label subsets as same as all positive waves belong to positive label subsets in the same way. Therefore the union of all label subsets is equal to S.

$$S_n = \bigcup_{z=1}^{j} \bigcup_{l=-10}^{-1} S_{zl} \tag{3.56}$$

,

$$S_p = \bigcup_{z=1}^{j} \bigcup_{l=1}^{10} S_{zl} \tag{3.57}$$

and,

$$S = S_p \cup S_n \tag{3.58}$$

Every sequence set does not have to include all wave labels therefore S_{zl} can be an empty set, but it exists.

The conditional probability of having a wave with label l with the condition of having sequence z can be defined as follow:

$$P_l = (W_n{:}\, Seq(W_n) = z | Lab(W_n) = l) = \frac{|S_{zl}|}{|S_z|} \tag{3.59}$$

Therefore, every label probability is greater than or equal to zero and less than or equal to one. The sum of the sequence probabilities is equal to one.

$$0 \leq P(W_n{:}\, Seq(W_n) = z | Lab(W_n) = l) \leq 1 \tag{3.60}$$

,

$$\sum_{l=-10}^{-1} P\big(Seq(W_n) = z \big| W_{n_l}\big) + \sum_{l=1}^{10} P\big(Seq(W_n) = z \big| W_{n_l}\big) = 1 \tag{3.61}$$

Furthermore, the outputs do not overlap. The last three properties prove that the array contains every label probability for the sequence z condition. Therefore, it is a probability vector, and it exists.

3.3.4. Label Growths

In chapter 3.3.2.1, we start to create subsets of our data, and finally, in chapter 3.3.2.8, we indicate the data until the last subsets. The label subsets contain various amounts of observations depending on its initial conditions. The amount of observations varies. Some of the subsets are the empty set, some of them contain less than seven observations, and some of them have more than a hundred observations. A variety of amount of observations urges different approaches.

For subsets that its distribution is available, the non-parametric Gaussian density function is fitted and the highest local peak point of the non-parametric density function is selected as the estimated growth (Scott, 1992).

Non-parametric density functions rely on kernel estimators. The kernel estimator is probably the most commonly used estimator and is certainly the most studied mathematically. It does, however, suffer from a slight drawback when applied to data from long-tailed distributions. Because the window width is fixed across the entire sample, there is a tendency for spurious noise to appear in the tails of the estimates; if the estimates are smoothed sufficiently to deal with this, then essential detail in the main part of the distribution is masked (Silverman, 1986).

For subsets with less than three waves or showing a long tail property, the altitude average of the set element waves is taken as the estimated growth with the following formula:

$$G^e(S_{zl}) = \frac{\sum_{i=1}^{n} G_i}{n} \tag{3.62}$$

Where $G^e(S_{zl})$ denotes growth estimation of label subset, G_i denotes the growth of wave i which is included on the subset S_{zl} and n denotes the total number of waves which is included in the subset S_{zl}.

This simple approach is assumed efficient for sets that do not have a long tail or do not have more than seven members because subsets have little probability of occurring than ones with more members. With this reasoning, we can say that regarding waves have little chance to be the next wave.

3.3.5. Final Estimations

3.3.5.1. Final Probabilities

Three types of possibilities have been identified in the last sections. The probability of continue or break of the wave is the first possibility as $P_C(W_m)$ and $P_B(W_m)$, and the sequence probabilities $P_z\big(Seq(W_m)\big)$ were discussed as the second. These two possibilities were calculated according to the Markov process. The last probability P_l was the label possibilities that are calculated respectfully to the hidden Markov process.

With all these initial conditions, final probabilities can be written as $\big(P_C(W_m)\big|P_z\big(Seq(W_m)\big)\ \big|P_l\big)$ and $P(P_B(W_m)\big|P_z\big(Seq(W_m)\big)\big|P_l)$. It only can be described as two different probability because two different Markov probability vector was created for continue condition and break condition.

The final number of probabilities changes each time because the Markov process generates a different number of possible wave sequences greater than zero, relative to the previous wave sequence or the current wave sequence.

According to the definitions, every possibility is conditional and it is based on the principle that the initial condition is fulfilled. Consequently, the final probabilities can be written as the product of all conditional probabilities which are $P(B)$, $P(C)$; $P(S_C)$, $P(S_B)$; $P(L)$ is equivalent of $P(B) \times\ P(S_B) \times\ P(L)$ or $P(C) \times\ P(S_C) \times\ P(L)$.

Currently, a probabilistic dataset with probability to occur, sequence indication and growth indication is captured. This dataset suggests many probabilities. Having one estimation for the variable growth (it is a derivative of price) is desirable. Hence, with the help of the created dataset, a decision tree with label leaves can be constructed for sequence and growth separately.

3.3.5.2. Final Growth

Growth estimation is calculated as the decision tree average benefit of the probabilistic dataset with taken account growth indicators. Thus, estimated growth can be presented with the following formula:

$$\bar{e}_g = \frac{\sum_{i=1}^{n}(P_i \times g_i)}{\sum_{i=1}^{n} P_i} \quad (3.63)$$

Where P_i and g_i denotes wave's probability of occurring and waves estimated growth respectfully.

3.3.5.3. Final Sequence

Growth estimation is calculated as the decision tree average benefit of the probabilistic dataset with taken account sequences. Thus, estimated growth can be presented with the following formula:

$$\bar{e}_s = \frac{\sum_{i=1}^{n}(P_i \times s_i)}{\sum_{i=1}^{n} P_i} \quad (3.64)$$

Where P_i and $s_i$denotes wave's probability of occurring and wave's sequence respectfully. When calculating according to this formula, the result can be a decimal number instead of an integer. That creates complications for the daily price dataset. Hence, the target sequence is rounded to integers.

3.4. Evaluation

Two evaluation methods are used and analyzed for the study. They are root mean square error (RMSE) and mean absolute percentage error (MAPE). This chapter follows with an introduction for both evaluation methods.

3.4.1. Root Mean Square Error (RMSE)

The root mean square error (RMSE) is a widely used method to compare forecast accuracies. In the early 1980s' Carbone and Armstrong (1982) asked 145 forecasting experts what error measures they preferred when generalizing the accuracy of different forecasting methods. Practitioners selected the Root Mean Square Error (RMSE) more frequently than any other tests, although it is not unit-free. Academicians had a strong preference for the RMSE.

Although there are other pieces of evidence (Armstrong & Collopy, 1992), prove that not unit-free methods can be misleading, RMSE was also included in this study as it is a widespread comparison method.

The average model-estimation error can be written generically as

$$\overline{e_{\gamma}} = \left[\sum_{i=1}^{n} w_i \, |e_i|^{\gamma} \Big/ \sum_{i=1}^{n} w_i\right]^{1/\gamma} \tag{3.65}$$

Where $\gamma \geq 1$ and w_i is a scaling assigned to each $|e_i|^{\gamma}$ according to its hypothesized influence on the total error (Willmott & Matsuura, 2005). The average error is most commonly taken with γ = 2; that is, as the root-mean-square error (RMSE) where

$$RMSE = \left[\frac{\sum_{i=1}^{n} |e_i|^2}{n}\right]^{1/2} \tag{3.66}$$

The stated rationale for squaring each error, $e_i = (\bar{P} - \bar{O})$ where $\bar{P}$ and $\bar{O}$ are predicted and observed values, is usually 'to remove the sign' so that the 'magnitudes' of the errors influence the average error measure (Willmott & Matsuura, 2005).

All forecasting methods are applied to natural logarithmic differences, as presented previously. Thus, all of them estimates natural logarithmic differences ($\ln(P_{t+n}/P_t)$), not the closing price itself. This transformation reversed to assess the estimated closing price ($\bar{e}_P$) as

$$\bar{e}_P = P_t \times e^{\ln(P_{t+n}/P_t)} \tag{3.67}$$

3.4.2. Mean Absolute Percentage Error (MAPE)

Mean Absolute Percentage Error (MAPE) is a unitless evaluation method because it is calculated by using the absolute error in each period divided by the observed values that are evident for that period. Averaging those fixed percentages is the second step for the final calculation. This approach is useful when the size or size of a prediction variable is significant in evaluating the accuracy of a prediction. MAPE indicates how much error in predicting compared with the real value (Khair, Fahmi, Al Hakim, & Rahim, 2017). MAPE can be generalized as the following formula:

$$MAPE = \frac{\sum_{i=1}^{n} \frac{|y_i - y_i'|}{y_i}}{n} \times 100\% \tag{3.68}$$

3.4.3. Forecast Length Adjustment for All Other Methods

By the nature of transformed data, the new predictor forecasts as further as forecasting length. Thus, the method predicts prices for a different amount of days each time, depends on the estimated sequence.

Example prediction lengths for one wave is presented in the figure below:

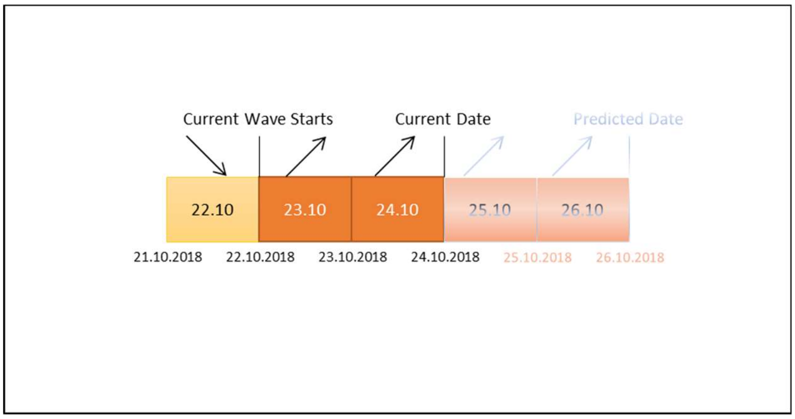

Figure 3.5 Forecast Length

When the market closes on 22.10.2018, the new wave starts because the growth sign turns to positive from negative. Positive growth continues two days until the end of 24.10.2018. After the market closes, the method tells us the current wave is going to continue, most likely until its sequence reaches four business days, which is equal to the end of 24.10.2018. It predicts two business days further.

Depending on the wave sequence, prediction lengths vary from one day to five days for this test dataset. The chart below presents the forecast length for each prediction for the test dataset.

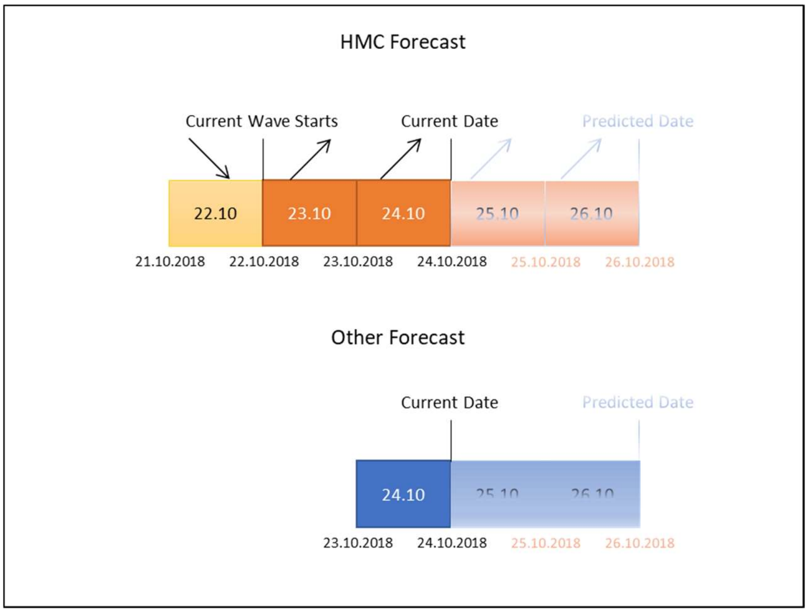

Figure 3.6 Forecast Length Adjustment

CHAPTER 4

EMPIRICAL STUDY

This chapter is dedicated to applying previously discussed and introduced methodologies and sharing the results of them. It contains six subsections as; Data to introduce the data that used in the study, transformed data to apply transformation which is introduced in chapter 3.3.1, probabilities and corresponding estimated growth of sets to present probabilities, sequences and growths for subsets which presented in chapter 3.3.3, forecast length for forecast length for each prediction during the test, ARIMA application to show how ARIMA criteria applied and selected for the data and finally Prediction results to present results for all forecasting methods that examined.

4.5. Data

The Navigator Company's daily stock prices have been chosen for the study, as mentioned before. The Navigator Company is listed on the Euronext Lisbon Stock Exchange.

4.5.1. Obtaining Data

www.investing.com has been used for obtaining daily prices from 01.01.1996 to 31.12.2019. The website has services like providing historical stock prices, portfolio creation and tracking, financial news feed and broker information. investing.com provides a gateway to obtain historical prices for python programming tools, which has been used in this study for major calculations and visualizing the data as well as obtaining historical stock prices.

The chart below presents daily closing prices and volume for regarding dates.

