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Predicting and Distinguishing Bankruptcy: An Application of a Market And Hybrid Model to US Publicly Listed Firms from 2008 to 2018

Bernardo Rui Vaz Silva

Master in Finance

Supervisor:
Prof. Joaquim Paulo Viegas Ferreira de Carvalho,
Visiting Assistant Professor, ISCTE Business School
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Resumo

A avaliação da probabilidade de falência tem sido um tema-chave abordado por investigadores e académicos ao longo do último meio século. A falência de empresas consideráveis como a Enron ou a WorldCom, aliada ao rigoroso ambiente regulamentar desencadeado pelas diretrizes de Basileia II, fomentou ainda mais o interesse pelo tema.

Além disso, na sequência da crise financeira, as agências de notação de crédito (ANC) foram criticadas por endereçarem notações inflacionadas e não anteciparem corretamente os incumprimentos. Ademais, as principais ANC não avaliam todas as empresas, e a nossa intenção é proporcionar ao investidor individual a melhor opção disponível para estimar autonomamente a probabilidade de falência.

Neste estudo analisou-se se um modelo baseado em dados de mercado, o KMV, e um modelo híbrido, o CHS, diferenciam o evento de falência e, caso isso seja verificado, qual deles melhor distingue entre empresas falidas e não falidas. Para tal, recorremos a uma amostra de 354 empresas cotadas nos EUA, divididas em empresas falidas e não falidas, aplicando a técnica estatística "ROC", num período de 10 anos.

Os nossos resultados sugerem que o modelo KMV é ligeiramente superior ao modelo CHS, maximizando a área sob a curva (AUC). Além disso, o primeiro proporcionou um ponto de corte de probabilidade mais elevado que distingue ambos os tipos de empresas. Os nossos resultados indiciam que o KMV é a melhor opção disponível para um investidor individual avaliar a probabilidade de incumprimento, dado os resultados alcançados e a facilidade de aplicação em comparação com o modelo CHS.

Classificação JEL: G33: G32

Palavras Chave: Falência, Modelização do Risco de Crédito, Análise ROC, Modelo KMV, Modelo CHS

Abstract

Assessing the probability of bankruptcy has been a key topic approached by researchers and academics throughout the last half century. The bankruptcy of considerable firms, such as Enron or WorldCom, coupled with the rigorous regulatory environment triggered by Basel II guidelines, fostered even further the interest in the topic.

Moreover, in the outcome of financial crisis, Credit Rating Agencies were criticized for addressing inflated ratings and not properly anticipating defaults. Besides, leading CRA's do not assess the creditworthiness of all firms, and our intention is to provide to individual investor the best option available to autonomously estimate the probability of bankruptcy

We analyse if either a market-based model, KMV, or a hybrid model, CHS, are able to properly anticipate the event of bankruptcy, and in case this is verified, which of them better distinguish between bankrupt and non-bankrupt firms. In order to do so, we resort to a sample of 354 US publicly listed firms, divided into bankrupt and non-bankrupt firms, and applied the ROC technique to assess our results, for a 10-year period.

Our results prove that KMV model is slightly superior to the CHS model at maximizing the Area Under the Curve (AUC). Besides, it provided a higher optimal probability's cut off point that distinguish both type of firms. Our results indicate that the KMV model is the best option available for an individual investor to assess the probability of default, given the results achieved and the easiness of application when compared to the CHS model.

JEL Classification: G33: G32

Keywords: Bankruptcy, Credit Risk Modelling, ROC analysis, KMV Model, CHS Model

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1. Introduction

Credit risk modelling and predicting bankruptcy have been key topics in finance since the second half of the 20th century. The increased number of bankruptcies, including the likes of Enron, WorldCom or Chrysler, coupled with the appearance of securitization products and the implementation of Basel II regulation fostered even further the research on credit risk modelling. Moreover, the role of leading Credit Rating Agencies (CRA), entities responsible for addressing creditworthiness of a firm or product, gained importance in the same period. However, many authors accounted CRAs for contributing to the credit bubble, which led to the 2008 financial crisis, and for being too slow in recognizing early warning signs on entities that went bankrupt. Given this ambiguity surrounding CRAs, as well as the fact that these entities do not rate all publicly listed firms, it arises the need to provide to individual investors tools to measure credit risk, more concretely the probability of bankruptcy.

The topic of credit risk modelling linked to the probability of bankruptcy was characterized, in the beginning of the 20th century, by accounting-based models, which presented some flaws, as being sample specific, based on a reduced number of firms and only considering backward measures. Merton (1974) developed its first market-based model, a theoretical approach based on the capital structure of a company, which had the advantage of being forward looking and not sample specific. Since then, researchers compared both type of models and, by the beginning of 21st century, many reached to the conclusion that models containing a mix of both market and accounting variables improved bankruptcy's accuracy prediction, in what we deem as hybrid models.

In this paper, we take into consideration an extension of the original Merton Model (1974), the Moody's KMV, and a hybrid model, the CHS model, developed by Campbell et al. (2008). We contribute to the existing literature review, which is being mainly focused on comparing market-based with accounting-based models, or models belonging to the same division. The comparison between the original CHS Model and KMV model is made by ranking their relative ability of distinguishing bankruptcy on a 1 year horizon, among a sample of 354 US publicly listed firms, 118 bankrupt and 252 non-bankrupt, for a period of 10 years (2008 to 2018), resorting to the Receiver Operating Characteristic's (ROC) statistical technique. By the end, we achieved to the conclusion that both models have an outstanding ability of discriminating bankruptcy, however, the KMV model is slightly better than CHS, in terms of distinguishing both type of firms and providing more meaningful optimal cut off points.

The reminder of this work is organized as follows. Section 2 provides a literature overview on the divisions of credit risk and its importance; the role of CRAs; and the main models of credit risk linked to bankruptcy's prediction. Section 3 is focused in the objectives of this paper; as well as the rationale behind the model's selection, formulation of hypotheses and the steps taken to choose the data sample. Section 4 discusses the methodologies regarding the KMV and CHS model, as well as the statistical technique used to compare models, the ROC. Section 5 shows the empirical results reached while Section 6 presents the main conclusion achieved as well as some advices for future researches.

2. Literature Review

2.1. Credit Risk: Definitions, Importance, Participants and Modelling

The assessment of credit risk and prediction of corporate failure has been a subject of study and research throughout the last half century. Multiple definitions can be found in the literature regarding the concept of credit risk. In a nutshell, it can be defined as the risk that a lender faces when a borrower does not have the ability to meet its financial obligations. For example, Spuchľáková et. al (2015:675) refer that *"Credit Risk or Default Risk involves inability or unwillingness of a customer or counterparty to meet commitments in relation to lending, trading, hedging, settlement and other financial transactions";* in the same reasoning, but directed to corporates, Crosbie et. al (2003:5) consider that *"Default risk is the uncertainty surrounding a firm's ability to service its debts and obligations"*.

2.1.1 Credit Risk: Divisions and Scope

As per Crosbie et al. (2003), credit risk elements can be gathered into two major groups[1]: 1) standalone risk, where only one firm is considered and 2) portfolio risk, where multiple firms are taken into account. Both groups have their subdivisions, as per the following:

1. Standalone risk
    a. Default probability: the probability that the counterparty is going to fail in its obligations
    b. Loss Given Default: the loss incurred in the case of default
    c. Migration risk: the probability and value impact of a change in default probability
2. Portfolio risk
    a. Default Correlation: the degree to which the default probabilities of multiple firms are related
    b. Exposure: the proportion of the portfolio exposed to the default risk of each counterparty.

Besides, either concerning standalone or portfolio risk, credit risk's scope can be directed to a company as whole or to a specific financial instrument. In fact, Credit Rating Agencies

[1] For a different breakdown of credit risk, please refer, for instance to Basel Committee on Banking Supervision (2001)

(CRA's), institutions that are deemed as specialists in measuring creditworthiness in the form of rating grades, divide their rating assessments into two categories (Langohr et al, 2006):

1. Issuer rating: where the issuer as a whole is considered, independently of a particular debt instrument. It is also known as counterparty risk rating.
2. Instrument rating: deals with the performance of one particular instrument and the primary purpose is to assess if weather or not the issuer will be able to deliver on the terms of the specific security that is has issued.

In this paper, the focus will be directed into standalone risk, more concretely to the component "probability of default" of firms, while taking into consideration their creditworthiness as whole rather than a specific instrument.

2.1.2 Credit Risk: Augmented importance in the 2nd half of 20th century and role on the 2007-2008 financial crisis.

Credit risk necessarily involves the failure of a firm towards its payment commitments, which, in extreme cases, can lead to declaration of bankruptcy / insolvency and the possible shutdown of business operations. Piesse et al (2006) refers that the failure of one firm affects a various number of stakeholders, such as shareholders, lenders and employees, whereas a simultaneous failure of multiple firms is a concern to policy makers, given the wider economic impact. Moreover, and considering the process of globalization, multiple failures at the same time in a specific region may have consequences in other geographical areas, in a so called "domino effect" (Jackson and Wood, 2013).

Specifically, Altman and Saunders (1997) argue that the interest in credit risk increased significantly in the second half of the 20th century due to several factors, including the global increase in the number of bankruptcies, the more competitive margins on loans, and dramatic growth of off the balance sheet instruments, such as credit risk derivatives, with inherent default risk exposure. In fact, a few years later, Aziz and Dar (2006) refer that the bankruptcies of major players, such as Enron or WorldCom, further contributed to the growing awareness around credit risk. Besides, the implementation of Basel II regulations fostered the interest in the area, as it incorporates three pillars to establish minimum capital requirements for financial institutions, included credit risk. Concerning the 2007-2008 financial crisis, Saunders and Allen (2010) consider that some schemes were in the core of the crisis, including securitization of non-standard mortgage assets, syndication of loans and increased use of derivatives, such as Credit Default Swaps (CDS), which allowed for the risks' removal from the balance sheet of

Financial Institutions, while passing it to other players in the financial system. With more complex instruments appearing, little attention was given to monitor the activities of borrowers to whom the instruments were linked. This combination of factors led to an unstainable rise of consumer and corporate indebtedness passing undetected, leading to a credit bubble and posterior financial crisis.

2.1.3 Credit Risk in practice: the case of Credit Rating Agencies (CRA)

As per the definition by the Security and Exchange Commission (SEC), Credit Rating Agencies (CRA) are *"(...) organizations that provide an assessment of the creditworthiness of a company or a financial instrument.* In 1975, SEC defined some of the CRA as Nationally Recognized Statistical Rating Organizations (NRSRO's), immediately declaring that only the ratings of NRSROs were valid to broker-dealers capital requirements. A few years later, in 1990, SEC announced that only rating of such organizations is valid to define safety requirements for money market mutual funds (White, 2010). More recently, Basel II guidelines allowed banks, in certain cases, to rely on the rating assessment provided by CRA to measure credit risk, further contributing to the importance of these institutions (Basel Committee on Banking Supervision, 2006).

CRA's uses letters ranging from AAA to the D (default) to grade issuers and to rate companies, countries, provinces, water authorities, territories, municipalities and structured financing products (Booth and De Bruin, 2019). Table 2.1 condenses all the rating grades from the 3 most important CRA's, with an adjacent small interpretation,

Table 2. 1 Credit Rating Agencies: Grades and Respective Interpretation

Interpretation		**Moody's**		**Standard & Poor's**		**Fitch**	
		Long Term	**Short term**	**Long Term**	**Short term**	**Long Term**	**Short term**
Investment Grade Ratings	Highest credit quality	Aaa		AAA		AAA	
	High credit quality	Aa1 Aa2 Aa3	Prime-1	AA+ AA AA-	A1+	AA+ AA AA-	F1
	Strong payment capacity	A1 A2 A3	Prime-2	A+ A A-	A1		
	Adequate payment capacity	Baa1 Baa2	Prime-3	BBB+ BBB	A2 A3	BBB+ BBB	F2 F3
	Last rating in investment-grade	Baa3		BBB-		BBB-	
Speculative Grade Ratings	Speculative Credit risk developing, due to economic changes	Ba1 Ba2 Ba3	Not Prime	BB+ BB BB-	B	BB+ BB BB-	B
	Highly speculative, credit risk present, with limited margin safety	B1 B2 B3		B+ B B-		B+ B B-	
	High default risk, capacity depending on favourable conditions	Caa1 Caa2 Caa3		CCC+ CCC CCC- CC	C	CCC+ CCC CCC- CC	C
Default	Possible partially recover	Ca, C	Not Prime	C,D	D	C,D	D

Source: Own representation, interpretation based on Elkhoury (2009)

Given the augmented importance and role in the financial system, CRA activities were subject of study, and some critics were pointed out. Bolton et al. (2012) summarizes the multiple conflict of interests that CRA faces, by questioning how CRAs should act, if the main source of income arises from firms whose products are being rated, which are, in their best interest, willing to receive a better rating. Moreover, legal challenges also arise, as CRA's claim that *"(...) their rating are independent expressions of opinion (...)"*(Stolper, 2009:1266)*,* thus making it very difficult for being accounted for any misevaluation or mismeasuring.

Furthermore, and following the previous referred bankruptcies of Enron and WorldCom, many have accused CRA's of being too slow to recognize the weakening of the firms' conditions, allegation supported by the fact that only 5 days before Enron's declaration of bankruptcy, the 3 main CRA's classified the firm's bonds as "investment grade" (White, 2010). Besides, in the remnant of 2007-2008 financial crisis, it was possible to observe that CRA's assigned inflated ratings to Structured Finance Products (SFP), whose defaults were at the core of the crisis (Mathis et al., 2009; Stolper, 2009). Bolton et al. (2012) argue that the majority of CRAs profit growth arose from the assessment of the SFP that had defaulted, clearly showing how CRAs let themselves being carried away by the conflict of interests.

2.1.4 Credit Risk Modelling: Events Considered and Divisions

As per previously indicated in section 1.1, the purpose here is to reflect about the probability of default of firms, considering their overall creditworthiness. About this respect, Crosbie et al. (2003) state that prior to failure, no one can undoubtfully discriminate between firms that will default and those who will not. As such, and despite the possibility of not gathering all the reality's complexity, modelling is important to measure the likelihood or probability of default.

Default is a broad concept and deemed as *"(..) one of the more ambiguous notions in law*" (Langohr & Langohr, 2006, p. 24). However, as per Ganguin and Bilardello (2005), there are two broad types of event of default: one more serious, the "failure to pay" of principal and interest, and one less, which is a breach of covenant. Nonetheless, as practitioners tried to be exhaustive, bankruptcy and insolvency can also be defined as events of default (Ganguin and Bilardello, 2005).

Due to the extensive concept of default, multiple events were considered in literature, to measure and model it. Beaver (1966) uses the term failure, which includes four events: bankruptcy, bond default, an overdrawn bank account or non-payment of preferred stock dividend. Furthermore, in other benchmark papers in modelling default, Altman (1968) and

Ohlson (1980) also present a research on predicting corporate failure, but narrowing it only to the event of bankruptcy. Campbell et al (2008), in an attempt to measure how companies fail in their financial obligations, consider not only firms that filed for bankruptcy, but also those delisted, and/or were awarded with a "D" ("default") grade by a leading credit agency. Last but not least, Duffie et al. (2007) in a estimation of a model for US Listed Industrials, use a sample of "exit" firms, which had several different categories, such as: bankruptcy, dividend omission, missed principal and/or interest payments, any failure to meet exchange listing requirements, among other exits. This clearly exemplifies the wide range of events that are judged in modelling credit /default risk.

