Managing complexity: A problem of chaos in fisheries policy

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Abstract: This work intends to present chaos theory (and dynamical systems such as the theories of complexity), in terms of interpretation of ecological phenomena. The chaos theory applied in the context of ecological systems, especially in the context of fisheries has allowed the recognition of the relevance of this kind of theories to explain fishing phenomena and fisheries policies. It has permitted new advances in the study of marine systems, contributing to the preservation of fish stocks. This paper deals with the way how to manage fisheries taking chaos in account of the problem.

Key words: chaos theory; dynamical systems; complex adaptive systems

1. Introduction

Managing fisheries has brought increasingly new contextualization in terms of the limits of analysis. Fisheries have been analysed in contexts of uncertainty. But more recently, some works considering chaos in fisheries have been presented. This work intends to have chaos theory and complexity in consideration, and the way ecological problems may have theoretical contextualization in this field, considering chaos methodologies to solve fisheries problems in particular.

A general consideration of chaos problem and chaos theory and some considerations about the status of fisheries in this context are given firstly; Then, the study shows the role of dynamical theories for the consequent analysis of ecological problems; Finally, the fisheries case is studied, highlighting the importance of the application of the chaos theory to the fisheries policies, introducing new factors of analysis, complementing the usual views of fisheries management and perspectives of analysis based on Clark and Munro's documents and later on game theoretical frameworks.

2. A problem of chaos and complexity

Chaos got rapidly a developing field. Much of the progress in this area was revealed just since the 1970s. This means that many facets of chaos are distant from been understood or determined yet. It is important to note that nowadays chaos is extremely difficult to identify in real world information to be workable. It is possible to

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find its usage in mathematical computer problems' solution and laboratory research. As soon as the idea of nonlinearity¹ is introduced into theoretical models, chaos gets obvious. A very complex structure is observed in field data. Simple patterns can be found and approximated, while complex patterns are another matter. In any event, we can't just grab a nice little set of data, apply a simple test or two, and declare "chaos" or "no chaos". (Williams, 1997). Chaos occurs in deterministic, nonlinear or dynamical systems.

The chaos theory involves multiple interactions and supposes the existence of an enormous number of interrelations, with direct developments in vast fields of study. It got an important role in the context of recent theoretical developments in non-equilibrium theories. The word "chaos" assumes the idea of the existence of turbulence and disorder, and an unwanted chance or even the idea of an "abyss". The predisposition to a profound change in the direction of a phenomenon generates an own force, which is understood as a depth change that results from small changes in their initial conditions. The chaos is, from this point of view, something extremely sensitive to the initial conditions. However, it is interesting to note that the chaotic system normally seems to develop itself in a very smooth and orderly way, although inside changes may be complex and paradoxical.

Recent developments in the dynamical systems theories, which require the existence of an inherent complexity of the systems themselves that are based on a set of large inside interactions, led in some cases to understand and highlight self-organizing systems, revealing strong strengths and reinforcing their internal cohesion factors.

Given the non-linearity conditions of the nature phenomena, theories, which are based on the dynamics of non-equilibrium, seem to explain the spatial and temporal heterogeneity observed in ecological systems quite well. The disturbances and heterogeneity are interdependent factors that create opportunities for re-colonization and determine the structure of communities. For reflection about the effects of the disturbances in the systems, it is important to note that the ability to recover the ecological systems depends, to some extent, on the existence of refuge areas both for flora and fauna, acting as reservoirs of re-colonizers, after the disturbances occurred in the ecosystem.

Many live resources, particularly many marine resources have suffered drastic reductions motivated by their overexploitation. Populations of many species have been led to the rupture and close to the extinction. How can humanity modify this state of things? Is there a line of evolution that helps to explain this kind of events? How can we shape these facts in this perspective? Is there any ways to invert these trends, or simply to find the principles that underline the facts? May the main agents in this process be questioned? Do the supranational institutions just emit simple indicative rules?

3. Chaos in dynamical systems' theories

A dynamical system represents moving, changing or evolving in time. For this reason, chaos deals with dynamical systems theory (the study of phenomena that vary with time) or nonlinear dynamics (the study of nonlinear movement or evolution).