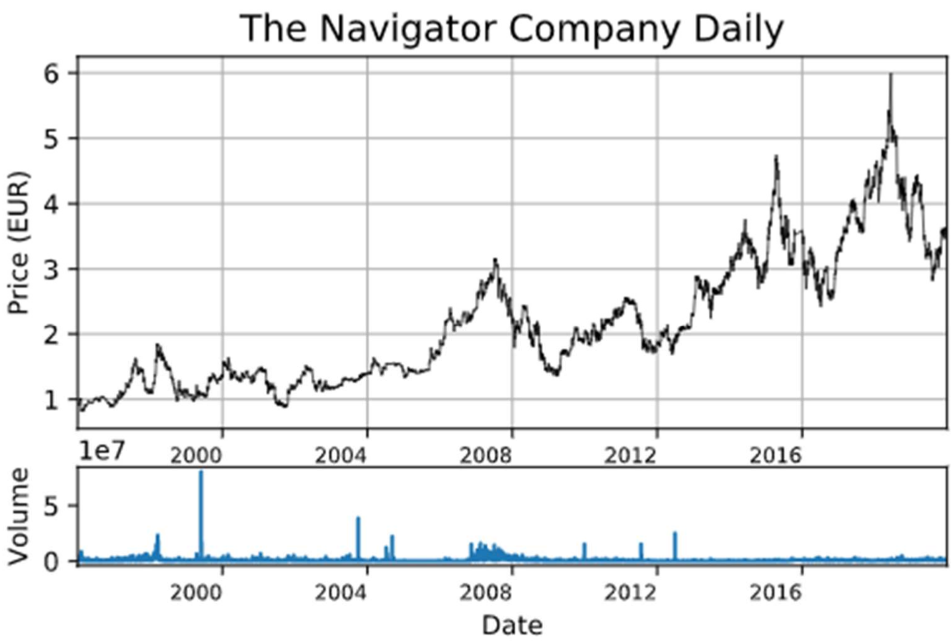

Figure 4.1The Navigator Company Closing Prices and Volume

Sharing all six thousand-twenty-two observation is insufficient because of the space occupation. Thus, for further understanding of the data, the key statistics table created as follows.

Definition	Open	High	Low	Close	Volume
count	6022	6022	6022	6022	6022
mean	2.167635	2.190017	2.143343	2.167579	797133.5
std	1.010219	1.020117	0.998371	1.009677	1770980
min	0.82	0.83	0.76	0.81	0
25%	1.36	1.38	1.34	1.36	202597.5
50%	1.92	1.94	1.89	1.92	432760
75%	2.83	2.85	2.79	2.83	847462.5
max	5.99	6.05	5.93	5.99	80520000

Table 4.1. Key Statistics for Prices

4.6. Transformed Data

Respectfully to the presented method in chapter 3.3.1, let the transformed data be presented in the following chapters. The chapter will follow with presenting sequences, altitudes and labels with the relationship between each other.

4.6.1. Sequence

First, the transformation for dates from 14.11.2019 to 26.11.2019 will be demonstrated. Nine days price table for regarding dates are presented in the next table:

Date	Open	High	Low	Close	Volume	Currency
14.11.2019	3.58	3.58	3.54	3.56	295650	EUR
15.11.2019	3.58	3.62	3.57	3.62	623410	EUR
18.11.2019	3.64	3.64	3.56	3.57	601400	EUR
19.11.2019	3.58	3.63	3.57	3.59	785950	EUR
20.11.2019	3.6	3.6	3.52	3.54	630270	EUR
21.11.2019	3.53	3.54	3.47	3.51	510770	EUR
22.11.2019	3.5	3.54	3.49	3.53	387440	EUR
25.11.2019	3.58	3.65	3.57	3.61	1220000	EUR
26.11.2019	3.63	3.63	3.57	3.62	866770	EUR

Table 4.2 Sample Price

There would be five waves created respectfully to sequence formula and definition for regarding dates. As we can observe in Table 4.2, in the first, second and third days sign of return has changed for each day. Thus, different waves would be suitable for each day. On the date 19.11.2019, two days have negative returns in a row. Thus, one wave would fit into regarding two dates. After date 21.11.2019, consecutive four days have positive returns. Thus, one wave with sequence four would be adequate for regarding dates. The last five wave's sequence can be presented as in the next table. Prices have assumed dropping or rising from the last closing price. Thus, the start date of the wave presents a closing price of the date before.

wave	start date	Sequence(days)
1	14.11.2019	1
2	15.11.2019	1
3	18.11.2019	1
4	19.11.2019	2
5	21.11.2019	3

Table 4.3. Sequences

After a brief demonstration above, the following explanations present all waves between 01.01.1996 and 31.12.2019, as in the study.

While two thousand four hundred fifty-six waves created for regarding dates, one thousand fifty-nine of them has sequence one. This is approximately equivalent to forty-three percent of all waves. As expected, the largest number of waves have this sequence.Five hundred sixty-five of the waves have sequence two, three hundred thirty-eight of them have sequence three. Hundred eighty-three of them have sequence four. Since the average number for each sequence group is one hundred forty-four, sequence clusters for one, two, three and four have more wave than the average number of cluster groups.

One hundred fifteen of them have sequence five. Seventy-four of them have sequence six. Forty-three of them has sequence seven. Thirty-one of them have sequence eight. Seventeen of them have sequence nine. Eleven of them have sequence ten. Twenty waves have sequence vary between eleven to seventeen. Sequence wave clusters with more than four sequences are below the average amount of one hundred forty-four.While the minimum sequence of all waves is one as expected, the maximum sequence observed as seventeen in the transformed data. Low sequences are more populated than high sequences in general. The bar chart below presents the wave amount for each sequence group.

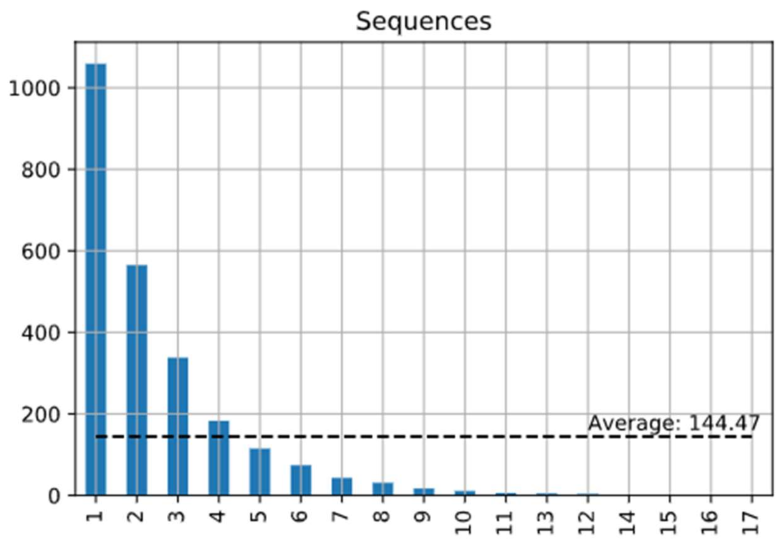

Figure 4.2 Sequences

4.6.2. Altitude

According to the sample closing prices in the previous chapter, corresponding altitudes of the last five waves are presented in the table below.

wave	start date	Altitude(natural log)
1	14.11.2019	0.0167135
2	15.11.2019	-0.013908
3	18.11.2019	0.0055866
4	19.11.2019	-0.022536
5	21.11.2019	0.030858

Table 4.4. Altitude

As can be easily noticed, each altitude sign is different from the previous altitude sign with a sense of negativity and positivity because the wave's altitudes are equal to the sum of the growths with the same sign.

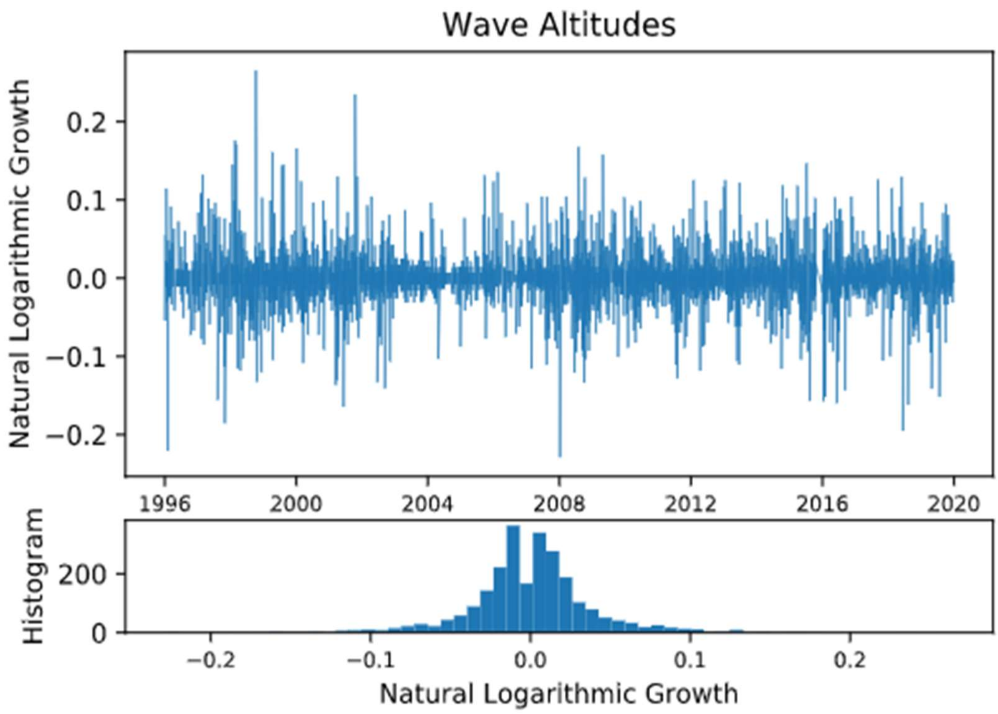

Figure 4.3Altitudes

The preceding figure helps to visualize how altitudes change over time. It is presented with the histogram to understand its distribution briefly. While the following table presents a brief key statistics of altitudes, the next figure shows the linear relationship between sequences and altitudes:

Description	Value
Count of waves	2456
Mean of Altitudes	0.000563
Standard Deviation of Altitudes	0.039982
Minimum Altitude	-0.228715
Lower 25 Percentile of Altitudes	-0.017094
Median of Altitudes	0.000141
Higher 75 Percentile of Altitudes	0.017544
Maximum Altitude	0.264693
Skewness	0.031997
Kurtosis	4.6741

Table 4.5. Key Statistics of Altitudes

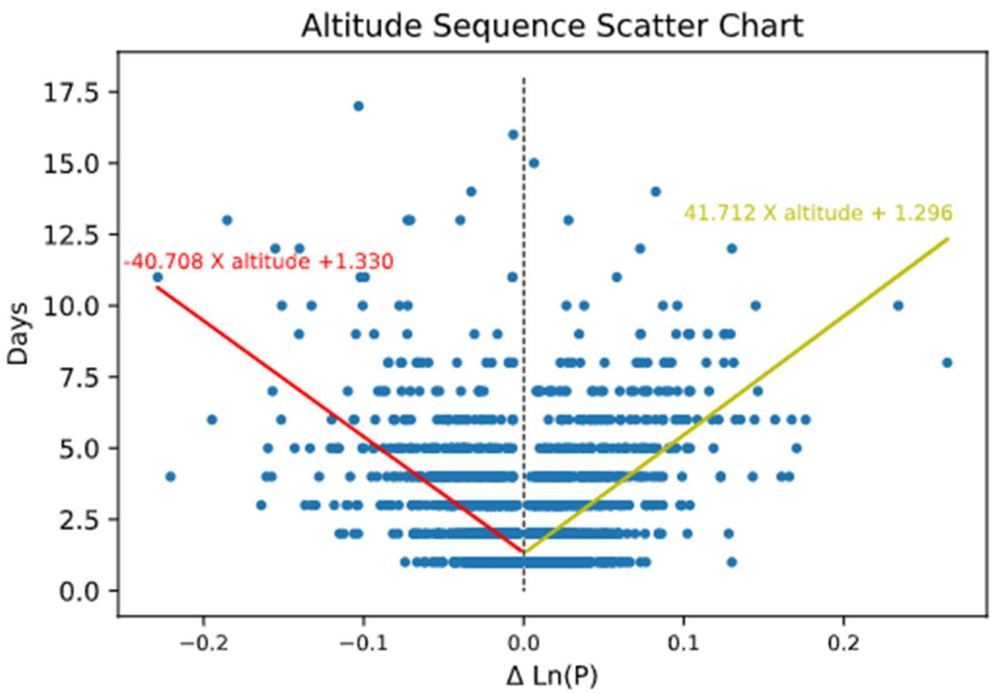

Figure 4.4 Altitudes and Sequences

On the positive side, the slope is 41,712 for the linear fit. The sequence is expected to be 41,712 days for the altitude to increase one unit. On the negative side, the linear slope is -40,708. The sequence is expected to be 41,712 days for the altitude to decrease one unit.