Multiple divisions and separations among different type of models can be made For instance, Jackson and Wood (2013) identified 25 different methods to "predict" corporate insolvency, in a sample of over 350 papers in the last five decades, with an extended breakdown possible if many more variations within the same method were counted. On the other hand, Fernandes (2007) narrows the models into three main approaches: for enterprises with traded debt equity and debt, Structural Models and Reduced-Form Models, whereas for private firms, Accounting-Based Credit Scoring Models are the most common alternative.

In this paper we propose the following division: Accounting Based Models, Market Based Models and Hybrid Models. In the accounting-based models, the bulk of inputs are based in financial statements; in the market-based models the majority is built around market data whereas hybrid models, as per what the name indicates, uses both type of inputs. In the next section, the main models on each division will be described.

2.2. Credit Risk Modelling

2.2.1 Accounting based models.

In an intuitive way, ratios derived from financial statements allow for the enhancement of certain characteristics such as Profitability and Solvency, that can be used to compare firms and assess its creditworthiness (Trigueiros, 2019). For instance, the current ratio (current assets / current liabilities) was first utilized in the field, as early as the beginning of 20th century, to evaluate creditworthiness (Beaver, 1966). Beaver (1966) argues that despite not being the only predictors of failure, financial ratios can bring added value to the field. By using a statistical technique denominated Univariate Classification Analysis, he tries to provide empirical validation for the usefulness of those ratios, proving that the Cash Flow / Total Debt is the best to discriminate the event of failure, with a lower percentage of misclassifications across 5 years.

However, Jayasekera (2018) criticise not only the non-usage of a dispersion measure to assess the strength and reliability of the analysis, jeopardizing Beaver's differentiation process, but also the binary state of the results: failure and non-failure.

Furthermore, Altman (1968) disapproves the univariate approach, as it resorts only to a single indicator. In fact, a firm may show a Cash Flow / Total Debt ratio that may classify it as "bankrupt" in Beaver's classification, however the same firm can have liquidity levels that allow to cope with its obligations. As such, he suggests another statistical technique, the Multivariate Discriminant Analysis (MDA). This approach discriminates a sample of firms, belonging to the same industry (manufactures), with roughly the same size, in two groups (bankrupt and non-bankrupt) considering their characteristics (financial ratios), deriving linear combinations between them, resulting into a single score: the "Z Score". The best model correctly predicted 96% and 94% of bankruptcies at one-year horizon, in the original and a secondary sample, respectively. However, Z scores between the range of 1.81 and 2.99 represented the area where the largest number of misclassifications occurred, described as "the zone of ignorance", which is a flaw to the obtained results. Altman revised its Z score model several times, to expand the scope to non-manufacturing firms, to include different variables and to improve the discriminatory power for longer horizons (Altman, 2013).

On the other hand, Ohlson (1980) disapproves the use of MDA, given that it assumes certain statistical requirements, such as the equality of the variance-covariance matrix between group of firms. Moreover, he criticizes the sample selection of Beaver (1966) and Altman (1968), since they are based in characteristics such as size and sector, used for matching purposes on the development of ordinal discriminatory devices. He argues that "*it would seem to be more fruitful actually to include variables as predictors rather than to use them for matching purposes*." (Ohlson, 1980, p. 112). Moreover, he also points out that the usage, of ordinal discriminatory devices, bring little intuition interpretation. As such, he develops a model following a logistic specification, with 9 variables, able to accurately predict 87.6% of bankruptcies and 92.6% of non-bankruptcies, considering the cut-off point which minimizes misclassifications. Ohlson (1980) logistic model was pioneer in the field since it brought a probabilistic feature to the analysis, while using a larger sample to derive it (105 bankrupt and 2,058 firms). Zmijewski (1984) further contributed to the probabilistic models, by using a probit model, with fewer rations than Ohlson (1980), only 3, with similar sample's firms and results.

Many argue that the models based in accounting variables, similar to those of Beaver (1966), Altman (1968) or Ohlson (1980), have their shortcomings. Vassalou and Xing (2004) state that as accounting models use information from financial statements, the focus is on the past performance rather than on future prospects. Hillegeist et al. (2004) refer the issue with the going concern basis on which financial statements are prepared, on which firms are expected to survive, intuitively restrains the analysis and usefulness of these information. Agarwal and Taffler (2008) point out that it may also exists a discrepancy between the true value and the book values, mainly the asset one, due to conservatism and historical cost accounting. Furthermore, Hillegeist et al. (2004) and Vassalou and Xing (2004) criticize accounting models for not considering asset volatility, an important measure for assessing whether or not the repayment of debts is possible, whereas Agarwal and Taffler (2008) doubt about a wider application of those models, derived from samples. However, accounting models remain crucial, as they are the only way to statistically measure default risk for private owned firms (Das et al., 2009).In fact, Altman (2013) specifically adapted his Z score for this reason. Moreover, Agarwal and Taffler (2008) argue that failure is a culmination of adverse conditions, which can be captured precisely by financial statements, thus being an important and rationale input.

2.2.2 Market Based Models

In 1974, Robert Merton deviate from the traditional financial ratios analysis, by adapting the option pricing theory developed by Black and Scholes (1973) to the study of corporate liabilities, in what is considered the groundwork of modern credit risk modelling (Lando, 2009). Merton (1974) demonstrates that since corporate liabilities can be considered as contingent claims on the value of the firm's assets, those liabilities can be estimated according to the option pricing theory. Merton (1974) objective is to accurately measure the price corporate debt, and by explicitly accounting for default, he concluded that borrowers may ask firms for a spread over the default risk free rate.

Merton (1974) model is the first in credit risk deemed as structural, since it links the default risk to the firms' asset value process and capital structure (Santos, 2018). Despite being in the genesis of the structural models, the initial empirical results of the Merton Model were discouraging, given the high number and limiting nature of the assumptions used to derive it (Kealhofer, 2003). In fact, apart from the simple capital structure, Merton (1974) relied on other basic assumptions to derive its models, which were challenged by many authors in literature:

Black and Cox (1976) allowed for default to happen in every point in time; Vasicek (1984) expanded to scope to more complex capital structure; whereas Leland (1994) included bankruptcy costs and taxes.

Apart from other academics, Merton model is also the groundwork for the development of models used by both CRA and Financial Institutions in their credit risk related activities. For example, Merton Model foundations are used to not only derive the *CreditGrades* model, jointly developed by Goldman Sachs, J.P Morgan Chase, Deutshe Bank and the Risk Metrics Group; but also the KMV model, built by an agency with the same name, later acquired by Moody's in 2002.

Structural models have two equal features at the core: they assume that the value of the firm's activities, or asset value, randomly move through time, given an expected return and volatility (Leland, 2015); and that default occurs when than asset related value fall below a certain threshold, often called default barrier (Dionne and Laajimi, 2012). Nonetheless, and concerning the second feature, Dionne and Laajimi (2012) refer that default barriers can be divided into two categories, depending on when and on what triggers default:

i. the exogenous/endogenous dichotomy, on which exogenous models impose a pre-specified default barrier in time, such as Merton (1974) or Vasicek (1984); and endogenous models, that presume that equity holders chose when to default, to maximize its claim, such as Black and Cox (1976) or Leland (1994) model;
ii. the default event, since some assume that the firm fails whenever the asset value falls below the nominal of debt, such as Merton (1974), Vasicek (1984) or KMV, and other undertake the idea that firms fail default when only the cash flow is insufficient to face debts repayments, such as Anderson and Sundaresan (1996) .

The KMV model maintains the foundations of the Merton model intact, while adapting its assumptions and relying on empirical observations to better reflect the real-world dynamics. Besides, the emphasis of the KMV model is predict probability of default, whereas the Merton Model focus is on the debt's valuations (Kealhofer, 2003). In the KMV model, which consider a exogenous barrier, linked to the asset value, the default probability is solely determined by a leverage measure, the distance to default (DD), which consists on *"(…) the number of standard deviations of annual asset growth by which the asset level (or expected asset level at a given time horizon) exceeds the firm's liabilities*" (Duffie et al., 2007, p. 638). In fact, Crosbie et al. (2003) and Sun et al. (2012) refer three main steps to apply the model, starting on estimating asset value and volatility, in order to be inputted into the DD, which is then mapped into an empirical distribution function.

A leading alternative to the structural models, in the market based models, are the reduced form or intensity models, firstly developed by Jarrow and Turnbull (1995). In this type of models, there is no intent of considering the asset value of the firm, and the focus is rather on modelling other factors influencing default, but typically leaving aside the search for what exactly triggered the event. (Lando, 2009). In reduced from models, the default is associated with an unexpected or random event, and time to default is assumed to be a stopping time at a first jump generated by an independent Poison process, with a random intensity process (Pereira, 2013).

2.2.3 Hybrid Models

Shumway (2001) denominates the traditional accounting models as "static", since they estimate single period classifications, either in the form of a discriminant score (Beaver, 1966) (Altman, 1968) or probability (Ohlson, 1980; Zmijewski, 1984), resorting to multiple-period bankruptcy data. Given this timing mismatch, the estimates produced are biased and inconsistent, by ignoring the fact that firms change overtime. In order to overcome these issues, Shumway (2001), using a sample from 1962 to 1992, with approximately 300 bankruptcies, estimates a discrete time hazard model, which main advantage lays on specifically accounting for time. In this model, the dependent variable is the time spent by a firm in the healthy group, which allows to analyse how much time a firm is at risk before declaring bankruptcy, which may take years or only one, while capturing sudden deteriorations in the credit profile . Moreover, thorough exploiting each firm's time series data, it is possible to include not only annual observations as time-varying covariates, but also monthly and even daily data, such as macroeconomic variables. Besides, Shumway (2001) refers that since each year is considered as a separate observation, a hazard model has much more data available than the equivalent static model, which would produce much more efficient out of sample estimates. Shumway (2001) proves that the equivalent hazard version of Altman's (1968) and Zmijewski's (1984) outperforms the static/original one, while also reaching to the conclusion that a hazard model composed by 3 market and 2 accounting variables, an hybrid model, produces more accurate estimates

Following Shumway (2001), Chava and Jarrow (2004) develop a discrete hazard model using monthly and yearly observations, while accounting for industry effects. In their research, a model derived from monthly observations which uses industry effects, improves the accuracy at predicting bankruptcy, while also demonstrating that accounting variables add little predicted power when market variables are included.

By having as objective the confirmation of the general perceived idea that investors charge a premium for bearing default risk, Campbell et al. (2008) estimate a dynamic panel model using a logistic specification, following the econometric approach used by Shumway (2001) and Chava and Jarrow (2004). Campbell et el. (2008), in line with Chava and Jarrow (2004), consider monthly observations and make significant alterations to the existing explanatory variables in hybrid models, 8 in total, by inserting new or modifying the existing variables. Moreover, Campbell et al. (2008) use a wider indicator of failure than the previous authors, by not only including bankruptcy but also incorporating delisting of the stock exchange and/or firms awarded with a grade "D" (Default") by a leading CRA. By including the indicator "failure", Campbell et al (2008) pursue to capture either firms that perform so poorly that their stocks were delisted, a frequent event prior to bankruptcy, or firms that avoided filing for bankruptcy and negotiated with creditors out of the court. Campbell et al (2008) prove that, for different periods, either when predicting bankruptcy or predicting "failure", their model has superior explanatory power than a model which uses Shumway (2001) and Chava and Jarrow (2004) variables.

The hazard models are the latest generation of modelling and is dominating literature in the past few years (Duffie et al., 2007). Interestingly, many academics, by recurring to this econometric technique, tried to imbed in their models and studies the DD measure, present in the structural models, as an explanatory variable. Hillegeist et al. (2004) proves that a hazard model only containing the DD measure contains more significantly more information about probability of bankruptcy than the respective hazard model composed by accounting based measures present in Altman (1968) and Ohlson (1980); Bharath and Shumway (2008) and Campbell et al. (2008) proves that, in the presence of other explanatory variables, including leverage and volatility, the DD measure adds relatively little information; in other hand, Duffie et al.(2007) proves that the DD measure, when included with other market based variable, stock return and other 2 macroeconomic variables, S&P 500 returns and US interest rates, improves the predictive power of the model.

3. Objectives, Model and Data Selection, and Formulation of Hypotheses

3.1. Objectives of the paper

The objective of this paper is to compare two different types of model, one market based, the Moody's KMV, which falls under the structural model category, and a hybrid one, developed by Campbell, Hilscher and Szilagyi (2008), the CHS model. In that sense, we would be able to provide to anyone interested in autonomously estimate the probability of bankruptcy with a rationale choice.

The comparison will be made considering which one better discriminates the event "bankruptcy", taking into consideration a one-year horizon. The analysis will be done resorting to the Receiving Operating Characteristic (ROC) indicator, which helps to interpret the probabilistic forecasts for binary classification, "default" and "not default", between two or more models. The ROC curve allows to assess how much a model is capable of distinguishing classes and the details will be provided in the next section.

3.2. Model Selection

Firstly, it is important to state that an attempt to derive new models, by including more variables and/or changing coefficients, is not sought. Moreover, the choice among different types of model was not random.

Despite its intuition and easiness in terms of application for both private and public firm, an accounting-based model will not be object of study, given that:

i) The more traditional accounting-based models, such as Beaver (1966), Altman (1968), Ohlson (1980) and Zmijewski (1984) tend to be sample specific, given the econometric approach used to derive the models. Besides, and more concretely to Beaver (1966) and Altman (1968), the reduced number of firms considered further highlights on the sample specificity problem.

ii) Secondly, it is proved by academic researchers, such as Bharath & Shumway (2008), Campbell et al., (2008, 2011), Chava and Jarrow (2004), Das et al. (2009) and Trujillo-Ponce et al. (2014), that a mix of market based variables and accounting based models improve the accuracy of models.

As such we opted to choose hybrid models, developed by authors such as Shumway (2001), Chava and Jarrow (2004), Campbell et. al (2008), which consider not only both type of variables, but also use much more data to derive them, given the panel data feature, which, in theory, overpass the sample specificity problem of the "static" accounting models. The last feature is particularly important in our case, since, as previously stated, we do not seek models' adaptions to include different coefficients or variables, and our study can be deemed as an out of sample application of an hybrid model. The choice for the CHS model, developed by Campbell et. al (2008) was due to a better overall fitness in explanatory power than its equivalent hazard models, developed by Shumway (2001) and Chava and Jarrow (2004).

Last but not least, it was decided to test a theoretical model, based on market-based variables, which do not have the sample specificity constraint. The choice for a structural model, rather than a reduced form model, the other main division among market based category, is due to its superior theoretical attractiveness: structural models uses a capital structure to explain default probability, linking its value to the financial condition of an enterprise, whereas the reduced form models uses default intensity, an exogenous estimation. Among structural models, the choice fell for a proxy of the commercial model, Moody's KMV, using adaptions found in literature, as further explained in the next section. The Moody's KMV was chosen rather than the CreditGrades model, the other equivalent commercial model in terms of importance, due to the different objectives between them: whereas the aim of the KMV is to accurately model default probabilities, by making use of its default database, the goal of the CreditGrades model is rather measuring perfectly credit spreads and timely indicate when a firms' credit becomes impaired, by using historical market spreads (Finger, 2002).