Dynamical systems fall into one of two categories, depending on whether the system loses energy. A conservative dynamical system has no friction and it doesn't lose energy over time. In contrast, a dissipative dynamical system has friction. It loses energy over time and therefore always approaches some asymptotic or

¹ Nonlinear means that output isn't directly proportional to input, or that a change in one variable doesn't produce a proportional change or reaction in the related variable(s).

limiting condition. That asymptotic or limiting state, under certain conditions, is where chaos occurs (Williams, 1997).

The dynamical systems theories have been applied to numerous areas of knowledge. In the 1980's, several exact sciences (physics, chemistry or biology, for example) and some social sciences (economics or management or even the sociology) still had their own objects of study and their own methods of analysis and each one of them was different from others. The science has been branched and specialized, so that each one uses to have its own world. Recently, new forms of analysis, looking for an integrated study have emerged (Filipe, 2006).

The chaos theory and complexity theory itself reflect the phenomena that in many activities (such as fisheries) they are translated into dynamic forms of analysis and reflect a very complex and widespread reality, specific of complex systems. That reality falls within a range of situations integrated in a broader context, which is expressed not only in the theory itself, but also in terms of their own realities (fisheries, for example), dynamic, complex and often chaotic features in their essence.

The chaos theory stresses that the world does not necessarily work as a linear relationship with perfectly defined or with direct relations in terms of expected proportions between causes and effects. The chaos occurs when a system is very sensitive to the initial conditions. These initial conditions are the measured values for a given initial time. The presence of chaotic systems in nature seems to place a limit on our ability to apply physical deterministic laws to predict movements with any degree of certainty. Indeed, one of the most interesting subjects in the study of chaotic systems is the question that whether the presence of chaos may or may not produce ordered structures and patterns on a wider scale. In the past, the dynamic systems showed up completely unpredictable, and the only ones who could aspire to be understood were those that were represented by linear relationships, which are not the rule. On the contrary, there are some situations clearly isolated.

Today, with the help of computers, it is possible to make extremely complex calculations and to understand the occurrence of chaos better.

As Williams (1997) says, phenomena happen over time as at discrete (separate or distinct) intervals² or as continuously³. Discrete intervals can be spaced evenly in time or irregularly in time. Continuous phenomena might be measured continuously. However, we can measure them at discrete intervals⁴. Special types of equations apply to each of those two ways in which phenomena happen over time. Equations for discrete time changes are difference equations and could be solved by iteration. In contrast, equations based on a continuous change (continuous measurements) are differential equations. The term "flow" is often associated with differential equations⁵.

Differential equations are often the most accurate mathematical way to describe a smooth continuous evolution. However, some of these equations are difficult or impossible to solve. In contrast, different equations usually can be solved right away. Furthermore, they are often acceptable approximations of differential equations (Williams, 1997)⁶. Olsen and Degn (1985) say that difference equations are the most powerful vehicle to understand chaos.

Examples are the occurrence of earthquakes, rainstorms or volcanic eruptions.

Examples are air temperature and humidity or the flow of water in perennial rivers.

For example, we may measure air temperature only once per hour, over many days or years.

To some authors (Bergé, et al., 1984), a flow is a system of differential equations. To others (Rasband, 1990), a flow is the solution of differential equations.

This is not true in general. Usually, different equations are much more difficult to solve than differential equations. For the particular ones used in chaos theory, often the difference equations are easier to solve than differential equations.

Many scientists see, with particular interest, chaos theory as a way to explain the environment. Therefore, the chaos theory stresses the fundamental laws of nature and natural processes and requires a course for a constant evolution and recreation of nature. The chaos theory allows realizing the endless alternative ways leading to a new form or new ways that will be disclosed and that eventually emerge from the chaos as a new structure. The reality is a process in which structure and chaos rotate between form and deformation in an eternal cycle of death and renewal. Conditions of instability seem to be the rule and, in fact, a small inaccuracy in the conditions of departure tends to grow to a huge scale. Basically, two insignificant changes in the initial conditions for the same system tend to end in two situations different completely. This situation is known as the "butterfly wing effect". A small movement of the wings of a butterfly can have huge consequences. It is the microscopic turbulence that have effects in a macroscopic scale—an effect called by Grabinski (2004) as "hydrodynamics". Mathematically, the "butterfly wing effect" corresponds to the effect of chaos, which can be expressed as follows.