4.6.3. Label

According to the sample closing prices in chapter 4.2.1, corresponding labels of the last five waves are presented in the table below.

wave	start date	label
1	14.11.2019	5
2	15.11.2019	-5
3	18.11.2019	1
4	19.11.2019	-7
5	21.11.2019	8

Table 4.6. Labels

Since labels are products of altitudes, each label sign is different from the previous label sign with a sense of negativity and positivity as same as altitudes.

While absolute values of labels grow, the corresponding sequence for the wave also shows a tendency to grow. This situation occurs because of the nature of waves. Labels are classified altitudes and altitudes are the sum of consecutive growths. The following figure helps to visualize the change of the group sizes. Bubble sizes are arranged according to the population of the sequence and label peers.

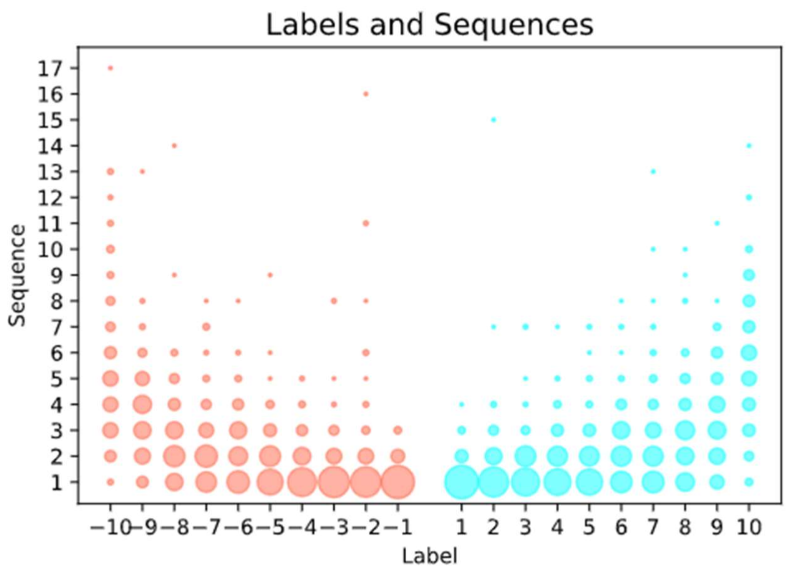

Figure 4.5 Labels and Sequences

4.7. Probabilities and Corresponding Estimated Growths for Sets

Different wave sets are defined and discussed in the methodology part. For each forecast, the break or continue probability, sequence probability, label probability and expected growth corresponding to these sets are determined. Relative probability table for the forecast in date22/11/2019 added to the appendix.

The four label subset groups that are most likely to occur on 22/11/2019 presented as follows as the histogram, non-parametric Gaussian kernel density function, and estimated growth accordingly.

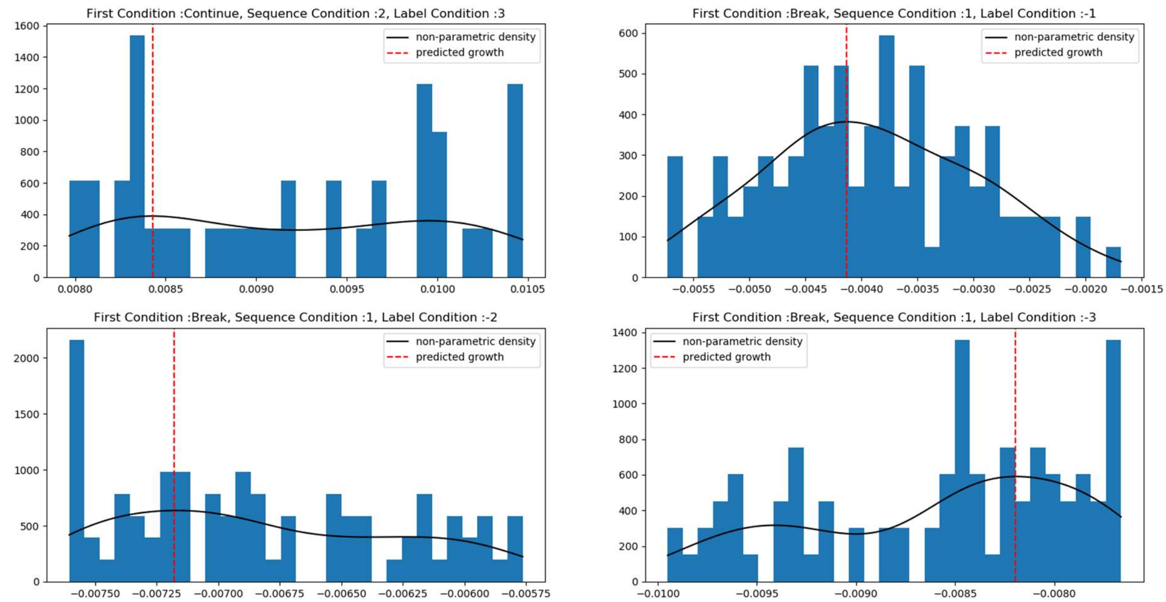

Figure 4.6 Non-Parametric Density Functions

Growth estimates have been calculated by non-parametric density function for 112 of label subsets where the total label subsets amount is 320 in regarding date. The arithmetic mean has made the rest of the growth estimation for the subset growth.

4.8. Forecast Length

Chapter 3.4.3 mentioned that all forecast lengths for each predictor were adjusted as the new predictor’s forecast length. With the new predictor, 776 business days prediction has been made in 301 times, with an average of 2,578 business days ahead forecasts during the test. Forecast lengths are illustrated with the bar chart in the figure below.

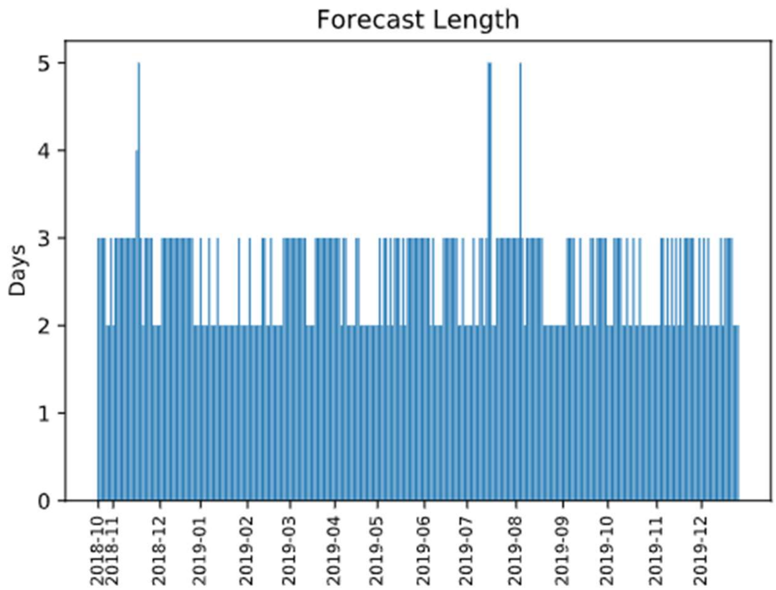

Figure 4.7 Forecast Length

4.9. ARIMA Application

4.9.1. Data Preparation

The Navigator company stocks were obtained for the study. Stocks are trading on the Lisbon stock exchange. In this work, only daily closing prices in between 01.01.1996 and 31.12.2019 will be used.

We can use the ADF test to analyze stationarity for the closing price (MacKinnon, 2010). ADF test is applied to daily closing prices, and the following results are obtained:

Augmented Dickey-Fuller Stationary Test Results for Daily Natural Growth:	Values
ADF Test Statistic	-1.593110
P-Value	0.487054
# Lags Used	33
# Observations Used	5988
Critical Value (1%)	-3.431443
Critical Value (5%)	-2.862023

Table 4.7. Augmented Dickey-Fuller Stationary Test

P-value is higher than the critical value %05. Thus, we do not reject the null hypothesis of the Augmented Dickey-Fuller test. We can say; The time series of closing prices are not stationary. There is a unit root, as we suspected.

4.9.2. Data Transformation

ADF test result shows us our data is not stationary. As discussed before for the ARMA process, the time series should be stationary. To obtain a stationary time-series, the first difference over natural logarithmic daily closing prices can be used (Dritsaki, 2018). Thus, Transformed values which can be called natural logarithmic growth for each day's closing price obtained by the following formula,

$$G_t = \ln(X_t) - \ln(X_{t-1}) \tag{4.1}$$

Where G_t shows daily natural logarithmic growth and X_t presents daily closing price for day t. Thus, this transformation allows us to apply the ADF test for stationarity once again. Daily natural logarithmic growth data's 23 years chart, histogram and density graphs as follows.

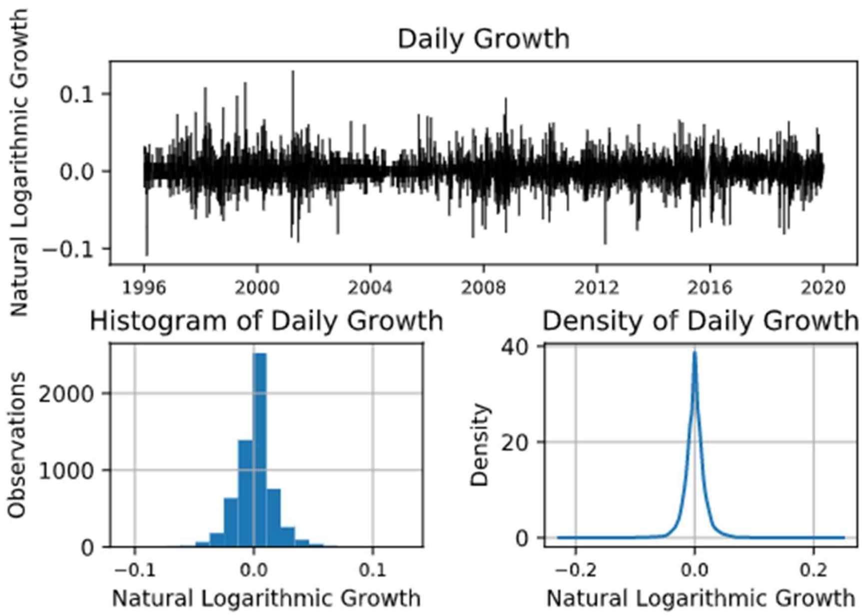

Figure 4.8 Growth Charts

ADF test applied to transformed data to be sure if results do not have a unit root. Therefore they are stationary.

Augmented Dickey-Fuller Stationary Test Results for Daily Natural Growth:	**Values**
ADF Test Statistic	-76.141877
P-Value	0.000000
# Lags Used	0
# Observations Used	6020
Critical Value (1%)	-3.431437
Critical Value (5%)	-2.862020

Table 4.8. Augmented Dickey-Fuller Stationary Test for Growth

P-value is greater than critical values. Thus, we reject the null hypothesis of the Augmented Dickey-Fuller test. We can say; The time series of transformed closing prices, which is Daily Natural Growth, is stationary and ready to be processed by ARMA.

4.9.3. Model Selection

After obtaining stationary data, we can try to estimate p and q parameters for ARMA and find the best model for the data. Therefore, captured data slashed and split into two categories which are training and testing groups. Training data selected as the first 95 percentile of all given data which includes the first 5719 observations. The testing data selected as the last five percentile of all assigned data which contains the final 302 observations. For choosing a good model, ACF and PACF graphs will help to determine p and q parameters. Therefore, they have created from training data as follows:

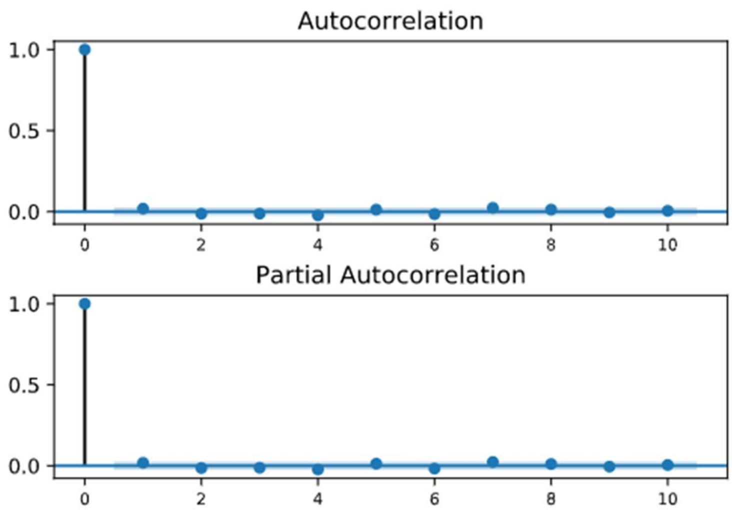

Figure 4.9 Autocorrelation Functions

ACF Graph shows us p parameter for the AR process can be suggested as 0, 1 or 2 because the level for the first and second lags is highly close to 0.05 line and further investigation required.