3.3. Formulation of Hypotheses

Considering the objective of the thesis as well as the models in study, our results may fall in one of the two following hypotheses:

1. *None of the models differentiate the event of bankruptcy.*

In the case that both models are unable to distinguish between bankrupt and non-bankrupt firms, we would not be able to meet our goal of providing the best option available to anyone interested in autonomously estimate the probability of bankruptcy. Furthermore, that would not be in line with the current results found in literature: not only Tserng et al. (2011) and Agarwal and Taffler (2008), among others, resorting to the ROC analysis, proved that KMV model differentiates bankruptcy, but KMV model is also a widely commonly used tool to assess its probability. Moreover, and regarding the CHS model, Campbell et al. (2008) proved that its model, at predicting bankruptcy in one-year horizon, has a sound and suitable accuracy ratio of 86.2%, which indicates its differentiation power.

2. *One model is better at discriminating the event than the other.*

This could be either the case that:

2.1 *One model discriminates the event and the other does not*

2.2. *Both models distinguish bankrupt and non-bankrupt firms, but one has a better predictive power.*

In the case KMV model is superior, this would be a confirmation of the ideas followed by Hillegeist et al. (2004); Sun et al. (2012) and Vassalou and Xing (2004), which argue that a market-based model is superior to models containing accounting variables. Moreover, it would reconfirm the statement a theoretical model produces more accurate results than a sample derived model, such as CHS, in out of sample analysis. In the other hand, if CHS model is superior, this would prove the theoretical assumption followed by Shumway, Chava and Jarrow, CHS that a dynamic panel model, which uses a logit specification, indeed is capable of providing appropriate and fit out of sample estimates.

3.4. Data Selection

In order to address the discriminating power of both models, entities of two different types must be chosen: the "bankrupt" firms and the "non-bankrupt" or "healthy" firms. The "*Bloomberg"* Terminal was used for the purpose of selecting both type of firms, as well as for gathering all the inputs necessary to the application of the models, resorting to the respective Excel Add In, "Spreadsheet Builder". [2] The only exceptions were the risk free rate, which was directly collected from the "US Department of the Treasury" website, and data from the S&P 500 index, sprightly withdrawn from the Terminal.

The first restriction for the data selection was that firms must belong to the same country, to help our posterior analysis. In that sense, the option fell on US firms, as they are located in the country with the highest number of publicly listed companies. Besides, we opted to analyse companies' constituents of the Russell 3000 (RAY) index, which gather the 3000 largest US

[2] A list for all the Bloomberg's Spreadsheet Builder functions used to construct the variables and its respective definition as per the terminal, is provided in Appendix A.

traded stocks, representing about 98% of all US-Incorporated Equity Securities. The choice for the RAY index was due to high number of constituents, with different financial profiles and characteristics, which, in theory, could facilitate the process of finding bankrupt firms.

Given the above, we started by collecting all firms listed in RAY as of 31/12/2007, excluding firms belonging to the "Financial" and "Utilities" sector. After that, we have applied to that set of companies the *Bloomberg* function concerning corporate actions, "CACT", filtered by the event "bankruptcy", in the period 31/12/2007 to 31/12/2018. The result displayed 166 entities, which either filled for the chapter 7 or 11 according to the US Bankruptcy Legislation. Chapter 7 is known as "liquidation" bankruptcy, and firms that fill for this chapter already are past the stage of reorganization and must sell assets to pay creditors. On the other hand, Chapter 11 is commonly referred as "reorganization" bankruptcy, as it allows firms to reorganize its debt, by changing terms of payments and to remerge as a healthy organization. Only 5 out of the 166 entities in our original sample filled for the most serious form, the chapter 7.

Furthermore, we have found 35 entities for which no market data was available, thus making impossible to apply the models, leading to the idea that many were delisted even prior to the declaration of bankruptcy, a fact indicated by Campbell et al. (2008). Moreover, we found out that 13 entities filed for chapter 11 more than once, indicated that the process of reorganization was not the best one, and on the second declaration, market or accounting data were not available. Therefore, after excluding entities for which information was not available, given the two above mentioned situations, we were left with 118 "bankrupt" entities for which the 2 models were possible to apply. In order to apply the models, data from the year preceding the year of the declaration was selected, i.e., if a firm declared bankruptcy in 2008, data from 2007 was retrieved, as the prediction at one year horizon is sought.

The criteria to select "healthy firms" was that for each correspondent "bankrupt" entity, it would have to correspond at twice as many stable firms. Since we are evaluating the model's ability to distinguish the event of bankruptcy, it is logical to choose healthy firms with similar characteristics to those that went bankrupt. The sample for healthy firm fell on those that, throughout the sample period from 31/12/2007 to 31/12/2018 were always listed on RAY, in order to avoid the issue of missing information found before. Moreover, and resorting to *Bloomberg*, information on the Net Income and Market Cap for the last trading day of each year was selected, to each firm. With that information, and using Excel, we randomly selected firms between the first and third quartile, leaving aside firms with such high profitability and capitalization levels that are not comparable to our bankruptcy sample.

If a selected “healthy firm”, with profitability and capitalization levels between the first and third quartile, as of, for example, 31/12/2007, the data needed to apply the models was from 2007. This allowed for the comparison to those firms that went bankrupt in 2008, for which 2007’s figures were also selected

In the end, we were left with 236 healthy firms, with the distribution of bankrupt and healthy firms, per year, shown in Table 3.1. The more representative year in the sample is 2009 (27.1%), as many firms declared bankruptcy given the rough economic context, started in the last months of 2008, with the stock market crash and financial crisis. The Ticker Codes of the entities, respective year and Type can be found in Appendix B.

Table 3. 1 Number of Bankrupt and Healthy Firms, per year

Year	**Type**		**Total Number**
	Bankrupt (number, %	**Healthy (number, %**	
2010	**6**	**12**	**18**
	5.1%	*5.1%*	*5.1%*
2009	**32**	**64**	**96**
	27.1%	*27.1%*	*27.1%*
2010	**8**	**16**	**24**
	6.8%	*6.8%*	*6.8%*
2011	**10**	**20**	**30**
	8.5%	*8.5%*	*8.5%*
2012	**13**	**26**	**39**
	11.0%	*11.0%*	*11.0%*
2013	**10**	**20**	**30**
	8.5%	*8.5%*	*8.5%*
2014	**6**	**12**	**18**
	5.1%	*5.1%*	*5.1%*
2015	**11**	**22**	**33**
	9.3%	*9.3%*	*9.3%*
2016	**10**	**20**	**30**
	8.5%	*8.5%*	*8.5%*
2017	**4**	**8**	**12**
	3.4%	*3.4%*	*3.4%*
2018	**8**	**16**	**24**
	6.8%	*6.8%*	*6.8%*
Total	**118**	**236**	**354**

4. <u>Methodology</u>

4.1. Moody's KMV Model

In line with what we previously stated, the Moody's KMV is an extension of the Merton (1974) structural model, adapting it to more realistic assumptions. As such, the first part of this section will present the methodology behind the Merton Model (1974), while the second shows how the Moody's KMV extends the scope of the original model and how can it be applied by practitioners and academics.

4.1.1 Merton Model (1974) Foundations

Merton (1974), in order to currently pricing corporate debt, resorted to the option pricing theory, which allowed to account for default as an input. Merton (1974) relied on some basic assumptions in terms of capital structure and on the process of bankruptcy, where a firm's assets have only two types of claimants: Bondholders (Dt), which solely possess zero-coupon bonds (ZCB), maturing at time T, and Equity holders (Et), which only have common stocks. As such, the value of the firm's assets (Vt) can be interpreted as the sum of the company's debt (Dt) and the company's equity (Et).

$$Vt = Et + Dt \tag{1}$$

At the time that the ZCB matures (T), the firm is committed to pay its respective face value (Xt). If at T, the firm is not able to do so, it will declare bankruptcy where the bondholders will take over the firm's assets (Vt), while the equity holders will receive nothing. Considering this simple case of capital structure and bankruptcy process, which only involves the passage of the ownership from shareholders to bondholders, it is possible to evaluate a firm's equity through an European Call Option on the firm's asset: at T, the equity holders have the right to buy back the firm's asset (Vt), by paying to the bondholders the face value of the ZCB (Xt), equivalent to the strike price in option pricing theory.

Since Equity can be seen as a European Call Option on the Firm's Asset, the put call parity from Black and Scholes (1973) follows:

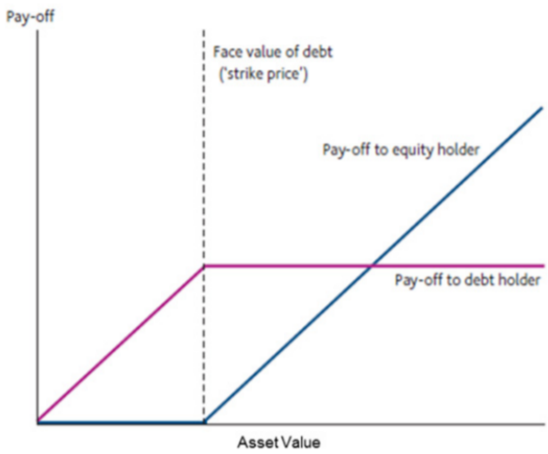
$$Vt + Pt = Xt + Et \Leftrightarrow Vt = Et + (Xt - Pt) \tag{2}$$

From the above equation, and comparing it to (1), one may reach to the conclusion that the company's debt (Dt) can be viewed as the difference between a default free debt (Xt) minus a put option (Pt) with a strike price of precisely Xt.

$$Dt = Xt - Pt \tag{3}$$

Analysing the payoff at maturity and recurring to Figure 4.1 we may perceive that both claimants are better off if the asset value (Vt) is greater than Xt, which is the same to say that the call option was exercised.

Figure 4. 1 Payoff of bond and equity holders at T



Source: Chatterjee, S., & Blake, A. (2015, pp.15). Modelling credit risk. Bank of England, Centre for Central Banking Studies.

- If Vt>X: the call option is exercised, the bondholders will receive the face value of debt (Xt) and the equity holders receive the difference between the asset value (Vt) and the face value of debt (Xt).
- If Vt<X: the call option is not exercised and the put option is, meaning that the firm default. Given the limited liability nature of equity, the equity holders receive nothing, whereas the bond holders, receives the asset value (Vt), which nonetheless is lower than what it was promised (Xt). This downside protection is provided by the implicit put option that bondholders benefit from. (Chatterjee and Blake, 2015)

Apart from the capital structure and bankruptcy rules, Merton (1974) expand the assumptions in order to formulate a model:

1. There are no transaction and bankruptcy costs taxes
2. Trading in assets is continuously in time.
3. There is a sufficient number of investors, so there is the common belief among them that one can sell and buy an asset as much as he/she wants
4. No difference in the borrowing and lending rates

5. Short sales of all assets are allowed, with full use of the proceeds
6. The proposition I of Modigliani-Miller theorem, which states that the value of the firm is independent of its capital structure, is assumed.
7. The term structure of interest rate is flat and certain, for instance, the price of a risk-free bond which pays 1 dollar at time T is given by:

$$P(r,t,T) = e^{-r(T-t)} \tag{4}$$

where r stands for the risk-free rate.

8. The dynamics of the asset value are described by a diffusion-type stochastic process, a Geometric Brownian Motion, defined by:

$$dVt = (\mu V - C)dt + \sigma Vtdz \tag{5}$$

where Vt is the asset value, μ is the expected rate of return of the asset value, equal to the risk free rate (r) and σ^2 is the immediate variance of the that return, both assumed constant; C is the total dollar pay-outs by the firm per unit of time , o either equity holders or bondholders and *dz* is a is a standard Gauss-Wiener process.

Since Equity can be seen as a European Call Option on the firm's asset, and under the assumption that a firm's asset value follows a Geometric Brownian motion, following the Black and Scholes (1973) formula, Equity Value is given by:

$$Et = Vt * N(d1) - X * e^{-r(T-t)} * N(d2) \tag{6}$$

in which d1 and d2 stands for:

$$d1 = \frac{\ln\left(\frac{Vt}{X}\right) + (r + 0.5 * \sigma_v^2) * (T-t)}{\sigma_v * \sqrt{(T-t)}}) \tag{7}$$

$$\begin{aligned} d2 &= \frac{\ln\left(\frac{Vt}{X}\right) + (r - 0.5 * \sigma_v^2) * (T-t)}{\sigma_v * \sqrt{(T-t)}} \\ &= d1 - \sigma_v * \sqrt{(T-t)} \end{aligned} \tag{8}$$

where:

- Et= Equity Market Value
- Vt= Asset Market Value
- Xt= Face Value of the ZCB (Strike Price)

- r= Risk Free Rate
- σ_v= Asset Volatility
- T-t= Maturity
- N(.) stands for the standard normal density function

Resorting to the payoff structure on Figure 1, the probability of default is linked to the likelihood that the put option is exercised rather than the call option. Therefore, and under Black and Sholes approach, the probability that the put option is exercised is simply given by:

$$PD\ (T,t) = N\left(-\frac{\ln\left(\frac{Vt}{Xt}\right) + (r - 0.5 * \sigma_v^2) * (T - t)}{\sigma_v * \sqrt{(T - t)}}\right) = N(-d2) \qquad (9)$$

The term d2 is commonly referred as the Distance to Default (DD), that is, the number the number of standard deviations that the asset value (Vt) is away from the face value of the ZCB (Xt), considering its respective volatility (σ_v), assumed to be constant over the period (T-t). and expected rate of return, which is equal to the risk-free rate (r).

4.1.2 Moody's KMV Extension

The Moody's KMV model main input is the Expected Default Frequency (EDF), which is reflected into the probability of default of a certain obligor, in the next year [3]. Crosbie et al. (2003) describes three essentials steps to determine the EDF of a firm: 1) estimate asset value and volatility; 2) calculate the Distance to Default (DD) and 3) calculate the probability of default. The next section will describe how those steps are taken, while emphasizing the differences between KMV and Merton model, utilizing two papers which describe the original KMV model: Crosbie et al. (2003) and Sun et al (2012).

a) Estimate the Asset Value and Volatility

Looking at Equation (9), which is the probability of default in the original Merton model, we may observe that, given the assumption of simple capital structure, the variable the Asset Value (Vt) would be simply the sum of the Fixed Asset face value of the ZCB and Equity. Moreover, and under a risk neutral world, the asset volatility (σ_v), would be linked to the risk-free rate (r), as the expected asset return would be precisely equal to that rate.

[3] The Moody's KMV EDF can be applied to predict default in more than one-year time, by applying a specific term structure. The scope of this thesis is predicting default in one-year horizon; thus, the term structure will not be analysed. Please refer to or Crosbie et. al (2003) or Sun et. al (2012) for further information on the topic.

In fact, Moody's KMV model extends the capital structure of Merton by, including all sorts of short term and long-term liabilities, as well as common, preferred, and convertible stock. AS such, under a more complex capital structure, estimating the asset value and the respective volatility is the first step to calculate the PD, which challenge was not firstly identified by Moody's.