Given the initial conditions, $x_1, x_2, x_3, \dots, x_N$, it is possible to calculate the final condition given by:

final result = $f(x_1, x_2, x_3, ..., x_N)$

If the initial conditions x_i have a margin of error (variation), the final result will be influenced by the existence of this margin. If these margins in x_i are small as the margin of error in the final result, a non-chaotic situation occurs. Otherwise, if the margins of error in x_i are small, but the final result has a big variation, there is a chaotic situation. Therefore, small variations in initial conditions can lead to a major effect in the final outcome. Sometimes, small changes in x_i have exponential effects on the final result due to the passage of time.

This effect can be demonstrated mathematically⁷ using the Lyapunov Exponent⁸ (Grabinski, 2008). Given the initial value x_0 and ε being its (arbitrarily small) variation, we are conducted to an initial value between x_0 and x_0 + ε . The general form of Lyapunov indicator is presented by:

$$
x_{n+1} = f(x_n)
$$

that after N iterations leads to a value for x_N between $f^N(x_0)$ and $f^N(x_0)$, being the difference between these two values:

$$
f^N(x_{0+\epsilon}) - f^N(x_0) \Xi \epsilon . e^{N\lambda(x_0)}
$$

where λ is a parameter. Dividing both sides by the variation ε and assuming limit $\varepsilon \rightarrow 0$, we have a differential quotient. Making its logarithm and assuming limit $N \rightarrow \infty$, we get the final definition of the Lyapunov exponent:

$$
\lambda(x_0) = \lim_{N \to \infty} \frac{1}{N} \log \left| \frac{df^N(x_0)}{dx_0} \right|
$$

and there is chaos when $\lambda > 0$.

Through this function, the chaos exists when arbitrary small variations in initial conditions grow exponentially with a positive exponent.

Grabinski (2008) also says that the nonlinearity is the main characteristic of a chaotic situation. Mathematically, the nonlinear functions to be considered chaotic should be based on variables with some resistance. The author also argues that it is not enough to describe the chaotic situations, such as turbulence, but it is necessary to find ways to cope with the nonlinearity better. A smooth flow of a river (non-chaotic) that can be

⁷ Several statistics may indicate chaos and can express how chaotic a system is. One of the most important statistics to measure magnitude of chaos is at present Lyapunov exponents. Other statistics could be presented as the Kolmogorov-Sinai entropy or the mutual information or redundancy.

A Lyapunov exponent is a number that reflects the rate of divergence or convergence, averaged over the entire attractor, of two neighbouring phase space trajectories. Trajectory divergence or convergence has to follow an exponential law, for the exponent to be definable.

described in quantities, like the flow velocity, can reach a chaotic behavior with variations of many situations. The best example is a waterfall where the speed of the flow reaches a certain point. In a smoothly flowing river, it is easy to calculate or predict the flow velocity of the river at any point. However, to calculate it in a river with a waterfall, it is necessary to introduce chaos. In an attempt to make this calculation, man has focused on the construction of super computers that have shown to be useless due the infinity of factors that may cause turbulence in the flow of the river. Thus, the analysis of frequency on the change of flow's velocity is much more promising than the analysis of velocities themselves.

Moreover, Grabinski (2008) shows the situation in which there is chaos on a microscopic scale, but not on a macroscopic scale—the "hydrodynamics". An example is a glass of water resting on a table, a not chaotic event. A slight disturbance on the table causes a small flow on a macroscopic level in the water. However, a microscopic observation reveals a great agitation of millions of molecules, a chaotic event. This is a situation where there is chaos on the microscopic scale, but a smooth flow on the macroscopic scale. Mathematically Grabinski (2008) presents hydrodynamics equations which combine the chaos theory with business situations. For that, he presents the function about the value of a company (v) that depends on two variables, the revenue (r) and the number of employees (n) . Its general form is:

$$
v(r, n) = v_0 + a_{10} r + a_{01} n + a_{11} r n + a_{20} r^2 + a_{02} n^2 + ...
$$

where a_{ii} represents general parameters. For $n = 0$ (no employees) or $r = 0$ (no revenue) the company doesn't exist because the function value is equal to 0. So, some terms of the function must be removed $(v_0 = a_{10} = a_{01} = a_{20} = a_{02}$ $= \dots = 0$). The general form comes as:

$$
v(r, n) = a_{11} \cdot r \cdot n + a_{21} \cdot r^2 \cdot n + a_{12} \cdot r \cdot n^2 + a_{22} \cdot r^2 \cdot n^2 + ...
$$

Now to the case of symmetry, r and n could be negative. A negative employee means that the employee is paying to work, and negative revenue means that the company is paying the customer to consume. So the previous formula can lead to negative results if r and n change signs simultaneously. Only these terms are allowed for the sums of the powers of r and v are even numbers. Thus, the general expression of the equation is:

 $v(r, n) = a_{11} \cdot r \cdot n + a_{22} \cdot r^2 \cdot n^2 + ...$

4. Chaos, dynamical systems' theory and ecology

Chaos (deterministic chaos) deals with long-term evolution—How something changes over a long time. A chaotic time series looks irregular. Two of chaos's important practical implications are that long-term predictions under chaotic conditions are worthless, and complex behaviour can have simple causes. Chaos is difficult to identify in real world data, because the available tools generally were developed for idealistic conditions that are difficult to fulfil in practice (Williams, 1997).

The ecology where many things are random and uncertain, in which everything interacts with everything at the same time is, itself, a fertile area for a cross search to the world explanations (Filipe, et al., 2005).

Lansing (2003) states that the initial phase of the research of nonlinear systems was based on the deterministic chaos, and it was later redirected to new outbreaks of research focusing on the systems properties, which are self-organizing—called anti-chaos. It also says that the study of complex adaptive systems, which is discussed in the context of non-linear dynamic systems, has become a major focus of interest resulting from the interdisciplinary research in the social sciences and the natural sciences.

The theory of systems in general represents the natural world as a series of reservoirs and streams governed

by various feedback processes. However, the mathematical representations were ignoring the role of these adjustment processes.

The theory of complex adaptive systems is part of the theory of systems, although it has in specific account the diversity and heterogeneity of systems, rather than representing them only by reservoirs. It explicitly considers the role of adaptation on the control of the dynamics and of the responses of these heterogeneous reservoirs. This theory allows ecologists to analyze the reasons inherent to the process at the lower levels of the organization that lead to patterns at higher levels of organization and ecosystems. The adaptive systems represent one of the means to understand how the organization is produced to a large scale and how it is controlled by processes that operate at lower levels of organization. According to Lansing (2003), there came to be a general idea involving physical and mathematical complexity that is hidden behind very simple systems.

Considering a system composed by many interactive parts, if it is sufficiently complex, it may not be practical or even not be possible to know the details of each interaction place. Moreover, the interactions can generate local non-linear effects that often become impossible to find a solution even for simple systems. However, diverting us from causal forces that move the individual elements, if we focus on the system behavior as a whole, we can highlight certain global behavior standards. However, these behavior standards may hide an associated cost—It can not be expected to understand the causes at the level of individual behavior.

Indeed, the systems do not match the simple decomposition of the whole into parts, and therefore do not correspond to the mere sum of the parts, as living systems are not the juxtaposition of molecules and atoms. Since the molecule to the biosphere, the whole is organized and each level of integration leads to properties that can not be analyzed only from mechanisms that have explanatory value in the lower levels of integration. This corresponds to the appearance of new features to the level of the set that does not exist at the level of the constituent elements. Lansing (2003) believes that the adoption in the social sciences of the idea that complex global patterns can emerge with new properties from local interactions had a huge impact here.