PACF Graph gives us a clue, q parameter for the MA process can be suggested in between 0 to 2 because the change in levels occur dramatically and values are very close to critical level 0.05. Further investigation is required.

4.9.4. Parameter Estimation

ACF and PACF do not help enough to be sure about ARMA(p,q) orders. Thus, Akaike's Information Criterion involves for preselected parameters and the following Statistics for mentioned orders obtained as follows:

Arima Log Likelihood, AIC and BIC comparison table:

Dep. Variable	Model	Log Likelihood	AIC	BIC
Daily Natural Growth	ARMA(0, 0)	15312.14	-30620.28	-30606.977
Daily Natural Growth	ARMA(0, 1)	15313.132	-30620.265	-30600.31
Daily Natural Growth	ARMA(0, 2)	15313.643	-30619.287	-30592.68
Daily Natural Growth	ARMA(0, 3)	15314.041	-30618.082	-30584.825
Daily Natural Growth	ARMA(0, 4)	15315.603	-30619.206	-30579.297
Daily Natural Growth	ARMA(1, 0)	15313.106	-30620.212	-30600.257
Daily Natural Growth	ARMA(1, 1)	15314.393	-30620.786	-30594.179
Daily Natural Growth	ARMA(2, 0)	15313.653	-30619.305	-30592.699

Table 4.9. ARMA Model Comparison

According to the results in table 14, the ARMA (1,1) process has the lowest value of AIC (-30620.786). Thus, p and q parameters were captured by 1 and 1. Summary statistics for the ARMA(1,1) model presented in the next table.

Model:						**ARMA(1, 1)**
Log Likelihood						15314.393
S.D. of innovations						0.017
AIC						-30620.786
BIC						-30594.179
HQIC						-30611.524
	coef	std err	z	P>|z|	[0.025	0.975]
const	0.0003	0	1.184	0.236	0	0.001
ar.L1.log	-0.783	0.121	-6.469	0	-1.02	-0.546
ma.L1.log	0.8005	0.117	6.872	0	0.572	1.029
			Real	Imaginary	Modulus	Frequency
AR.1			-1.277	+0.0000j	1.277	0.5
MA.1			-1.2491	+0.0000j	1.2491	0.5

Table 4.10. ARMA(1,1) Statistics

4.9.5. Model Checking

ARMA(1,1) process results obtained for training data with given parameters and the formula above. Residual distribution follows given PDF below:

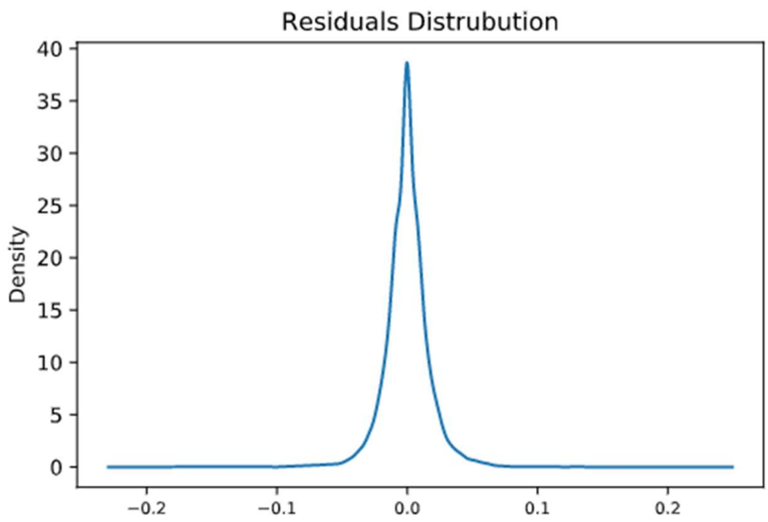

Figure 4.10 ARMA(1,1) Residuals Density

Results for ACF and PACF were not precise and AICs were close in previous chapters; therefore, analyzing residuals will provide a better understanding of the model.

Thus Ljung-Box test is applied to ARMA(1.1) residuals and the following p-value table for mentioned lags and chart for p values obtained.

	lag:1	lag:2	lag:3	lag:4	lag:5	lag:6	lag:7	lag:8	lag:9	lag:10
P_val	0.013	0.014	2.953	4.245	4.458	5.170	7.204	8.531	8.900	9.158

Table 4.11. Ljung-Box Test

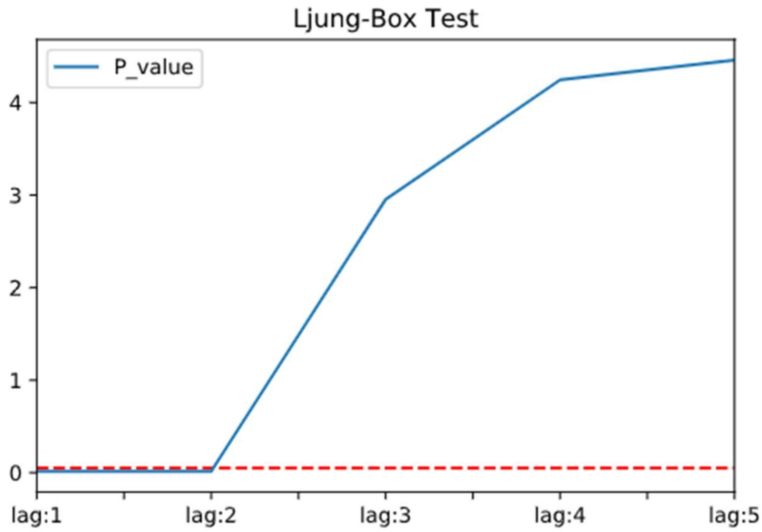

Figure 4.11 Ljung-Box Test

In the Ljung-box test, p-value "-0.0013" obtained for the first lag and value "-0.0014" obtained for lag order 2. Results give us a clue: there can still be autocorrelation in residuals. Therefore, Parameters need to be changed and adjusted for a higher degree of AR process.

Raising AR degree one and using ARMA (2,0) instead of ARMA(1,1) would be more appropriate even if AIC criteria is higher than ARMA(1,1) process.

Model:						**ARMA(2, 0)**
Log Likelihood						15313.653
S.D. of innovations						0.017
AIC						-30619.305
BIC						-30592.699
HQIC						-30610.043
	coef	std err	z	P>|z|	[0.025	0.975]
const	0.0003	0	1.190	0.234	0	0.001
ar.L1.log	0.0186	0.013	1.409	0.159	-0.007	0.045
ar.L2.log	-0.0138	0.013	-1.046	0.296	-0.040	0.012
Roots						
			Real	Imaginary	Modulus	Frequency
AR.1			0.6735	-8.4758j	8.5025	-0.2374
AR.2			0.6735	+8.4758j	8.5025	0.2374

Table 4.12. ARMA(2,0) Statistics

After concluding parameters of p and q as 2 and 0, the AR process and the MA process can be written as follow. AR(2) process with coefficients 0.0186 and -0,0138:

$$\tilde{z_t} = a_t + 0.0186a_{t-1} - 0.0138a_{t-2} \tag{4.2}$$

MA(0) process with coefficient 0; therefore, it is not included in the general formula.In mixed ARMA(2,0) process constant will be 0.0003. Thus, the general formula for mixed AR(2) and MA(0) processes, which is ARMA(2,0), will be:

$$\tilde{z_t} = +0.0186a_{t-1} - 0.0138a_{t-2} + 0.0003 + \varepsilon_t \tag{4.3}$$

4.10. Prediction Results

The new predictor, ARIMA, linear regression, decision tree regression and support vector regression were introduced separately. Obtaining evaluation scores of all methods would be appropriate to identify the performances and have a proper comparison. For this study, two evaluation methods, which are Root Mean Square Error (RMSE) and Mean absolute percentage error (MAPE), applied to all machine learning predictions.

Since the original dataset contains daily stock prices from 01.01.1996 to 31.12.2019, it includes six thousand twenty-two observations. Although, after wave transformation applied, the total number of observations dropped to two thousand four hundred fifty-six. The large dataset was the necessity to obtain healthy results, especially after sub-grouped the waves. The test dataset does not have the same obligation to evaluate the accuracy of predictions.

Most machine learning algorithms divide the original dataset into two as eighty percent training and twenty percent test datasets. For this study, the test dataset contains the last five percent of transformed data, which is equivalent to three hundred one predictions, starts from 23.10.2018. Thus, all 2019 is included in the test.

For the testing phase, the extended window method was used. Therefore, after one prediction was tested, while the next prediction is testing; The tested variable was included in the training dataset and no variables were removed from the training dataset as well.

4.10.1. Comparison and Test Results

For each method presented, the same test runs simultaneously and the prediction length for the methods adjusted according to new predictors forecasting length. Thus, every method is tested for the same forecasting length. The results chart can be found in the appendix for each method.

The following comparison charts are created according to prediction results. The prediction results for the corresponding predictors presented on the right-hand top legend. Background color adjusted according to the left-hand bottom legend explanation. Since the new predictor's forecasting length varies, the prediction made days are used as x-axis dates instead of predicted dates to prevent overlap on figures.

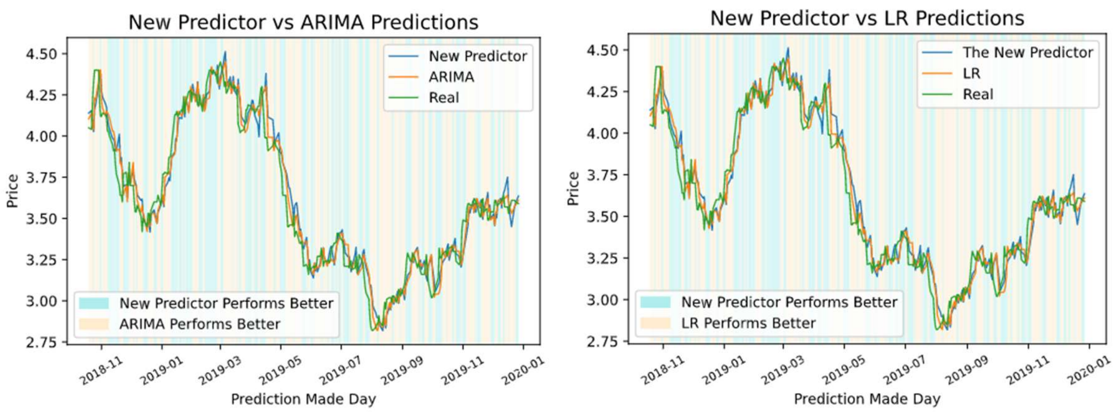

Figure 4.12 The New Predictor Versus ARIMA and Linear Regression

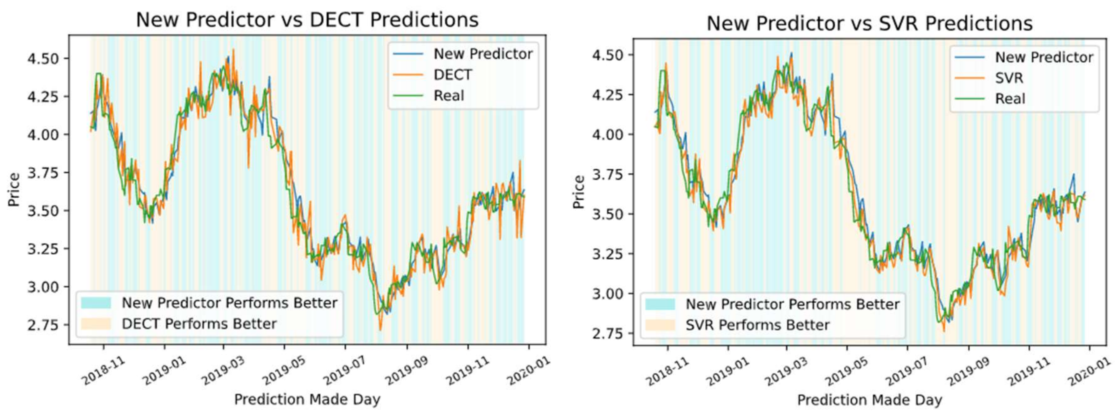

Figure 4.13 The New Predictor Versus Decision Tree Regression and Support Vector Regression

4.10.2. RMSE Scores

RMSE calculated for the new predictor, ARIMA, linear regression, decision tree regression, and support vector regression predictions with the presented methods. The following table presents scores respectfully.

	New Predictor	ARIMA	LR	DTR	SVR
RMSE	0.108497	0.098542	0.098984	0.120516	0.099658

Table 4.13. Root Mean Square Error Comparison

As shown in Table 4.15, among five predictors, ARIMA performs best, linear regression second, support vector regression third, new predictor fourth and decision tree regression worst according to RMSE scores.

4.10.3. MAPE Scores

RMSE calculated for the new predictor, ARIMA, linear regression, decision tree regression, and support vector regression predictions with the presented methods. The following table presents scores respectfully.