Jones et al. (1984), in order to estimate asset value and volatility, propose that under the Ito's Lemma, the instant standard deviation of equity (σ_E) is given by:

$$\sigma_E = \sigma_v * \frac{\partial E}{\partial V} * \frac{Vt}{Et} \tag{10}$$

Jones et al, (1984) suggest simultaneously solving Equation (10) and the European call option formula (Equation 6), to estimate Vt and σ_v. However, d1 and d2 both depend on the non-observable variables. So, to arrive to a numerical solution, Jones et al. (1984) propose using the Markov chain to approximate solutions or the method of finite differences. Nonetheless, Moody's KMV does not rely in this method, as not only market leverage moves around far too much for Equation (10) to produce reasonable results (Crosbie et al., 2003), but also due to the fact that equity volatility can be estimated in different ways and it is difficult to estimate for recent listed firms (Sun et al., 2012).

As such, Moody's KMV estimates the asset value and volatility by an iterative procedure, which uses an initial guess of σ_v to calculate a set of Vt, from the rearranging of the call option formula. The returns of the set of Vt is used to calculate a new σ_v, which will be used to compute a new set of Vt, that in turn generate a new set of returns and a new σ_v. The procedure continues until two consecutive iterations converge in terms of σ_v value that will then be used to calculate the final set of Vts

Unfortunately, Moody's does not disclose what should be the initial value, and two different approaches can be found in academic literature. Vassalou and Xing (2004) sets the initial value equal to the equity volatility σ_E, which may be difficult to estimate, considering the constraints pointed out by Sun et. al (2012). In other hand, Löffler and Posch (2007) use other initial value of σ_v, which respective procedure is described next, as it is the one utilized in this paper:

1. The initial values for the Asset Value (V) are obtained through the sum of daily market value of equity (E) and quarterly book value of liabilities (X). The book value of liabilities

(X) is considered as current liabilities plus half of the non-current portion. This result in a set of around 260 daily values of Vt, in line with the trading days.

2. With the daily values of V, compute the respective log asset return, which respective standard deviation is used for setting the initial value for the asset volatility (σ_v)
3. Compute daily values for d1 and d2, through equations (7) and (8), assuming the asset volatility calculated in the previous step, while also considering the other inputs. (V, X, and r).
4. Input d1 and d2 on the calculation of a new set of daily asset value (V), following a rearranged Black and Scholes (1973) call option formula:

$$V = \frac{E + X * e^{-r(T-t)} * N(d2)}{N(d1)} \tag{11}$$

5. Use the new set of daily asset value to compute a new set of log asset return to compute a new asset volatility.
6. Repeat the process until convergence. In order to check for convergence, the changes in the asset value, from one interaction to the next, is evaluated: if the sum of squared errors between the asset value of two consecutive interactions is less than $10^{-10,}$ the process is stopped.

After applying its iterative approach, Moody's uses large property database, and adjusts the final value asset volatility, by combining it in a Bayesian way with the country, industry, and size averages, to produce a more powerful and accurate estimate (Crosbie et al, 2003).

b) Calculate the Distance to Default (DD)

In the original Merton model, the default point, that is, the threshold of liabilities that the firm's asset must hit to be considered defaulted, was simply considered as the face of value of the ZCB. Nonetheless, in the real world, where different classes of liabilities exist and that change over time, other default point must be considered. Sun et al (2012) considers its measurement as "tricky", whereas Crosbie et al. (2003) assumes that it is indeed a random variable. Firms tend to change the level of its liabilities near default, in order to stay afloat (Crosbie et al., 2003). Moreover and by analysing Moody's large database, it is possible to observe that firms may not default when the level of its liabilities it is higher than the asset value, and in other hand, some default even before the level of asset is higher than the short-term liabilities (Sun et al., 2012). In order to partially address these dynamics, for non-financial firms, the Default Point (X*) is set at 100% short term liabilities plus 50% of long-term liabilities, which we will use, since we are only dealing with these types of firms. For financial firms, a percentage of total adjusted liabilities is used, depending on the subsector.

Apart from adapting the default point, Moody's KMV calibrates the expected rate of return of assets (μ) to account for possible cash outflows to service debt, dividends, etc. (Crosbie et al, 2003). In literature, the estimation of the expected rate of return is vast: Löffler and Posh (2011) uses the CAPM to derive it; Hillegeist et al. (2008) adapts it to include dividends, which is a way of cash pay-outs included in the actual Moody's KMV model; Bharath and Shumway (2008) uses the previous year stock return to measure, bounded between the risk free rate and 100%, Vassalou and Xing (2004) uses the mean of the change in the final set of the asset value. In this paper we will use the Vassalou and Xing (2004) approach, for simplicity reasons.

As previously stated, the DD in the original Merton model was equal to the parameter d2. Given the inclusion of a different default point (X*) and expected rate of return (μ), the DD in Moody's KMV model is given by:

$$DD = \frac{\ln\left(\frac{Vt}{X^*}\right) + (\mu - 0.5 * \sigma_v^2) * (T - t)}{\sigma_v * \sqrt{(T - t)}} \tag{12}$$

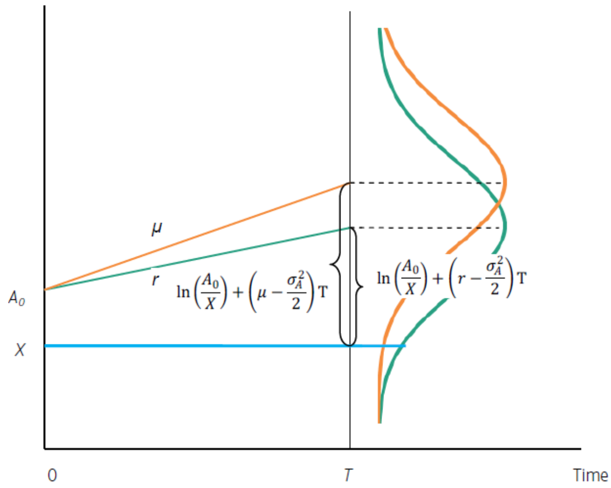
Since the DD is measured using the expected rate of return (μ), one might consider better if a proxy of this variable is used to estimate the asset value and volatility. Following Sun et al. (2012, pp.23) "(…) *asset volatility of the firm is not changed by adjustments made to the firm's asset value drift, as per the Girsanov Theorem",* which allows the usage of the risk-free rate (r), rather than a proxy of μ, to estimate asset value and volatility, without jeopardizing the results.

c) Calculate the Probability of Default.

In the Merton model, as a result of the Geometric Brownian Motion to model the asset value's dynamics, the mapping of the DD follows a normal distribution. However, Moody's KMV instead of approximating the DD with a normal distribution, it constructs an empirical one, based in historical data between the registered DDs and default rates. As per Crosbie et al (2003), the empirical distribution captures the uncertainty around liabilities adjustments that usually occur near default, issue referred back in the second step (calculating the DD). As a result, Moody's Empirical distribution has much wider tails than the Normal Distribution, and for instance, a firm with a DD of 4 maps to 0.4% of PD with the Empirical Distribution, whereas using the Normal Distribution, the PD is essentially 0% (Sun et. al 2012) (Crosbie et al, 2003).

Figure 4.2 demonstrates the differences between Moody's KMV and Merton Model, in terms of inputs and distributions used:

Figure 4.2 Comparison between Moody's KMV and Merton Model



Source: Sun et. al (2012) *Public Firm Expected Default Frequency (EDFTM) Credit Measures: Methodology, Performance, and Model Extensions,* Moody's Analytics Report.

Given not only the fact that the empirical distribution is Propriety Information, but also the non-disclosure on how the asset volatility is adjusted in terms of sector, size, country, etc, make it impossible to any practitioner to directly apply the Moody's KMV model. Nevertheless, in Bharath and Shumway (2008, p. 1346) words, it is possible to apply "feasible" Moody's KMV models, and this is what we seek in this paper. We intend to compute our DD measure and map it in a Normal Distribution, like Vassalou and Xing (2004) or Tserng et al. (2011), as it is the best option available. However, other academics, such as Bharath and Shumway (2008), Campbell et al. (2008), Duffie et al. (2007) or Hillegeist et al (2004) prefer to focus in the ranking ability of the DD measure, since if the measure ranks firms accurately, it is straightforward to map the DD using historical data, like Moody KMV does.

A practical example of the application of the KMV model, including the application of the Interactive Approach described in section 4.1.2 a.) can be found in Appendix C.

4.2. CHS Model

By having as a starting point the confirmation of the general idea that investors charge a premium for bearing default risk, Campbell et al (2008) estimate a *"(...) dynamic panel model using a logistic specification (...)"*(Campbell et al., 2008, p. 2900), following the econometric approach used by Shumway (2001) and Chava and Jarrow (2004). By using the default probabilities derived from their model, they allocate stocks within different portfolios according to its distress risk. Campbell et al. (2008) reach the conclusion that the equity market had not correctly priced distress risk, as financial distressed firms do not have high average returns. In this paper, given its objective, the focus will be in the construction of the model and we leave aside the asset pricing feature.

As previously said in the literature review section, Campbell et al. (2008), in order to build a measure of financial distress, uses two indicators of: i) a narrower one, "bankruptcy", which only includes firms that have filed for bankruptcy under chapter 7 and/or 11, according to the US Legislation; ii) a wider one, "failure", considering not only bankruptcy but also firms that have delisted and/or have a "D" (Default) grade by a leading CRA. The sample period, referring to the indicator "bankruptcy", is from 1963-1998, while for the broader indicator "failure", the sample period is from 1963-2003.

Campbell et al. (2008) estimate the dynamic panel model using a logit specification, following the idea developed by Shumway (2001) and also Chava and Jarrow (2004). According to Shumway (2001), hazard models or dynamic panel models can be estimated using a logit program, and the statistical inferences are possible, since both hazard and logit models have the same asymptotic likelihood function. Moreover, and following the procedure of Chava and Jarrow (2004), that improved accuracy in predicting bankruptcy, Campbell et al. (2008) consider monthly rather than yearly observations, extending even further the span of them, with predictor variables up to 1.7 million "firm months". Giving the logistic specification, Campbell et al (2008) define a firm's probability of failure over the next month as:

$$P_{t-1}(Y_{it} = 1) = \frac{1}{1 + \exp(-\alpha - \ \beta_{xi,t-1})} \tag{13}$$

Where $Y_{i,t}$ equals 1 if the firm fails in month t, $xi, t-1$ is a vector of explanatory variables known at the end of the previous month and $\alpha + \beta_{xi,t-1}$ is a linear combination of those variables.

a) Explanatory variables: modifications and introductions

Campbell et al. (2008) expand the existing literature in terms of explanatory variables, by modifying some of them and inserting new ones. In terms of the profitability and leverage ratios used either by Shumway (2001) and Chava and Jarrow (2004), it is assessed weather or not dividing net income and total liabilities by the market value of assets rather than the book value of assets, helps improving the explanatory power. The rationale behind these modifications is that those new variables are more sensitive to new information about firms' prospects, given the partial market-based nature. Thus, Net Income to Total Assets (NITA) and Total Liabilities to Total Assets (TLTA) variables used in previous papers are substituted into two new ones: The Net Income to Total Market Value of Assets (NITMTA) and Total Liabilities to Total Market Value of Assets (TLMTA). Moreover, they add further time lags on the variables, reaching to the conclusion that lagged variables regarding profitability (NITMA) and stock returns (EXRET) enter significantly in the regression, weather lagging other variables do not, giving place to two new variables, which replace the non-lagged ones (NITMAAVG and EXRETAVG). Indeed, one might expect that a long history of losses and / or successive decline in stock market can bring added value in predicting bankruptcy.

Additionally, 3 news explanatory variables are introduced: The Market to Book Ratio (MB), Cash and Cash Equivalents to Market Value of Assets (CASHMTA) and the Price per share (PRICE). The MB is introduced to evaluate the relative value of equity placed either by shareholders or accountants, on which a higher value indicates a discrepancy between these evaluations, eventually meaning a sign of distress. CASHMTA is added to capture liquidity: a higher CASHMTA means the company has liquid assets available to make interest payments and possibly avoiding bankruptcy. Last but not least, Campbell et al. (2008) introduce PRICE as a variable, since exceptional lower prices might be relevant, as stock exchanges usually delist firms with prices lower 1$, which is line with the fact that delisting is present in the indicator "failure".

b) Models estimated and explanatory power

Campbell et al. (2003) estimate six different hazard models, considering the indicators "failure" and "bankruptcy", for different periods and using different variables, either modified or introduced. In table 4.1, the first three columns represent models estimated using Shumway (2001) and Chava and Jarrow (2004) variables, while the last three columns represent models estimated using Campbell et al. (2008) modified and introduced variables.

Table 4. 1 Models estimated by Campbell et al (2008) and respective explanatory power

Variables used	Shumway (2001) and Chava et al (2004)			Campbell et al (2008)		
Indicator	Bankruptcy	Failure	Failure	Bankruptcy	Failure	**Failure**
Sample Period	1963-1998	1963-1998	1963-2003	1963-1998	1963-1998	**1963-2003**
Mc Fadden Pseudo R^2	0.26	0.258	0.27	0.2999	0.296	**0.312**

Source: Campbell, J. Y., Hilscher, J., & Szilagyi, J. (2008). In search of distress risk. *Journal of Finance, 63(6)*, 2899–2939

The way Campbell et. al. (2008) asses the efficiency of models is through the McFadden's pseudo R^2 coefficient, on which higher the value, the better the explanatory power. As such, and from the above table, one can observe that the model estimated through the usage of the broader indicator "failure", during the period 1963-2003, with the variables proposed by Campbell et al. (2008), has the greater explanatory power among the 6 models estimated, in what is named as the "best model".