The ecological systems are comparable to the systems self-organized, as they are open systems which arise far from thermodynamic equilibrium. On self-organized and self-regulated systems, the reciprocal interactions within the system between the structures and the processes contribute to the regulation of its dynamics and the maintenance of its organization, partly due to the phenomena of feedback (Lévêque, 2002). These systems seem to develop themselves in accordance with the properties referred to the anti-chaotic systems. Indeed, we have auto-regulated systems that channel different initial conditions for the same stage, instead of what is happening with chaotic systems, which are very sensitive to initial conditions (Kauffman, 1993). These systems would be relatively robust for a particular type of disturbance, to which the components of the system fit, creating a meta-stability that depends not only on the internal interactions within the system, but also on external forces that can regulate and strengthen the internal factors of cohesion (Lévêque, 2002).

Scoones (1999) argues that there should be concluded a new commitment in research on the ecological new thinking, and he develops its search precisely in the area of ecology around the concepts of chaotic dynamics and systems of non-equilibrium. In turn, Levin (2003) shows that in the study of complex adaptive systems, anti-chaos involves the understanding of how the cooperation, alliances and networks of interactions emerge from individual behaviors and how it generates a feed-back effect to influence these behaviors within the spontaneous order and ecosystems self-organization.

5. Dynamical systems, chaos theory and fisheries

In order to frame some methodological developments, it must be mentioned, first of all, that some characteristics associated with some species support strategic survival features which are exploited by the present theory. Its aim is to find the reasons and the way in which these strategies could be developed and the resulting consequences. The species use their biological characteristics resulting from evolutionary ancient processes to establish defense strategies.

However, given the emergence of new forms of predation, species got weaker and weaker, because they are not prepared with mechanisms for effective protection for such situations. In fisheries, there is a predator—man, with new fishing technologies, who can completely destabilize the ecosystem. By using certain fisheries technologies, such as networks of siege, allowing the capture of all individuals of the population who are in a particular area of fishing, the fishers cause the breakdown of certain species, particularly the pelagic ones, normally designated by schooling species.

To that extent, with small changes in ecosystems, this may cause the complete deterioration of stocks and the final collapse of ecosystems, which in extreme cases can lead to extinction. These species are concentrated in high density areas in small space. These are species that tend to live in large schools (Filipe, et al., 2005).

Usually, large schools allow the protection against large predators. The mathematical theory, which examines the relationship between schools and predators, due to Brock and Riffenburgh (Clark, 1974), indicates that the effectiveness of predators is a reverse function of the size of the school. Since the amount of fish that a predator can consume has a maximum average value, overcoming this limit, the growth of school means a reduction in the rate of consumption by the predator. Other aspects that are defensive for the school such as intimidation or confusing predators are also an evidence of greater effectiveness of schools.

However, this type of behavior has allowed the development of very effective fishing techniques. With modern equipment for detecting schools (sonar, satellites, etc.) and with modern artificial fibers' networks (strong, easy to handle and quick placement), fishing can keep up advantageous for small stocks (Bjorndal, 1987; Mangel & Clark, 1983).

As soon as schools become scarce, stocks become less protected. Moreover, the existence of these modern techniques prevents an effect of stock in the costs of businesses, as opposed to the so-called search fisheries, for which a fishery involves an action of demand and slow detection. Therefore, the existence of larger populations is essential for fishermen because it reduces the cost of their detection (Neher, 1990). However, the easy detection by new technologies means that the costs are not more sensitive to the size of the stock (Bjorndal & Conrad, 1987).

This can be extremely dangerous due to poor biotic potential of the species subject to this kind of pressure. The reproductive capacity requires a minimum value, below which the extinction is inevitable. Since the efficiency of the school is proportional to its size, the losses due to the effects of predation are relatively high for low levels of stocks. This implies non-feedback in the relation stock-recruitment, which causes a break in the curves of income-effort, so that an infinitesimal increase on fishing effort leads to an unstable condition that can lead to its extinction (Filipe, et al., 2008).

However, considering the fishing as a broader issue, we may consider the modeling of the fish stocks on the basis of an approach associated with the theory of chaos, instead considering the usual prospect based on classical models. Indeed, the issue can be placed within this framework from two different prisms: the traditional vision and the vision resulting from theories of non-equilibrium. Around the traditional Newtonian view, the facts can be modeled in terms of linear relationships: involving the definition of parameters, identifying relevant variables and using differential equations to describe the processes that change slowly over time. For a given system, it should then carry out measurements in a context that remains stable during various periods.