	New Predictor	ARIMA	LR	DTR	SVR
MAPE	0.023370	0.020878	0.020974	0.025838	0.021112

Table 4.14. Mean Absolute Percentage Error Comparison

As shown in Table 4.3, among five predictors, ARIMA performs best, linear regression second, support vector regression third, the new predictor fourth and decision tree regression worst according to MAPE scores.

4.10.4. Best Performer for Each Prediction

Besides MAPE and RMSE, let us compare the results, prediction by prediction. The test period has been selected as 301 predictions in the study, as mentioned before. Among 301 predictions: The decision tree regression estimates closest to real 89 times; the new predictor forecast closest to real 82 times; support vector regression forecast closest to real 64 times; ARIMA has the best prediction 41 times; linear regression has the best prediction 24 times.

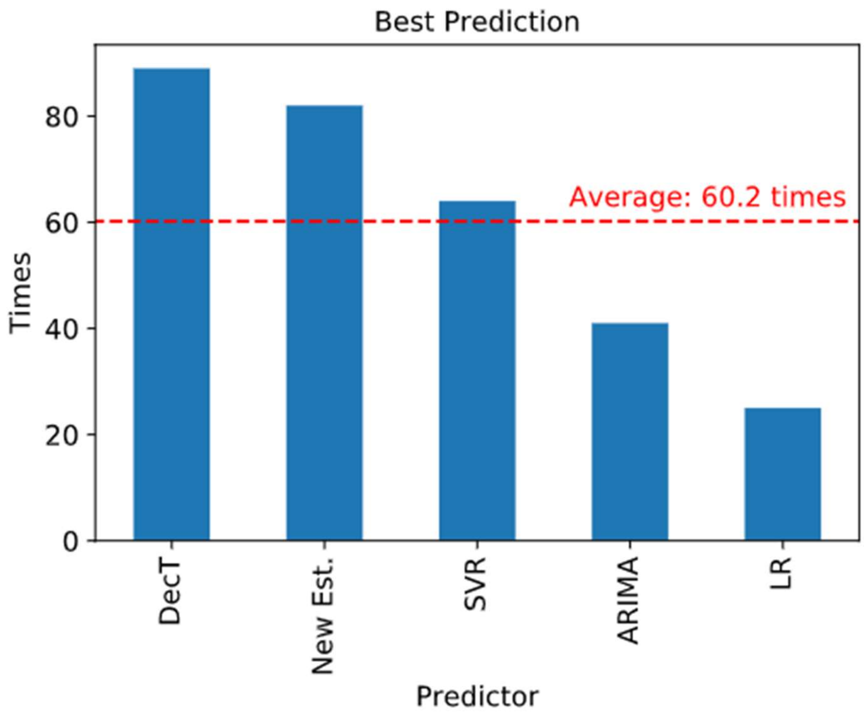

Figure 4.14 Prediction by Prediction Scores

CHAPTER 5

CONCLUSION

After comparing the new predictor, results show that there are better options on stock price estimations. Namely, ARIMA performs best upon other methods. Nevertheless, there is evidence that the new Markovian model performs better than some other models. The model applied only on stock prices, with other financial instruments or with a different type of time series; it might perform better.

Although stock markets are efficient, ideally unpredictable (Fama, 1970), different type or combined models, such as neural networks, with multi-factor variables may provide better results with multivariate data feed than one-dimensional data by reducing unexpected impulse.

Hidden Markov Chains and Markov Chains are considered memoryless processes. Even though it has many benefits in many fields, when investors, executives, producers, and consumers are related, they keen to keep patterns in mind for more than one step ahead.

ARIMA is a well-defined model that is studied by many mathematicians and practicians. Even to compete on scores on estimations with ARIMA and have better results than other well-defined estimators are promising. From my perspective, there are a few points in the study that might be constructed better. I want to continue with points where I faced limitations and suggestions for further studies.

5.11. Limitations and Suggestions for Further Studies

5.11.1. Combined Methods

Recurrent neural networks (RNN) are able to take multivariate data into account. This study showed that while the newly developed Markovian model failed to beat the ARIMA in the RMSE and MAPE comparisons, it gave accurate predictions more times than ARIMA. Those two methods or more can be processed with RNN, and non-linear and linear methods can be combined for better predictions.

5.11.2. Transformation With Threshold

Suggested transformations for the study contain many waves that have a wavelength (sequence) one. The proposed transformation creates noise for the data. Different kinds of transformation methods can be proposed. Phetchanchai, Selamat, Rehman, and Saba (2010) proposed to index time series with volatility threshold and name it Zigzag-Perceptual indexing. This method can be used to create waves instead of grouping daily returns directly. This method will prevent or reduce the generation of noise relative to the current transformation.

5.11.3. Non-Parametric Density Function

In chapter 3.3.4, Fitting non-parametric density functions were introduced. Some of the label subsets, especially ones that have a low amount of members, do not show the feature of Gaussian distribution, have long tails or do not have an efficient number of members. Thus, the model can not fit density functions properly. For those subsets, arithmetic means applied, and the average value is captured as an associated growth rate. Better interpretation over all subsets can be made.

5.11.4. Selective Estimates

In this study, the Decision tree was constructed related to Markov chain transmission probabilities. Instead of a decision tree expected benefit method, a selective process with logic operators might be developed, and the model can be forced to give prediction only when it is appropriate.

Relatively to the study, selecting the wave with the highest probability has been tried. When the model predicts according to maximum probability, results show that its estimations are very aggressive and volatile, sometimes it predicts better than ARIMA and from the current version itself, but when it fails, it fails too much. The situation affects evaluation scores dramatically.

The figure below presents the comparison of results if the model does not build a decision tree; instead, it selects the wave that has the highest probability to occurs as an estimation. Note that the mentioned method has not been wholly introduced and presented in the previous chapters. Thus, the following figure may be for informational purposes only, not part of the methods itself.

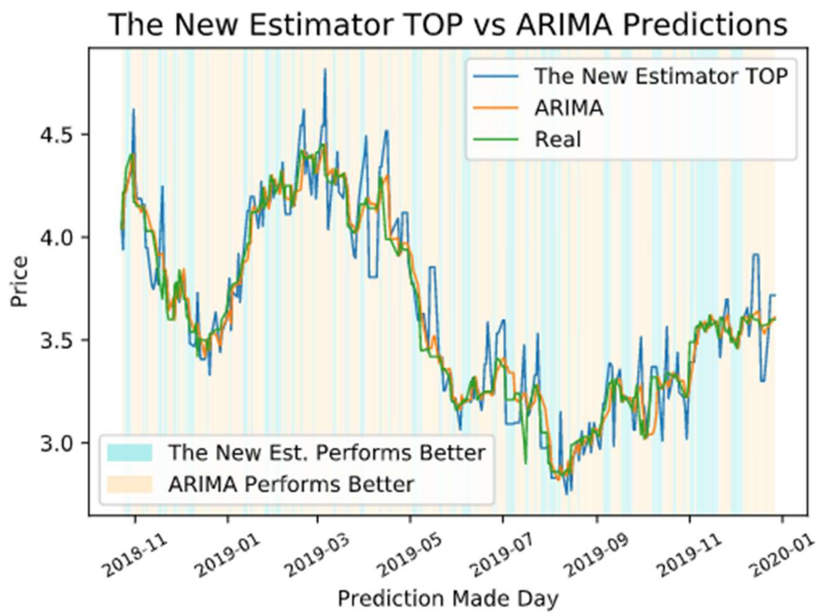

Figure 5.1 The New Predictor TOP Versus ARIMA*

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APPENDIX

Prediction Date	Prediction Length	Predicted Date	New Predictor	ARIMA	LR	SVR	DECT	Real
23.10.2018	3	26.10.2018	4.062	4.052	4.053	4.025	4.008	4.22
24.10.2018	3	29.10.2018	4.027	4.041	4.043	4.014	4.126	4.33
25.10.2018	3	30.10.2018	4.166	4.228	4.230	4.207	4.181	4.4
26.10.2018	3	31.10.2018	4.187	4.221	4.220	4.197	4.171	4.4
29.10.2018	2	31.10.2018	4.306	4.330	4.331	4.305	4.252	4.4
30.10.2018	2	1.11.2018	4.368	4.400	4.401	4.376	4.300	4.21
31.10.2018	3	5.11.2018	4.390	4.401	4.402	4.374	4.350	4.12
1.11.2018	2	5.11.2018	4.272	4.214	4.214	4.182	4.295	4.12
2.11.2018	3	7.11.2018	4.238	4.172	4.175	4.142	4.305	4.14
5.11.2018	3	8.11.2018	4.196	4.122	4.123	4.094	4.052	4.13
6.11.2018	3	9.11.2018	4.176	4.151	4.152	4.124	4.185	4.11
7.11.2018	3	12.11.2018	4.108	4.141	4.142	4.115	4.160	4.03
8.11.2018	3	13.11.2018	4.068	4.131	4.132	4.104	4.146	4.03
9.11.2018	3	14.11.2018	4.043	4.111	4.112	4.084	4.109	4
12.11.2018	3	15.11.2018	3.988	4.032	4.033	4.004	4.127	3.95
13.11.2018	3	16.11.2018	3.980	4.031	4.033	4.004	3.877	3.91
14.11.2018	3	19.11.2018	3.970	4.001	4.002	3.975	4.101	3.92
15.11.2018	3	20.11.2018	3.914	3.952	3.953	3.925	3.980	3.83
16.11.2018	4	22.11.2018	3.909	3.912	3.913	3.885	3.946	3.77
19.11.2018	5	26.11.2018	4.011	3.921	3.922	3.896	3.965	3.7
20.11.2018	3	23.11.2018	3.844	3.832	3.833	3.806	3.750	3.64
21.11.2018	2	23.11.2018	3.869	3.841	3.843	3.816	3.860	3.64
22.11.2018	3	27.11.2018	3.774	3.772	3.772	3.746	3.746	3.6
23.11.2018	3	28.11.2018	3.695	3.643	3.644	3.616	3.640	3.77
26.11.2018	3	29.11.2018	3.705	3.700	3.702	3.677	3.640	3.78
27.11.2018	3	30.11.2018	3.622	3.602	3.602	3.577	3.656	3.7
28.11.2018	2	30.11.2018	3.707	3.769	3.771	3.748	3.790	3.7
29.11.2018	2	3.12.2018	3.737	3.781	3.780	3.760	3.780	3.84
30.11.2018	2	4.12.2018	3.708	3.702	3.702	3.677	3.683	3.7
3.12.2018	2	5.12.2018	3.795	3.839	3.841	3.817	3.832	3.7
4.12.2018	3	7.12.2018	3.743	3.703	3.702	3.677	3.699	3.61
5.12.2018	3	10.12.2018	3.718	3.701	3.703	3.676	3.680	3.54

6.12.2018	3	11.12.2018	3.701	3.701	3.702	3.677	3.699	3.54
7.12.2018	3	12.12.2018	3.644	3.612	3.613	3.587	3.610	3.58
10.12.2018	3	13.12.2018	3.616	3.542	3.543	3.517	3.612	3.53
11.12.2018	3	14.12.2018	3.610	3.541	3.542	3.518	3.476	3.51
12.12.2018	3	17.12.2018	3.618	3.580	3.581	3.559	3.669	3.42
13.12.2018	3	18.12.2018	3.525	3.531	3.532	3.508	3.431	3.5
14.12.2018	3	19.12.2018	3.489	3.511	3.512	3.488	3.685	3.49
17.12.2018	3	20.12.2018	3.438	3.422	3.423	3.398	3.396	3.45
18.12.2018	3	21.12.2018	3.491	3.500	3.502	3.478	3.391	3.53
19.12.2018	3	24.12.2018	3.463	3.491	3.491	3.470	3.469	3.53
20.12.2018	3	27.12.2018	3.417	3.451	3.452	3.428	3.455	3.47
21.12.2018	3	28.12.2018	3.522	3.530	3.531	3.509	3.503	3.55
24.12.2018	3	31.12.2018	3.551	3.531	3.531	3.510	3.570	3.6
27.12.2018	2	31.12.2018	3.470	3.472	3.472	3.448	3.470	3.6
28.12.2018	2	2.01.2019	3.536	3.550	3.551	3.529	3.550	3.64
31.12.2018	2	3.01.2019	3.584	3.600	3.600	3.580	3.513	3.58
2.01.2019	3	7.01.2019	3.620	3.640	3.641	3.619	3.622	3.77
3.01.2019	2	7.01.2019	3.570	3.582	3.582	3.558	3.630	3.77
4.01.2019	2	8.01.2019	3.686	3.749	3.751	3.729	3.732	3.78
7.01.2019	2	9.01.2019	3.720	3.771	3.770	3.750	3.825	3.77
8.01.2019	3	11.01.2019	3.745	3.781	3.782	3.757	3.749	3.87
9.01.2019	2	11.01.2019	3.728	3.771	3.772	3.747	3.769	3.87
10.01.2019	2	14.01.2019	3.821	3.810	3.812	3.787	3.797	3.89
11.01.2019	2	15.01.2019	3.878	3.870	3.871	3.848	3.845	3.96
14.01.2019	3	17.01.2019	3.901	3.891	3.891	3.867	3.940	4.13
15.01.2019	2	17.01.2019	3.983	3.960	3.961	3.937	3.909	4.13
16.01.2019	2	18.01.2019	3.986	3.971	3.971	3.947	3.989	4.15
17.01.2019	2	21.01.2019	4.059	4.129	4.131	4.107	4.179	4.12
18.01.2019	2	22.01.2019	4.112	4.151	4.151	4.127	4.042	4.13
21.01.2019	2	23.01.2019	4.106	4.121	4.122	4.095	4.100	4.16
22.01.2019	2	24.01.2019	4.162	4.131	4.132	4.104	4.030	4.15
23.01.2019	2	25.01.2019	4.210	4.161	4.162	4.135	4.212	4.24
24.01.2019	2	28.01.2019	4.115	4.151	4.152	4.125	4.148	4.14
25.01.2019	2	29.01.2019	4.227	4.240	4.242	4.215	4.305	4.2
28.01.2019	3	31.01.2019	4.157	4.142	4.142	4.115	4.180	4.27
29.01.2019	2	31.01.2019	4.204	4.200	4.203	4.174	4.193	4.27
30.01.2019	2	1.02.2019	4.279	4.300	4.301	4.276	4.270	4.28
31.01.2019	2	4.02.2019	4.245	4.272	4.272	4.245	4.231	4.21
1.02.2019	2	5.02.2019	4.313	4.281	4.282	4.254	4.260	4.26
4.02.2019	3	7.02.2019	4.210	4.212	4.213	4.184	4.290	4.25