Campbell et al. (2011), in a revised paper and when estimating a model with the indicator failure for an extended period of 1963-2008, demonstrate that the Pseudo R^2 increase up to 31.6% and the accuracy ratio was 95.5% on the best "model" [4]. The accuracy ratio compares the number of correct predictions, that is, includes pairs with high probabilities followed by failure and pairs with low probabilities and followed by subsequent "survivals", divided by the number of incorrect predictions.

c) Construction of the Variables

The CHS model is composed by 8 variables (excluding the constant), 4 of which directly dependent on accounting-based measures (NITMAAVG, TLMTA, CASHMTA, MB) and 4 solely dependent on market-based indicators (EXRETAVG, SIGMA, RSIZE, PRICE)

As per previously referred, Campbell et al (2008) (2011) decide to scale the accounting based measures of profitability, leverage and liquidity, by the respective Market Value of Assets (MVA), which is simply given by the sum of the Book Value of Liabilities and the Market Capitalization. Thus, NIMTA, TLTMA and CASHMTA are given by:

$$NIMTA = \frac{Net\ Income}{MVA} \tag{14}$$

$$TLMTA = \frac{Total\ Liabilites}{MVA} \tag{15}$$

[4] The accuracy ratio analysis was only introduced in the last version of the paper: 2011, with no comparison made with the other models besides the best one.

$$CASHMTA = \frac{Cash\ and\ ST\ Investments}{MVA} \tag{16}$$

Moving on to the last variable dependent on accounting measure, MB is simply the division between the Market Capitalization and the Book Value of Equity ("BE"). Nonetheless, the Book Value of Equity is adjusted for small values, which may result in large and misleading values of MB. When BE is negative, the value is replaced by 1, before taking the following transformation:

$$Book\ Value\ of\ Equity\ (Adjust) == E + 0.1 * (Market\ Cap - BE) \tag{17}$$

As a result, the MB is given by:

$$MB = \frac{Market\ Capitalization}{Book\ Value\ of\ Equity\ (Adjust)} \tag{18}$$

Concerning solely market-based variables, EXRET is the monthly log excess return of each firm's stock price relative to the well-known S&P 500, whereas the RSIZE is measured by the log of each firm market capitalization relative to the total market capitalization of SP&500. Therefore, EXRET and RSIZE are given by:

$$EXRET = \ln(1 + \ return\ firm_{1m}) - \ln(1 + return\ S\&P\ 500_{1m}) \tag{19}$$

$$RSIZE = ln\left(\frac{Firm's\ Market\ Capitalization}{Total\ S\&P\ 500\ Market\ Capitalization}\right) \tag{20}$$

The variable SIGMA, that measures the volatility of daily equity returns, is computed as the annualized 3 months standard deviation centred around 0, where N is the number of days of the last three months.

$$SIGMA = \sqrt{\left(252 * \frac{1}{N-1} * \sum_{daily} r^2\right)} \tag{21}$$

Furthermore, and as previously stated, Campbell et al. (2008) found out that NIMTA and EXRET lagged variables enter significantly in the regression. Thus, they impose geometrically declining weights in each lag, which gives more relative importance to recent values. NIMTA is lagged quarterly, and EXRET monthly. As a result:

$$NIMTAAVG = \frac{1-\emptyset^3}{1-\emptyset^{12}} * \left(NIMTA_{last\ quarter} + \cdots + \emptyset^9 NIMTA_{first\ quarter}\right) \tag{22}$$

$$EXRETAVG = \frac{1-\emptyset}{1-\emptyset^{12}} * (EXRET_{last\ month} + \cdots + \emptyset^{11} EXRET_{first\ month}) \tag{23}$$

d) Forecasting at longer horizon

The hazard model estimated by Campbell et al. (2008; 2011), given the monthly observations, predicts bankruptcy over the next month. However, it is admitted that an investor's focus will certainly not only be immediate failure but is also interested in knowing in advance firms that are most likely to fail. Analogously, "*Although probably quite accurate, it may not be useful to predict a heart attack with a person clutching their hand to their chest*" (Campbell et al., 2011, p. 2)

As such, and using once again a logit specification, Campbell et al (2008; 2011) estimate the conditional probability of bankruptcy in 6 months, 1, 2 and 3 years, allowing the coefficients on the variables to vary depending on the horizon.

Logically, as the horizon increases, the coefficients and its respective significance levels, as well as the overall fit of the regression, decline. Table 4.2 contains the information on the coefficients and overall fitness measure present in the revised paper, Campbell et al (2011):

Table 4. 2 Forecasting at Longer Horizon: Variables Coefficients and Overall Fitness

		1 month	1 year	3 years
Variables and Coefficients	NITMAAVG	-29.00	-20.12	-11.93
	TLMTA	3.51	1.60	0.73
	CASHMTA	-2.49	-2.27	-1.85
	EXRETAVG	-8.02	-7.88	-3.50
	SIGMA	1.69	1.55	1.43
	RSIZE	0.138	-0.005	-0.133
	MB	0.05	0.07	0.115
	PRICE	-0.974	-0.09	0.219
	CONSTANT	-8.63	-8.87	-10.03
Model's Fitness	**Pseudo R^2**	**0.316**	**0.118**	**0.041**
	Accuracy Ratio	**0.955**	**0.862**	**0.737**

***Source**:* Campbell, J. Y., Hilscher, J., & Szilagyi, J. (2011). Predicting Financial Distress and the Performance of Distressed Stocks. *Journal of Investment Management*, Vol. 9, 1–21.

In one month, horizon, all variables are statistically significant (at 1%) and almost all enter with the expected sign: firms with lower profitability, higher leveraged, with less liquidity, smaller, with more volatile returns and with lower price per share are more likely to fail. However, RSIZE enters with a positive sign, which is counterintuitive, but this is likely to be due to the high correlation with PRICE. Nonetheless, at one- and three-years horizon, the sign is positive and intuitive: the larger the firm, the less likely to fail.

At one and three-years horizon, the variables enter with the expected sign and are statistically significant at 1%, with the exception being PRICE: at one year it is not statistically significant at 1%, but only at 5%, while at three years, it is again statistically significant at 1%, but enters with a contra intuitive positive sign. Again, Campbell et al. (2011) justifies this fact with the high level of correlation with RSIZE and suggests the possibility of unmodeled nonlinearities. In terms of overall fitness, both pseudo R^2 and accuracy ratio decrease with longer horizon. Nonetheless, with an accuracy ratio of 86.2% and a pseudo R^2 at 11.8% (1-year horizon), the predictability ability is still high and acceptable.

Following table 4, the linear combination among all variables and respective coefficients, for estimating failure at one year, as per the revised paper Campbell et al (2011) is given by:

$$Y = -20.12*NIMTAAVG + 1.60*TLMTA - 2.27*CASHMTA - 7.88*EXRETAVG + 1.55*SIGMA - 0.005*RSIZE + 0.07*MB - 0.09*PRICE - 8.87 \quad (24)$$

The linear combination presented in equation (24) is directly applied into the logit specification present in equation (13) in order to be transformed into a probability of distress in one-year horizon. For a practical example on the application of CHS model, please refer to Appendix D.

4.3. Receiver Operating Characteristic (ROC)

The Receiver Operating Characteristic (ROC) curve statistical technique was first utilized during World War UU by the US army, that resorted to this measure to improve the rate of detection of enemy's aircraft. Since then, it has been utilized in a wide range of fields, such as medicine, psychology and more recent in the credit risk modelling.

ROC curve analysis resorts to a contingency table or confusion matrix in order to summarize the performance of a model in its ability to distinguish between two different classes. In credit risk, the ROC curves may be put in place to assess the discriminatory power concerning bankruptcy's prediction. As such, a contingency table or confusion matrix measures the number of predicted bankruptcies (or non-bankruptcies) and compare those with the actual number of bankruptcies (or non-bankruptcies).

Based on Table 4.3, the errors that a model can produce are the False Negatives (FN), when there is predicted non bankruptcy and the company actually goes bankrupt, also named as Type I error; and False Positives (FP), where there is predicted a bankruptcy and the company does not fail, also known as Type II error. A perfect model would have zero cases of FP and FN, indicating that the total number of bankrupt firms would fit in the True Positive (TP) cell, whereas the total number of non-bankrupt firms would fall in the True Negative (TN) cell.

Table 4. 3 Contingency Table / Confusion Matrix

		Predicted Class	
		Non Bankruptcy	**Bankruptcy**
Actual Class	**Non Bankrputcy**	True Negative (TN)	False Positive (FP)
	Bankruptcy	False Negative (FN)	True Positive (TP)

Considering the confusion matrix, it is possible to compute the True Positive Rate (TPR), the True Negative Rate (TNR), the False Positive Rate (FPR) and the False Negative Rate (FNR):

$$TPR = \frac{True\ Positives\ (TP)}{True\ Positives\ (TP) + False\ Negatives (FN)} \tag{25}$$

$$TNR = \frac{True\ Negatives\ (TN)}{True\ Negatives\ (TN) + False\ Positives\ (FP)} \tag{26}$$

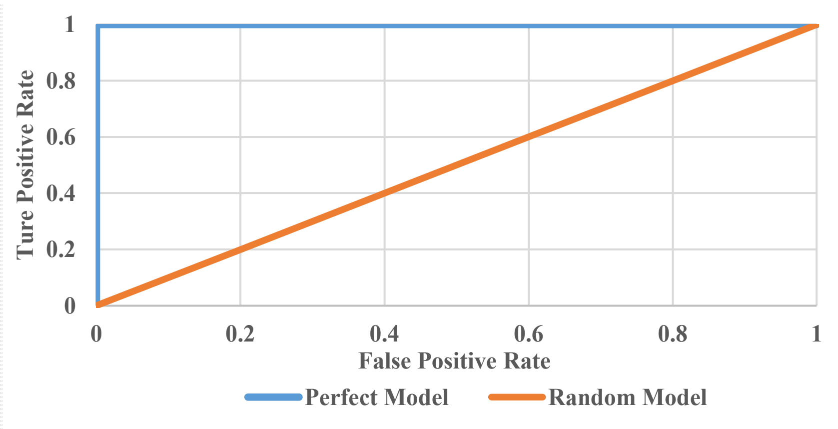
$$FNR = 1 - TPR = \frac{False\ Negatives\ (FN)}{True\ Positives\ (TP) + False\ Negatives (FN)} \tag{27}$$

$$FPR = 1 - TNR = \frac{False\ Positives\ (FP)}{True\ Negatives\ (TN) + False\ Positives\ (FP)} \tag{28}$$

The TPR is often called as "Sensitivity", while the TNR is frequently denominated as "Specificity". In our case, a perfect model would a Sensitivity and Specificity equal to 1, as there were no false classifications. Sensitivity and Specificity may be calculated considering a particular cutoff point, that is, a specific level of probability that differ bankrupt and non-bankrupt firms. However, and considering as example our study, as we consider different cutoff points, different relatives performances will be assessed, i.e., cut off "x" might result in a TPR and TNR higher for KMV model rather than CHS model, whereas cut off "y" might result in a TPR and TNR higher for CHS model rather than KMV model. Stein (2007) argues that contingency tables and indices derived from them, such as the ratios above demonstrated, can be challenging due to the arbitrary nature of the cutoff points.

The ROC curve overpasses the arbitrary issue, by plotting the FP rate on the *x*-axis against the TP rate in the *y*-axis, for all the possible probabilities' cutoff points. Figure 4.3 shows an example of a ROC curve graph. The graph is always plotted in a square, as both axes are bounded in the area [0,1]. Moreover, the graph always shows two extreme plots: i) the plot (0,0), which describes the scenario of a model predicting all entities as non-bankrupt, meaning a specificity of 1 (or FPR of 0) and sensibility of 0 ; ii) the plot (1,1) which describes the scenario that the model predicts all entities as bankrupt, meaning a specificity of 0 (or a FPR of 1) and a sensibility of 1. After all the plots being computed in the *x* and *y* axis, those are joined by interpolation in order for a curve to be created. In figure 4.3, two different curves are displayed: one referring to a perfect model (in blue) and other concerning a random model (in orange)

Figure 4. 3 ROC curves: perfect model and random model



From Figure 4.3 it is possible to infer that the a perfect model always yield a TPR of 1 a FPR of 0, for each cutoff point, thus the curve *"(...) runs vertically from the (0, 0) point to the (0, 1) point and then horizontally to the (1, 1) point of the square."* (Tourassi, 2018). Similarly, the curve of a random model produces as many true positives as of false ones, for each cutoff point, corresponding to the diagonal. In practice, a model will produce a curve that lies between the area of the random and perfect model, and the closer the curve to the upper left corner, the better the discriminatory power of the model, as it produces more TP than FP.

When trying to compare two or more models, the ROC curves produced may be very similar to each other. As such, it is complicated to reach to any conclusion by just visualizing the curves. In order to overpass this issue, it is usually computed the Area Under the Curve (AUC), to objectively measure the ability of a model to distinguish between classes. The AUC formula is as follows:

$$AUC = \int_0^1 TPR(FPR)d(FPR) \tag{29}$$

Referring back to Figure 4.3, and resorting to Equation 29, the AUC for a perfect model is equal to 1, while the AUC for a random model is only 0.5. Hosmer and Lemeshow (2000) concludes that a model with in AUC that falls in the interval of 0.7 to 0.79, the model has a an acceptable discrimination; if AUC is between 0.8 and 0.89, the model is deemed as having an excellent discrimination and if the AUC is greater than 0.9, the model is considered to have an outstanding discrimination.

In case one or both models discriminate the event, the optimal cut off point must be estimated. The optimal cut off point is the level of probability that could be used as benchmark to differentiate between bankrupt and non-bankrupt firms. There is no "golden" rule in choosing the optimal cut off point, and it depends on the researcher's criteria: he/she may want to maximize sensitivity, that is, to choose a cut-off point that identify all enterprises that went bankrupt, at the expense of specificity, which may lead to some "healthy" firms being identified as bankrupt. Nonetheless, if the goal is to maximize both sensitivity and specificity, and to have a more balanced cut off point, one may apply the Youden Index[5]:

$$Youden\ Index = Sensitivity + Specificity - 1 \tag{30}$$

[5] Please refer to Unal (2017) or Tourassi (2018) for different approaches to estimate the optimal cut off point.

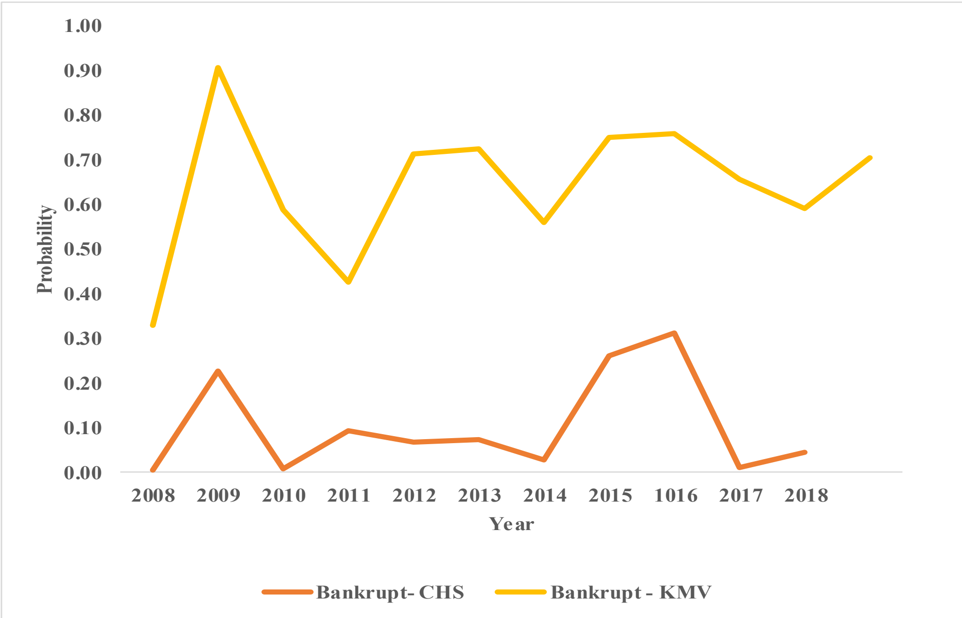
After the Index’s calculation is made for every possible cut off point, the one with the maximum amount should be selected as the optimal one.