Moreover, we may have models based on the chaos theory. These models are based on non-linear relationships and are very close to several disciplines, particularly in the branch of mathematics that study the invariant processes of scale, the fractals, and in a huge range of other subjects in the area of self spontaneous creation of order—the theory of disasters or complex systems, for example.

The first way is largely used by the majority of biologists, economists and environmentalists, scientists and technical experts that conduct studies in marine search and senior technicians from state and transnational agencies in the area of fisheries. It treats nature as a system, which has a regular order. But today, there are many responsible for fisheries management who also base their decisions on models of chaos.

The classical models center on a particular system and depend on a local analysis, studying several species, age, class, sub-regions of the marine eco-niche, the various ports and their discharges, depending on the account of an even wider range of other factors. Probably, the classic expression of linearity on the dynamics of the population (the principle that nature is orderly, balanced and that has a dynamic balance) is due to Maynard Smith (1968), which argues that the populations either remain relatively constant or regularly vary around an alleged point of balance. In the specific case of commercial fisheries, biologists believe that the fishing effort is often relevant to explain the deviations of actual populations' values for the model. They say that, specially based on studies made in the last decade, fish stocks sustainability should be ensured by the control made through fisheries regulation.

Moreover, some people see nature as not casual and unpredictable. The natural processes are complex and dynamic, and the causal relations and sequential patterns may extend so much in time that may seem to be non-periodical. The data appear as selected random works, disorderly, not causal in their connections and chaotic. The vision provided by nature leads to consider the fish stocks, time, the market and the various processes of fisheries management as likely to be continuously in imbalance rather than behave in a linear fashion and in a constant search for internal balance. It is this perspective that opens the way for the adoption of the chaos theory in fisheries. However, the models of chaos do not deny, for themselves, some of the linearity resulting from the application of usual bionomic models. What is considered is that there is no condition to implement all significant variables in a predictive model. Moreover, in finding that a slight change in initial conditions caused by a component of the system may cause major changes and deep consequences in the system itself (Filipe, et al., 2009). So, the application of the chaos theory to fishing is considered essential, by many researchers. The chaos theory depends on a multitude of factors, all major (and in the prospect of this theory, all are very important at the outset) on the basis of the wide range of unpredictable effects that they can cause.

Considering the fished value function of a company (v) depending on two variables, the fishing effort (r) and the fish stock (n) , a simple model for fisheries, analogous to the presented in section 3, can be built:

$$
v(r, n) = v_0 + a_{10} \cdot r + a_{01} \cdot n + a_{11} \cdot r \cdot n + a_{20} \cdot r^2 + a_{02} \cdot n^2 + \dots
$$

being a_{ij} general parameters. Now it makes no sense to consider negative values for the variables. For $n = 0$ (no fish stock) or $r = 0$ (no fishing effort), the company fished value doesn't exist because the function value is equal to $0.$

Consequently, $v_0 = a_{10} = a_{01} = a_{20} = a_{02} = ... = 0$, and now: $v(r, n) = a_{11} \cdot r \cdot n + a_{21} \cdot r^2 \cdot n + a_{12} \cdot r \cdot n^2 + a_{22} \cdot r^2 \cdot n^2 + ...$

6. Concluding remarks

Chaos theory got its own space among sciences and has become itself to be an outstanding science. However, there is much left to be discovered. Anyway, many scientists consider that chaos theory is one of the most important developed sciences on the twentieth century.

Aspects of chaos are shown up everywhere around the world, and chaos theory has changed the direction of science, studying chaotic systems and the way they work.

We can not say yet if chaos theory may give us solutions to problems that are posed by complex systems. Nevertheless, understanding the way chaos discusses the characteristics of complexity and analyzes open and closed systems and structures is an important matter of present discussion.

On the fisheries analysis, it is interesting to see that overfishing may cause a problem of irreversibility in the recovering of several species, after certain stages for the stocks. Anyway, to analyze the specific situation of each case, it is necessary to obtain enough data to analyze the kind of function which is specific for that particular case, and it must be analyzed the situation for certain phases of fishing and it must be seen the consequences for these species.

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