5.02.2019	2	7.02.2019	4.270	4.261	4.262	4.234	4.293	4.25
6.02.2019	2	8.02.2019	4.329	4.320	4.321	4.295	4.477	4.19
7.02.2019	2	11.02.2019	4.249	4.252	4.252	4.224	4.240	4.18
8.02.2019	2	12.02.2019	4.201	4.192	4.193	4.163	4.110	4.25
11.02.2019	2	13.02.2019	4.185	4.181	4.183	4.154	4.116	4.15
12.02.2019	3	15.02.2019	4.254	4.250	4.252	4.225	4.205	4.22
13.02.2019	3	18.02.2019	4.167	4.152	4.152	4.124	4.148	4.23
14.02.2019	2	18.02.2019	4.196	4.171	4.173	4.144	4.282	4.23
15.02.2019	2	19.02.2019	4.251	4.221	4.222	4.195	4.150	4.32
18.02.2019	3	21.02.2019	4.271	4.231	4.232	4.205	4.350	4.4
19.02.2019	2	21.02.2019	4.374	4.320	4.321	4.295	4.270	4.4
20.02.2019	2	22.02.2019	4.410	4.420	4.421	4.396	4.420	4.39
21.02.2019	2	25.02.2019	4.376	4.401	4.402	4.374	4.366	4.38
22.02.2019	2	26.02.2019	4.398	4.391	4.393	4.363	4.300	4.4
25.02.2019	2	27.02.2019	4.364	4.381	4.383	4.353	4.388	4.36
26.02.2019	3	1.03.2019	4.432	4.401	4.402	4.373	4.390	4.32
27.02.2019	3	4.03.2019	4.343	4.362	4.363	4.333	4.393	4.41
28.02.2019	3	5.03.2019	4.287	4.312	4.313	4.283	4.242	4.43
1.03.2019	3	6.03.2019	4.361	4.321	4.323	4.293	4.324	4.45
4.03.2019	3	7.03.2019	4.437	4.410	4.411	4.384	4.421	4.37
5.03.2019	3	8.03.2019	4.485	4.431	4.431	4.404	4.447	4.3
6.03.2019	3	11.03.2019	4.512	4.451	4.452	4.423	4.457	4.33
7.03.2019	3	12.03.2019	4.378	4.372	4.373	4.343	4.316	4.26
8.03.2019	3	13.03.2019	4.307	4.302	4.304	4.272	4.430	4.27
11.03.2019	3	14.03.2019	4.360	4.331	4.333	4.303	4.357	4.33
12.03.2019	3	15.03.2019	4.260	4.262	4.263	4.233	4.320	4.3
13.03.2019	2	15.03.2019	4.303	4.271	4.273	4.243	4.370	4.3
14.03.2019	2	18.03.2019	4.364	4.330	4.332	4.304	4.420	4.29
15.03.2019	2	19.03.2019	4.275	4.302	4.302	4.274	4.335	4.31
18.03.2019	2	20.03.2019	4.246	4.291	4.293	4.263	4.170	4.26
19.03.2019	3	22.03.2019	4.345	4.311	4.312	4.284	4.289	4.07
20.03.2019	3	25.03.2019	4.249	4.262	4.263	4.234	4.260	4.05
21.03.2019	3	26.03.2019	4.203	4.232	4.233	4.203	4.215	4.02
22.03.2019	3	27.03.2019	4.120	4.073	4.074	4.043	4.154	4.03
25.03.2019	3	28.03.2019	4.089	4.051	4.054	4.023	4.074	4.04
26.03.2019	3	29.03.2019	4.074	4.021	4.022	3.995	3.736	4.08
27.03.2019	3	1.04.2019	4.078	4.031	4.032	4.005	3.960	4.16
28.03.2019	3	2.04.2019	4.100	4.041	4.042	4.016	4.000	4.16
29.03.2019	3	3.04.2019	4.139	4.081	4.082	4.056	4.213	4.19
1.04.2019	3	4.04.2019	4.186	4.160	4.161	4.136	4.093	4.18

2.04.2019	3	5.04.2019	4.218	4.161	4.161	4.136	4.159	4.16
3.04.2019	3	8.04.2019	4.214	4.191	4.192	4.165	4.194	4.16
4.04.2019	2	8.04.2019	4.150	4.181	4.182	4.155	4.187	4.16
5.04.2019	3	10.04.2019	4.124	4.161	4.162	4.134	4.214	4.14
8.04.2019	3	11.04.2019	4.052	4.161	4.162	4.134	4.195	4.18
9.04.2019	2	11.04.2019	3.996	4.122	4.122	4.094	4.153	4.18
10.04.2019	2	12.04.2019	4.148	4.141	4.142	4.114	4.156	4.24
11.04.2019	2	15.04.2019	4.187	4.181	4.182	4.155	4.180	4.29
12.04.2019	2	16.04.2019	4.256	4.240	4.241	4.215	4.267	4.3
15.04.2019	3	18.04.2019	4.278	4.291	4.291	4.265	4.368	3.99
16.04.2019	3	23.04.2019	4.380	4.301	4.302	4.275	4.330	3.99
17.04.2019	2	23.04.2019	4.224	4.202	4.203	4.173	4.143	3.99
18.04.2019	2	24.04.2019	4.115	3.994	3.995	3.963	4.079	3.91
23.04.2019	2	25.04.2019	4.100	3.991	3.994	3.964	3.934	3.94
24.04.2019	2	26.04.2019	4.017	3.912	3.913	3.885	3.921	3.97
25.04.2019	2	29.04.2019	3.944	3.941	3.942	3.915	3.890	3.97
26.04.2019	2	30.04.2019	3.978	3.971	3.971	3.947	4.053	3.94
29.04.2019	2	2.05.2019	4.020	3.971	3.972	3.946	4.010	3.91
30.04.2019	2	3.05.2019	3.909	3.941	3.942	3.916	3.928	3.84
2.05.2019	2	6.05.2019	3.884	3.911	3.912	3.885	3.895	3.77
3.05.2019	3	8.05.2019	3.836	3.842	3.843	3.815	3.797	3.64
6.05.2019	2	8.05.2019	3.788	3.772	3.773	3.745	3.665	3.64
7.05.2019	3	10.05.2019	3.765	3.712	3.713	3.686	3.684	3.56
8.05.2019	3	13.05.2019	3.726	3.642	3.643	3.616	3.729	3.45
9.05.2019	2	13.05.2019	3.691	3.601	3.602	3.577	3.614	3.45
10.05.2019	3	15.05.2019	3.682	3.561	3.562	3.537	3.584	3.46
13.05.2019	2	15.05.2019	3.566	3.452	3.452	3.427	3.468	3.46
14.05.2019	3	17.05.2019	3.469	3.461	3.462	3.438	3.510	3.52
15.05.2019	3	20.05.2019	3.574	3.461	3.461	3.439	3.440	3.39
16.05.2019	3	21.05.2019	3.589	3.510	3.511	3.489	3.490	3.42
17.05.2019	2	21.05.2019	3.594	3.521	3.521	3.499	3.540	3.42
20.05.2019	3	23.05.2019	3.430	3.392	3.392	3.368	3.465	3.35
21.05.2019	2	23.05.2019	3.434	3.420	3.422	3.398	3.420	3.35
22.05.2019	3	27.05.2019	3.380	3.401	3.401	3.379	3.350	3.36
23.05.2019	3	28.05.2019	3.331	3.351	3.352	3.329	3.298	3.31
24.05.2019	3	29.05.2019	3.315	3.351	3.352	3.329	3.418	3.21
27.05.2019	3	30.05.2019	3.386	3.361	3.361	3.340	3.284	3.22
28.05.2019	3	31.05.2019	3.307	3.311	3.312	3.289	3.180	3.2
29.05.2019	3	3.06.2019	3.242	3.212	3.212	3.189	3.171	3.16
30.05.2019	3	4.06.2019	3.248	3.221	3.222	3.199	3.148	3.23

31.05.2019	3	5.06.2019	3.181	3.201	3.201	3.180	3.210	3.19
3.06.2019	3	6.06.2019	3.139	3.161	3.162	3.140	3.180	3.21
4.06.2019	3	7.06.2019	3.223	3.230	3.231	3.211	3.230	3.2
5.06.2019	3	10.06.2019	3.182	3.191	3.191	3.171	3.175	3.21
6.06.2019	2	10.06.2019	3.227	3.210	3.211	3.190	3.236	3.21
7.06.2019	3	12.06.2019	3.176	3.201	3.201	3.181	3.205	3.27
10.06.2019	2	12.06.2019	3.233	3.211	3.211	3.190	3.209	3.27
11.06.2019	2	13.06.2019	3.297	3.300	3.301	3.281	3.264	3.32
12.06.2019	2	14.06.2019	3.255	3.271	3.271	3.251	3.195	3.26
13.06.2019	2	17.06.2019	3.321	3.320	3.321	3.300	3.279	3.21
14.06.2019	3	19.06.2019	3.263	3.261	3.261	3.240	3.260	3.24
17.06.2019	3	20.06.2019	3.218	3.211	3.212	3.189	3.088	3.24
18.06.2019	3	21.06.2019	3.256	3.240	3.241	3.220	3.220	3.25
19.06.2019	3	24.06.2019	3.286	3.241	3.241	3.221	3.216	3.23
20.06.2019	3	25.06.2019	3.318	3.241	3.241	3.220	3.239	3.25
21.06.2019	3	26.06.2019	3.331	3.251	3.251	3.230	3.255	3.31
24.06.2019	3	27.06.2019	3.216	3.231	3.231	3.210	3.270	3.35
25.06.2019	2	27.06.2019	3.268	3.251	3.251	3.230	3.249	3.35
26.06.2019	2	28.06.2019	3.319	3.310	3.311	3.291	3.337	3.36
27.06.2019	3	2.07.2019	3.350	3.350	3.351	3.331	3.397	3.41
28.06.2019	2	2.07.2019	3.404	3.361	3.361	3.340	3.434	3.41
1.07.2019	2	3.07.2019	3.417	3.410	3.411	3.390	3.472	3.37
2.07.2019	2	4.07.2019	3.434	3.411	3.411	3.390	3.427	3.37
3.07.2019	2	5.07.2019	3.360	3.371	3.372	3.349	3.410	3.34
4.07.2019	3	9.07.2019	3.344	3.371	3.372	3.349	3.410	3.21
5.07.2019	2	9.07.2019	3.281	3.341	3.342	3.319	3.265	3.21
8.07.2019	2	10.07.2019	3.267	3.341	3.342	3.319	3.265	3.21
9.07.2019	3	12.07.2019	3.270	3.212	3.212	3.189	3.198	3.2
10.07.2019	3	15.07.2019	3.235	3.211	3.212	3.189	3.198	3.24
11.07.2019	2	15.07.2019	3.176	3.201	3.201	3.180	3.230	3.24
12.07.2019	3	17.07.2019	3.205	3.201	3.201	3.180	3.230	3.19
15.07.2019	5	22.07.2019	3.283	3.240	3.241	3.221	3.340	3.2
16.07.2019	5	23.07.2019	3.325	3.241	3.241	3.221	3.340	3.27
17.07.2019	2	19.07.2019	3.188	3.191	3.192	3.170	3.189	3.17
18.07.2019	2	22.07.2019	3.157	3.161	3.162	3.140	3.170	3.2
19.07.2019	3	24.07.2019	3.197	3.171	3.171	3.150	3.160	3.28
22.07.2019	3	25.07.2019	3.239	3.200	3.201	3.181	3.230	3.24
23.07.2019	3	26.07.2019	3.295	3.270	3.271	3.251	3.306	3.23
24.07.2019	3	29.07.2019	3.311	3.281	3.281	3.261	3.175	3.16
25.07.2019	3	30.07.2019	3.238	3.241	3.242	3.220	3.253	3.09