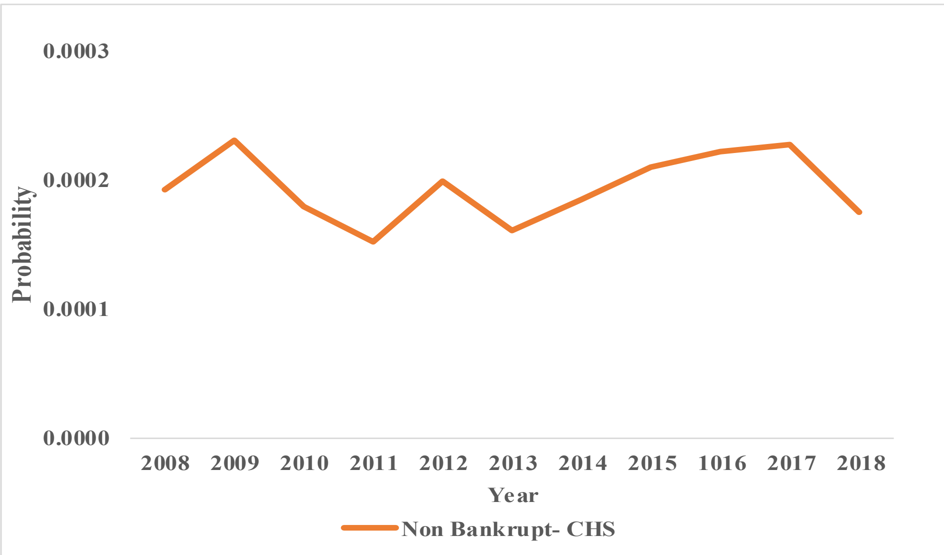
5. Empirical Results

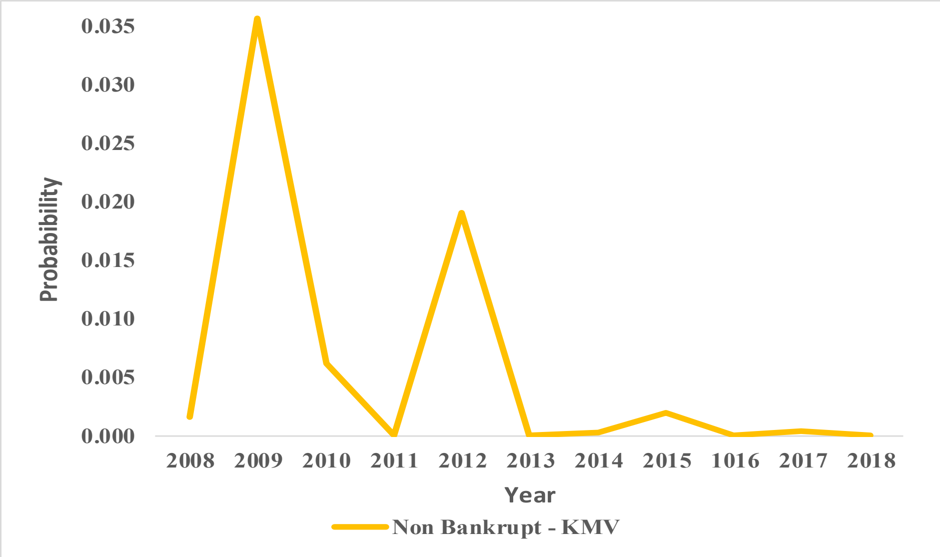
After applying the 2 models, and before resorting to the ROC statistical technique, it was analysed the probabilities generated by each model. All the empirical results, including the ROC analysis, were achieved through the usage of the statistical software, *SPSS*.

Our results commenced with the assumption that the output from both models are non-parametric as shown by Figure 5.1, which display the probabilistic mean for both models and for both type of firms.

Figure 5. 1 Graphical Comparison of Probabilities' means, for each model and group







Firstly, we analysed if in fact both models produce relatively higher probabilities to bankrupt firms rather than to non-bankrupt ones. As such, and similarly to the “t-test” for parametric samples, we opted for the “Mann-Whitney” test to observe if both models produce different probabilities’ means for the two different groups in study. The Mann-Whitney test has as null hypotheses the mean among different groups not being statistically significant, while the alternative hypotheses tests if the means among different groups are statistically significant. The test’s application for KMV model in in Table 5.1. and for CHS is in Table 5.2

Table 5. 1 Probabilities’ comparison between bankrupt and non-bankrupt firms (KMV)

Year	Type		ET (p)
	Bankrupt $\bar{x} \pm s$	**Non-Bankrupt** $\bar{x} \pm s$	
2008	0,330403± 0,292068	0,001542± 0,005115	-3,201 (0,000)
2009	0,908276± 0,081870	0,035753± 0,077441	-7,959 (0,000)
2010	0,589891± 0,284527	0,006195± 0,013616	-3,923 (0,000)
2011	0,428119± 0,349972	0,000005±0,000019	-4,702 (0,000)
2012	0,715337± 0,265389	0,019103±0,071976	-4,954 (0,000)
2013	0,726987± 0,317196	0,000016±0,000070	-4,775 (0,000)
2014	0,559942± 0,345966	0,000324±0,001113	-3,602 (0,000)
2015	0,752280± 0,252886	0,001940±0,005598	-4,735 (0,000)
2016	0,760974± 0,150685	0,000010±0,000027	-4,511 (0,000)

2017	0,658547± 0,220796	0,000373±0,000988	-2,727 (0,004)
2018	0,591966± 0,354669	0,000005±0,000021	-4,506 (0,000)
Total	0,705349± 0,290287	0,012512±0,049084	-15,174 (0,000)

Legend: $\bar{X}$**:** sample's mean; s**:** sample's standard deviation; **ET**: Statistical test; **p**: *p-value*

Table 5. 2 Probabilities' comparison between bankrupt and non-bankrupt firms (CHS)

Year	Type		ET (p)
	Bankrupt $\bar{x} \pm s$	Non-Bankrupt $\bar{x} \pm s$	
2008	0,004332±0,007093	0,000193±0,000086	-2,341 (0,018)
2009	0,228603±0,300028	0,000231±0,000243	-7,959 (0,000)
2010	0,008822±0,007036	0,000179±0,000079	-3,919 (0,000)
2011	0,095021±0,247975	0,000153±0,000080	-3,783 (0,000)
2012	0,068033±0,080062	0,000200±0,000105	-5,035 (0,000)
2013	0,073743±0,099596	0,000161±0,000068	-4,399 (0,000)
2014	0,028991±0,051128	0,000185±0,000063	-3,373 (0,000)
2015	0,260344±0,360936	0,000211±0,000105	-4,621 (0,000)
2016	0,314268±0,444511	0,000222±0,000124	-4,311 (0,000)
2017	0,009893±0,002093	0,000228±0,000105	-2,717 (0,004)
2018	0,045783±0,05697	0,000175±0,000067	-3,919 (0,000)
Total	0,140425±0,260405	0,000201±0,000150	-14,881 (0,000)

Legend: $\bar{X}$**:** sample's mean; s**:** sample's standard deviation; **ET**: Statistical test; **p**: *p-value*

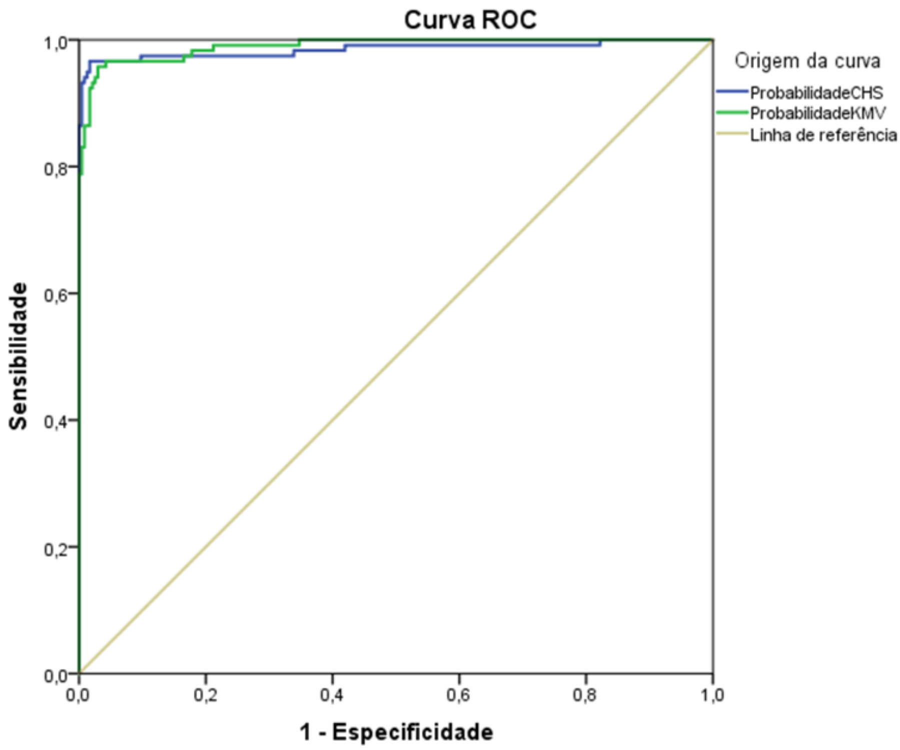
From the above tables, it is possible to conclude that both models are able, at least in terms of probabilities' means, to differentiate among different groups, as shown by the low p-values across all periods. However, in 2008 and 2017, the differences in the KMV model are more statistically significant than the CHS model since, in those periods, the latter model produces p-values above 0, which only occurs in those two years for both models.

Furthermore, the KMV model produced in all periods higher probabilities' means than the CHS model for the "bankrupt" and "non bankrupt" group. The differences between the models, especially in in the "Bankrupt" group, may be surprisingly at first glance. However, many authors such as Crosbie and Bohn (2003) consider default as a rare event. Based on that, and considering that CHS model resorts to a logistic regression to derive probabilities, authors such as King and Zeng (2003) argue that this type of statistical technique can sharply underestimate the probability of rare events.

Resorting again to the Mann-Whitney test, we analyse if both models indeed produce different probabilities among themselves, a hypothesis raised given the results achieved and presented on table 5.1 and 5.2[6]. Apart from 2016, in the "bankrupt" group, and 2010, in the "non-bankrupt" group, the models produce different probabilities between each other, which are statistically significant.

Despite understanding from the statistical techniques above described that indeed the models produce different probabilities between them and between groups, the application of the ROC analysis is crucial to validate these results. Figure 5.2. summarizes the application of the ROC analysis, being presented the respective curves: the blue line represents the CHS Model and the green one the KMV Model.

Figure 5. 2 Comparison of ROC Curves between CHS and KMV Model



From Figure 5.2, one can infer that both models produce more than satisfactory curves, close to the upper left corners, which indicate a considerable discriminatory power. Moreover, both curves are close to each other and only the analysis of the respective AUCs allow to

[6] Please refer to Appendix E for the Table summarizing the application of the Mann- Whitney test.

understand which model better discriminates bankruptcy. Table 5.3. summarize the AUC of each model, as well as its standard deviation and statistical significance.

Table 5. 3 AUC results

Variables	AUC	Standard error*	*p-value***	Asymptotic 95% Confidence Interval	
				Lower Bound	Upper Bound
CHS Model	0,985	0,008	0,000	0,969	1,000
KMV Model	0,990	0,004	0,000	0,981	0,998

Legend: * under the nonparametric assumption; ** H0= True AUC= 0.5 (no discriminatory power)

From the table above, it is possible to confirm that both models discriminate the event of bankruptcy, considering the sample and the period chosen, have an outstanding discriminatory power as shown by the AUC of 0.990 and 0.985 for KMV and CHS models, respectively. Considering these results, the null hypotheses of the test, which state that the AUC is equal to 0.5 is rejected for both models, as shown by the respective *p-values.* Moreover, KMV model is better at discriminating than the CHS model, which follows the hypotheses number 2.1) raised in section number 3.

Given the models' AUC, which shows discriminatory power, the next step is to calculate the optimal cut off resorting to the Youden's Index, presented in the previous section. In tables 5.4. and 5.5 are presented the top 10 cut off points that maximize the Youden Index

Table 5.4 Sensitivity / Specificity / Youden's Index: Top 10 cut off points (KMV)

Bankrupt if greater or equal to	Sensibility	1-Specificity	Specificity	Youden's Index
0.08400905	0.958	0.03	0.97	**0.928**
0.0528887	0.966	0.042	0.958	**0.924**
0.0699081	0.958	0.034	0.966	**0.924**
0.0602574	0.958	0.038	0.962	**0.92**
0.05115555	0.966	0.047	0.953	**0.919**
0.09619095	0.949	0.03	0.97	**0.919**
0.05538245	0.958	0.042	0.958	**0.916**
0.113039	0.941	0.025	0.975	**0.916**
0.0498503	0.966	0.051	0.949	**0.915**
0.0487921	0.966	0.055	0.945	**0.911**

Table 5. 5 Sensitivity/ Specificity/ Youden's Index: Top 10 cut off points (CHS)

Bankrupt if greater or equal to	Sensibility	1-Specificity	Specificity	Youden's Index
0.00057965	0.966	0.017	0.983	**0.949**
0.00054525	0.966	0.021	0.979	**0.945**
0.00052195	0.966	0.025	0.975	**0.941**
0.00061325	0.958	0.017	0.983	**0.941**
0.00049485	0.966	0.03	0.97	**0.936**
0.0006609	0.949	0.013	0.987	**0.936**
0.0007219	0.941	0.008	0.992	**0.933**
0.00048485	0.966	0.034	0.966	**0.932**
0.000638	0.949	0.017	0.983	**0.932**
0.00048165	0.966	0.038	0.962	**0.928**

As expected, given the distribution of probabilities presented earlier, the optimal cut off point for the KMV model (8.4%) is greater than the one for CHS model (0.058%). If someone is examining the probability of bankruptcy through CHS model, without knowing *"a priori"* the optimal cut off point, he/she may consider that a value close to 0.058% is not distressing enough and it is likely to assume that the entity is healthy. In other hand, if the assessment is made through the KMV model and a probability near the 8.4% threshold is given for a certain company, it leaves room to some uncertainty, being a more plausible cut off point than the CHS Model.

Although only producing slightly higher AUC than CHS model, the KMV model provides probabilities that are higher for entities that went bankrupt, as further evidenced by the optimal cutoff point analysis. Moreover, and in line with the CHS model, the KMV model also assess relatively lower probabilities for healthy firms. As such, we consider that when assessing the probability of bankruptcy, KMV is a more rationale choice than the CHS model. However, the results in this paper partially confirms the idea that a dynamic panel model, which resorts to a wide range of entities to be estimated, indeed is capable of providing accurate out of sample estimate when predicting bankruptcy.

We understand that the analysis between the KMV and CHS model is not commonly found in literature. However, our results can be interpreted to be in line with the ones achieved by Hillegeist et al. (2004), which reached to the conclusion that an hazard market based model, which measure similarly follow ours, provides more information about the probability of bankruptcy than the typical accounting models. Despite the CHS model overpassing some issues of the accounting models studied by Hillegeist et al. (2004), our study still proves that a market based model overpass a model containing accounting measures, such as the CHS.

More interestingly is the comparison with the study performed by Campbell et al. (2008), which proved that their best model (CHS) performs better than a model solely containing the only input of the KMV model, the "DD" measure. However, the inclusion of the same measure to the existing 8 variables, brings a slight improvement to the overall model's fitness. In our paper, our results proves that the KMV model outperforms the CHS model, contrarily to what is stated by Campbell et al. (2008). However, here we resorted to the original KMV model, which recur to a normal distribution to derive probabilities, whereas Campbell et al. (2008) includes the DD measure in a hazard model, using a logistic specification to derive the probabilities.

6. Conclusion

The main goal of this paper is to compare two different types of models, one market-based and a hybrid-based model, in order to understand which of them better discriminates the event of bankruptcy, at a 1-year horizon. To do so, we select 356 US publicly traded firms, 118 that went bankrupt and 256 that went not, in a 10-year period, from 2008 to 2018.

The non-selection for a model from the other main division in credit risk modelling, the accounting based, relates not only to its shortcomings, like the sample specificity or backward measures, but it also proved by many authors in literature that a mix of accounting based and market based variables improves the accuracy of models. As such, we opt for the original model developed by Campbell et al. (2008), an hybrid model, which overpasses the sample specificity issue while considering both types of variables, being the best of its type in terms of accuracy. Moreover, we compared it with the theoretical KMV model, a benchmark in credit risk modelling, by resorting to a ROC analysis, a statistical technique increasingly used in the bankruptcy's field. Our study contributes to a rational model's choice by an individual investor, interested in assessing bankruptcy's likelihood in its decision concerning a certain company, not rated by a Credit Rating Agency (CRA).