26.07.2019	3	31.07.2019	3.203	3.231	3.232	3.210	3.305	3.05
29.07.2019	3	1.08.2019	3.141	3.161	3.162	3.140	3.180	3.01
30.07.2019	3	2.08.2019	3.093	3.091	3.092	3.070	3.153	2.9
31.07.2019	3	5.08.2019	3.093	3.051	3.052	3.030	3.050	2.84
1.08.2019	3	6.08.2019	3.029	3.011	3.012	2.991	2.937	2.82
2.08.2019	3	7.08.2019	2.969	2.902	2.902	2.881	2.872	2.82
5.08.2019	5	12.08.2019	2.924	2.841	2.842	2.821	2.738	2.84
6.08.2019	3	9.08.2019	2.899	2.821	2.821	2.802	2.853	2.86
7.08.2019	2	9.08.2019	2.896	2.821	2.821	2.802	2.836	2.86
8.08.2019	3	13.08.2019	2.890	2.890	2.891	2.873	2.913	2.91
9.08.2019	3	14.08.2019	2.850	2.861	2.861	2.843	2.880	2.87
12.08.2019	3	15.08.2019	2.819	2.841	2.841	2.822	2.820	2.85
13.08.2019	3	16.08.2019	2.900	2.910	2.911	2.893	2.910	2.93
14.08.2019	3	19.08.2019	2.865	2.871	2.871	2.853	2.890	2.99
15.08.2019	3	20.08.2019	2.835	2.851	2.851	2.832	2.846	2.95
16.08.2019	3	21.08.2019	2.914	2.930	2.931	2.913	2.894	3.01
19.08.2019	3	22.08.2019	2.963	2.990	2.991	2.974	2.999	3.02
20.08.2019	2	22.08.2019	2.944	2.951	2.951	2.932	2.916	3.02
21.08.2019	2	23.08.2019	3.003	3.010	3.011	2.992	3.057	2.99
22.08.2019	2	26.08.2019	3.030	3.021	3.021	3.003	3.076	3.03
23.08.2019	2	27.08.2019	2.978	2.991	2.991	2.971	3.000	3.03
26.08.2019	2	28.08.2019	3.035	3.030	3.031	3.012	2.951	2.99
27.08.2019	2	29.08.2019	3.058	3.031	3.031	3.012	2.982	3.03
28.08.2019	2	30.08.2019	2.984	2.991	2.991	2.971	2.938	3.07
29.08.2019	2	2.09.2019	3.035	3.030	3.031	3.011	3.029	3.06
30.08.2019	2	3.09.2019	3.075	3.070	3.071	3.052	3.035	3.03
2.09.2019	2	4.09.2019	3.037	3.061	3.061	3.042	3.037	3.08
3.09.2019	2	5.09.2019	3.007	3.031	3.031	3.011	3.090	3.1
4.09.2019	3	9.09.2019	3.083	3.080	3.081	3.061	3.065	3.16
5.09.2019	3	10.09.2019	3.119	3.100	3.101	3.082	3.065	3.24
6.09.2019	3	11.09.2019	3.154	3.120	3.121	3.101	3.095	3.29
9.09.2019	3	12.09.2019	3.178	3.160	3.161	3.142	3.149	3.24
10.09.2019	2	12.09.2019	3.249	3.240	3.241	3.222	3.277	3.24
11.09.2019	2	13.09.2019	3.243	3.290	3.291	3.272	3.219	3.29
12.09.2019	3	17.09.2019	3.233	3.241	3.241	3.220	3.239	3.27
13.09.2019	2	17.09.2019	3.291	3.290	3.291	3.270	3.289	3.27
16.09.2019	2	18.09.2019	3.324	3.311	3.311	3.291	3.345	3.2
17.09.2019	2	19.09.2019	3.262	3.271	3.271	3.250	3.319	3.23
18.09.2019	2	20.09.2019	3.216	3.201	3.202	3.179	3.090	3.21
19.09.2019	3	24.09.2019	3.247	3.230	3.231	3.210	3.177	3.18

20.09.2019	3	25.09.2019	3.191	3.211	3.211	3.190	3.193	3.16
23.09.2019	2	25.09.2019	3.244	3.221	3.221	3.200	3.150	3.16
24.09.2019	3	27.09.2019	3.172	3.181	3.181	3.160	3.198	3.22
25.09.2019	3	30.09.2019	3.140	3.161	3.162	3.140	3.185	3.28
26.09.2019	3	1.10.2019	3.120	3.151	3.151	3.130	3.156	3.18
27.09.2019	3	2.10.2019	3.211	3.220	3.221	3.201	3.460	3.11
30.09.2019	3	3.10.2019	3.271	3.280	3.281	3.262	3.312	3.02
1.10.2019	2	3.10.2019	3.207	3.182	3.181	3.160	3.160	3.02
2.10.2019	2	4.10.2019	3.162	3.111	3.112	3.089	3.110	3.04
3.10.2019	2	7.10.2019	3.107	3.022	3.022	3.000	2.979	3.04
4.10.2019	3	9.10.2019	3.057	3.040	3.041	3.021	2.978	3.08
7.10.2019	3	10.10.2019	3.098	3.041	3.041	3.022	3.029	3.14
8.10.2019	3	11.10.2019	3.111	3.051	3.051	3.031	2.970	3.32
9.10.2019	3	14.10.2019	3.171	3.080	3.081	3.062	3.075	3.26
10.10.2019	2	14.10.2019	3.191	3.140	3.141	3.122	3.089	3.26
11.10.2019	2	15.10.2019	3.230	3.319	3.320	3.304	3.393	3.28
14.10.2019	3	17.10.2019	3.261	3.261	3.261	3.242	3.308	3.3
15.10.2019	2	17.10.2019	3.298	3.281	3.282	3.259	3.320	3.3
16.10.2019	2	18.10.2019	3.327	3.291	3.291	3.270	3.320	3.27
17.10.2019	3	22.10.2019	3.344	3.301	3.301	3.280	3.210	3.32
18.10.2019	2	22.10.2019	3.245	3.271	3.271	3.250	3.230	3.32
21.10.2019	2	23.10.2019	3.330	3.340	3.341	3.320	3.308	3.33
22.10.2019	3	25.10.2019	3.300	3.321	3.321	3.300	3.237	3.3
23.10.2019	2	25.10.2019	3.354	3.331	3.331	3.309	3.529	3.3
24.10.2019	2	28.10.2019	3.380	3.331	3.331	3.310	3.250	3.3
25.10.2019	2	29.10.2019	3.285	3.301	3.302	3.279	3.269	3.24
28.10.2019	2	30.10.2019	3.268	3.301	3.302	3.279	3.269	3.22
29.10.2019	2	31.10.2019	3.235	3.241	3.242	3.219	3.203	3.23
30.10.2019	2	1.11.2019	3.206	3.221	3.222	3.199	3.196	3.28
31.10.2019	2	4.11.2019	3.241	3.231	3.231	3.210	3.290	3.44
1.11.2019	2	5.11.2019	3.282	3.280	3.281	3.261	3.240	3.49
4.11.2019	2	6.11.2019	3.380	3.439	3.441	3.423	3.350	3.48
5.11.2019	3	8.11.2019	3.422	3.490	3.491	3.472	3.438	3.56
6.11.2019	3	11.11.2019	3.457	3.481	3.481	3.459	3.516	3.59
7.11.2019	2	11.11.2019	3.551	3.570	3.571	3.549	3.618	3.59
8.11.2019	3	13.11.2019	3.532	3.561	3.561	3.539	3.608	3.58
11.11.2019	2	13.11.2019	3.605	3.591	3.592	3.568	3.550	3.58
12.11.2019	3	15.11.2019	3.535	3.541	3.542	3.518	3.620	3.62
13.11.2019	2	15.11.2019	3.589	3.580	3.582	3.558	3.560	3.62
14.11.2019	3	19.11.2019	3.538	3.561	3.562	3.538	3.650	3.59

15.11.2019	2	19.11.2019	3.618	3.620	3.622	3.598	3.610	3.59
18.11.2019	3	21.11.2019	3.565	3.571	3.572	3.548	3.590	3.51
19.11.2019	2	21.11.2019	3.610	3.591	3.592	3.568	3.635	3.51
20.11.2019	3	25.11.2019	3.535	3.541	3.542	3.518	3.540	3.61
21.11.2019	3	26.11.2019	3.496	3.511	3.512	3.488	3.470	3.62
22.11.2019	3	27.11.2019	3.556	3.531	3.532	3.508	3.420	3.54
25.11.2019	3	28.11.2019	3.621	3.610	3.611	3.589	3.722	3.56
26.11.2019	3	29.11.2019	3.658	3.621	3.621	3.599	3.620	3.49
27.11.2019	2	29.11.2019	3.543	3.542	3.542	3.518	3.530	3.49
28.11.2019	2	2.12.2019	3.580	3.561	3.562	3.537	3.568	3.51
29.11.2019	3	4.12.2019	3.496	3.491	3.492	3.468	3.653	3.54
2.12.2019	2	4.12.2019	3.529	3.511	3.512	3.488	3.602	3.54
3.12.2019	3	6.12.2019	3.456	3.461	3.462	3.438	3.460	3.61
4.12.2019	2	6.12.2019	3.525	3.540	3.541	3.519	3.586	3.61
5.12.2019	3	10.12.2019	3.504	3.531	3.531	3.509	3.547	3.58
6.12.2019	2	10.12.2019	3.596	3.610	3.611	3.589	3.707	3.58
9.12.2019	2	11.12.2019	3.623	3.611	3.611	3.589	3.570	3.6
10.12.2019	2	12.12.2019	3.562	3.581	3.582	3.558	3.570	3.62
11.12.2019	2	13.12.2019	3.620	3.601	3.602	3.578	3.580	3.62
12.12.2019	2	16.12.2019	3.656	3.621	3.621	3.598	3.603	3.64
13.12.2019	3	18.12.2019	3.670	3.621	3.622	3.598	3.600	3.57
16.12.2019	2	18.12.2019	3.749	3.641	3.642	3.618	3.580	3.57
17.12.2019	3	20.12.2019	3.592	3.601	3.602	3.578	3.545	3.53
18.12.2019	3	23.12.2019	3.542	3.571	3.572	3.547	3.533	3.57
19.12.2019	3	24.12.2019	3.492	3.551	3.552	3.528	3.609	3.58
20.12.2019	3	27.12.2019	3.451	3.531	3.532	3.508	3.503	3.61
23.12.2019	2	27.12.2019	3.569	3.570	3.572	3.548	3.829	3.61
24.12.2019	2	30.12.2019	3.592	3.581	3.581	3.559	3.322	3.6
27.12.2019	2	31.12.2019	3.636	3.611	3.611	3.588	3.602	3.59