Our results show that the KMV model is slightly superior to the CHS model at discriminating bankruptcy, as denoted by the AUC that each of the models produced. Moreover, and when analysing the optimal cut off point that maximizes both True Events, we reached the conclusion that KMV produces a significantly higher probability threshold that distinct bankrupt and non-bankrupt firms (8.4%) than CHS Model (0.058%), thus being more rationale and intuitive. Furthermore, and from our experience of applying the models. we conclude that despite recurring to a complex Interactive Approach to reach to a Probability of Bankruptcy, the KMV model is less demanding, as it requires less inputs than CHS model and less intermediate calculations to assess the independent variables. Moreover, given that KMV model is widely used as benchmark, there is considerable information on how to develop an interactive approach using a wide range of software.

Last but not least, we suggest some measures to be considered in future researches. Firstly, the industry/sector of bankrupt firms could be taken into account when selecting the respective non bankrupt sample, i.e., if there are 3 bankrupt firms belonging to the manufacturing sector, at least 6 non-bankrupt firms of the same field should be chosen. Secondly, and apart from profitability and size metrics, we recommend that future researches consider the inclusion of additional measures such as gearing ratios, as it is crucial to assess the KMV model, and/or leverage and liquidity ratios, part of the CHS model, to select the non-bankrupt firms. This consideration, coupled with the contemplation of industry/sectors, would increase even further the similarity between groups, and in theory, it would turn tougher the task of differentiating and distinguish bankrupt of non-bankrupt firms. Finally, we suggest a complementary analysis of the optimal cut off points resulted from the ROC analysis. By resorting to a secondary sample, the models could be applied and considering the probabilities reached, observe how many bankrupt firms would fall above (success) or below (unsuccess) the optimal cut off point. Similarly, the same rationale could be applied to non-bankrupt firms in order to calculate accuracy ratios for both groups. This would allow to observe weather or not the optimal cut off points provides sound results for secondary samples.

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Appendix

Appendix A: List of Bloomberg's function needed to apply the models and its definition

Function	Full Name	Definition	Applied in CHS	Applied in KMV
CASH_&_ST_INVESTMENTS	Cash and Short-Term Investments	Total Amounts of cash and short-term investments at the period end date. Excluding financials: Cash & Near Cash Items + Marketable Securities & Other Short-term Investments	Yes	No
BS_TOT_LIAB2	Total Liabilities	Sum of all current and non-current liabilities. Calculated as Current Liabilities + Long Term Borrowings + Other Long-Term Liabilities	Yes	No
TOTAL_EQUITY	Total Equity	Firm's total assets minus its total liabilities. Calculated as: Common Equity+ Minority Interest+ Preferred Equity	Yes	No
NET_INCOME	Net Income/Net Profit (Losses)	Amount of Profit the company made after paying all of its expenses. It is known as bottom-line or net profit	Yes	No
CUR_MKT_CAP	Current Market Cap	total current market value of all of a company's outstanding shares stated in the pricing currency. Capitalization is a measure of corporate size	Yes	Yes
PX_LAST	Last Price	Last price of the security	Yes	No
BS_CUR_LIAB	Current Liabilities	the summation of Accounts Payable, Short-term borrowings, and Other Short-Term Liabilities	No	Yes
NON_CUR_LIAB	Non-Current Liabilities	Sum of Long-Term Borrowings and Other Long-term Liabilities	No	Yes

Appendix B: List of Entities subject of study (Breakdown by Year and Type)

Ticker Code	Year	Bankrupt (Y/N)	Ticker Code	Year	Bankrupt (Y/N)	Ticker Code	Year	Bankrupt (Y/N)
BRLCQ US Equity	2008	Y	CHTRQ US Equity	2009	Y	CACI UN Equity	2009	N
WCIMQ US Equity	2008	Y	NBFAQ US Equity	2009	Y	CATO UN Equity	2009	N
USBE US Equity	2008	Y	IDARQ US Equity	2009	Y	CBT UN Equity	2009	N
VSUNQ US Equity	2008	Y	SUTMQ US Equity	2009	Y	CCMP UW Equity	2009	N
CCTYQ US Equity	2008	Y	SGICQ US Equity	2009	Y	CMP UN Equity	2009	N
CSKEQ US Equity	2008	Y	NOBLQ US Equity	2009	Y	CMTL UW Equity	2009	N
CNC UN Equity	2008	N	ABWTQ US Equity	2009	Y	COLM UW Equity	2009	N
EXP UN Equity	2008	N	ASYTQ US Equity	2009	Y	CPA UN Equity	2009	N
FMC UN Equity	2008	N	SORCQ US Equity	2009	Y	CW UN Equity	2009	N
HL UN Equity	2008	N	ERPLQ US Equity	2009	Y	DBI UN Equity	2009	N
HLF UN Equity	2008	N	TXCOQ US Equity	2009	Y	DFODQ UN Equity	2009	N
KMT UN Equity	2008	N	RHDCQ US Equity	2009	Y	DRQ UN Equity	2009	N
KOP UN Equity	2008	N	VSTNQ US Equity	2009	Y	EBS UN Equity	2009	N
LII UN Equity	2008	N	MTLQQ US Equity	2009	Y	EXP UN Equity	2009	N
LSTR UW Equity	2008	N	BLGM US Equity	2009	Y	FLO UN Equity	2009	N
NC UN Equity	2008	N	EBHIQ US Equity	2009	Y	GGG UN Equity	2009	N
PDCE UW Equity	2008	N	LEARQ US Equity	2009	Y	GVA UN Equity	2009	N
WCC UN Equity	2008	N	EPEXQ US Equity	2009	Y	HTLD UW Equity	2009	N
APXSQ US Equity	2009	Y	AURDQ US Equity	2009	Y	HXL UN Equity	2009	N
TRXAQ US Equity	2009	Y	ALTUQ US Equity	2009	Y	IDCC UW Equity	2009	N
SSCCQ US Equity	2009	Y	CJHBQ US Equity	2009	Y	ITGR UN Equity	2009	N
MWYGQ US Equity	2009	Y	CTDBQ US Equity	2009	Y	LII UN Equity	2009	N
BGPTQ US Equity	2009	Y	ADM UN Equity	2009	N	MCS UN Equity	2009	N
SPSNQ US Equity	2009	Y	ASNA UW Equity	2009	N	MNRO UW Equity	2009	N
MCOAQ US Equity	2009	Y	BECN UW Equity	2009	N	MSA UN Equity	2009	N
MECAQ US Equity	2009	Y	BLKB UW Equity	2009	N	MTSC UW Equity	2009	N
FLTWQ US Equity	2009	Y	BMS UN Equity	2009	N	MYGN UW Equity	2009	N
CEMJQ US Equity	2009	N	BRKR UW Equity	2009	Y	NL UN Equity	2009	N

Ticker Code	Year	Bankrupt (Y/N)	Ticker Code	Year	Bankrupt (Y/N)	Ticker Code	Year	Bankrupt (Y/N)
NTRI UW Equity	2009	N	WST UN Equity	2009	N	RZTIQ US Equity	2011	Y
ODFL UW Equity	2009	N	WW UN Equity	2009	N	JHTXQ US Equity	2011	Y
PRGS UW Equity	2009	N	XRM US Equity	2010	Y	ESLRQ US Equity	2011	Y
PVH UN Equity	2009	N	RVHLQ US Equity	2010	Y	SDTHQ US Equity	2011	Y
PXD UN Equity	2009	N	RMIXQ US Equity	2010	Y	SLPHQ US Equity	2011	Y
PZZA UW Equity	2009	N	TRMAQ US Equity	2010	Y	SYMSQ US Equity	2011	Y
RAMP UW Equity	2009	N	BLIAQ US Equity	2010	Y	GMRRQ US Equity	2011	Y
RES UN Equity	2009	N	PHHMQ US Equity	2010	Y	LEE US Equity	2011	Y
RMD UN Equity	2009	N	MIPIQ US Equity	2010	Y	AME UN Equity	2011	N
ROG UN Equity	2009	N	GAPTQ US Equity	2010	Y	AOS UN Equity	2011	N
ROL UN Equity	2009	N	AIR UN Equity	2010	N	CBT UN Equity	2011	N
SEB UA Equity	2009	N	BGG UN Equity	2010	N	CNC UN Equity	2011	N
SKX UN Equity	2009	N	CCMP UW Equity	2010	N	DRQ UN Equity	2011	N
SLGN UW Equity	2009	N	HEI UN Equity	2010	N	ESL UN Equity	2011	N
SONC UW Equity	2009	N	HUBG UW Equity	2010	N	FINL UW Equity	2011	N
SSD UN Equity	2009	N	IART UW Equity	2010	N	FLO UN Equity	2011	N
SWKS UW Equity	2009	N	KOP UN Equity	2010	N	GEF UN Equity	2011	N
TG UN Equity	2009	N	NYT UN Equity	2010	N	IVC UN Equity	2011	N
TPX UN Equity	2009	N	OII UN Equity	2010	N	NATI UW Equity	2011	N
TRK UN Equity	2009	N	POOL UW Equity	2010	N	NEU UN Equity	2011	N
TSCO UW Equity	2009	N	REV UN Equity	2010	N	ODFL UW Equity	2011	N
TTEC UW Equity	2009	N	ROL UN Equity	2010	N	PBI UN Equity	2011	N
TTWO UW Equity	2009	N	SON UN Equity	2010	N	RGLD UW Equity	2011	N
TUP UN Equity	2009	N	TSCO UW Equity	2010	N	SHOO UW Equity	2011	N
TYL UN Equity	2009	N	VSAT UW Equity	2010	N	TSCO UW Equity	2011	N
WCC UN Equity	2009	N	WAB UN Equity	2010	N	UNFI UW Equity	2011	N
WDFC UW Equity	2009	N	BGPIQ US Equity	2011	Y	WWE UN Equity	2011	N
WERN UW Equity	2009	N	AMIEQ US Equity	2011	Y	XRAY UW Equity	2011	N

Ticker Code	Year	Bankrupt (Y/N)	Ticker Code	Year	Bankrupt (Y/N)	Ticker Code	Year	Bankrupt (Y/N)
TRIDQ US Equity	2012	Y	OSK UN Equity	2012	N	JBLU UW Equity	2013	N
EVEIQ US Equity	2012	Y	OXM UN Equity	2012	N	LPNT UW Equity	2013	N
TBSIQ US Equity	2012	Y	SCL UN Equity	2012	N	NUAN UW Equity	2013	N
ENERQ US Equity	2012	Y	SD UN Equity	2012	N	PWR UN Equity	2013	N
GRBEQ US Equity	2012	Y	SEE UN Equity	2012	N	RMD UN Equity	2013	N
PNCLQ US Equity	2012	Y	STRA UW Equity	2012	N	SFL UN Equity	2013	N
RDDYQ US Equity	2012	Y	TECH UW Equity	2012	N	TECH UW Equity	2013	N
DYNIQ US Equity	2012	Y	TR UN Equity	2012	N	TTEK UW Equity	2013	N
KVPHQ US Equity	2012	Y	TXRH UW Equity	2012	N	TYL UN Equity	2013	N
ATPAQ US Equity	2012	Y	UHAL UW Equity	2012	N	UAA UN Equity	2013	N
BNVIQ US Equity	2012	Y	XRAY UW Equity	2012	N	VIVO UW Equity	2013	N
OSGIQ US Equity	2012	Y	LNETQ US Equity	2013	Y	WOR UN Equity	2013	N
THQN GR Equity	2012	Y	PWAVQ US Equity	2013	Y	ZBRA UW Equity	2013	N
ALK UN Equity	2012	N	SCHSQ US Equity	2013	Y	LEU US Equity	2014	Y
AN UN Equity	2012	N	GEOKQ US Equity	2013	Y	DOLNQ US Equity	2014	Y
CCMP UW Equity	2012	N	CEDCQ US Equity	2013	Y	CWTRQ US Equity	2014	Y
CRS UN Equity	2012	N	CPICQ US Equity	2013	Y	KIDBQ US Equity	2014	Y
ENDP UW Equity	2012	N	XIDEQ US Equity	2013	Y	EGLE US Equity	2014	Y
FINL UW Equity	2012	N	FBNIQ US Equity	2013	Y	BAXSQ US Equity	2014	Y
FNSR UW Equity	2012	N	GHSEQ US Equity	2013	Y	AIT UN Equity	2014	N
GPOR UW Equity	2012	N	SVNTQ US Equity	2013	Y	ANF UN Equity	2014	N
HAE UN Equity	2012	N	ADTN UW Equity	2013	N	AWI UN Equity	2014	N
HBI UN Equity	2012	N	DCI UN Equity	2013	N	CTB UN Equity	2014	N
HEI UN Equity	2012	N	DIN UN Equity	2013	N	FOSL UW Equity	2014	N
HL UN Equity	2012	N	GRA UN Equity	2013	N	MOG/A UN Equity	2014	N
LPNT UW Equity	2012	N	HIBB UW Equity	2013	N	MSM UN Equity	2014	N
MATW UW Equity	2012	N	HUBG UW Equity	2013	N	RES UN Equity	2014	N
MMS UN Equity	2012	N	MD UN Equity	2013	N	TTC UN Equity	2014	N

Ticker Code	Year	Bankrupt (Y/N)	Ticker Code	Year	Bankrupt (Y/N)	Ticker Code	Year	Bankrupt (Y/N)
TTEC UW Equity	2014	N	ODFL UW Equity	2015	N	JW/A UN Equity	2016	N
USG UN Equity	2014	N	OI UN Equity	2015	N	LII UN Equity	2016	N
WAB UN Equity	2014	N	SANM UW Equity	2015	N	MEI UN Equity	2016	N
WTSLQ US Equity	2015	Y	SCSC UW Equity	2015	N	MSM UN Equity	2016	N
CACH US Equity	2015	Y	UNFI UW Equity	2015	N	OII UN Equity	2016	N
RSHCQ US Equity	2015	Y	URBN UW Equity	2015	N	REV UN Equity	2016	N
CDVIQ US Equity	2015	Y	WOR UN Equity	2015	N	SEB UA Equity	2016	N
BPZRQ US Equity	2015	Y	WST UN Equity	2015	N	SONC UW Equity	2016	N
SRCTQ US Equity	2015	Y	ZINCQ US Equity	2016	Y	SWM UN Equity	2016	N
KWKAQ US Equity	2015	Y	RJETQ US Equity	2016	Y	XPER UW Equity	2016	N
COCOQ US Equity	2015	Y	PSUNQ US Equity	2016	Y	ULTRF US Equity	2017	Y
SOGCQ US Equity	2015	Y	SUNEQ US Equity	2016	Y	HGGGQ US Equity	2017	Y
ANRZQ US Equity	2015	Y	AROPQ US Equity	2016	Y	CBRI US Equity	2017	Y
HEROQ US Equity	2015	Y	SD US Equity	2016	Y	RTKHQ US Equity	2017	Y
AROC UN Equity	2015	N	WRESQ US Equity	2016	Y	AAN UN Equity	2017	N
BKS UN Equity	2015	N	HEROQ US Equity	2016	Y	AVY UN Equity	2017	N
CAL UN Equity	2015	N	PRXIQ US Equity	2016	Y	DFODQ UN Equity	2017	N
CPRT UW Equity	2015	N	ESINQ US Equity	2016	Y	JBL UN Equity	2017	N
DKS UN Equity	2015	N	ALOG UW Equity	2016	N	MOG/A UN Equity	2017	N
ESND UW Equity	2015	N	BIG UN Equity	2016	N	RGLD UW Equity	2017	N
FELE UW Equity	2015	N	BJRI UW Equity	2016	N	SAM UN Equity	2017	N
GRA UN Equity	2015	N	CMP UN Equity	2016	N	WYNN UW Equity	2017	N
ITGR UN Equity	2015	N	EME UN Equity	2016	N	XCOOQ US Equity	2018	Y
JBL UN Equity	2015	N	EQT UN Equity	2016	N	CVOVQ US Equity	2018	Y
JBLU UW Equity	2015	N	FOE UN Equity	2016	N	BONTQ US Equity	2018	Y
MDP UN Equity	2015	N	GGG UN Equity	2016	N	OREXQ US Equity	2018	Y
MGLN UW Equity	2015	N	HAIN UW Equity	2016	N	CCO US Equity	2018	Y
MTX UN Equity	2015	N	HMSY UW Equity	2016	N	REXXQ US Equity	2018	N

Ticker Code	Year	Bankrupt (Y/N)
SHLDQ US Equity	2018	Y
PQUEQ US Equity	2018	Y
BCPC UW Equity	2018	N
CHE UN Equity	2018	N
DY UN Equity	2018	N
FDP UN Equity	2018	N
JBL UN Equity	2018	N
JW/A UN Equity	2018	N
MMSI UW Equity	2018	N
MWA UN Equity	2018	N
NATI UW Equity	2018	N
PBH UN Equity	2018	N
PWR UN Equity	2018	N
RBC UN Equity	2018	N
TKR UN Equity	2018	N
TXRH UW Equity	2018	N
VAR UN Equity	2018	N
WBC UN Equity	2018	N

Appendix C: KMV Model Application, including Interactive Approach (Example Based on CTDBQ US Equity, 2009, Bankrupt)

Before Interactive Approach:

Dates	Equity	Liabilities	Risk free rate	Returns Equity	Assets value (k)	Asset value (k+1)	Log returns K	Asset Volatility iter k (Standard deviation)	Asset volatility k+1		d1	d2
								0.184695848				
31/12/2007	543.6276	1651.237	0.0334		2194.86505	2240.878767		Sum of squared errors	834323.1055		1.814074	1.629378
01/01/2008	543.6276	1651.237	0.03255	0	2194.86505	2239.415027	0				1.809471	1.624776
02/01/2008	543.6276	1651.237	0.0317	0	2194.86505	2237.953309	0	Botão 1			1.804869	1.620173
03/01/2008	509.321	1651.237	0.0313	-0.065185988	2160.55845	2200.810375	-0.015753834	Miu	-0.001559514		1.717407	1.532712
04/01/2008	453.9027	1651.237	0.0306	-0.115195608	2105.14015	2139.75529	-0.025984685				1.572928	1.388233
07/01/2008	459.1806	1651.237	0.0311	0.011560739	2110.41805	2146.376204	0.002504011	Distance to Default	-0.560051169		1.589193	1.404497
08/01/2008	419.5961	1651.237	0.0309	-0.090151015	2070.83355	2102.485028	-0.018934848				1.485591	1.300895
09/01/2008	445.9858	1651.237	0.0304	0.060994531	2097.22325	2130.745734	0.012663	Probability of Default	0.712277732		1.551445	1.366749
10/01/2008	422.2351	1651.237	0.0304	-0.054724845	2073.47255	2104.551089	-0.011389446				1.489779	1.305084
11/01/2008	411.6792	1651.237	0.0291	-0.025317863	2062.91665	2090.56099	-0.005103931				1.455107	1.270411
14/01/2008	401.1233	1651.237	0.029	-0.025975544	2052.36075	2078.571277	-0.005130115				1.426789	1.242093
15/01/2008	401.1233	1651.237	0.0287	0	2052.36075	2078.05378	0				1.425165	1.240469
16/01/2008	427.513	1651.237	0.0286	0.063715836	2078.75045	2107.297599	0.012776252				1.493798	1.309102
17/01/2008	432.7909	1651.237	0.0281	0.012270004	2084.02835	2112.271971	0.002535759				1.50482	1.320124
18/01/2008	401.1233	1651.237	0.0269	-0.07598584	2052.36075	2074.955677	-0.015312011				1.415419	1.230723
21/01/2008	401.1233	1651.237	0.0249	0	2052.36075	2071.527316	0				1.40459	1.219895
22/01/2008	401.1233	1651.237	0.0229	0	2052.36075	2068.1139	0				1.393762	1.209066
23/01/2008	424.874	1651.237	0.0219	0.057523793	2076.11145	2092.928045	0.011505933				1.450644	1.265948
24/01/2008	430.152	1651.237	0.024	0.01234598	2081.38945	2102.343091	0.002539027				1.475761	1.291065
25/01/2008	435.4299	1651.237	0.0234	0.012195184	2086.66735	2107.15068	0.002532548				1.486225	1.301529
28/01/2008	456.5417	1651.237	0.023	0.047346224	2107.77915	2129.637691	0.010066634				1.538563	1.353867
29/01/2008	440.7078	1651.237	0.0233	-0.035297972	2091.94525	2112.794259	-0.007540483				1.499361	1.314665
30/01/2008	382.6505	1651.237	0.023	-0.141260031	2033.88795	2047.406036	-0.028145167				1.34535	1.160654
31/01/2008	385.2895	1651.237	0.0211	0.00687296	2036.52695	2047.169697	0.001296674				1.342083	1.157387

After Interactive Approach:

Dates	Equity	Liabilities	Risk free rate	Returns Equity	Assets value (k)	Asset value (k+1)	Log returns K	Asset Volatility iter k (Standard deviation)	Asset volatility k+1		d1	d2
								0.294500785				
31/12/2007	543.6276	1651.237	0.0334		2192.052706	2192.052706		Sum of squared errors	9.52746E-11		1.222675	0.928174
01/01/2008	543.6276	1651.237	0.03255	0	2190.543571	2190.543571	-0.000688695				1.21745	0.92295
02/01/2008	543.6276	1651.237	0.0317	0	2189.039835	2189.039835	-0.000686703	Botão 1			1.212232	0.917732
03/01/2008	509.321	1651.237	0.0313	-0.065185988	2147.335519	2147.335519	-0.019235236	Miu	-0.002570468		1.145559	0.851059
04/01/2008	453.9027	1651.237	0.0306	-0.115195608	2078.013864	2078.013863	-0.032815216				1.031756	0.737255
07/01/2008	459.1806	1651.237	0.0311	0.011560739	2085.495481	2085.49548	0.003593903	Distance to Default	-1.104907868		1.045657	0.751156
08/01/2008	419.5961	1651.237	0.0309	-0.090151015	2035.024682	2035.024681	-0.02449852				0.961791	0.667291
09/01/2008	445.9858	1651.237	0.0304	0.060994531	2067.718231	2067.718231	0.015937748	Probability of Default	0.865400244		1.014211	0.719711
10/01/2008	422.2351	1651.237	0.0304	-0.054724845	2037.52643	2037.52643	-0.014709158				0.964265	0.669765
11/01/2008	411.6792	1651.237	0.0291	-0.025317863	2021.617972	2021.617972	-0.007838371				0.933235	0.638735
14/01/2008	401.1233	1651.237	0.029	-0.025975544	2007.706848	2007.706847	-0.006904968				0.909449	0.614949
15/01/2008	401.1233	1651.237	0.0287	0	2007.178076	2007.178075	-0.000263406				0.907536	0.613036
16/01/2008	427.513	1651.237	0.0286	0.063715836	2041.108391	2041.10839	0.016763196				0.964118	0.669617
17/01/2008	432.7909	1651.237	0.0281	0.012270004	2046.963322	2046.963322	0.0028644				0.972146	0.677645
18/01/2008	401.1233	1651.237	0.0269	-0.07598584	2004.025167	2004.025167	-0.021199647				0.896086	0.601586
21/01/2008	401.1233	1651.237	0.0249	0	2000.561959	2000.561958	-0.001729621				0.883422	0.588921
22/01/2008	401.1233	1651.237	0.0229	0	1997.141408	1997.141408	-0.001711258				0.87082	0.576319
23/01/2008	424.874	1651.237	0.0219	0.057523793	2026.188697	2026.188696	0.014439677				0.916456	0.621955
24/01/2008	430.152	1651.237	0.024	0.01234598	2036.505834	2036.505834	0.005078974				0.940832	0.646332
25/01/2008	435.4299	1651.237	0.0234	0.012195184	2042.20114	2042.201139	0.002792703				0.948278	0.653777
28/01/2008	456.5417	1651.237	0.023	0.047346224	2068.126627	2068.126626	0.01261497				0.989755	0.695254
29/01/2008	440.7078	1651.237	0.0233	-0.035297972	2048.722441	2048.722441	-0.009426787				0.958764	0.664263
30/01/2008	382.6505	1651.237	0.023	-0.141260031	1972.943932	1972.943931	-0.037689591				0.829768	0.535267
31/01/2008	385.2895	1651.237	0.0211	0.00687296	1973.239546	1973.239545	0.000149823				0.823825	0.529324

Interactive Approach: Excel Macro based on Löffler & Posch (2007)

```
Public Sub KMV_it()
Do While Range("J4") > 10 ^ -10

'Copy asset values from iteration k+1 to iteration k

Range("F4:F268") = (Range("G4:G268"))

Loop

End Sub
```

Appendix D: CHS Model Application (Example Based on CTDBQ US Equity, 2009, Bankrupt)

								31/03/2008	30/06/2008	30/09/2008	31/12/2008			Value	Ponderador
		Dates	31/03/2008	30/06/2008	30/09/2008	31/12/2008	Market Value Total Assets (MTA)	3586.432	3323.4025	3107.2642	2775.1092		NITMAAG	-0.149661937	-20.12
CTDBQ US Equity	Total Assets	BS_TOT_ASSET	3749.168	3377.877	3303.4441	2432.97	Net Income	-8.273	-251.55	27.986	-737.982		TLMTA	0.984436216	1.6
	Cash and Short Term Investments	CASH_&_ST_INVESTMENTS	128.507	62.871	17.099	18.634	Total Liabilities	3147.3889	3001.644	2896.6511	2731.918		CASHMTA	0.006714691	-2.27
	Total Liabilities	BS_TOT_LIAB2	3147.3889	3001.644	2896.6511	2731.918	Cash and Cash Equivelents	128.507	62.871	17.099	18.634		MB	8.275571361	0.07
	Total Equity	TOTAL_EQUITY	601.7791	376.2329	406.793	-298.948	Market Cap	439.0431	321.7585	210.6131	43.1912	Neg Eq adj	SIGMA	0.196528012	1.55
	Net Income/Net Profit (Losses)	NET_INCOME	-8.273	-251.55	27.986	-737.982	Total Equity	601.7791	376.2329	406.793	-298.948	1	PRICE	-1.832581464	-0.09
							BE Adjusted	585.5055	370.78546	387.17501	5.21912		EXRETAAV	-0.247311897	-7.88
							S&P Market Cap				8129635.64		RSIZE	-12.14538989	-0.005
							Weight (Net Income)	0.0666	0.1333	0.2666	0.5333		Constant		-8.87
	Current Market Cap		Last Price			S&P Dados	NITMA	-0.00230675	-0.0756905	0.00900664	-0.26592899				
Dates	CUR_MKT_CAP	PX_LAST	Retorno diario	Retorno Mensal	PX_LAST	Monthly Return		0.0666	0.1333	0.2666	0.5333		Logit	-1.240560794	-1.2405608
31/12/2007	543.6276	2.06			1468.36								Probability	0.224	
01/01/2008	543.6276	2.06	0		#N/A N/A										
02/01/2008	543.6276	2.06	0		1447.16										
03/01/2008	509.321	1.93	-0.063106796		1447.16										
04/01/2008	453.9027	1.72	-0.10880829		1411.63		EXRET Calculations								
07/01/2008	459.1806	1.74	0.011627907		1416.18		Period	Weight	EXRET	Multiply					
08/01/2008	419.5961	1.59	-0.086206897		1390.19		1	0.017328	-0.25896275	-0.00448731					
09/01/2008	445.9858	1.69	0.062893082		1409.13		2	0.021832	-0.24774655	-0.0054088					
10/01/2008	422.2351	1.6	-0.053254438		1420.33		3	0.027507	0.41748483	0.01148376					
11/01/2008	411.6792	1.56	-0.025		1401.02		4	0.034656	-0.24578384	-0.00851788					
14/01/2008	401.1233	1.52	-0.025641026		1416.25		5	0.043664	0.24721152	0.01079424					
15/01/2008	401.1233	1.52	0		1380.95		6	0.055013	-0.2765794	-0.01521546					
16/01/2008	427.513	1.62	0.065789474		1373.2		7	0.069312	-0.30547637	-0.02117318					
17/01/2008	432.7909	1.64	0.012345679		1333.25		8	0.087328	0.08411227	0.00734536					
18/01/2008	401.1233	1.52	-0.073170732		1325.19		9	0.110026	-0.13297582	-0.0146308					
21/01/2008	401.1233	1.52	0		1325.19		10	0.138625	-0.83886783	-0.11628805					
22/01/2008	401.1233	1.52	0		1310.5		11	0.174656	-0.36403441	-0.06358079					
23/01/2008	424.874	1.61	0.059210526		1338.6		12	0.220053	-0.12557417	-0.02763297					
24/01/2008	430.152	1.63	0.01242236		1352.07										
25/01/2008	435.4299	1.65	0.012269939		1330.61		SIGMA Calculations								
28/01/2008	456.5417	1.73	0.048484848		1353.97		Count	66							
29/01/2008	440.7078	1.67	-0.034682081		1362.3		standard deviation	0.117068474							
30/01/2008	382.6505	1.45	-0.131736527		1355.81		variance	0.013705027							
31/01/2008	385.2895	1.46	0.006896552	-0.291262136	1378.55	-0.061163475	days	2.818181818							
01/02/2008	419.5961	1.59	0.089041096		1395.42		sigma	0.196528012							

Appendix E: Application of Mann-Whitney Test, different probabilities between models.

Year	Comparison between CHS and KMV	
	Bankrupt	Non-Bankrupt
	ET (p)	ET (p)
2008	-2,562 (0,010)	-2,257 (0,024)
2009	-5,801 (0,000)	-5,337 (0,000)
2010	-3,361 (0,000)	-0,151 (0,880)
2011	-2,117 (0,034)	-5,530 (0,000)
2012	-4,333 (0,000)	-2,326 (0,020)
2013	-3,553 (0,000)	-5,032 (0,000)
2014	-2,722 (0,006)	-3,559 (0,000)
2015	-2,791 (0,005)	-3,652 (0,000)
2016	-1,814 (0,070)	-5,412 (0,000)
2017	-2,309 (0,021)	-2,314 (0,021)
2018	-3,256 (0,000)	-5,093 (0,000)
Total	-10,557 (0,000)	-6,662 (0,000)

Legend: ET – Statistical Test; ***p*** **–** *p-value*