Table 0.1 Every Prediction

State Indication	Continue or Breake Probability	Sequence Probability	Label Indication	Label Probability	Final State Probability	State Indication	Continue or Breake Probability	Sequence Probability	Label Indication	Label Probability	Final State Probability
Continue, Total Sequence 2	56.429%	39.873%	-10	0.000%	0.000%	Continue, Total Sequence 3	56.429%	20.253%	-10	0.000%	0.000%
			-9	0.000%	0.000%				-9	0.000%	0.000%
			-8	0.000%	0.000%				-8	0.000%	0.000%
			-7	0.000%	0.000%				-7	0.000%	0.000%
			-6	0.000%	0.000%				-6	0.000%	0.000%
			-5	0.000%	0.000%				-5	0.000%	0.000%
			-4	0.000%	0.000%				-4	0.000%	0.000%
			-3	0.000%	0.000%				-3	0.000%	0.000%
			-2	0.000%	0.000%				-2	0.000%	0.000%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	5.300%	1.193%				1	2.959%	0.338%
			2	9.187%	2.067%				2	5.325%	0.609%
			3	13.781%	3.101%				3	4.142%	0.473%
			4	12.367%	2.783%				4	7.692%	0.879%
			5	12.721%	2.862%				5	8.284%	0.947%
			6	13.074%	2.942%				6	15.385%	1.758%
			7	12.367%	2.783%				7	13.018%	1.488%
			8	9.894%	2.226%				8	18.343%	2.096%
			9	8.834%	1.988%				9	16.568%	1.893%
			10	2.473%	0.557%				10	8.284%	0.947%
Continue, Total Sequence 4	56.429%	16.456%	-10	0.000%	0.000%	Continue, Total Sequence 5	56.429%	4.430%	-10	0.000%	0.000%
			-9	0.000%	0.000%				-9	0.000%	0.000%
			-8	0.000%	0.000%				-8	0.000%	0.000%
			-7	0.000%	0.000%				-7	0.000%	0.000%
			-6	0.000%	0.000%				-6	0.000%	0.000%
			-5	0.000%	0.000%				-5	0.000%	0.000%
			-4	0.000%	0.000%				-4	0.000%	0.000%
			-3	0.000%	0.000%				-3	0.000%	0.000%
			-2	0.000%	0.000%				-2	0.000%	0.000%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	1.190%	0.111%				1	0.000%	0.000%
			2	3.571%	0.332%				2	0.000%	0.000%
			3	2.381%	0.221%				3	1.852%	0.046%
			4	5.952%	0.553%				4	3.704%	0.093%
			5	3.571%	0.332%				5	5.556%	0.139%
			6	11.905%	1.105%				6	5.556%	0.139%
			7	10.714%	0.995%				7	9.259%	0.231%
			8	16.667%	1.548%				8	16.667%	0.417%
			9	26.190%	2.432%				9	22.222%	0.556%
			10	17.857%	1.658%				10	35.185%	0.880%
Continue, Total Sequence 6	56.429%	8.228%	-10	0.000%	0.000%	Continue, Total Sequence 7	56.429%	3.797%	-10	0.000%	0.000%
			-9	0.000%	0.000%				-9	0.000%	0.000%
			-8	0.000%	0.000%				-8	0.000%	0.000%
			-7	0.000%	0.000%				-7	0.000%	0.000%
			-6	0.000%	0.000%				-6	0.000%	0.000%
			-5	0.000%	0.000%				-5	0.000%	0.000%
			-4	0.000%	0.000%				-4	0.000%	0.000%
			-3	0.000%	0.000%				-3	0.000%	0.000%
			-2	0.000%	0.000%				-2	0.000%	0.000%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	2.381%	0.111%				5	7.143%	0.153%
			6	2.381%	0.111%				6	7.143%	0.153%
			7	7.143%	0.332%				7	7.143%	0.153%
			8	11.905%	0.553%				8	0.000%	0.000%
			9	26.190%	1.216%				9	17.857%	0.383%
			10	50.000%	2.321%				10	46.429%	0.995%

State Indication	Continue or Breake Probability	Sequence Probability	Label Indication	Label Probability	Final State Probability	State Indication	Continue or Breake Probability	Sequence Probability	Label Indication	Label Probability	Final State Probability
Continue, Total Sequence 8	56.429%	3.797%	-10	0.000%	0.000%	Continue, Total Sequence 9	56.429%	3.165%	-10	0.000%	0.000%
			-9	0.000%	0.000%				-9	0.000%	0.000%
			-8	0.000%	0.000%				-8	0.000%	0.000%
			-7	0.000%	0.000%				-7	0.000%	0.000%
			-6	0.000%	0.000%				-6	0.000%	0.000%
			-5	0.000%	0.000%				-5	0.000%	0.000%
			-4	0.000%	0.000%				-4	0.000%	0.000%
			-3	0.000%	0.000%				-3	0.000%	0.000%
			-2	0.000%	0.000%				-2	0.000%	0.000%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	0.000%	0.000%				5	0.000%	0.000%
			6	5.882%	0.126%				6	0.000%	0.000%
			7	5.882%	0.126%				7	0.000%	0.000%
			8	11.765%	0.252%				8	9.091%	0.162%
			9	5.882%	0.126%				9	0.000%	0.000%
			10	70.588%	1.513%				10	90.909%	1.623%
Breake, Total Sequence 1	43.571%	44.338%	-10	0.566%	0.109%	Breake, Total Sequence 2	43.571%	21.689%	-10	4.286%	0.405%
			-9	2.264%	0.437%				-9	8.214%	0.776%
			-8	4.906%	0.948%				-8	14.643%	1.384%
			-7	7.358%	1.422%				-7	16.071%	1.519%
			-6	8.302%	1.604%				-6	11.429%	1.080%
			-5	11.321%	2.187%				-5	13.929%	1.316%
			-4	14.340%	2.770%				-4	10.000%	0.945%
			-3	16.415%	3.171%				-3	7.857%	0.743%
			-2	15.660%	3.025%				-2	7.857%	0.743%
			-1	18.868%	3.645%				-1	5.714%	0.540%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	0.000%	0.000%				5	0.000%	0.000%
			6	0.000%	0.000%				6	0.000%	0.000%
			7	0.000%	0.000%				7	0.000%	0.000%
			8	0.000%	0.000%				8	0.000%	0.000%
			9	0.000%	0.000%				9	0.000%	0.000%
			10	0.000%	0.000%				10	0.000%	0.000%
Breake, Total Sequence 3	43.571%	13.436%	-10	12.575%	0.736%	Breake, Total Sequence 4	43.571%	7.678%	-10	20.619%	0.690%
			-9	16.168%	0.946%				-9	29.897%	1.000%
			-8	16.168%	0.946%				-8	13.402%	0.448%
			-7	11.976%	0.701%				-7	9.278%	0.310%
			-6	15.569%	0.911%				-6	11.340%	0.379%
			-5	8.982%	0.526%				-5	6.186%	0.207%
			-4	6.587%	0.386%				-4	4.124%	0.138%
			-3	5.389%	0.315%				-3	2.062%	0.069%
			-2	2.994%	0.175%				-2	3.093%	0.103%
			-1	3.593%	0.210%				-1	0.000%	0.000%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	0.000%	0.000%				5	0.000%	0.000%
			6	0.000%	0.000%				6	0.000%	0.000%
			7	0.000%	0.000%				7	0.000%	0.000%
			8	0.000%	0.000%				8	0.000%	0.000%
			9	0.000%	0.000%				9	0.000%	0.000%
			10	0.000%	0.000%				10	0.000%	0.000%

State Indication	Continue or Breake Probability	Sequence Probability	Label Indication	Label Probability	Final State Probability	State Indication	Continue or Breake Probability	Sequence Probability	Label Indication	Label Probability	Final State Probability
Breake, Total Sequence 5	43.571%	4.415%	-10	34.426%	0.662%	Breake, Total Sequence 6	43.571%	3.263%	-10	40.625%	0.578%
			-9	29.508%	0.568%				-9	21.875%	0.311%
			-8	14.754%	0.284%				-8	12.500%	0.178%
			-7	6.557%	0.126%				-7	6.250%	0.089%
			-6	6.557%	0.126%				-6	6.250%	0.089%
			-5	1.639%	0.032%				-5	3.125%	0.044%
			-4	3.279%	0.063%				-4	0.000%	0.000%
			-3	1.639%	0.032%				-3	0.000%	0.000%
			-2	1.639%	0.032%				-2	9.375%	0.133%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	0.000%	0.000%				5	0.000%	0.000%
			6	0.000%	0.000%				6	0.000%	0.000%
			7	0.000%	0.000%				7	0.000%	0.000%
			8	0.000%	0.000%				8	0.000%	0.000%
			9	0.000%	0.000%				9	0.000%	0.000%
			10	0.000%	0.000%				10	0.000%	0.000%
Breake, Total Sequence 7	43.571%	1.344%	-10	53.333%	0.312%	Breake, Total Sequence 8	43.571%	1.344%	-10	50.000%	0.293%
			-9	20.000%	0.117%				-9	14.286%	0.084%
			-8	0.000%	0.000%				-8	0.000%	0.000%
			-7	26.667%	0.156%				-7	7.143%	0.042%
			-6	0.000%	0.000%				-6	7.143%	0.042%
			-5	0.000%	0.000%				-5	0.000%	0.000%
			-4	0.000%	0.000%				-4	0.000%	0.000%
			-3	0.000%	0.000%				-3	14.286%	0.084%
			-2	0.000%	0.000%				-2	7.143%	0.042%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	0.000%	0.000%				5	0.000%	0.000%
			6	0.000%	0.000%				6	0.000%	0.000%
			7	0.000%	0.000%				7	0.000%	0.000%
			8	0.000%	0.000%				8	0.000%	0.000%
			9	0.000%	0.000%				9	0.000%	0.000%
			10	0.000%	0.000%				10	0.000%	0.000%
Breake, Total Sequence 9	43.571%	0.576%	-10	66.667%	0.167%	Breake, Total Sequence 10	43.571%	0.576%	-10	100.000%	0.251%
			-9	0.000%	0.000%				-9	0.000%	0.000%
			-8	16.667%	0.042%				-8	0.000%	0.000%
			-7	0.000%	0.000%				-7	0.000%	0.000%
			-6	0.000%	0.000%				-6	0.000%	0.000%
			-5	16.667%	0.042%				-5	0.000%	0.000%
			-4	0.000%	0.000%				-4	0.000%	0.000%
			-3	0.000%	0.000%				-3	0.000%	0.000%
			-2	0.000%	0.000%				-2	0.000%	0.000%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	0.000%	0.000%				5	0.000%	0.000%
			6	0.000%	0.000%				6	0.000%	0.000%
			7	0.000%	0.000%				7	0.000%	0.000%
			8	0.000%	0.000%				8	0.000%	0.000%
			9	0.000%	0.000%				9	0.000%	0.000%
			10	0.000%	0.000%				10	0.000%	0.000%

State Indication	Continue or Breake Probability	Sequence Probability	Label Indication	Label Probability	Final State Probability	State Indication	Continue or Breake Probability	Sequence Probability	Label Indication	Label Probability	Final State Probability
Breake, Total Sequence 11	43.571%	0.192%	-10	60.000%	0.050%	Breake, Total Sequence 12	43.571%	0.192%	-10	100.000%	0.084%
			-9	0.000%	0.000%				-9	0.000%	0.000%
			-8	0.000%	0.000%				-8	0.000%	0.000%
			-7	0.000%	0.000%				-7	0.000%	0.000%
			-6	0.000%	0.000%				-6	0.000%	0.000%
			-5	0.000%	0.000%				-5	0.000%	0.000%
			-4	0.000%	0.000%				-4	0.000%	0.000%
			-3	0.000%	0.000%				-3	0.000%	0.000%
			-2	40.000%	0.033%				-2	0.000%	0.000%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	0.000%	0.000%				5	0.000%	0.000%
			6	0.000%	0.000%				6	0.000%	0.000%
			7	0.000%	0.000%				7	0.000%	0.000%
			8	0.000%	0.000%				8	0.000%	0.000%
			9	0.000%	0.000%				9	0.000%	0.000%
			10	0.000%	0.000%				10	0.000%	0.000%
Breake, Total Sequence 13	43.571%	0.384%	-10	75.000%	0.125%	Breake, Total Sequence 14	43.571%	0.192%	-10	0.000%	0.000%
			-9	25.000%	0.042%				-9	0.000%	0.000%
			-8	0.000%	0.000%				-8	100.000%	0.084%
			-7	0.000%	0.000%				-7	0.000%	0.000%
			-6	0.000%	0.000%				-6	0.000%	0.000%
			-5	0.000%	0.000%				-5	0.000%	0.000%
			-4	0.000%	0.000%				-4	0.000%	0.000%
			-3	0.000%	0.000%				-3	0.000%	0.000%
			-2	0.000%	0.000%				-2	0.000%	0.000%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	0.000%	0.000%				5	0.000%	0.000%
			6	0.000%	0.000%				6	0.000%	0.000%
			7	0.000%	0.000%				7	0.000%	0.000%
			8	0.000%	0.000%				8	0.000%	0.000%
			9	0.000%	0.000%				9	0.000%	0.000%
			10	0.000%	0.000%				10	0.000%	0.000%
Breake, Total Sequence 16	43.571%	0.192%	-10	0.000%	0.000%	Breake, Total Sequence 17	43.571%	0.192%	-10	100.000%	0.084%
			-9	0.000%	0.000%				-9	0.000%	0.000%
			-8	0.000%	0.000%				-8	0.000%	0.000%
			-7	0.000%	0.000%				-7	0.000%	0.000%
			-6	0.000%	0.000%				-6	0.000%	0.000%
			-5	0.000%	0.000%				-5	0.000%	0.000%
			-4	0.000%	0.000%				-4	0.000%	0.000%
			-3	0.000%	0.000%				-3	0.000%	0.000%
			-2	100.000%	0.084%				-2	0.000%	0.000%
			-1	0.000%	0.000%				-1	0.000%	0.000%
			1	0.000%	0.000%				1	0.000%	0.000%
			2	0.000%	0.000%				2	0.000%	0.000%
			3	0.000%	0.000%				3	0.000%	0.000%
			4	0.000%	0.000%				4	0.000%	0.000%
			5	0.000%	0.000%				5	0.000%	0.000%
			6	0.000%	0.000%				6	0.000%	0.000%
			7	0.000%	0.000%				7	0.000%	0.000%
			8	0.000%	0.000%				8	0.000%	0.000%
			9	0.000%	0.000%				9	0.000%	0.000%
			10	0.000%	0.000%				10	0.000%	0.000%

Table 0.2 Probabilities